The more we know, the less we agree: Public announcements, higher-order expectations, and rational inattention

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Abstract

Why do announcements of public information set off a frenzy of trading? Usually, public information brings beliefs closer to each other. With less disagreement, there should be less reason to trade. I highlight two novel properties of certain Gaussian information structures and show that they explain hectic and informative trading around announcements. First, announcements can lead to contrarian higher-order expectations. That is, the more optimistic an agent is regarding the fundamental value, the more pessimistic she is regarding the expectations of others. Second, the announcements can polarize higher-order expectations. That is, the announcement can increase the dispersion of expectations regarding the expectations of others. An information structure with both properties is the equilibrium outcome in a Grossman-Stiglitz type asset pricing model, where agents are subject to rational inattention.

1 Introduction

Decisions are often determined by higher-order expectations (agents' opinion about other agents' opinions), rather than the agents' own opinions about the fundamentals of the econ-

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omy. Public information, in such an environment, aside from providing information about the uncertain fundamentals, has an important secondary role in helping each agent to guess the private information of other agents. In this paper, I focus on the economic consequences of the effect of public information on higher-order expectations.

The main contribution of this paper is the composite of three results. First, I highlight that public information leads to *contrarian* and *polarized higher-order expectations* in certain Gaussian information structures. If higher-order expectations are contrarian, then the more optimistic an agent is regarding the fundamental value, the more pessimistic she is regarding the expectations of others. If higher-order expectations are polarized, then the public announcement increases the dispersion of expectations regarding the opinion of others, even if it decreases the dispersion of expectations regarding the fundamental value. Second, I show that an information structure with these two properties is the equilibrium outcome in a Grossman-Stiglitz type asset pricing model, where each agent chooses which elements of the uncertain asset value to learn about. Third, I show that these properties explain why public announcements induce hectic and informative trading in financial markets.

Regarding the first result, suppose that an agent forms her expectation about the expectation of another agent. I show in a general Gaussian environment that public information leads to contrarian and polarized higher-order expectations if the correlation between the private information sets of the two agents is sufficiently weak. I also show that polarized higher-order expectations imply contrarian higher-order expectations, but not the other way around.

To see the intuition behind the result, consider the simplest example where agents' private information is uncorrelated. Before a decision maker forms an expectation about the fundamental, each agent in a group forecasts the decision maker's expectation. The fundamental is the sum of two factors, $\theta = \theta_1 + \theta_2$ and a decision maker observes a noisy signal on the first factor, $z = \theta_1 + \varepsilon$. In contrast, each agent in the group observes a noisy signal on the second factor, $x^i = \theta_2 + \varepsilon^i$. The public signal, observed by all, is a noisy version of the fundamental, $y = \theta + \eta$. All factors and noise terms are pairwise uncorrelated zero-mean normal variables. Without a public announcement, agents in the group agree in their forecasts, because their private signals do not reveal any information on the decision maker's signal. However, there is disagreement with a public announcement. For example, an agent with a high private signal on the first factor relative to the announcement concludes that most probably, the other factor is low. Consequently, the decision maker's signal and fundamental expectation must also be low. An agent with a low private signal relative to the announcement reaches the opposite conclusion. Thus, the announcement leads to polarized second-order expectations. In addition, an agent with higher private signal is still more optimistic about the

fundamental value, even if she is more pessimistic about the decision maker's expectation. That is, second-order expectations are also contrarian.

In order to show that the result has economically important consequences,[1] I introduce rational inattention into a differential information, noisy rational expectations model of financial markets.[2] Similarly to Allen, Morris and Shin (2006), I use a version with overlapping generations of traders. There are early traders and late traders. There are two rounds of trading among early traders, and then they sell all their assets to late traders. In the third trading round only late traders trade, before they liquidate their risky assets for the fundamental value. Thus, early traders are interested in the valuation of late traders, which implies that second-order expectations matter. Between the two rounds of trading of early traders, a public signal is released.

Following Sims (2003) and the emerging literature on rational inattention, each trader allocates her limited research capacity among the available sources of information. The fundamental value of the asset depends on two factors representing the outcome of an old project and the outcome of a new project. The new project is new in the sense that early traders cannot gather information about it. However, late traders can decide on the relative quality of their signals about the old and the new project. Thus, the correlation between the private information sets of early and late traders is endogenous. It is high, if late traders decide to learn about the old project, but it is low, if they decide to learn only about the new project.

I show that under mild conditions, late traders decide to learn only about the new project. Thus, private information sets of early and late traders are uncorrelated and higher-order expectations are contrarian and polarized. This is an equilibrium by the following logic. Under this choice of late traders, the last period price contains information only about the new factor, while past prices contain information only about the old factor. If past prices contain relatively large amount of information, because early traders trade actively on their private information, then late traders indeed prefer to focus their limited resources on the new factor. I show that for any parameters, a sufficiently precise public signal announced with sufficiently high probability implies that this is the case.

Contrarian and polarized higher-order expectations in the model imply that public announcements induce large and informative trading activity. This is consistent with a vast body of empirical work. This is a success, given the difficulty of similar models to explain

[1] In a companion paper, Kondor (2009), I modify the Morris-Shin (1998) currency crisis model to show that more public information can increase the chance of a speculative attack. The mechanism relies on polarization in higher-order expectations due to the announcement.

[2] The basic model of noisy rational expectations was developed in steps by Grossman and Stiglitz (1980), Diamond and Verrecchia (1980) and Hellwig (1981).

these well established stylized facts.[3] Polarization helps because the increased disagreement of early traders translates to active speculative trading after the announcement. Contrarian higher-order expectations help because they imply active portfolio rebalancing after the announcement. Before the announcement, a major motivation for trade is to speculate on the content of the scheduled public announcement. As the announcement is correlated with the fundamental value, this portfolio formation is mainly influenced by fundamental expectations. After the announcement, early traders liquidate these speculative positions and form portfolios based on their expectation about the next period price. Thus, the new positions are mainly influenced by second-order expectations. Under contrarian higher-order expectations, these two portfolios are very different as they have sensitivity to private information of the opposite sign.

This paper fits in the recent flow of papers analyzing the effect of higher-order expectations in various contexts. The most closely related part of this literature[4] analyzes environments where various groups of agents act sequentially and the pay-off of early actors depends on the actions of groups acting later (e.g. Allen, Morris and Shin (2006), Makarov and Rytchkov (2007) Banerjee, Kaniel and Kremer (2009), Goldstein, Ozdenoren and Yuan (2008), Angeletos, Lorenzoni and Pavan (2007)). Thus, early actors have to guess the information of agents acting later. Applications include financial markets, currency attacks and the interaction between stock prices and real investment. The novelty of this paper is that I focus on agent's choice of information and the resulting polarized and contrarian higher-order expectations.

The application presented here is closely related to the recent literature on rational inattention following a seminal paper of Sims (2003). In particular, Van Nieuwerburgh and Veldkamp (2009a), Peng (2004), Peng and Xiong (2006) and Mondria (2009) consider applications on financial markets where prices are endogenous.[5] The application presented is novel in the sense that I use a dynamic model. This is a substantive change as the mechanism of the model relies on the relative information content of past and present prices. Thus, this

[3] In Section 4.2, I examine the stylized facts of the empiricial literature related to trading patterns around public announcements. I also discuss the extent to which existing models can or cannot explain these facts.

[4] Other papers concentrate on whether imperfect information of decision makers leads to a unique equilibrium in coordination games (e.g. Morris and Shin (1998), Angeletos and Werning (2006), Hellwig, Mukherji and Tsyvinski (2006)), and on "Beauty contest" environments where the pay-off of agents is a weighted sum of the deviation of their actions from an optimal level and of the deviation of their actions from the average action of others (e.g. Morris and Shin (2002), Woodford (2002), Hellwig (2002), Angeletos and Pavan (2007)).

[5] See also Veldkamp (2006) for a model of financial markets where information structure is endogenized by explicitly modeling the market for information, and Maćkowiak and Wiederholt (2009), Hellwig and Veldkamp (2009) and Van Nieuwerburgh and Veldkamp (2009b) for applications of rational inattention in other contexts.

mechanism could not arise in the static models of the cited papers.

There is also a literature on the possibility of announcements leading to polarized first-order expectations (e.g. Kandel and Pearson (1995), Kim and Verrecchia (1997), Rabin and Shrag (1999), Dixit and Weibull (2007), Acemoglu, Chernozhukov and Yildiz (2009)). As the standard assumptions of common priors and positively associated signals and fundamentals rule out polarization in first-order expectations, this literature explores various ways to relax these assumptions. Instead, I highlight that polarization in higher-order expectations is consistent with these standard assumptions. Furthermore, I argue that in some contexts polarization in higher-order (but not in first-order) expectations is sufficient to explain puzzling stylized facts.

The structure of this paper is as follows. In the next section I present conditions for polarized and contrarian higher-order expectations in Gaussian information structures. In section 3, I present the financial application and derive the equilibrium. In section 4, I discuss the empirical implications and the related literature regarding the volume and information content of trading around announcements. Finally I conclude.

2 Polarized and contrarian higher-order expectations

In this section, I analyze the effect of public announcements on expectations of different order in a general Bayesian-Gaussian model of learning. I characterize information structures with the property of *polarized and contrarian higher-order expectations*. I define these properties below. My strategy is to present all the results with relatively little explanation first, and to provide the intuition with the help of an example at the end of this section.

Note that this is a statistical exercise, where the only action of agents is to form expectations and their only objective is to make these expectations as accurate as possible. I leave the economic consequences to the rest of the paper where I introduce a model of financial markets to analyze the implications of the presented statistical observations.

Let us consider a continuum of agents divided in T groups. In each group there is a unit mass of agents. There is a fundamental variable of interest, θ. The public information on θ is represented by the public signal y, while the private information of agent i in group t is represented by private signals x_t^i.[6] All signals and θ are jointly normally distributed, all signals are pairwise positively correlated to θ, and the covariance structure is symmetric[7] in

[6] For incorporating a larger number of private and public signals redefine $x_t^i \equiv E(\theta|\mathcal{I}_t^i)$ and $y \equiv E(\theta|\mathcal{P}_t)$ where $\mathcal{I}_t^i$ is the collection of all private signals of agent i in group t and $\mathcal{P}_t$ is the collection of all public signals. Assume that θ has a diffuse prior. Then all the results and proofs in this section are also valid for that more general case.

[7] Besides the analytical convenience, the symmetry imposed on the covariance structure has the advantage

the sense that

$$\begin{aligned} var\left(x_t^i\right) &= \sigma_x^2 > 0 \\ var\left(y\right) &= \sigma_y^2 > 0 \\ corr(x_t^i, \theta) &= \rho_{x,\theta} > 0 \\ corr\left(x_t^i, y\right) &= \rho_{x,y} \geq 0 \\ corr\left(x_t^i, x_t^j\right) &= \rho_{x,x} \geq 0 \\ corr\left(x_t^i, x_u^i\right) &= \rho_{x,x'} \geq 0 \end{aligned}$$

for all t and $u \neq t$ and $i \neq j$. That is, each private signal has the same variance, each private signal in group 1 has the same correlation to θ, and we have a single correlation coefficient for signals within each group and another one for the correlation between signals across groups.

I am interested in the effect of the public signal y on expectations of different order. For this purpose, I require agents in each group 1 to form fundamental expectations on θ,

$$f_1^n\left(x_1^i\right) \equiv E\left(\theta | x_1^i\right) = a_\theta^n x_1^i \tag{1}$$

$$f_1^a\left(x_1^i, y\right) \equiv E\left(\theta | x_1^i, y\right) = a_\theta^a x_1^i + c_\theta^a y \tag{2}$$

without and with observing the public announcement, respectively. The objects f_1^a and f_1^n are first-order expectations. I require agents in group $t > 1$ to form expectations on the expectation of an agent in group $t-1$,

$$\begin{aligned} f_t^n\left(x_t^i\right) &\equiv E\left(f_{t-1}^n\left(x_{t-1}^i\right) | x_t^i\right) \\ f_t^a\left(x_t^i, y\right) &\equiv E\left(f_{t-1}^a\left(x_{t-1}^i, y\right) | x_t^i, y\right) \end{aligned}$$

Thus, an agent in group t forms an expectation of the order t.

I say that public announcement leads to contrarian higher-order expectations if

$$\frac{\partial f_t^a\left(x_t^i, y\right)}{\partial x_t^i} \frac{f_{t-1}^a\left(x_{t-1}^i, y\right)}{\partial x_{t-1}^i} < 0$$

but

$$\frac{\partial f_t^n\left(x_t^i\right)}{\partial x_t^i} \frac{\partial f_{t-1}^n\left(x_{t-1}^i\right)}{\partial x_{t-1}^i} \geq 0.$$

For example, if second-order expectations are contrarian then the most optimistic agents

that the order of the groups does not matter. Comparing the expectation of an agent in group t, and of an agent in group u, is the same as comparing the expectation of a given agent, whose group is once ordered in place t and once in place u.

regarding the fundamental value are also the most pessimistic agents regarding the expectations of others.

The next proposition states that all higher-order expectations are contrarian if the correlation between private information sets of agents in different groups is sufficiently weak.

Proposition 1 *If*

$$\rho_{x'x} < \rho_{x,y}^2 \tag{3}$$

then a public announcement leads to contrarian higher-order expectations of all order $t > 1$.

Proof. Observe that

$$a_x^a \equiv \frac{\partial E\left(x_{t-1}^j | x_t^i, y\right)}{\partial x_t^i} = \frac{\rho_{x,x'} - \rho_{x,y}^2}{1 - \rho_{x,y}^2} \tag{4}$$

$$a_x^n \equiv \frac{\partial E\left(x_{t-1}^j | x_t^i\right)}{\partial x_t^i} = \rho_{x,x'}. \tag{5}$$

Thus, the observation

$$\frac{\partial f_t^a\left(x_t^i, y\right)}{\partial x_t^i} = \left(a_x^a\right)^{t-1} a_\theta^a$$

implies the result. ■

I say that the given information structure leads to a polarized t-th order expectation, if

$$\int_0^1 \left| f_t^n\left(x_t^i\right) - \overline{f_t^n} \right| di < \int_0^1 \left| f_t^a\left(x_t^i, y\right) - \overline{f_t^a} \right| di,$$

where $\overline{f_t^n} \equiv \int_0^1 f_t^n\left(x_t^i\right) di$, $\overline{f_t^a} \equiv \int_0^1 f_t^a\left(x_t^i, y\right) di$ are the average forecast in the given sample without and with the announcement, respectively. That is, there is polarization if the dispersion of forecasts in group t increases after the announcement.

It is a well-known result that, under fairly general distributional assumptions, a public signal cannot push first-order expectations further apart. Broadly speaking, to rule this out, it is sufficient if the fundamental and the signals of a given agent are all positively associated to each other.[8] This is a commonly assumed property in models of asymmetric information since Milgrom (1981).[9] Intuitively, under this condition agents with different private signals

[8] More precisely, under common priors, if the conditional densities $\psi\left(y|\theta, x^i\right), \psi\left(x^i|\theta\right)$ and $\psi\left(x^i|\theta, y\right)$ for $i = 1, 2$, satisfy the monotone likelihood ratio property, then

$$E\left(\theta|y, x^1\right) > E\left(\theta|x^1\right) \geq E\left(\theta|x^2\right) > E\left(\theta|y, x^2\right)$$

cannot hold.

[9] See Kandel and Pearson (1995), Kim and Verrecchia (1997), Rabin and Shrag (1999), Dixit and Weibull (2007), Acemoglu et al. (2009) for various relaxations of these assupmtions leading to polarization in first-order expectations.

rank the possible public announcements the same way according to how favorable their information content is. In this sense, private information does not change the interpretation of public signals. Although my definition of polarization is weaker,[10] the next proposition shows that, given common priors and the Gaussian structure, a similar condition rules out even this weaker concept of polarization.

Proposition 2 *Let*

$$a_\theta^a, c_\theta^a \geq 0 \tag{6}$$

then the public signal does not cause polarization in the first order of expectations.

Proof. Using Bayes' Rule[11] gives

$$f_1^a\left(x_t^i, y\right) = E\left(\theta | x_1^i, y\right) = E\left(\theta | x_1^i\right) + \frac{Cov\left(y, \theta | x_1^i\right)}{Var\left(y | x_1^i\right)}\left(y - E\left(y | x_1^i\right)\right)$$

which, together with the assumptions of the proposition, implies

$$0 \leq a_\theta^a = a_\theta^n - c_\theta^a \frac{\rho_{x,y}\sigma_y}{\sigma_x} < a_\theta^n.$$

The linearity of conditional expectations of jointly normal variables gives the result. ■

However, polarization in higher-order expectations is consistent with a structure where (6) holds. The next proposition shows that weak correlation between private signals across groups implies that public announcement polarizes higher order expectations regardless of its content. Note also that condition (7) for polarization implies condition (3) for contrarian higher-order expectations.

Proposition 3 *Suppose that the condition in (6) holds. If and only if*

$$\rho_{x,x'} < \frac{\rho_{x,y}^2 a_\theta^a}{\left(a_\theta^n\left(1 - \rho_{x,y}^2\right) + a_\theta^a\right)}, \tag{7}$$

then there is polarization in expectations of any order of $t > 1$. This is true for any announcement y.

[10] I do not require the mean of posteriors to move into opposite directions.

[11] Bayes' Rule implies that if $\mathbf{v}_\theta$ and $\mathbf{v}_s$ are vectors of variables which are jointly normally distributed with the vector of expected values $\boldsymbol{\mu}_\theta, \boldsymbol{\mu}_s$, respectively and covariance matrix

$$\begin{bmatrix} \Sigma_\theta & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_s \end{bmatrix},$$

then

$$\mathbf{v}_\theta | \mathbf{v}_s \tilde{} N\left(\boldsymbol{\mu}_\theta + \Sigma_{\theta,s}\Sigma_s^{-1}\left(\mathbf{v}_s - \boldsymbol{\mu}_s\right), \Sigma_\theta - \Sigma_{\theta,s}\Sigma_s^{-1}\Sigma_{s,\theta}\right)$$

(see Brunnermeier, 2001).

Proof. Note that

$$\left|\frac{\partial f_t^n\left(x_t^i\right)}{\partial x_t^i}\right| = \left|\left(a_x^n\right)^{t-1} a_\theta^n\right| \text{ and } \left|\frac{\partial f_t^a\left(x_t^i, y\right)}{\partial x_t^i}\right| = \left|\left(a_x^a\right)^{t-1} a_\theta^a\right|$$

where a_x^n and a_x^a are given by (4)-(5). As $a_\theta^n > a_\theta^a > 0$ holds, $|a_x^n a_\theta^n| < |a_x^a a_\theta^a|$ implies $|a_x^n| < |a_x^a|$ and $\left|\left(a_x^n\right)^{t-1} a_\theta^n\right| < \left|\left(a_x^a\right)^{t-1} a_\theta^a\right|$ for all t. Note also that $\left|\frac{\partial f_t^n\left(x_t^i\right)}{\partial x_t^i}\right| < \left|\frac{\partial f_t^a\left(x_t^i, y\right)}{\partial x_t^i}\right|$ implies polarization in the t-th order of higher-order expectations. Substituting in the definition of a_x^n and a_x^a into $|a_x^n a_\theta^n| < |a_x^a a_\theta^a|$ and simple algebraic manipulation gives (7) and concludes the proof. ■

To see the intuition behind Propositions 1,2 and 3, let us consider the following extreme example reiterated from the introduction where $\rho_{x,x'} = 0$.

Example 1 *Suppose θ is the fundamental value of a firm and it has two components: $\theta = \theta_1 + \theta_2$. Suppose that θ_1 and θ_2 are independent and have zero mean and the same variance σ_θ^2. Group 1 contains a single decision maker observing a private signal on the performance of the first component: $x_1 = \theta_1 + \varepsilon$. Group 2 contains a unit mass of speculators, each observing a private signal about the performance of the second component, $x_2^i = \theta_2 + \varepsilon^i$. Both ε and ε^i are zero mean noise terms with variance σ_ε^2. The public signal is $y = \theta + \eta$.*

Observe that before the public announcement, speculators have no private information on the private information of the decision maker, so their forecasts of the forecast of the decision maker are identical:

$$f_2^n\left(x_2^i\right) = E\left(E\left(\theta|x_1\right)|x_2^i\right) = E\left(a_\theta^n x_1|x_2^i\right) = 0.$$

Thus, there is no dispersion in the forecast of the second group at all:

$$\int_0^1 \left|f_2^n\left(x_2^i\right) - \overline{f_2^n}\right| di = 0.$$

However, after the announcement of y, a speculator i's forecast changes to

$$\begin{aligned} f_2^a\left(x_2^i, y\right) &= E\left(E\left(\theta|x_1, y\right)|x_2, y\right) = E\left(a_\theta^a x_1 + c_\theta^a y|x_2^i, y\right) = \\ &= a_\theta^a E\left(x_1|x_2^i, y\right) + c_\theta^a y = a_\theta^a a_{x_1}^a x_2^i + c_\theta^a y, \end{aligned}$$

where

$$a_{x_1}^a = -\frac{\left(\sigma_\theta^2\right)^2}{\sigma_\theta^2\left(\sigma_\theta^2 + \sigma_\eta^2\right) + \sigma_\varepsilon^2\left(\sigma_\theta^2 + \sigma_\theta^2 + \sigma_\eta^2\right)} < 0.$$

That is, a speculator with a high private signal on θ_2 relative to the announcement concludes

that most probably, the other factor is low. Consequently, the decision maker's signal and her fundamental expectation must also be low. A speculator with low private signal on θ_2 reaches the opposite conclusion. In the presesence of the public signal, the private signals are informative about the forecast of the decision maker. Thus, the announcement polarizes second-order expectations:

$$\int_0^1 \left| f_2^a\left(x_2^i, y\right) - \overline{f_2^a} \right| di = \left| a_{x_1}^a a_\theta^a \right| \int_0^1 \left| \varepsilon^i \right| di > 0.$$

Moreover, there are contrarian second-order expectations, since

$$\frac{\partial f_1^a\left(x_1, y\right)}{\partial x_1} \frac{\partial f_2^a\left(x_2^i, y\right)}{\partial x_2^\iota} = a_{x_1}^a a_\theta^a < 0.$$

In this example, private signals alone are informative about the fundamental, but they are not informative about the signals of others. That is, the unconditional correlation between the decision maker's private signal and an agent's private signal is zero. Because the public signal is correlated to the private signals of each agent, the correlation conditional on the public signal is not zero. In fact, it is negative. The fact that it is negative implies contrarian higher-order expectations. The fact that it is not zero implies that each private signal becomes informative about other private signals. This polarizes second-order expectations, while disagreement in first-order expectations still decreases. Propositions 1 and 3 show that the a similar intuition works as long as the unconditional correlation between private signals across groups, $\rho_{x,x'}$, is relatively small.

Observe that polarization in higher-order expectations differs from polarization in first-order expectations, because of the critical role of the strength of connection across private information sets. When an agent in group t forecasts the forecast of an agent in group $t-1$, she has to forecast the private signal of that agent. When her private signal, x_t^i, is informative about the private signal x_{t-1}^j, then the forecasts in group t vary a lot. Polarization occurs when the public signal increases this informativeness. Thus, it is possible that a private signal will not change the interpretation of the public announcement, if it is interpreted as a signal about the fundamental value, but it does change its interpretation, if it is interpreted as a signal about the valuation of others.

One might wonder why these properties have not received any attention in the previous literature. There are two likely reasons. The first is that interest in models where higher-order expectations play an important role is relatively recent. The second is that even in such models the information structure is virtually always assumed to be of the form where both private and public signals are noisy observations of the fundamental: $x_t^i =$

$\theta + \varepsilon^i, y = \theta + \eta$. I will refer to this structure as the *standard information structure.* These informational assumptions impose a rigid structure on the correlation structure of signals and θ. In particular,

$$cov\left(x_t^i, x_u^i\right) = cov\left(x_t^i, x_t^j\right) = cov\left(x_t^i, y\right) = var\left(\theta\right) \tag{8}$$

for any two agent i, j and two groups t, u. This structure is inconsistent with conditions (3) and (7) in Propositions 1 and 3.

In this section, I have characterized a statistical property of normally distributed variables. In the next section, I argue that the statistical property highlighted in this section has important economic consequences by modifying a standard workhorse model of financial markets with learning.

3 An OLG, noisy rational expectations model with rational inattention

In this section, I modify a standard, CARA-Normal, noisy rational expectations model with differential information.[12] Traders are subject to rational inattention, that is, each trader optimally allocates her limited research capacity among the available sources of information. The main point of this section is that in equilibrium traders endogenously choose an information structure where public announcement leads to contrarian and polarized higher-order expectations.

3.1 The set-up

There is a single type of risky asset with an uncertain fundamental value θ. There are two groups of traders and three trading periods, $t = 0, 1, 2$. In both groups there is a unit mass of traders. Traders in the first group trade among themselves in period 0 and 1 and sell all of their remaining assets in period 2. I refer these traders as early traders. Traders in the second group take all positions of the first group, trade among each other in period 2 and liquidate for θ at the end of the period. They are the late traders. Just as in Allen, Morris and Shin (2006) and in the myopic version of Brown and Jennings (1989), traders in group 1 maximize their end of period utility in each period.[13] The utility of a trader i trading in

[12] The static version was developed in steps by Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981), while dynamic versions include Brown and Jennings (1989) and He and Wang (1995).

[13] In a previous version of this paper, I solve the model assuming that traders in the first group are non-myopic: they maximize the expectation of their end-of-period-1 utility in both periods 0 and 1. The

period t, is given by $U_t^i(W_t^i) = -e^{-\gamma W_t^i}$, where γ is the absolute risk-aversion parameter and W_t^i is her monetary wealth at the end of the given trading period with

$$W_2^i = d_2^i(\theta - p_2), \tag{9}$$

$$W_t^i = d_t^i(p_{t+1} - p_t) \tag{10}$$

for $t = 0, 1$, where p_t is the price in period t and d_t^i is the demand of a given trader. In each period, traders submit demand curves to an auctioneer to buy up the random supply of assets:

$$u_t \sim N\left(0, \frac{1}{\delta_t}\right).$$

At the beginning of period 0, with probability π, it is announced that a public signal, y, will be released at the beginning of period 1. That is, each trader in period 0 knows whether there will be an announcement at the time she has to form her portfolio, but she does not know the content of the signal. The public signal gives information about the fundamental value:

$$y = \theta + \eta.$$

I focus on the differences in trading patterns of early traders with and without announcement. Traders base their portfolio decision on their private information set $\mathcal{I}^i$ and public information set $\mathcal{P}_t$. $\mathcal{I}^i$ contains private signals which traders receive at the moment of their entry, while $\mathcal{P}_t$ contains all available public signals released by period t, i.e., past and present prices and the potential public announcement. Where necessary, I will distinguish between public information sets with and without announcement with the superscript a, n, respectively. Traders update their beliefs by Bayes' Rule. Prices, p_t, are determined by market clearing in each period.

The main departure from previous models is in the information structure. Traders can decide on the quality and composition of their private information. Collecting private information is subject to the following structure. The fundamental value of the asset is given by

$$\theta = \theta_o + \theta_n$$

where θ_o and θ_n are interpreted as the value of the "old project" and the value of the "new project", respectively. The "new project" is new, because there is no private information available on it in period 0, when early traders enter and receive their private information. Thus, each early trader i can learn only about the current project. Her private information

qualitative results do not change. Allen, Morris and Shin (2006) reach a similar conclusion.

is represented by a private signal of the form:

$$x^i = \theta_o + \varepsilon^i.$$

In contrast, each late trader can learn about both projects. I allow each late trader to observe conditionally independent private signals on each project:

$$\begin{aligned} z_o^i &= \theta_o + \upsilon_o^i \\ z_n^i &= \theta_n + \upsilon_n^i. \end{aligned}$$

All factors and noise terms are i.i.d. and normally distributed

$$\begin{aligned} \theta_o &\sim N\left(0, \frac{1}{\omega_o}\right), \theta_n \sim N\left(0, \frac{1}{\omega_n}\right), \varepsilon^i \sim N\left(0, \frac{1}{\alpha^i}\right), \\ \upsilon_o^i &\sim N\left(0, \frac{1}{\alpha_o^i}\right), \upsilon_n^i \sim N\left(0, \frac{1}{\alpha_n^i}\right). \end{aligned}$$

Traders choose how to allocate their limited research capacity among different sources of information before the trading periods, in period $t = -1$. Following the recent literature on rational inattention, I formalize this choice based on information theory. That is, a trader i chooses the distribution of her private information set, $\mathcal{I}^i$, to maximize ex ante expected utility

$$\max_{\Psi^i(\mathcal{I}^i)} E\left(\pi \max_{d_t^i(\mathcal{I}^i, \mathcal{P}_t^a)} E\left(-e^{-\gamma W_t^i} | \mathcal{I}^i, \mathcal{P}_t^a\right) + (1-\pi) \max_{d_t^i(\mathcal{I}^i, \mathcal{P}_t^n)} E\left(-e^{-\gamma W_t^i} | \mathcal{I}^i, \mathcal{P}_t^n\right)\right) \tag{11}$$

subject to (9)-(10), where $t = 0$ for early traders and $t = 2$ for late traders and $\Psi^i(\cdot)$ is the cumulative distribution function of the private information set, $\mathcal{I}^i$, of trader i.[14] Given our assumed structure of information collection, for each agent, the choice of the $\Psi^i(\mathcal{I}^i)$, is reduced to the choice of their respective private signals, α^i or α_o^i, α_n^i. The choice is subject to the *information-processing constraint* limiting the total reduction of uncertainty by the observation of private signals:

$$H([\theta_o, \theta_n]) - H\left([\theta_o, \theta_n] | \mathcal{I}^i\right) \leq \frac{\kappa}{2} \tag{12}$$

where $H(\cdot)$ is the entropy of a random variable measuring uncertainty and $\frac{\kappa}{2}$ is a positive

[14] Note that as each early trader is myopic, she does not take into account her utility in period 1 at her information choice. However, the argument in Lemma 2 shows that this is an innocuous assumption, because her equilibrium information choice will remain the same even if she considers a convex combination of her utility in periods 0 and 1.

constant interpreted as the capacity limit. If $\psi(\cdot)$ is the density function of the random variable, entropy is defined as

$$H(\cdot) \equiv -E(\ln \psi(\cdot)).$$

Entropy is used in economics and statistics in a wide range of applications, because of its many attractive properties.[15] One of its useful properties is its especially simple form for normal variables. If $X \sim N(0, \Sigma)$ an n dimensional normal variable, then

$$H(X) = \frac{1}{2} \ln\left((2\pi e)^n \det(\Sigma)\right). \tag{13}$$

After substituting in (13) into (12),taking the exponential, using the Bayes' Rule and the imposed signal structure, the information-processing constraint of early traders and late traders simplify to

$$\frac{(\alpha^i + \omega_o)}{\omega_o} \leq e^{\kappa} \tag{14}$$

and

$$\frac{(\alpha_o^i + \omega_o)(\alpha_n^i + \omega_n)}{\omega_n \omega_o} \leq e^{\kappa}, \tag{15}$$

respectively.

The main purpose of the given information structure is to endogenize the correlation between the information set of traders trading in different time periods. Most CARA-Normal models of financial markets (e.g. Hellwig (1980), Diamond and Verrecchia (1981), Brown and Jennings (1989), He and Wang (1995), Allen, Morris and Shin (2006)) impose the standard information structure implying condition (8). As I pointed out, such structures do not allow for polarized higher-order expectations or contrarian higher-order expectations. If late traders decide to learn only about the old project, then the resulting information structure is similar to the standard one. However, if late traders choose to learn only about the new project, the information set of early and late traders becomes perfectly independent, just as described in Example 1. With the information choice in period $t = -1$, I allow for any of these extreme cases or anything in between to arise in equilibrium.

I view the set up of the model as an abstract description of a financial market with many professional traders. The assumptions of short-horizon traders and the OLG structure correspond to professional traders' needs to impress their investors by short-term results. Otherwise they would face capital withdrawals. I think of the information choice in $t = -1$ as the professional trader's decision on how to organize the research activity at her firm. I see it as an ex-ante investment in human and physical capital which will produce proprietary

[15] See Veldkamp (2009) for a detailed discussion.

information by the trading periods. The information-processing constraint describes her choice of allocating limited resources across research activities on various factors of the value of the asset.

I look for a rational expectations equilibrium defined as follows.

Definition 1 *An endogenous learning rational expectations equilibrium is a collection of demand functions,* $d_t^i(\mathcal{I}^i, \mathcal{P}_t)$, *price functions* $p_t = P_t(\theta_o, \theta_n, u_t, \mathcal{P}_t)$ *and information choice,* $\alpha^i, \alpha_o^i, \alpha_n^i$,

1. *for the equilibrium prices and information choice,* $d_t^i(\mathcal{I}^i, \mathcal{P}_t)$ *maximizes* $E(U_t^i(W_t^i)|\mathcal{I}^i, \mathcal{P}_t)$ *for any trader* i *trading in period* $t = 0, 1, 2$,

2. *prices clear the market in each trading period,*

3. *for the equilibrium distribution of prices and public signals, the information choice,*$\alpha^i, \alpha_o^i, \alpha_n^i$, *solves the maximization problem (11) subject to (12).*

For expositional purposes, I also define the rational expectations equilibrium for a fixed information structure.

Definition 2 *An exogenous learning rational expectations equilibrium for fixed* $\alpha^i = \alpha, \alpha_o^i = \alpha_o, \alpha_n^i = \alpha_n$ *is a collection of demand functions,* $d_t^i(\mathcal{I}^i, \mathcal{P}_t)$, *and price functions,* $p_t = P_t(\theta_o, \theta_n, u_t, \mathcal{P}_t)$ *that*

1. *For the equilibrium prices and fixed information structure,* $d_t^i(\mathcal{I}^i, \mathcal{P}_t)$ *maximizes* $E(U_t^i(W_t^i)|\mathcal{I}^i, \mathcal{P}_t)$ *for any trader* i *trading in period* $t = 0, 1, 2$,

2. *prices clear the market in each trading period.*

In the next section, I derive the exogenous learning rational expectations equilibrium of the model. Then I proceed to the endogenous learning case by solving for the optimal information choice of traders. The proofs all statements are in the Appendix.

3.2 Equilibrium with exogenous learning

I fix the distribution of private signals at the same arbitrary level for each early trader,$\alpha^i = \alpha$, and for each late trader, $\alpha_o^i = \alpha_o, \alpha_n^i = \alpha_n$. Given the information structure, I derive the equilibrium price and demand functions for the two branches of the event-tree: the one with the announcement and the one without the announcement. Given that the analysis of the branch without the announcement is equivalent to the analysis of the branch with the

announcement if the announcement is non-informative (if $\beta = 0$), I derive only a general case applicable to any of the branches. I distinguish between equilibrium coefficients of each branch only if it is necessary to avoid confusion. In that case, I use the superscripts a, n for the case of announcement and no announcement, respectively.

Finding the exogenous learning rational expectations equilibrium entails three, fairly standard steps. First, I conjecture that prices are given by the functions

$$p_2 = a_2\theta_n + g_2\theta_o + c_2 y + h_2 q_1 + k_2 q_0 - e_2 u_2 \tag{16}$$

$$p_1 = a_1\theta_o + c_1 y + h_1 q_0 - e_1 u_1 \tag{17}$$

$$p_0 = a_0\theta_o - e_0 u_0 \tag{18}$$

where $a_t, c_t, e_t, h_2, h_1, k_2$ are undetermined coefficients, while q_t is a price signal defined as follows. The price p_t together with the public signal and past prices is informationally equivalent to the price signal, q_t, and the public signal and past price signals, where

$$q_2 \equiv \frac{1}{a_2 + g_2}\left(p_2 - c_2 y - h_2 q_1 - k_2 q_0\right) = (1-\varphi)\,\theta_n + \varphi\theta_o - \frac{e_2}{a_2 + g_2} u_2 \tag{19}$$

$$q_1 \equiv \frac{1}{a_1}\left(p_1 - c_1 y - h_1 q_0\right) = \theta_o - \frac{e_1}{a_1} u_1 \tag{20}$$

$$q_0 \equiv \frac{1}{a_0} p_0 = \theta_o - \frac{e_0}{a_0} u_0 \tag{21}$$

and

$$\varphi \equiv \frac{g_2}{a_2 + g_2}. \tag{22}$$

I define τ_t^2 as the precision of q_t:

$$\frac{1}{\tau_2^2} \equiv \frac{e_2^2}{(a_2 + g_2)^2 \delta_2^2} \text{ or } \frac{\tau_t}{\delta_t} = \frac{a_2 + g_2}{e_2}$$

$$\frac{1}{\tau_1^2} \equiv \frac{e_1^2}{(a_1)^2 \delta_1^2} \text{ or } \frac{\tau_1}{\delta_1} = \frac{a_1}{e_1}$$

$$\frac{1}{\tau_0^2} \equiv \frac{e_0^2}{(a_0)^2 \delta_0^2} \text{ or } \frac{\tau_0}{\delta_0} = \frac{a_0}{e_0}.$$

Second, given the conjectured price functions, I derive the optimal strategies of traders. To ease on the notation, I denote the coefficients in different conditional expectations of early and late traders as follows:

$$
\begin{aligned}
E\left(\theta|z_o^i, z_n^i, y, q_2, q_1, q_0\right) &= a_\theta z_n^i + g_\theta z_o^i + b_\theta q_2 + c_\theta y + k_\theta q_0 + h_\theta q_1 && (23)\\
E\left(\theta_n|x^i, y, q_1, q_0\right) &= a_n x^i + b_n q_1 + c_n y + h_n q_0 && (24)\\
E\left(\theta_o|x^i, y, q_1, q_0\right) &= a_o x^i + b_o q_1 + c_o y + h_o q_0 && (25)\\
E\left(\theta_o|x^i, q_0\right) &= a_{o0} x^i + b_{o0} q_0. && (26)
\end{aligned}
$$

From the first order conditions of the maximization problem of late traders in period 2 and early traders in period 1 and 0, demand functions are

$$
\begin{aligned}
d_2^i &= \frac{E\left(\theta|z_n^i, z_o^i, y, q_2, q_1, q_0\right) - p_2}{\gamma var\left(\theta|z_n^i, z_o^i, y, q_2, q_1, q_0\right)} && (27)\\
d_1^i &= \frac{E\left(p_2|x^i, y, q_1, q_0\right) - p_1}{\gamma var\left(p_2|x^i, y, q_1, q_0\right)} && (28)\\
d_0^i &= \frac{E\left(p_1|x^i, q_0\right) - p_0}{\gamma var\left(p_1|x^i, q_0\right)}. && (29)
\end{aligned}
$$

Third, I impose the market clearing conditions and find the undetermined coefficients which simultaneously satisfy all three market clearing conditions. In particular, the market clearing conditions determine the coefficients of the price functions, (16)-(18), as the function of the primitives and φ and τ_t, $t = 0, 1, 2$. Furthermore, they also determine functions $F_t(\cdot)$, $t = 0, 1, 2$ that

$$\tau_t = F_t\left(\tau_2, \tau_1, \tau_0, \varphi\right). \tag{30}$$

As the argument for the three periods involves analogous steps, I show here only the case of period 1 and delegate the cases of period 2 and 0 to the Appendix. The market clearing condition in period 1 is

$$\int_0^1 E\left(p_2|x^i, y, q_1, q_0\right) di - p_1 = \gamma var\left(p_2|x^i, y, q_0, q_0\right) u_1, \tag{31}$$

which I rewrite as

$$
\begin{aligned}
a_2\left(a_n\theta_o + b_n q_1 + c_n y + h_n q_0\right) + g_2\left(a_o\theta_o + b_o q_1 + c_o y + h_o q_0\right) + c_2 y + h_2 q_1 + k_2 q_0 - \left(a_1 q_1 + c_1 y + h_1 q_0\right) = \\
= \gamma var\left(p_2|x^i, y, q_1, q_0\right) u_1.
\end{aligned} \tag{32}
$$

Collecting the coefficients of y and q_0 implies

$$c_1 = a_2 c_n + g_2 c_o + c_2 \tag{33}$$

$$h_1 = a_2 h_n + g_2 h_o + k_2. \tag{34}$$

Substituting in (33), (34) and $q_1 = \theta_o - \frac{e_1}{a_1} u_1$ into (32) and collecting the coefficients of $\theta_o + \theta_w$ and u_1 imply

$$a_1 = a_2 (a_n + b_n) + g_2 (a_o + b_o) + h_2 \tag{35}$$

and

$$\frac{a_2 a_n + g_2 a_o}{\gamma var (p_2 | x^i, y, q_1, q_0)} = \frac{\tau_1}{\delta_1}. \tag{36}$$

I show in the Appendix that g_2 and a_2 are determined by the primitives and τ_t and φ. Consequently, the undetermined coefficients enter the left hand side of (36) only through values of τ_t and φ. Thus, for a given τ_2, τ_1 and φ, it is sufficient to find τ_1 which solves

$$\tau_1 = F_1 (\tau_2, \tau_1, \tau_0, \varphi) \tag{37}$$

where

$$F_1 (\tau_2, \tau_1, \tau_0, \varphi) \equiv \delta_1 \frac{a_2 a_n + g_2 a_o}{\gamma var (p_2 | x^i, y, q_1, q_0)}. \tag{38}$$

The last equation characterizing the equilibrium of a given branch is

$$\varphi = F_3 (\tau_0, \tau_1, \tau_2, \varphi,)$$

where

$$F_3 (\tau_0, \tau_1, \tau_2, \varphi,) \equiv \frac{g_\theta}{a_\theta + g_\theta}$$

by (48),(47) and (22).

To sum up, finding the fixed point $[\tau_2^n, \tau_1^n, \tau_0^n, \varphi^n]$ of the four dimensional system

$$\begin{aligned} F_t (\tau_2^n, \tau_1^n, \tau_0^n, \varphi^n) |_{\beta=0} &= \tau_t^n \\ F_3 (\tau_2^n, \tau_1^n, \tau_0^n, \varphi^n) |_{\beta=0} &= \varphi^n \end{aligned}$$

where $t = 0, 1, 2$, and finding the fixed point $[\tau_2^a, \tau_1^a, \tau_0^a, \varphi^a]$ of the four dimensional system

$$\begin{aligned} F_t (\tau_2^a, \tau_1^a, \tau_0^a, \varphi^a) &= \tau_t^a \\ F_3 (\tau_2^a, \tau_1^a, \tau_0^a, \varphi^a) &= \varphi^a \end{aligned}$$

where $t = 0, 1, 2$, are equivalent to finding the equilibrium demand and price functions for the branch without and with announcement, respectively.

The following proposition states that a linear equilibrium always exists.

Proposition 4 *For any fixed information choice,$\alpha, \alpha_o, \alpha_n$, a linear, exogenous learning rational expectations equilibrium exists.*

3.3 Equilibrium with endogenous learning

In this section, I derive the equilibrium of the full model where each trader chooses the distribution of her private signals in period $t = -1$ subject to her rational inattention constraint. I argue that, under fairly general conditions, late traders choose to learn only about the new factor. This results in uncorrelated private information sets of early and late traders. As I showed in Proposition 1 and 3, this is sufficient for public announcements leading to contrarian and polarized higher-order expectations. Thus, the main result of this section is that the existence of information structures resulting in contrarian and polarized higher-order expectations is not only a theoretical curiosity. These structures arise as an equilibrium outcome, if agents are allowed to choose what to learn about. In the next section, I also show that polarized and contrarian higher-order expectations explain empirical patterns of trading and information flow in our financial markets context.

First, I rewrite the objective function (11) of the rational inattention problem in a simpler form. Using trader i's optimal demand in periods 0 and 2 in (29), (27) and known results of probability theory helps us to simplify the objective function (11) as follows.

Lemma 1 *The information choice problem of an early trader in period $t = -1$ is equivalent to*

$$\max_{\alpha^i} -\sqrt{var\left(p_1 | x^i, y, p_1, p_0\right)} \tag{39}$$

subject to (14).

The information choice of the late trader solves the problem

$$\max_{\alpha_n^i, \alpha_o^i} -\sqrt{var\left(\theta | z_o^i, z_n^i, y, p_2, p_1, p_0\right)} \tag{40}$$

subject to (15).

Given that more information should decrease the conditional variance of the fundamental and early traders can learn only about the old project, we naturally anticipate that (14) binds, pinning down the value of α^i. The next Lemma shows that this is indeed the case.

Lemma 2 *In any endogenous learning rational expectations equilibrium*

$$\alpha^i = \omega_o \left(e^\kappa - 1\right).$$

For our purposes, the information choice of late traders is more informative. We are especially interested in conditions leading to an equilibrium where the private information sets of early and late traders are separated, because late traders learn only about the new project. Why would late traders focus only on the new project when their pay-off depends on both? The intuition relies on the fact that past prices can only incorporate information which early traders know and trade on. Thus, past prices can reveal information only about the old project. If sufficient information is contained about the old project in past prices and not too much information is contained about the new project in current prices, then the late trader prefers to devote her limited research capacity to the new project only. The next Lemma formalizes this intuition.

Lemma 3 *The choice of*

$$\alpha_o^i = 0,\ \alpha_n^i = \omega_n \left(e^\kappa - 1\right)$$

is individually optimal at a given branch of announcement or no announcement, if $\varphi = 0$, *and* $(\tau_0^2 + \tau_1^2)$ *is sufficiently large compared to* τ_2^2 *at the given branch.*

Note that Lemma 3 describes only individually optimal choices as a function of equilibrium variables. Such an equilibrium might not exist. Consider the branch with no announcement. If only this branch matters, because $\pi = 0$, then learning only about the new project is not an equilibrium strategy by the following logic. If late traders learn only about the new project, the second period price, p_2, does not incorporate any new information on the old project. As early traders' private information is orthogonal to the value of the new project, and, hence, to p_2 in this case, early traders do not trade in period $t = 1$. Consequently, the price in period 1 does not incorporate any new information on the old project either. Similarly, this implies that early traders do not trade in period $t = 0$ and p_0 does not incorporate information on the old project. This results in $\tau_1 = \tau_0 = 0$ and violates the conditions of the Lemma 3. In Example 1, I used a very similar argument to show that there is no disagreement among early traders if there is no announcement and no correlation between private information sets.

The next Proposition contains one of the main results of the paper. It shows that for any given set of parameters, if there is sufficiently high probability of a sufficiently precise announcement, then there is an equilibrium where late traders learn only about the new project.

Proposition 5 *For any fixed set of parameters, there are $\bar{\beta}$ and $\bar{\pi} \in (0,1)$ that if $\beta > \bar{\beta}$ and $\pi > \bar{\pi}$, then there is an equilibrium where*

$$\alpha_o^i = 0,\ \alpha_n^i = \omega_n \left(e^{\kappa} - 1\right),\ \alpha^i = \omega_o \left(e^{\kappa} - 1\right),\ \varphi = 0.$$

The intuition is in line with the logic of Lemma 3. A more precise announcement leads to more trading resulting in more information incorporated into prices in both periods 0 and 1. That is, τ_0 and τ_1 are high. To understand why more precise announcement implies more trading, observe that in period 0, one motivation to trade is to speculate on the value of the public announcement. If the public announcement is sufficiently precise, this motivation is strong and leads to intense trading. In period 1, early traders trade if their private information on the old project helps to forecast the new projects and, hence, p_2. As Example 1 demonstrates, because the public announcement correlates with both factors, the announcement makes early traders' private signal informative about late traders' private signal. With a more precise announcement, this effect strengthens.

There is no consensus yet in the literature of REE models with rational inattention on how to specify the information-processing constraint. In particular, Van Nieuwerburgh and Veldkamp (2009a) include the price in the constraint, which would correspond to a formulation of

$$H\left([\theta_o, \theta_n]\right) - H\left([\theta_o, \theta_n] | \mathcal{I}^i, \mathcal{P}_t\right) \leq \frac{\kappa}{2}. \tag{41}$$

Thus, in their model, agents can decide how much to learn from prices and learning from prices is just as costly as learning from private signals. In contrast, I follow Peng (2004), Peng and Xiong (2006) and Mondria (2009) by including only the private signal in the information-processing constraint. In Peng (2004), and Peng and Xiong (2006) this formulation is implied by the assumption that traders cannot learn from prices at all. Just as in my specification, in Mondria (2009), traders do learn from prices, but they can learn from prices for free, i.e., it does not count towards their capacity. In my set-up, the two formulations would not give the same results. As Lemma 3 shows, the result in Proposition 5 critically depends on the intuition that agents do not want to spend resources on pieces of information which they can learn from public information. I argue that the adequate formulation of the information-processing constraint depends on the context. If the information-processing constraint is interpreted as the cognitive limitation of a decision maker who interprets various signals in isolation, probably (41) is more sensible. After all, understanding prices might not be a much easier intellectual exercise than understanding private information. However, I

interpret the information-processing constraint as the reflection of an institutional traders' ex ante problem of setting up a research unit with limited resources. In this context, there is a difference between public and private information. First of all, public information is available for free. Second, analyses of public information are available in various forms on the public domain. In contrast, gathering proprietary information is costly and it has to be analyzed by proprietary models. This relative cost difference is reflected in an extreme form in formalization (12).

4 Trading volume and information content in trades

In this section, I argue that the main implications of the model are consistent with stylized facts on the effect of new public information on the trading pattern and on the amount of information incorporated in prices through trading. In the first part of this section, I show that the contrarian and polarized higher-order expectations implied by the model result in hectic and informative trading around announcements. In the second part, I confront these implications with the existing theoretical and empirical results in the literature.

4.1 Model implications

Probably, the simplest way to measure the change in information content of prices, is the ratio of the precision of the price signals in the period of the potential announcement,

$$C \equiv \frac{\left(\tau_1^a\right)^2}{\left(\tau_1^n\right)^2}.$$

The precision of the price signal measures the additional information content of prices over and above the direct information contained in the public signal and in past prices. This additional information is the fraction of private information of traders which is aggregated into prices through the market clearing process. Given that agents private information sets are the same in the two branches, the ratio measures the effect of the announcement on the efficiency of this information aggregation process.

For measuring trading volume around announcements, I rewrite individual demand in

period 1 as

$$
\begin{aligned}
d_1^i &= \frac{E\left(p_2|x^i, y, q_1, q_0\right) - p_1}{\gamma var\left(p_2|x^i, y, q_1, q_0\right)} = \\
&= \frac{a_2 E\left(\theta_n|x^i, y, q_1, q_0\right) + g_2 E\left(\theta_o|x^i, y, q_1, q_0\right) + c_2 y + h_2 q_1 + k_2 q_0 - \left(a_1 q_1 + c_1 y + h_1 q_0\right)}{\gamma var\left(p_2|x^i, y, q_1, q_0\right)} = \\
&= \frac{\left(a_2 a_n + a_o g_2\right) x^i + \left(a_2 b_n + g_2 b_o + h_2 - a_1\right) q_1}{\gamma var\left(p_2|x^i, y, q_1, q_0\right)} = \\
&= \frac{\tau_1}{\delta_1}\left(x^i - q_1\right) = \frac{\tau_1}{\delta_1}\varepsilon^i + u_1
\end{aligned} \tag{42}
$$

using (20), (28) and (33)-(36). This shows that the trader evaluates the difference between her private signal x^i and the price signal, q_1, which is a noisy aggregator of the private signal of others, and holds the risky asset proportionally to the difference. As the proportionality is determined by $\frac{\tau_1}{\delta_1}$, $\frac{\tau_1}{\delta_1}$ is also a measure of the aggressiveness of the trader. The last equation shows that the trader's position is the sum of two parts. Each trader holds her share of the risky asset supply, u_1, and supplements this with a speculative position, $\frac{\tau_1}{\delta_1}\varepsilon^i$, depending on her aggressiveness and her private information. Similarly, the demand in period 0 simplifies to

$$
d_0^i = \frac{E\left(p_1|x^i, q_0\right) - p_0}{\gamma var\left(p_1|x^i, q_0\right)} = \frac{\tau_0}{\delta_0}\left(x^i - q_0\right) = \frac{\tau_0}{\delta_0}\varepsilon^i + u_0. \tag{43}
$$

Given expressions (43) and (42), I introduce two measures of trading volume around the public announcement. First,

$$
V^{11} \equiv \frac{\frac{1}{2}E_{u_1}\left(\int_0^1 |d_1^i| di\right)}{\frac{1}{2}E_{u_1}\left(\int_0^1 |d_1^i| di\right)|_{\beta=0}} = \frac{|\tau_1^a|}{|\tau_1^n|}
$$

measures the proportion of the average absolute position of traders in period 1 with and without announcement. It is apparent that this measure of trading volume is a monotonic function of the measure of the changing informativeness of trades around announcements as $V^{11} = \sqrt{C}$. The second trading volume measure,

$$
\begin{aligned}
V^{01} \equiv \frac{1}{2}E_{u_1-u_0}\left(\int_0^1 \left|d_1^i - d_0^i\right| di\right) = \frac{1}{2}E_{u_1-u_0}\left(\int_{-\infty}^{\infty}\left|\left(\frac{\tau_1^a}{\delta_1} - \frac{\tau_0^a}{\delta_0}\right)\varepsilon^i\right| \phi\left(\varepsilon^i\right) d\varepsilon^i\right) = \\
= \left|\frac{\tau_1^a}{\delta_1} - \frac{\tau_0^a}{\delta_0}\right| \frac{1}{\alpha\sqrt{2\pi}}
\end{aligned}
$$

measures the change of positions before and after the announcement. It is apparent that all three measures depend only on the coefficients $\frac{\tau_t}{\delta_t}$, $t = 0, 1$.

The two measures of trading volume are representative of the two major classes of empirical strategies to measure the effect of an announcement on trading activity.[16] V^{11} is a cross-sectional type of measure corresponding to empirical strategies where the effect of announcement is assessed by comparing periods with and without the announcement. In contrast, V^{01} is a time-series type of measure corresponding to event-study type of empirical strategies where activity in the pre-announcement period is compared to activity in the post-announcement period. In some sense, V^{11} is a purer measure of the effect of the announcement because it keeps the trading incentives constant. It compares the trading patterns of two sets of early traders trading in period 1. In contrast, the time-series version, V^{01}, not only measures the effect of the announcement, but also the effect of changing incentives for trade between period 0 and 1. Most importantly, recall that a period 0 trader speculates on the content of the announcement, y. Given that her private information influences her guess on y, she builds up a speculative position on y, as shown by the presence of the c_1 coefficient in (51). In period 1, y becomes common knowledge, and traders liquidate these speculative positions.

As a benchmark, it is useful to recall that in the standard, differential information model of Diamond and Verrecchia (1981) and Hellwig (1980) (just as in representative agent models) a public announcement changes prices but does not generate trade. The reason is that the announcement affects demand through two channels. It decreases disagreement among traders and increases the precision of their estimates of θ. The former decreases the numerator of the standard demand function,

$$d^i = \frac{E\left(\theta|\mathcal{I}^i, \mathcal{P}\right) - p}{Var\left(\theta|\mathcal{I}^i, \mathcal{P}\right)},$$

while the latter decreases the denominator. Under the standard information structure, the two forces exactly cancel out. The next Lemma reproduces this observation within our model if $\alpha_n = 0$, and $\omega_n, \delta_1, \delta_2 \to \infty$. That is, when each trader learns only about the old project and the importance of the new project is very small, the final payoff depends almost only on the old project. Moreover, if $\delta_1, \delta_2 \to \infty$, the payoff is virtually the same in each period, because prices are close to fully revealing.

Lemma 4

1. *Suppose that late traders gather private information only about the old project,* $\alpha_n^i = 0$,

[16] For example, Evans and Lyons (2008) and Love and Payne (2008) measure the change of trading volume around announcements in the spirit of V^{11}, while Green (2004) uses both V^{01} and V^{11} types of measures.

then

$$\lim_{\omega_n \to \infty} \frac{\tau_2^a}{\delta_2} = \lim_{\omega_n \to \infty} \frac{\tau_2^n}{\delta_2} = \frac{\alpha}{\gamma}$$

2. *Regardless of the choice of late traders,*

$$\lim_{\omega_n, \delta_1, \delta_2 \to \infty} \frac{\tau_0^a}{\delta_0} = \lim_{\omega_n, \delta_1, \delta_2 \to \infty} \frac{\tau_0^n}{\delta_0} = \frac{\alpha}{\gamma}.$$

3. *Suppose that late traders gather some private information about the old project,* $\alpha_o^i > 0$, *then*

$$\lim_{\omega_n, \delta_2 \to \infty} \frac{\tau_1^a}{\delta_1} = \lim_{\omega_n, \delta_2 \to \infty} \frac{\tau_1^n}{\delta_1} = \frac{\alpha}{\gamma}.$$

He and Wang (1995) points out that the introduction of multiple trading periods does not change this conclusion. Even in a multiple period version of Hellwig (1980), all of our measures, V^{01}, V^{11}, and C would be zero.

Returning to our set-up, it is useful to define

$$\lambda^i \equiv \frac{1}{\kappa} \ln \frac{(\alpha_o^i + \omega_o)}{\omega_o} = 1 - \frac{1}{\kappa} \ln \frac{(\alpha_n^i + \omega_n)}{\omega_n}, \tag{44}$$

the fraction of attention which a late trader i devotes to the old project. With some abuse of notation, I denote by $V^{01}(\lambda), V^{11}(\lambda)$, and $C(\lambda)$, the values of the measures of volume and the measure of information content under an exogenously given $\lambda^i = \lambda$ informational choice of each late traders.

Starting with a numerical example, Figure 1 helps us to understand how volume and the information content of prices change under different information choices. Figure 1 shows a typical plot of three curves: the curve with the asterisks and the one with the squares depict the aggressiveness of traders, $\frac{\tau_t^a}{\delta_t}$, in periods $t = 0, 1$, keeping the precision of the announcement at a constant positive level. The dashed curve shows the aggressiveness in period 1 when there is no announcement: $\frac{\tau_1^n}{\delta_1}$. On the x-axis, I measure the information choice of late traders parameterized by λ.

The right hand side of the figure shows that when late traders learn only about the old project ($\lambda = 1$), all three coefficients, $\frac{\tau_t^a}{\delta_t}$ $t = 0, 1$ and $\frac{\tau_1^n}{\delta_1}$ are close to each other. That is, our trading volume measures, $V^{01}(1), V^{11}(1)$, are small and there is little change in informativeness of prices, $C(1)$, due to the announcement. The figure also illustrates that as λ decreases, the three curves fan out. Although the ratio between $\frac{\tau_1^a}{\delta_1}$ and $\frac{\tau_1^n}{\delta_1}$ and the distance between $\frac{\tau_1^a}{\delta_1}$ and $\frac{\tau_0^a}{\delta_0}$ might not be minimal at $\lambda = 1$, both reach their respective maxima at

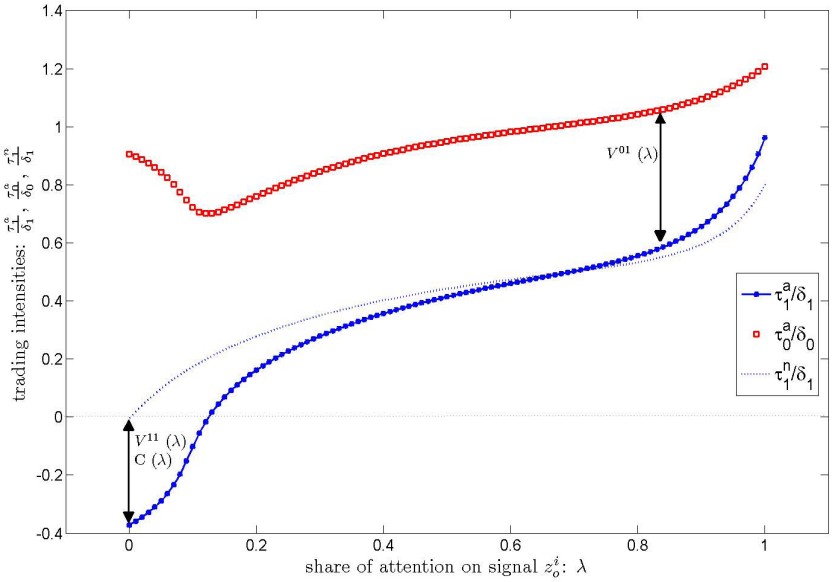

Figure 1: Trading intensities with announcement, $\frac{\tau_t^a}{\delta_t}$ in $t = 0, 1$ and the trading intensity without announcement, $\frac{\tau_1^n}{\delta_1}$, in period 1, as a function of the fraction of attention which late traders devote to the old project, $\lambda^i = \lambda$. The curve with asterisks, the curve with squares and the dashed curve are $\frac{\tau_1^a}{\delta_1}$, $\frac{\tau_0^a}{\delta_0}$ and $\frac{\tau_1^n}{\delta_1}$, respectively. The distance $\left|\frac{\tau_1^a}{\delta_1} - \frac{\tau_0^a}{\delta_0}\right|$ is proportional to V^{01} measure of trading volume and the proportion $\frac{|\tau_1^a|}{|\tau_1^n|}$ is proportional to V^{11} measure of trading volume and C, the information content channeled into prices through trading. Parameters are $\delta_0 = \gamma = \omega_o = \beta = 1$, $\omega_n = \delta_1 = \delta_2 = 3$, $\kappa = \ln 2$.

$\lambda = 0$.[17] This implies that the measures $V^{01}(\lambda), V^{11}(\lambda)$, and $C(\lambda)$ are also reaching their maxima at $\lambda = 0$, which corresponds to the equilibrium under the conditions of Proposition 5.

Consider first the argument behind the comparative statics of the measures $V^{11}(\lambda)$ and $C(\lambda)$. Rewriting the market clearing conditions,(45) and (31), as

$$\begin{aligned} \int E\left(\theta | z_n^i, z_o^i, y, q_2, q_1, q_0\right) di - \gamma var\left(\theta | z_n^i, z_o^i, y, q_2, q_1, q_0\right) &= p_2 \\ \int E\left(p_2 | x^i, y, q_1, q_0\right) di - \gamma var\left(p_2 | x^i, y, q_0, q_0\right) &= p_1 \end{aligned}$$

shows that the price, p_2, is a noisy measure of the average fundamental expectation of late traders, while the price, p_1, is a noisy measure of the average second order expectation of early traders. Thus, the numerator of the demand function, (28), is a noisy measure of the second-

[17] Lemma 4 shows that all three curves will be arbitrarily close, if $\omega_n, \delta_2, \delta_1$ are sufficiently large, at any $\lambda \neq 0$.

order disagreement between trader i and the average investor. When $\lambda = 1$, both early and late traders observe signals on the same factor, so their private information sets are highly correlated. Thus, condition (7) is not satisfied and the public announcement decreases the disagreement among early traders. The new information still decreases conditional variance of agent i's estimation of p_2, $var\left(p_2|\mathcal{I}^i, \mathcal{P}_t\right)$. Thus, the intuition I pointed out for the standard case still holds. Even if the two opposing forces affecting the demand function do not cancel each other out exactly, they result in a small change across $\frac{\tau_1^a}{\delta_1}$ and $\frac{\tau_1^n}{\delta_1}$. In contrast, when $\lambda = 0$, condition (7) is satisfied, the announcement causes polarization in second-order expectations. Thus, the numerator of the demand function increases while the denominator decreases, leading to a large change between $\frac{\tau_1^a}{\delta_1}$ and $\frac{\tau_1^n}{\delta_1}$.

Analytically, I prove in the following Proposition that as $\lambda = 0$ implies $\frac{\tau_1^n}{\delta_1} = 0$ for any parameters, volume, $V^{11}(\lambda)$, and information content, $C(\lambda)$, are maximal when late traders learn only about the new project.

Proposition 6 *For any parameter values*

$$0 = \arg\max_{\lambda} V^{11}(\lambda) = \arg\max_{\lambda} C(\lambda).$$

To understand the comparative statics of the measure $V^{01}(\lambda)$, recall that this measure is mainly influenced by the different incentives to form a portfolio in periods 0 and 1. Demand in period 0 is largely influenced by speculation on the content of the public announcement. Given that y is a noisy version of θ this speculation is driven by fundamental expectations. In contrast, in period 1, y is common knowledge, thus the portfolio formation is driven by expectations of p_2, that is, second-order expectations. When $\lambda = 1$, late traders and early traders learn about the same factor, which results in a informational structure close to the standard case characterized by (8). Condition (3) is not satisfied, and second-order expectations are not contrarian. That is, the most optimistic agents still have the largest positive positions. The small rebalancing shown by the difference between $\frac{\tau_1^a}{\delta_1}$ and $\frac{\tau_0^a}{\delta_1}$ is largely due to the fact that second-order expectations overweight the public signal and underweight the private signal compared to fundamental expectation as higlighted by Allen, Morris and Shin (2006). However, when $\lambda = 0$, the rebalancing is much more radical. Condition (3) is satisfied, second-order expectations are contrarian, implying that the most optimistic agents about the public announcement are also the most pessimistic about the price p_2. Thus, they fully liquidate their speculative position on y, and take a position of the opposite sign to speculate on p_2. This implies a large trading volume measured by $V^{01}(0)$.

Although experimentation with a wide set of parameter values shows that $V^{01}(\lambda)$ is maximal at $\lambda = 0$, I prove only a weaker analytical statement in the next proposition. In

particular, the choice $\lambda = 0$ maximizes $V^{01}(\lambda)$, at least if δ_1, δ_2 and ω_n are sufficiently large.

Proposition 7 *There are thresholds $\bar{\delta}_1, \bar{\delta}_2, \bar{\omega}$ that if $\delta_1 > \bar{\delta}_1, \delta_2 > \bar{\delta}_2, \omega_n > \bar{\omega}_n$ then*

$$0 = \arg\max_{\lambda} V^{01}(\lambda).$$

In line with Lemma 4, the parameter restriction of the proposition implies that for any $\lambda > 0$, the structure of the model is sufficiently close to the standard, one-period model of Diamond and Verrecchia (1981) and Hellwig (1980).

4.2 Stylized facts and the previous literature on trading around announcements

The stylized fact that the trading volume of stocks increases around earnings announcements has been known for decades (e.g. Beaver (1968), Bamber (1987), Ziebart (1990), Kandel and Pearson (1995)). As Lemma 4 illustrates, this was considered a puzzle because neither in representative agent models, nor in standard differential information models should the price adjustment caused by the new common information be accompanied by abnormal trading volume. Motivated by this fact, Kim and Verrecchia (1991) introduce heterogeneous risk-aversion and information precision, while He and Wang (1995) introduce residual uncertainty for the final pay-off into a dynamic version of the standard model. They point out that small modifications on the standard framework does lead to some trading volume. However, in these models only the measure V^{01}, but not V^{11} or C would increase significantly. That is, trading volume increases because agents build up speculative positions before the announcement which they liquidate after observing the announcement. Informative trading does not increase after the announcement, because disagreement decreases.

In contrast, Kandel and Pearson (1995) argue that only a model where public announcement polarizes agents' valuation of the asset is consistent with the data. This picture is enforced by more recent literature using high-frequency data sets (e.g. Fleming and Remolona (1999), Green (2004), Evans and Lyons (2008), Love and Payne (2008)). This literature shows that around public announcements (macroeconomic news), new information is incorporated into prices by two channels. Apart from the direct price effect of the public announcement, information is also incorporated indirectly, through informative trading after the announcement. That is, C and V^{11} are high. These papers also demonstrate that the stylized fact is true across various markets. In particular, Fleming and Remolona (1999) and Green (2005) focus on the market for US treasuries, while Evans and Lyons (2008) and Love and Payne (2008) analyze markets of major currencies. The implications of the current

model are consistent with these results.

In line with the empirical results, the majority of the literature is settled on the conclusion that these observations can be explained only by models where agents do not agree on interpretation of the public announcement as the signal about the fundamental asset value. That is, agents look at the same piece of information, but do not see the same piece of information. Two groups of papers achieve this differently. Following Varian (1989), Harris and Raviv (1994) and Kandel and Person (1995) relax the common prior assumption of the standard model. In contrast, Kim and Verrechia (1997) assume that agents observe a private signal about the error term of the public signal.

The financial application in this paper contributes to the literature of trading around announcements in two ways. First, I highlight that a public announcement can polarize market participants' valuation of the asset, even if it does not polarize their fundamental expectation. This is so because market participants often expect to resell their assets, hence their valuation depends on future price, that is, on higher-order expectations. Thus, even if each agent sees the public announcement as the same piece of information if it is interpreted as a signal about the fundamental value, she can still interpret it differently, if it is interpreted as a signal of the opinion of others. This is a useful result because conditions for polarization conditions for polarization in higher-order expectations are weaker than conditions in polarization in first-order expectations.

Second, I propose a model where the information structure leading to polarized asset valuations endogenously arises from the ex-ante optimal choice of agents. This is a step forward from the models where either the particular priors or the particular information structure is picked arbitrarily in order to generate polarization. In this sense, the model can be read as the missing microfoundation for information structures resulting in differential interpretation of the public announcement.

5 Conclusion

In this paper, I have characterized Gaussian information structures with two properties. I have shown that when the connection between private signals is sufficiently weak, a public announcement leads to contrarian and polarized higher-order expectations regardless of the content of the announcement. I illustrated the economic relevance of these properties by a noisy rational expectations model of financial markets where agents are subject to rational inattention. I have argued that information structures with these properties endogenously arise if we let agents choose what to learn about. I have also shown that these properties can explain stylized facts of trading patterns around announcements like high trading volume

and informative trading.

I believe that this observation has economic implication in a wide range of context. As another example, in a companion paper, Kondor (2009), I analyze a version of the speculative currency attack model of Morris and Shin (1998) where the central bank has imperfect knowledge of the state of the economy. To assess the probability of a devaluation, speculators have to second guess the expectation of the central bank. I show that the fact that a public announcement can polarize higher-order expectations implies that generating and disclosing more public information can destabilize the exchange rate system.

As further research, empirical analyses on the relative effect of announcements on trading patterns across markets with different characteristics (e.g. frequency of announcements, the importance of private information etc.) could help to establish the relative importance of the presented mechanism across different markets.

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Appendix

A.1 Price and demand function coefficients in periods 0 and 2

From (27) and (23), the market clearing condition in period 2 is

$$D_2 = \frac{a_\theta \theta_n + g_\theta \theta_o + b_\theta q_2 + c_\theta y + h_\theta q_1 + k_\theta q_0 - p_2}{\gamma var(\theta | z_n^i, z_o^i, y, q_2, q_1, q_0)} = u_2. \tag{45}$$

Using (19) gives

$$a_\theta \theta_n + g_\theta \theta_o + b_\theta q_2 + c_\theta y + h_\theta q_1 + k_\theta q_0 - \gamma var\left(\theta | z_n^i, z_o^i, y, q_2, q_1, q_0\right) u_2 = q_2 \left(a_2 + g_2\right) + c_2 y + h_2 q_1 + k_2 q_0$$

Setting the coefficients of random variables on the two sides of the equation to equal gives

$$c_2 = c_\theta, \;\; h_2 = h_\theta, \;\; k_2 = k_\theta \tag{46}$$

and

$$a_\theta \theta_n + g_\theta \theta_o - \gamma var\left(\theta | z_n^i, z_o^i, y, q_2, q_1, q_0\right) u_2 = \left(a_2 + g_2 - b_\theta\right) q_2.$$

Substituting in $q_2 = \frac{a_2}{g_2 + a_2}\theta_n + \frac{g_2}{a_2 + g_2}\theta_o - \frac{e_2}{a_2 + g_2} u_2$ and collecting the coefficients of θ_n, θ_o and u_2 give

$$g_2 = \frac{\left(a_\theta + g_\theta + b_\theta\right) g_\theta}{a_\theta + g_\theta} \tag{47}$$

$$a_2 = \frac{\left(a_\theta + g_\theta + b_\theta\right) a_\theta}{a_\theta + g_\theta}, \tag{48}$$

and

$$\frac{a_\theta + g_\theta}{\gamma var\left(\theta | z_n^i, z_o^i, y, q_2, q_1, q_0\right)} = \frac{a_2 + g_2}{e_2}.$$

Note that $\frac{a_2 + g_2}{e_2} = \frac{\tau_2}{\delta_2}$ by definition and the undetermined coefficients enter this expression only through φ and τ_t, $t = 0, 1, 2$. Thus, I rewrite the last equation as

$$\tau_2 = F_2\left(\tau_2, \tau_1, \tau_0, \varphi\right). \tag{49}$$

where

$$F_2\left(\tau_2, \tau_1, \tau_0, \varphi\right) \equiv \delta_2 \frac{a_\theta + g_\theta}{\gamma var\left(\theta | z_n^i, z_o^i, y, q_2, q_1, q_0\right)}. \tag{50}$$

The market clearing condition in period 0 is

$$E\left(p_1|x^i,q_0\right)-p_0=\gamma var\left(p_1|x^i,q_0\right)u_0,$$

or

$$\left(a_1+c_1\right)\left(a_{o0}\theta_o+b_{o0}q_0\right)+h_1q_0-a_0q_0=\gamma var\left(p_2|x^i,q_0\right)u_0$$

which, after substituting in $q_0=\theta_o-\frac{e_0}{a_0}u_0$ and collecting the coefficients of θ_o and u_1, implies

$$a_0=\left(a_1+c_1\right)\left(a_{o0}+b_{o0}\right)+h_1$$

and

$$\frac{\left(a_1+c_1\right)a_o}{\gamma var\left(p_1|x^i,q_0\right)}=\frac{\tau_0}{\delta_0}. \tag{51}$$

I rewrite this expression as

$$\tau_0=F_0\left(\tau_2,\tau_1,\tau_0,\varphi\right) \tag{52}$$

where

$$F_0\left(\tau_2,\tau_1,\tau_0,\varphi\right)=\frac{\left(a_1+c_1\right)a_{o0}}{\gamma var\left(p_1|x^i,q_0\right)}\delta_0. \tag{53}$$

A.2 Conditional expectations, variances and the functions, $F_t\left(\cdot\right)$

The variance-covariance matrix of the vector[$\theta,z_n,z_o,q_2,q_1,q_0,y$] is

$$\begin{pmatrix}
\frac{1}{\omega_n}+\frac{1}{\omega_o} & \frac{1}{\omega_n} & \frac{1}{\omega_o} & \frac{(1-\varphi)}{\omega_n}+\frac{\varphi}{\omega_o} & \frac{1}{\omega_o} & \frac{1}{\omega_o} & \frac{1}{\omega_n}+\frac{1}{\omega_o} \\
\frac{1}{\omega_n} & \frac{1}{\omega_n}+\frac{1}{\alpha_n} & 0 & \frac{(1-\varphi)}{\omega_n} & 0 & 0 & \frac{1}{\omega_n} \\
\frac{1}{\omega_o} & 0 & \frac{1}{\omega_o}+\frac{1}{\alpha_o} & \frac{\varphi}{\omega_o} & \frac{1}{\omega_o} & \frac{1}{\omega_o} & \frac{1}{\omega_o} \\
\frac{(1-\varphi)}{\omega_n}+\frac{\varphi}{\omega_o} & \frac{(1-\varphi)}{\omega_n} & \frac{\varphi}{\omega_o} & \frac{(1-\varphi)^2}{\omega_n}+\frac{\varphi^2}{\omega_o}+\frac{1}{\tau_2^2} & \frac{\varphi}{\omega_o} & \frac{\varphi}{\omega_o} & \frac{(1-\varphi)}{\omega_n}+\frac{\varphi}{\omega_o} \\
\frac{1}{\omega_o} & 0 & \frac{1}{\omega_o} & \frac{\varphi}{\omega_o} & \frac{1}{\omega_o}+\frac{1}{\tau_1^2} & \frac{1}{\omega_o} & \frac{1}{\omega_o} \\
\frac{1}{\omega_o} & 0 & \frac{1}{\omega_o} & \frac{\varphi}{\omega_o} & \frac{1}{\omega_o} & \frac{1}{\omega_o}+\frac{1}{\tau_0^2} & \frac{1}{\omega_o} \\
\frac{1}{\omega_n}+\frac{1}{\omega_o} & \frac{1}{\omega_n} & \frac{1}{\omega_o} & \frac{(1-\varphi)}{\omega_n}+\varphi\frac{1}{\omega_o} & \frac{1}{\omega_o} & \frac{1}{\omega_o} & \frac{1}{\omega_n}+\frac{1}{\omega_o}+\frac{1}{\beta}
\end{pmatrix}.$$

By the projection theorem, the coefficient vector $[a_\theta, g_\theta, b_\theta, h_\theta, k_\theta, c_\theta]$ is

$$A \begin{pmatrix} \alpha_n \left(\alpha_o + \omega_o + \tau_0^2 + \tau_1^2 + \varphi\tau_2^2 \left(2\varphi - 1\right)\right) \\ \alpha_o \left(\alpha_n + \omega_n + \tau_2^2 \left(2\varphi - 1\right)\left(\varphi - 1\right)\right) \\ \tau_2^2 \left(\left(1-\varphi\right)\left(\alpha_o + \omega_o + \tau_0^2 + \tau_1^2\right) + \varphi\left(\alpha_n + \omega_n\right)\right) \\ \tau_1^2 \left(\alpha_n + \omega_n + \tau_2^2 \left(2\varphi - 1\right)\left(\varphi - 1\right)\right) \\ \tau_0^2 \left(\alpha_n + \omega_n + \tau_2^2 \left(2\varphi - 1\right)\left(\varphi - 1\right)\right) \\ \beta\left(\alpha_o + \omega_o + \alpha_n + \omega_n + \tau_0^2 + \tau_1^2 + \tau_2^2 \left(2\varphi - 1\right)^2\right) \end{pmatrix}$$

where

$$A = \frac{1}{\beta(\alpha_o+\omega_o+\alpha_n+\omega_n)+(\alpha_n+\omega_n)(\alpha_o+\omega_o)+\left(\tau_1^2+\tau_0^2\right)(\beta+\alpha_n+\omega_n)+\tau_2^2\left(\varphi^2(\beta+\alpha_n+\omega_n)+\left(\beta+\alpha_o+\omega_o+\left(\tau_1^2+\tau_0^2\right)\right)(1-\varphi)^2-2\beta\varphi(1-\varphi)\right)}.$$

The conditional variance of θ is

$$Var\left(\theta|z_n, z_o, q_2, q_1, q_0, y\right) = \left(\alpha_o + \omega_o + \alpha_n + \omega_n + \tau_0^2 + \tau_1^2 + \tau_2^2 \left(2\varphi - 1\right)^2\right) A. \tag{54}$$

Similarly, the coefficient vector of the conditional expectation of early traders on θ_n after observing the public signal, $[a_n, b_n, h_n, c_n]$, is

$$B \begin{pmatrix} -\beta\alpha \\ -\beta\tau_1^2 \\ -\beta\tau_0^2 \\ \beta\left(\alpha + \omega_o + \tau_0^2 + \tau_1^2\right) \end{pmatrix}$$

where

$$B = \frac{1}{\beta(\alpha+\omega_o+\omega_n)+\omega_n(\alpha+\omega_o)+\left(\tau_0^2+\tau_1^2\right)(\beta+\omega_n)}$$

and on θ_o ,$[a_o, b_o, h_o, c_o]$, is

$$\begin{pmatrix} \alpha\left(\beta + \omega_n\right) \\ \tau_1^2\left(\beta + \omega_n\right) \\ \tau_0^2\left(\beta + \omega_n\right) \\ \beta\omega_n \end{pmatrix} B.$$

The variance-covariance matrix of θ_n, θ_o conditional on early traders' information set is

$$Var\left(\theta_n, \theta_o|x^i, q_1, q_0, y\right) = B \begin{pmatrix} \beta + \alpha + \omega_o + \tau_0^2 + \tau_1^2 & -\beta \\ -\beta & \beta + \omega_n \end{pmatrix}.$$

In period 0, the coefficients of the conditional expectation of $\theta_o, [a_{o0}, b_{o0}]$ are

$$\begin{pmatrix} \frac{\alpha}{\alpha+\omega_o+\tau_0^2} \\ \frac{\tau_0^2}{\alpha+\omega_o+\tau_0^2} \end{pmatrix}$$

and

$$Var\left(\theta_n, \theta_o | x^i, q_0\right) = \begin{pmatrix} \frac{1}{\omega_n} & 0 \\ 0 & \frac{1}{\alpha+\omega_o+\tau_0^2} \end{pmatrix}. \tag{55}$$

Substituting in expressions for a_θ, g_θ into (50) gives

$$F_2\left(\tau_0, \tau_1, \tau_2, \varphi\right) \equiv \delta_2 \frac{\alpha_o(\alpha_n+\omega_n)+\alpha_n(\alpha_o+\omega_o)+\alpha_n\left(\tau_1^2+\tau_0^2\right)+\tau_2^2(2\varphi-1)(\alpha_o(\varphi-1)+\varphi\alpha_n)}{\left(\alpha_o+\omega_o+\alpha_n+\omega_n+\left(\tau_1^2+\tau_0^2\right)+\tau_2^2(2\varphi-1)^2\right)\gamma}.$$

Similarly, using (48),(47),(19) and expressions for a_n, a_o, substitution into (38) gives

$$F_1\left(\tau_0, \tau_1, \tau_2, \varphi\right) \equiv \delta_1 \frac{a_2 a_n + g_2 a_o}{\gamma var(p_2|x^i, y, q_1, q_0)} =$$
$$= \delta_1 \frac{1}{(a_\theta+g_\theta+b_\theta)\gamma} \frac{(1-\varphi)a_n+\varphi a_o}{var((1-\varphi)\theta_n+\varphi\theta_o|x^i, y, q_1, q_0)+\frac{1}{\tau_2^2}} =$$
$$= \frac{\delta_1}{(a_\theta+g_\theta+b_\theta)\gamma} \frac{\alpha(\varphi\omega_n+\beta(2\varphi-1))}{(1-\varphi)^2\left(\beta+\alpha+\omega_o+\tau_0^2+\tau_1^2\right)+\varphi^2(\beta+\omega_n)-\varphi(1-\varphi)\beta+\frac{\beta(\alpha+\omega_o+\omega_n)+\omega_n(\alpha+\omega_o)+\left(\tau_0^2+\tau_1^2\right)(\beta+\omega_n)}{\tau_2^2}}.$$

Also, using (20) and the expressions for a_o, substitution into (51) results in

$$F_0\left(\tau_0, \tau_1, \tau_2, \varphi\right) \equiv \frac{(a_1+c_1)a_{o0}}{\gamma var(p_1|x^i, q_0)}\delta_0 = \delta_0 \frac{\alpha}{\gamma} \frac{(a_1+c_1)}{(a_1+c_1)^2+\left(\alpha+\omega_o+\tau_0^2\right)\left(\frac{a_1^2}{\tau_1^2}+c_1^2\left(\frac{1}{\beta}+\frac{1}{\omega_n}\right)\right)},$$

where a_1 and c_1 are given by (35) and (33).

Finally,

$$F_3\left(\tau_0, \tau_1, \tau_2, \varphi, \right) \equiv \frac{g_\theta}{a_\theta + g_\theta} = \tag{56}$$
$$= \frac{\alpha_o\left(\alpha_n+\omega_n-\tau_2^2(2\varphi-1)(1-\varphi)\right)}{\alpha_o(\alpha_n+\omega_n)+\alpha_n(\alpha_o+\omega_o)+\alpha_n\left(\tau_1^2+\tau_0^2\right)-\tau_2^2(2\varphi-1)\alpha_o(1-\varphi)+\tau_2^2(2\varphi-1)\varphi\alpha_n}.$$

A.3 Proof of Proposition 4

Define the functions

$$\hat{F}_3(\tau_0,\tau_1,\tau_2,\varphi) = \left\{ \begin{array}{ccc} F_3 & if & a_\theta, g_\theta \geq 0 \\ 0 & if & g_\theta < 0 \\ 1 & if & a_\theta < 0 \end{array} \right\} \tag{57}$$

$$\hat{F}_2(\tau_0,\tau_1,\tau_2,\varphi) = \left\{ \begin{array}{ccc} F_2 & if & a_\theta + g_\theta \geq 0 \\ 0 & if & g_\theta + a_\theta < 0 \end{array} \right\},$$

and

$$\hat{F}_1(\tau_0,\tau_1,\tau_2,\varphi) = \tag{58}$$

$$= \left(\begin{array}{ccc} F_1 & if & g_\theta + a_\theta \geq 0 \\ \frac{\delta_1 \frac{\alpha_o(\varphi\omega_n+\beta(2\varphi-1))\delta_1\tau_2^2}{b_\theta\gamma}}{\left((1-\varphi)^2\left(\beta+\alpha_o+\omega_o+\tau_0^2+\tau_1^2\right)-\varphi(1-\varphi)\beta\right)\tau_2^2+\beta(\alpha_o+\omega_o+\omega_n)+\omega_n(\alpha_o+\omega_o)+\left(\tau_2^2\varphi^2+\tau_0^2+\tau_1^2\right)(\beta+\omega_n)} & if & g_\theta + a_\theta < 0 \end{array} \right).$$

Observe that $[F_0, \hat{F}_1, \hat{F}_2, \hat{F}_3]$ are continuous functions on $\Re^4$, all bounded from below and above. Thus, the finite constants defined as

$$\begin{aligned} \bar{\tau}_2 &= \arg \sup_{[\tau_0,\tau_1,\tau_2,\varphi]} \hat{F}_2 \\ \bar{\tau}_1 &= \arg \sup_{[\tau_0,\tau_1,\tau_2,\varphi]\in S} \hat{F}_1 \\ \underline{\tau_1} &= \arg \inf_{[\tau_0,\tau_1,\tau_2,\varphi]\in S} \hat{F}_1 \\ \bar{\tau}_0 &= \arg \sup_{[\tau_0,\tau_1,\tau_2,\varphi]\in S} F_0 \end{aligned}$$

exist. For the existence proof, my strategy is to show that a fixed point $[\tau_0^*, \tau_1^*, \tau_2^*, \varphi^*]$ of the system $[F_0, \hat{F}_1, \hat{F}_2, \hat{F}_3]$ exists, and that $[\tau_0^*, \tau_1^*, \tau_2^*, \varphi^*]$ is also a fixed point of the original $[F_0, F_1, F_2, F_3]$ system. I use the Brouwer fixed point theorem. Define the compact set

$$S = \left\{ \begin{array}{c} [\tau_0,\tau_1,\tau_2,\varphi]: \\ 0 \leq \varphi \leq 1 \\ 0 \leq \tau_0 \leq \bar{\tau}_0 \\ \underline{\tau_1} \leq \tau_1 \leq \bar{\tau}_1 \\ 0 \leq \tau_2 \leq \bar{\tau}_2 \end{array} \right\}.$$

Observe that by definition, $[F_0, \hat{F}_1, \hat{F}_2, \hat{F}_3]$ is continuous on S and maps S to S. Thus,

the fixed point $[\tau_0^*, \tau_1^*, \tau_2^*, \varphi^*] \in S$ exists. Also, if at the fixed point, $a_\theta > 0$ and $g_\theta > 0$, then $[\tau_0^*, \tau_1^*, \tau_2^*, \varphi^*]$ is also a fixed point of the original system $[F_0, F_1, F_2, F_3]$.

First, consider the possibility that $a_\theta + g_\theta < 0$. This would imply that $\tau_2^* = 0$. However, this cannot be a fixed point as $F_2(\tau_0, \tau_1, 0, \varphi) > 0$, for any τ_0 and τ_1 and φ. Second, suppose that $a_\theta + g_\theta > 0$, but $a_\theta < 0$. This implies that $\varphi^* = 1$. However, if $\varphi = 1$, then $a_\theta > 0$. Similarly, suppose that $g_\theta < 0$. This implies $\varphi^* = 0$ which is in contradiction with $g_\theta < 0$.

A.4 Proof of Lemma 1

I show only the derivation of (40). The argument for (39) is analogous.

Note that by (27) and the law of iterated expectations, we can rewrite the objective function in (11) as

$$\begin{aligned} E\left(\max_{d_2^i(\mathcal{I}^i,\mathcal{P}_2)} E\left(-e^{-\gamma W_2^i}|\mathcal{I}^i, \mathcal{P}_2\right)\right) &= E\left(\exp\left(-\frac{1}{2}\frac{\left(E\left(\theta|\mathcal{I}^i, \mathcal{P}_2\right) - p_2\right)^2}{var\left(\theta|\mathcal{I}^i, \mathcal{P}_2\right)}\right)\right) = \\ &= E\left(E\left(\exp\left(-\frac{1}{2}\frac{\left(E\left(\theta|\mathcal{I}^i, \mathcal{P}_2\right) - p_2\right)^2}{var\left(\theta|\mathcal{I}^i, \mathcal{P}_2\right)}\right)|p_2\right)\right) \quad (59) \end{aligned}$$

Keeping in mind that conditional expectations of normal variables are linear and that linear functions of normal variables are normal and using twice the property that for any normal variable with $X\tilde{}N(\mu, \sigma^2)$

$$E\left(\exp\left(\frac{X^2}{2}\right)\right) = \frac{1}{\sqrt{1+\sigma^2}}\exp\left(-\frac{\mu^2}{2\left(1+\sigma^2\right)}\right)$$

holds, I rewrite (59) as

$$\begin{aligned} &E\left(E\left(\exp\left(-\frac{1}{2}\frac{\left(E\left(\theta|\mathcal{I}^i, \mathcal{P}_2\right) - p_2\right)^2}{var\left(\theta|\mathcal{I}^i, \mathcal{P}_2\right)}\right)|p_2\right)\right) = \\ &= E\left(\frac{1}{\sqrt{1+\sigma_A^2}}\exp\left(-\frac{\mu_A^2}{2\left(1+\sigma_A^2\right)}\right)\right) = \frac{1}{\sqrt{1+\sigma_A^2}}\frac{1}{\sqrt{1+\sigma_B^2}}E\left(\exp\left(-\frac{\mu_B^2}{2\left(1+\sigma_B^2\right)}\right)\right) \end{aligned}$$

where

$$\begin{aligned}
\sigma_A^2 &= \frac{var\left(E\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)|p_2\right)}{var\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)}, \\
\mu_A &= E\left(\frac{E\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)-p_2}{\sqrt{var\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)}}|p_2\right) = \frac{E\left(E\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)|p_2\right)-p_2}{\sqrt{var\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)}} = \frac{E\left(\theta|p_2\right)-p_2}{\sqrt{var\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)}}, \\
\sigma_B^2 &= Var\left(\frac{\mu_A}{\sqrt{1+\sigma_A^2}}\right) = \frac{\frac{var(E(\theta|p_2)-p_2)}{var(\theta|\mathcal{I}^i,\mathcal{P}_2)}}{1+\sigma_A^2}, \\
\mu_B &= E\left(\frac{\mu_A}{\sqrt{1+\sigma_A^2}}\right) = \frac{E\left(\left(\theta|p_2\right)-p_2\right)}{\sqrt{1+\sigma_A^2}} = 0.
\end{aligned}$$

Thus, (59) is equivalent to

$$\begin{aligned}
&\frac{1}{\sqrt{1+\sigma_A^2}}\frac{1}{\sqrt{1+\frac{\frac{var(E(\theta|p_2)-p_2)}{var(\theta|\mathcal{I}^i,\mathcal{P}_2)}}{1+\sigma_A^2}}} = \frac{1}{\sqrt{1+\sigma_A^2+\frac{var(E(\theta|p_2)-p_2)}{var(\theta|\mathcal{I}^i,\mathcal{P}_2)}}} = \\
&= \frac{1}{\sqrt{1+\frac{var(E(\theta|\mathcal{I}^i,\mathcal{P}_2)|p_2)}{var(\theta|\mathcal{I}^i,\mathcal{P}_2)}+\frac{Var(E(\theta|p_2)-p_2)}{var(\theta|\mathcal{I}^i,\mathcal{P}_2)}}} = \\
&= \frac{\sqrt{var\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)}}{\sqrt{var\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)+var\left(E\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)|p_2\right)+Var\left(E\left(\theta|p_2\right)-p_2\right)}}. \qquad (60)
\end{aligned}$$

Now, we use three times the law of total variance that for any random variables X, Y

$$var(X) \equiv E[var(X|Y)] + var[E(X|Y)].$$

Substituting in first $X = \theta$, $Y = \{\mathcal{I}^i, \mathcal{P}_2\}$, then $X = \theta$, $Y = p_2$, finally, $X = E(\theta|\mathcal{I}^i, \mathcal{P}_2)$ $Y = p_2$, give

$$\begin{aligned}
var(\theta) &\equiv var(\theta|\mathcal{I}^i,\mathcal{P}_2) + var[E(\theta|\mathcal{I}^i,\mathcal{P}_2)] \\
var(\theta) &= var\left(\theta|p_2\right) + var\left(E\left(\theta|p_2\right)\right) \\
var[E(\theta|\mathcal{I}^i,\mathcal{P}_2)] &= var[E(\theta|\mathcal{I}^i,\mathcal{P}_2)|p_2] + var\left(E(\theta|p_2)\right)
\end{aligned}$$

which we can combine into

$$var\left(\theta|p_2\right) = var(\theta|\mathcal{I}^i,\mathcal{P}_2) + var[E(\theta|\mathcal{I}^i,\mathcal{P}_2)|p_2].$$

Thus, we can rewrite (60) as

$$\frac{\sqrt{var\left(\theta|\mathcal{I}^i,\mathcal{P}_2\right)}}{\sqrt{var\left(\theta|p_2\right)+Var\left(E\left(\theta|p_2\right)-p_2\right)}}.$$

Observing that the denominator is independent from $\mathcal{I}^i$ gives the result.

A.5 Proof of Lemma 2, Lemma 3 and Proposition 5

Using (17) and (55), observe that

$$\begin{aligned} Var\left(p_1|x^i,q_0\right) &= Var\left(a_1\theta_o+c_1y-e_1u_1|x^i,q_0\right)= \\ &= \frac{\left(a_1+c_1\right)^2}{\alpha^i+\omega_o+\tau_0^2}+c_1^2\left(\frac{1}{\beta}+\frac{1}{\omega_n}\right)+\frac{a_1^2}{\tau_1^2} \end{aligned}$$

which is monotonically decreasing in α^i. Given that trader i takes the coefficients of the price, a_1, c_1, e_1 as given, this concludes the proof of Lemma 2.

From (54),

$$\frac{\partial Var\left(\theta|\mathcal{I}^i,\mathcal{P}_t\right)}{\partial\alpha_o^i}=$$

$$=-\frac{\left(\alpha_n^i+\omega_n+\tau_2^2+2\varphi^2\tau_2^2-3\varphi\tau_2^2\right)^2}{\left(\beta(\alpha_o^i+\omega_o+\alpha_n^i+\omega_n)+(\alpha_n^i+\omega_n)(\alpha_o^i+\omega_o)+\left(\tau_1^2+\tau_0^2+\tau_2^2\varphi^2\right)(\beta+\alpha_n^i+\omega_n)+\tau_2^2\left((1-\varphi)^2\left(\beta+\alpha_o^i+\omega_o+\left(\tau_1^2+\tau_0^2\right)\right)-2\beta\varphi(1-\varphi)\right)\right)^2}<0$$

$$\frac{\partial Var\left(\theta|\mathcal{I}^i,\mathcal{P}_t\right)}{\partial\alpha_n^i}=$$

$$=-\frac{\left(\alpha_o^i+\omega_o+\tau_0^2+\tau_1^2+2\varphi^2\tau_2^2-\varphi\tau_2^2\right)^2}{\left(\beta(\alpha_o^i+\omega_o+\alpha_n^i+\omega_n)+(\alpha_n^i+\omega_n)(\alpha_o^i+\omega_o)+\left(\tau_1^2+\tau_0^2+\tau_2^2\varphi^2\right)(\beta+\alpha_n^i+\omega_n)+\tau_2^2\left((1-\varphi)^2\left(\beta+\alpha_o^i+\omega_o+\left(\tau_1^2+\tau_0^2\right)\right)-2\beta\varphi(1-\varphi)\right)\right)^2}<0$$

which shows that larger α_o^i and α_n^i are always beneficial for the agent. This implies that (15) always binds and

$$\alpha_n^i=\frac{\omega_n\omega_o e^{\kappa}}{\left(\omega_o+\alpha_o^i\right)}-\omega_n \tag{61}$$

in any equilibrium.

We want to show that $\alpha_o^i=0$ for all i is an equilibrium for sufficiently large β and π. It is clear from (56) that if $\alpha_o^i=0$ for all i, then $\varphi^a=\varphi^n=0$. Also, from (53) and (38), $\varphi^n=\beta=0$ implies $\tau_0^n=\tau_1^n=0$. Substituting in (61) into the objective function, results an objective function with only one decision variable α_o^i. Observe that under $\varphi^a=\varphi^n=\tau_0^n=\tau_1^n=0$, this objective function is continuous in π and α_o^i in $\pi\in[0,1]$ and $\alpha_o^i\in[0,\omega_o\left(e^{\kappa}-1\right)]$ and

the marginal expected utility in any of the branches,

$$\frac{\partial\left(-\sqrt{Var\left(\theta|\mathcal{I}^i,\mathcal{P}_t\right)}\right)}{\partial\alpha_o^i}\Big|_{\alpha_n^i=\frac{\omega_n\omega_o e^\kappa}{\left(\omega_o+\alpha_o^i\right)}-\omega_n}= \tag{62}$$

$$=\frac{1}{2\sqrt{Var\left(\theta|\mathcal{I}^i,\mathcal{P}_t\right)}}\frac{\frac{1}{\left(\omega_o+\alpha_o^i\right)^2}\left(\left(\omega_n\omega_o e^\kappa+\left(\omega_o+\alpha_o^i\right)\tau_2^2\right)^2-\omega_n\omega_o e^\kappa\left(\alpha_o^i+\omega_o+\tau_0^2+\tau_1^2\right)^2\right)}{\left(\beta\left(\alpha_o^i+\omega_o+\frac{\omega_n\omega_o e^\kappa}{\left(\omega_o+\alpha_o^i\right)}\right)+\omega_n\omega_o e^\kappa+\left(\tau_1^2+\tau_0^2\right)\left(\beta+\frac{\omega_n\omega_o e^\kappa}{\left(\omega_o+\alpha_o^i\right)}+\tau_2^2\right)+\tau_2^2(\beta+\alpha_o^i+\omega_o)\right)^2},$$

is finite in $\alpha_o^i\in[0,\omega_o\left(e^\kappa-1\right)]$. Thus, if $\alpha_o^i=0$ maximizes expected utility in the branch with announcement, that there is a $\bar{\pi}$ such that $\pi>\bar{\pi}$ implies that $\alpha_o^i=0$ for all i is an equilibrium.

From (62), a necessary condition for $\alpha_o^i=0$ to maximize

$$-\sqrt{Var\left(\theta|\mathcal{I}^i,\mathcal{P}_t^a\right)}\Big|_{\alpha_n^i=\frac{\omega_n\omega_o e^\kappa}{\left(\omega_o+\alpha_o^i\right)}-\omega_n} \tag{63}$$

is that

$$\omega_o\left(\omega_n e^\kappa+\left(\tau_2^a\right)^2\right)^2-\omega_n e^\kappa\left(\omega_o+\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2\right)^2<0,$$

that is, an increase in α_o^i at $\alpha_o^i=0$ decreases utility in the branch with announcement. This holds if

$$\omega_o\left(\frac{\omega_n e^\kappa+\left(\tau_2^a\right)^2}{\sqrt{\omega_n\omega_o e^\kappa}}-1\right)<\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2.$$

Also, the sign of (63) changes at most once in $\alpha_o^i\in[0,\omega_o\left(e^\kappa-1\right)]$. Thus, a sufficient condition is that at $\alpha_o^i=0$, the objective function is larger than at $\alpha_n^i=0$. That is,

$$\frac{\omega_o+\omega_n e^\kappa+\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2+\left(\tau_2^a\right)^2}{\beta(\omega_o+\omega_n e^\kappa)+e^\kappa\omega_n\omega_o+\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)(\beta+e^\kappa\omega_n)+\left(\tau_2^a\right)^2\left(\beta+\omega_o+\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)\right)}<$$

$$<\frac{\omega_o e^\kappa+\omega_n+\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2+\left(\tau_2^a\right)^2}{\beta(\omega_o e^\kappa+\omega_n)+\omega_n\omega_o e^\kappa+\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)(\beta+\omega_n)+\left(\tau_2^a\right)^2\left(\beta+\omega_o e^\kappa+\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)\right)},$$

or

$$-\omega_n\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)^2-\omega_n\omega_o\left(e^\kappa+1\right)\left(\tau_1^a+\tau_0^a\right)+\omega_o\left(\left(\tau_2^a\right)^4+\left(\tau_2^a\right)^2\omega_n+e^\kappa\omega_n\left(\omega_n-\omega_o+\left(\tau_2^a\right)^2\right)\right)<0,$$

which must hold for

$$\tau_1^a+\tau_0^a>\frac{\omega_n\omega_o(e^\kappa+1)+\sqrt{(\omega_n\omega_o(e^\kappa+1))^2+4\omega_n\omega_o\left(\left(\tau_2^a\right)^4+\left(\tau_2^a\right)^2\omega_n+e^\kappa\omega_n\left(\omega_n-\omega_o+\left(\tau_2^a\right)^2\right)\right)}}{2\omega_n}.$$

Thus,

$$\tau_1^a + \tau_0^a >$$
$$> \max\left(\frac{\omega_n\omega_o(e^\kappa+1)+\sqrt{(\omega_n\omega_o(e^\kappa+1))^2+4\omega_n\omega_o\left(\left(\tau_2^a\right)^4+\left(\tau_2^a\right)^2\omega_n+e^\kappa\omega_n\left(\omega_n-\omega_o+\left(\tau_2^a\right)^2\right)\right)}}{2\omega_n}, \frac{\omega_o\left(\omega_n e^\kappa+\left(\tau_2^a\right)^2\omega_o-\sqrt{e^\kappa\omega_o\omega_n}\right)}{\sqrt{e^\kappa\omega_o\omega_n}}\right)$$

is a sufficient condition for $\alpha_o^i = 0$ to be individually optimal. This concludes the proof of Lemma 3.

To conclude the proof of Proposition 5, I show that $\beta \to \infty$ implies $\left(\tau_1^a\right)^2 \to \infty$, while $\left(\tau_2^a\right)^2$ converges to a finite constant. By definition

$$\lim_{\beta\to\infty} \frac{F_1\left(\tau_1^a, \tau_2^a\right)}{\tau_1^a} = \frac{1}{\delta}$$

thus,

$$\lim_{\beta\to\infty} \frac{F_1\left(\tau_1^a, \tau_2^a\right)}{\tau_1^a}\Big|_{\alpha_o^i=0,\varphi=0} =$$
$$= \lim_{\beta\to\infty} \frac{-\alpha^i\beta\left(\tau_2^a\right)^2}{\left(\alpha_n^i\left(\omega_o+\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2\right)+\left(\tau_2^a\right)^2\left(\left(\omega_o+\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2\right)\right)\right)\gamma}$$
$$\frac{(\beta+\omega_o)\left(\tau_2^a\right)^2+\beta\left(\omega_o+\alpha_n^i+\omega_n\right)+\omega_o\left(\alpha_n^i+\omega_n\right)+\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)\left(\beta+\alpha_n^i+\omega_n+\left(\tau_2^a\right)^2\right)}{(\beta+\alpha^i+\omega_o)\left(\tau_2^a\right)^2+\beta(\alpha^i+\omega_o+\omega_n)+\omega_n(\alpha^i+\omega_o)+\left(\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2\right)\left(\beta+\omega_n+\left(\tau_2^a\right)^2\right)} =$$
$$= \lim_{\beta\to\infty} \frac{\alpha^i\beta\left(\tau_2^a\right)^2}{-\tau_1^a\left(\alpha_n^i\left(\omega_o+\left(\tau_1^a\right)^2\right)+\left(\tau_2^a\right)^2\left(\omega_o+\left(\tau_1^a\right)^2\right)\gamma\right)} \frac{\alpha_n^i+\omega_n+\omega_o+\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2+\left(\tau_2^a\right)^2}{\alpha^i+\omega_n+\omega_o+\left(\tau_0^a\right)^2+\left(\tau_1^a\right)^2+\left(\tau_2^a\right)^2} = \frac{1}{\delta}$$

which implies that $\left(-\left(\tau_1^a\right)^2\right) \to \infty$. While $F_2\left(\left(\tau_0^a\right)^2, \left(\tau_1^a\right)^2, \left(\tau_2^a\right)^2, \varphi\right)$ is independent of β and

$$\lim_{\left(\tau_1^a\right)^2\to\infty} F_2\left(\tau_0^a, \tau_1^a, \tau_2^a, \varphi\right) =$$
$$= \lim_{\left(\tau_1^a\right)^2\to\infty} \frac{\alpha_o^i\left(\alpha_n^i+\omega_n\right)+\alpha_n^i\left(\alpha_o^i+\omega_o\right)+\alpha_n^i\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)+\left(\tau_2^a\right)^2\alpha_o^i}{\left(\alpha_o^i+\omega_o+\alpha_n^i+\omega_n+\left(\left(\tau_1^a\right)^2+\left(\tau_0^a\right)^2\right)+\left(\tau_2^a\right)^2\right)\gamma} = \alpha_n^i.$$

A.6 Proofs of Lemma 4 and Proposition 6 and 7

Note that $\lim_{\delta_t\to\infty} \tau_t = \infty$ for $t = 1, 2$. It is true as

$$\tau_t = F_t\left(\tau_0, \tau_1, \tau_2, \varphi\right)$$

and

$$\lim_{\tau_1\to\infty}\frac{F_t\left(\tau_0,\tau_1,\tau_2,\varphi\right)}{\delta_t},\lim_{\tau_2\to\infty}\frac{F_t\left(\tau_0,\tau_1,\tau_2,\varphi\right)}{\delta_t}$$

are finite for $t=1,2$. Note also that $\alpha_o>0$ implies $\varphi>0$ and $\alpha_o=0$ implies $\varphi=0$. Then, the proposition is a consequence of the following facts which one can check by directly calculating the limits. For any fixed α_o, α_n and φ

$$\begin{aligned}\lim_{\omega_n\to\infty}F_2\left(\tau_0,\tau_1,\tau_2,\varphi\right) &= \frac{\alpha_o}{\gamma}\\ \lim_{\tau_1\to\infty}\lim_{\tau_2\to\infty}\lim_{\omega_n\to\infty}F_0\left(\tau_0,\tau_1,\tau_2,\varphi\right) &= \frac{\alpha}{\gamma}.\end{aligned}$$

For $\alpha_o,\varphi>0$

$$\lim_{\tau_2\to\infty}\lim_{\omega\to\infty}F_1\left(\tau_0,\tau_1,\tau_2,\varphi\right)=\frac{\alpha}{\gamma}.$$

This concludes the statements in Lemma 4.

Also, by direct substitution

$$\begin{aligned}&F_1\left(\tau_0,\tau_1,\tau_2,0\right)|_{\alpha_o=0}=\\ &=\frac{-\alpha\beta\left(\tau_0^2\tau_2^2+\tau_1^2\tau_2^2+\beta\alpha_n+\beta\omega_n+\beta\omega_o+\alpha_n\omega_o+\omega_n\omega_o+\beta\tau_0^2+\beta\tau_1^2+\beta\tau_2^2+\tau_0^2\alpha_n+\tau_1^2\alpha_n+\tau_0^2\omega_n+\tau_1^2\omega_n+\tau_2^2\omega_o\right)\tau_2^2}{\gamma\left(\omega_o+\tau_0^2+\tau_1^2\right)\left(\alpha_n+\tau_2^2\right)\left(\tau_0^2\tau_2^2+\tau_1^2\tau_2^2+\alpha\beta+\alpha\omega_n+\beta\omega_n+\beta\omega_o+\omega_n\omega_o+\beta\tau_0^2+\alpha\tau_2^2+\beta\tau_1^2+\beta\tau_2^2+\tau_0^2\omega_n+\tau_1^2\omega_n+\tau_2^2\omega_o\right)}<0.\end{aligned}$$

and

$$F_1\left(\tau_0,\tau_1,\tau_2,0\right)|_{\alpha_o=0,\beta=0}=0 \tag{64}$$

and

$$F_1\left(\tau_0,\tau_1,\tau_2,0\right)|_{\alpha_o>0,\beta=0}>0. \tag{65}$$

For Proposition 6 and 7 recall that the equilibrium $(\tau_0,\tau_1,\tau_2,\varphi)$ is a solution of the fixed point problem described by the continuous functions $\left[F_0\left(\cdot\right),\hat{F}_1\left(\cdot\right),\hat{F}_2\left(\cdot\right),\hat{F}_3\left(\cdot\right)\right]$ defined in (57)-(58). Then Lemma 4 implies that for sufficiently large $\delta_1,\delta_2,\omega_n$, $V^{01}\left(\lambda\right)$ is arbitrarily close to zero for all $\lambda>0$, but $V^{01}\left(0\right)>0$. This proves Proposition 7. Also, (64) implies that $V^{11}\left(0\right)$ is arbitrarily close to ∞ while (65) implies that $V^{11}\left(0\right)$ is finite. This gives Proposition 6.