# STANDARD AND POORS

# PROBABILITY AND STATISTICS

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Aaron Tenenbein
Professor of Statistics
Faculty Director, Executive MBA Programs
Director of the Actuarial Science Program
Stern School of Business
New York University

# Topics

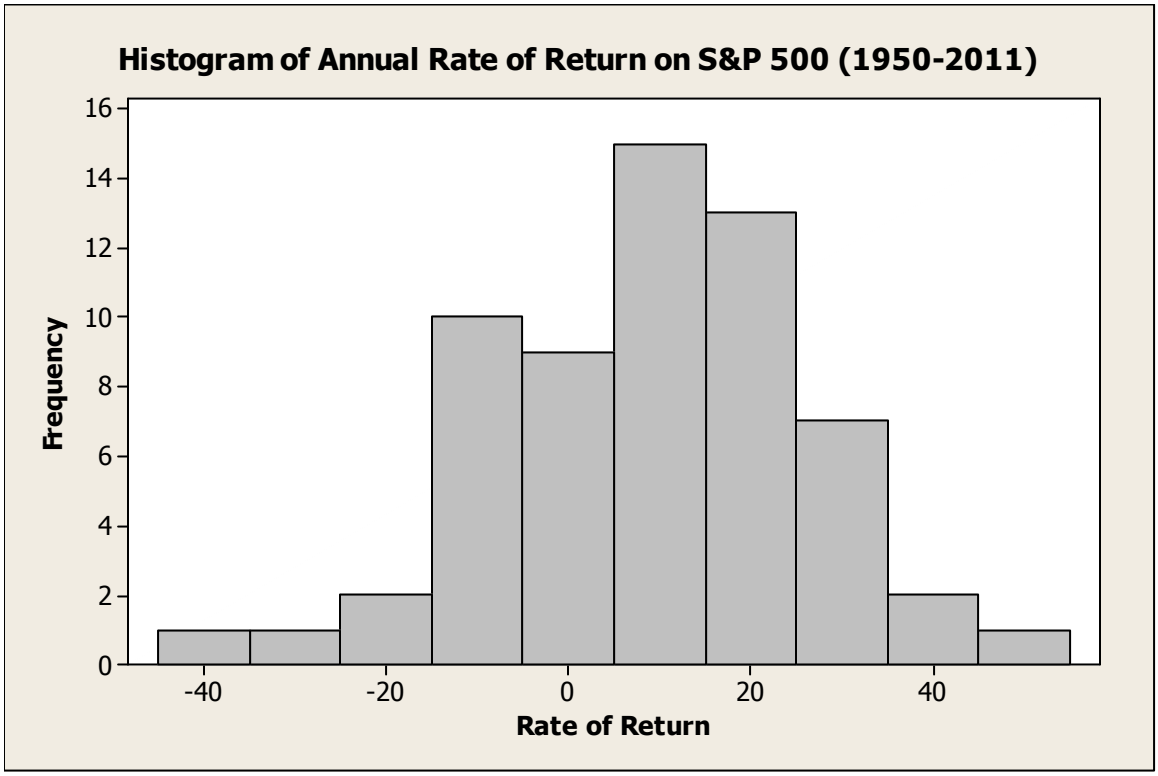
1) Normal distributions and computing probabilities with them

2) Regression models
    - Regression line
    - T-statistics
    - Std dev around the line
    - Outlier detection
    - R-square

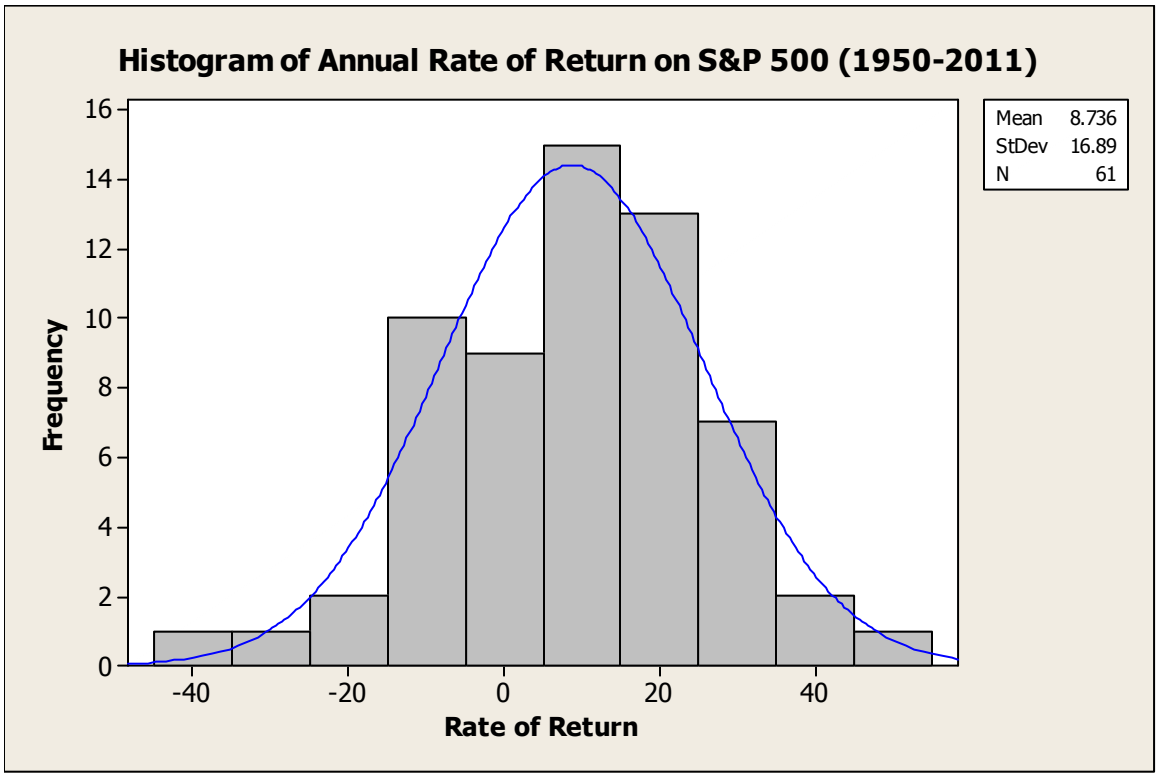
3) Brief introduction to idea of logistic regression

4) Practice examples

Annual Returns on S&P500 Index
(Jan 1950-Jan 2011)



HISTOGRAM
(Normal Curve Superimposed)

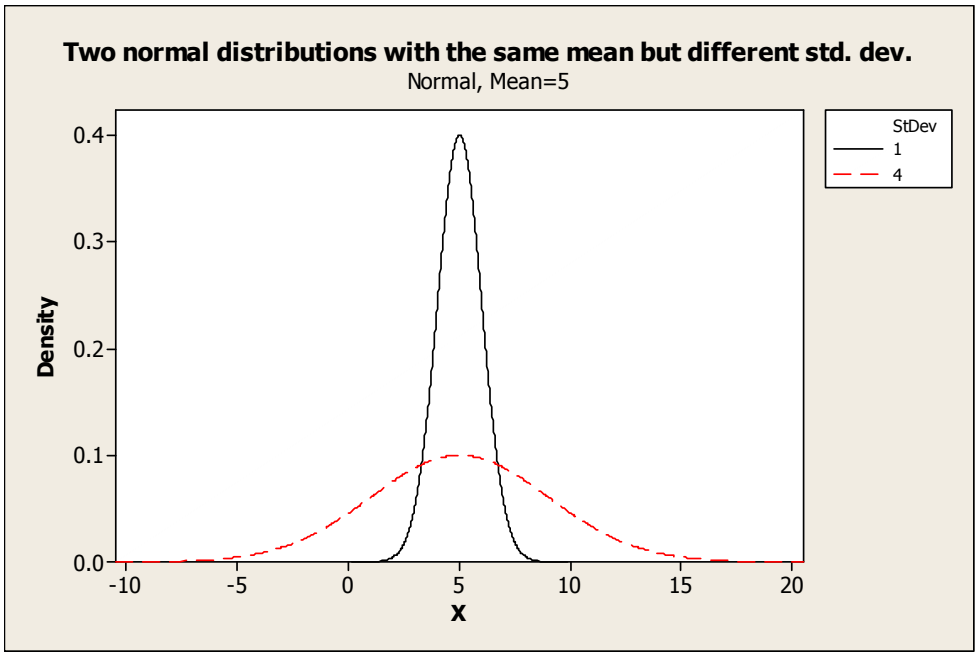


Normal Distribution (also called Gaussian distribution)

Symmetric bell-shaped curve which describes how the histogram of the values of the variable looks

Governed entirely by two numbers

1) Mean (Center of the curve)
2) Standard deviation (measures spread of the curve around the center)



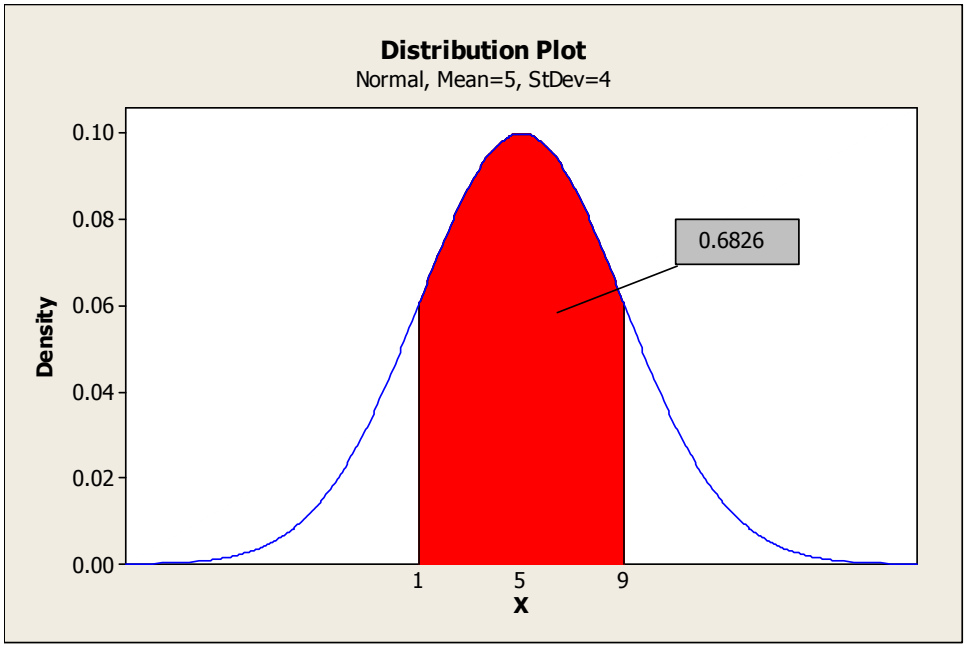

Variance: It is the square of the standard deviation. (Alternatively, the standard deviation is the square root of the variance)

Conceptually, the variance measures the same thing as the standard deviation, viz. the spread of the normal curve

Three useful properties of <u>any</u> normal curve, regardless of the value of the mean or standard deviation

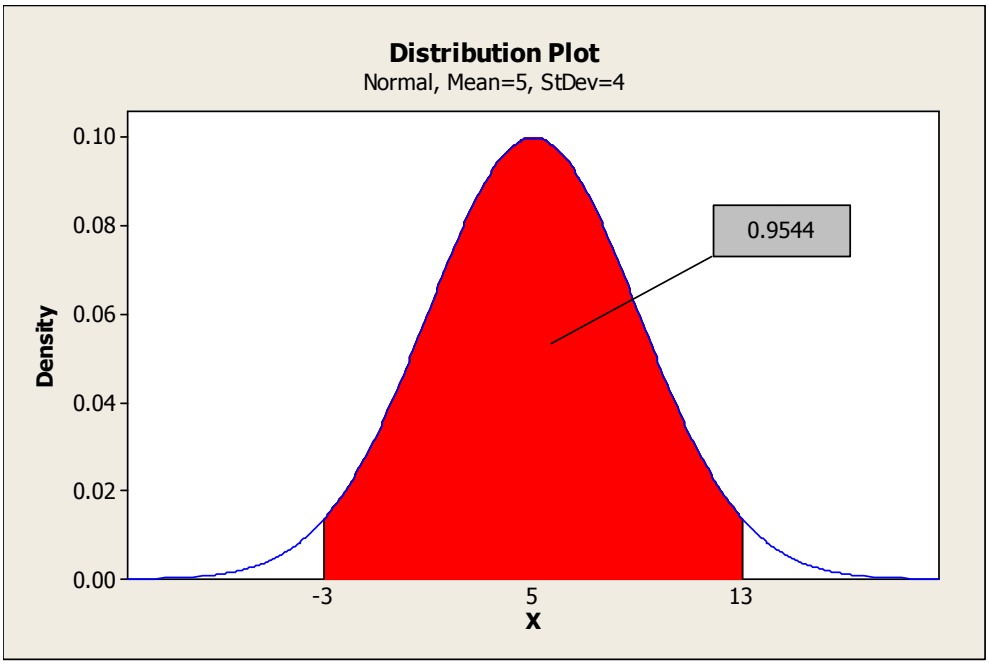
1) Approximately 68% of the values will fall within 1 standard deviation of the mean

Example: Normal curve with mean = 5 and std.dev = 4



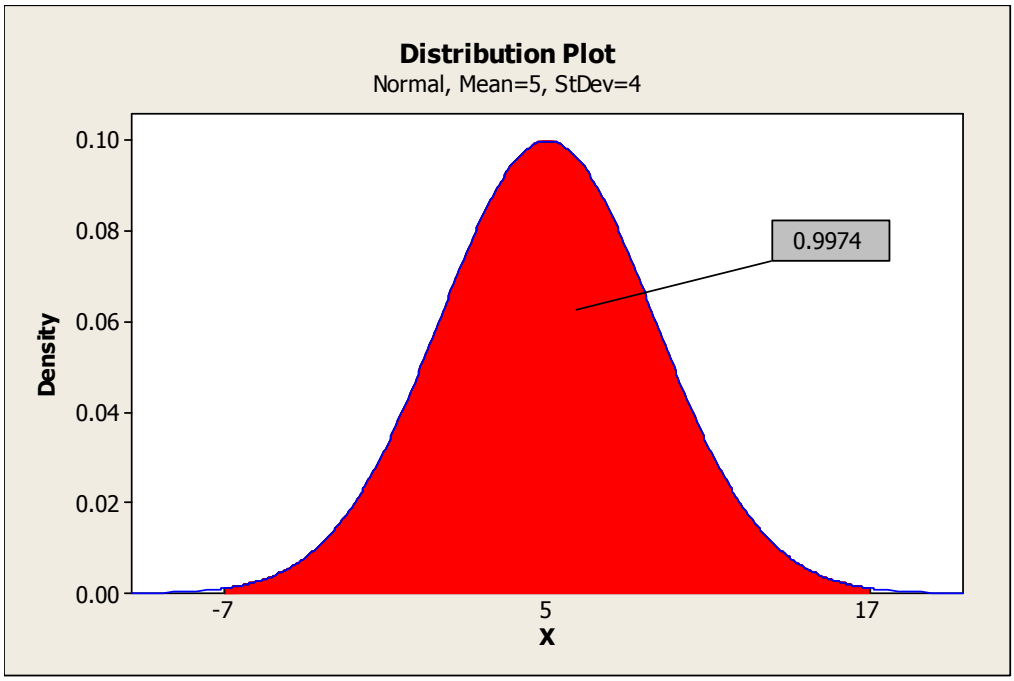
2) Approximately 95% of the values will fall within 2 standard deviations of the mean

Example: Normal curve with mean = 5 and std.dev = 4



3) Approximately all (~100%) the values will fall within 3 standard deviations of the mean

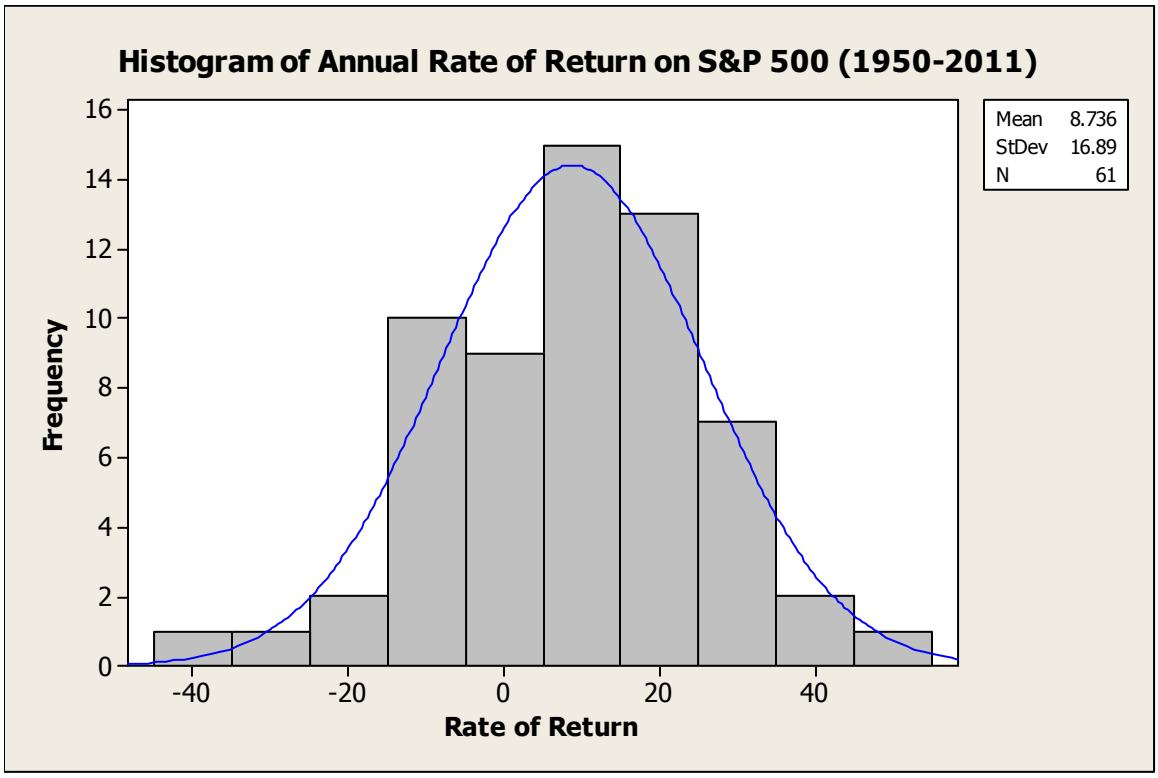
Example: Normal curve with mean = 5 and std.dev = 4



# EXAMPLE

## INFORMATION ON ANNUAL RATE OF RETURN ON THE STANDARD AND POORS INDEX (1950-2011)

**Mean = 8.74%**
**Median = 9.44%**
**Standard Deviation = 16.89%**
**Minimum (2008-2009) = -35.61%**
**Maximum (1954-1955) = 47.29%**
**% within 1SD of the mean = 62% (Normal is 68%)**
**% within 2SD of the mean = 95% (Normal is 95%)**
**% within 3SD of the mean = 100% (Normal is 99.74%)**
**% beyond 3SD of the mean = 0% (Normal is 0.26%)**
**Coefficient of Skewness = - 0.30 (Normal Value =0)**
**Coefficient of Kurtosis = - 0.10 (Normal Value = 0)**



What if we are interested in some other areas under the normal curve?

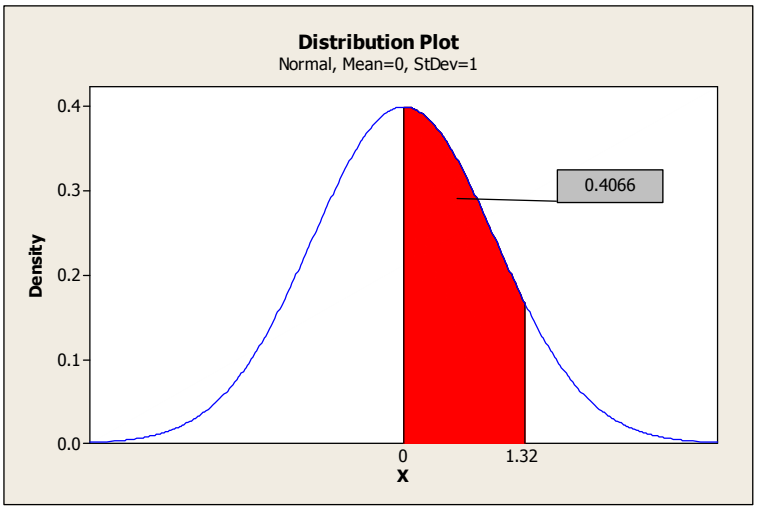
## Standard Normal Distribution

A normal distribution with mean = 0 and standard deviation = 1 (Hence, variance = 1)

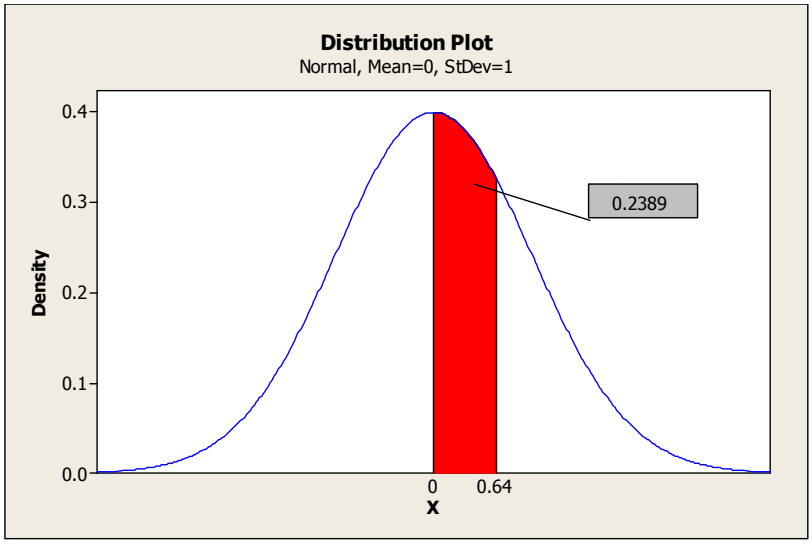
Denoted by the letter Z

Using the Z Table

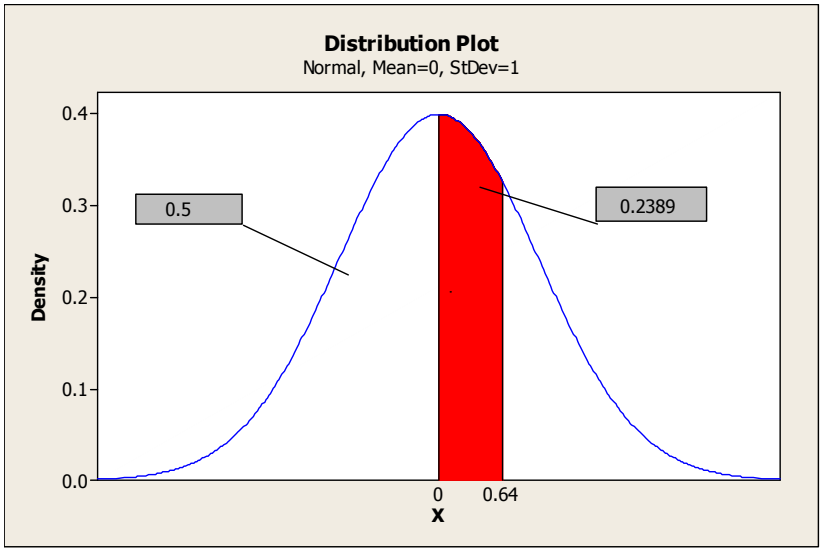
1) P(0<Z<1.32) = ?



2) P(0<Z<0.64) = ?

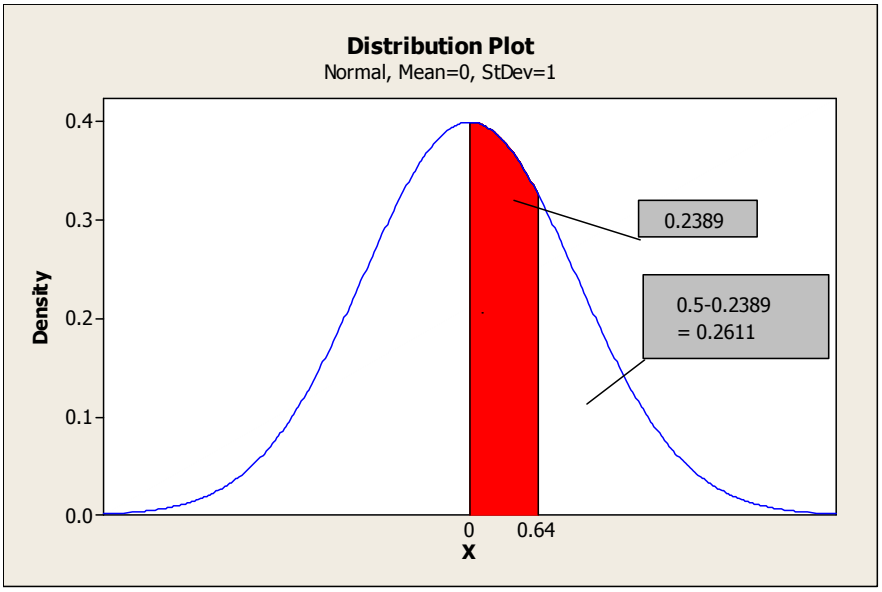



3) P(Z<0.64) = ?



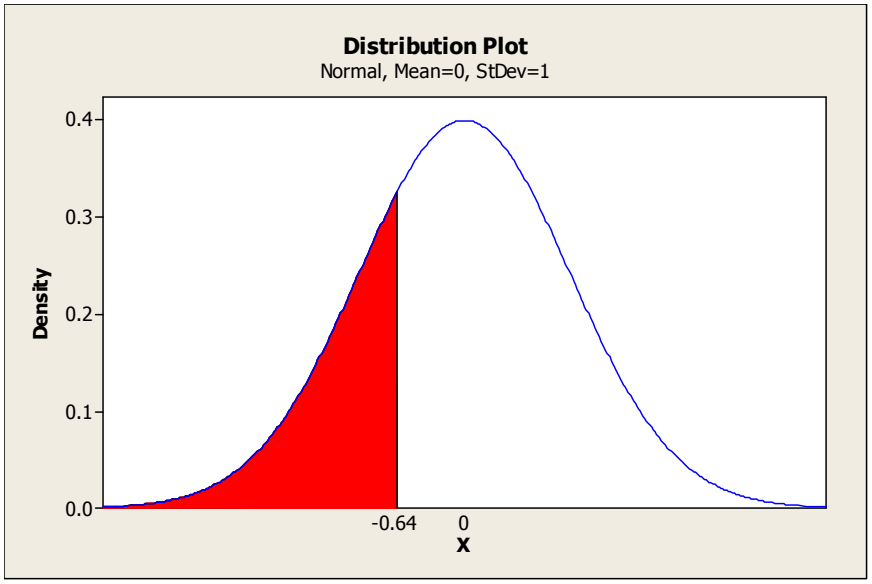

Thus, P(Z<0.64) = 0.5 + 0.2389 = 0.7389

4) P(Z>0.64) = ?




Thus, P(Z>0.64) = 0.5 – 0.2389 = 0.2611

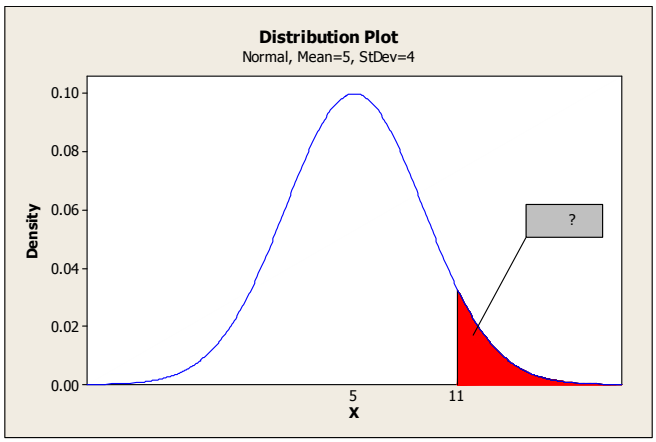
5) P(Z< -0.64) = ?




P(Z< -0.64) = P(Z>0.64) = 0.5 – 0.2389 = 0.2611

Computing probabilities for arbitrary normal distributions

<u>Example 1</u>: Let R denote the annual return on the S&P 500 index and assume that it is normally distributed with mean 5% and standard deviation 4%. What is P(R > 11%) ?

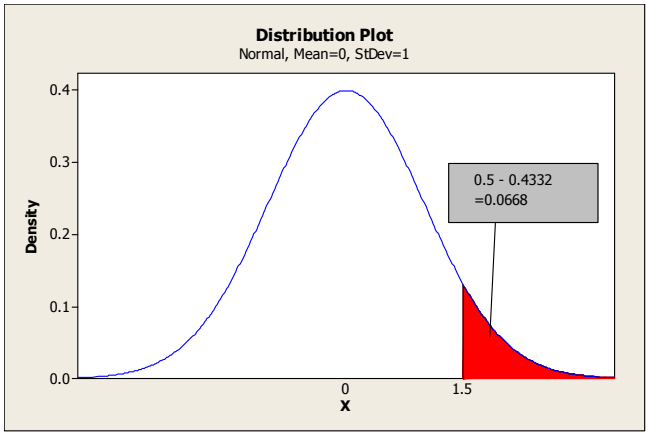



The Z-value: $Z = (R-5)/4 = (11-5)/4 = 1.5$

In general, the Z value is computed as

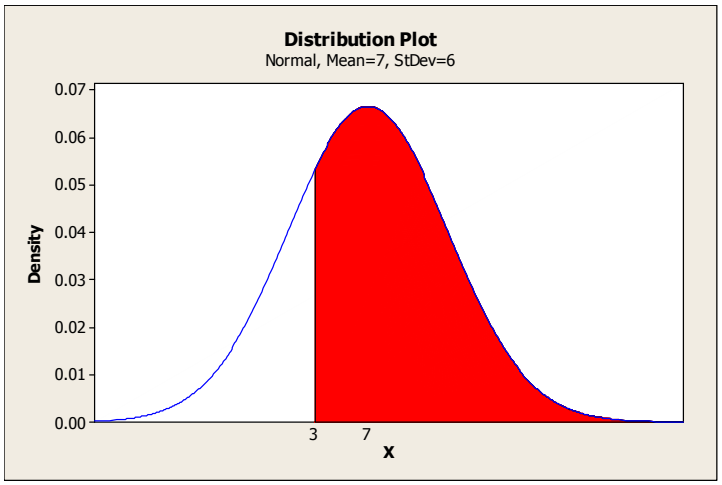
$$Z = (\text{Variable} - \text{Mean})/\text{Std. deviation}$$

The Z values follow the standard normal distribution which enables us to compute probabilities of interest using the Z table

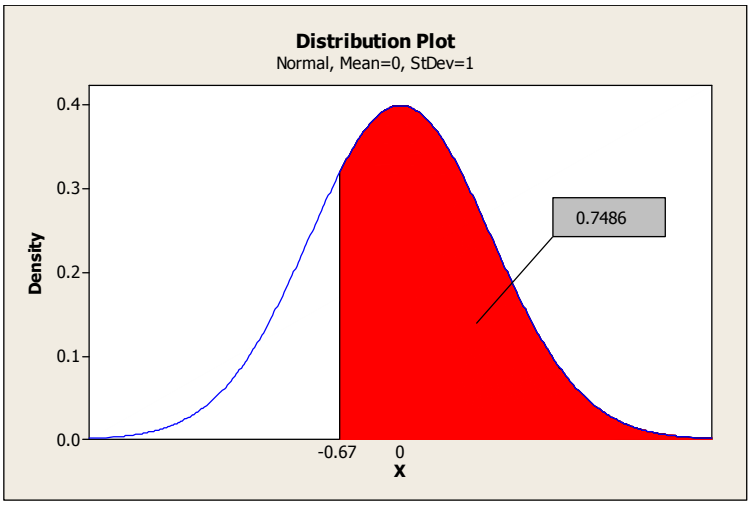



<u>Solution of Example 1</u>: Thus, P(R>11%) = P(Z>1.5) = 0.0668

Example 2: Let R denote the annual return on a mutual fund and assume that it is normally distributed with mean 7% and standard deviation 6%. What is P(R > 3%) ?

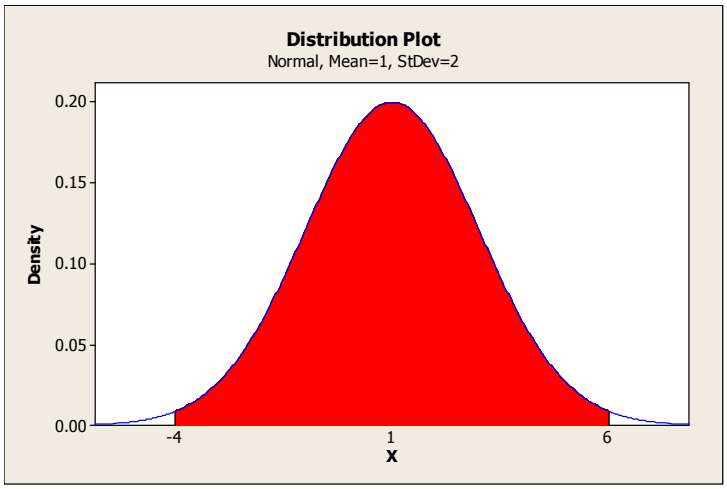



Z value =(R-7)/6 = (3-7)/6 = -0.67

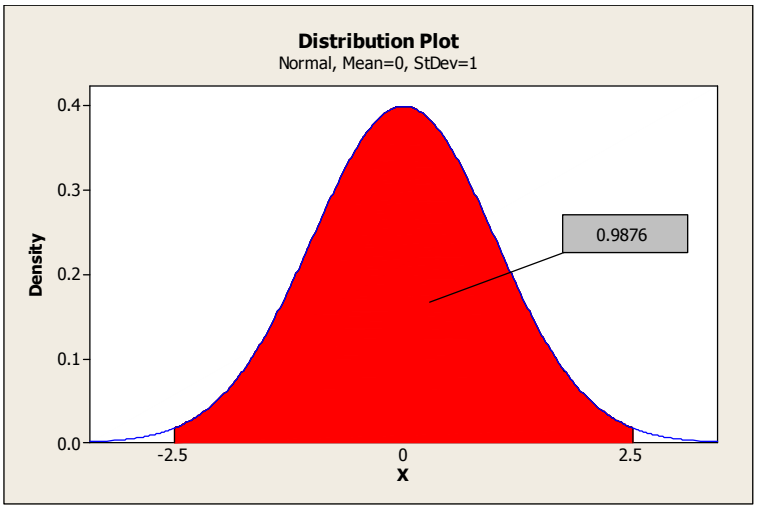



Solution of Example 2: Thus, P(R>3) = P(Z> -0.67) = 0.7486

Example 3: Let R denote the annual return on a mutual fund and assume that it is normally distributed with mean 1% and standard deviation 2%. What is P(-4%<R < 6%) ?




Z-values: (R-1)/2 = (-4-1)/2 = -2.5 and (R-1)/2 = (6-1)/2 = 2.5



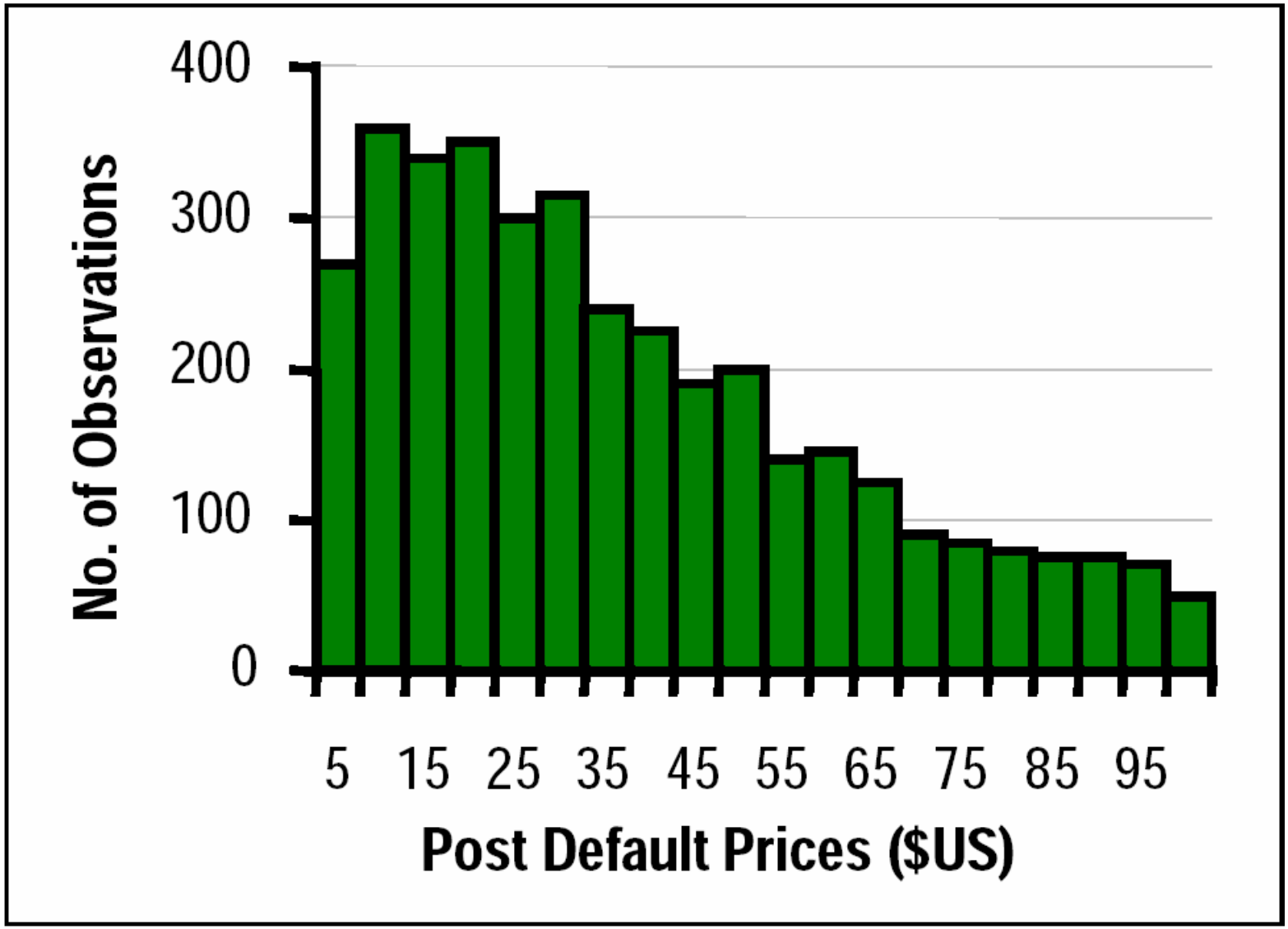

Solution of Example 3: P(-4<R<6) = P(-2.5<Z<2.5) = 0.9876

Outliers: Values which are unusually large or unusually small with respect to the rest of the values in a data set

If we believe that our variable is well described by a normal curve, then a commonly used rule to detect/describe outliers is as follows:

> *A value that is more than 3 standard deviations from the mean is an outlier*

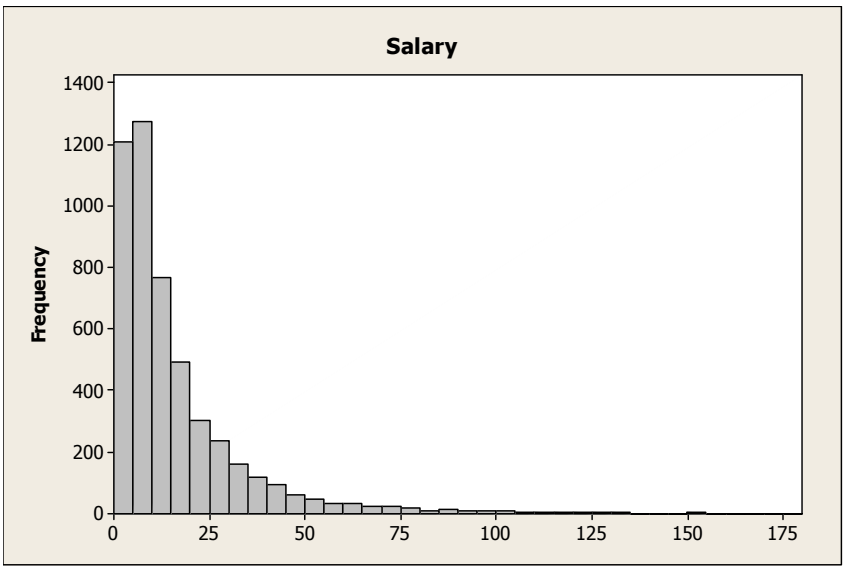
Example: It is known from experience that 1st quarter revenues of companies in the pharmaceutical sector are normally distributed with mean \$8 million and standard deviation \$2 million
The pharmaceutical company ABC posts a 1st quarter revenue of \$16 million. Since the value of 16 is more than 3 standard deviations above the mean of 8, ABC's 1st quarter revenue is an outlier (on the high side)




Distribution of Recovery Rates for Straight Bond Issues, 1982-2002
Source: Moody's, ML

Things to be careful of when dealing with non-normal distributions

A. Not all variables that one encounters in practice are described by normal curves. Sometimes, the distributions of positive variables (example, annual salary) may be right skewed




Mean = 17 Median 10 Std. Dev = 22

*i) The mean will overstate the "central" value (But median will be robust to the lack of symmetry)*

*ii) The normal distribution statement of 2 or 3 std. deviations from the mean can be inaccurate*

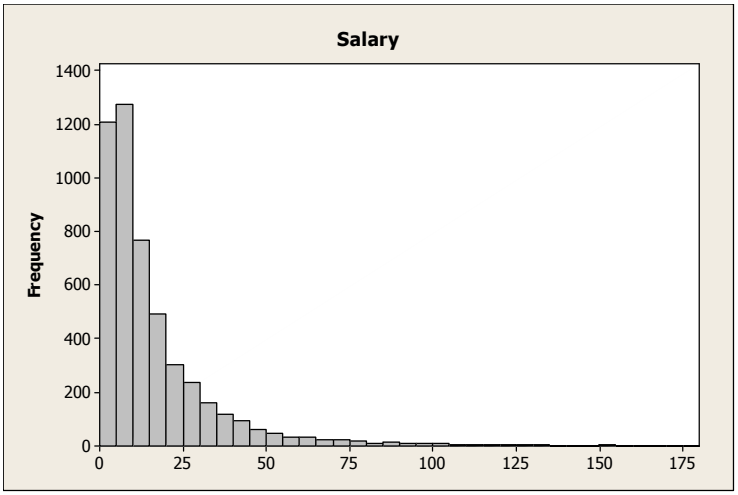
In the above example, Mean+3*std.dev = 83

However, approx. 2% of the observations are larger than 83

## Potential "fix" for positive variables that are not normally distributed

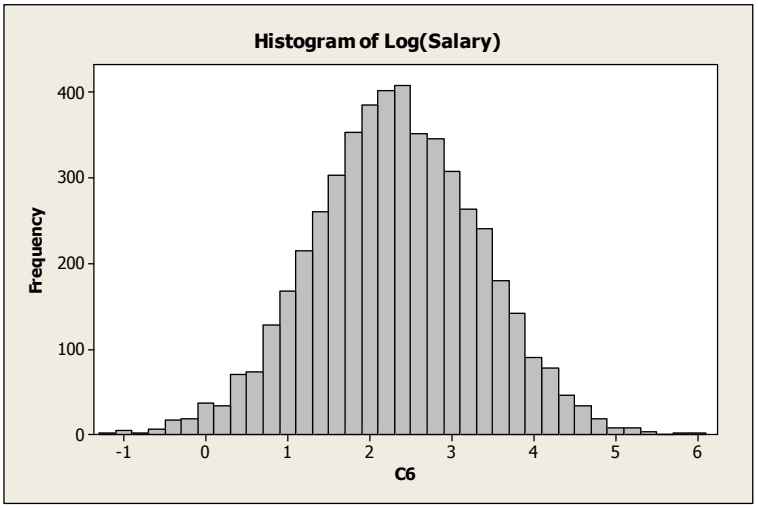
Convert data to logarithms, which has the effect of "shrinking" large values

| x | Log(x) |
|---|---|
| 5 | 1.6094 |
| 50 | 3.9120 |
| 5000 | 8.5172 |
| 5000000 | 15.4249 |




Log-Normal Distribution

Histogram of log(salary) values

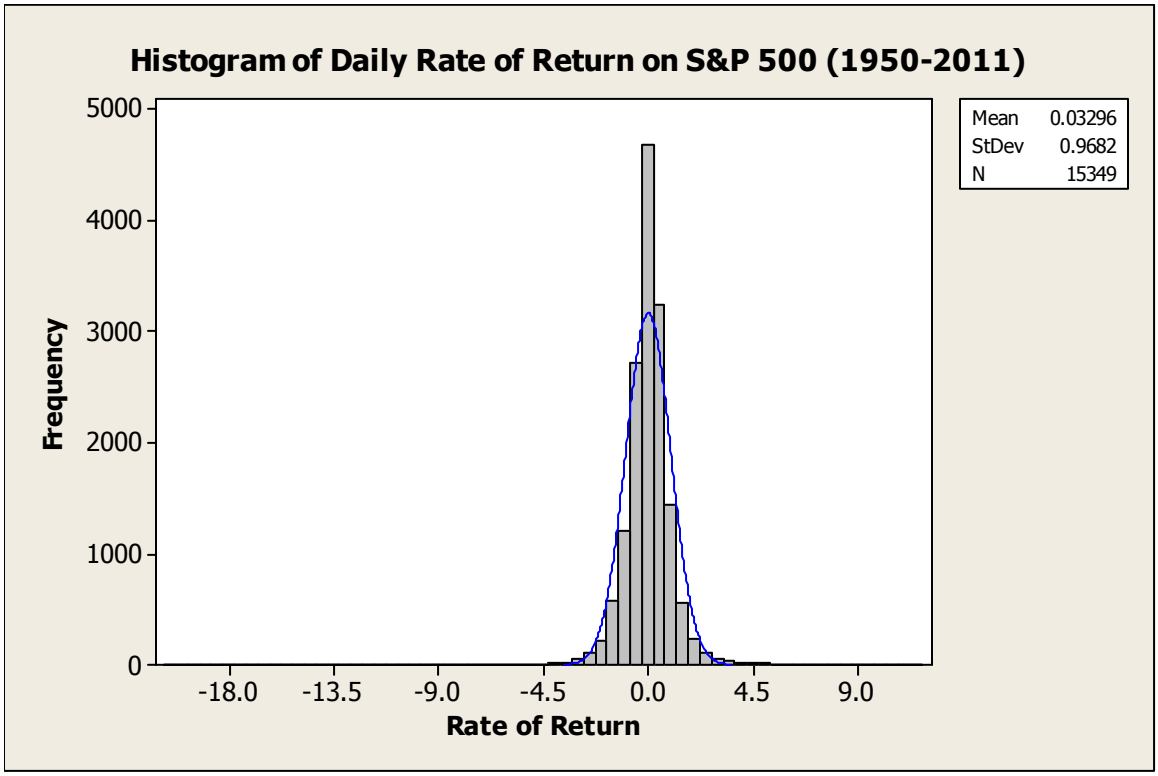


Things to be careful of when dealing with non-normal distributions

B. Not all variables that one encounters in practice are described by normal curves. Sometimes, the distributions of variables (example, daily rates of return of financial instruments) may be symmetric but are subject to large outliers on either side. A measure of this tendency is called Kurtosis. A distribution with a high level of Kurtosis is sometimes referred to as a heavy tailed distribution.

**INFORMATION ON DAILY RATE OF RETURN ON THE STANDARD AND POORS INDEX (1950-2011)**

**Mean = 0.033%**
**Median = 0.046%**
**Standard Deviation = 0.968%**
**Minimum (10/16/1987) = -20.47%**
**Maximum (10/10/2008) = 11.58%**
**% within 1SD of the mean = 79% (Normal is 68%)**
**% within 2SD of the mean = 95% (Normal is 95%)**
**% within 3SD of the mean = 98.6% (Normal is 99.74%)**
**% beyond 3SD of the mean = 1.4% (Normal is 0.26%)**
**Coefficient of Skewness = -0.66 (Normal Value =0)**
**Coefficient of Kurtosis = 22.30 (Normal Value =0)**




This is a case where the normal distribution is not valid. The curve is fairly symmetrical, but there is a lot of Kurtosis, because of the high percentage of outliers. 1.4% of the daily returns are more than 3 standard deviations away from the mean which is more than 5 times as many that would be expected to appear in a normal distribution.

# Regression

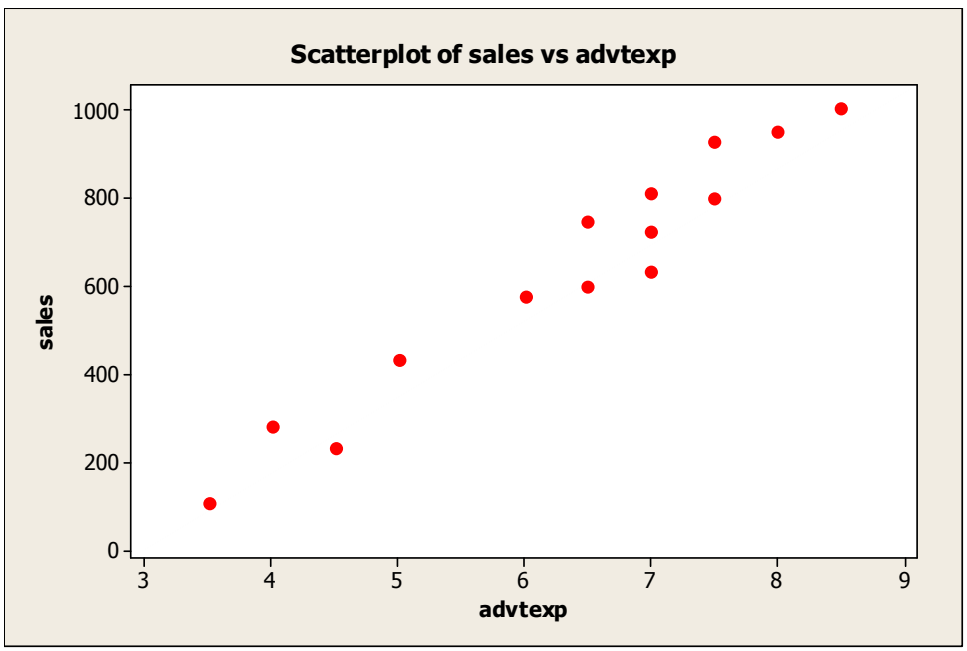
Modeling the relationship between the variable of interest and other variable(s)

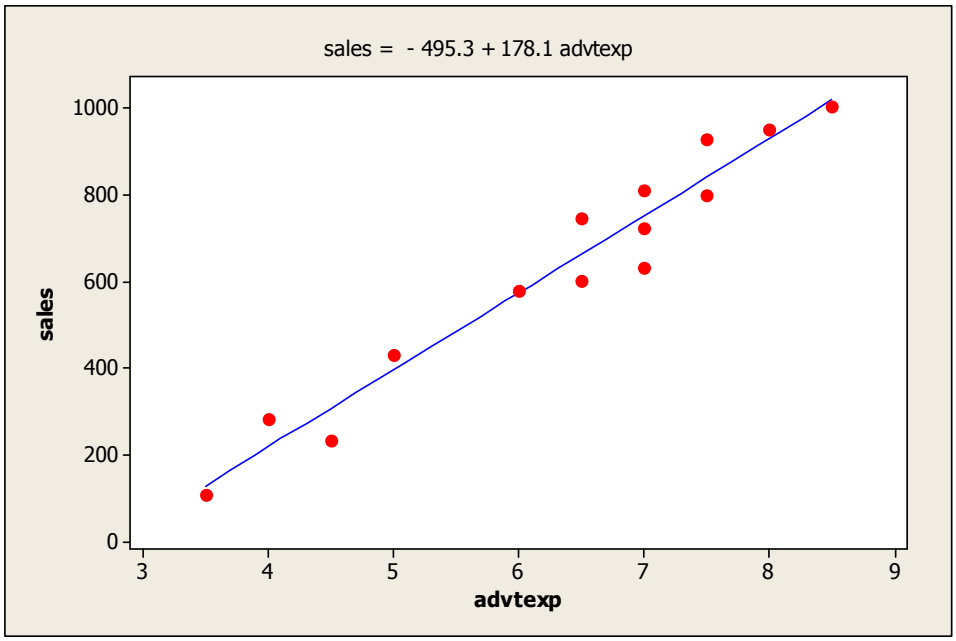
We will look at some useful numbers that are provided by a regression model and study how to interpret/use these numbers

a) Regression line
b) T-statistic of the coefficient
c) Standard deviation of values around the line
d) R-square

Advertising Data in 1000's of dollars

| No. | Advt Exp (X) | Sales (Y) |
|---|---|---|
| 1 | 5 | 432 |
| 2 | 7 | 723 |
| 3 | 6 | 578 |
| 4 | 6.5 | 600 |
| 5 | 8 | 950 |
| 6 | 3.5 | 106 |
| 7 | 4 | 282 |
| 8 | 4.5 | 232.6 |
| 9 | 6.5 | 746 |
| 10 | 7 | 812.4 |
| 11 | 7.5 | 800 |
| 12 | 7.5 | 929 |
| 13 | 8.5 | 1004 |
| 14 | 7 | 632 |






Sales = -495 + 178*AdvtExp

*The regression line provides a "benchmark" value (a prediction) for sales if you provide a value of Advt Expenditure*

<u>Example</u>: If Advt Exp = 5.5, then the regression line predicts that the sales value is

Sales = -495 + 178*(5.5) = 484

*The regression line allows you to back out the X value (Advt Exp) that corresponds to a given value of Y (Sales)*

<u>Example</u>: If Sales = 620, then from the regression line, the value of Advt Exp that corresponds to Sales = 620 is found as follows

620 = -495 + 178*(Advt Exp)

1115 = 178*(Advt Exp)

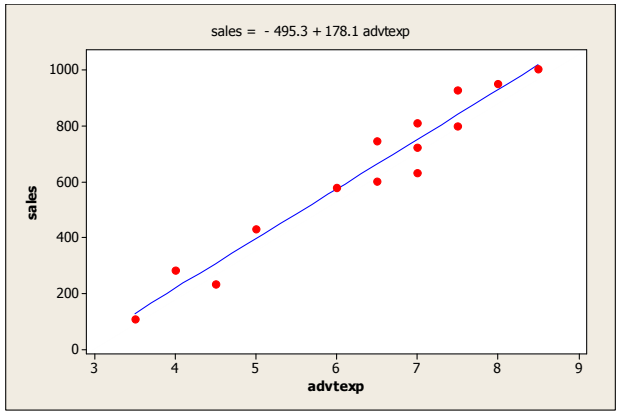
Thus, Advt Exp = 1115/178 = 6.26

*<u>The units in which the data have been measured affects the magnitude of the coefficients in the regression line</u>*

Our sales/Advt exp data is in thousands $

| No. | Advt Exp (X) | Sales (Y) |
|---|---|---|
| 1 | 5 | 432 |
| 2 | 7 | 723 |
| 3 | 6 | 578 |
| . | . | . |

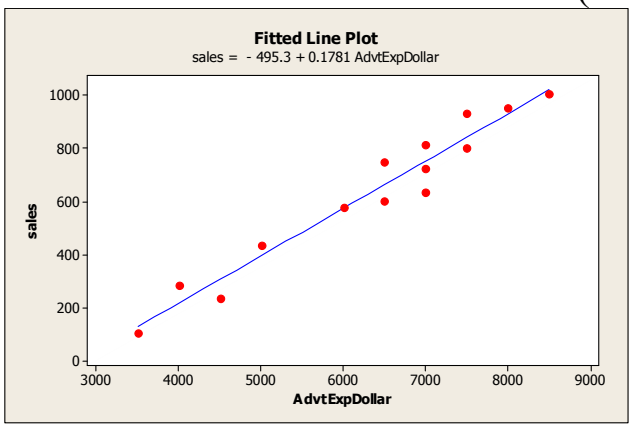
Regression Line from this data is Sales = -495 + 178*AdvtExp




Supposing the Advt Exp data had been entered in $ amounts

| No. | Advt Exp (X) | Sales (Y) |
|---|---|---|
| 1 | 5000 | 432 |
| 2 | 7000 | 723 |
| 3 | 6000 | 578 |
| . | . | . |

Regression from this data Sales= -495+0.178*(Advt Exp in $)



*Thus, using the magnitude of the coefficient in the regression model to verify whether the X variable (Advt exp) is useful in predicting the Y variable (sales) is misleading*

Solution: Look at the t-statistic of the coefficient. The t-statistic is a "unit-free" number

Computed as: t-statistic = Coefficient/Fudge Factor

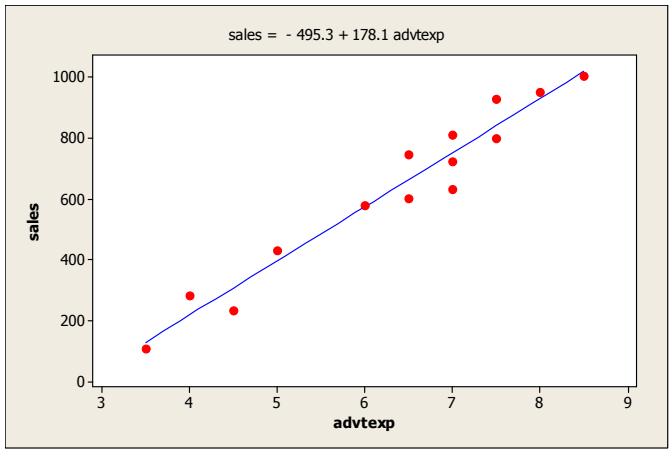
In the Sales and Advt. Exp example, the t-statistic of the coefficient of Advt. Exp is 15.

The value of 15 for the t-statistic remains *unchanged, regardless* of whether Advt Exp is measured in *thousands of $* or in *just $*

In practice, t-statistic values outside (-2,2) are considered to indicate that the corresponding coefficient is sufficiently different from zero and that the X variable is useful to predict the Y variable

*The standard deviation of the Y values around the regression line*

Sales and Advt Exp data




Sales = -495 + 178*AdvtExp

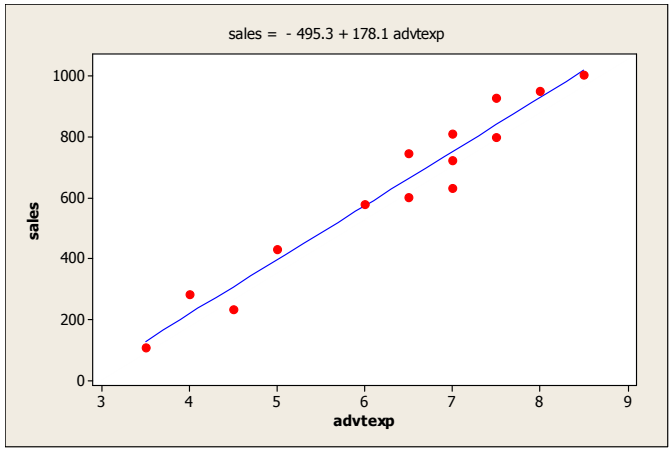
The standard deviation of the Sales values around the regression line is 65

This standard deviation, in conjunction with the line and the normal distribution

a) gives us an idea of the range of Y (sales) values to expect for a given value of X (advt exp)

b) helps us identify unusual observations (outliers)

*Standard deviation around the line (contd)*




Sales = -495 + 178*AdvtExp

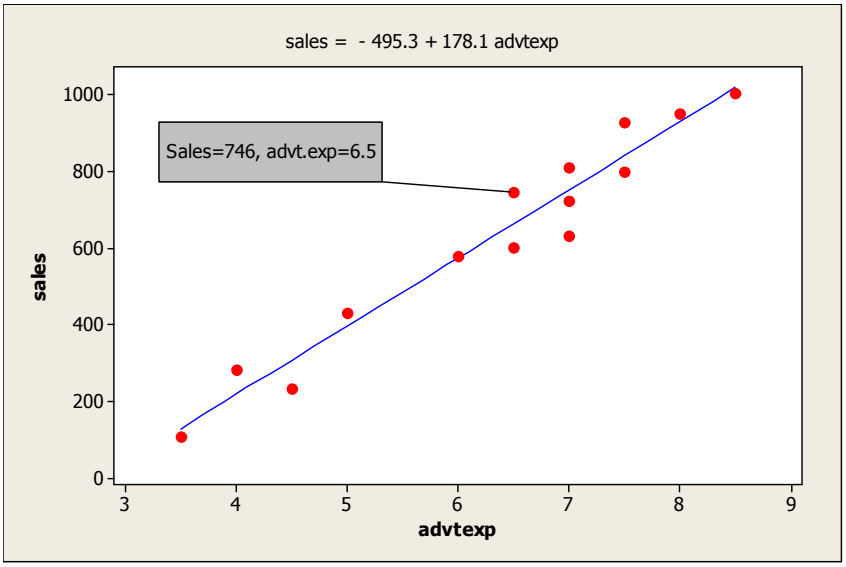
The standard deviation of the Sales values around the regression line is 65

For X (advt exp)=5.5, we saw that the predicted (benchmark) value of Y (sales) is = -495 + 178*(5.5) = 484

Thus, the standard deviation of 65, in conjunction with the normal curve tells us that when Advt Exp =5.5, 95.44% of the values of sales will fall in the range (484-2*65, 484+2*65)
i.e. in the range (354, 614)

Similarly, when advt.exp=5.5, the normal curve says that 99.74% of the values of sales will fall in the range (484-3*65, 484+3*65)
i.e. in the range (289, 679)

## *Detecting outliers using the standard deviation*




Sales = -495 + 178*AdvtExp

The standard deviation of the Sales values around the regression line is 65

Consider the observation of (sales=746, advt.exp=6.5). Is the sales value of 746 unusual relative to the line at advt.exp=6.5?

We compute the Z-value for sales=746, using the regression line evaluated at advt.exp=6.5 as the mean

Regression line when advt.exp is 6.5 is=-495+178*(6.5) = 662

Z-value = (746-662)/65 = 1.29

This is not an unusually large Z value and hence the value of Sales=746, when advt.exp=6.5, is not unusually large

## *R-square Value*

Every regression output will also provide a R-square value

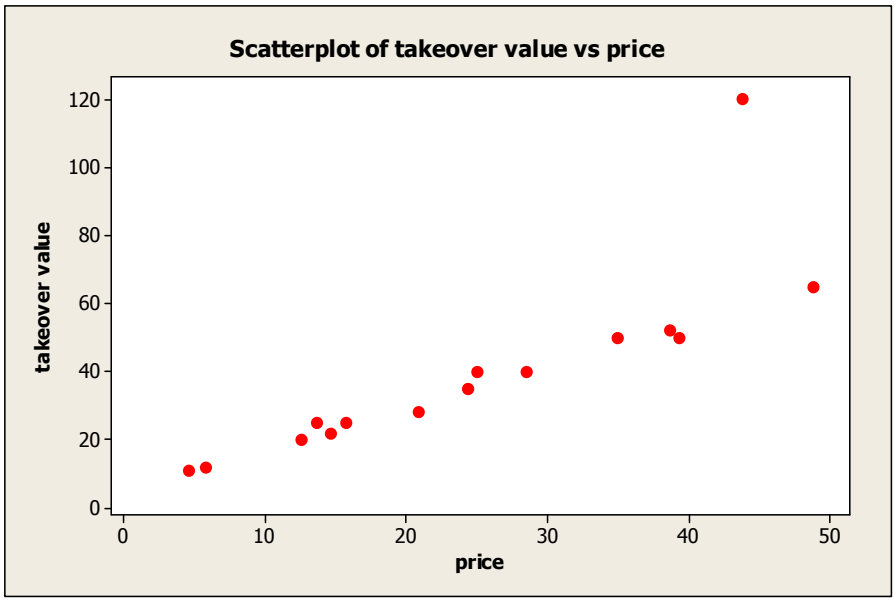
R-square is number between 0% and 100% that measures how useful the X variable is in explaining the Y variable.

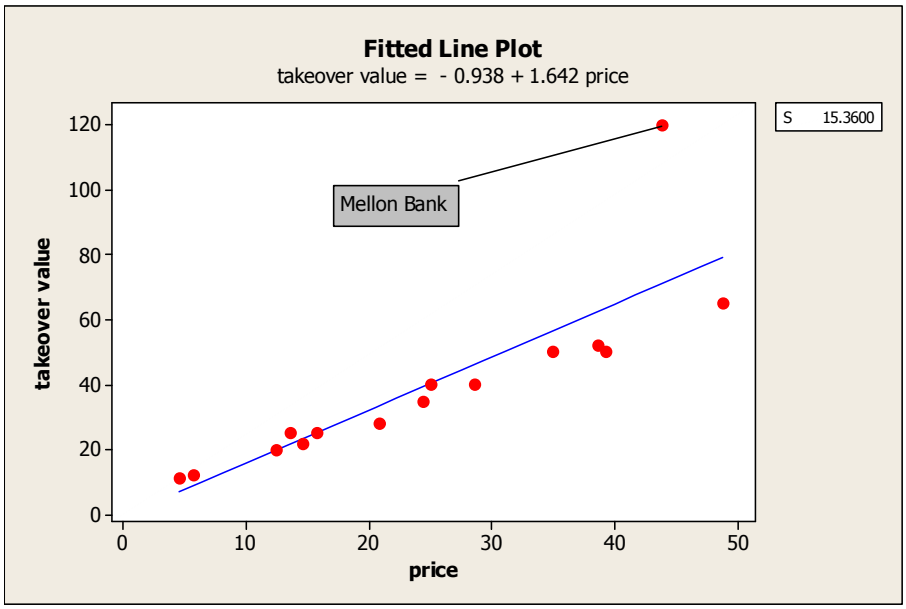
In the sales and advt exp example, the R-square is 95%

# An example of detecting Outliers in Regression

Data source: WSJ June 1997

| Stock | price | takeover value |
|---|---|---|
| Aames Financial | 13.625 | 25 |
| Alumax | 38.625 | 52 |
| Bay Networks | 24.375 | 35 |
| Bowater | 48.750 | 65 |
| Digital Equipment | 34.875 | 50 |
| Genovese Drug Stores | 15.750 | 25 |
| Georgia Gulf | 28.500 | 40 |
| Glenayre Technologies | 14.625 | 22 |
| Group Long Distance | 4.625 | 11 |
| Inland Steel | 25.000 | 40 |
| Integon | 12.500 | 20 |
| John Alden Financial | 20.875 | 28 |
| Lehman Brothers | 39.250 | 50 |
| Little Switzerland | 5.750 | 12 |
| Mellon Bank | 43.750 | 120 |






Regression line is Takeover value = -0.938+1.642*(Price)

The standard deviation of takeover values around the line is 15.36

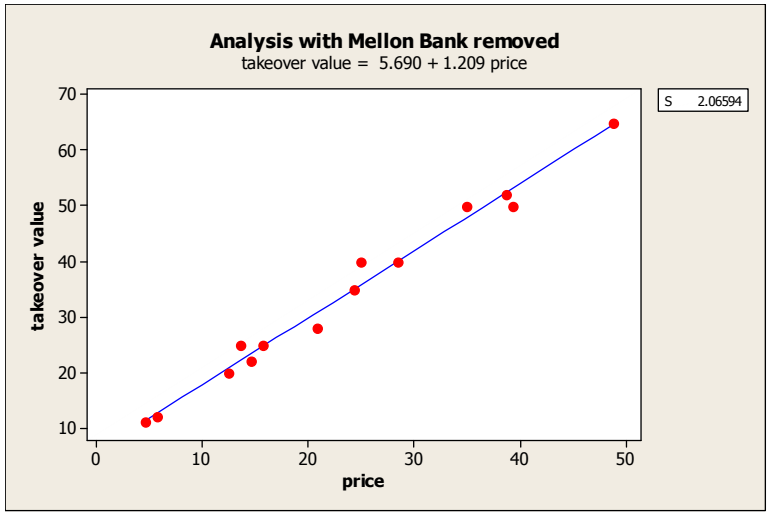
"Distance" of actual Mellon Bank takeover value from the regression line

Predicted MB takeover value = -0.938+1.642*(43.75) = 70.89

Z-value for actual MB = (120-70.89)/15.36 = 3.2

Thus, Mellon Bank is an outlier (an unusually high takeover value relative to what is to be expected given its price)

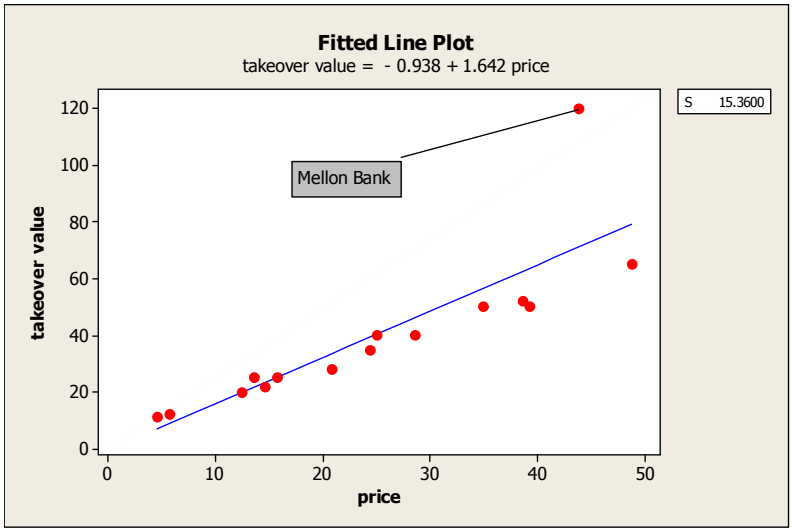
We may choose to remove the outlier and run analysis on the remaining data to get more robust estimates of the line




The regression line here is Takeover value = 5.69+1.209*Price

The standard deviation of takeover values around the line is 2.06

Original Analysis with MB data




Regression line is Takeover value = -0.938+1.642*(Price)

The standard deviation of takeover values around the line is 15.36

Ex: “The relationship between default rates and recovery”
http://www2.standardandpoors.com/spf/pdf/media/DefaultRatesAndRecovery_26_Jan07.pdf

Y =U.S. speculative-grade annual recovery rates over 1988-2005
X = Corresponding speculative-grade annual default rates

From Table 1 in the report, the estimated model is

Y= 0.55-2.206*X

The t-statistic for X is -3.47

R-sq = 0.43 (43%)

The standard deviation of Y (recovery rates) values around the regression line is 0.074

Suppose the speculative-grade default rate is 4.7% (i.e. 0.047)

The predicted recovery rate is 0.55-2.206*(0.047) = .446 (44.6%)

# Multiple regression

Data source: Zagat 2006 299 restaurants sampled

| Restaurant | Cost | Service | Decor |
|---|---|---|---|
| Aqua Grill | 54 | 22 | 19 |
| Meskerem | 23 | 16 | 9 |
| . | . | . | . |

.

The regression equation is

Cost = - 33.0 + 1.33*Decor + 2.72*Service

Just as in regression with 2 variables, in multiple regression we have

i) t-statistics of coefficients
ii) standard deviation of the Y values around the line
iii) R-square

As before, the regression line can be used to

a) Predict the Y value for given values of the X variables
b) Back out the value of one X variable, given a Y value and values of the remaining X variables

$$\text{Cost} = -33.0 + 1.33*\text{Decor} + 2.72*\text{Service}$$

What is the predicted cost for a restaurant that has décor rating of 17 and service rating of 21?

Cost = - 33.0 + 1.33*(17) + 2.72*(21) = 46.73

What is the décor rating that corresponds to a cost of 38 and a service rating of 16?

38 = -33+1.33*Décor+2.72*16

Décor = 27.48/1.33 = 20.66

## *T-statistics*

The coefficients in the regression line are not unit free and looking at their magnitude may be misleading. However, the t-statistic of each coefficient is unit free.

In this example, the regression line is

$$\text{Cost} = -33.0 + 1.33*\text{Décor} + 2.72*\text{Service}$$

The regression output tells us that the t-statistic of the coefficient of décor (i.e. of 1.33) is 8.33 and that the t-statistic of the coefficient of service (i.e. of 2.72) is 11.87

Since the t-stat of décor is outside (-2,2), we conclude that décor is useful in predicting cost, in the presence of the other X variable (service)

Similarly, since the t-stat of service is outside (-2,2), we conclude that service is useful in predicting cost, in the presence of the other X variable (décor)

## *Std. dev. of values around the regression line*

The regression output also gives us the standard deviation of the Y values around the regression line. In our example, the std. dev. of cost values around the line is 8.5

As before, this standard deviation, in conjunction with the line and the normal distribution

a) gives us an idea of the range of Y (cost) values to expect for a given value of X variables (décor, service)

c) helps us identify unusual observations (outliers)

For example, the restaurant *Per Se* has a décor rating of 27, a service rating of 28 and its cost is 201. Is this unusually high relative to what the regression line would predict?

Predicted cost = - 33.0 + 1.33*27 + 2.72*28 = 79.07

Thus, the Z-value for *Per Se's* cost is

Z-value=(Actual cost-predicted cost)/Std Dev = (201-79.07)/8.5
= 14.34

As earlier, the R-square of the regression model is a numerical measure of how well the X variables help predict the Y variable. In this example the R-square is 70.7%

We do not have to be limited to only 2 explanatory variables, for example:

| Restaurant | Price | Service | Decor | Food |
|---|---|---|---|---|
| Aqua Grill | 54 | 22 | 19 | 26 |
| Meskerem | 23 | 16 | 9 | 20 |
| . | . | . | . | |

The regression equation is
Cost = - 33.1 + 1.34 Decor + 2.71 Service + 0.008 Food

The regression equation can be used as before to generate predictions of Y or back out an X value.

The regression output yields the t-statistics for the coefficients of Décor, Service and Food as 8.3, 9.2 and 0.03 respectively.

The t-statistic for the coefficient of Food lies in (-2,2) and hence the data does not provide evidence that Food is useful to predict Cost, *in the presence of the other X variables (here, Décor and Service)*

*Cautionary note about R-square values in multiple regressions*

When only décor and service are used, the regression equation is

$$\text{Cost} = -33.0 + 1.33*\text{Décor} + 2.72*\text{Service}$$

and the corresponding R-square value is 70.7%

When Food is added as a variable to the model, the regression equation is

$$\text{Cost} = -33.1 + 1.34\ \text{Decor} + 2.71\ \text{Service} + 0.008\ \text{Food}$$

and the corresponding R-square value is 71%

*Fact:* If you add an X variable to a model, the R-square value of the resulting model will increase, or at worst stay the same. Conversely, if you drop an X variable from the model, the resulting R-square value will decrease, or at best stay the same

Ex: “The link between default and recovery rates” (Altman, Brady, Resti and Sironi, 2003)

Y = (BRR) Weighted average recovery rate of all corporate bond defaults over 1982-2001

Three X variables:
BDR: Weighted average default rate on bonds in high yield market
BDRC: One year change in default rate
BOA: Total amount of high yield bonds outstanding for the year (in trillions of dollars)

BRR=0.514-1.358*BDR-1.93*BDRC-0.164*BOA

The t-statistics for the three X variables are -2.52, -3.18 and -2.13 respectively

The R-sq value is 0.764 (76.4%)

The standard deviation of the Y (BRR) values around the regression line is 0.06

Suppose we know that BDR = 0.04, BDRC = -0.01 and BOA = 0.2

The predicted BRR is

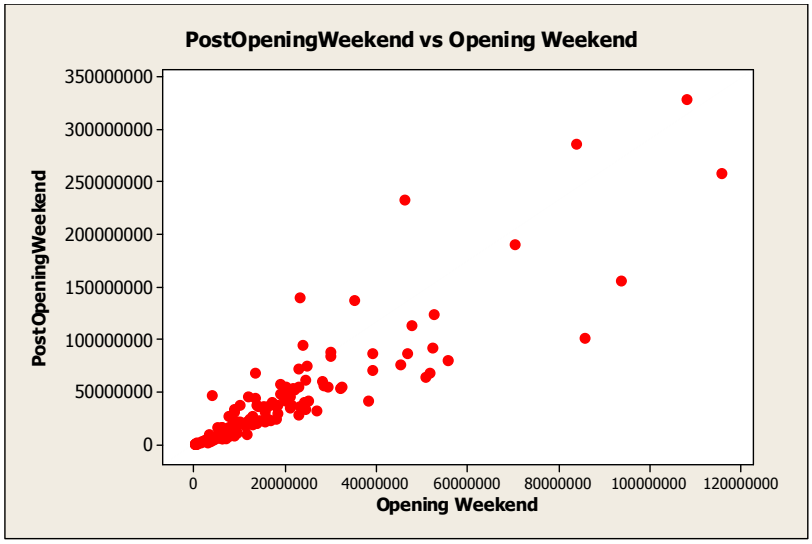
=0.514-1.358*0.04-1.93*(-0.01)-0.164*(0.2) = 0.463

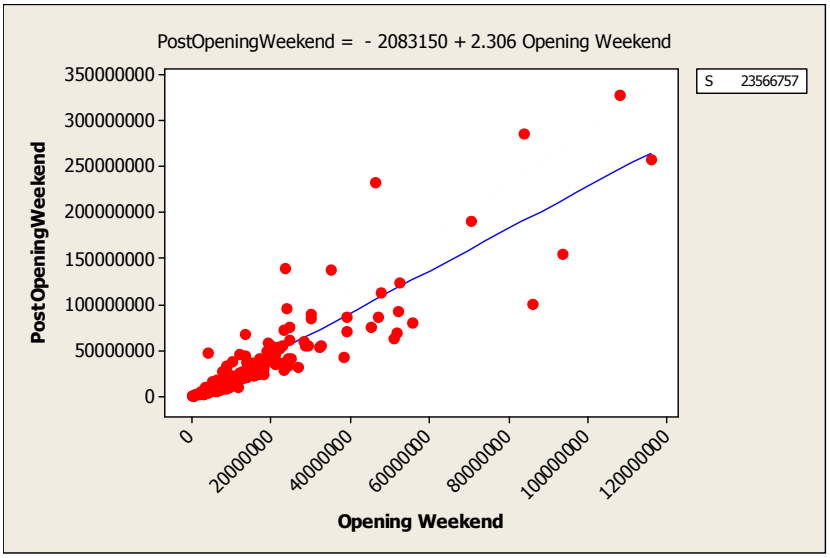
We expect that 95.44% of the BDR values for the given values of the 3 X variables will fall within 0.463 +/- 2*(0.06) = (0.343, 0.583)

# Logarithmic Transformations

Data source: IMDB.com  151 Movies that opened on at least 100 screens in 2004

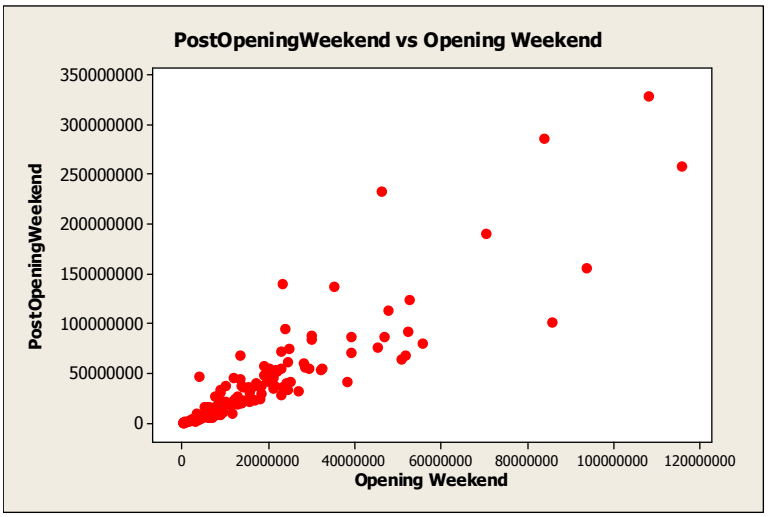
| Movie | Opening weekend $ | Post Opening weekend $ |
|---|---|---|
| 50 First Dates | 45107871 | 75668961 |
| Ocean's Twelve | 39153380 | 86378254 |
| . | . | . |
| . | . | . |

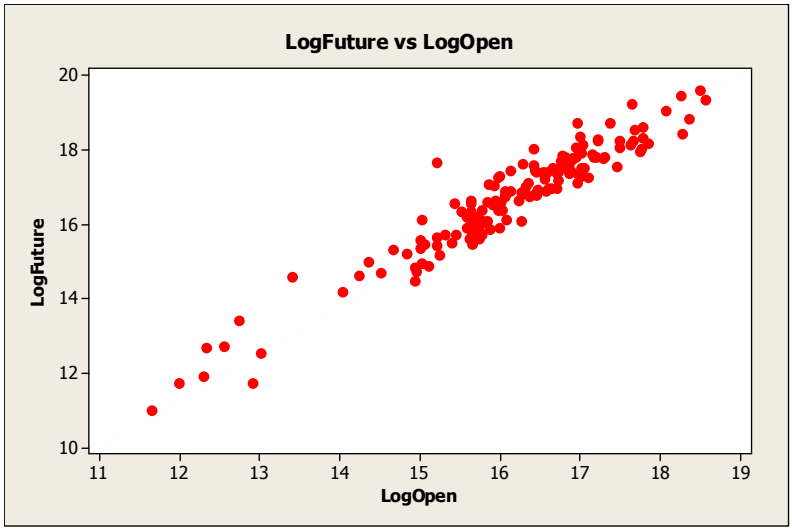


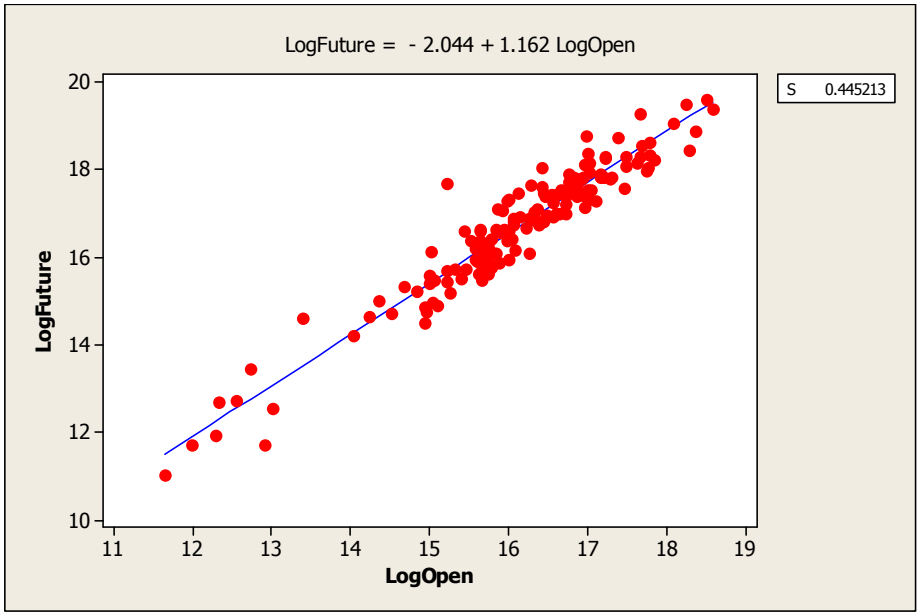
# Logarithms

| Movie | Opening weekend $ | Post Opening weekend $ | Log(Opening) | Log(Post Opening) |
|---|---|---|---|---|
| 50 First Dates | 45107871 | 75668961 | 17.6245 | 18.1419 |
| Ocean's Twelve | 39153380 | 86378254 | 17.4830 | 18.2742 |
| . | . | . | . | . |
| . | . | . | . | . |



Regression line fit to the log transformed data




Regression line is Log(future) = -2.044 + 1.162*Log(Open)

Suppose a movie has opening weekend revenue of $20,000,000

Log(20,000,000) = 16.81

Thus, predicted log(future) = -2.044 + 1.162*16.81 = 17.49

Back-transforming the predicted log(future) value of 17.49 to predicted future value in $ (using the exponential function) gives predicted future value = 39,397,779 $

# LOGISTIC REGRESSION

Leakage in the O-ring of the space shuttle
Temperature at the time of flight

Data available before Challenger's fatal flight:

Temperatures at which there were zero O-ring failures:
66, 67, 67, 67, 68, 69, 70, 70, 72, 73, 75, 76, 76, 78, 79, 80, 81

Temperatures at which there were O-ring failures:
52, 56, 58, 63, 70, 70, 75

$$P(\text{O - ring failure at certain Temp}) = ?$$

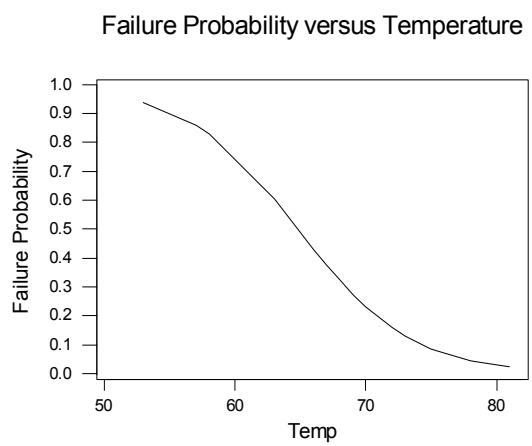
$$\log_e \frac{P(\text{O - ring failure at certain Temp})}{1 - P(\text{O - ring failure at certain Temp})} = a + b * \text{Temp}$$

$$\log_e \frac{P(\text{O - ring failure at certain Temp})}{1 - P(\text{O - ring failure at certain Temp})} = 15 - 0.23 * \text{Temp}$$

$$\log_e \frac{P(\text{O - ring failure at certain Temp})}{1 - P(\text{O - ring failure at certain Temp})} = 15 - 0.23 * \text{Temp}$$

This implies that

$$P(\text{O - ring failure at certain Temp}) = \frac{\exp(15 - 0.23 * \text{Temp})}{1 + \exp(15 - 0.23 * \text{Temp})}$$



# LOGISTIC REGRESSION FOR MODELING DEFAULT

(Modelling credit risk for Small and Medium Sized Enterprises, Altman et al)

$$\log_e \frac{P(\text{Default})}{1 - P(\text{Default})} = -4.28 - 0.18 * \text{Ebitda/Total Assets}$$
$$+ 0.01 * \text{Short Term Debt/Equity Book Value}$$
$$- 0.08 * \text{Retained Earnings/Total Assets}$$

Practice Problem with solutions

a) A regression model is built to relate the Annualized Investment Risk of a country to its Credit Rating. (The data is from 1995). The resulting model is

Investment Risk = 58-0.4*(Credit rating)

The standard deviation of the investment risk values around the regression line is 10.

i) Based on the regression model, predict the annualized investment risk of a country with a credit rating of 75
Solution: 28

ii) If a country has a predicted investment risk of 35 based on the regression model, what would its corresponding credit rating be?
Solution: 57.5

iii) Argentina has a credit rating of 31.8 and an annualized investment risk of 87. How many standard deviations is this actual investment risk from the risk predicted by the model above for Argentina?
Solution: 4.172

iv) Based on the regression line and using the normal distribution, what is the interval such that approximately all (to be precise, 99.74% ) countries with a credit rating of 60 will have investment risk values falling in this interval?
Solution: 34 +/- 3*(10) = (4, 64)

b) A variable Y is normally distributed with mean 3 and standard deviation 5.

i) What is $P(Y>0.85)$?
<u>Solution:</u> $P(Y>0.85) = P(Z>-0.43) = 0.5+0.1664 = 0.6664$

ii) What is $P(Y< - 0.85)$?
<u>Solution:</u> $P(Y< - 0.85) = P(Z< - 0.77) = 0.5 – 0.2794 = 0.2206$

# NORMAL CURVE AREAS

## AREAS UNDER THE STANDARD NORMAL CURVE FROM 0 TO Z

| Z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |