The more we know, the less we agree: Public announcements, higher-order expectations and rational inattention

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Environments with higher-order expectations

- Decisions often influenced by higher-order expectations (agents' opinion about the opinion of others) rather than the agents' own opinion about fundamentals
- In such environments: public announcements have double role:
  1. information about the fundamental
  2. information about the opinion of others
- In financial markets: $\Rightarrow$ frenzy of trading around announcements

In this paper:

1. In some Gaussian structures, public announcement leads to
   A. **Polarized HOE**: increased dispersion in higher order expectations (not first)
   B. **Contrarian HOE**:the more optimistic about fundamental $\Rightarrow$ the more pessimistic about others' expectation about the fundamental
2. standard, Grossman-Stiglitz, OLG-dynamic model with rational inattention: the two properties arise endogenously
3. Two properties explain hectic, volatile, informative trading around public announcements

Polarized and Contrarian HOEs:Example

- Apple=old project+new project: $\theta = \theta_o + \theta_n$,
- early, $x_o^j = \theta_o + \varepsilon^j$, and late analyst, $x_n = \theta_n + \upsilon$
  - without public signal private signals are uninformative:

$$\int_0^1 \left| E(E(\theta|x_n)|x_o^j) - \bar{E}(E(\theta|x_n)|x_o^j) \right| dj =$$

$$= \int_0^1 \left| E(\hat{a}_\theta x_n|x_o^j) - \bar{E}(\hat{a}_\theta x_n|x_o^j) \right| dj = 0$$

  - with public signal, $y = \theta + \eta$, higher $x_o^j \rightarrow$ lower guess of x_n: contrarian and polarized HOEs

$$\int_0^1 \left| E(E(\theta|x_n, y)|x_o^j, y) - \bar{E}(E(\theta|x_n)|x_o^j, y) \right| dj =$$

$$= \int_0^1 \left| E(a_\theta x_n + c_\theta y|x_o^j, y) - \bar{E}(a_\theta x_n + c_\theta y|x_o^j, y) \right| dj =$$

$$= \left| a_{x_n} a_\theta \right| \int_0^1 \left| \varepsilon^j \right| dj > 0$$

Polarized and Contrarian HOEs: Statistics

- 2 groups of agents: $1, 2$ unit mass of agents in each group (in paper T groups)
- group 2 forecasts θ fundamental, group 1 forecasts the forecast of group 2
- $y, x_1^j, x_2^j, \theta \tilde{} N(0, \cdot)$ general, but symmetric covariance structure:

$$\sigma_x^2, \sigma_\theta^2, \sigma_y^2, \rho_{x,x}, \rho_{x,x'}, \rho_{x,y}, \rho_{\theta,x}, \rho_{\theta,y} > 0$$

where $\rho_{x,x'}$ across groups

Observations

1. (Milgrom, 1980): Suppose ”positive signals”:

$$a_\theta \equiv \frac{\partial E\left(\theta|x_1^j, y\right)}{\partial x_1^j}, c_\theta \equiv \frac{\partial E\left(\theta|x_1^j, y\right)}{\partial y} > 0$$

then there is no polarization in first-order expectations:

$$\int_0^1 \left| E\left(\theta|x_1^j, y\right) - \overline{E\left(\theta|x_1^j, y\right)} \right| dj < \int_0^1 \left| E\left(\theta|x_1^j\right) - \overline{E\left(\theta|x_1^j\right)} \right| dj$$

Observations

2. Add $\rho_{x,x'} < \frac{\rho_{x,y}^2 a_\theta}{\frac{\sigma_\theta \rho_{\theta,x}}{\sigma_x}\left(1-\rho_{x,y}^2\right)+a_\theta}$, then **polarized HOE**:

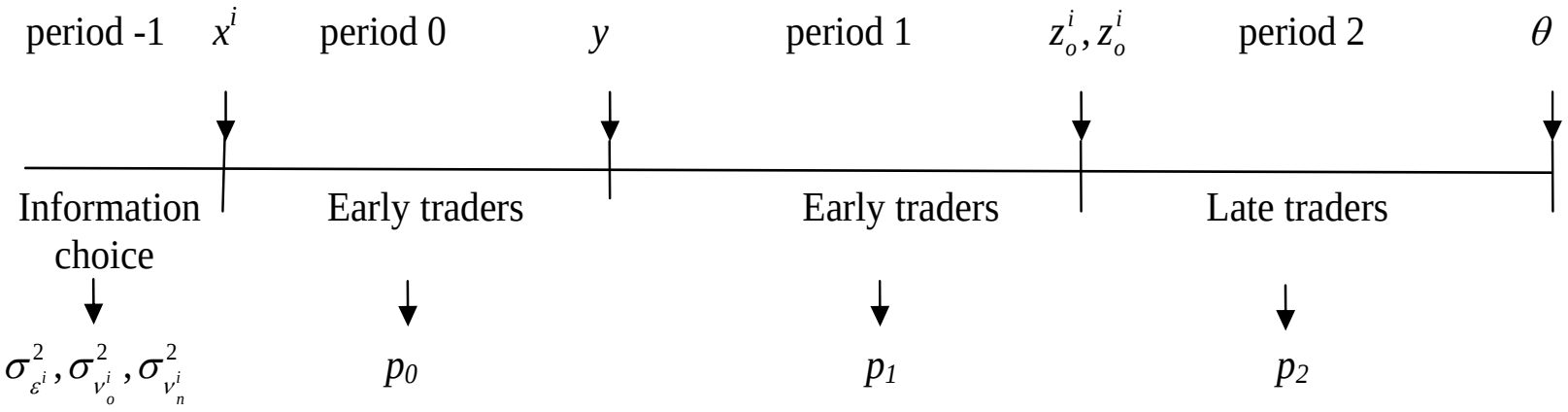
$$\int_0^1 \left| E(E\left(\theta|x_2^i,y\right)|x_1^j,y) - \overline{E(E\left(\theta|i_2,y\right)|x_1^j,y)} \right| di >$$
$$\int_0^1 \left| E(E\left(\theta|x_2^i\right)|x_1^j) - \overline{E(E\left(\theta|x_2^i\right)|x_1^j)} \right| di$$

3. Same condition implies **contrarian HOE**:

$$\frac{\partial E\left(\theta|x_1^j,y\right)}{\partial x_1^j} \frac{\partial E(E\left(\theta|x_2^i,y\right)|x_1^j,y)}{\partial x_1^j} < 0$$

Announcements on financial markets

- Multi period, CARA-Normal Grossman-Stiglitz with endogenous choice of information
- two, unit mess, groups of myopic traders, OLG structure
- noisy supply of assets, u_t, in each period



- fundamental value: $\theta = \theta_o + \theta_n$ (old project, new project)
- each trader receive private signals
  - Early traders:
$$x^i = \theta_o + \varepsilon^i$$
  - Late traders:
$$\begin{aligned} z_o^i &= \theta_o + \upsilon_o^i \\ z_n^i &= \theta_n + \upsilon_n^i. \end{aligned}$$
- w.p. π there is an announcement in period 1:$y = \theta + \eta$
- in period 0, each trader finds out whether there will be an announcement
- all factors and noise terms are i.i.d normal

Information choice

- In period -1 agents allocate attention across factors: early traders pick $\sigma^2_{\varepsilon^i}$, late traders pick $\sigma^2_{\upsilon^i_o}, \sigma^2_{\upsilon^i_n}$

  to maximize the expectation of

$$\pi \max_{d^i_t\left(\mathscr{I}^i, \mathscr{P}^a_t\right)} E\left(-e^{-\gamma W^i_t} | \mathscr{I}^i, \mathscr{P}^a_t\right) + (1-\pi) \max_{d^i_t\left(\mathscr{I}^i, \mathscr{P}^n_t\right)} E\left(-e^{-\gamma W^i_t} | \mathscr{I}^i, \mathscr{P}^n_t\right)$$

- early trader can learn only about current project (no information yet about next project)
- late trader can decide which project to learn about

Information processing constraint

- The choice is subject to the *information processing constraint*:

$$H([\theta_o, \theta_n]) - H\left([\theta_o, \theta_n] | \mathscr{I}^i\right) \leq \frac{\kappa}{2}$$

- $H(\cdot)$ is entropy of a random variable
- for late traders this translates to:

$$\ln \frac{\sigma^2_{\theta_n} + \sigma^2_{\upsilon^i_n}}{\sigma^2_{\upsilon^i_n}} \frac{\sigma^2_{\theta_o} + \sigma^2_{\upsilon^i_o}}{\sigma^2_{\upsilon^i_o}} \leq \kappa,$$

- or fraction of attention on old project:

$$\lambda^i \equiv \frac{1}{\kappa} \ln \frac{\sigma^2_{\theta_o} + \sigma^2_{\upsilon^i_o}}{\sigma^2_{\upsilon^i_o}}$$

The RE equilibrium for given information structure

- by standard methods (1. assume linear prices, 2. derive optimal strategies, 3. find fixed point of coefficients)
- REE equilibrium portfolio holdings in period 1 and 0

$$d_1^i = \frac{E\left(p_2|x^i, y, q_1, q_0\right) - p_1}{\gamma var\left(p_2|x^i, y, q_1, q_0\right)} = \frac{\tau_1}{\delta_1}\left(x^i - q_1\right)$$

$$d_0^i = \frac{E\left(p_1|x^i, q_0\right) - p_0}{\gamma var\left(p_1|x^i, q_0\right)} = \frac{\tau_0}{\delta_0}\left(x^i - q_0\right)$$

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- q_t: residual information in price p_t, over and above information in public signal and past prices

$$\begin{aligned} q_1 &= \theta_o - \frac{\delta_1}{\tau_1} u_1 \\ q_0 &= \theta_o - \frac{\delta_0}{\tau_0} u_0 \end{aligned}$$

- finding the equilibrium amounts to finding the fixed point in $\frac{\tau_t}{\delta_t}$

Trading volume and information content of prices

- change of positions between periods 0 and 1 (time-series)

$$V^{01} = \frac{1}{2} E_u \left(\int \left| d_{1,i} - d_{0,i} \right| di \right) = \left| \frac{\tau_1^a}{\delta_1} - \frac{\tau_0^a}{\delta_0} \right| \frac{\sigma_\varepsilon}{\sqrt{2\pi}}$$

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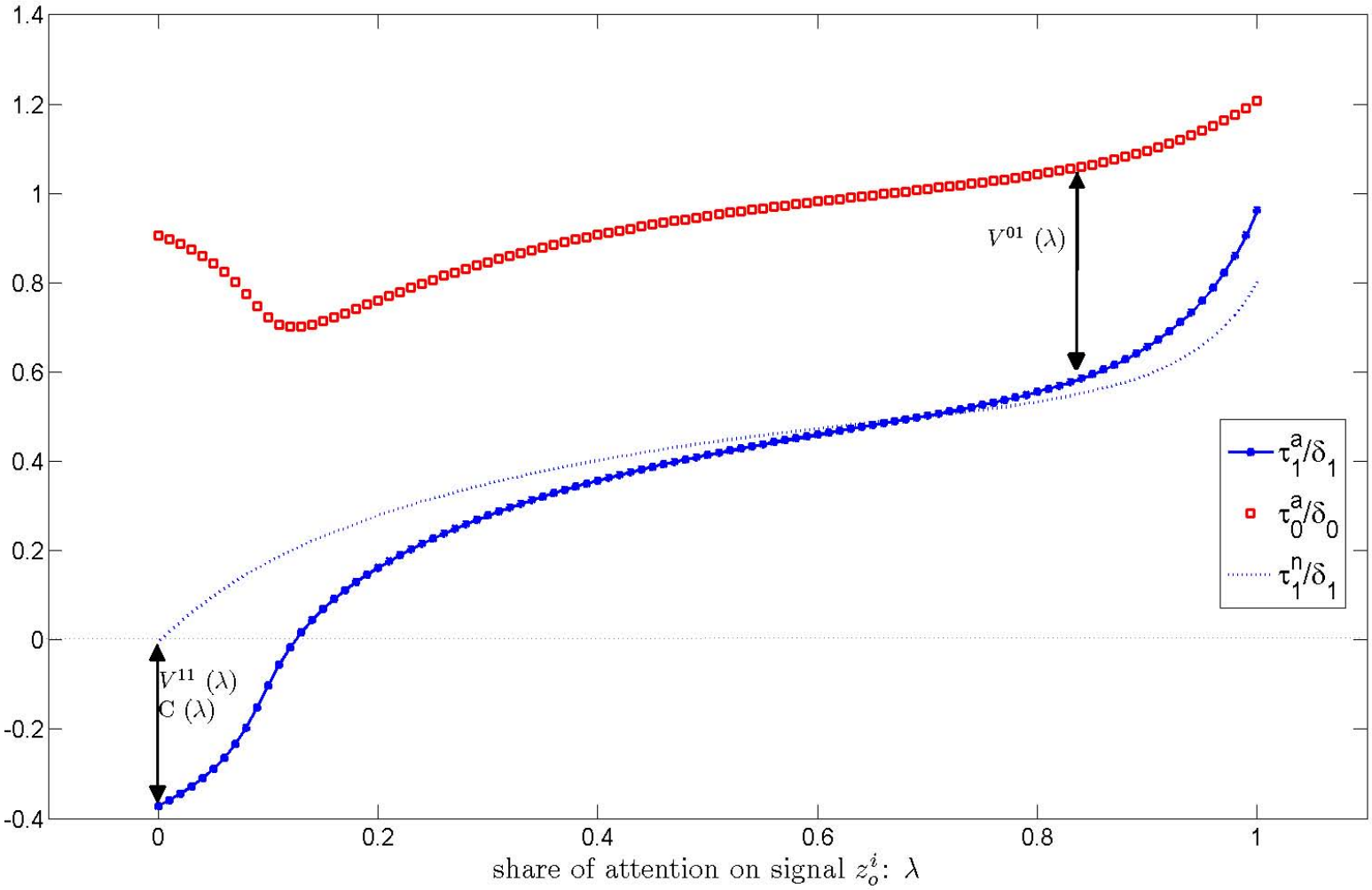
- or positions in period 1 with and without announcement (cross-section)

$$V^{11} = \frac{E_u \left(\int \left| d_{1,i}^a \right| di \right)}{E_u \left(\int \left| d_{1,i}^n \right| di \right)} = \frac{\left| \frac{\tau_1^a}{\delta_1} \right|}{\left| \frac{\tau_1^n}{\delta_1} \right|}$$

- information content of prices: compare variance in q_t with and without announcement: $C \equiv \frac{\left(\tau_1^a\right)^2}{\left(\tau_1^n\right)^2}$

Trading intensities with different informational choices



Remarks

- large V^{11}, C because of polarized second-order expectations:
  - holding is disagreement over variance

$$d^i = \frac{E(\theta|\mathscr{I}^i, \mathscr{P}_t) - p}{\gamma Var(\theta|\mathscr{I}^i, \mathscr{P}_t)}$$

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  - here: go into same directions, large effect

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- large V^{01}, because of contrarian second-order expectations:
  - early traders in period 0, bet on y, forecast depends on first-order expectations
  - early traders in period 1, bet on $p_2 \approx$ expectation of late traders, forecast depends on second-order expectations

Attention allocation: REE with endogenous learning

- trading volume is large when private information sets are independent across periods
- why would a late trader specialize on the other factor?
- simple (but slightly wrong) intuition: learn about θ_n, because past price already reveals a lot on θ_o

- However: in REE equilibrium
  - current prices also reveal information: if other late traders learn about θ_n, you might not want to
  - past prices are informative only if early traders trade aggressively
  - early traders' trading intensity depends on information content in p_2 (which depends on the attention allocation of late traders)
- **Result:** For any fixed set of parameters, there are $\overline{\sigma_\eta^2}, \overline{\pi}$ that if $\sigma_\eta^2 < \overline{\sigma_\eta^2}$ and $\pi > \overline{\pi}$ than late traders learn **only** about θ_n

Matching stylized facts

- Known that trading volume increases around announcements
- puzzling: should not in a representative agent model or in standard noisy REE model
- with some modification (He-Wang (1995), Kim-Verrecchia (1991)), some volume but mostly of the type V^{01}

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- Kandel-Pearson (1995): polarized expectations necessary to match facts
  - enforced by high-frequency evidence: announcement is followed by hectic and informative trading (V^{11}, C are large)
  - Varian (1989), Harris-Raviv (1993), Kandel-Pearson (1995): models with heterogeneous priors to generate polarized first-order expectations
- But:
  - polarization in higher-order expectations is sufficient
  - it is generated endogenously by rational inattention

Related literature

- higher-order expectations in sequential environments (e.g. Allen-Morris-Shin, 2006, Makarov-Rytchkov, 2006, Banerjee-Kaniel-Kremer, 2008, Goldstein-Ozdenoren-Yuan, 2009, Angeletos-Lorenzoni-Pavan, 2007)
- rational inattention (Sims, 2002) and prices: Peng-Xiong (2006), Hellwig-Veldkamp(2009), Nieuwerburgh-Veldkamp (2008a/b), Mondria (2009)
- public announcement and trading volume (previous slide)
- polarized expectations in other contexts: Rabin-Schrag (1998), Gerber-Green (2002), Dixit-Weibull (2007)

Conclusion

- Separated private information sets $\Rightarrow$ public announcement leads to polarized and contrarian HOEs
  - public announcement connects information sets $\Rightarrow$ makes private signals relevant to guess other private signals
- In an OLG-dynamic model rational inattention $\Rightarrow$ separated private information sets
  - late traders prefer to learn privately on elements they cannot learn publicly
  - past prices reveal information of past traders publicly
- contrarian and polarized HOE implies hectic and informative trading around announcements