Soft Shareholder Activism

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ABSTRACT

This paper studies the conditions under which voice and exit are effective forms of shareholder activism. Different from the existing literature, voice is interpreted as a strategic transmission of information from an activist investor to an opportunistic manager. The analysis provides several results. First, voice and exit exhibit complementarity. Second, transparency reduces the credibility of voice and thereby harms shareholder value. Third, the value of the firm can decrease with the quality of the activist's private information and increase with the cost of obtaining this information. Forth, managerial myopia unambiguously benefits shareholders if and only if the activist can voice herself. Finally, voice can be effective even when the activist is biased, and sometimes, even more effective than when the activist is unbiased.

Keywords: Shareholder Activism, Voice, Exit, Corporate Governance, Communication, Transparency, Cheap-Talk.

JEL Classification Numbers: D74, D82, D83, G34

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Introduction

Shareholder activism is an important channel of corporate governance. It is the act of disciplining an otherwise opportunistic management. Discipline can be achieved in several ways. Most notably, investors can seek formal control of the company by accumulating a significant number of voting shares or by winning board seats in contested director elections. With sufficient control, the activist can command the incumbent management to follow a particular strategy or simply replace it. Obtaining formal control, however, can be very costly.[1] To the extent that activist investors are not fully reimbursed for these and other expenses, these tactics could be under-used from a social point of view. With the frustration of not being able to obtain control, the activist may lose trust in the company and exit by selling her holdings in the firm.

Formal control, however, is not necessary for an effective implementation of shareholder activism. Investors can send letters, make phone calls or even meet face to face with senior executives and board members, and express their view how to unlock what they believe is a hidden value. This can include the common activist goals of spinning off a division of the company or a share buyback, but it can also be a recommendation on a strategic, financial, operational or organizational change with consequences for the company's long-run viability. If investors have useful insights, the board and management may consider their advice.

Anecdotally, this informal engagement, whether it is taking place "behind-the-scenes" or it appears in the public media, becomes increasingly common. "Companies have been under so much pressure. Now you can walk in the door and just have a conversation about what to do to make the company better," says Harlan Zimmerman of Cevian, a large European activist hedge fund.[2] A recent survey of 25 senior executives and 25 activist investors concludes that the majority of activists view communication as their most effective strategy in achieving desired results, and that companies and shareholders often cooperate outside of the media glare.[3] More systematically, McCahery, Sautner, and Starks (2011) survey institutional investors and find that 55% of them would be willing to engage in discussions with the firm's executives. They

[1] These costs consist of the fees of hiring lawyers, proxy advisors and solicitors, corporate governance experts, investment banks, public relations and advertising firms. Gantchev (2012) estimates that on average the cost of a US public activist campaign ending in a proxy fight is $10.5 millions, roughly two thirds of its gross returns.

[2] See "Ready, set, dough - Activist investors are limbering up to make trouble once more" Economist 12/02/2010. As a recent example, following press reports about Yahoo's dealings with potential buyers, the WSJ reported ("At Yahoo, Top Investor Frets", 11/10/2011) that large shareholders of Yahoo contacted the company's board to express concerns over a possible transaction.

[3] See http://www.mergermarket.com/pdf/Shareholder_Activism_Insight_2010.pdf for a 2010 report made by law firm Schulte Roth & Zabel and research firm Mergermarket.

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conclude that behind-the-scenes shareholder activism may be more prevalent than previously thought.[4]

This paper explores the various ways through which shareholders can exercise activism when obtaining formal control is not feasible or too costly. It focuses on two primary mechanisms: *voice* and *exit*. Voice is the attempt of investors to persuade their opportunistic management to follow certain strategies. The effectiveness of voice crucially depends on its credibility and publicity. Different from the existing literature, voice is modeled literally as the communication of private information. Exit, on the other hand, is the activist's decision to sell her entire stake in the firm. By exiting, the activist signals dissatisfaction which could depress the share price and pressure managers to be more accountable to shareholders. The implementation of voice and exit does not require formal control and entails very little or no direct costs on the activist. I therefore refer to this type of activism as "soft". In this respect, the objective of this paper is to study the conditions under which soft shareholder activism is an effective form of corporate governance.

In order to study this topic I develop a model. In the model, the manager of a public firm has the formal authority over its long run investment policy. The manager is not perfectly informed about the investment opportunities, and due to the separation of ownership and control, his policy does not necessarily maximize the value of shareholders. These frictions create room for shareholder activism. Specifically, among shareholders there is an activist investor with private information that complements the manager's knowledge.[5] The activist can voice her opinion by sending at no cost a non-verifiable message to the manager. The activist can also exit by selling her entire stake in the firm before the long-term value of the firm is realized. The activist exits in order to satisfy her liquidity needs or because she believes the share is over-valued. While the activist's motives cannot be distinguished by the market, prices are set fairly given the public information, including the activist's decision to exit.

[4] Carleton, Nelson, and Weisbach (1998) study letters TIAA-CREF sends to their portfolio companies and find that they are usually successful at inducing firms to make governance related changes. Becht, Franks, Mayer, and Rossi (2009) provide evidence on "behind the scenes" communication as a form of (profitable) shareholder activism of the Hermes UK Focus Fund. Becht, Franks, and Grant (2010) use proprietary data collected from five activist funds and show that private interventions are extensive and profitable.

[5] There is a broad literature on how corporate insiders may learn value-relevant information from outsiders. Among many, Holmstrom and Tirole (1993) argue that stock prices provide information about the manager's actions and are therefore useful for managerial incentive contracts. Levit and Malenko (2011) analyze nonbinding voting for shareholder proposals and show that the information that is conveyed by voting outcome can affect corporate decision makers. Marquez and Yilmaz (2008) examine tender offers where shareholders have information about the firm value that the raider does not have. In Dow and Gorton (1997), Foucault and Gehrig (2008), and Goldstein and Guembel (2008), firms use information in stock prices to make investment decisions.

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The first result of the paper characterizes the interaction between voice and exit and shows that these mechanisms exhibit *complementarity*. That is, the option to exit enhances the positive effect that voice has on the value of the firm. Importantly, when management is highly opportunistic, voice is an idle mechanism of governance unless exit is possible. Different from the existing literature, with voice, exit is a powerful form of shareholder activism even if the manager has no direct utility from the short-term stock price. Instead, the channel through which exit exercises discipline is by improving the ability of the activist to credibility communicate with the opportunistic manager.

To understand this result, note that inevitably the activist will manipulate some of her information in order to overcome the inherent conflict of interests between shareholders and the manager. Worrying about its credibility, the opportunistic manager will often ignore the activist's advice. This dynamic limits the amount of information that can be revealed by the activist in any equilibrium. The option to exit, however, enables the activist to dispose her holdings in the firm at times she believes that the share is over-priced. As a result, with the option to exit, the activist is less sensitive to the long-run performances of the firm and is more willing to compromise with inefficient managerial decisions. This increases the credibility of the activist's voice in the eyes of the manager and allows for an informed deliberation. With more information the manager can make better decisions.[6] Overall, voice is more effective with exit than without it.

In practice, the activist does not have to voice herself secretly. Instead, the activist can make the letters to management public, ensuring that other market participants are aware of her demand. The second set of results relates to the role of *transparency* in shareholder activism. Transparency can relate to the activist's message or the manager's decision. I show that these two types of transparency generate exactly the same set of equilibria and hence are equivalent. Moreover, all else equal, voice is less effective as a mechanism of shareholder activism with transparency than without it. Nevertheless, when the activist can choose between public and private (or both) forms of communication, the adverse effect of transparency disappears.

Intuitively, the activist would like to get the highest price possible for her shares when she decides to exit and sell her holdings. With transparency, the activist cannot resist the temptation to send messages that inflate the short term price of the company's shares. Knowing that information will be manipulated, the activist loses credibility and her voice becomes less effective even when she does not exit. For this reason, with transparency, voice can be less

[6] Interestingly, the analysis below demonstrates that the most informative equilibrium is not always the equilibrium that maximizes the expected value of the firm.

effective with exit than without it. In other words, with transparency, voice and exit may exhibit *substitution*. In this respect, the analysis suggests that private engagements are more likely to be successful than public engagements, and it is crucial to allow activist investors to have private channels of communication with their management.

Communication of private information or trading based on private information are the key channels through which activism is exercised in the present model. I show that the ability of the activist to communicate with the opportunistic manager and increase the value of the firm can *decrease* with the quality of her private information. Moreover, when information is costly, the amount of information the activist acquires in equilibrium and the effectiveness of her voice can *increase* with the cost of information. To see the intuition, note that while high quality of private information increases the amount of information that is potentially communicated by the activist, it can also exacerbate the adverse selection problem that is created when the activist exits. Since exit and voice complement each other, the total effect of the quality of the activist's private information on her ability to voice herself credibly can be negative. For the same reason, when the cost of information is low, the activist will acquire a significant amount of information and inevitably harm her ability to influence the manager. Overall, since the cost of acquiring information (per share) tends to decrease with the number of shares, the analysis suggests that small share-holdings can be a commitment by the activist not to acquire too much private information and consequently be more effective when exercising soft shareholder activism.

The structure of executive compensation and the desire to demonstrate talent often make managers of public companies sensitive to the short-term performances of the stock price. Managerial myopia has a non-trivial effect on the effectiveness of soft shareholder activism. I show that when the activist cannot voice herself, the effect of exit and managerial myopia on shareholder value is ambiguous and can be negative. By contrast, when the activist can communicate with the manager, managerial myopia and exit unambiguously benefit shareholders. Intuitively, since exit depresses the short term price of the stock, the myopic manager tries to minimize the likelihood the activist exits. With voice, this objective is translated into stronger incentives to follow the activist's advice. The enhanced ability of the activist to influence the manager's decisions increases shareholder value.

Finally, activist investors may not share the same objective as other shareholders of the firm.[7] I study the effect of opportunistic activism on voice and exit, and show two results. First,

[7] Anabtawi and Stout (2008) provide a comprehensive discussion of common conflicts of interest between

since a biased activist may exit for reasons which are not necessarily related to low valuation of the firm, regardless of the direction or magnitude of the this bias, the adverse selection problem upon exit is less significant when the activist is biased than when she is unbiased. Second, when the manager's bias is significant, voice is an effective mechanism of shareholder activism if and only if the activist is biased as well. Essentially, a biased activist functions as an intermediary who trades off the manager's opportunism with shareholder value.

The paper proceeds as follows. The remainder of this section discusses the relationship to the existing literature. Section I and Section II present the baseline model and its analysis, respectively. Section III analyzes the effect of transparency on soft shareholder activism. Section IV extends the baseline model by considering the quality of the activist's private information and her decision to acquire private information. Section V and Section VI introduce managerial myopia and opportunistic activism, respectively. Section VII discusses the empirical implications of the analysis and Section VIII concludes. All proofs are collected in the Appendix.

Relation to the Literature

Traditional models of shareholder activism focus on the benefits of corrective actions through direct intervention (For example, Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997), Maug (1998), Kahn and Winton (1998), Bolton and von Thadden (1998), and Faure-Grimaud and Gromb (2004)). These studies share the idea that large shareholders are able to exercise *formal control* and thereby either force the company's management to improve the value of the firm or do it themselves. By contrast, the present study emphasizes that even without formal control, investors can exercise *real control* by communicating information and thereby persuading the management of the company to make better decisions.

In this respect, closely related are studies by Levit and Malenko (2011) and Cohn and Rajan (2011). Levit and Malenko (2011) investigate non-binding voting for shareholder proposals as a mechanism through which shareholders of public companies can voice their opinions about governance and strategic related issues. They show that because of strategic voting, this mechanism often fails to aggregate and convey shareholder views when the interests of the manager and shareholders are not aligned. Instead, the presence of a biased activist investor

activist investors and other shareholders. Examples include short-termist goals of certain types of activists and self dealing involving other companies in activists' portfolios.

who can launch a proxy fight to replace the incumbent management may enhance the advisory role of non-binding voting, but only if the activist herself is biased. Similar to Levit and Malenko (2011), the present paper shows that shareholders can persuade their management to take a value enhancing decision by communicating information. Different from their work, however, the present paper focuses on the interaction of voice and investors' trading decision, and shows that communication can be an effective form of shareholder activism even when there is no binding threat of discipline in the background. Cohn and Rajan (2011) study a model in which a board arbitrates between an activist investor and a manager, and focus on the interaction between internal and external governance. Similar to the present work, in their model the blockholder makes a recommendation for a strategic change in the company that she cannot implement herself. However, in their model, strategic communication of information and trading are assumed away.

The current study is also related to Admati and Pfleiderer (2009), Edmans (2009), Edmans and Manso (2011), who point out that exit can be an effective form of governance in itself. Key for their result is that the manager has direct utility from the short term stock price, the channel through which exit matters. By contrast, the present study shows that even when managers are not myopic in that sense, exit is an important mechanism of governance since it enhances the ability of the activist to credibly communicate with the manager. Related, Dasgupta and Piacentino (2011) and Edmans and Manso (2011) consider exit and voice simultaneously. However, similar to the traditional literature of voice, these papers assume that investors can take binding actions that either reduce the agency problem with the manager or increase the value of the firm directly. Dasgupta and Piacentino (2011) also point out that exit can complement voice. Different from the present study, their argument relies on the assumptions that managers are myopic and voice is a costly action that reduces the manager's private benefits.

The present paper also contributes to the literature on governance and liquidity. Bhide (1993) argues that because blockholders add value through voice, and voice and exit are mutually exclusive, liquidity is harmful as it allows a shareholder to leave rather than intervene. The present paper argues the opposite. It shows that liquidity complements voice. In this respect, Maug (1998) and Kahn and Winton (1998) demonstrate that liquidity facilitates block formation in the first place, as activist shareholders can buy additional shares at a price that does not incorporate the gains from intervention. Different from the present study, conditional on the size of the activist's holdings, liquidity discourages intervention in those models. Faure-Grimaud and Gromb (2004) show that liquidity encourages intervention as it increases stock

price informativeness, and if the activist is forced to sell prematurely due to a liquidity shock, the price she receives will partially reflect the gains from intervention. Thus, in their paper liquidity directly increases the expected gains from intervention. By contrast, in the present paper, liquidity alleviates the adverse selection problem when the activist trades, and thereby reduces the sensitivity of the activist to inefficient decisions made by the manager. This effectively reduces the disagreement between the activist and the manager. Thus, in the present paper, liquidity enhances the credibility of voice and thereby improves the value of the firm.

I Baseline Model - Setup

Consider a public firm whose long term value to shareholders depends on its business strategy as well as on the fundamentals. Denote the business strategy by $a \in \{A, R\}$. I refer to decision a as an investment in a project and say that the project is approved when $a = A$ and is rejected otherwise. The value of the firm to shareholders is given by

$$v(\theta, a) = \theta \cdot \left(1_{\{a=A\}} - 1_{\{a=R\}}\right) \tag{1}$$

where $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ is a random variable whose probability density function f is continuous and has full support. I assume that $\underline{\theta} \in [-\infty, 0)$ and $\overline{\theta} \in (0, \infty]$. Thus, from shareholders' point of view, it is optimal to approve the project if and only if $\theta \geq 0$.

Shareholders own the cash flow rights of the firm, but the manager has the formal authority over the firm's investment policy. The manager and shareholders have conflicting preferences with respect to the investment policy. Specifically, the manager's preferences are represented by,

$$u_M = v(\theta + \beta, a) \tag{2}$$

where $\beta \in (0, -\underline{\theta})$ is the non-pecuniary private benefit the manager obtains from investment in the project. It follows from (2) that the manager approves the project if and only if he believes that $\theta \geq -\beta$. If the manager knows θ, he would inefficiently invest in the project when $\theta \in [-\beta, 0]$. Thus, the larger is β, the greater is the conflict of interests between shareholders and their manager.

The ownership structure of the firm consists of dispersed shareholders and an activist investor. Dispersed shareholders have no ability or incentives to discipline the incumbent management and hence remain passive. The focus of the analysis is on the ability of the activist

investor to influence the manager's decision over the project. The activist, however, does not have and cannot obtain formal control. Thus, the manager cannot be forced to take any action. Presumably, the cost of initiating and executing a proxy contest or a hostile takeover is too high. Instead, I study the ability of the activist to communicate her own view to the manager and thereby persuade him toward one action or the other.

To emphasize this channel of communication, let us assume that at the outset the activist obtains private and perfect information about θ. The key assumption of the model is that the activist's information is incremental to the manager's information. Specifically, I assume that all other market participants, including the manager, are uninformed about θ. Before the manager makes his decision about the project, the activist sends him a *private* message $m \in \left[\underline{\theta}, \overline{\theta}\right]$ about θ. The activist's private information is non-verifiable and her recommendation m cannot be backed-up with hard information. Moreover, the content of m does not affect the activist's payoff directly but only through its effect on the manager's decision. Thus, there are no private benefits or costs from communication per-se. Formally, the communication is modeled as "Cheap Talk" a la Crawford and Sobel (1982). Denote by $\mu\left(m \mid \theta\right) \in [0,1]$ the probability that the activist sends message m conditional on her private information about θ, and by $a\left(m\right) \in \{A, R\}$ be the manager's decision whether to accept the project conditional on observing message m.

After communicating her message to the manager the activist can trade with a competitive risk neutral market maker. Unless the activist is hit by a liquidity shock, she is free to choose whether to exit or keep her holdings in the firm. With probability $\delta \in [0,1]$, however, the activist is forced to sell her entire stake in firm in order to accommodate her liquidity needs. Denote the activist's decision to sell her entire stake in firm by $\sigma = 1$, and her decision to keep it by $\sigma = 0$. The activist cannot buy shares or commit to an exit strategy. The analysis will demonstrate that the activist *strategically exits* when based on her private information she believes the firm is over-valued. The decision of the activist to exit is observed by the market maker. However, the market maker does not observe the message the activist sent the manager or the needs of the activist for liquidity. These are the private information of the activist. Moreover, at the time of trade, neither the activist nor the market maker observe the manager's decision. Based on all the available public information, the market maker sets the short term price of the firm's share to be the expected value of the company. I denote this price

by $p(\sigma)$. Overall, the activist's preferences are given by,[8]

$$u_A = \sigma p(\sigma) + (1-\sigma)\, v(\theta, a) \tag{3}$$

To summarize, there are four periods in the model. Initially, before the activist observes her liquidity needs but after she becomes informed about θ, the activist communicates with the manager, trying to persuade him to follow her advice by sending a private message m. At period 1, the manager decides whether to approve the project, taking into account the activist's message. The manager's decision is unobservable by other market participants. At period 2, the activist observes her liquidity needs and based on her private information about θ and communication with the manager, she decides whether to sell her holdings in the firm. The market maker observes the order flow and determines the stock price accordingly. Finally, at period 3, the outcome of the project is realized and becomes public. All agents are risk neutral and preferences are common knowledge.

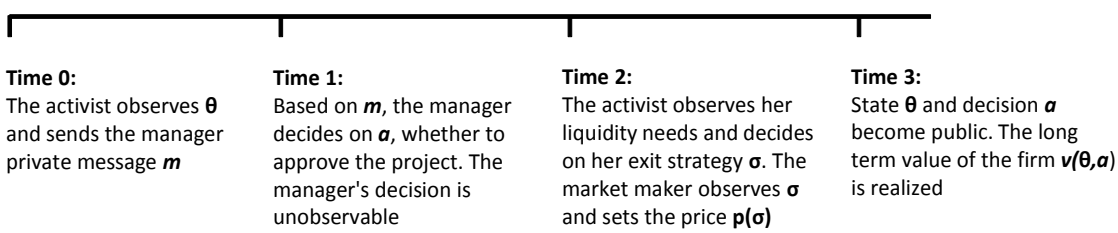

Figure 1 - Timeline

Solution Concept

A Perfect Bayesian Equilibrium in our analysis consists of four parts: the activist's communication strategy $\mu(m|\theta) \in [0,1]$, the manager's decision on the project $a(m) \in \{A, R\}$, the activist's trading strategy $\sigma(\theta, m) \in \{0,1\}$, and the market maker's pricing decision $p(\sigma) \in \mathbb{R}$. In any equilibrium the following must hold:

1. For any σ, the market maker sets the price p equals to the expected value of the firm taking the activist's communication strategy $\mu(\cdot \mid \theta)$ and the manager's project approval strategy $a(\cdot)$ as given.

[8] To save on notation, whenever the activist is subject to a liquidity shock $\sigma = 1$.

2. For any θ and m the activist's trading decision $\sigma(\theta, m)$ maximizes the value of her holdings in the firm taking the manager's approval strategy $a(m)$ and the market maker's pricing policy $p(\cdot)$ as given.

3. For any message m the approval decision $a(m)$ maximizes the manager's expected utility, taking the activist's communication strategy $\mu(\cdot \mid \theta)$, trading strategy $\sigma(\cdot, m)$, and the market maker's pricing policy $p(\cdot)$ as given.

4. For any θ, if message m is in the support of $\mu(\cdot \mid \theta)$, then m maximizes the expected utility of the activist given the manager's approval strategy $a(\cdot)$, her own trading strategy $\sigma(\theta, \cdot)$, and market maker's pricing policy $p(\cdot)$.

II Analysis of the Baseline Model

I solve the model backward and start with several definitions and observations. Suppose in equilibrium the activist strategically exits if and only if $\theta \in \Upsilon$ and the manager accepts the project with probability one if $\theta \in \Theta$ and otherwise he rejects it for sure. The market maker uses this information to price the shares of the company. In equilibrium, the value of the share conditional on the activist's trading decision satisfies,

$$p(\sigma, \Theta, \Upsilon) = \begin{cases} \frac{\mathbb{E}\left[v(\theta, 1_{\{\theta \in \Theta\}})\right] + \rho \Pr[\theta \in \Upsilon] \mathbb{E}\left[v(\theta, 1_{\{\theta \in \Theta\}}) | \theta \in \Upsilon\right]}{1 + \rho \Pr[\theta \in \Upsilon]} & \textit{if } \sigma = 1 \\ \mathbb{E}\left[v(\theta, 1_{\{\theta \in \Theta\}}) | \theta \notin \Upsilon\right] & \textit{if } \sigma = 0 \end{cases} \tag{4}$$

where $\rho \equiv \frac{1-\delta}{\delta} \in (0, \infty)$. Note that Υ and Θ themselves may depend on the share price. The explicit formulation of the price when the activist does not exit plays no role in the analysis until managerial myopia in considered in Section IV. Thus, in what follows and whenever there is no risk of confusion, σ is omitted from the notation of price and p is simply the short term stock price conditional on the activist's decision to exit. Since Θ and Υ uniquely determine the outcome of the game, I will often say that two equilibria are *equivalent* if they have identical sets Θ and Υ.

Suppose the activist sends the manager message m. Recall the manager's decision to approve the project is unobservable. Therefore, from the manager's point of view, the probability of exit and the short term share price are independent of his actual decision. They only depend on the (equilibrium) expectations of which action the manager eventually undertakes. Therefore,

expression (2) implies that the manager approves the project if and only if

$$\mathbb{E}_{\mu}\left[\theta|m\right]+\beta\geq 0 \tag{5}$$

where $\mathbb{E}_{\mu}\left[\cdot|m\right]$ is the manager's expectation of θ conditional on message m and the activist's communication strategy μ.

Lemma 1 *In any equilibrium the manager accepts the project if and only if* $\mathbb{E}_{\mu}\left[\theta|m\right]+\beta\geq 0$.

Lemma 1 implies that as long as action a is unobservable the short term stock price does not have a *direct* effect on the manager's incentives to approve the project. This observation holds even if the manager has direct utility from the short term stock price, for example, through short term compensation. Nevertheless, I will show that the short term stock price may have an *indirect* effect on the manager's incentives to approve the project through the channel of communication. This feature is one aspect by which this model departs from the existing literature.

In what follows we focus attention on equilibria in which a meaningful (but possibly noisy) communication between the activist and the manager is feasible. In these equilibria the manager incorporates the activist's advice into his decision making and hence voice is a meaningful mechanism of shareholder activism. Formally,

Definition 1 *An equilibrium is* ***"responsive"*** *if and only if there exist* $m_A\neq m_R$ *and* $\theta_A\neq\theta_R$ *such that* $\mu\left(m_A\left|\theta_A\right.\right)>0$, $\mu\left(m_R\left|\theta_R\right.\right)>0$, *and* $a\left(m_A\right)\neq a\left(m_R\right)$.

In words, voice is an effective form of shareholder activism if and only if there are at least two different messages the activist sends the manager such that for one message, denoted by m_A, the manager responds by approving the project and for the other message, denoted by m_R, the manager responds by rejecting the project.[9] Equivalent requirements to Definition 1 is either $\Theta\neq\left[\underline{\theta},\overline{\theta}\right]$ or the existence of messages $m_A\neq m_R$ and communication strategy μ such that

$$\mathbb{E}_{\mu}\left[\theta|m_R\right]<-\beta\leq\mathbb{E}_{\mu}\left[\theta|m_A\right] \tag{6}$$

[9] In principle, an equilibrium can be informative yet non-responsive. This would be the case if despite the revelation of information the manager does not condition his decision on the activist's message. Since the communication between the manager and the activist is private, any information that does not affect the manager's decision has no effect on the equilibrium outcome as well.

Note that Definition 1 is invariant to the exit strategy of the activist. However, as I will demonstrate below, the existence of a responsive equilibrium crucially depends on the ability of the activist to exit.

Denote by $V(\Theta, \Upsilon)$ the ex-ante long term value of shareholders. Since conditional on Θ the set Υ has no effect on shareholder value, $V(\Theta, \Upsilon)$ can be explicitly written as $\mathbb{E}\left[v\left(\theta, 1_{\{\theta \in \Theta\}}\right)\right]$. For example, in the first best equilibrium $\Theta^{FB} = \left[0, \overline{\theta}\right]$ and the manager accepts the project if and only if $\theta \geq 0$. Thus, $V\left(\Theta^{FB}, \Upsilon\right) \equiv \mathbb{E}\left[|\theta|\right]$.

An equilibrium is more *efficient* if it generates a higher ex-ante value of the firm. When multiple of equilibria exist, we let $\overline{V}$ and $\underline{V}$ be shareholders's ex-ante value under the most and least efficient equilibrium, respectively. Throughout the analysis the focus will be on the most efficient equilibrium.

The next definition will be helpful when I study the interaction between voice and exit.

Definition 2 *Voice and exit are* ***"complement" ("substitute")*** *mechanisms of governance if and only if* $\overline{V}_{Voice,Exit} - \overline{V}_{NoVoice,Exit} > (<) \overline{V}_{Voice,NoExit} - \overline{V}_{NoVoice,NoExit}$.

In words, if the efficiency gains from the activist's option to exit are higher when voice is allowed than when it is restricted, then voice and exit are complement mechanisms of governance. Otherwise, voice and exit substitute each other.

Finally, I restrict attention to a subset of equilibria of the game in which $\Theta = \left[\tau, \overline{\theta}\right]$ for some $\tau \in \left[\underline{\theta}, \overline{\theta}\right]$. I name equilibria within this subset as *threshold* equilibria and often say that the manager follows a threshold strategy τ. Note that the efficiency of a threshold equilibrium decreases with $|\tau|$. The next result shows that in our search for efficiency the focus on threshold equilibria is without the loss of generality.

Lemma 2 *For any responsive equilibrium there is a threshold equilibrium which is more efficient.*

A Benchmarks

In order to better understand the role and the effect of exit on voice, I consider two benchmarks. In the first benchmark the activist can exit and sell her holdings to the market maker, but she cannot voice her opinion to the manager. In the other benchmark the activist can voice herself but she cannot exit.

Benchmark I - Exit Without Voice

Suppose the activist cannot communicate with the manager. According to Lemma 1, if communication is not allowed then regardless of the activist's ability or incentives to exit, the manager approves the project if and only if

$$\beta \geq -\mathbb{E}[\theta] \tag{7}$$

where $\mathbb{E}[\cdot]$ is the unconditional expectations with respect to θ. If condition (7) holds (does not hold) then in equilibrium the manager approves (rejects) the project with probability one and $\Theta_{NoVoice,Exit} = \left[\underline{\theta}, \overline{\theta}\right]$ ($\Theta_{NoVoice,Exit} = \varnothing$). In that case, the activist strategically sells her shares in the firm if and only if $\theta \leq p$ ($\theta \geq -p$) and hence $\Upsilon_{NoVoice,Exit} = [\underline{\theta}, p]$ ($\Upsilon_{NoVoice,Exit} = [-p, \overline{\theta}]$).

The market maker is competitive and hence the price is set fairly and reflects the expected value of the share conditional on the sell order. Specifically, for any real function $g(\cdot)$ define

$$\varphi(g(\theta), p) \equiv \frac{\mathbb{E}[g(\theta)] + \rho \Pr[g(\theta) \leq p] \mathbb{E}[g(\theta) | g(\theta) \leq p]}{1 + \rho \Pr[g(\theta) \leq p]} \tag{8}$$

where the expectations are taken with respect to θ. Thus, substituting the $\Theta_{NoVoice,Exit}$ and $\Upsilon_{NoVoice,Exit}$ into expression (4) implies that the price in this benchmark is given by the solution of $p = \varphi(\theta, p)$ when $\beta \geq -\mathbb{E}[\theta]$ and by the solution of $p = \varphi(-\theta, p)$ otherwise. To ease the exposition, hereafter, I assume that condition (7) holds. That is, without more information, the manager will follow his bias and approve the project.[10]

The following lemma summarizes the equilibrium when communication is not feasible, which corresponds to the non-responsive equilibrium of the game. As in any "Cheap Talk" game, this equilibrium always exists, even when communication is allowed.

Lemma 3 (Non-Responsive Equilibrium) *A non-responsive equilibrium always exists. In any non-responsive equilibrium the manager accepts the project with probability one and the activist exits if and only if $\theta \leq p_{NR}$. The share price conditional on exit is given by $p_{NR} \equiv \min_p \{\varphi(\theta, p)\}$ and it decreases with ρ.*

Without voice, the option to exit does not change the manager's incentives to invest in the project. This is because the activist does not observe the manager's decision to take the project. Nevertheless, both the activist and the market maker correctly anticipate the approval

[10] Most proofs also consider $\beta < -\mathbb{E}[\theta]$. In this case the manager by default rejects the project. The activist's challenge is convincing the manager to accept the project despite the manager's bias toward the project.

of the project by the manager. Importantly, the market maker understands that for a given price p, the activist exits if and only if $v(\theta, a) = \theta < p$. Therefore, the market maker forms the "worst case" beliefs on the value of the firm conditional on the activist's exit. This is the reason why the price equals $\min_p \{\varphi(\theta, p)\}$. Finally, since in this benchmark the option of the activist to exit has no effect on the manager's decision to approve the project, it is irrelevant from efficiency point of view, that is, $\overline{V}_{NoVoice,Exit} = \overline{V}_{NoVoice,NoExit} = \mathbb{E}[\theta]$.

Benchmark II - Voice Without Exit

Suppose the activist cannot exit her investment in the firm. The activist incurs no cost by voicing her opinion and hence her utility is proportional to the firm's value. Consistent with maximizing shareholders' long term value, the activist would like the manager to accept the project if and only if $\theta \geq 0$. Let us define

$$z(x) \equiv \begin{cases} z : \mathbb{E}[\theta | \theta < -z] = -x & \textit{if } x > -\mathbb{E}[\theta] \\ -\overline{\theta} & \textit{if } x \leq -\mathbb{E}[\theta] \end{cases} \tag{9}$$

and note that $z(x) < x$, $z(x)$ strictly increases in x, and $\lim_{x \to -\underline{\theta}} z(x) = -\underline{\theta}$. The function z will be useful in the characterization of equilibria.

Lemma 4 *Suppose the activist cannot exit. A responsive equilibrium exists if and only if $\beta \leq z^{-1}(0)$. In any responsive equilibrium the manager follows the first best decision rule.*

Lemma 4 suggests that without the option to exit, voice is ineffective when the conflict of interests between the manager and shareholders is significant. In those cases, the manager does not find the activist's advice credible, and therefore, ignores it. When $\beta \leq z^{-1}(0) = -\mathbb{E}[\theta | \theta < 0]$ the conflict of interests is small and the activist has enough credibility to influence the manager's decision. Since the activist is unbiased, she will always use her advisory power to persuade the manager to implement the first best. With respect to welfare, Lemma 4 implies

$$\overline{V}_{Voice,NoExit} = \begin{cases} \mathbb{E}[|\theta|] & \textit{if } \beta \leq z^{-1}(0) \\ \mathbb{E}[\theta] & \textit{else} \end{cases} \tag{10}$$

B Voice and Exit - Soft Shareholder Activism

In this subsection I analyze the strategic communication between the activist and the manager in light of the possibility of exit. The following lemma provides the first hint for the interaction between voice and exit in a responsive equilibrium.

Lemma 5 *In any responsive equilibrium $p^* > 0$ and the activist exits strategically if and only if $|\theta| \leq p^*$.*

The activist has incentives to exit and sell her holdings only if the share price that is set by the market maker over-values the firm. Lemma 5 suggests that in any responsive equilibrium the share price upon exit must be strictly positive. To see why, note that in any responsive equilibrium, by definition, the value of the firm depends on the activist's voice. The activist can dictate the action that is taken by the manager by sending the appropriate message. If on the contrary the share price (upon exit) is negative, then the activist is strictly better off by keeping her holdings in the firm ($\Upsilon = \varnothing$) and guiding the manager through the optimal decision rule ($\Theta = \Theta^{FB}$).[11] In this case, the value of the firm is strictly positive with probability one. While the market maker does not observe the private message m or the managerial decision a, in equilibrium, the market maker has rational expectations about the manager's approval strategy and the activist's communication and exit strategies. The market maker understands that a negative price implies an efficient decision rule. Therefore, setting a negative price is inconsistent with the fair share value of the company, $V\left(\Theta^{FB}, \varnothing\right) = \mathbb{E}\left[|\theta|\right] > 0$, yielding a contradiction. Since the share price upon exit is strictly positive, if $|\theta| \leq p^*$ then the activist is strictly better off by selling her shares, a property which must be taken into account by the market maker when setting the price.

Since the stock price affects the incentives of the activist to exit, it may also feed back into the quality of communication between the activist and the manager. As I explain below, when the activist plans to exit she is less sensitive to the fundamentals of the firm and can communicate more freely with the manager. In equilibrium, all of these forces must be consistent with each other. Lemma 6 presents the set of communication strategies that the activist has incentives to follow, taking into account the market maker's pricing decision.

[11] If $p^* = 0$ and $\theta = 0$ the activist is indifferent and hence may exit. But since $\theta = 0$ is a zero probability event, it will not change any of the results even if the activist chooses to exit in this case.

Lemma 6 *Suppose the manager follows a threshold strategy τ. The implied share price conditional on exit is $\pi(\tau) \equiv \min_{p \geq -|\tau|}\{\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)\}$. Let $\underline{\tau} < 0$ be the unique negative solution of $\pi(\tau) + \tau = 0$ and $\overline{\tau} > 0$ the unique positive solution of $\pi(\tau) - \tau = 0$, then:*

(i) If $\tau \in [\underline{\tau}, \overline{\tau}]$ then $\pi(\tau) > 0$.

(ii) If and only if $\tau \in [\underline{\tau}, \overline{\tau}]$ then $\tau \in [-\pi(\tau), \pi(\tau)]$.

To understand Lemma 6, suppose in equilibrium the manager approves the project if and only if $\theta \geq \tau$ and the share price upon exit is give by $p > 0$. As was mentioned before, the activist exits strategically if and only if $\theta \in [-p, p]$. In equilibrium, the market maker's expectations are consist with the manager and the activist's behavior, and the share price is set equal to $\pi(\tau)$. Indeed, in any equilibrium where $\Theta = \left[\tau, \overline{\theta}\right)$ and $\Upsilon = [-p, p]$ the price must be the solution of $p = \varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)$. Following the same line of reasoning as in Lemma 3, the solution of this equation is the worst case scenario, $\min_{p \geq -|\tau|}\{\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)\}$. Thus, if the manager follows a threshold strategy τ then the short term price of the stock must be $\pi(\tau)$. Part (i) of Lemma 6 proves that as long as $\tau \in [\underline{\tau}, \overline{\tau}]$ the implied share price is strictly positive as required by Lemma 5. Part (ii) of Lemma 6 guarantees that if $\tau \in [\underline{\tau}, \overline{\tau}]$ the activist indeed finds it optimal to follow a communication strategy that implements $\Theta = \left[\tau, \overline{\theta}\right]$. To see why, note that when the activist strategically exits she becomes indifferent with respect to the long term value of the firm. Thus, the activist has weak incentives to follow any threshold $\tau \in [-\pi(\tau), \pi(\tau)]$, even if $\tau \neq 0$. Figure 2 illustrates the determination of upper bound $\overline{\tau}$ and lower bound $\underline{\tau}$.

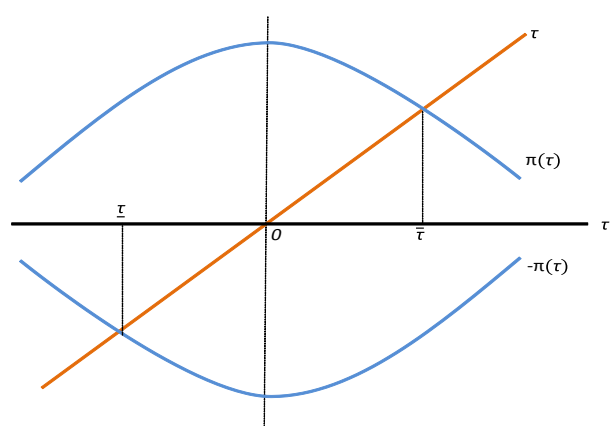

Figure 2

An immediate conclusion that follows from Lemma 6 is that there is no responsive equilibrium with threshold $\tau \notin [\underline{\tau}, \overline{\tau}]$. The next proposition fully characterizes the equilibrium by imposing an additional condition which requires that the manager has incentives to follow the proposed decision rule τ.

Proposition 1 *If and only if* $\tau \in [\underline{\tau}, \min\{-z(\beta), \bar{\tau}\}]$ *then a responsive equilibrium with threshold* τ *exists. A responsive equilibrium exists if and only if* $\beta \leq z^{-1}(-\underline{\tau})$.

The manager of the firm is biased toward the approval of the project. Therefore, the manager always follows the recommendation of the activist to approve the project. The binding incentive constraint is convincing the manager to reject the project. As long as the bias is not too high, there exists $\tau \in [\underline{\tau}, \bar{\tau}]$ such that conditional on $\theta < \tau$ the manager believes that rejection will be optimal, that is, $\mathbb{E}[\theta|\theta < \tau] + \beta \leq 0$. The higher is the bias, the lower must be threshold τ in order to satisfy this constraint. Thus, in addition to requiring $\tau \in [\underline{\tau}, \overline{\tau}]$, Proposition 1 reveals that a threshold strategy τ can be sustained by a responsive equilibrium only if $\tau \leq -z(\beta)$.

Given Lemma 2, it immediately follows from the first part of Proposition 1 that a responsive equilibrium exists if and only if $\beta \leq z^{-1}(-\underline{\tau})$.[12] Since $z^{-1}(\cdot)$ is an increasing function and $\underline{\tau} < 0$ the comparison between Proposition 1 and Lemma 4 reveals that voice is more effective with exit than without it. Counter to the intuition, the option of the activist to exit and sell her holdings enhances her ability to communicate with the manager and thereby affects his decision. The reason is that the activist becomes less sensitive to the performances of the firm. Indeed, if the activist believes the share is overpriced, she has the option to exit and therefore she becomes indifferent with respect to the decision of the manager to approve the project. This property facilitates the ability of the activist to credibly convey information and thereby overcome the disagreement with the manager.

Proposition 1 also implies that the most efficient threshold that can be supported by an equilibrium is $\tau^* = \max\{-z(\beta), 0\}$. Thus, similar to Lemma 4, the first best $\tau^{FB} = 0$ is obtained if and only if $\beta \leq z^{-1}(0)$. However, when $\beta \in (z^{-1}(0), z^{-1}(-\underline{\tau})]$ the second best threshold is $\tau^{SB} = -z(\beta) < 0$. Thus, in equilibrium, the project is over-accepted by the manager. The next corollary provides the welfare analysis.[13]

[12] Implicitly, the analysis considers only pure strategy equilibria. In the Appendix I show that for any mixed strategy responsive equilibrium there is a pure strategy responsive equilibrium which is more efficient. Thus, Proposition 1 continues to hold even if one considers mixed strategy equilibria.

[13] Despite the activist's opportunistic exit behavior, for any threshold τ her expected utility is identical to value of the firm, $\mathbb{E}\left[v(\theta, 1_{\{\theta \geq \tau\}})\right]$. This is because in equilibrium the share is priced fairly. On the other hand, the

Corollary 1 (Welfare) *The least efficient equilibrium is non-responsive and the most efficient equilibrium is responsive. Moreover,* $\overline{V}_{Voice,Exit}$ *decreases in* β *and* ρ *and it satisfies,*

$$\overline{V}_{Voice,Exit} = \begin{cases} \mathbb{E}\left[v(\theta, 1_{\{\theta \geq \max\{-z(\beta),0\}\}})\right] & \text{if } \beta \leq z^{-1}\left(-\underline{\tau}\right) \\ \mathbb{E}\left[\theta\right] & \text{else} \end{cases} \tag{11}$$

Note that $\mathbb{E}\left[v(\theta, 1_{\{\theta \geq \max\{-z(\beta),0\}\}})\right] > \mathbb{E}\left[\theta\right]$ if and only if $\mathbb{E}\left[\theta | \theta < \max\left\{-z\left(\beta\right), 0\right\}\right] < 0$ which always holds. The comparison between expressions (10) and (11) reveals that exit can facilitate communication and thereby efficiency. Table 1 summarizes the ex-ante long run value of the firm that is obtained by the most efficient equilibrium under the different regimes.

	No Voice	**Voice Without Exit**	**Voice With Exit**
$\beta \in [0, z^{-1}(0)]$	$\mathbb{E}\left[\theta\right]$	$\mathbb{E}\left[\lvert\theta\rvert\right]$	$\mathbb{E}\left[\lvert\theta\rvert\right]$
$\beta \in (z^{-1}(0), z^{-1}(-\underline{\tau})]$	$\mathbb{E}\left[\theta\right]$	$\mathbb{E}\left[\theta\right]$	$\mathbb{E}\left[v(\theta, 1_{\{\theta \geq -z(\beta)\}})\right]$
$\beta \in (z^{-1}(-\underline{\tau}), \infty)$	$\mathbb{E}\left[\theta\right]$	$\mathbb{E}\left[\theta\right]$	$\mathbb{E}\left[\theta\right]$

Table 1 - The Ex-ante Long Run Value of the Firm

It immediately follows from Definition 2 and Table 1 that voice and exit exhibit *complementarity.*

More generally, Corollary 1 implies that the value of the firm increases with δ (decreases with ρ), where δ is the frequency of the activist's liquidity shocks. Recall that because of the adverse selection between the activist and the market maker, exit generally reflects bad news on the company and its investment opportunities. Higher δ relaxes the adverse selection problem since the activist is more likely to exit for reasons which are unrelated to the value of the firm. All else equal, the negative price impact of exit diminishes. The option to strategically exit at better terms makes the activist less sensitive to the long run performances of the firm, and therefore, allows the activist to communicate more freely with the manager. The improved quality of communication enhances the efficiency of the manager's decision making. In the Appendix I show that when $\delta \to 0$ then $\underline{\tau} \uparrow 0$ and $\bar{\tau} \downarrow 0$. Thus, the existence of a responsive equilibrium converges to the same conditions of the benchmark case of voice without exit when

manager's expected utility is given by $\mathbb{E}\left[v(\theta, 1_{\{\theta \geq \tau\}})\right] + \left(2\Pr\left[\theta \geq \tau\right] - 1\right)\beta$. Thus, the manager would benefit by shifting threshold τ^{SB} downward to $-\beta$. Nevertheless, the most efficient equilibrium is Pareto efficient and hence can be sustained as a valid outcome of the game.

the frequency of liquidity shocks vanishes.

An interesting comparative static that is missing from Corollary 1 is whether shareholders are better off if the distribution of θ shifts upward in a first-order degree stochastic dominance manner. On the one hand, the manager is biased toward the approval of the project and he needs a stronger evidence that θ is low in order to reject the project. Thus, $z(\beta)$ is strictly higher for any β, and $\left|\tau^{SB}\right|$ increases. This implies a loss of efficiency. On the other hand, since θ has shifted upward, rejection is less likely to be the efficient decision in the first place. So the total effect on welfare is ambiguous. In the Appendix I provide examples where the direction of change in shareholders' welfare can be either positive or negative as a response to such perturbation of the density function f.

Interestingly, Proposition 1 implies that the value of the firm does not monotonically increase with the amount of information that is exchanged between the activist and the manager.

Corollary 2 *A fully revealing equilibrium exists if and only if $\beta \leq -\underline{\tau}$. If it exists, there is a threshold equilibrium which is strictly more efficient.*

Recall that $\underline{\tau} \uparrow 0$ as $\delta \to 0$. Thus, the condition in Corollary 2 implies that without exit a full revelation of information is not feasible in equilibrium. By contrast, with exit a fully revealing equilibrium exists if and only if the bias of the manager is sufficiently small. Nevertheless, even if it exists, a fully revealing equilibrium is never the most efficient equilibrium. To understand why, recall that the binding incentive constraint is convincing the manager to reject the project. If information is fully revealed then $\Theta_{Full_Revelation} = \left[-\beta, \overline{\theta}\right]$ and the manager sub-optimally approves the project when $\theta \in [-\beta, 0]$. The activist can do strictly better by introducing noise into the communication with the manager. She can reveal whether θ is greater or smaller than $\min\{-z(\beta), 0\}$, but not the exact value of θ. By pooling very low realizations of θ with intermediate realizations of θ, the activist is able to persuade the manager to reject the project even when $\theta \in [-\beta, \min\{-z(\beta), 0\}]$. Since $-z(\beta) > -\beta$, the implemented threshold under the latter strategy is strictly more efficient. Figure 3 summarizes the observations above by plotting the manager's decision rule under different equilibria.

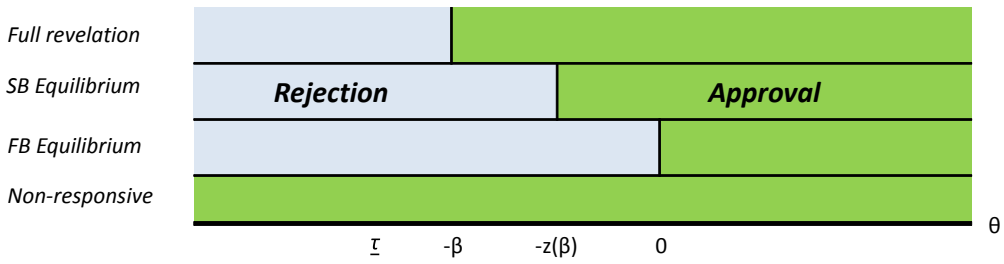

Figure 3 - Revelation of Information and Welfare Implications

The next corollary shows the comparative static of the likelihood of exit in equilibrium.

Corollary 3 (The Likelihood of Exit) *At the most efficient equilibrium the ex-ante probability of exit decreases in β and ρ unless $\beta = z^{-1}(-\underline{\tau}(\rho))$ and $\Pr[\theta \leq p_{NR}(\rho)] > \Pr[|\theta| \leq -\underline{\tau}(\rho)]$.*

One might expect that when conflict of interests between shareholders and the manager becomes more significant the activist is more likely to exit. However, the likelihood of exit is determined by the over-valuation of the stock price relative to the firm's long term value. Consider the most efficient responsive equilibrium. The second best is given by a threshold $-z(\beta) < 0$. According to Lemma 6, as long as $\beta < z^{-1}(-\underline{\tau})$ the price is given by $p^* = \pi(-z(\beta))$ where $-z(\beta) \in (-p^*, p^*)$. Recall that the activist strategically exits if and only if $\theta \in (-p^*, p^*)$ and that $-z(\beta)$ is decreasing in β. When β increases, instead of rejecting the project when $\theta = -z(\beta)$ the manager is now approving it. Since in this range the activist exits whether or not the manager takes the efficient decision, the activist exit strategy can change only if the share price changes. Indeed, the market maker anticipates that manager will follow a less efficient decision making and hence reduces the share price. This latter effect reduces the incentives of the activist to exit. If $\beta = z^{-1}(-\underline{\tau})$ and β increases then the most efficient equilibrium becomes non-responsive. Since the activist completely loses her ability to influence the manager, this change discretely shifts the ex-ante probability of exit by a magnitude that is proportional to $\Pr[\theta \leq p_{NR}] - \Pr[|\theta| \leq -\underline{\tau}]$. Finally, when $\beta > z^{-1}(-\underline{\tau})$ the equilibrium is non-responsive and Lemma 3 implies that the ex-ante probability of exit is invariant to β.

Corollary 3 also shows that almost everywhere the likelihood of exit increases with the frequency of the activist's liquidity shocks. Indeed, with a higher δ (lower ρ) the market maker is less exposed to adverse selection and sets the price at a higher level. Therefore, the activist

has more opportunities to exit and the likelihood of exit increases with δ. Interestingly, when $\beta = z^{-1}(-\underline{\tau})$ and δ increases, the switch from a responsive equilibrium to a non-responsive equilibrium can discretely decrease the likelihood of the exit. Thus, in this setting, a relaxed adverse selection problem can in fact reduce the likelihood of trade.

III Soft Shareholder Activism and Transparency

The baseline model assumes *"No-Transparency"*, that is, the activist and the market maker do not observe the decision made by the manager, and the message the activist sends the manager is private and unobservable to the market maker. In this section I relax these assumptions and study the effect of transparency on voice and exit.

Transparency of Managerial Actions

It is important to distinguish between two kinds of transparency: *"Activism-Transparancy"* and *"Market-Transparnacy"*. Under Activism-Transparency only the activist observes the manager's decision before she decides whether to exit. Under Market-Transparency both the activist and the market maker observe the manager's decision. Under either of these regimes I maintain the assumption that the communication between the activist and the manager is private.

The definition of a responsive equilibrium is trivially extended to the new setup. Regardless of the kind of transparency, a non-responsive equilibrium always exists and its analysis coincides with the benchmark of exit without voice. Indeed, in a non-responsive equilibrium the activist's message does not affect the manager's decision and his decision is fully predicted by the market, even without observing it directly. In anticipation of the manager's behavior, the activist exits under exactly the same circumstances that are described by Lemma 3. The next lemma shows that under Activism-Transparency the set of responsive equilibria does not change as well.

Lemma 7 *The sets of equilibria under No-Transparency and Activism-Transparency are identical.*

Activism-Transparency can change the set of equilibria if and only if it changes the circumstances under which the manager approves the project. Recall that the manager has no direct utility from the short term share price. Therefore, by itself, exit does not affect the manager's decision making. Since in any responsive equilibrium the activist can perfectly predict the

decision of the manager,[14] whether or not it is observed by the activist has no effect on the set of equilibria.[15]

Unlike Activism-Transparency, Market-Transparency has a significant effect on soft shareholder activism. The market maker observes the manager's decision to approve the project and uses this information to set the fair price. Let p_a be the share price conditional on exit and the manager's decision $a \in \{A, R\}$. In equilibrium, the activist takes p_a as given. Thus, if the activist persuades the manager to approve the project, she exits if and only if $p_A \geq \theta$. If the activist persuades the manager to reject the project, she exits if and only if $p_R \geq -\theta$. The next proposition describes the set of equilibria under Market-Transparency.

Proposition 2 *There is $\beta^* < z^{-1}(-\underline{\tau})$ such that a responsive equilibrium under Market-Transparency exists if and only if $\beta \leq \beta^*$, where $\beta^* < z^{-1}(0)$ for certain density functions of θ and values of ρ. Moreover, any responsive equilibrium under Market-Transparency satisfies $p_A = p_R > 0$ and it is also an equilibrium under No-Transparency.*

The comparison between Proposition 2 and Proposition 1 implies that Market-Transparency limits the ability of the activist to communicate with the manager. Since the most efficient equilibrium is a responsive equilibrium, Market-Transparency also harms shareholder value. Interestingly, the observation that β^* can be smaller than $z^{-1}(0)$ suggests that the value of the firm under Market-Transparency is lower than when the activist cannot trade at all. In this sense, voice and exit can exhibit substitution under Market-Transparency.

Intuitively, in a responsive equilibrium the activist is able to persuade the manager which action to take. Therefore, the share price conditional on exit must be invariant to the decision made by the manager. Otherwise, the activist will arbitrage the difference between these prices by sending the appropriate message. The incentives of the activist to persuade the manager to approve the project are distorted, they are affected by the activist's desire to inflate the short term price of the stock. Consequently, the credibility of her advice and the effectiveness of her voice are diminished. These are reflected by the additional constraint that in any responsive equilibrium it must be that $p_A = p_R$. Overall, under Market-Transparency, a

[14] With Activism-Transparency there are off-equilibrium events. However, since the manager's utility is invariant to σ and p, the activist's off-equilibrium beliefs or actions cannot change the set of equilibria.

[15] Activism-Transparency can also be interpreted as the timing of trade relative to the manager's decision making. To the extent that the market maker does not observe whether the activist trades before or after the manager has made his decision, the set of equilibria does not change even if the activist can time her trade.

responsive equilibrium is less likely to exist.[16]

Transparency of Voice

Suppose the manager's actions are not observable but the communication between the activist and the manager is not private. Then, the market maker observes the message m that the activist sends to the manager. The new framework is titled as *"Voice-Transparency"*. The objective of this section is to understand whether shareholder activism is more effective when voice is made public.

As in Section II, a non-responsive equilibrium always exists. In this equilibrium, the manager ignores any message from the activist and his decision is fully predictable. The activist may or may not decide to exit based on her observation of θ. However, she may still try to send a message that provides information to the market about θ. Without the ability to influence the manager's decision to approve the project, the activist is left with the objective of getting the highest price conditional on exit. The incentive to inflate the short term stock price prevents any revelation of information by the activist in such equilibrium. Therefore, a non-responsive equilibrium always exists when communication is public, and its characterization is given by Lemma 3. The next proposition proves that because a public message gives the activist incentives to manipulate prices directly, shareholder activism is severely impeded.

Proposition 3 *The sets of equilibria under Voice-Transparency and Market-Transparency are identical.*

In any responsive equilibrium under Voice-Transparency the market maker infers from the activist's message whether the manager will approve or reject the project. In the proof of Proposition 3 I show that the incentives of the activist to inflate the share price are sufficiently strong, and therefore, additional information cannot be revealed in equilibrium. This observation implies the equivalence between Voice-Transparency and Market-Transparency.

Suppose the activist cannot commit to a particular channel of communication but she can *choose* between sending a private message to the manager, communicating publicly, or both. In this case, any equilibrium under Voice-Transparency can be implemented: the manager ignores

[16] Admati and Pfleiderer (2009) also demonstrate that transparency of corporate decisions does not necessarily improve welfare. However, in their paper the channel through which transparency is harmful is fundamentally different: it is the consequence of a distorted exit strategy by the blockholder.

any private messages from the activist, and the activist always randomizes when sending private messages. Thus, the option to communicate privately does not impede activism. Interestingly, the option to communicate privately extends the set of equilibria to the one that is obtained under No-Transparency. In light of Proposition 2, the option to communicate privately has social value. Basically, there is always an equilibrium in which the manager and the market maker ignore any public message sent by the activist, and the activist always randomizes when sending public messages.

IV Activist's Information Endowment

The main channels through which activism is exercised in the present model are communication of private information and trading based on private information. It is therefore natural to wonder how the quality of the activist's private information affects her ability to influence the manager's decisions. In order to explore this question, I modify the baseline model and assume that the activist perfectly observes θ with probability $\lambda \in (0, 1]$. With the complement probability the activist is entirely uninformed about θ. Whether the activist is informed or uninformed is her own private information. Parameter λ captures the quality of the activist's private information, and higher λ implies higher quality. To simplify the analysis I assume throughout this section that $\mathbb{E}[\theta] \geq 0$.

Suppose in equilibrium the price upon exit is $p^*(\lambda)$ and the threshold for approval of the project is τ, where $\tau = \underline{\theta}$ implies that the equilibrium is non-responsive. Since $\mathbb{E}[\theta] \geq 0$, without additional information, the uninformed activist and the manager agree on the action to be taken. This implies that the uninformed activist can reveal its type to the manager.[17] If $p^*(\lambda) < \mathbb{E}[\theta]$ the uninformed activist tries to persuade the manager to approve the project and she exits only if she needs liquidity. If $p^*(\lambda) \geq \mathbb{E}[\theta]$ the uninformed activist has weak incentives to persuade the manager to approve the project and she exits with probability one. The informed activist would recommend on threshold strategy τ as long as $\tau \in [-p^*(\lambda), p^*(\lambda)]$ and exit if and only if $v\left(\theta, 1_{\{\theta \geq \tau\}}\right) \leq p^*(\lambda)$.

For any $\tau \in \left[\underline{\theta}, \overline{\theta}\right]$ and $x \geq 0$ let $\underline{\tau}(x)$ be the unique negative solution of $\pi(\tau, x) + \tau = 0$

[17] Alternatively, the uninformed activist randomizes over messages in M_A.

where $\pi(\tau, x) \equiv \min_{p \geq -|\tau|}\{\varphi(\tau, p, x)\}$ and

$$\varphi(\tau, p, x) \equiv \frac{\mathbb{E}\left[v\left(\theta, 1_{\{\theta \geq \tau\}}\right)\right] + \rho \Pr\left[v\left(\theta, 1_{\{\theta \geq \tau\}}\right) \leq p\right] \mathbb{E}\left[v\left(\theta, 1_{\{\theta \geq \tau\}}\right) | v\left(\theta, 1_{\{\theta \geq \tau\}}\right) \leq p\right] + x\mathbb{E}[\theta]}{1 + \rho \Pr\left[v\left(\theta, 1_{\{\theta \geq \tau\}}\right) \leq p\right] + x} \tag{12}$$

Similar to Lemma 3, a non-responsive equilibrium always exists and in this equilibrium $p^*(\lambda) = \pi\left(\underline{\theta}, \frac{1-\lambda}{\lambda}\right)$. The next result generalizes Proposition 1 for any $\lambda \in (0, 1]$.

Proposition 4 *A responsive equilibrium exists if and only if* $\beta \leq z\left(-\max\left\{\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right), \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)\right\}\right)^{-1}$.

(*i*) *If* $\underline{\tau}(0) > -\mathbb{E}[\theta]$ *then* $p^*(\lambda) = \pi\left(\tau, \frac{1-\lambda}{\lambda}\right) < \mathbb{E}[\theta]$ *and* $p^*(\lambda)$ *decreases with* λ.

(*ii*) *If* $\underline{\tau}(0) \leq -\mathbb{E}[\theta]$ *then* $p^*(\lambda) = \pi\left(\tau, \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) \geq \mathbb{E}[\theta]$ *and* $p^*(\lambda)$ *increases with* λ.

A corollary of Proposition 4 is that $p^*(\lambda) = \min\left\{\pi\left(\tau, \frac{1-\lambda}{\lambda}\right), \pi\left(\tau, \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)\right\}$. Basically, $p^*(\lambda)$ is a weighted average of the prior $\mathbb{E}[\theta]$ and the information that is embedded in the informed activist's decision to exit. If $p^*(\lambda) < (\geq) \mathbb{E}[\theta]$ then this information yields conditional expectations which are lower (greater) than $\mathbb{E}[\theta]$, and hence, $p^*(\lambda)$ decreases (increases) with the likelihood that the activist is informed. The condition $\underline{\tau}(0) > -\mathbb{E}[\theta]$ implies that δ is relatively low and hence the adverse selection problem is severe in this case.

When the equilibrium is non-responsive the expected value of the firm is $\mathbb{E}[\theta]$ regardless of λ. However, when the equilibrium is responsive, the expected value of the firm is given by $\overline{V} = \lambda\mathbb{E}\left[v\left(\theta, 1_{\{\theta \geq \min\{-z(\beta), 0\}\}}\right)\right] + (1-\lambda)\mathbb{E}[\theta]$. Thus, conditional on the existence of a responsive equilibrium, the expected value of the firm increases with λ. Let $\lambda^* \in [0, 1]$ be the (highest) level of λ that maximizes the value of the firm. The next proposition shows that there are circumstances under which the expected value of the firm decreases with the quality of the activist's private information.

Proposition 5 *Suppose* $\mathbb{E}[\theta] > 0$. *If and only if* $\underline{\tau}(0) > -z(\beta) > -\mathbb{E}[\theta]$ *then* $\lambda^* < 1$.

Since the activist is unbiased and her expected utility in a responsive equilibrium is exactly the expected value of the firm, one might expect that the value of the firm will increase with the quality of her private information. Seemingly, with more information, the activist has more opportunities to persuade the manager to take an informative action that is consistent with maximizing the value of shareholders. Proposition 5 shows that this intuition can be misleading.

In order to understand this result note that higher quality of private information can also intensify the adverse selection problem that the informed activist creates when she trades with the market maker. The intensified adverse selection problem restricts the ability of the activist to exit. Since exit and voice complement each other, the quality of the activist's private information can in fact limits her ability to voice herself credibly. Specifically, Proposition 5 states three conditions. First, the condition $\underline{\tau}(0) > -\mathbb{E}[\theta]$ implies that the adverse selection problem is severe and $p^*(\lambda)$ decreases with λ. Second, according to Proposition 1, the condition $-z(\beta) > -\mathbb{E}[\theta]$ implies that a responsive equilibrium does not exist when $\lambda = 1$. Last, the condition $-z(\beta) > -\mathbb{E}[\theta]$ requires that the manager is not too biased. Under these conditions there is $\lambda^* < 1$ such that if and only if $\lambda < \lambda^*$ a responsive equilibrium exists. Since conditional on the existence of a responsive equilibrium the value of the firm increases with λ, it follows that $\lambda^* < 1$.

Endogenous Acquisition of Information

A natural extension of the model is to consider the decision of the activist to acquire private information. Suppose that at the outset the activist is uninformed about θ but she can pay a fixed amount $c \geq 0$ and perfectly observe θ with probability one. The cost of acquiring information is distributed according to cumulative distribution function G with full support over $[0, \infty)$ and it is the activist's private information. Moreover, c is independent of the activist's liquidity shocks and fundamentals of the firm. The activist's decision to acquire information is unobserved by the market maker and the manager. Let $\lambda \in [0, 1]$ be the probability that the market maker and the manager believe that the activist is informed in equilibrium. Unlike in the analysis above, here λ will be endogenous.

Fix λ, if the activist acquires information then she gets on expectations (per share)

$$\delta p^*(\lambda) + (1-\delta)\,\mathbb{E}\left[\max\left\{v\left(\theta, 1_{\{\theta \geq \tau\}}\right), p^*(\lambda)\right\}\right] - \frac{c}{\alpha} \tag{13}$$

where $\alpha \in (0, 1]$ the size of the activist's holdings. If the activist does not acquire information she gets

$$\delta p^*(\lambda) + (1-\delta)\max\left\{\mathbb{E}[\theta], p^*(\lambda)\right\} \tag{14}$$

Note that expressions (13) and (14) increase with the price of the share upon exit. Since λ is determined in equilibrium and independent of the realization of c, in any equilibrium there is $c^* > 0$ such that the activist acquires information if and only if $c \leq c^*$. Therefore, $\lambda = G(c^*)$

where type c^* must be indifferent between acquiring information and remaining uninformed. Thus, c^* is given by the solution of the following equation,

$$\frac{c}{\alpha(1-\delta)} = \mathbb{E}\left[\max\left\{v\left(\theta, 1_{\{\theta\geq\tau\}}\right), p^*\left(G\left(c\right)\right)\right\}\right] - \max\left\{\mathbb{E}\left[\theta\right], p^*\left(G\left(c\right)\right)\right\} \tag{15}$$

A simple algebra shows that the right hand side of (15) is bounded and it increases in $p^*(G(c))$ if and only if $p^*(G(c)) < \mathbb{E}[\theta]$. Since $p^*(\lambda)$ decreases in λ if and only if $p^*(\lambda) < \mathbb{E}[\theta]$, the right hand side of (15) always decreases with c. Therefore, given τ, a solution for (15) always exists and is unique.

The level of information acquisition in a non-responsive equilibrium, which always exists, is obtained by the solution of (15) when $p^*(G(c)) = \pi\left(\tau, \frac{1-G(c)}{G(c)}\right)$ and $\tau = \underline{\theta}$. The sufficient and necessary condition for the existence of a responsive equilibrium is the same as in Proposition 4, but instead of taking λ exogenously, λ is given by $G(c^*)$ where c^* is the solution of (15) when $p^*(G(c)) = \min\left\{\pi\left(\tau, \frac{1-G(c)}{G(c)}\right), \pi\left(\tau, \frac{1}{\delta}\frac{1-G(c)}{G(c)}\right)\right\}$ and $\tau = \min\{0, -z(\beta)\}$.[18]

Proposition 6 *Suppose $\underline{\tau}(0) > -z(\beta) > -\mathbb{E}[\theta]$. There are $G_1 \overset{FDSD}{\prec} G_2$ such that $G_1(c_1^*) < G_2(c_2^*)$ and a responsive equilibrium exists if $c \sim G_2$ but it does not exist when $c \sim G_1$.*

Proposition 6 demonstrates that in spite of having lower cost of information acquisition, in equilibrium, the activist acquires less information and is less effective in voicing herself credibly. Without a commitment mechanism, the reduction in the cost of acquiring information exacerbates the adverse selection problem since the activist will be tempted to acquire a significant amount of information. As Proposition 5 shows, this can harm the ability of the activist to persuade the manager to take actions and ultimately harm the value of the firm. Thus, when private information is relatively cheap, the only feasible equilibrium is a non-responsive equilibrium. Since information is less valuable if the activist cannot use it to influence the manager, the activist ends up acquiring less private information.

An alternative interpretation of Proposition 6 regrades the size of the activist's holdings. Since a larger stake is associated with a lower cost per share of information acquisition, Proposition 6 suggests that the voice of small blockholders can be more effective than the voice of

[18] It is worth pointing out that c^* does not depend on β when the equilibrium is non-responsive but it decreases with β if and only if $\underline{\tau}(0) > -\mathbb{E}[\theta]$ when the equilibrium is responsive. Also, the effect of δ on c^* is ambiguous. All else equal, higher δ increases the price upon exit. This effect increases the value of information (the option to exit when the stock is over-valued). On the other hand, higher δ implies that the activist is more likely to be forced to exit regardless of her private information. This reduces the incentives to acquire information.

large blockholders. Small share-holdings is a commitment tool to remain relatively uninformed, which increases the effectiveness of voice due to a weaker adverse selection problem. This is could be another explanation why some investors choose to limit the size of their initial holdings in the firm.

V Managerial Myopia

Managers are often sensitive to the short run performances of their company's stock price. This could either be because of their compensation package which has stocks and options, or because of their career concerns and incentives to demonstrate executive talent. Either way, managerial myopia can play a significant role in the context of soft shareholder activism. To study the effect of myopia on the interaction of exit and voice, I modify the preferences of the manager and assume

$$u_M = \omega p(\sigma) + v(\theta + \beta, a) \tag{2'}$$

where $\omega \geq 0$ is the relative weight the manager puts on the short term stock price.

For what follows it is useful to note that exit always conveys bad news for the company and hence $p(\sigma = 0) > p(\sigma = 1)$ for any Θ and Υ.[19] Thus, when $\omega > 0$ the option to exit opens up the possibility that the activist threatens to sell her holdings in the company if the manager does not defer to her view.

I start with the observation that under No-Transparency the set of equilibria is invariant to managerial myopia. Indeed, since both the activist and the market maker do not observe the manager's action, the realized short term stock price is independent of the actual decision taken by the manager. Therefore, the manager's incentives to approve the project are invariant to the stock price and ω, and the analysis of Section II continues to hold. Hereafter, I assume Activism-Transparency.

Consider the benchmark cases with managerial myopia and under Activism-Transparency. When exit is not allowed, for the same reasons that are mentioned above, the set of equilibria is characterized by Lemma 4 and it is invariant to managerial myopia ($\overline{V}_{Voice,NoExit}(\omega) = \overline{V}_{Voice,NoExit}(0)$ for any ω). The next lemma considers the benchmark case of exit without

[19] To see why $p(\sigma = 0) > p(\sigma = 1)$, note that by definition, $\theta \in \Upsilon$ implies $v\left(\theta, 1_{\{\theta \in \Theta\}}\right) \leq p(\sigma = 1)$ or else the activist is strictly better off by not exiting. Therefore, $\mathbb{E}\left[v\left(\theta, 1_{\{\theta \in \Theta\}}\right) | \theta \in \Upsilon\right] \leq p(\sigma = 1)$ as well. From expression (4) it follows that $\mathbb{E}\left[v\left(\theta, 1_{\{\theta \in \Theta\}}\right) | \theta \in \Upsilon\right] \leq \mathbb{E}\left[v\left(\theta, 1_{\{\theta \in \Theta\}}\right)\right]$. This in turn implies $\mathbb{E}\left[v\left(\theta, 1_{\{\theta \in \Theta\}}\right) | \theta \in \Upsilon\right] \leq \mathbb{E}\left[v\left(\theta, 1_{\{\theta \in \Theta\}}\right) | \theta \notin \Upsilon\right]$ and hence $p(\sigma = 0) > p(\sigma = 1)$ as required.

voice. The analysis of this benchmark corresponds to the set of non-responsive equilibria with managerial myopia.

Lemma 8 (Non-Responsive Equilibrium With Managerial Myopia) *A non-responsive equilibrium exists for any* $\omega \geq 0$. *There is* $\hat{\omega} \in (0, \infty)$ *such that if and only if* $\Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}]$ *and* $\omega > \hat{\omega}$ *then in any non-responsive equilibrium the manager rejects the project with a strictly positive probability.*

With myopia, a non-responsive equilibrium does not have to be unique and sometimes the equilibrium must involve mixed strategies. When comparing with Lemma 3, the set of non-responsive equilibria with myopia dramatically changes. The reason is that when the manager considers his decision to approve the project, he realizes the activist's exit strategy depends on this decision. Therefore, even though the market maker cannot condition the price on the manager's decision, by changing the probability the activist exits, the manager also changes the price of the stock indirectly. In particular, if the manager's myopia is significant ($\omega > \hat{\omega}$) and the activist is relatively more likely to exit when the project is approved than when it is rejected ($\Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}]$), the attempt of the manager to minimize the likelihood the activist exits can crowd out his inherent bias toward the project's approval. Indeed, under those circumstances, despite the manager's bias for the project and the assumption that $\beta \geq -\mathbb{E}[\theta]$, an equilibrium in which the manager approves the project with probability one does not exist.

Lemma 8 has welfare implications. If $\mathbb{E}[\theta] \in (-\beta, 0)$ then myopia increases the likelihood the manager rejects the project and therefore has positive effect on shareholders' welfare. However, if $\mathbb{E}[\theta] > 0$ then myopia actually harms shareholders. Thus, with myopia exit exerts power on the manager even when voice is idle. The next proposition shows that with exit and voice, the effect of myopia is always positive.

Proposition 7 *The set of responsive equilibria strictly increases with* ω *and decreases with* β. *Moreover, a threshold equilibrium exists if and only if* $\beta \leq \bar{\beta} \equiv \max_{\tau \in [\underline{\tau}, \bar{\tau}]} \Psi(\tau; \omega, \rho)$, *and the first best is obtained in equilibrium if and only if* $\beta \leq \underline{\beta} \equiv \Psi(0; \omega, \rho)$ *where*

$$\Psi(\tau; \omega, \rho) \equiv \frac{\omega}{2} \frac{\rho}{1+\rho} \Pr[\theta < -\pi(\tau) | \theta < \tau] (\mathbb{E}[|\theta| \,|\, |\theta| > \pi(\tau)] - \pi(\tau)) - \mathbb{E}[\theta | \theta < \tau] \tag{16}$$

Proposition 7 demonstrates that any responsive equilibrium without managerial myopia is

also a responsive equilibrium with managerial myopia.[20] Intuitively, with myopia the manager has stronger incentives to follow the activist's advice. If the manager deviates and ignores the activist's recommendation then the activist is strictly more likely to exit. As was explained above, exit always conveys bad news about the company. Thus, by ignoring the activist's recommendation the manager depresses the short term price of the stock, to which he is sensitive. Different from the analysis of non-responsive equilibria in Lemma 8, the manager's decision depends on the activist's message and therefore the threat of exit exercises stronger discipline on the manager to follow the proposed action. For this reason, there are responsive equilibria with managerial myopia which do not exist without managerial myopia. This can bee seen by noting that $\underline{\beta} > z^{-1}(0)$ and $\bar{\beta} > z^{-1}(\underline{\tau})$. In conclusion, managerial myopia increases the effectiveness of voice.[21]

VI Opportunistic Activism

In many cases activist investors are suspected of having a secret agenda that is conflicted with maximizing the value of the firm. For example, hedge funds are often blamed from being opportunistic and pursuing short-term goals which are inconsistent with the long-term value of the firm. Mutual funds often have business ties with their portfolio companies. They administrate their corporate pension plans and therefore could be biased toward the management's agenda. In order to study this type of opportunism I modify the activist's preferences and assume

$$u_A = \sigma p(\sigma) + (1-\sigma)\, v(\theta + \gamma, a) \tag{3'}$$

where $\gamma \neq 0$ is the activist's bias. Note that when the activist exits she is indifferent with respect to the performances of the firm. Only when the activist keeps her holdings in the firm her bias matters. For example, if the hedge fund has cross-holdings in the target firm and a potential set of acquirers, its deviation from maximizing target shareholders' value persists only as long as the fund keeps her holdings in the target.

The analysis starts with the observation that without voice, the price upon exit is higher

[20] Since Lemma 2 does not apply in a setup with managerial myopia, Proposition 4 does not rule out the existence of non-threshold responsive equilibria when $\beta > \bar{\beta}$. The term $p(\sigma = 0) - p(\sigma = 1)$, which plays a key role in the incentives of the manager's to follow the activist's advice, does not monotonically increase with the "efficiency" of the manager's decision rule. Therefore, it is possible that with more efficient decision rule the manager has fewer incentives to follow the activist's advice.

[21] In the Appendix I show that only when $\beta \leq z^{-1}(0)$ then with managerial myopia there are special circumstances under which voice and exit exhibit substitution.

when the activist is biased than when it is unbiased. This is true regardless of the direction or magnitude of the bias. The reason for this seemingly counter-intuitive result comes from the observation that a biased activist will exit under different circumstances than an unbiased activist. That is, the activist's private benefit relaxes the adverse selection problem that is embedded in her exit decision. To see why, recall that when the activist is unbiased, the market maker ascribes the "worst case beleifs" upon exit and hence the solution of the equation $p = \varphi(\theta, p)$ is $\min_p \{\varphi(\theta, p)\}$. With the bias, however, the activist would like to exit if and only if $|\theta + \gamma| \leq p \Leftrightarrow \theta \in [-p - \gamma, p - \gamma]$. Essentially, taking the price as given, the bias shifts the range of exit where the direction of the shift depends on the sign of the bias. This implies that the biased activist may strategically exit even if the share is under-valued or keep her holdings when the share is over-valued. Either way, this dynamic pushes the price upon exit upward.

The activist's bias has an indirect effect on manager's decision making by changing the activist's ability to credibly convey information. The next proposition shows that if $\beta > z^{-1}(-\underline{\tau})$ then a responsive equilibrium exists if and only if the activist is biased for the project's approval.

Proposition 8 *Let γ^* be the lowest bias (in absolute terms) of the activist for which threshold $\tau^* \equiv \max\{-z(\beta), 0\}$ is implementable in equilibrium. Then, $\gamma^* \in (0, z(\beta)]$ if $\beta > z^{-1}(-\underline{\tau})$ and $\gamma^* = 0$ otherwise. Moreover, γ^* increases in β and ρ.*

Proposition 8 implies that when the manager is sufficiently biased, voice can be effective only if the activist is biased in the same direction as the manager. The smallest bias that is necessary to sustain a responsive equilibrium is never as large as the manager's bias. Thus, the optimal bias is always in between the manager and shareholders, and it increases with the manager's bias. Overall, a biased activist can improve welfare by communicating more information. One way to interpret this result is that ties between large shareholders and senior management are not necessarily harmful. For example, mutual funds that benefit from administrating the pension plan of their portfolio company might have an inherent bias toward the incumbent management. This result is in contrast to models in which voice is modeled as a costly action taken by the activist. In these models, in order to provide the activist with incentives to incur the personal cost of voice, the optimal bias would be the opposite to the bias of the manager.[22]

[22] By contrast to the analysis in Section II, when the bias of the manager is very large, there is a responsive equilibrium with voice and exit in which the biased activist never exits for strategic reasons even though she has the option to.

Myopic Activism

Suppose the activist discounts the long term value of the firm in the following sense,

$$u_A = \sigma p + (1 - \sigma)\, \xi v\,(\theta, a) \tag{3"}$$

where $\xi \in [0, 1]$. In that case, the activist would like to exit as long as $\frac{p}{\xi} > v\,(\theta, a)$. Therefore, all else equal, the activist is more likely to exit. Similar to the analysis of Section II, this implies that prices will be higher and hence more information can be communicated. Similar to the reasoning that was presented above, the price is higher not only because more information is communicated, but also because exit by the myopic activist is not the worst case scenario.

VII Empirical Implications

The main premise of the theoretical analysis in this paper is that voice is an informal communication between investors and the manager of the firm. To the extent that the number of meetings, emails, letters, or phone calls between management and investors can be observed, the model offers new testable predictions about the frequency of this type of engagement.

The model predicts the frequency of engagement should be negatively related to the longevity of the large investor, where longevity is the likelihood that the activist is not subject to a short-term liquidity shock $(1 - \delta)$. This prediction is a reflection of the complementarity between voice and exit: shorter longevity implies that the adverse selection problem when trading with the market maker is weaker, and hence, all else equal, the investor can exit at better terms and be more effective when talking to the management. This prediction is consistent with Solomon and Soltes (2012) who study the frequency of meetings between senior management and investors and show that investors who have greater turnover in their holdings gain greater access to management.

The model also predicts that high frequency of engagement should be observed when the conflict of interests between shareholders and the manager is not significant (low β) and the short term component in the manager's compensation package is relatively important (high ω). In both cases the analysis predicts that voice is more effective (Proposition 1 and Proposition 7, respectively), and hence, one would expect to see more engagement between investors and management.

The analysis considers both private and public engagements between activist investors and

firms, and concludes that private engagements (lack of transparency) are more likely to be effective. Thus, the model predicts that private engagements should be observed more often than public engagements. If one can control for the decision of the activist to run a public (rather than private) campaign, then, all else equal, private engagements should have a stronger positive effect on the performances of the firm than public engagements. Consist with this observation, Becht, Franks, and Grant (2010) find that for board and payout changes, and restructuring events other than takeovers, the returns to the activist are higher when the engagement is private.

The communication between investors and the manager of the firm is often informal and private. It is therefore difficult to measure the magnitude and quality of this informal engagement using publicly available data. To the extent that the frequency of engagement or its quality are not observed, the model in this paper offers some indirect empirical predictions. The model predicts that the effectiveness of soft shareholder activism generally decreases with the longevity of the large shareholders. That is, there is an inverse relationship between the longevity of the investor and the value of the firm. This is in contrast to other models of shareholder activism. For example, Admati and Pfleiderer (2010) predict exactly the opposite. Moreover, different from other models of exit, soft shareholder activism can be effective even when managers are not myopic. Thus, by studying the effect of exit on the performances of firms with a negligible amount of short-term executive compensation, one can indirectly identify the effect or prevalence of private engagement and communication.

Finally, the model predicts that even small investors who are mainly active in middle markets and do not have the capacity of obtaining control through hostile takeovers or proxy fights (and in particular, those with holdings below 5%) can have a significant effect on the value of the firm. Alternatively, when the firm has a controlling shareholder and a change of control is practically impossible, the model predicts that smaller blockholders can still play an active role and enhance the value of the firm.[23] These predictions are in contrast with other models of intervention which builds on the ability of investors to obtain formal control and force management to take actions.

[23] Using proprietary data, Becht, Franks, and Grant (2010) give an example of a successful private engagement between an activist fund and the management of a company whose largest shareholder was a family holding over 50% of the voting rights. While the fund owned less than 2% of the company, it was able to significantly change the strategy of the company and consequently realized a significant abnormal returns on its investment.

VIII Concluding Remarks

This paper offers a new perspective on shareholder activism by focusing on what can be achieved when costly formal control cannot be obtained or exercised by shareholders. Two primary mechanisms are analyzed, voice and exit. Departing form the majority of the existing literature on shareholder activism, voice is modeled as a strategic transmission of information. This form of informal communication is a reflection of investors' attempt to exercise activism by sending letters, calling senior executives, and meeting with board members, thereby providing their input and ultimately changing the strategic course of the company. The paper analyzes the conditions under which "soft" shareholder activism is an effective form of corporate governance. It highlights the synergetic nature of the interaction between voice and exit, the role of transparency in shareholder activism, and the effect of managerial myopia and opportunistic activism on investors' ability to voice themselves effectively.

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Appendix

Proofs of Section II

Unless stated otherwise, all the results of Section I are proved for the general case where the activist's utility function is given by $u_A(\theta, a, p, \sigma) = \sigma p + (1-\sigma) v(\theta + \gamma, a)$ and $\gamma \in \left(-\overline{\theta}, -\underline{\theta}\right)$.

Proof of Lemma 2. If a responsive equilibrium is non-threshold there are $\theta_1 < \theta_2$ such that $\theta_1 \in \Theta$ and $\theta_2 \notin \Theta$. Since in a responsive equilibrium the activist can dictate a, according to (3), if $\theta \in [\theta_1, \theta_2]$ then she must be indifferent between approval and rejection and hence the activist exits with probability one in this interval. Let p be the price upon exit in this equilibrium, then $[\theta_1, \theta_2] \subset \Upsilon$. Therefore, a non-threshold responsive equilibrium exists only if the activist can exit ($\rho < \infty$). Let m_i be the message that is sent when the activist observes θ_i and $\Theta_i \equiv \{\theta : \mu(m_i|\theta) > 0\}$ the set of θ for which the activist sends message m_i. By definition, $\theta_i \in \Theta_i$. Note that $\mathbb{E}[\theta|m_i] = \mathbb{E}[\theta|\theta \in \Theta_i]$ and Lemma 1 implies

$$\mathbb{E}[\theta|\theta \in \Theta_1] \geq -\beta \geq \mathbb{E}[\theta|\theta \in \Theta_2] \tag{A1}$$

Consider an alternative equilibrium for which the communication strategy is identical to the original equilibrium, with the sole exception $\Theta_i' \equiv \Theta_i \cup \{\theta_j\} \setminus \{\theta_i\}$. Under the new strategy, the activist sends message m_i when he observes θ_j, $i \neq j$. Since $\theta_1 < \theta_2$ then $\mathbb{E}[\theta|\theta \in \Theta_1'] > \mathbb{E}[\theta|\theta \in \Theta_1]$ and $\mathbb{E}[\theta|\theta \in \Theta_2] > \mathbb{E}[\theta|\theta \in \Theta_2']$. Thus, given Lemma 1 and (A1), the manager will have incentives to follow the activist's recommendation under the new strategy. Note that if the activist would adopt this communication strategy, then the difference in expected value is $\Delta = (\theta_2 - \theta_1) - (\theta_1 - \theta_2) > 0$. Thus, the price under the new strategy is higher and the activist finds it weakly optimal to follow the new communication strategy, yielding an equilibrium which is strictly more efficient, as required. One can repeat this procedure as long as the equilibrium is non-threshold, eventually, converging to a threshold equilibrium. ■

Lemma A.1 (Non-Responsive Equilibrium) *Let $p_{NR}(\gamma)$ be the share price conditional on "exit" in equilibrium without voice, then:*

(i) *If $\mathbb{E}[\theta] \geq -\beta$ then for any γ and in any equilibrium the manager accepts the project with probability one and the activist exits if and only if $\theta + \gamma \leq p_{NR}(\gamma)$ where $p_{NR}(\gamma)$ is given by the solution of $p = \varphi(\theta, p - \gamma)$. If $\gamma = 0$ the equilibrium is unique and*

$p_{NR}(0) \equiv \min_p \{\varphi(\theta, p)\}$.

(*ii*) *If* $\mathbb{E}[\theta] < -\beta$ *then for any* γ *and in any equilibrium the manager rejects the project with probability one and the activist exits if and only if* $\theta + \gamma \geq -p_{NR}(\gamma)$ *where* $p_{NR}(\gamma)$ *is given by the solution of* $p = \varphi(-\theta, p + \gamma)$. *If* $\gamma = 0$ *the equilibrium is unique and* $p_{NR}(0) \equiv \min_p \{\varphi(-\theta, p)\}$.

(*iii*) $p_{NR}(\gamma) > p_{NR}(0)$ *for any* $\gamma \neq 0$.

(*iv*) $p_{NR}(\gamma, \rho)$ *decreases in* ρ.

Proof of Lemma A.1. As follows from the discussion that precedes Lemma 3, without voice the manager accepts the project in equilibrium if and only if $\mathbb{E}[\theta] \geq -\beta$. If the manager accepts (rejects) the project, then the value of the activist's holdings is given by $\theta + \gamma$ $(-\theta - \gamma)$ and hence she has strict incentives to exit if and only if $\theta + \gamma < p_{NV}(\gamma)$ $(-\theta - \gamma < p_{NV}(\gamma))$. The market maker anticipates this behavior and hence conditional on exit it prices the share as $\varphi_\theta(p - \gamma)$ $(\varphi_{-\theta}(p + \gamma))$ where $\varphi_\theta(p)$ is given by (8). In equilibrium, therefore, the price must satisfy $p = \varphi_\theta(p - \gamma)$ $(p = \varphi_{-\theta}(p + \gamma))$. Note that this equation has a solution for any γ as $\lim_{p \to \pm\infty} \varphi_\theta(p) = \mathbb{E}[\theta]$ $(\lim_{p \to \pm\infty} \varphi_{-\theta}(p) = \mathbb{E}[-\theta])$. Finally, when $\gamma = 0$, the equation $p = \varphi_\theta(p)$ $(p = \varphi_{-\theta}(p))$ has a a unique fixed point given by $\min_p \{\varphi_\theta(p)\}$ $(\min_p \{\varphi_{-\theta}(p)\})$. This follows from Proposition 1 in Archaya et al. (2010).

Consider the third part and suppose $\mathbb{E}[\theta] \geq -\beta$, a similar argument applies when $\mathbb{E}[\theta] < -\beta$. We argue that $\gamma \neq 0 \Rightarrow p_{NV}(\gamma) - \gamma \neq p_{NV}(0)$. Suppose on the contrary that there is $\gamma \neq 0$ such that $p_{NV}(\gamma) - \gamma = p_{NV}(0)$. If the manager accepts the project then $p_{NV}(\gamma) = \varphi_\theta(p_{NV}(\gamma) - \gamma) = \varphi_\theta(p_{NV}(0)) = p_{NV}(0)$. Therefore, $p_{NV}(0) = p_{NV}(\gamma)$ implying $p_{NV}(\gamma) - \gamma \neq p_{NV}(0)$, a contradiction. Therefore, $\varphi_\theta(p_{NV}(\gamma) - \gamma) \neq \varphi_\theta(p_{NV}(0))$. Since $\varphi_\theta(p_{NV}(0))$ is the unique minimum of $\varphi_\theta(p)$, for any $\gamma \neq 0$ we have $p_{NV}(\gamma) = \varphi_\theta(p_{NV}(\gamma) - \gamma) > \varphi_\theta(p_{NV}(0)) = p_{NV}(0)$ implying $p_{NV}(\gamma) > p_{NV}(0)$. Finally, the comparative static of $p_{NV}(\gamma)$ with respect to ρ follows from the observation that $\varphi_\theta(p)$ and $\varphi_{-\theta}(p)$ decrease in ρ. ■

Proof of Lemma 3. Lemma 3 is a special case of Lemma A.1 where $\gamma = 0$ and $\mathbb{E}[\theta] \geq -\beta$, and hence follows directly. ■

Lemma A.2 (Voice Without Exit) *Suppose the activist cannot exit and let,*

$$k(x) \equiv \begin{cases} -\underline{\theta} & \text{if } x \geq -\mathbb{E}[\theta] \\ k : \mathbb{E}[\theta | \theta > -k] = -x & \text{if } x < -\mathbb{E}[\theta] \end{cases}$$

(*i*) *In any responsive equilibrium the project is accepted if and only if* $\theta + \gamma \geq 0$.

(*ii*) *A responsive equilibrium exists if and only if* $\gamma \in [z(\beta), k(\beta)]$.

(*iii*) *If exists, a non-responsive equilibrium is the most efficient equilibrium if and only if* $\beta \leq -\mathbb{E}[\theta]$ *and* $\gamma \notin [z(0), k(0)]$.

Proof of Lemma A.2. We start with several properties of the functions k and z: $k(x)$ and $z(x)$ increase in x, $x \in [z(x), k(x)]$, if $x > -\mathbb{E}[\theta]$ then $[z(x), k(x)] = [z(x), \infty]$, if $x < -\mathbb{E}[\theta]$ then $[z(x), k(x)] = [-\infty, k(x)]$, and if $x = -\mathbb{E}[\theta]$ then $[z(x), k(x)] = [-\infty, \infty]$.
By Definition 1, if a responsive equilibrium exists then the activist can dictate the action taken by the manager. Since the activist cannot exit, she has strict incentives to persuade the manager to accept the project when $\theta + \gamma > 0$ and to reject it when $\theta + \gamma < 0$. This establishes the first part.
Note that for any γ and $\beta > 0$ the condition $\gamma \in [z(\beta), k(\beta)]$ is equivalent to

$$-\mathbb{E}[\theta | \theta < -\gamma] \geq \beta \geq -\mathbb{E}[\theta | \theta \geq -\gamma] \tag{A2}$$

Suppose condition (A2) holds and consider an equilibrium in which the activist sends message m_A if $\theta + \gamma \geq 0$ and a message $m_R \neq m_A$ otherwise. Conditional on $m = m_A$ the manager believes $\theta + \gamma \geq 0$. According to Lemma 1, the manager accepts the project if and only if $\mathbb{E}[\theta | \theta + \gamma \geq 0] + \beta \geq 0$. Conditional on $m = m_R$ the manager believes that $\theta + \gamma < 0$. According to Lemma 1, the manager rejects the project if and only if $\mathbb{E}[\theta | \theta + \gamma < 0] + \beta \leq 0$. Thus, if condition (A2) holds the manager will follow the activist's recommendation. Given the manager's expected behavior, it is in the best interest of the activist to follow the proposed communication strategy, so this is indeed a responsive equilibrium. To see the other direction, suppose a responsive equilibrium holds. Let M_R be the set of all messages such that $a(m) < 1$ and M_A be the set of all messages such that $a(m) = 1$. Since the equilibrium is responsive, neither set is empty. According to Lemma 1, if $a(m) = 0$ then $\mathbb{E}_\mu[\theta | m] + \beta \leq 0$ and if $a(m) \in (0, 1)$ then $\mathbb{E}_\mu[\theta | m] + \beta = 0$. Therefore, integrating over all $m \in M_R$ it follows that

$\mathbb{E}_\mu[\theta|M_R] + \beta \leq 0$. Similarly, integrating over all $m \in M_A$ implies $\mathbb{E}_\mu[\theta|M_A] + \beta \geq 0$. Recall the activist has incentives to induce $a(m) < 1$ if and only if $\theta + \gamma \leq 0$. For this reason, $\mathbb{E}_\mu[\theta|M_R] = \mathbb{E}[\theta|\theta + \gamma \leq 0]$ and $\mathbb{E}_\mu[\theta|M_A] = \mathbb{E}[\theta|\theta + \gamma > 0]$. Overall, condition (A2) holds. This establishes the second part.

Last, in any responsive equilibrium shareholders' welfare is given by $\mathbb{E}\left[\left(1_{\{\theta+\gamma\geq 0\}} - 1_{\{\theta+\gamma<0\}}\right)\cdot\theta\right]$. In any non-responsive equilibrium the manager's decision is independent of θ. If the manager accepts the project then

$$\mathbb{E}\left[\left(1_{\{\theta+\gamma\geq 0\}} - 1_{\{\theta+\gamma<0\}}\right)\cdot\theta\right] > \mathbb{E}[\theta] \Leftrightarrow 0 > \mathbb{E}[\theta|\theta < -\gamma] \Leftrightarrow \gamma > z(0)$$

If the manager rejects the project then

$$\mathbb{E}\left[\left(1_{\{\theta+\gamma\geq 0\}} - 1_{\{\theta+\gamma<0\}}\right)\cdot\theta\right] > -\mathbb{E}[\theta] \Leftrightarrow \mathbb{E}[\theta|\theta > -\gamma] > 0 \Leftrightarrow k(0) > \gamma$$

Note that $\beta > -\mathbb{E}[\theta] \Rightarrow k(\beta) = \infty$ and $z(0) < z(\beta)$. When $\beta > -\mathbb{E}[\theta]$ the manager accepts the project in a non-responsive equilibrium. Hence, if a responsive equilibrium exists, $\gamma > z(\beta) > z(0)$ and it is always the most efficient equilibrium. If $\beta = -\mathbb{E}[\theta]$ then any action can be taken by the manager in a non-responsive equilibrium. Thus, from the above calculations it is clear that any responsive equilibrium is more efficient than any non responsive equilibrium if and only if $\gamma \in [z(0), k(0)]$. If $\beta < -\mathbb{E}[\theta]$ then the manager rejects the project in a non-responsive equilibrium. Note that $\beta < -\mathbb{E}[\theta] \Rightarrow z(\beta) = -\infty$ and $k(0) < k(\beta)$. Note that $\mathbb{E}[\theta] < -\beta < 0$ and hence $z(0) = -\infty$ as well. Therefore, a responsive equilibrium is the most efficient one if and only if $\gamma \in [z(0), k(0)] = (-\infty, k(0)]$. This concludes the claim. ■

Proof of Lemma 4. Lemma 4 is a special case of Lemma A.2 where $\gamma = 0$ and $\mathbb{E}[\theta] \geq -\beta$, and hence follows directly. ■

Proof of Lemma 5. Suppose a responsive equilibrium exists. By definition, there are $m_A \neq m_R$ such that $a(m_A) = 1$ and $a(m_R) = 0$. Suppose by the way of contradiction $p^* \leq 0$. Since the action a is not observable and the message m is private, the market maker prices the stock conditional on exit, based on its expectation of the activist's communication strategy in equilibrium, and the manager's approval strategy in equilibrium. Therefore, in equilibrium, from the activist's point of view the price conditional on exit is fixed. Since $p^* \leq 0$ then the activist does not find it optimal to exit for any θ. Indeed, if $\theta \geq 0$ then the activist is better of

by sending message m_A and keeping his holdings. If $\theta < 0$ the activist is better of by sending m_R and keeping his holdings. Either way, the activist never exits strategically (or only in cases where $\theta = 0$ which is a probability zero event). Thus, $p = \mathbb{E}\left[v\left(\theta, 1_{\{\theta \geq 0\}}\right)\right] = \mathbb{E}\left[|\theta|\right] > 0$, a contradiction. If $p^* > 0$ then whenever $|\theta| < p^*$ the activist is strictly better off by exiting. Note that the argument does not hold for $\gamma \neq 0$. If $\gamma \neq 0$ then the price can be negative, in which case there is no strategic exit. Indeed, with a bias, the activist exits if and only if $|\theta + \gamma| \leq p$. ■

Proof of Lemma 6. Extending Proposition 1 in Archaya et al. (2010) for a random variable $v(\theta, 1_{\{\theta \geq \tau\}})$, one can show that $\pi(\tau)$ is the unique solution of the equation $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right) = p$. Therefore, $\pi(\tau)$ is well defined. Consider several properties of $\pi(\tau)$. First, note that $\pi(0) \equiv \min_p \{\varphi(|\theta|, p)\} > 0$, $\pi(\underline{\theta}) = \min_p \{\varphi(\theta, p)\} \in \mathbb{R}$, and $\pi\left(\overline{\theta}\right) = \min_p \{\varphi(-\theta, p)\} \in \mathbb{R}$. Second, the random variable $v(\theta, 1_{\{\theta \geq \tau'\}})$ FOSD $v(\theta, 1_{\{\theta \geq \tau''\}})$ if and only if $|\tau'| < |\tau''|$. It follows from (8) that for any given p, $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)$ strictly decreases in $|\tau|$. Therefore, $\pi(\tau)$ strictly decreases in $|\tau|$ as well, and $\tau = 0$ is the unique global maximizer of $\pi(\tau)$. Third, note that

$$\begin{aligned}\left\{\theta : v(\theta, 1_{\{\theta \geq \tau\}}) \leq p\right\} &= \{\theta : \tau < \theta \wedge \theta \leq p\} \cup \{\theta : \theta < \tau \wedge -\theta \leq p\} \\ &= \{\theta : \tau < \theta \leq p\} \cup \{\theta : -p < \theta < \tau\}\end{aligned}$$

and one can verify that for any $p \geq -|\tau|$ we have $\left\{\theta : v(\theta, 1_{\{\theta \geq \tau\}}) \leq p\right\} = [\min\{-p, \tau\}, \max\{p, \tau\}]$. Overall, (8) can be rewritten as

$$\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right) = \frac{-\int_{\underline{\theta}}^{\tau} \theta dF(\theta) + \int_{\tau}^{\overline{\theta}} \theta dF(\theta) + \rho\left[\int_{\min\{-p,\tau\}}^{\tau} -\theta dF(\theta) + \int_{\tau}^{\max\{p,\tau\}} \theta dF(\theta)\right]}{1 + \rho \Pr\left[\theta \in [\min\{-p, \tau\}, \max\{p, \tau\}]\right]}$$

and hence $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)$ is continuous in $\tau \in \left[\underline{\theta}, \overline{\theta}\right]$. Since $\pi(\tau)$ is the unique minimum of $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)$, it is continuous in τ as well.

Note that $\pi(0) > 0$, $\pi(\tau) + \tau$ increases in $\tau < 0$, and $\pi(\tau) - \tau$ decreases in $\tau > 0$. Moreover, $\lim_{\tau \downarrow \underline{\theta}} \pi(\tau) = \min_p \varphi(\theta, p) < \mathbb{E}[\theta]$, and therefore, $\lim_{\tau \downarrow \underline{\theta}} (\pi(\tau) + \tau) < \mathbb{E}[\theta] + \underline{\theta}$. Similarly, $\lim_{\tau \uparrow \overline{\theta}} \pi(\tau) = \min_p \varphi(-\theta, p) < -\mathbb{E}[\theta]$, and therefore, $\lim_{\tau \uparrow \overline{\theta}} (\pi(\tau) - \tau) < -\mathbb{E}[\theta] - \overline{\theta}$. Under the assumptions $\max\{0, -\mathbb{E}[\theta]\} < \overline{\theta}$ and $\underline{\theta} < \min\{0, -\mathbb{E}[\theta]\}$ we get $\lim_{\tau \downarrow \underline{\theta}} (\pi(\tau) + \tau) < 0$ and $\lim_{\tau \uparrow \overline{\theta}} (\pi(\tau) - \tau) < 0$. By the intermediate value theorem, $\underline{\tau}$ and $\overline{\tau}$ are well defined.[24] Suppose

[24] Note that if $\max\{0, -\mathbb{E}[\theta]\} > \overline{\theta} > 0$ then it is not guaranteed that $\lim_{\tau \uparrow \overline{\theta}} (\pi(\tau) - \tau) < 0$. If $\lim_{\tau \uparrow \overline{\theta}} (\pi(\tau) - \tau) \geq 0$ then $\overline{\tau} = \overline{\theta}$. Similarly, if $\min\{0, -\mathbb{E}[\theta]\} < \underline{\theta} < 0$ then it is not guaranteed that

$\tau \in [\underline{\tau}, 0]$. Since $\pi(\tau) + \tau$ is an increasing function over $\mathbb{R}^-$ and $\underline{\tau} < 0$ then it follows that $\pi(\tau) + \tau > 0$ implying $\pi(\tau) > 0$ and $\tau \in [-\pi(\tau), 0] \subset [-\pi(\tau), \pi(\tau)]$. Suppose $\tau \in [0, \bar{\tau}]$. Since $\pi(\tau) - \tau$ is a decreasing function over $\mathbb{R}^+$ then it follows that $\pi(\tau) - \tau > 0$ implying $\pi(\tau) > 0$ and $\tau \in [0, \pi(\tau)] \subset [-\pi(\tau), \pi(\tau)]$ as required. Suppose $\tau \in [-\pi(\tau), \pi(\tau)]$ then $\pi(\tau) - \tau \geq 0$ and $\pi(\tau) + \tau \geq 0$ implying $\tau \in [\underline{\tau}, \bar{\tau}]$. ■

Proof of Proposition 1. Suppose $\beta \leq z^{-1}(-\underline{\tau})$ and $\tau^* \in [\underline{\tau}, \min\{-z(\beta), \bar{\tau}\}]$, and note that since $\beta \leq z^{-1}(-\underline{\tau})$ then the interval is not empty. Consider an equilibrium in which the activist sends message m_A if $\theta \geq \tau^*$ and a message $m_R \neq m_A$ otherwise. In this equilibrium the manager is advised to accept the project if and only if $m = m_A$. If exists, the price must solves $p = \varphi\left(v(\theta, 1_{\{\theta \geq \tau^*\}}), p\right)$, and following Proposition 1 in Archaya et al. (2010), the unique solution exists and is given by $\pi(\tau^*)$. Since $\beta \geq -\mathbb{E}[\theta]$ then $\beta \geq -\mathbb{E}[\theta|\theta \geq \tau]$ for any τ the manager follows the recommendation to accept the project. Conditional on $m = m_R$ the manager believes that $\theta < \tau^*$ and therefore follows the recommendation and rejects the project if and only if $\mathbb{E}[\theta|\theta \leq \tau^*] \leq -\beta$. Note that $\mathbb{E}[\theta|\theta \leq \tau^*] \leq \mathbb{E}[\theta|\theta \leq -z(\beta)]$ and by definition $\mathbb{E}[\theta|\theta \leq -z(\beta)] = -\beta$. Thus, the manager has incentives to follow the recommendation and reject the project. Last, since $\tau^* \in [\underline{\tau}, \min\{-z(\beta), \bar{\tau}\}] \subset [\underline{\tau}, \bar{\tau}]$ then according to Lemma 6, $\tau^* \in [\underline{\tau}, \bar{\tau}]$ implies $\pi(\tau^*) > 0$ and $\tau^* \in [-\pi(\tau^*), \pi(\tau^*)]$. Therefore, the activists finds it (weakly) optimal to follow this communication strategy. Overall, such a responsive equilibrium exists.

Given Lemma 2 it is sufficient to focus on threshold equilibria. Suppose by the way of contradiction that there exists a responsive equilibrium where $\Theta = [\tau^*, \infty)$ and $\tau^* \notin [\underline{\tau}, \bar{\tau}]$. Recall the price must satisfy $p^* = \pi(\tau^*)$. However, by Lemma 6, $\tau^* \notin [-\pi(\tau^*), \pi(\tau^*)]$. If $\tau^* < -\pi(\tau^*)$ then for any $\theta \in [\tau^*, -\pi(\tau^*)]$ the activist is strictly better off by keeping his holdings in the firm and sending the manager a message that leads to rejection of the project. If $\tau^* > \pi(\tau^*)$ then for any $\theta \in [\pi(\tau^*), \tau^*]$ the activist is strictly better off by keeping his holdings in the firm and sending the manager a message that leads to approval of the project. Either way, $\Theta \neq [\tau^*, \infty)$ leading to a contradiction.

Suppose by the way of contradiction that there exists a responsive equilibrium where $\Theta = [\tau^*, \infty)$ and $\tau^* \in (-z(\beta), \bar{\tau}]$. According to Lemma 1, $\mathbb{E}[\theta|m] + \beta \leq 0$ for any $m \in M_R$. By integrating over all messages in M_R we get $\mathbb{E}[\theta|M_R] + \beta \leq 0$ as well, where $\mathbb{E}[\theta|M_R] = \mathbb{E}[\theta|\theta \leq \tau^*]$. By definition, $\mathbb{E}[\theta|\theta \leq -z(\beta)] = -\beta$. Therefore, $\tau^* > -z(\beta)$ implies $\mathbb{E}[\theta|\theta \leq \tau^*] > \mathbb{E}[\theta|\theta \leq -z(\beta)]$

$\lim_{\tau \downarrow \underline{\theta}} (\pi(\tau) + \tau) < 0$. If $\lim_{\tau \downarrow \underline{\theta}} (\pi(\tau) + \tau) \geq 0$ then $\underline{\tau} = \underline{\theta}$. Overall, the same analysis follows.

yielding $\mathbb{E}[\theta|\theta \leq \tau^*] + \beta > 0$, a contradiction. ■

Lemma A.3 *For any mixed strategy responsive equilibrium there is a pure strategy responsive equilibrium which is more efficient.*

Proof of Lemma A.3. For any responsive equilibrium let

$$M^*_{mix} \equiv \left\{ m : a^*(m) \in (0,1) \wedge \int \mu^*(m|\theta) f(\theta) d\theta > 0 \right\}$$

where $a^*(m)$ is the probability the manager accepts the project conditional on observing message m. Similarly we define M^*_A and M^*_R. Since the equilibrium is responsive $\arg\min a^*(m) \neq \arg\max a^*(m)$. Suppose M_{mix} is not empty and let p^* be the share price upon exit in this equilibrium. Note that if $m \in M^*_{mix}$ then $\mathbb{E}[\theta|m] + \beta = 0$ and the manager is indifferent between approval and rejection. We consider two cases.

First, suppose the set M^*_A is empty. According to Lemma 1, $\mathbb{E}[\theta|m] + \beta \leq 0$ for any $m \in M^*_R$ and hence $\mathbb{E}[\theta|M^*_R \cup M^*_{mix}] + \beta \leq 0$. Since M^*_A is empty then $\mathbb{E}[\theta|M^*_R \cup M^*_{mix}] = \mathbb{E}[\theta]$. Since by assumption $\mathbb{E}[\theta] + \beta \geq 0$, it must be that $\mathbb{E}[\theta|M^*_R \cup M^*_{mix}] + \beta = 0$ and hence $\mathbb{E}[\theta] + \beta = 0$. Since $\mathbb{E}[\theta] > \mathbb{E}[\theta|\theta < 0]$ then $\beta = -\mathbb{E}[\theta]$ implies $\beta < -\mathbb{E}[\theta|\theta < 0]$. According to Proposition 1, there is a responsive equilibrium with threshold $\tau = 0$. Thus, the first best can be obtained by a pure strategy responsive equilibrium as required.

Second, suppose the set M^*_A is not empty and consider the following new approval strategy: if $m \in M^*_{mix}$ then $a'(m) = 0$ and otherwise $a'(m) = a^*(m)$. Obviously, the corresponding set M'_{mix} is empty, but M'_A and M'_R are not. Let us show that under the new approval strategy there is an equilibrium in pure strategies which is more efficient. I do it in three parts.

First, consider the manager's incentives to follow the new approval strategy. If $m \in M^*_{mix}$ then the manager is indifferent between approval and rejection. Therefore, the manager will follow the new approval strategy. If $m \in M^*_{mix}$ the incentives constraints under the original equilibrium continue to hold.

Consider the market maker's pricing strategy for the new approval decision of the manager. If $m \in M^*_{mix}$ then $\mathbb{E}[\theta|m] = -\beta < 0$. Thus, decreasing $a^*(m) > 0$ to $a'(m) = 0$ increases the expected value of the firm. If $m \notin M^*_{mix}$ then the manager decision does not change. Thus, the market maker must set the price upon exit at a higher level and $p^* < p'$.

Consider the activist's incentives to implement the approval strategy. Suppose $m \in M^*_{mix}$.

Since $a^*(m) \in (0,1)$ and M_A^* is not empty then it must be that $\theta \leq \max\{0, p^*\}$, the circumstances under which the activist does not have strict incentives to approve the project. This means that the activist is either indifferent or has strict incentives to reject the project. Since $\max\{0, p^*\} \leq \max\{0, p'\}$ then the activist has weak incentives to reject the project as required. If $m \notin M_{mix}^*$ then the incentives constraint as in the original equilibrium continue to hold. Note that since $p^* < p'$ then for some $m \notin M_{mix}^*$ the activist has weak preferences instead of strict preference to make the original recommendation. Moreover, similar to Lemma 5 it must be that $p' > 0$ and similar to Lemma 6 the price p' is well defined. Overall, the new approval strategy can be supported by a pure strategy and responsive equilibrium which is more efficient. ■

Proof of Corollary 1. We start by arguing that any responsive equilibrium is more efficient then a non-responsive equilibrium. Recall that in a non-responsive equilibrium the manager accepts the project with probability one and hence the ex-ante value of the firm is $\mathbb{E}[\theta]$. A responsive equilibrium generates expected value higher than $\mathbb{E}[\theta]$ if and only if $\mathbb{E}[\theta|M_R] < 0$. According to Lemma 1, $m \in M_R \Leftrightarrow \mathbb{E}[\theta|m] < -\beta$. Therefore, it is necessary that $\mathbb{E}[\theta|M_R] < -\beta < 0$, as required.

According to Proposition 1, if $\beta > z^{-1}(-\underline{\tau})$ then a responsive equilibrium does not exist. Therefore, $\bar{V} = \underline{V} = \mathbb{E}[\theta]$. Suppose $\beta < z^{-1}(-\underline{\tau})$. Recall from the proof of Proposition 1 that $v(\theta, 1_{\{\theta \geq \tau\}})$ decreases in $|\tau|$ and that the most efficient equilibrium has $\tau^* = \max\{-z(\beta), 0\}$. Since $z(\beta)$ is an increasing function and $\tau^* \leq 0$, then $|\tau^*|$ increases with β. Therefore, $\bar{V}$ decreases in β as well. If $\beta = z^{-1}(-\underline{\tau})$ then V drops from $\mathbb{E}\left[v(\theta, 1_{\{\theta \geq \underline{\tau}\}})\right]$ to $\mathbb{E}[\theta]$ as β increases.

Consider the comparative static with respect to ρ. It is immediate to see that if $\beta < z^{-1}(-\underline{\tau})$ then ρ has no effect on $\bar{V}$. Suppose $\beta = z^{-1}(-\underline{\tau})$. In this case $\bar{V} = \mathbb{E}\left[v(\theta, 1_{\{\theta \geq \underline{\tau}\}})\right] > \mathbb{E}[\theta]$. Recall from the proof of Lemma 6 that $\pi(\tau)$ is the unique solution of $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right) = p$ for any τ. Since $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)$ decreases in ρ, so does $\pi(\tau)$. Moreover, $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right)$ decreases in $|\tau|$ and so does $\pi(\tau)$. Recall from Lemma 6 that $\pi(\underline{\tau}) + \underline{\tau} = 0$ and $\underline{\tau} < 0$. Therefore, at $\tau = \underline{\tau}$, $\pi(\tau)$ increases with τ. This implies that $\underline{\tau}$ increases in ρ as well. Therefore, $\bar{V}$ drops to $\mathbb{E}[\theta]$.

Suppose $\lim_{\rho \to \infty} \underline{\tau} < 0$. Since $\pi(\underline{\tau}) = -\underline{\tau}$ and $\pi(\tau)$ is the unique solution of $\varphi\left(v(\theta, 1_{\{\theta \geq \tau\}}), p\right) = p$ for any τ, then $\lim_{\rho \to \infty} \pi(\underline{\tau}) = \mathbb{E}[\theta | \theta \in [\underline{\tau}, \pi(\underline{\tau})]]$, a contradiction. Therefore, it must be that $\underline{\tau} \to 0$. A similar arguments proves that $\lim_{\rho \to \infty} \bar{\tau} = 0$ as well. Therefore, $[\underline{\tau}, \bar{\tau}]$ shrinks to zero as $\rho \to \infty$. ■

Example - F Changes in a FDSD Manner. Let $\zeta(\Delta) \equiv \theta + \Delta$, $\Delta > 0$. Note that x such that $\mathbb{E}[\zeta|\zeta \leq -x] = -\beta$ satisfies $x = z(\beta + \Delta) - \Delta$. Let $x(\beta, \Delta) \equiv z(\beta + \Delta) - \Delta$ and Suppose $\beta > \beta_0$ where $z(\beta_0) = 0$. Note that $\beta_0 > 0$ and is unique. Then, for any $\Delta \geq 0$,

$$\begin{aligned}
W(\Delta) &\equiv \mathbb{E}\left[v\left(\zeta(\Delta), 1_{\{\zeta(\Delta) > -x(\beta,\Delta)\}}\right)\right] \\
&= -\Pr[\zeta(\Delta) < -x(\beta,\Delta)]\,\mathbb{E}[\zeta(\Delta)|\zeta(\Delta) \leq -x(\beta,\Delta)] \\
&\quad + \Pr[\zeta(\Delta) > -x(\beta,\Delta)]\,\mathbb{E}[\zeta(\Delta)|\zeta(\Delta) > -x(\beta,\Delta)] \\
&= \mathbb{E}[\zeta(\Delta)] - 2\Pr[\zeta(\Delta) < -x(\beta,\Delta)]\,\mathbb{E}[\zeta(\Delta)|\zeta(\Delta) \leq -x(\beta,\Delta)] \\
&= \mathbb{E}[\zeta(\Delta)] + 2\Pr[\zeta(\Delta) < -x(\beta,\Delta)]\,\beta \\
&= \mathbb{E}[\theta] + \Delta + 2\Pr[\theta < -z(\beta + \Delta)]\,\beta
\end{aligned}$$

Since $\frac{\partial z(\beta+\Delta)}{\partial \Delta} = \frac{1}{f(-z(\beta+\Delta))}\frac{F(-z(\beta+\Delta))}{\beta+\Delta-z(\beta+\Delta)} > 0$ then

$$\begin{aligned}
\frac{\partial W(\Delta)}{\partial \Delta} &= 1 - 2\beta f(-z(\beta+\Delta))\frac{\partial z(\beta+\Delta)}{\partial \Delta} \\
&= 1 - 2F(-z(\beta+\Delta))\frac{\beta}{\beta+\Delta-z(\beta+\Delta)}
\end{aligned}$$

Thus,

$$\frac{\partial W(\Delta)}{\partial \Delta}|_{\Delta=0} = 1 - 2F(-z(\beta))\frac{\beta}{\beta - z(\beta)}$$

and as $\beta \downarrow \beta_0$ then

$$\frac{\partial W(\Delta)}{\partial \Delta}|_{\Delta=0} \to 1 - 2F(-z(\beta_0))\frac{\beta_0}{\beta_0 - z(\beta_0)} = 1 - 2F(0)$$

Thus, depending on how zero is positioned relative to the median of θ, an increase in Δ can decrease or increase the welfare of shareholders. ■

Proof of Corollary 2. A fully revealing equilibrium is a responsive equilibrium with threshold $-\beta$. According to Proposition 1 it exists if and only if $-\beta \in [\underline{\tau}, \min\{-z(\beta), \bar{\tau}\}]$. Recall $\beta > z(\beta)$ and hence it is always the case $-\beta < -z(\beta)$. Therefore, A fully revealing equilibrium exists if and only if $-\beta \in [\underline{\tau}, \bar{\tau}]$. Note that $\min\{-z(\beta), 0\} \in [-\beta, 0]$ and hence if $-\beta \in [\underline{\tau}, \bar{\tau}]$ then $\min\{-z(\beta), 0\} \in [\underline{\tau}, \bar{\tau}]$ as well. Therefore, according to Proposition 1, whenever a fully revealing equilibrium exists, there also exists an equilibrium with threshold $\min\{-z(\beta), 0\}$.

Since $|\min\{-z(\beta),0\}| < |-\beta|$ then this equilibrium is strictly more efficient. ■

Proof of Corollary 3. If the equilibrium is responsive then at the most efficient equilibrium $\eta_R = \delta + (1-\delta)\Pr\left[v(\theta, 1_{\{\theta \geq \tau^{SB}\}}) \leq \pi\left(\tau^{SB}\right)\right]$ and if the equilibrium is non-responsive then $\eta_{NR} = \delta + (1-\delta)\Pr[\theta \leq p_{NR}]$. Note that according to Lemma 6, $\tau^{SB} \in \left[-\pi\left(\tau^{SB}\right), \pi\left(\tau^{SB}\right)\right]$. Therefore, $\Pr\left[v(\theta, 1_{\{\theta \geq \tau^{SB}\}}) \leq \pi\left(\tau^{SB}\right)\right] = \Pr\left[|\theta| \leq \pi\left(\tau^{SB}\right)\right]$. We define $\Delta\eta \equiv \frac{\eta_R - \eta_{NR}}{1-\delta} = \Pr[|\theta| \leq -\underline{\tau}] - \Pr[\theta \leq p_{NR}]$.

Consider the comparative static with respect to β. If $\beta \in [0, z^{-1}(0)]$ the equilibrium is responsive and $\tau^{SB} = 0$. Therefore, $\frac{\partial \eta_R}{\partial \beta} = 0$. If $\beta \in [z^{-1}(0), z^{-1}(-\underline{\tau})]$ the equilibrium is responsive and $\tau^{SB} = -z(\beta)$. Since $\pi\left(\tau^{SB}\right)$ decreases with β then $\frac{\partial \eta_R}{\partial \beta} < 0$ as well. Finally, if $\beta > z^{-1}(-\underline{\tau})$ the equilibrium is non-responsive. Since p_{NR} is independent of β then $\frac{\partial \eta_{NR}}{\partial \beta} = 0$. Overall, if $\beta \neq z^{-1}(-\underline{\tau})$ then $\frac{\partial \eta}{\partial \beta} \leq 0$. It is left to compare $\eta_R - \eta_{NR}$ when $\beta = z^{-1}(-\underline{\tau})$ (i.e. $\tau^{SB} = \underline{\tau}$). By definition, $\eta_{NR} - \eta_R = -(1-\delta)\Delta\eta$. Thus, $\eta_{NR} - \eta_R < 0$ if and only if $\Pr[|\theta| \leq -\underline{\tau}] > \Pr[\theta \leq p_{NR}]$.

Consider the comparative static with respect to δ. Note that $\frac{\partial \underline{\tau}}{\partial \delta} = -\frac{\frac{\partial \pi(\underline{\tau},\delta)}{\partial \delta}}{\frac{\partial \pi(\underline{\tau},\delta)}{\partial \underline{\tau}}+1} < 0$ and $\frac{\partial p_{NR}}{\partial \delta} > 0$. If $\beta > z^{-1}(-\underline{\tau}(\delta)) \Leftrightarrow \underline{\tau}(\delta) > -z(\beta)$ then the equilibrium is non-responsive. Thus $\eta = \eta_{NR}$ for small values of δ and $\frac{\partial \eta_{NR}}{\partial \delta} = 1 - \Pr[\theta \leq p_{NR}] + (1-\delta)\Pr[\theta = p_{NR}]\frac{\partial p_{NR}}{\partial \delta} > 0$. Suppose $\underline{\tau}(\delta) < -z(\beta)$ then the equilibrium is responsive and $\frac{\partial \eta_R}{\partial \delta} = 1 - \Pr\left[|\theta| \leq \pi\left(\tau^{SB}\right)\right] + (1-\delta)\Pr\left[\theta \in \left\{\pi\left(\tau^{SB}\right), -\pi\left(\tau^{SB}\right)\right\}\right]\frac{\partial \pi\left(\tau^{SB}\right)}{\partial \delta}$. Since $\pi(\tau)$ is a global minimum, it follows from its definition that $\frac{\partial \pi(\tau)}{\partial \delta} > 0$ for any $\tau \in [\underline{\tau}, \bar{\tau}]$. Therefore, $\frac{\partial \eta_R}{\partial \delta} > 0$ as well. Overall, if $\underline{\tau}(\delta) \neq -z(\beta)$ then $\frac{\partial \eta}{\partial \delta} > 0$. It is left to compare $\eta_R - \eta_{NR}$ when $\underline{\tau}(\delta) = -z(\beta)$ (i.e. $\tau^{SB} = \underline{\tau}$). By definition, $\eta_{NR} - \eta_R = -(1-\delta)\Delta\eta$. Thus, $\eta_{NR} - \eta_R < 0$ if and only if $\Pr[|\theta| \leq -\underline{\tau}] > \Pr[\theta \leq p_{NR}]$. ■

Proofs of Section III

Proof of Proposition 2. We start by proving that $p_A = p_R > 0$ in any responsive equilibrium under Market-Transparency. The proof of this argument holds regardless of the manager's short term compensation. First note that there is no responsive equilibrium in which $\max\{p_A, p_R\} \leq 0$. In that case, the activist would like the manager to accept the project if and only if $\theta > 0$, and would never exit unless she has a liquidity shock. This would imply that $p_A = \mathbb{E}[\theta|\theta > 0] > 0$ and $p_R = \mathbb{E}[-\theta| - \theta < 0] > 0$, yielding a contradiction. Suppose on the contrary a responsive equilibrium exists with $p_A \neq p_R$. Neither M_A nor M_R are empty, and from the activist's point

of view, these prices are exogenous. If $\theta > \max\{p_A, p_R\}$ then the activist strictly prefers the manager to accept the project. If $\theta < -\max\{p_A, p_R\}$ the activist strictly prefers the manager to reject the project. Either way, the activist will not exit unless she has liquidity needs. If $\theta \in [-\max\{p_A, p_R\}, \max\{p_A, p_R\}]$ the activist strictly prefers the manager to accept the project when $p_A > p_R$ and strictly prefers the manager to reject the project if $p_A < p_R$. In the former case the activist will exit for sure and get p_A. In the latter case she will exit for sure and get p_R. Either way, the activist will exit. Consider how the market maker would price the stock conditional on the activist's decision to exit and the manager's decision a. If $p_A > p_R$ then the manager rejects the project if and only if $\theta < -p_A$ and the activist never exits conditional on the manager's rejection. Therefore, $p_R = -\mathbb{E}[\theta|\theta < -p_A] > p_A$ which contradicts the presumption that $p_A > p_R$. If $p_A < p_R$ then the manager accepts the project if and only if $\theta > p_R$ and the activist never exit conditional on the manager's approval. Therefore, $p_A = \mathbb{E}[\theta|\theta > p_R] > p_R$ which contradicts the presumption that $p_A < p_R$. Either way, in any responsive equilibrium $p_A = p_R > 0$ as required.

Next, we prove that a responsive equilibrium under Market-Transparency exists if and only if there are p and Θ that satisfy

$$\Theta \supseteq [p, \overline{\theta}] \text{ and } \Theta \cap [\underline{\theta}, -p] = \varnothing \tag{A3}$$

$$p = \varphi_A(p, \Theta) = \varphi_R(p, \Theta) > 0 \tag{A4}$$

$$\mathbb{E}[\theta|\theta \notin \Theta] \leq -\beta \leq \mathbb{E}[\theta|\theta \in \Theta] \tag{A5}$$

where

$$\varphi_a(p, \Theta) \equiv \begin{cases} \frac{\mathbb{E}[\theta|\theta\in\Theta]+\rho\Pr[\theta<p|\theta\in\Theta]\mathbb{E}[\theta|\theta\in[-p,p]\cap\Theta]}{1+\rho\Pr[\theta<p|\theta\in\Theta]} & \text{if } a = A \\ \frac{\mathbb{E}[-\theta|\theta\notin\Theta]+\rho\Pr[\theta>-p|\theta\notin\Theta]\mathbb{E}[-\theta|\theta\in[-p,p]\setminus\Theta]}{1+\rho\Pr[\theta>-p|\theta\notin\Theta]} & \text{if } a = R \end{cases} \tag{A6}$$

Suppose a responsive equilibrium exits. As was argued above, it is necessary that $p_A = p_R = p^* > 0$, $\Theta^* \supseteq [p^*, \overline{\theta}]$, and $\Theta^* \cap [\underline{\theta}, -p^*] = \varnothing$. Therefore, condition (A3) must hold. Condition (A4) holds as well since in equilibrium it must be $p_a^* = \varphi_a(p_a^*, \Theta^*)$. Last, since the equilibrium is responsive, $-\beta \leq \mathbb{E}[\theta|m]$ for any $m \in M_A$ and $\mathbb{E}[\theta|m] \leq -\beta$ for any $m \in M_R$. By integrating over all messages, it follows that $\mathbb{E}[\theta|M_A] = \mathbb{E}[\theta|\theta \in \Theta^*]$ and $\mathbb{E}[\theta|M_R] = \mathbb{E}[\theta|\theta \notin \Theta^*]$. Therefore, condition (A5) holds as required. Suppose conditions (A3), (A4), and (A5) hold for some p and Θ. Consider an equilibrium in which the manager sends m_A if $\theta \in \Theta$ and $m_R \neq m_A$ otherwise. As was explained above and since condition (A3) holds, the activist finds it weakly

optimal to follow this communication strategy. At the same time, since condition (A5) holds the manager finds it optimal to follow the activist's recommendation. Finally, (A4) ensures that under the proposed communication strategy and approval rule the implied exit strategy is consistent with $p_A = p_R > 0$. Thus, a responsive equilibrium indeed exists.

Next, according to the proofs of Proposition 1 and Lemma 6, a responsive equilibrium under No-Transparency exists if and only if there are p and Θ such that conditions (A3) and (A5) hold, but instead of condition (A4), p and Θ satisfy $p = \varphi_{A\cup R}(p, \Theta)$ where

$$\varphi_{A\cup R}(p,\Theta) \equiv \frac{\mathbb{E}\left[v\left(\theta, 1_{\{\theta\in\Theta\}}\right)\right] + \rho \Pr\left[\theta \in [-p,p]\right] \mathbb{E}\left[v\left(\theta, 1_{\{\theta\in\Theta\}}\right) | \theta \in [-p,p]\right]}{1 + \rho \Pr\left[\theta \in [-p,p]\right]} \tag{A7}$$

We argue that if $p = \varphi_A(p,\Theta) = \varphi_R(p,\Theta)$ then $p = \varphi_{A\cup R}(p,\Theta)$ as well. Indeed, $\varphi_a(p,\Theta)$ are conditional expectations and for any p and Θ that satisfy (A3), the following must hold

$$\begin{aligned} \varphi_{A\cup R}(p,\Theta) \quad = \quad & \frac{\Pr\left[\theta\in\Theta\right] + \rho \Pr\left[\theta \in [-p,p]\cap\Theta\right]}{1+\rho\Pr\left[\theta\in[-p,p]\right]}\varphi_A(p,\Theta) \\ & + \frac{\Pr\left[\theta\notin\Theta\right] + \rho \Pr\left[\theta \in [-p,p]\setminus\Theta\right]}{1+\rho\Pr\left[\theta\in[-p,p]\right]}\varphi_R(p,\Theta) \end{aligned} \tag{A8}$$

Thus, any responsive equilibrium under Market-Transparency is also an equilibrium under No-Transparency.

We show that if a responsive equilibrium under Market-Transparency exists then $\beta < z^{-1}(-\underline{\tau})$. Obviously, $\beta \leq z^{-1}(-\underline{\tau})$ since otherwise there is a responsive equilibrium under No-Transparency which contradicts Proposition 1. It is left to prove that $\beta \neq z^{-1}(-\underline{\tau})$. Suppose on the contrary a responsive equilibrium under Market-Transparency exists and $\beta = z^{-1}(-\underline{\tau})$. Conditions (A3) and (A5) hold and by definition of $z(\cdot)$ it must be $\Theta = \left[\underline{\tau}, \overline{\theta}\right]$. Recall $\underline{\tau} = -\pi(\underline{\tau})$ where $\pi(\underline{\tau}) = p_{A\cup R}$, and since $p_A = p_R = p_{A\cup R}$ then $\pi(\underline{\tau}) = p_R$. Therefore, $\Theta = \left[-p_R, \overline{\theta}\right]$. If so, the project is rejected if and only if $\theta < -p_R \Leftrightarrow -\theta > p_R$ which implies that conditional on rejection the value of the firm is strictly greater than p_R. This yields a contradiction since p_R must reflect the fair value of the firm conditional on the project being rejected. Thus, if a responsive equilibrium exists, $\beta < z^{-1}(-\underline{\tau})$.

Note that if a responsive equilibrium under Market-Transparency exists for some $\beta_0 < z^{-1}(-\underline{\tau})$, then it exists for any $\beta \in [0, \beta_0]$. Indeed, if we keep the same communication rule and exit rule it is only left to verify the manager has the same incentives to follow the activist's recommendation. That is, that condition (A5) holds for any $\beta \in [0, \beta_0]$. Note that, $\beta \leq \beta_0$ implies $\mathbb{E}\left[\theta | \theta \notin \Theta\right] \leq -\beta$. Since $\beta \leq \beta_0 < z^{-1}(-\underline{\tau})$ then $-z(\beta) < \underline{\tau}$ for any $\beta \in [0, \beta_0]$ and hence

$-\beta = \mathbb{E}[\theta|\theta < \underline{\tau}]$. Since condition (A3) must hold and the price must be greater than $\underline{\tau}$ it follows that $\mathbb{E}[\theta|\theta < \underline{\tau}] < \mathbb{E}[\theta|\theta \in \Theta]$. We conclude that condition (A5) holds for any $\beta \in [0, \beta_0]$ and this completes the argument. Therefore, there is $\beta^* < z^{-1}(-\underline{\tau})$ such that a responsive equilibrium under Market-Transparency exists if and only if $\beta \leq \beta^*$.

Finally, suppose f is symmetric around zero and $\overline{\theta} = \infty = -\underline{\theta}$. We prove that $\beta^* = z^{-1}(0)$. Conditions (A4) and (A6) imply

$$\frac{\int_p^\infty (\theta - p)\, dF(\theta)}{\int_{\Theta \cap [-p,p]} (\theta - p)\, dF(\theta)} = \frac{\int_{-\infty}^{-p} (\theta + p)\, dF(\theta)}{\int_{\Theta^c \cap [-p,p]} (\theta + p)\, dF(\theta)} \tag{A9}$$

Note that the symmetry of f around zero implies $\int_p^\infty (\theta - p)\, dF(\theta) = -\int_{-\infty}^{-p} (\theta + p)\, dF(\theta)$ and hence (A.9) holds if and only if

$$\int_{\Theta \cap [-p,p]} (\theta - p)\, dF(\theta) + \int_{\Theta^c \cap [-p,p]} (\theta + p)\, dF(\theta) = 0 \tag{A10}$$

Note that (A10) can be rewritten as $\int_{[-p,p]} \theta dF(\theta) = p\left[\int_{\Theta \cap [-p,p]} dF(\theta) - \int_{\Theta^c \cap [-p,p]} dF(\theta)\right]$. The symmetry of f around zero implies $\int_{[-p,p]} \theta dF(\theta) = 0$ and since $\int_p^\infty dF(\theta) = \int_{-\infty}^{-p} dF(\theta)$ then it must be that $\int_{\Theta^c} dF(\theta) = \int_\Theta dF(\theta)$, which implies $\int_{\Theta^c} dF(\theta) = \frac{1}{2}$. Given condition (A5), if a responsive equilibrium exists then $\beta \leq -\mathbb{E}[\theta|\theta \notin \Theta]$. Suppose on the contrary that a responsive equilibrium exists and $\beta > z^{-1}(0)$. This implies $-\mathbb{E}[\theta|\theta < 0] < \beta$ and hence $\mathbb{E}[\theta|\theta < 0] > \mathbb{E}[\theta|\theta \notin \Theta]$. Since $\int_{-\infty}^0 dF(\theta) = \frac{1}{2} = \int_{\Theta^c} dF(\theta)$ then $\int_{-\infty}^0 \theta dF(\theta) > \int_{\Theta^c} \theta dF(\theta)$. At the same time, since $\int_{-\infty}^0 dF(\theta) = \frac{1}{2} = \int_{\Theta^c} dF(\theta)$ then if $\Theta^c \cap [0, \infty) \neq \varnothing$ then it must be $\int_{\Theta^c} \theta dF(\theta) > \int_{-\infty}^0 \theta dF(\theta)$, a contradiction. Similarly, if $\Theta^c \cap [0, \infty) = \varnothing$ then it must be that $\Theta^c = [0, \infty)$ which implies $\int_{\Theta^c} \theta dF(\theta) = \int_{-\infty}^0 \theta dF(\theta)$, a contradiction. Thus, $\beta^* \leq z^{-1}(0)$. Next, we will show that a responsive equilibrium exists when $\beta = z^{-1}(0)$. Consider an equilibrium where $\Theta = [0, \infty)$. The manager has incentives to follow this strategy since $\beta^* = z^{-1}(0)$ and the activist trivially will find it optimal. It is therefore left to verify that condition (A4) is satisfied. Note that according to (A6) and under the assumption that f is symmetric around zero, $\varphi_A(p, \Theta) = \varphi_R(p, \Theta)$ for all p. Therefore, this constraint is not binding. The price will satisfy $\varphi_{A \cup R}(p, \Theta) = p$ which is the $\min_p \{\varphi(|\theta|, p)\}$. Overall, a responsive equilibrium exists for all $\beta \leq z^{-1}(0)$ and hence $\beta^* = z^{-1}(0)$.

We argue that if $\mathbb{E}[\theta|\theta > 0] + \mathbb{E}[\theta|\theta < 0] < 0$ and $\overline{\theta} = \infty = -\underline{\theta}$ then there exist $\delta' \in (0, 1)$ and $\beta' < z^{-1}(0)$ such that under Market-Transparency a responsive equilibrium does not exist. Suppose on the contrary that for any $\delta \in (0, 1)$ and $\beta \in (0, z^{-1}(0))$ a responsive equilibrium

under Market-Transparency exists, and in this equilibrium, the manager accepts the project if and only if $\theta \in \Theta^{\delta,\beta}$. Let μ be the unique solution of $\mathbb{E}[\theta|\theta > x] + \mathbb{E}[\theta|\theta < x] = 0$ and note that $\mu > 0$ and $\mathbb{E}[\theta|\theta \leq \mu] < 0$. Let $\beta' = z^{-1}(-\mu) > 0$ and therefore $\beta' \in (0, z^{-1}(0))$. For any $\beta \in (z^{-1}(-\mu), z^{-1}(0))$ and $\delta \in (0,1)$ the set $\Theta^{\delta,\beta}$ satisfies condition (A3). Condition (A5) implies $\mathbb{E}\left[\theta|\theta \notin \Theta^{\delta,\beta}\right] \leq -\beta = \mathbb{E}[\theta|\theta < -z(\beta)]$ for any δ. Since $z^{-1}(-\mu) < \beta$ then $-z(\beta) < \mu$. Therefore,

$$\mathbb{E}\left[\theta|\theta \notin \Theta^{\delta,\beta}\right] < \mathbb{E}[\theta|\theta \leq \mu] < 0 \tag{A11}$$

Note that as $\delta \to 1$ conditions (A4) and (A6) imply

$$\mathbb{E}\left[\theta|\theta \notin \Theta^{\delta\to 1,\beta}\right] + \mathbb{E}\left[\theta|\theta \in \Theta^{\delta\to 1,\beta}\right] = 0 \tag{A12}$$

in conjunction with condition (A5) and $\beta > 0$ we get

$$\mathbb{E}\left[\theta|\theta \notin \Theta^{\delta\to 1,\beta}\right] < 0 < \mathbb{E}\left[\theta|\theta \in \Theta^{\delta\to 1,\beta}\right] \tag{A13}$$

Since $\mathbb{E}[\theta|\theta > \mu] + \mathbb{E}[\theta|\theta < \mu] = 0$ and given (A12) and (A13) then

$$0 < \mathbb{E}[\theta|\theta > \mu] < \mathbb{E}\left[\theta|\theta \in \Theta^{\delta\to 1,\beta}\right] \tag{A14}$$

Note that (A14) implies that $[\mu,\infty) \setminus \Theta^{\delta\to 1,\beta}$ is not empty (as otherwise $[\mu,\infty) \subset \Theta^{\delta=1,\beta}$ which contradicts (A14)). Note that $\sup\left\{[\mu,\infty) \setminus \Theta^{\delta=1,\beta}\right\} > \mu > \sup\left\{\Theta^{\delta=1,\beta} \setminus [\mu,\infty)\right\}$. Thus, relative to the pool $[\mu,\infty)$, the set $\Theta^{\delta=1,\beta}$ removes $\theta > \mu$ and adds $\theta < \mu$. This implies that $\mathbb{E}[\theta|\theta > \mu] > \mathbb{E}\left[\theta|\theta \in \Theta^{\delta=1,\beta}\right]$, a contradiction. We conclude that for δ sufficient close to one, a responsive equilibrium cannot exists, proving that $\beta^* < z^{-1}(0)$ is feasible. ■

Proof of Proposition 3. Let M be set of all (public) messages that are sent with a strictly positive probability in equilibrium under Voice-Transparency, and $p(m)$ the share price conditional on the decision of the activist to exit and public message m.

Consider a non-responsive equilibrium under Market-Transparency. This equilibrium also exists under Voice-Transparency when the market and the manager completely ignore the activist's message, and the activist's message is indeed non-informative. Consider a non-responsive equilibrium under Voice-Transparency. By definition, the manager ignores the activist's message and accepts the project with probability one (Lemma 3). To the extent that there is a strictly positive probability the activist will exit, either because the share is over-priced or for liquidity

reasons, the activist chooses $m \in \arg\max_{m \in M} p(m)$. Therefore, $p(m) = p$ for any $m \in M$. This implies that the price does not response to any information that is revealed by the activist, if any. Therefore, this non-responsive equilibrium is identical to equilibrium in which no information is revealed by the activist. Overall, the set of non-responsive equilibria under either form of transparency is identical.

Consider a responsive equilibrium under Market-Transparency and let Θ be the manager's decision rule. Following Proposition 2, $p_A = p_R$ where p_a is the solution of $p = \varphi_a(p, \Theta)$. Consider an equilibrium under Voice-Transparency in which the message sends message m_A if $\theta \in \Theta$ and message $m_R \neq m_A$ otherwise. Integrating over condition (A5), the manager's has incentives to follow the recommendation and approves the project if and only if $m = m_A$. The market maker observes m and hence conditional on $m = m_A$ ($m = m_R$) it infers that $\theta \in \Theta$ ($\theta \notin \Theta$) and the manager will accept (reject) the project. Therefore, $p(m_a)$ must be the solution of $p = \varphi_a(p, \Theta)$. Effectively, the market maker has exactly the same information set as it would have under Market-Transparency. It is left to verify the activist has incentives to follow the proposed communication strategy. We suppose that off-equilibrium any message $m \notin \{m_A, m_R\}$ is interpreted as if $\theta \in \Theta$ with probability $\frac{1}{2}$ and $\theta \notin \Theta$ otherwise. Thus, since $p(m_A) = p(m_R)$ any other message yield exactly the same price conditional on exit. Thus, the incentives of the activist to send message m_A or m_R are solely determined by his incentives to change the manager's decision and not to change the price conditional on exit. Since this strategy is incentives compatible under Market-Transparency, it has to be incentives compatible in this equilibrium as well.

Consider a responsive equilibrium under Voice-Transparency. Let $a(m) \in \{A, R\}$ be the manager's decision given message $m \in M$. Let M_A and M_R be the set of messages that lead to approval and rejection in equilibrium, respectively. By definition, neither set is empty. We start by arguing that $m \in \arg\max_{m \in M_R} p(m) \cup \arg\max_{m \in M_A} p(m)$ is never sent with a strictly positive probability. Since $\rho < \infty$ there is a strictly positive probability that the activist exits, and hence it is a dominating strategy to send a message in $\arg\max_{m \in M_R} p(m) \cup \arg\max_{m \in M_A} p(m)$. This implies that there are exactly two different prices conditional on exit, one for $m \in M_R$ and one for $m \in M_A$. We let these prices be p_R and p_A, respectively. Essentially, there are only to effective messages: one that leads to approval and price p_A and one that leads to rejection and prices p_R. For the same argument that is given in the beginning of the proof of Proposition 2, it must be that $p_A = p_R > 0$. Overall, it is immediate to see that any set Θ and Υ that emerge as equilibrium under Voice-Transparency, can also emerge as equilibrium under

Market-Transparency. ■

Proofs of Section IV

Lemma A.4 *Let $\underline{\tau}(x) < 0$ be the unique negative solution of $\pi(\tau, x) + \tau = 0$ and $\overline{\tau}(x) > 0$ the unique positive solution of $\pi(\tau, x) - \tau = 0$. Then:*

(i) *If $\tau \in [\underline{\tau}(x), \overline{\tau}(x)]$ then $\pi(\tau, x) > 0$.*

(ii) *If and only if $\tau \in [\underline{\tau}(x), \overline{\tau}(x)]$ then $\tau \in [-\pi(\tau, x), \pi(\tau, x)]$.*

(iii) *Suppose $\tau \in [\underline{\tau}(x), \overline{\tau}(x)]$. If and only if $\pi(\tau, x) < \mathbb{E}[\theta]$ then $\pi(\tau, x)$ increases in x.*

(iv) *If there is $x_0 \geq 0$ such that $\pi(\tau, x_0) < (=, >) \mathbb{E}[\theta]$ then $\pi(\tau, x) < (=, >) \mathbb{E}[\theta]$ for all x.*

(v) *If $\underline{\tau}(0) > (=, <) - \mathbb{E}[\theta]$ then $\frac{\partial \underline{\tau}(x)}{\partial x} < (=, >) 0$ and $\underline{\tau}(x) > (=, <) - \mathbb{E}[\theta]$ for all x.*

Proof of Lemma A.4. Extending Proposition 1 in Archaya et al. (2010), one can show $\pi(\tau, x)$ is the unique solution of the equation $\varphi(\tau, p, x) = p$. Therefore, $\pi(\tau, x)$ is well defined. Similar to the proof of Lemma 6, the following properties of $\pi(\tau, x)$ follow. First, since $\mathbb{E}[\theta] > 0$ then $\pi(0, x) > 0$. Second, $\pi(\tau, x)$ strictly decreases in $|\tau|$. Third, $\varphi(\tau, p, x)$ is continuous in τ. Forth, part (i) and (ii) of the Lemma follow.
Consider part (iii). Since $\tau \in [\underline{\tau}(x), \overline{\tau}(x)]$ then $\tau \in [-\pi(\tau, x), \pi(\tau, x)]$ and $\pi(\tau, x) > 0$. Thus, $v(\theta, 1_{\{\theta \geq \tau\}}) \leq \pi \Leftrightarrow |\theta| \leq \pi$ and $\varphi(\tau, \pi, x)$ can be rewritten

$$\varphi(\tau, \pi, x) = \frac{\mathbb{E}\left[v(\theta, 1_{\{\theta \geq \tau\}})\right] + \rho\left[\int_{-\pi}^{\tau}(-\theta)\, dF(\theta) + \int_{\tau}^{\pi}(\theta)\, dF(\theta)\right] + x\mathbb{E}[\theta]}{1 + \rho \Pr[\theta \in [-\pi, \pi]] + x}$$

We know $\pi - \varphi(\tau, \pi, x) = 0$ which is equivalent to

$$\frac{\mathbb{E}\left[\pi - v(\theta, 1_{\{\theta \geq \tau\}})\right] + \rho\left[\int_{-\pi}^{\tau}(\pi + \theta)\, dF(\theta) + \int_{\tau}^{\pi}(\pi - \theta)\, dF(\theta)\right] + x\mathbb{E}[\pi - \theta]}{1 + \rho \Pr[\theta \in [-\pi, \pi]] + x} = 0 \qquad \text{(A15)}$$

Using the implicit function theorem,

$$\frac{\partial \pi}{\partial x} = -\frac{\frac{\mathbb{E}[\pi-\theta](1+\rho \Pr[\theta\in[-\pi,\pi]]+x) - \left(\mathbb{E}\left[\pi - v(\theta, 1_{\{\theta\geq\tau\}})\right] + \rho\left[\int_{-\pi}^{\tau}(\pi+\theta)dF(\theta) + \int_{\tau}^{\pi}(\pi-\theta)dF(\theta)\right] + x\mathbb{E}[\pi-\theta]\right)}{(1+\rho \Pr[\theta\in[-\pi,\pi]]+x)^2}}{\frac{\left(1+\rho\int_{-\pi}^{\pi} dF(\theta)+x\right)(1+\rho \Pr[\theta\in[-\pi,\pi]]+x) - \left(\mathbb{E}\left[\pi - v(\theta, 1_{\{\theta\geq\tau\}})\right] + \rho\left[\int_{-\pi}^{\tau}(\pi+\theta)dF(\theta) + \int_{\tau}^{\pi}(\pi-\theta)dF(\theta)\right] + x\mathbb{E}[\pi-\theta]\right)\rho\frac{\partial \Pr[\theta\in[-\pi,\pi]]}{\partial\pi}}{(1+\rho \Pr[\theta\in[-\pi,\pi]]+x)^2}}$$

using (A15) I get $\frac{\partial \pi}{\partial x} = -\frac{\mathbb{E}[\pi-\theta]}{1+\rho\int_{-\pi}^{\pi} dF(\theta)+x}$ and note that $\frac{\partial \pi}{\partial x} > 0$ if and only if $\pi < \mathbb{E}[\theta]$.
Consider part (iv). There are three cases. First, suppose $\pi(\tau, x_0) < \mathbb{E}[\theta]$. Note that $\varphi(\tau, \pi(\tau, x_0), x) < \mathbb{E}[\theta]$ for any x since $\varphi(\tau, p, x)$ is just a weighted average where $\mathbb{E}[\theta]$ is one of the components. Suppose on the contrary that there is $x \neq x_0$ such that $\pi(\tau, x) \geq \mathbb{E}[\theta]$. Since $\varphi(\tau, \pi(\tau, x), x) = \pi(\tau, x)$ then $\varphi(\tau, \pi(\tau, x), x) \geq \mathbb{E}[\theta]$. Therefore, $\varphi(\tau, \pi(\tau, x_0), x) < \mathbb{E}[\theta] \leq \varphi(\tau, \pi(\tau, x), x)$ which contradict the fact that $\varphi(\tau, \pi(\tau, x), x)$ is a global minimum of $\varphi(\tau, p, x)$. Second, suppose $\pi(\tau, x_0) > \mathbb{E}[\theta]$. Note that $\varphi(\tau, \pi(\tau, x_0), x) > \mathbb{E}[\theta]$ for any x since $\varphi(\tau, p, x)$ is just a weighted average where $\mathbb{E}[\theta]$ is one of the components. Suppose on the contrary that there is $x \neq x_0$ such that $\pi(\tau, x) \leq \mathbb{E}[\theta]$. Since $\varphi(\tau, \pi(\tau, x), x) = \pi(\tau, x)$ then $\varphi(\tau, \pi(\tau, x), x) \leq \mathbb{E}[\theta]$. This implies that $\varphi(\tau, \pi(\tau, x), x_0) \leq \mathbb{E}[\theta]$ as well. Since $\varphi(\tau, \pi(\tau, x_0), x_0) = \pi(\tau, x_0)$ and by assumption, $\pi(\tau, x_0) > \mathbb{E}[\theta]$ then $\varphi(\tau, \pi(\tau, x_0), x_0) > \mathbb{E}[\theta] \geq \varphi(\tau, \pi(\tau, x), x_0)$. This contradicts the fact that $\varphi(\tau, \pi(\tau, x_0), x_0)$ is a global minimum of $\varphi(\tau, p, x_0)$. Third, suppose $\pi(\tau, x_0) = \mathbb{E}[\theta]$. Note that $\varphi(\tau, \pi(\tau, x_0), x) = \mathbb{E}[\theta]$ for any x since $\varphi(\tau, p, x)$ is just a weighted average where $\mathbb{E}[\theta]$ is one of the components. Suppose on the contrary that there is $x \neq x_0$ such that $\pi(\tau, x) \neq \mathbb{E}[\theta]$. Since $\varphi(\tau, \pi(\tau, x), x) = \pi(\tau, x)$ then $\varphi(\tau, \pi(\tau, x), x) \neq \mathbb{E}[\theta]$. Therefore, $\varphi(\tau, \pi(\tau, x), x_0) \neq \mathbb{E}[\theta]$ as well. If $\varphi(\tau, \pi(\tau, x), x_0) < \mathbb{E}[\theta]$ then this contradicts the fact that $\varphi(\tau, \pi(\tau, x_0), x_0) = \mathbb{E}[\theta]$ is a global minimum of $\varphi(\tau, p, x_0)$. If $\varphi(\tau, \pi(\tau, x), x_0) > \mathbb{E}[\theta]$ then $\varphi(\tau, \pi(\tau, x), x) > \mathbb{E}[\theta]$ however, $\varphi(\tau, \pi(\tau, x_0), x) = \mathbb{E}[\theta]$. This contradicts the fact that $\varphi(\tau, \pi(\tau, x), x)$ is a global minimum of $\varphi(\tau, p, x)$.
Consider part (v). Since $\underline{\tau}(x)$ is defined by $\pi(\underline{\tau}(x), x) + \underline{\tau}(x) = 0$ then by the implicit function theorem $\frac{\partial \underline{\tau}(x)}{\partial x} = -\frac{\frac{\partial \pi(\underline{\tau}(x), x)}{\partial x}}{\frac{\partial \pi(\underline{\tau}(x), x)}{\partial \tau}+1}$. Note that since $\underline{\tau}(x) < 0$ then $\frac{\partial \pi(\underline{\tau}(x), x)}{\partial \tau} > 0$. Therefore, $\frac{\partial \underline{\tau}(x)}{\partial x} > 0$ if and only if $\frac{\partial \pi(\underline{\tau}(x), x)}{\partial x} < 0$. Suppose $\underline{\tau}(0) > (=, <) - \mathbb{E}[\theta]$ this means that $\pi(\underline{\tau}(0), 0) < (=, >) \mathbb{E}[\theta]$. According to part (iv) we have $\pi(\underline{\tau}(x), x) < (=, >) \mathbb{E}[\theta]$ for all x. According to part (iii) $\pi(\tau, x)$ increases (does not change, decreases) with x. Therefore, $\frac{\partial \underline{\tau}(x)}{\partial x} < (=, >) 0$ as reqired. This also implies $\underline{\tau}(x) > (=, <) - \mathbb{E}[\theta]$ for all x. ■

Proof of Proposition 4. Suppose $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) > \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$. Since $\frac{1-\lambda}{\lambda} < \frac{1}{\delta}\frac{1-\lambda}{\lambda}$, Lemma A.4 implies $\underline{\tau}(x) > -\mathbb{E}[\theta]$ for any $x \geq 0$. This also implies that $\pi(\min\{-z(\beta), 0\}), x) \leq \mathbb{E}[\theta]$ for any $x \geq 0$. If a responsive equilibrium exits then the price upon exit is smaller than $\mathbb{E}[\theta]$ and the uninformed activist exits only with probability δ. Thus, the price at the most efficient equilibrium is given by $\pi\left(\min\{-z(\beta), 0\}), \frac{1-\lambda}{\lambda}\right)$ and is decreasing with λ. Based on Lemma A.4 this can be an equilibrium if and only if $\min\{-z(\beta), 0\} \in \left[\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right), 0\right]$. Similar to the argument in Proposition 1, a responsive equilibrium exists if and only if $-z(\beta) \geq \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$.

Suppose $\underline{\tau}\left(\frac{1-\lambda}{\lambda}\right) < \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$. Since $\frac{1-\lambda}{\lambda} < \frac{1}{\delta}\frac{1-\lambda}{\lambda}$, Lemma A.4 implies $-\mathbb{E}[\theta] > \underline{\tau}(x)$ for any $x \geq 0$. This also implies that $\pi(\min\{-z(\beta), 0\}), x) > \mathbb{E}[\theta]$ for any $x \geq 0$. If a responsive equilibrium exits then the price upon exit is greater than $\mathbb{E}[\theta]$ and the uninformed activist exits only with probability one. Thus, the price at the most efficient equilibrium is given by $\pi\left(\min\{-z(\beta), 0\}), \frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$ and is increasing in λ. Based on Lemma A.4 this can be an equilibrium if and only if $\min\{-z(\beta), 0\} \in \left[\underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right), 0\right]$. Similar to the argument in Proposition 1, a responsive equilibrium exists if and only if $-z(\beta) \geq \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$. ■

Proof of Proposition 5. If the equilibrium is not responsive then information is not revealed, and based on Lemma 3 the manager approves the project with probability one. This implies that the value of the firm is $\mathbb{E}[\theta]$. Suppose the equilibrium is responsive. Since $\mathbb{E}[\theta] > 0$ the uninformed activist and the manager agree on the action to be taken. If $p^* \leq \mathbb{E}[\theta]$ then the uninformed activist has strict incentives to make sure that $a = A$ and she exists only because of her liquidity needs. If instead $p^* > \mathbb{E}[\theta]$ then the uninformed activist has weak incentives to make sure that $a = A$ and she exists with probability one. At the most efficient equilibrium, the value of the firm is

$$\lambda\mathbb{E}\left[v(\theta, 1_{\{\theta \geq \min\{0, -z(\beta)\}\}})\right] + (1-\lambda)\mathbb{E}[\theta] \tag{A16}$$

Note that Proposition 1 continues to hold under the assumption that $\mathbb{E}[\theta] > 0$ since the uninformed activist has incentives to send a message $m \in M_A$ and the informed activist has incentives to follow threshold strategy τ. Since the manager always follow the recommendation to approve the project, the sufficient and necessary conditions for the existence of a responsive equilibrium remain the same as in Proposition 1.

Suppose $-z(\beta) \geq \underline{\tau}(0)$. Then, a responsive equilibrium with threshold $-z(\beta)$ exists. Given expression (A16), as λ decreases, if a responsive equilibrium continues to exist, then the value of the firm decreases. If at some point a responsive equilibrium no longer exists, then the value of the firm jumps downward to $\mathbb{E}[\theta]$. Therefore, $\lambda^* = 1$.

Suppose $-z(\beta) \leq -\mathbb{E}[\theta]$. First, if $-\mathbb{E}[\theta] > \underline{\tau}(0)$ then based on Lemma A.4 part (v) then $\frac{\partial \underline{\tau}(x)}{\partial x} > 0$. Thus for any $\lambda < 1$ we have $\underline{\tau}(0) < \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$ and $\underline{\tau}(0) < \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$. Based on Lemma A.4, a responsive equilibrium exists if and only if $-z(\beta) \geq \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$ and $p^* \geq \mathbb{E}[\theta]$ or $-z(\beta) \geq \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$ and $p^* < \mathbb{E}[\theta]$. Either way, if $-z(\beta) < \underline{\tau}(0)$ then a responsive equilibrium does not exist for any λ and hence $\lambda^* = 1$. If $-z(\beta) \geq \underline{\tau}(0)$ then as λ decreases the likelihood a responsive equilibrium decreases, as well as the expression in (A16). Therefore, $\lambda^* = 1$. Second,

if $-\mathbb{E}[\theta] \leq \underline{\tau}(0)$ then a responsive equilibrium never exists for $\lambda = 1$. Note that based on Lemma A.4 part (v), since $-\mathbb{E}[\theta] \leq \underline{\tau}(0)$ then $\underline{\tau}(x) \geq -\mathbb{E}[\theta]$ for all x. This implies that a responsive equilibrium does not exists for any λ. Therefore $\lambda^* = 1$. Note that if $-z(\beta) = -\mathbb{E}[\theta] = \underline{\tau}(0)$ then $\underline{\tau}(0)$ is invariant to λ but expression (A16) increases with λ, and hence, $\lambda^* = 1$.

Suppose $\underline{\tau}(0) > -z(\beta) > -\mathbb{E}[\theta]$. Based on Proposition 1 there is no responsive equilibrium when $\lambda = 1$. Based on Lemma A.4, we know that if $\lambda < 1$ then $\underline{\tau}(0) > \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right)$ and $\underline{\tau}(0) > \underline{\tau}\left(\frac{1-\lambda}{\lambda}\right)$. Moreover, since $\lim_{\lambda=0}\underline{\tau} = -\mathbb{E}[\theta]$ there is unique $\underline{\lambda} \in (0,1)$ such that $\underline{\tau}(0) = -z(\beta)$. Since $\underline{\tau}(0) = -z(\beta) > -\mathbb{E}[\theta]$ it has to be that $\underline{\tau} = \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) = -p^* > -\mathbb{E}[\theta]$. Thus, if $\lambda \in (0, \underline{\lambda})$ then $-z(\beta) > \underline{\tau}\left(\frac{1}{\delta}\frac{1-\lambda}{\lambda}\right) > -\mathbb{E}[\theta]$ and a responsive equilibrium exists. Given expression (A16) it must be $\lambda^* = \underline{\lambda} < 1$. ■

Proof of Proposition 6. I start by arguing that for the existence of a responsive equilibrium it is sufficient to consider the second best threshold, $\min\{-z(\beta), 0\}$. Recall that for any x, $\pi(\tau, x)$ increases in τ if and only if $\tau < 0$. If $\pi(\tau, x) < \mathbb{E}[\theta]$ then the right hand side of (15) increases in p and hence c^* increases in τ. Since the left hand side of (15) is higher, it must be that the right hand side of (15) is higher as well. This implies that $p^*(G(c))$ increases with τ. Similarly, if $\pi(\tau, x) > \mathbb{E}[\theta]$ then the right hand side of (15) decreases in p and hence c^* decreases in τ. Since the left hand side of (15) is smaller, it must be that the right hand side of (15) is smaller as well. This implies that $p^*(G(c))$ increases with τ, when $\tau < 0$. Overall $p^*(G(c))$ increases with τ and it is sufficient to consider threshold $\tau = \min\{-z(\beta), 0\}$.

Next, note that since $\underline{\tau}(0) > -\mathbb{E}[\theta]$ then $\underline{\tau}(x)$ is decreasing in x and all relevant prices are smaller than $\mathbb{E}[\theta]$. Moreover, $\lim_{x\to\infty}\underline{\tau}(x) = -\mathbb{E}[\theta]$. Therefore, there is $x_2 > 0$ such that $\underline{\tau}(x_2) = -z(\beta)$. Define $c_2 \equiv (1-\delta)\mathbb{E}[\max\{|\theta|, z(\beta)\}]$ and let G_2 be such that $-z(\beta) = \underline{\tau}\left(\frac{1-G_2(c_2)}{G_2(c_2)}\right)$. Note that G_2 is well defined (even if it is not unique). Since $0 > -z(\beta) = \underline{\tau}\left(\frac{1-G_2(c_2)}{G_2(c_2)}\right)$ then $\pi\left(-z(\beta), \frac{1-G_2(c_2)}{G_2(c_2)}\right) = -\underline{\tau}\left(\frac{1-G_2(c_2)}{G_2(c_2)}\right)$. Therefore, c_2 must be the unique solution of

$$\frac{c}{1-\delta} = \mathbb{E}\left[\max\left\{|\theta|, \pi\left(\min\{-z(\beta), 0\}, \frac{1-G_2(c)}{G_2(c)}\right)\right\}\right] - \mathbb{E}[\theta]$$

We conclude that for G_2 there is a responsive equilibrium in which the price upon exit equals $z(\beta) < \mathbb{E}[\theta]$ and $c_2^* = c_2$.

Consider a non-responsive equilibrium for the same G_2. Since $\theta \leq |\theta|$ and $\pi\left(-z(\beta), \frac{1-G_2(c)}{G_2(c)}\right) > \pi\left(-\infty, \frac{1-G_2(c)}{G_2(c)}\right)$ for any c it follows that the r.h.s of (15) is uniformly lower. This implies that

that c_2^* shifts to a strictly lower level, c_{NR}. That is, the activist is less likely to acquire information.

Consider any G_1 such that $G_1 \overset{FDSD}{\prec} G_2$. I argue that a responsive equilibrium does not exist. Suppose on the contrary it does. Note that for any c, $\pi\left(\min\{-z(\beta),0\}, \frac{1-G_1(c)}{G_1(c)}\right) < \pi\left(\min\{-z(\beta),0\}, \frac{1-G_2(c)}{G_2(c)}\right)$. Which implies that $G_1(c_1^*) > G_2(c_2^*)$. However, in this case, $-z(\beta) = \underline{\tau}\left(\frac{1-G_2(c_2^*)}{G_2(c_2^*)}\right) < \underline{\tau}\left(\frac{1-G_1(c_1^*)}{G_1(c_1^*)}\right)$ and hence a responsive does not exist, a contradiction. In a non responsive equilibrium, if G_1 is sufficiently close to G_2 then c_1^* converges to $c_{NR} < c_2^*$ and it is possible to find G_1 sufficiently close to G_2 such that $G_1(c_1^*) \approx G_2(c_{NR}) < G_2(c_2^*)$. ■

Proofs of Section V

Proof of Lemma 8. Consider a non-responsive equilibrium in which the manager chooses $a^* \in \{A, R\}$. The activist observes the actual decision a and strategically exits if and only if $v(\theta, a) < p^*(1)$. Since the market maker does not observe a, in equilibrium $p^*(0) = \mathbb{E}[v(\theta, a^*) | v(\theta, a^*) > p^*(1)]$. If the manager chooses action a his expected utility is

$$\omega\left[\left(\delta + (1-\delta)\Pr\left[v(\theta,a) < p^*(1)\right]\right)p^*(1) + (1-\delta)\Pr\left[v(\theta,a) > p^*(1)\right]p^*(0)\right] + \mathbb{E}\left[v(\theta+\beta,a)\right]$$

and the manager chooses $a = A$ if and only if

$$\frac{\rho}{1+\rho}\frac{\omega}{2}\left(\Pr\left[-\theta < p^*(1)\right] - \Pr\left[\theta < p^*(1)\right]\right)\left(p^*(0) - p^*(1)\right) + \mathbb{E}[\theta] + \beta \geq 0 \tag{A17}$$

and recall that $p^*(0) - p^*(1) > 0$ and by assumption $\beta \geq -\mathbb{E}[\theta]$. Note that if $a^* = A$ then $p^*(0) = \mathbb{E}[\theta | \theta > p^*(1)]$ where $p^*(1) = p_{NR}$ as given by Lemma 3. If $a^* = R$ then $p^*(0) = \mathbb{E}[-\theta | -\theta > p^*(1)]$ where $p^*(1) = \pi(\infty) \equiv \lim_{\tau\to\infty}\pi(\tau)$. There are several cases to consider. First, if $\Pr[-\theta < p_{NR}] \geq \Pr[\theta < p_{NR}]$ then a non-responsive equilibrium with $a^* = A$ always exists. Second, suppose $\Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}]$. A non-responsive equilibrium in which the manager accepts the project exists if and only if

$$\omega \leq \hat{\omega} \equiv \frac{\mathbb{E}[\theta] + \beta}{\frac{\rho}{1+\rho}\frac{1}{2}\left(\Pr[\theta < p_{NR}] - \Pr[-\theta < p_{NR}]\right)\left(\mathbb{E}[\theta | \theta > p_{NR}] - p_{NR}\right)}$$

and note that $\hat{\omega} > 0$. Third, suppose $\Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}]$ and $\omega > \hat{\omega}$. There is no equilibrium in which the manager accepts the project with probability one. Consider an equilibrium in which $a^* = R$. According to (A17), such equilibrium exists if and only if

$\Pr[-\theta < \pi(\infty)] - \Pr[\theta < \pi(\infty)] < 0$ and

$$\omega \geq \omega_0 \equiv \frac{\mathbb{E}[\theta] + \beta}{\frac{\rho}{1+\rho}\frac{1}{2}\left(\Pr[\theta < \pi(\infty)] - \Pr[-\theta < \pi(\infty)]\right)\left(\mathbb{E}[-\theta| - \theta > \pi(\infty)] - \pi(\infty)\right)}$$

It is left to show that if $\omega > \hat{\omega}$ and either $\omega < \omega_0$ or $\Pr[-\theta < \pi(\infty)] - \Pr[\theta < \pi(\infty)] \geq 0$ then there is a mixed strategy equilibrium. Under a mixed strategy equilibrium, the manager accepts the project with probability $x \in (0,1)$ and let the price be $p_x^*(\sigma)$. Since the market maker does not observe a, it follows that $p_x^*(1) = xp_{NR} + (1-x)\pi(\infty)$ and $p_x^*(0) = x\mathbb{E}[\theta|\theta > p_x^*(1)] + (1-x)\mathbb{E}[-\theta| - \theta > p_x^*(1)]$. The manager must be indifferent and hence (A17) must holds with equality for the assumed x. Since with $x = 1$ the l.h.s of (A17) is negative (there is no equilibrium in which $a = A$ with probability one) and with $x = 0$ it is positive (there is no equilibrium in which $a = R$ with probability one), there is $x \in (0,1)$ that solves this condition. ■

Proof of Proposition 7. Consider a responsive equilibrium and let M_A and M_R be the set of messages that lead to approval and rejection of the project, respectively. Without the loss of generality, suppose $M_A \cup M_R = \left[\underline{\theta}, \overline{\theta}\right]$. In a responsive equilibrium neither set is empty. Regardless of the message, the activist observes the actual decision a and strategically exits if and only if $v(\theta, a) < p^*(1)$. Since the market maker does not observe a or m, in equilibrium $p^*(0) = \mathbb{E}\left[v(\theta, 1_{\{\theta\in\Theta\}}|v(\theta, 1_{\{\theta\in\Theta\}} > p^*(1)\right]$. If the manager chooses action a his expected utility conditional on message m is

$$\omega\left[(\delta + (1-\delta)\Pr[v(\theta,a) < p^*(1)|m])\, p^*(1) + (1-\delta)\Pr[v(\theta,a) > p^*(1)|m]\, p^*(0)\right] + \mathbb{E}[v(\theta+\beta, a)|m]$$

Therefore, the manager chooses $a = A$ if and only if

$$\frac{\omega}{2}\frac{\rho}{1+\rho}\left(\Pr[-\theta < p^*(1)|m] - \Pr[\theta < p^*(1)|m]\right)[p^*(0) - p^*(1)] + \mathbb{E}[\theta + \beta|m] \geq 0 \qquad \text{(A18)}$$

Note that if $m \in M_A$ then $\theta \geq -p^*(1)$ with probability one. Otherwise, the activist is strictly better off sending message $m \in M_R$ and keeping her holdings in the firm. Therefore, for $m \in M_A$ the manager follows the recommendation and approves the project if and only if

$$\frac{\omega}{2}\frac{\rho}{1+\rho}\Pr[\theta > p^*(1)|m]\,[p^*(0) - p^*(1)] + \mathbb{E}[\theta + \beta|m] \geq 0 \qquad \text{(A18a)}$$

Similarly, if $m \in M_R$ then $\theta \leq p^*(1)$ with probability one. Therefore, for $m \in M_R$ the manager follows the recommendation and rejects the project if and only if

$$\frac{\omega}{2}\frac{\rho}{1+\rho}\Pr\left[-\theta > p^*(1)\,|m\right]\left[p^*(0) - p^*(1)\right] - \mathbb{E}\left[\theta + \beta|m\right] \geq 0 \tag{A18b}$$

Note that if both conditions (A18a) and (A18b) hold for some $\omega_0 \geq 0$ then since $p^*(0) > p^*(1)$ they hold for any $\omega > \omega_0$. Therefore, a responsive equilibrium under ω_0 (and not necessarily a threshold equilibrium) is also a responsive equilibrium under any $\omega > \omega_0$. By the same reasoning as above, the set of responsive equilibria decreases with β.

Suppose $\beta \leq \bar{\beta}$ and consider $\tau \in [\underline{\tau}, \bar{\tau}]$ where the activist randomly chooses message in M_A (M_R) and sends it to the manager if $\theta \in \left[\tau, \overline{\theta}\right]$ ($\theta \notin \left[\tau, \overline{\theta}\right]$). Based on Lemma 6, $\tau \in [-\pi(\tau), \pi(\tau)]$, $p^*(1) = \pi(\tau) > 0$, and $p^*(0) = \mathbb{E}\left[|\theta| \,|\, |\theta| > \pi(\tau)\right]$. Therefore, the activist has (weak) incentives to follow the proposed communication strategy. Consider the manager's incentives to follow threshold τ. The manager rejects the project upon $m \in M_R$ if and only if (A18b) holds, where the conditioning event is $\theta < \tau$. Condition (A18b) holds if and only if $\beta \leq \Psi(\tau; \omega, \rho)$. Since $\beta \leq \bar{\beta}$ condition (A18b) holds for any $\tau \in [\underline{\tau}, \bar{\tau}]$. The manager approves the project upon $m \in M_A$ if and only if (A18a) holds, where the conditioning event is $\theta \geq \tau$. If there is $\tau_0 \in [\underline{\tau}, \bar{\tau}]$ such that $\Psi(\tau_0; \omega, \rho) = \beta$ then for $\tau = \tau_0$ condition (A18a) holds as well. If there is no such τ_0 then from the continuity of $\Psi(\tau; \omega, \rho)$ in τ and since $\max_{\tau \in [\underline{\tau}, \bar{\tau}]}\{\Psi(\tau; \omega, \rho)\} \geq \beta$ then $\min_{\tau \in [\underline{\tau}, \bar{\tau}]} \Psi(\tau; \omega, \rho) > \beta$ as well. It immediately follows that (A18a) holds for $\tau = 0$. Overall, if $\beta \leq \bar{\beta}$ then a threshold equilibrium exists. This argument also proves that if $\beta \leq \underline{\beta}$ then there is a responsive equilibrium with $\tau = 0$, the first best. Suppose $\beta > \bar{\beta}$ and that on the contrary a threshold equilibrium exists. Per Lemma 6, it is necessary that $\tau \in [\underline{\tau}, \bar{\tau}]$. Since the manager follows the recommendation to reject the project, $\Psi(\tau; \omega, \rho) \geq \beta$. However, by assumption $\beta > \max_{\tau \in [\underline{\tau}, \bar{\tau}]}\{\Psi(s; \omega, \rho)\}$ and hence a contradiction. A similar argument proves that if $\beta > \underline{\beta}$ then the first best cannot be obtained. ■

Corollary 4 *With managerial myopia, voice and exit exhibit substitution if and only if* $\beta \leq z^{-1}(0)$, $\mathbb{E}[\theta] < 0$, $\omega > \hat{\omega}$, *and* $\Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}]$.

Proof. Note that if $\beta > z^{-1}(0)$ then without exit the only equilibrium is non-responsive and $\overline{V}_{Voice,NoExit} = \mathbb{E}[\theta] = \overline{V}_{NoVoice,NoExit}$. Since a non-responsive equilibrium always exists when voice is allowed, $\overline{V}_{Voice,Exit} \geq \overline{V}_{NoVoice,Exit}$. Since $\Psi(\tau; \omega, \rho)$ strictly increases with ω, there is

$\omega^* \in [0, \infty)$ such that the first best is obtained in equilibrium with voice and exit if and only if $\omega \geq \omega^*$. This implies that when $\beta > z^{-1}(0)$ then $\overline{V}_{Voice,Exit} > \overline{V}_{NoVoice,Exit}$ and voice and exit exhibit complementarity. When $\beta \leq z^{-1}(0)$ the first best is obtained with and without voice, that is, $\overline{V}_{Voice,NoExit} = \mathbb{E}[|\theta|] = \overline{V}_{Voice,Exit}$. Since $\overline{V}_{NoVoice,NoExit} = \mathbb{E}[\theta]$ then voice and exit exhibit complementarity if and only if $\overline{V}_{NoVoice,Exit}(\omega) \leq \mathbb{E}[\theta]$. It follows from Lemma 8, that $\overline{V}_{NoVoice,Exit}(\omega) > \mathbb{E}[\theta]$ if and only if $\mathbb{E}[\theta|\theta < 0] < -\beta < \mathbb{E}[\theta] < 0$, $\omega > \hat{\omega}$, and $\Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}]$. Under those circumstances voice and exit exhibit substitution. ■

Proofs of Section VI

Proof of Proposition 8. Following Proposition 1, regardless of γ the most efficient threshold that can be obtained in any equilibrium is $\tau^* = \min\{0, -z(\beta)\}$. If $\beta \leq z^{-1}(-\underline{\tau})$ then τ^* is implementable in equilibrium when the activist is unbiased. Therefore, $\gamma^* = 0$ if and only if $\beta \leq z^{-1}(-\underline{\tau})$. Hereafter we assume $\beta > z^{-1}(-\underline{\tau})$ and focus on the smallest bias γ that implements $\tau^* = -z(\beta)$ in equilibrium.

We start with the following observation. Let β_0 be a solution of $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right] = 0$ if exists, and else define $\beta_0 = \infty$. Since $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]$ strictly decreases in β and $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right] \geq \mathbb{E}[\theta]$ for any $\beta > z^{-1}(0)$, it follows that $\beta_0 = \infty$ if $\mathbb{E}[\theta] \geq 0$ and else β_0 is unique by the intermediate value theorem. Note that $\beta_0 > z^{-1}(-\underline{\tau})$. According to Proposition 1 and Lemma 6, if $\beta = z^{-1}(-\underline{\tau})$ then the stock price conditional on exit in a responsive equilibrium is $\pi(\underline{\tau}) = -\underline{\tau} > 0$. Since $\pi(\tau) > 0$ implies $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > \tau\}}\right)\right] > 0$ for any τ, then $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > \underline{\tau}\}}\right)\right] > 0$ which proves $\beta_0 > z^{-1}(-\underline{\tau})$.

Let $p_{\beta,\gamma}$ be the price conditional on exit in this equilibrium with threshold $\tau = -z(\beta)$. Note that $\Upsilon = \left\{\theta : v\left(\theta + \gamma, 1_{\{\theta > -z(\beta)\}}\right) \leq p_{\beta,\gamma}\right\}$. We argue that $\Upsilon = \varnothing$ if and only if $p_{\beta,\gamma} \leq 0$. If $p_{\beta,\gamma} > 0$ then $\Pr[\theta \in \Upsilon] > 0$. If $p_{\beta,\gamma} \leq 0$ the activist has strict incentives to send message $m \in M_R$ if $\theta < -\gamma$ and message $m \in M_A$ otherwise. Since the proposed equilibrium is responsive, neither set is empty, and the manager will follow this recommendation. This strategy will guarantee the activist a long term value that satisfies $\Pr\left[v\left(\theta + \gamma, 1_{\{\theta > -z(\beta)\}}\right) > 0\right] = 1$. Therefore, $\Upsilon = \varnothing$.

We argue that a responsive equilibrium with $\Upsilon = \varnothing$ and $\tau = -z(\beta)$ exists if and only if $\gamma = z(\beta)$ and $\beta \geq \beta_0$. Suppose such equilibrium exists. The market maker's pricing strategy must be consistent with $\Upsilon = \varnothing$. Therefore, the fair price of the stock is $p_{\beta,\gamma} = \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]$. Since $\Upsilon = \varnothing$ implies $p_{\beta,\gamma} \leq 0$ it is required that $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right] \leq 0$ and hence $\beta \geq \beta_0$. Moreover, note that in any responsive equilibrium without exit, the activist has strictly

incentives to implement threshold $-\gamma$. Therefore, it must be $\gamma = z(\beta)$. By construction, if $\gamma = z(\beta)$ and $\beta \geq \beta_0$ then there is an equilibrium with $\Upsilon = \varnothing$ and $\tau = -z(\beta)$. We conclude that if $\Upsilon = \varnothing$ and $\tau = -z(\beta)$ then $\gamma^* = z(\beta) \geq z(\beta_0)$.

We argue that if $\tau = -z(\beta)$ and $\Upsilon \neq \varnothing$ in a responsive equilibrium then $\Upsilon = [-p_{\beta,\gamma} - \gamma, p_{\beta,\gamma} - \gamma]$. Note that $\Upsilon \neq \varnothing$ implies $p_{\beta,\gamma} > 0$. Moreover, the biased activist has incentives to follow a communication strategy that implements threshold $-z(\beta)$ if and only if

$$-z(\beta) \in [-p_{\beta,\gamma} - \gamma, p_{\beta,\gamma} - \gamma] \tag{A19}$$

Noting that $v\left(\theta + \gamma, 1_{\{\theta > -z(\beta)\}}\right) \leq p$ if and only if $\theta \in [-p - \gamma, p - \gamma]$ completes the argument. Consider a responsive equilibrium where $\tau = -z(\beta)$ and $\Upsilon \neq \varnothing$. Given (A19) the price $p_{\beta,\gamma}$ must be strictly positive and the solution of

$$p = \frac{\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right] + \rho \Pr\left[\theta \in [-p - \gamma, p - \gamma]\right] \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right) | \theta \in [-p - \gamma, p - \gamma]\right]}{1 + \rho \Pr\left[\theta \in [-p - \gamma, p - \gamma]\right]} \tag{A20}$$

In what follows we prove that there is $p_0 > 0$ and γ_0 that satisfy $p_0 + \gamma_0 = z(\beta)$ and (A20). Let $\gamma = z(\beta) - p$ then for any $p > 0$ the r.h.s of (A20) becomes

$$\zeta(p, \beta) \equiv \frac{\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right] + \rho \Pr\left[\theta \in [-z(\beta), 2p - z(\beta)]\right] \mathbb{E}\left[\theta | \theta \in [-z(\beta), 2p - z(\beta)]\right]}{1 + \rho \Pr\left[\theta \in [-z(\beta), 2p - z(\beta)]\right]} \tag{A21}$$

and (A20) implies $p = \zeta(p, \beta)$. We note several properties of $\zeta(p, \beta)$ that follows from its definition. First, $\lim_{p \to 0} \zeta(p, \beta) = \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]$. Second, $\lim_{p \to \infty} \zeta(p, \beta)$ equals $\frac{\mathbb{E}\left[v(\theta, 1_{\{\theta > -z(\beta)\}})\right] + \rho \Pr[\theta > -z(\beta)] \mathbb{E}[\theta | \theta > -z(\beta)]}{1 + \rho \Pr[\theta > -z(\beta)]}$. Third, $\zeta(p, \beta)$ is continuous in p and β. Forth, $\zeta(p, \beta)$ decreases in p if and only if $2p - z(\beta) < \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]$ if and only if $p < p_{\min}(\beta) \equiv \frac{z(\beta) + \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]}{2}$. This means that $\zeta(p, \beta)$ has a unique minimum obtained at $p_{\min}(\beta)$. Since $v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right) > -z(\beta)$ then $p_{\min}(\beta) > 0$. We conclude that if $\beta < \beta_0$ then $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right] > 0$ and hence there exists a strictly positive solution to $p = \zeta(p, \beta)$. If $\beta \geq \beta_0$ then $\mathbb{E}[\theta] < 0$. Since $\mathbb{E}[\theta | \theta < -z(\beta)] < \mathbb{E}[\theta | \theta > -z(\beta)]$, this implies $\mathbb{E}[\theta | \theta < -z(\beta)] < 0$. Note that $\beta \geq \beta_0$ also implies $\mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right] < 0$. Therefore $\mathbb{E}[\theta | \theta > -z(\beta)] < \frac{\Pr[\theta < -z(\beta)]}{\Pr[\theta > -z(\beta)]} \mathbb{E}[\theta | \theta < -z(\beta)]$. Overall, since $\mathbb{E}[\theta | \theta < -z(\beta)] < 0$ and $\frac{\Pr[\theta < -z(\beta)]}{\Pr[\theta > -z(\beta)]} > 0$ then $\mathbb{E}[\theta | \theta > -z(\beta)] < 0$ as well. Last, $\lim_{p \to \infty} \zeta(p, \beta) < \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]$ if and only if $\mathbb{E}[\theta | \theta > -z(\beta)] < \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]$, which holds if and only if $0 > \mathbb{E}[\theta | \theta > -z(\beta)] + \mathbb{E}[\theta | \theta < -z(\beta)]$. Since both components are negative, and since $\zeta(p, \beta)$ has a unique minimum, that $\zeta(p, \beta) = p$ has no positive solution

if $\beta \geq \beta_0$. This immediately implies that if $\beta \geq \beta_0$ then $\Upsilon = \varnothing$ and hence $\gamma^* = z(\beta)$. We conclude that if $\beta < \beta_0$ then the equation $p = \zeta(p, \beta)$ has a strictly positive solution. We let $\gamma_0 = \min_{p>0, p=\zeta(p,\beta)} |z(\beta) - p|$. That is, if there are multiple solutions we choose the one that minimizes $|z(\beta) - p|$.

We argue that if $\beta < \beta_0$ then $\gamma^* = \gamma_0$. We let $\xi(\gamma, p, \beta)$ be the r.h.s of (A.18). Note that $\xi(z(\beta) - p, p, \beta) = \zeta(p, \beta)$ for any $p > 0$ and β in this range. Suppose on the contrary there is γ_1 such that $\gamma_1 + p_1 > z(\beta)$, $|\gamma_1| < |\gamma_0|$, and $p_1 = \xi(\gamma_1, p_1, \beta)$. We note several properties of $\xi(\gamma, p, \beta)$ that follows directly from its definition. First, $\lim_{p\to 0,\infty} \xi(\gamma, p, \beta) = \mathbb{E}\left[v\left(\theta, 1_{\{\theta > -z(\beta)\}}\right)\right]$. Second, $\xi(\gamma, p, \beta)$ is continuous in $p \geq 0$, γ, and $\beta \geq 0$ (one can see that by a direct calculation of $\mathbb{E}\left[v\left(\theta, 1_{\{\theta>-z(\beta)\}}\right) | \theta \in [-p-\gamma, p-\gamma]\right]$). Thus, from the intermediate value theorem, if $\mathbb{E}\left[v\left(\theta, 1_{\{\theta>-z(\beta)\}}\right)\right] \geq 0$ then for any β and γ there is a positive solution to $p = \xi(\gamma, p, \beta)$. Let us define $\kappa(\gamma) \equiv \gamma + p(\gamma)$ where $p(\gamma)$ is the highest solution of $p = \xi(\gamma, p, \beta)$. Note that $\kappa(\gamma)$ is continuous in γ and $\kappa(\gamma_1) = \gamma_1 + p_1 > z(\beta)$. From Proposition 1 we know that $\kappa(0) < z(\beta)$ for $\beta > z^{-1}(-\underline{\tau})$. Thus, from the intermediate value theorem there is γ_2 such that $\kappa(\gamma_2) = z(\beta)$, $|\gamma_2| < |\gamma_1| < |\gamma_0|$, and $p_2 = \xi(\gamma_2, p_2, \beta)$. This contradicts the fact that γ_0 is the lowest absolute value that satisfies $p + \gamma = z(\beta)$ and $p = \xi(\gamma, p, \beta)$.

Let $p^*_\beta = \arg\min_{p>0, p=\zeta(p,\beta)} |z(\beta) - p|$. We argue if $\beta \geq z^{-1}(-\underline{\tau})$ then $\frac{\partial p^*_\beta}{\partial \beta} < 0$. Since p^*_β satisfies $p = \zeta(p, \beta)$, the implicit function theorem implies $\frac{\partial p^*_\beta}{\partial \beta} = -\frac{-\frac{\partial \zeta(p,\beta)}{\partial \beta}|_{p=p^*_\beta}}{1-\frac{\partial \zeta(p,\beta)}{\partial p}|_{p=p^*_\beta}}$ and (A21) implies that $\frac{\partial \zeta(p,\beta)}{\partial \beta} < 0$ for any p. Thus, $\frac{\partial p^*_\beta}{\partial \beta} < 0$ if and only if $\frac{\partial \zeta(p,\beta)}{\partial p}|_{p=p^*_\beta} < 1$. Note that $\frac{\partial \zeta(p,\beta)}{\partial p}|_{p=p^*_\beta} = \frac{2\rho f\left(2p^*_\beta - z(\beta)\right)}{1+\rho\int_{-z(\beta)}^{2p^*_\beta - z(\beta)} f(\theta)}\left(p^*_\beta - z(\beta)\right)$. This implies that if there is β' for which $p^*_{\beta'} \leq z(\beta')$ then since $z(\beta)$ increases in β, for any $\beta > \beta'$ we have $p^*_\beta \leq z(\beta)$. For $\beta = z^{-1}(-\underline{\tau})$ we know from Proposition 1 that $p^*_{z^{-1}(-\underline{\tau})} = z(z^{-1}(-\underline{\tau})) = -\underline{\tau}$ and hence for any $\beta > z^{-1}(-\underline{\tau})$ we have $p^*_\beta \leq z(\beta)$ and $\frac{\partial p^*(\beta)}{\partial \beta} < 0$. This implies that γ^* increases with β.

Last, we show that when $\beta > z^{-1}(-\underline{\tau})$ then γ^* strictly increases in ρ. If $\beta \geq \beta_0$ then $\gamma^* = z(\beta)$ and is independent of ρ. Suppose $\beta \in (z^{-1}(-\underline{\tau}), \beta_0)$. By the implicit function theorem, $\frac{\partial p^*(\rho)}{\partial \rho} = -\frac{-\frac{\partial \zeta(p,\beta)}{\partial \rho}|_{p=p^*_\beta}}{1-\frac{\partial \zeta(p,\beta)}{\partial p}|_{p=p^*_\beta}}$ and $\frac{\partial \zeta(p,\beta)}{\partial \rho} < 0$. Therefore, $\frac{\partial p^*(\rho)}{\partial \rho} < 0 \Leftrightarrow \frac{\partial \zeta(p,\beta)}{\partial p}|_{p=p^*_\beta} < 1$. But we have shown above that $\frac{\partial \zeta(p,\beta)}{\partial p}|_{p=p^*_\beta} < 0$ for $\beta > z^{-1}(-\underline{\tau})$ which completes the proof. ■