

Course Summary: Credit Risk Models

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Note This is a *brief* summary of the main points covered in the Credit Risk Models material. It is not meant to be either comprehensive or to be a substitute for that material.

1 Introduction

1. Two main categories of credit risk models:
 - (a) Structural Models:
 - i. View equity and debt as contingent claims on the assets of a firm.
 - ii. Price these securities using techniques from option pricing theory.
 - iii. Implementation is carried out using equity market information (equity prices and volatility).
 - (b) Reduced-form models.
 - i. Posit a default process directly, possibly without reference to an underlying firm value process.
 - ii. parameters of the process are estimated from/calibrated to debt market instruments (bond spreads, CDS spreads).

2 Structural Models

1. Equity and debt are contingent claims on firm asset value.
2. A structural model involves three components:
 - (a) Process driving evolution of firm value.
 - (b) Initial capital structure of the firm.
 - (c) Conditions determining the default event and what happens in the event of default.

3. The Merton model:

- (a) Firm asset value V evolves according to a lognormal process.
- (b) Firm has only two securities outstanding:
 - i. Equity.
 - ii. Zero-coupon debt with face value D and maturity T .
- (c) Default can occur only at T .
 - i. Default occurs if $V_T < D$. Debt holders receive V_T and equity holders receive zero.
 - ii. If $V_T \geq D$, there is no default. Debt holders receive D , equity holders receive the residual amount $V_T - D$.

4. So debt holders' payoff is $\min\{V_T, D\}$.

5. This is equivalent to

$$D - \max\{D - V_T, 0\}.$$

That is, risky debt is equivalent to riskless debt minus a put option on firm value with strike D and maturity T .

6. The value of the put determines the spread on risky debt: anything that increases the value of the put increases the spread.

- (a) As an option on firm value, the value of the put depends on, among other things, current firm value V and the volatility of firm value σ , riskless interest rates r , maturity T , and the strike (of the put) D .
- (b) A change in any of these parameters affects put values, so affects spreads. For example:
 - i. If σ increases, the value of the put increases, so spreads increase.
 - ii. If V increases, the value of the put decreases, so spreads decrease.
 - iii. If r increases, the value of the put decreases, so spreads decrease.

7. Default is the event of the put being exercised. So the likelihood of default is the likelihood the put finishes in-the-money.

8. The driving variables in the Merton model are the firm value V and firm volatility σ , However, these are *unobservable*. So how do we implement the model? Answer: use equity prices.

- (a) Equity is a call option on firm value with strike D and maturity T .

- (b) The value E of equity and the volatility σ_E of equity are both functions of V and σ .
 - (c) So from observed values of E and σ_E , we can back out V and σ .
9. The KMV implementation of Merton's model proceeds in 4 steps:
- (a) Define the *default point* D to be equal to the firm's short-term debt plus one-half the firm's long-term debt.
 - (b) Use the default point D together with the firm equity value E and equity volatility σ_E to back out firm value V and firm asset volatility σ .
 - (c) Calculate the *distance-to-default* δ :

$$\delta = \frac{V - D}{\sigma V}$$

- (d) Use the empirical database to identify of all firms who were a distance δ from default, how many defaulted within a year. This is the firm's *expected default frequency* or EDF.
10. For example, suppose we have identified the following values: $D = 80$ million, $V = 190$ million, and $\sigma = 0.20$. Then, the firm's distance-to-default is

$$\delta = \frac{190 - 80}{0.20 \times 190} = \frac{110}{38} = 2.895.$$

11. Empirical investigation has shown that distance-to-default is a very good indicator of distress with a *cumulative accuracy profile* of 65%-90%.

3 Reduced-Form Models

1. Reduced-form models posit a default process directly without necessarily deriving it from an underlying firm value process.
2. There are two broad approaches to reduced-form usage:
 - (a) One is to see what default probabilities are consistent with observed spreads at each maturity, and use this to identify a term-structure of default probabilities.
 - (b) The other is to posit a specific functional form for the intensity process driving default (as a function of firm-specific and perhaps macro variables) and to estimate the parameters of the functional form from the data.

3. The former approach is commonly used in industry and by trading desks to see the behavior of default probabilities at a point in time. The approach does not enable us to determine why or how these probabilities change from day to day, since the factors driving the default process are not modeled.
4. The latter approach is commonly used in academic research to identify the factors driving the default process and how they matter.
5. Our focus in this study guide mainly on the former approach.
6. A one-period example:

(a) Define the following notation:

- i. The risk-free one-period rate of interest is r
- ii. The spread on a one-year risky bond (face value \$1) is s .
- iii. The price of the risky bond is B^* .
- iv. The probability of default is p_1 .
- v. The recovery rate in the event of default is δ .

(b) The expected payoff from the bond over the one period is

$$(1 - p_1) \cdot 1 + p_1\delta.$$

(c) The present value of this payoff is

$$\frac{1}{1 + r} [1 - p_1 + p_1\delta].$$

(d) This present value must be equal to the bond price B^* .

(e) But the bond price is also equal to the promised payoff discounted at the yield of $r + s$:

$$B^* = \frac{1}{1 + r + s}$$

(f) So we have

$$\frac{1}{1 + r + s} = \frac{1}{1 + r} [1 - p_1 + p_1\delta]. \tag{1}$$

(g) Equation (1) enables us to back out the probability of default from the bond spread s (given an assumption about the recovery rate δ):

$$p_1 = \frac{1}{1 - \delta} \left(1 - \frac{1 + r}{1 + r + s} \right).$$

- (h) For example, suppose $r = 2.50\%$, $s = 50$ basis points, and $\delta = 0.50$. Then, the implied one-year probability of default is

$$p_1 = \frac{1}{0.50} \left(1 - \frac{1.025}{1.030} \right) = 0.0097$$

or about 0.97%.

- (i) An approximation to this default probability is obtained using the so-called “credit triangle” which states that

$$s \approx p_1(1 - \delta),$$

or, in words,

$$\text{Bond spread} = \text{Prob of default} \times \text{Loss-given-default}$$

The credit triangle is obtained from equation (1) by using the approximation

$$\frac{1 + r}{1 + r + s} \approx 1 - s.$$

- (j) If we had used the credit triangle in the example, we would have obtained

$$p_1 \times (1 - 0.50) \approx 0.005,$$

so $p_1 \approx 0.01$ or 1%, pretty close to the value of 0.97%.

7. These ideas may easily be extended to two periods and beyond.
- (a) Once we know the one-period probability of default, we can use this in conjunction with two-period bond spreads to identify the probability of default in the second period.
 - (b) Bootstrapping similarly, we can identify the probabilities of default in the third period, fourth period, and so on.
 - (c) For details of the procedure and examples, see the slides.
8. Backing out default probabilities from CDS spread information involves a similar reasoning procedure. As an example, suppose we are given a one-year CDS spread of s . Assume that:
- (a) The spread is paid at the beginning of the year.
 - (b) If there is a default, any payoffs from the CDS are received at the end of the year.

- (c) The recovery rate on the reference obligation is δ .
 - (d) The one-year risk-free rate is r .
9. Let p_1 denote the one-year probability of default (to be identified).
 10. In a “fairly priced” CDS, the present value of the premium payments made must be equal to the present value of the protection received.
 11. The present value of the premium payments in this one-period model is simple:

$$PV(\text{Premium leg}) = s.$$

12. The protection leg pays $1 - \delta$ if there is a default, and zero if there is no default. Any payment is received at the end of the year. So the present value of the protection payments is

$$PB(\text{Protection leg}) = \frac{1}{1+r} p_1(1 - \delta).$$

13. Equating these present values, we can solve for p_1 from knowledge of r , s , and δ .
14. For example, suppose $r = 3.60\%$, $s = 60$ basis points, and $\delta = 0.4$. Then, the implied one-year probability of default is obtained by solving

$$0.006 = \frac{1}{1.036} \times p_1 \times 0.60,$$

or $p_1 = 0.01036$ or 1.036%.

15. Again, the basic ideas are readily extended to solve for two-year and higher default probabilities from longer-term CDS spreads. The slides elaborate.