Trading Frenzies and Their Impact on Real Investment

Itay Goldstein, Emre Ozdenoren, and Kathy Yuan*

July 24, 2009

Abstract

We study a model where a capital provider learns from the price of a firm's security in deciding how much capital to provide for new investment. This feedback effect from the financial market to the investment decision gives rise to trading frenzies, where speculators all wish to trade like others, generating large shifts in prices and firms' investments. Coordination among speculators is sometimes desirable for price informativeness and investment efficiency, but speculators' incentives push in the opposite direction, so that they coordinate exactly when it is undesirable. We analyze the determinants of coordination among speculators and study policy measures that affect patterns of coordination to improve price informativeness and investment efficiency.

*We thank Roland Benabou, Alex Edmans, Peter Kondor, Miklos Koren, Alessandro Pavan, Dimitri Vayanos, and seminar participants at CEU, LSE, MBS, Paris School of Economics, and UCL for their comments. Goldstein is from the University of Pennsylvania; e-mail: `itayg@wharton.upenn.edu`. Ozdenoren is from the London Business School and CEPR; e-mail: `eozdenoren@london.edu` and `emreo@umich.edu`. Yuan is from the London School of Economics and CEPR; e-mail: `K.Yuan@lse.ac.uk`.

1 Introduction

Trading frenzies in financial markets occur when many speculators rush to trade in the same direction leading to large price changes. Financial economists have long been searching for the sources of trading frenzies, asking what causes strategic complementarities in speculators' behavior. After all, the price mechanism in financial markets naturally leads to strategic substitutes, whereby the change in price caused by speculators' trades pushes others to trade in the opposite direction. We argue in this paper that the potential effect that financial-market trading has on the real economy may provide the mechanism for trading frenzies to arise.

For example, consider two prime events of the recent financial crisis – the fall of Bear Stearns and the fall of Lehman Brothers. In both events, the shares of the firms were subject to a massive 'run' by short sellers, which most likely contributed to the collapse of these firms, given that their deteriorating stock prices made it impossible for them to raise new capital.[1] In the presence of such a real effect, speculators know that the decrease in price caused by the 'run' on a firm's stock will decrease the firm's value. They then want to join the massive short-selling when it happens, so that they can profit from having a short position on a firm whose value is about to decline. This can ignite a frenzy.

Building on a recent literature that studies the feedback effect from financial markets to real investments (reviewed below), we develop a model that studies this phenomenon. In our model, a capital provider has to decide how much capital to provide to a firm for the purpose of making new real investment. The decision of the capital provider depends on his assessment of the productivity of the proposed investment. In his decision, the capital provider uses the available private information and also the information conveyed by the price of the firm's traded asset as determined in the financial market. The reliance of capital provision on financial-market prices establishes the feedback effect that the financial market has on the real economy. In the financial market, many small speculators trade a security, whose payoff is correlated with the cash flow obtained from the firm's investment. Each speculator makes a trading decision based on two signals: the first signal is independent across speculators (conditional on the realization of the productivity), while the second one

[1] Indeed, these events and others have led regulatory authorities in various countries to become very concerned about speculative trading and eventually to put restrictions on short selling activities. In executing a naked-shortsale-ban order on July 15, 2008, the SEC concluded that short sales have exacerbated a loss of investor confidence and caused further panic selling making counterparties to Bear Stearns unwilling to make secured funding (see: `http://www.sec.gov/rules/other/2008/34-58166.pdf`).

is correlated among them.[2]

The weights that speculators put on each of the two signals are determined by two strategic effects. The first effect is the usual one that arises due to the price mechanism. The sale (purchase) of securities by other speculators reduces (increases) the price and then the profit from selling (buying) the securities. This generates strategic substitutes – speculators wish to act differently from one another by reducing the weight they put on the correlated signal. The second effect arises due to the feedback effect from the price to the capital provision decision. A coordinated sale (purchase) by many speculators transmits negative (positive) information to the capital provider and leads to a reduction (an increase) in the amount of capital provided and in the amount of investment undertaken. This reduces (increases) the underlying value of the security and increases the profit from selling (buying).[3] The result is strategic complementarities that make speculators put a larger weight on the correlated signal.

This second effect is what causes a trading frenzy, leading speculators to put too much weight on their correlated information, and to trade in a coordinated fashion. When this effect dominates, our model generates a pattern that echoes the events mentioned above. Essentially, our model gives rise to a 'run' on a stock by many speculators, who are driven by common noise in their correlated signals (e.g. rumor), leading to a price decline, lack of provision of new capital, and collapse of real value.

Our paper analyzes when trading frenzies are expected to occur. We find that when there is small variance in noise/liquidity trading in the financial market, i.e., when liquidity dries up, speculators tend to put large weights on their correlated signals and thus to act in a coordinated fashion. This is because in these situations the ability to affect the capital provider's beliefs improves and hence the incentive to trade in a coordinated way to affect the capital provision decision increases. The information environment also plays an important role in shaping the incentive to coordinate. Generally speaking, there will be more coordination when speculators' correlated signals are sharper, when their uncorrelated signals are noisier,

[2]In our model, the correlation is perfect, but this is for expositional clarity and is not essential.

[3]The setup of the model assumes that speculators holding a long position in the security always benefit from more real investment, while the capital provider faces a tradeoff in choosing the investment level. As a result, the model generates symmetric implications for buy-side speculation pushing investment up and sell-side speculation pushing investment down. In this, our model is different from the model by Goldstein and Guembel (2008), which is discussed below. There, speculators who hold a long position are aligned with the manager who decides on real investment, and as a result buy-side and sell-side speculations are asymmetric. Our model may capture better a situation with conflict of interests, that is, where the provider of capital and speculators face different tradeoff in making their corresponding investment decisions.

when the capital provider has less precise information of his own, and when there is overall more uncertainty about the firm's productivity.

Interestingly, speculators' incentives to coordinate go in the opposite direction to efficiency considerations (from the point of view of the capital provider's investment decision).[4] Providing the most informative signal from the market to the capital provider entails higher coordination among speculators when there is a lot of liquidity trading and lower coordination when liquidity dries up. This is because, in liquid markets, coordination among speculators is beneficial in suppressing the noise in liquidity trading that reduces the informativeness of the price. In such markets, trading frenzies among speculators are actually desirable because they enable decision makers to detect some trace of informed trading in a market subject to large volume of liquidity and noise. On the other hand, when liquidity dries up, the importance of coordination among speculators declines, and the additional noise that coordination adds via the excess weight that speculators put on their correlated information (which translates into weight on common noise) makes coordination undesirable. Hence, given speculators' own incentives (as described above), they end up coordinating their trading too much in illiquid markets and too little in liquid markets.

This disparity between speculators' incentives and investment efficiency suggests a role for policy measures to improve the usefulness of financial markets in guiding investment decisions. One of the main policy tools available to policymakers is the control over the cost of capital for the capital provider. The government can affect the cost of capital by changing the interest rate and/or the availability of funds. Our analysis shows that such policy can have an important effect on the informativeness of prices and the efficiency of investments if it is made contingent on the realization of fundamentals. A policy that reduces the cost of capital when fundamentals are weak and increases it when fundamentals are strong reduces the sensitivity of investment to the perceived strength of the fundamental and thus weakens the real effect of financial-market trading. This mitigates the incentive of speculators to coordinate, and hence is useful when the financial market is illiquid. Conversely, a policy that reduces the cost of capital in good times and increases it in bad times would be desirable when the financial market has high volume of liquidity trading.

Other policy measures target the trading environment in the financial market. A very intuitive measure based on the discussion above is to affect the amount of liquidity/noise trading in the market. Increasing (decreasing) liquidity when it is low (high) will reduce (increase) coordination and improve efficiency. The government can also attempt to achieve

[4]Focusing on the investment decision for the efficiency criterion is appropriate if we think of the financial market as a zero-sum game.

more efficient levels of coordination by affecting the informational environment. Reduction in coordination can be achieved by releasing public information that reduces uncertainty about investment productivity, by restricting communication among speculators to reduce the correlation in their information, and by providing capital providers unique access to better information.[5]

As mentioned above, our paper builds on a small, but growing, branch of models in financial economics that consider the feedback effect from trading in financial markets to corporate investments. The basic motivation for this literature goes back to Hayek (1945), who posited that market prices provide an important source of information for various decision makers. Empirical evidence for this link is provided by Baker, Stein, and Wurgler (2003), Luo (2005), and Chen, Goldstein, and Jiang (2007). On the theoretical side, earlier contributions to this literature include Fishman and Hagerty (1992); Leland (1992); Khanna, Slezak, and Bradley (1994); Boot and Thakor (1997); Dow and Gorton (1997); Subrahmanyam and Titman (1999); and Fulghieri and Lukin (2001).

Several recent papers in this literature are more closely related to the mechanism in our paper. Ozdenoren and Yuan (2008) show that the feedback effect from asset prices to the real value of a firm generates strategic complementarities. In their paper, however, the feedback effect is modeled exogenously and is not based on learning. As a result, their paper does not deliver the implications that our paper delivers on the effect of liquidity and various information variables on coordination and efficiency. Khanna and Sonti (2004) also model feedback exogenously and show how a single trader can increase the value of his existing inventory in the stock by trading to affect the value of the firm. Goldstein and Guembel (2008) do analyze learning by a decision maker, and show that this might lead to manipulation of the price by a single potentially informed trader. Hence, the manipulation equilibrium in their paper is not a result of strategic complementarities among heterogeneously informed traders. Dow, Goldstein, and Guembel (2007) show that the feedback effect generates complementarities in the decision to produce information, but not in the trading decision.[6] In different contexts, Goldstein, Ozdenoren, and Yuan (2009) and Angeletos, Lorenzoni, and Pavan (2007)

[5]Our finding that releasing public signals unambiguously reduces coordination, differs from that in the existing literature. The reason is that in our paper the coordination incentive among speculators is endogenous and stems from the speculators' wish to affect the capital provider's inference. More public information reduces uncertainty and the room for speculators to affect the capital provider's inference becomes limited. Hence, there is less incentive for speculators to coordinate.

[6]Complementarities in the decision to produce information also arise due to other reasons in several other papers. For example see, Froot, Scharfstein, and Stein (1992); Hirshleifer, Subrahmanyam, and Titman (1994); Bru and Vives (2002); and Veldkamp (2006a and 2006b).

also study learning-based complementarities. More generally, our paper is the first one to derive a closed-form solution in a model with endogenous feedback, where prices aggregate information from heterogeneously informed agents and reflect the expected investment. As described in the body of the paper, we are able to achieve this methodological innovation by working with log-normal distributions.

The remainder of this paper is organized as follows. In Section 2, we present the model setup and characterize the equilibrium of the model. In Section 3, we solve the model. Section 4 analyzes the determinants of coordination among speculators in our model. In Section 5, we discuss the implications for the efficiency of investments and the volatility of prices and investments. In Section 6, we discuss policy implications. Section 7 concludes.

2 Model

The model has one firm and a traded asset. There is a capital provider who has to decide how much capital to provide to the firm for the purpose of making an investment. There are three dates, $t = 0, 1, 2$. At date 0, speculators trade in the asset market based on their information about the fundamentals of the firm. At date 1, after observing the asset price and receiving private information, the capital provider of the firm decides how much capital the firm can have and the firm undertakes investment accordingly. Finally, at date 2, the cash flow is realized and agents get paid.

2.1 Investment

The firm in this economy has access to a production technology, which at time $t = 2$ generates cash flow $\tilde{F}I$. Here, I is the amount of investment financed by the capital provider, and $\tilde{F} \geq 0$ is the level of productivity. Let $\tilde{f}$ denote the natural log of productivity, $\tilde{f} = \ln \tilde{F}$. We assume that $\tilde{f}$ is unobservable and drawn from a normal distribution with mean $\bar{f}$ and variance σ_f^2. We use τ_f to denote $1/\sigma_f^2$. Focusing on the natural log of the productivity parameter is important for the tractability of our model and is part of the methodological contribution of our paper.

At time $t = 1$ the capital provider chooses the level of capital I. Providing capital is costly and the capital provider must incur a private cost of: $C(I) = \frac{1}{2}cI^2$, where $c > 0$. This cost can be thought of as the cost of raising the capital, which is increasing in the amount of capital provided, or as effort incurred in monitoring the investment (which is also increasing in the size of the investment). The capital provider's benefit increases in the cash flow

generated by the investment. For simplicity, we assume that he captures the full cash flow, i.e., he gets $\tilde{F}I$, but none of the results depends on this assumption. The capital provider chooses I to maximize the value of the cash flow from investing in the firm's production technology minus his cost of raising capital $C(I)$, conditional on his information set, $\mathcal{F}_l$, at $t = 1$:

$$I = \arg\max_I E[\tilde{F}I - C(I)|\mathcal{F}_l]. \tag{1}$$

The solution to this maximization problem is:

$$I = \frac{E[\tilde{F}|\mathcal{F}_l]}{c}. \tag{2}$$

The capital provider's information set, denoted by $\mathcal{F}_l$, consists of a private signal $\tilde{s}_l$ received at date 0 and the asset price observed at the date 0, P (we will elaborate on this next). That is, $\mathcal{F}_l = \{\tilde{s}_l, P\}$. The private signal $\tilde{s}_l$ is a noisy signal about $\tilde{f}$ with precision τ_l: $\tilde{s}_l = \tilde{f} + \sigma_l\tilde{\epsilon}_l$, where $\tilde{\epsilon}_l$ is distributed normally with mean zero and standard deviation one and $\tau_l = 1/\sigma_l^2$.

2.2 Speculative Trading

The traded asset is a derivative, whose payoff replicates the payoff of the firm's investment. That is, the payoff is $\tilde{F}I$, which is realized at the final date $t = 2$. The price of this risky asset at $t = 0$ is denoted by P.

There is a measure-one continuum of heterogeneously informed risk-neutral speculators indexed by $i \in [0, 1]$. Each speculator is endowed with two signals about $\tilde{f}$ at time 0. The first signal, $\tilde{s}_i = \tilde{f} + \sigma_s\tilde{\epsilon}_i$, is privately observed where $\tilde{\epsilon}_i$ is independently normally distributed across speculators with mean zero and unit variance. The precision of this signal is denoted as $\tau_s = 1/\sigma_s^2$. The second signal is $\tilde{s}_c = \tilde{f} + \sigma_c\tilde{\epsilon}_c$. This signal is observed by all speculators and $\tilde{\epsilon}_c$ is independently and normally distributed with mean zero and unit variance and $\tau_c = 1/\sigma_c^2$.[7]

[7] Our results remain the same but with expositional complexity in an alternative setup where the second signal is specified as a heterogenous private signal with a common noise component $\tilde{\epsilon}_c$ and an agent-specific noise component $\tilde{\epsilon}_{2i}$. That is, $\tilde{s}_{ci} = \tilde{f} + \sigma_c\tilde{\epsilon}_c + \sigma_{\epsilon 2}\tilde{\epsilon}_{2i}$, where $\tilde{\epsilon}_c$ and $\tilde{\epsilon}_{2i}$ are independently normally distributed variables with mean zero and variance one. In this setup, the second signal can be observed by the capital provider as well. Essentially, for our results to go through, speculators need to share some correlated information to facilitate coordination and this information cannot be entirely filtered out by the capital provider.

Each speculator can buy or sell up to a unit of the risky asset. The size of speculator i's position is denoted by $x(i) \in [-1, 1]$. This position limit can be justified by limited capital and/or borrowing constraints faced by speculators.[8] Due to risk neutrality, speculators choose their positions to maximize expected profits. For example, a speculator's profit from shorting one unit of the asset is given by $P - \tilde{F}I$, where $\tilde{F}I$ is the asset payoff and P is the price of the asset.

Formally, speculator i chooses $x(i)$ to solve:

$$\max_{x(i)\in[-1,1]} x(i)\, E\left[\tilde{F}I - P|\mathcal{F}_i\right], \tag{3}$$

where $\mathcal{F}_i$ denotes the information set of speculator i and consists of $\tilde{s}_i$ and $\tilde{s}_c$. Since each speculator has measure zero and is risk neutral, an informed speculator optimally chooses to either short up to the position limit, or buy up to the position limit. We denote the aggregate demand by speculators as $X = \int_0^1 x(i)\, di$, which is given by the fraction of speculators who buy the asset minus the fraction of those who short the asset.

2.3 Equilibrium

At date 0, conditional on his information, each speculator submits a market order to buy or sell a unit of the asset to a Walrasian auctioneer. The Walrasian auctioneer then obtains the aggregate demand by speculators X and also a noisy supply curve from uninformed traders, and sets a price to clear the market. The noisy supply of the risky asset is exogenously given by $Q(\tilde{\xi}, P)$, a continuous function of the exogenous demand shock $\tilde{\xi}$ and price P. The supply curve $Q(\tilde{\xi}, P)$ is strictly decreasing in $\tilde{\xi}$, and increasing in P, that is, it is upward slopping in price. The demand shock $\tilde{\xi} \in \mathbb{R}$ is independent of other shocks in the economy, and $\tilde{\xi} \sim N(0, \sigma_\xi^2)$. The shock $\tilde{\xi}$ can be interpreted as demand for the asset by liquidity traders, and so a high σ_ξ^2 characterizes a liquid market. As always, we denote $\tau_\xi = 1/\sigma_\xi^2$.

To solve the model in closed form, we assume that $Q(\tilde{\xi}, P)$ takes the following functional form:

$$Q(\tilde{\xi}, P) = 1 - 2\Phi\left(\frac{\tilde{\xi} - \ln(\delta P)}{\sigma_s}\right), \tag{4}$$

[8] The specific size of this position limit on asset holdings is not crucial for our results. What is crucial is that informed speculators cannot take unlimited positions; if they do, strategic interaction among informed speculators will become immaterial.

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution function. We now turn to the definition of equilibrium.

Definition 1: [Equilibrium with Market Orders] An equilibrium consists of a price function, $P(\tilde{f}, \tilde{\epsilon}_c, \tilde{\xi}) : \mathbb{R}^3 \to \mathbb{R}$, an investment policy for the capital provider $I(\tilde{s}_l, P) : \mathbb{R}^2 \to \mathbb{R}$, strategies for speculators, $x(\tilde{s}_i, \tilde{s}_c) : \mathbb{R}^2 \to [-1, 1]$, and the corresponding aggregate demand $X(\tilde{f}, \tilde{\epsilon}_c)$, such that:

- For speculator i, $x(\tilde{s}_i, \tilde{s}_c) \in \arg\max_{x(i)\in[-1,1]} x(i) E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right]$;
- The capital provider's investment is $I(\tilde{s}_l, P) = E\left[\tilde{F}|\tilde{s}_l, P\right]/c$.
- The market clearing condition for the risky asset is satisfied:

$$Q(\tilde{\xi}, P) = X(\tilde{f}, \tilde{\epsilon}_c) = \int x(\tilde{f} + \sigma_s\tilde{\epsilon}_i, \tilde{f} + \sigma_c\tilde{\epsilon}_c) d\Phi(\tilde{\epsilon}_i). \tag{5}$$

A *linear monotone equilibrium* is an equilibrium where $x(\tilde{s}_i, \tilde{s}_c) = 1$ if $\tilde{s}_i + k\tilde{s}_c \geq g$ for constants k and g, and $x(\tilde{s}_i, \tilde{s}_c) = -1$ otherwise. In words: in a monotone linear equilibrium, a speculator buys the asset if and only if a linear combination of her signals is above a cutoff g, and sells it otherwise. In the rest of the paper we focus on linear monotone equilibria.

3 Solving the Model

In this section, we explain the main steps that are required to solve our model. Restricting attention to a linear monotone equilibrium, we first use the market clearing condition to determine the asset price. We then characterize the information content of the asset price to derive the capital provider's belief on $\tilde{f}$ based on $\{P, \tilde{s}_l\}$ and solve for the optimal investment problem. Finally, given the capital provider's investment rule and the asset pricing rule, we solve for individual speculators' optimal trading decision.

In a linear monotone equilibrium, speculators short the asset whenever $\tilde{s}_i + k\tilde{s}_c \leq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \leq g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate selling can be characterized by: $\Phi\left(\left(g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. Conversely, they purchase the asset whenever $\tilde{s}_i + k\tilde{s}_c \geq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \geq g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate purchase can be characterized by $1 - \Phi\left(\left(g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. The net holding from speculators is then:

$$X\left(\tilde{f}, \tilde{\epsilon}_c\right) = 1 - 2\Phi\left(\frac{g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right). \tag{6}$$

The market clearing condition together with Equation (4) indicate that

$$1-2\Phi\left(\frac{g-(1+k)\,\tilde{f}-k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right)=1-2\Phi\left(\frac{\tilde{\xi}-\ln(\delta P)}{\sigma_s}\right). \tag{7}$$

Therefore the equilibrium price is given by

$$\delta P=\exp\left((1+k)\,\tilde{f}+k\sigma_c\tilde{\epsilon}_c-g+\tilde{\xi}\right)=\exp\left(\tilde{f}+k\tilde{s}_c-g+\tilde{\xi}\right), \tag{8}$$

which can be rewritten as

$$z(P)\equiv\frac{g+\ln(\delta P)}{1+k}=\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{1}{1+k}\tilde{\xi}=\left(\frac{1}{1+k}\right)\tilde{f}+\frac{k}{1+k}\tilde{s}_c+\frac{1}{1+k}\tilde{\xi}. \tag{9}$$

From the above equation, we can see that $z(P)$, which is a sufficient statistic for the information in P, provides some information about the realization of the productivity shock $\tilde{f}$. Yet, the signal $z(P)$ is not fully revealing of $\tilde{f}$, as it is also affected by the noise in the common signal $\tilde{\epsilon}_c$ and by the noisy demand $\tilde{\xi}$. Since the capital provider observes $z(P)$, he will use it to update his belief about the productivity. Note that $z\,(P)$ is distributed normally with a mean of $\bar{f}$ and a variance of $\sigma_p^2=(k/(1+k))^2\,\sigma_c^2+(1/(1+k))^2\,\sigma_\xi^2$. We denote the precision of $z(P)$ as a signal for $\tilde{f}$ as:

$$\tau_p=\frac{1}{\sigma_p^2}=\frac{(1+k)^2\,\tau_c\tau_\xi}{k^2\tau_\xi+\tau_c}. \tag{10}$$

After characterizing the information content of the price, we can derive the capital provider's belief on $\tilde{f}$. That is, conditional on observing $\tilde{s}_l$ and $z\,(P)$, the capital provider believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f}{\tau_f+\tau_l+\tau_p}\bar{f}+\frac{\tau_l}{\tau_f+\tau_l+\tau_p}s_l+\frac{\tau_p}{\tau_f+\tau_l+\tau_p}z\,(P) \tag{11}$$

and variance $1/\left(\tau_f+\tau_l+\tau_p\right)$. Then, using the capital provider's investment rule in Equation (1) and taking expectations, we can express the level of investment as:

$$\begin{aligned} I &= \frac{1}{c}E[\tilde{F}|\tilde{s}_l=s_l,P]=\frac{1}{c}E[\exp\left(\tilde{f}\right)|\tilde{s}_l=s_l,P] \\ &= \frac{1}{c}\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\,(P)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right). \end{aligned} \tag{12}$$

Given the capital provider's investment policy in (12) and the price in (8), we can now write speculator i's expected profit from buying the asset given the information that is available to him (shorting the asset would give the negative of this):

$$\begin{aligned} & E\left[\tilde{F}I-P|\tilde{s}_i,\tilde{s}_c\right] \\ = & \frac{1}{c}E\left[\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\,(P)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}+\tilde{f}\right)|\tilde{s}_i,\tilde{s}_c\right] \\ & -E\left[\frac{1}{\delta}\exp\left(\tilde{f}+k\tilde{s}_c-g+\tilde{\xi}\right)|\tilde{s}_i,\tilde{s}_c\right]. \end{aligned} \tag{13}$$

Note that we made use here of the fact that $\tilde{F} = \exp\left(\tilde{f}\right)$. This is where focusing on the natural log of the productivity parameter plays a key role. Using the properties of the exp function, we can express the value of the firm $\tilde{F}I$ as $\exp\left(\frac{\tau_f \bar{f} + \tau_l s_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)} + \tilde{f}\right)$, where the expression in parentheses is linear in $\tilde{f}$. This enables us to get a linear closed-form solution, which would otherwise be impossible in a model of feedback.

Conditional on observing $\tilde{s}_i$ and $\tilde{s}_c$, speculator i believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f}{\tau_f + \tau_s + \tau_c}\bar{f} + \frac{\tau_s}{\tau_f + \tau_s + \tau_c}\tilde{s}_i + \frac{\tau_c}{\tau_f + \tau_s + \tau_c}\tilde{s}_c \tag{14}$$

and variance $1/\left(\tau_f + \tau_s + \tau_c\right)$. Hence, substituting for $z\left(P\right)$ (from (9)) and taking expectations, Equation (13) can be rewritten as:

$$\begin{aligned} E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right] = \frac{1}{c}\exp\left(\begin{array}{c} \frac{\tau_f \bar{f} + \tau_p \frac{k}{1+k}\tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\tau_f + 2\tau_l + \tau_p\left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p}\left(\frac{\tau_f \bar{f} + \tau_s \tilde{s}_i + \tau_c \tilde{s}_c}{\tau_f + \tau_s + \tau_c}\right) \\ + \left(\frac{\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f + \tau_l + \tau_p}\right)^2 \sigma_l^2 \\ + \frac{1}{2}\left(\frac{\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2 \left(\frac{1}{1+k}\right)^2 \sigma_\xi^2 \end{array}\right) \\ -\frac{1}{\delta}\exp\left(\frac{\tau_f \bar{f} + \tau_s \tilde{s}_i + \tau_c \tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2\right). \end{aligned} \tag{15}$$

In equilibrium, a speculator who receives a private signal $\tilde{s}_i = g - k\tilde{s}_c$ must be indifferent between buying the asset or shorting it. That is,

$$E\left[P - \tilde{F}I|\tilde{s}_i = g - k\tilde{s}_c, \tilde{s}_c\right] = 0. \tag{16}$$

Substituting $\tilde{s}_i = g - k\tilde{s}_c$ into (15), and taking logs, the indifference condition of (16) becomes:

$$\begin{aligned} & \ln\frac{1}{c} + \left(\frac{\tau_f \bar{f} + \tau_p \frac{k}{1+k}\tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p\right)}{\tau_f + \tau_l + \tau_p}\left(\frac{\tau_f \bar{f} + \tau_s\left(g - k\tilde{s}_c\right) + \tau_c \tilde{s}_c}{\tau_f + \tau_s + \tau_c}\right)\right) \\ & + \left(\frac{\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f + \tau_l + \tau_p}\right)^2 \sigma_l^2 \\ & + \frac{1}{2}\left(\frac{\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2 \left(\frac{1}{1+k}\right)^2 \sigma_\xi^2 \\ = \; & \ln\frac{1}{\delta} + \frac{\tau_f \bar{f} + \tau_s\left(g - k\tilde{s}_c\right) + \tau_c \tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2. \end{aligned} \tag{17}$$

In a linear monotone equilibrium, this indifference condition must hold for all $\tilde{s}_c$. Hence, the coefficient for $\tilde{s}_c$ in the above expression must be zero. Using this, we solve for the speculator's cutoff strategy and characterize the equilibrium. The result is provided in the

following proposition. The proof of this proposition, as well as all other proofs, is in the Appendix.

PROPOSITION 1: There is a unique linear monotone equilibrium. In the equilibrium, the (strictly positive) weight k^* on the common signal is the unique real root of:

$$\begin{aligned} 0 &= -\left(\tau_s\tau_l + \left(\tau_f + \tau_s + \tau_c\right)\left(\tau_f + \tau_l + \tau_c\right)\right)k^3 + \tau_c\left(\tau_l - 2\tau_s - \tau_f - \tau_c\right)k^2 \\ &\quad + \tau_c\left(\tau_c - \tau_s\right)k + \tau_c^2 - \frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l + \left(\tau_f + \tau_l\right)\left(\tau_f + \tau_s + \tau_c\right)\right)k + \frac{\tau_c^2\tau_l}{\tau_\xi}. \end{aligned} \tag{18}$$

The weight speculators put on the common signal in equilibrium, k^*, captures the degree of coordination in their trading decisions. When k^* is high, speculators put a large weight on the common information when deciding whether to sell or buy the asset. This leads to large coordination among them and gives rise to a trading frenzy. In the upcoming sections, we develop a series of results on the determinants of coordination and its implications for the efficiency of the investment decision and for the volatility of prices.

4 The Determinants of Speculators' Coordination

The weight that speculators put on the common signal in this model is affected by the degree to which there are strategic complementarities or strategic substitutes among them. Strategic substitutes are generated by the usual price mechanism. Since the aggregation of speculators' orders affects the price, when many of them decide to short sell (buy) the asset, the price is low (high), and the profit from short selling (buying) is low. This creates an incentive for speculators to act differently from others – their incentive to short sell (or buy) the asset decreases if many others are expected to do so – and thus leads speculators to put less weight on the common signal in their trading decision. Strategic complementarities, on the other hand, are generated here by the feedback effect that prices have on the investment decision and thus on the real value of the firm. When many speculators decide to short sell (buy) the asset, the price declines (rises), and this transmits a negative (positive) signal to the capital provider that leads to a reduction (an increase) in the level of investment. Then, the value of the firm decreases (increases) and this increases the profit from short selling (buying). This creates an incentive for speculators to coordinate and act like each other, and thus to put more weight on the common signal. The resulting level of k^* reflects the sum of these two effects in addition to the raw effect that the precision of the signals (private and common) has on the weights they should receive. In the rest of this section, we isolate

the various determinants of coordination to understand the impact of each factor on the equilibrium level of coordination.

4.1 Impact of Learning by the Capital Provider

To get a clearer understanding of the two effects, let us start by shutting down one of them. In particular, suppose that there is no feedback effect from prices to real values, because the capital provider does not learn from the price. In this case, the capital provider's decision on how much capital to provide becomes (this equation is analogous to Equation (12) in the full model):

$$\begin{aligned} I &= \frac{1}{c}E[\tilde{F}|\tilde{s}_l = s_l] \\ &= \frac{1}{c}\exp\left(\frac{\tau_f}{\tau_f+\tau_l}\bar{f} + \frac{\tau_l}{\tau_f+\tau_l}s_l + \frac{1}{2(\tau_f+\tau_l)}\right). \end{aligned} \tag{19}$$

We again solve for the linear monotone equilibrium where speculators short sell the asset if and only if $\tilde{s}_i + k_{BM}\tilde{s}_c \leq g_{BM}$ (the subscript BM stands for 'benchmark'), and purchase the asset otherwise. Given the investment rule in (19), the expected profit for speculator i from buying the asset, given the information available to her, becomes (this equation is analogous to Equation (13) in the full model):

$$\begin{aligned} & E[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c] \\ = \; & E\left[\frac{1}{c}\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l}{\tau_f+\tau_l} + \frac{1}{2(\tau_f+\tau_l)}\right)\tilde{F}|\tilde{s}_i, \tilde{s}_c\right] - E\left[\frac{1}{\delta}\exp\left(\tilde{f} + k_{BM}\tilde{s}_c - g_{BM} + \tilde{\xi}\right)|\tilde{s}_i, \tilde{s}_c\right]. \end{aligned} \tag{20}$$

We know that a speculator observing $\tilde{s}_i = g_{BM} - k_{BM}\tilde{s}_c$ is indifferent between buying and shorting the asset. Following similar steps to those in the full model, we obtain:

$$\begin{aligned} & \ln\left(\frac{1}{c}\right) + \frac{\tau_f\bar{f}+\frac{1}{2}}{\tau_f+\tau_l} + \left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)\left(\frac{\tau_f\bar{f}+\tau_s\left(g_{BM}-k_{BM}\tilde{s}_c\right)+\tau_c\tilde{s}_c}{\tau_f+\tau_s+\tau_c}\right) \\ & + \left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)^2\frac{1}{2(\tau_f+\tau_s+\tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f+\tau_l}\right)^2\sigma_l^2 \\ = \; & \ln\frac{1}{\delta} + \frac{\tau_f\bar{f}+\tau_s\left(g_{BM}-k_{BM}\tilde{s}_c\right)+\tau_c\tilde{s}_c+\frac{1}{2}}{\tau_f+\tau_s+\tau_c} + k_{BM}\tilde{s}_c - g_{BM} + \frac{1}{2}\sigma_\xi^2. \end{aligned} \tag{21}$$

Finally, since the above equality must be satisfied for all $\tilde{s}_c$ we set the coefficient of $\tilde{s}_c$ to zero:

$$\left(\frac{\tau_f+2\tau_l}{\tau_f+\tau_l}\right)\left(\frac{1}{\tau_f+\tau_s+\tau_c}\right)(-k_{BM}\tau_s+\tau_c) = \left(\frac{1}{\tau_f+\tau_s+\tau_c}\right)(-k_{BM}\tau_s+\tau_c) + k_{BM}. \tag{22}$$

Then, we obtain the weight that speculators put on the common signal in the case of no feedback effect from price to real investment:

$$k_{BM} = \frac{\tau_l \tau_c}{(\tau_f + \tau_s + \tau_c)(\tau_f + \tau_l) + \tau_l \tau_s}. \tag{23}$$

The following proposition states the properties of k_{BM} in comparison with the equilibrium weight k^* in the full model.

PROPOSITION 2: If the capital provider does not learn from the price when making lending decisions, the weight speculators put on the common signal k_{BM} is strictly below the equilibrium weight k^* they put in the full model (with a feedback effect), which is lower, in turn, from τ_c/τ_s – the precision ratio of the two signals held by the speculators.

We can see that when we shut down the feedback effect from the price to real investment, the weight that speculators put on the common signal decreases. This is in line with our discussion above, according to which the feedback effect from prices to real investment is the source of complementarity in speculators' strategies, making them want to put more weight on the common signal. Hence, the feedback effect is the cause of trading frenzies in our model.

Moreover, as one would expect, without a feedback effect the substitutability among speculators' strategies reduces the weight that speculators put on the common signal below the ratio between the precision of the two signals held by the speculators, τ_c/τ_s, which is the weight that speculators would be expected to put on the common signal absent any strategic effects. Interestingly, even with the feedback effect, k^* is less than τ_c/τ_s highlighting the strength of the substitution effect from the price. Note, however, that τ_c/τ_s is not the optimal level of coordination. As we will see later, k^* is sometimes above and sometimes below the optimal level of coordination

4.2 Impact of Noise Trading

The comparison with the case of no feedback clarifies that the feedback effect from the price to the real investment has a crucial impact. It creates an incentive for speculators to coordinate to influence the decision of the capital provider. Clearly, in a model with feedback, the ability of speculators to transmit a message to the capital provider via the price depends on the amount of noise trading. As the following proposition states, this affects the weight speculators end up putting on the common signal.

Proposition 3: The equilibrium weight k^* that speculators put on the common signal in the presence of feedback effects (i.e., the full model) is decreasing in the variance of noise demand σ_ξ^2.

The intuition here goes as follows: With high variance in the noise demand, there is high variance in the market price for reasons that are not related to speculators' trades. As a result, the ability of speculators to impact the capital provider's decision via coordination diminishes. This reduces the incentive of speculators to act like each other and thus reduces the equilibrium level of k^*. Interpreting this result, we obtain the interesting implication that informed speculators are more likely to act in a coordinated fashion in illiquid markets, and so illiquid markets are those were trading frenzies can arise.

Finally, it is worth noting that changes in the position limits of speculators will have similar effects to changes in the variance of noise trading. For example, if speculators could choose positions in the range $[-2, 2]$ (instead of $[-1, 1]$, assumed in the paper), they would have more impact on the capital provider's decision for a given level of σ_ξ^2 and thus would coordinate their trades more in equilibrium. Hence, the effect of loosening speculators' trading constraints is similar to that of reducing the variance of noise trading (i.e., of reducing liquidity).

4.3 Impact of the Information Structure

We now establish comparative statics results on the effect of the informativeness of various signals on the equilibrium level of coordination. The results are summarized in the next proposition.

Proposition 4: The equilibrium level of coordination decreases in the precision of the prior and the private signals: $\partial k^*/\partial \tau_f < 0$ and $\partial k^*/\partial \tau_s < 0$. If the prior is not too precise (for small enough τ_f), the equilibrium level of coordination decreases in the precision of the capital provider's signal and increases in the precision of the common signal: $\partial k^*/\partial \tau_l < 0$ and $\partial k^*/\partial \tau_c > 0$.

The proposition shows that a more precise prior reduces the ability of speculators to coordinate. Since the capital provider relies more on the prior when it becomes more precise, the scope for speculators to affect the capital provider's belief is much more limited. Therefore, speculators have a lower incentive to act like other speculators, and they reduce the weight put on the common signal. The precision of speculators' idiosyncratic signals also has a negative effect on the degree of coordination in equilibrium. If each speculator

holds a very precise private signal about the fundamental, each bases the trading decision mostly on the private signal rather than on the noisy common signal, and hence there is less coordination.

For volatile enough underlying fundamentals, the proposition shows that if the capital provider's private information is less precise, or the common signal is more precise, speculators coordinate more in equilibrium. Intuitively, if the capital provider holds a precise signal, he relies less on the information revealed in the market price. In equilibrium, this gives speculators little incentive to coordinate since their ability to affect the capital provider's beliefs is limited. Hence, the equilibrium weight on the common signal is lower. Finally, the incentive to coordinate is largest when the common signal is very precise. In this case, speculators put a large weight on the common signal and the capital provider does not ignore the information revealed in the market price.

We also note that simulations indicate that the result that $\partial k^*/\partial \tau_c > 0$ holds no matter what the value of τ_f is. Simulations show, however, that the restriction of small τ_f is necessary for the result that $\partial k^*/\partial \tau_l < 0$. For large τ_f, an increase in τ_l leads to a larger k^*, that is, a higher degree of coordination among speculators. Overall, when the prior is very precise (higher τ_f), there is little room for speculators to coordinate their trading in order to affect the capital provider's beliefs. However, in this case, if the capital provider's private signal becomes more precise, he will gradually rely less on the prior, allowing more room for speculators to coordinate and affect the capital provider's actions.

5 Coordination, Efficiency, and Volatility

In this section, we explore the effect that coordination has on the efficiency of investment decisions and on market volatility. As our efficiency criterion, we use the ex ante expected net benefit of investment (i.e. expected net benefit before any of the signals are realized given the prior belief that $\tilde{f}$ is normally distributed with mean $\bar{f}$ and precision τ_f) from the perspective of the capital provider. This efficiency criterion is appropriate in the context of our model since the derivative market can be regarded as a zero-sum game. We keep the information structure the same as before, and in particular, in the interim stage we allow the capital provider to obtain information only from his private signal and the price. So our efficiency criterion is given by:

$$E_0\left[\max_I E\left[\tilde{F}I - \frac{1}{2}cI^2 \,|\tilde{s}_l = s_l, P\right]\right], \tag{24}$$

where a speculator purchases the asset if $\tilde{s}_i + k\tilde{s}_c \geq g$ and shorts it otherwise (for constant k and g) and P is the market clearing price. We denote the optimal level of coordination k_{OP} to be the one that maximizes (24).

The following proposition characterizes the optimal level of coordination, k_{OP}, and how it is linked to the accuracy of the information inferred from the market price, τ_p:

PROPOSITION 5: The optimal level of coordination that maximizes (24) is $k_{OP} = \tau_c/\tau_\xi$, which also maximizes τ_p. Ex ante efficiency increases in k for $k < k_{OP}$ and decreases for $k > k_{OP}$.

Essentially, the capital provider cares about the events in the security market only to the extent that it affects the quality of the information he has when making the investment decision. Hence, the level of coordination that is optimal is the one that maximizes the accuracy of the information in the market price. Examining the expression for τ_p in (10), we can see that there is a tradeoff in setting the level of coordination. The tradeoff arises because there are two sources of noise in the price, one coming from the noise demand and the other one from the noise in the common signal. A high level of coordination reduces the effect of the first source of noise – as speculative trading becomes more prominent than noise trading – and increases the effect of the second source of noise – as the weight on the common signal is higher. Therefore, the optimal level of coordination will be high when the potential damage from noise demand is high (τ_ξ is low) or when the potential damage from noise in the common signal is low (τ_c is high). One can easily verify from (10) that, on balance, optimal coordination is given by $k_{OP} = \tau_c/\tau_\xi$.

It is interesting to compare the optimal level of coordination characterized here with the level of coordination that is obtained in equilibrium. From Proposition 3 we know that in equilibrium speculators coordinate more when the variance in the noise demand is low (τ_ξ is high). A high τ_ξ implies that speculators' trades have more effect on the capital provider's decision, giving them more incentive to coordinate. Yet, this is exactly when coordination is not desirable for the efficiency of the investment. Hence, there is a sharp contrast between the profit incentives of speculators and the efficiency of the investment. Speculators coordinate more exactly when it is inefficient to do so. The following proposition summarizes the comparison between the optimal level of coordination and the equilibrium level of coordination.

PROPOSITION 6: There exists $\bar{\tau}_\xi$ such that $k_{OP} > k^*$ for $\tau_\xi < \bar{\tau}_\xi$ and $k_{OP} < k^*$ for $\tau_\xi > \bar{\tau}_\xi$.

The proposition says that speculators coordinate too much in illiquid markets and coor-

dinate too little in liquid markets. Interestingly, this implies that trading frenzies are only sometimes undesirable. When there is high variation in noise demand, price informativeness would improve if speculators coordinated their trades more to provide a signal that overcomes the effect of noise demand. Yet, it is exactly in this case that they find coordination less profitable.

We close this section by noting some of the implications of inefficient coordination levels. Deviations from the optimal level of coordination k_{OP} are manifested in our model by higher levels of excess volatility. The following proposition establishes the link between the level of coordination and excess volatility – volatility that does not come from the variability in fundamental – of price and investment.

PROPOSITION 7: (a) Excess volatility of asset price is minimized at $k = k_{OP}$ (where its value is $1/(\tau_c + \tau_\xi)$), decreases in k when $k < k_{OP}$ and increases in k when $k > k_{OP}$. In particular, when $k > k_{OP}$, excess volatility of asset prices is higher because prices are more sensitive to the noise component in speculators' common signal $\tilde{\epsilon}_c$. When $k < k_{OP}$, excess volatility of asset prices is higher because prices are more sensitive to the noise demand $\tilde{\xi}$.

(b) Similarly, excess volatility of investment is minimized at $k = k_{OP}$ (where its value is $1/(\tau_l + \tau_c + \tau_\xi)$), decreases in k when $k < k_{OP}$ and increases in k when $k > k_{OP}$.

This proposition indicates that the strategic interactions among speculators in the financial markets often lead to the excess (non-fundamental) volatility in prices as well as real activities. The source of this excess volatility could come from either too low coordination (that is, when the market is characterized by a high amount of trading by noise investors) or too high coordination (that is, when the market is illiquid and the noise in the correlated signals among speculators is high).

6 Policy Implications

As demonstrated earlier, the equilibrium level of coordination is either too high or too low, both of which lead to excess volatility in the asset market and the real economy and to loss of efficiency. To curb extreme and disruptive swings in these markets, the government may consider stepping in under certain conditions. Next, we describe two broad types of policy interventions: those that directly regulate the funding of the firms and those that oversee the trading of the security markets.

6.1 Funding Policies Contingent on Economic Conditions

One of the main policy tools available to the government is the control over the capital provider's cost of capital. The government can affect the cost of capital by changing the interest rate and/or the availability of funds. In our model, the focus is on the effect that such policies may have on the behavior of speculators in the security market.

Changes in the cost of capital that are not contingent on the realization of variables in the model will change the level of investment and the profits of speculators, but not their incentive to coordinate (i.e., the weights they put on correlated vs. idiosyncratic information). This is because the unconditional change in the cost of capital does not affect the ability of speculators to influence the decision of the capital provider. It affects the overall level of investment but not its dependence on messages from the financial market. However, if the cost of capital is made contingent on the realization of the fundamental, the capital provider will change the sensitivity of his investment policy to market signals, and this will make coordination more or less attractive for speculators. Therefore, for policies that affect the cost of capital to be effective in our context the government needs to condition the capital provider's cost of investment on some fundamental-related information that will be revealed ex post.

To illustrate this point we assume that the government can affect the cost of investment based on the ex post realization of the fundamental itself. This policy can be implemented, for example, as an ex post tax (or subsidy) to investment. The level of $\tilde{F}$ could be tied to the GDP or a general industry performance index which is less subject to manipulation by individual investors. More specifically, we assume that the government sets the capital provider's cost so that the net cost of investment is $C(I, \tilde{F}) = \frac{1}{2}c\tilde{F}^{\beta}I^2$ when the realized fundamental is $\tilde{F}$. In this case, a funding policy with a positive β leads to a higher cost of investment when productivity is high (large $\tilde{F}$.) Similarly, a funding policy with a negative β reduces the cost of investment for larger $\tilde{F}$.

We now analyze the new game with such state-contingent government policies in place. That is, the government chooses β to affect the capital provider's cost. After the policy choice, the capital provider and speculators play the same game as before. The equilibrium of the game with policy intervention is also defined just like before, except that the capital provider now maximizes $E[\tilde{F}I - C(I, \tilde{F})|\mathcal{F}_l] = E[\tilde{F}I - \frac{1}{2}c\tilde{F}^{\beta}I^2|\mathcal{F}_l]$. Once again we look for linear monotone equilibria where $x(\tilde{s}_i, \tilde{s}_c) = 1$ if $\tilde{s}_i + k\tilde{s}_c \geq g$ for constant k and g, and $x(\tilde{s}_i, \tilde{s}_c) = -1$ otherwise. Equilibrium price is still given by Equation (8). Following steps that are similar to the ones that we used in solving the standard model we solve for

the speculators' cutoff strategy and characterize the equilibrium of the game with policy intervention.

PROPOSITION 8: There is a unique linear monotone equilibrium of the game with policy intervention for β close to zero. In the equilibrium, the (strictly positive) weight $k(\beta)$ on the common signal is the unique real root of:

$$\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k(\beta)}{1+k(\beta)}+\frac{\tau_f+(2-\beta)\,\tau_l+\left(\left(\frac{1-\beta}{1+k(\beta)}+1\right)\right)\tau_p}{\tau_f+\tau_l+\tau_p}\left(\frac{-\tau_s k(\beta)+\tau_c}{\tau_f+\tau_s+\tau_c}\right)-\frac{-\tau_s k(\beta)+\tau_c}{\tau_f+\tau_s+\tau_c}-k(\beta)=0.$$

Utilizing the equilibrium condition in Proposition 8, we can characterize the comparative statistics of $k(\beta)$ with respect to β, a policy instrument controlled by the central planner. The following proposition presents the result.

PROPOSITION 9: For β close to zero, a policy with the cost of funding positively correlated with the fundamental ($\beta > 0$) leads to less coordination among speculators and a policy with the cost of funding negatively correlated with the fundamental ($\beta < 0$) leads to more coordination among speculators.

To understand this result intuitively, consider the case where the policy imposes a higher cost of funding when the fundamental is stronger ($\beta > 0$). In this case, investment is relatively more costly in a state of high productivity than in a state of low productivity, and the capital provider's investment decision is less sensitive to his belief about the productivity level. Consequently, the speculators' incentive to coordinate is smaller since learning through the price will have less of an impact in shaping the capital provider's investment decision. In the opposite case, when the policy imposes a lower cost of funding when the fundamental is stronger ($\beta < 0$), the capital provider's investment decision is more sensitive to his belief about the state of the fundamentals, and this increases the incentive for speculators to coordinate.

Recall from Proposition 6 that there exists $\bar{\tau}_\xi$ such that k^* is smaller than k_{OP} for τ_ξ less than $\bar{\tau}_\xi$ and the reverse is true for τ_ξ larger than $\bar{\tau}_\xi$. In the former case, there is too little coordination and in the latter case there is too much coordination. In either case moving coordination closer to the optimal level increases informational efficiency and lowers excess volatility. Combining this with Proposition 9 we obtain the next corollary:

COROLLARY 1: There exists $\bar{\tau}_\xi$ such that a policy that has positive correlation between the cost of funding and the productivity level ($\beta > 0$ for β small) when τ_ξ is larger than $\bar{\tau}_\xi$,

and negative correlation ($\beta < 0$ for β small) when τ_ξ is less than $\bar{\tau}_\xi$ improves efficiency and reduces excess volatility.

The above corollary implies that when $\tilde{\xi}$ has low variance (which can be interpreted as low market liquidity), the government should adopt policies where the cost of funding is positively correlated with the realized productivity shock. To understand this, recall that when market liquidity is low, speculators coordinate too much, because it is easier for them to impact the capital provider's beliefs through their impact on the price. By adopting such a state-contingent policy, the government diminishes the sensitivity of the capital provider's investment decision to the market signal. This makes coordination less desirable. Conversely, when market liquidity is high, the government should adopt policies where the cost of funding is negatively correlated with productivity since such a policy increases the sensitivity of the capital provider's investment decision to the market signal and makes coordination more desirable. Adopting such state-contingent funding policies would increase the information efficiency of asset prices, and decrease the excess volatility in the financial market and in the real economy.

6.2 Intervention in Security Trading

Another type of intervention is to increase the informational efficiency of market prices by changing the trading and information environment. One of the key determinants of strategic trading in the security market is the level of noise trading. To increase efficiency and curb excess volatility, the government may directly control noise trading. The following corollary states this policy implication which follows from Proposition 6.

Corollary 2: The equilibrium k^* is closer to k_{OP} if the government increases τ_ξ when $\tau_\xi < \bar{\tau}_\xi$ and decreases τ_ξ when $\tau_\xi > \bar{\tau}_\xi$.

That is, when the market is very liquid ($\tilde{\xi}$ has high variance), intervention should focus on absorbing this liquidity. Conversely, when the market is illiquid ($\tilde{\xi}$ has low variance), the government should step in and provide market liquidity. This market liquidity intervention can be in the form of buying and selling market indices. This policy encourages coordination among speculators when there is too little coordination and discourages coordination when there is too much coordination. By doing so it increases the informational efficiency of prices. Interestingly, in our model, liquidity is not always a good thing, and sometimes it is optimal to limit it. Specifically, this happens when too much noise trading reduces the ability of the capital provider to learn from the price and speculators fail to coordinate their trading to

overcome this effect. It should be noted that our conclusions are derived in a model where the information structure is given exogenously. It would be interesting to endogenize the precision of various signals and see how this is affected by the changes in the liquidity of the market.

Intervention can also target directly the information available to the speculators or to the capital provider as described by the following corollary (which follows immediately from Proposition 4):

Corollary 3: When there is too much (little) coordination, the government can move the equilibrium level of coordination towards k_{OP} by increasing (decreasing) τ_l and/or τ_s, or by decreasing (increasing) τ_c.

There are various ways by which the government can achieve these changes in the information environment. An increase in τ_l can be achieved by making more information about the firm's productivity available to capital providers (but not making it available to speculators at the same time). A decrease in τ_c can be achieved by restricting communication among speculators and thus reducing the correlation among their information sets. In an environment where the quality of the information held by speculators is heterogenous, the government may prevent a coordinated 'run' on a firm by imposing a transaction cost on trading which makes market participation less attractive to those holding less precise signals and thus improves the precision of private signals available to market participants (τ_s).

Finally, we consider the effect of making public announcements.

Corollary 4: By releasing public news, $\tilde{s}_n = \tilde{f} + \tilde{\epsilon}_n \sigma_n$, where $\tau_n = 1/\sigma_n^2$, to all market participants, the government can reduce the equilibrium level of coordination.

This result follows immediately from Proposition 4 since by releasing a public signal to all market participants, the precision of the prior increases to $(\tau_f + \tau_n)$. This implication about the effect of transparency contrasts the recent literature emphasizing that more public information might have a perverse effect on the information efficiency of the market.[9] In this literature, this might be due to the fact that more precise public information increases the ability of speculators to coordinate or lowers the incentive for speculators to act on their private information, causing the aggregate variables such as price or trading volume to be less informative. In our setting, transparency policy unambiguously lowers speculators'

[9]There is a large literature on transparency. Some recent works include Morris and Shin (2002, 2005), Heinemann and Cornand (2004), Woodford (2005), Svensson (2005), Hellwig (2005), Angeletos and Pavan (2007), and Amador and Weill (2007).

ability to coordinate and increases their incentive to act on their private information. This is because the public news becomes available to the credit provider, and makes him rely less on information from the market. This, in turn, makes speculators less interested in coordinating their trading (around common information that is available to them but not to the capital provider) and induces them to rely more on their private signals.

7 Conclusion

We study strategic interactions among speculators in financial markets and their real effects. Two opposite strategic effects exist. On the one hand, speculators wish to act differently from each other as a certain action by other speculators changes the price in a way that reduces the profit for other speculators from this action. On the other hand, due to the feedback effect from the price to the real investment, a certain action by speculators changes the real value of the firm in a way that increases the incentive of other speculators to take this action. This creates a basis for trading frenzies, where speculators rush to trade in the same direction, putting pressure on the price and on the firm's value. We characterize which effect dominates when and analyze the resulting level of coordination in speculators' actions.

The interaction among speculators affects the informational content of the price. Since prices affect real investment in our model, we can ask what level of coordination is most efficient for real investment. In general, speculators' incentives to coordinate go in opposite direction to the optimal level of coordination. Speculators want to coordinate more when there is low amount of noise trading, but this is when coordination is less desirable from an efficiency point of view. Hence, our model shows that there is always either too much or too little coordination, and this reduces the efficiency of investment and creates excess volatility in the price.

By analyzing the feedback mechanism between financial market trading and real investment activities, our model has implications for policy measures that can alter the level of coordination and improve efficiency. We consider changes to the cost of capital for the firm contingent on productivity fundamentals, and also measures that directly affect the trading environment by changing liquidity, transparency, and the precision of various sources of information.

Interestingly, our paper is also related to an old debate on whether speculators stabilize prices. The traditional view is that by buying low and selling dear, rational speculators stabilize prices. Hart and Kreps (1986) argue that when speculators can hold inventories and

there is uncertainty about preferences, speculative activity may cause excess price movement. Our paper contributes to this literature by pointing out that when speculative activity has an effect on real investments, speculators might coordinate on correlated sources of information, and create excess volatility in prices. In our model, this directly reduces efficiency.

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Appendix

Proof of Proposition 1: In the proposed equilibrium, (17) must hold for all $\tilde{s}_c$. Therefore, the coefficient of $\tilde{s}_c$ must be zero. That is:

$$\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k}{1+k}+\left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)\left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right)-\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}-k=0.$$

Substituting for τ_p and rearranging, this equation can be rewritten as:

$$\begin{aligned}&-\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)k^3+\tau_c\left(\tau_l-2\tau_s-\tau_f-\tau_c\right)k^2\\&+\tau_c\left(\tau_c-\tau_s\right)k+\tau_c^2-\frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l+\left(\tau_f+\tau_l\right)\left(\tau_f+\tau_s+\tau_c\right)\right)k+\frac{\tau_c^2\tau_l}{\tau_\xi}=0.\end{aligned}\tag{25}$$

Next we show that the above cubic equation can be solved for k and has a unique strictly positive root for k. To see this first consider the following function:

$$\begin{aligned}H(k)\;=\;&-\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)k^3+\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)k^2\\&+\tau_c\left(\tau_c-\tau_s\right)k+\tau_c^2.\end{aligned}\tag{26}$$

The discriminant for $H(k)=0$ is:

$$\begin{aligned}&4\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)^3\tau_c^2-\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)^2\tau_c^2\left(\tau_c-\tau_s\right)^2\\&-4\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)\tau_c^3\left(\tau_c-\tau_s\right)^3\\&+18\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)\left(\tau_c-\tau_s\right)\tau_c^3\\&+27\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)^2\tau_c^4\end{aligned}$$

which can be rewritten as:

$$\tau_c^3\begin{pmatrix}32\tau_c^4\tau_f+64\tau_c^4\tau_l+91\tau_c^3\tau_f^2+184\tau_c^3\tau_f\tau_l+80\tau_c^3\tau_f\tau_s+32\tau_c^3\tau_l^2\\+160\tau_c^3\tau_l\tau_s+86\tau_c^2\tau_f^3+174\tau_c^2\tau_f^2\tau_l+152\tau_c^2\tau_f^2\tau_s+60\tau_c^2\tau_f\tau_l^2\\+326\tau_c^2\tau_f\tau_l\tau_s+72\tau_c^2\tau_f\tau_s^2+4\tau_c^2\tau_l^3+104\tau_c^2\tau_l^2\tau_s+144\tau_c^2\tau_l\tau_s^2\\+27\tau_c\tau_f^4+54\tau_c\tau_f^3\tau_l+72\tau_c\tau_f^3\tau_s+27\tau_c\tau_f^2\tau_l^2+162\tau_c\tau_f^2\tau_l\tau_s\\+68\tau_c\tau_f^2\tau_s^2+90\tau_c\tau_f\tau_l^2\tau_s+152\tau_c\tau_f\tau_l\tau_s^2+28\tau_c\tau_f\tau_s^3\\+71\tau_c\tau_l^2\tau_s^2+56\tau_c\tau_l\tau_s^3+4\tau_f^2\tau_s^3+4\tau_f\tau_l\tau_s^3+4\tau_f\tau_s^4+8\tau_l\tau_s^4\end{pmatrix}>0.$$

Therefore, the equation $H(k)=0$ has a unique real root. Since $H(-\infty)=\infty$, $H(\infty)=-\infty$ and $H(0)=\tau_c^2>0$ the only real root occurs for $k>0$. Next consider the last two terms of Equation (25):

$$-\frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l+\left(\tau_f+\tau_l\right)\left(\tau_f+\tau_s+\tau_c\right)\right)k+\frac{\tau_c^2\tau_l}{\tau_\xi}\tag{27}$$

Note that when $k < 0$ Equation (27) is strictly positive, so there can not be a real root of Equation (25) for $k < 0$. Moreover, Equation (27) has a strictly negative derivative when $k > 0$ so the left side of Equation (25) decreases faster than $H(k)$ and thus crosses zero only once.

After characterizing k, we note that in the constructed linear equilibrium, the value of g is given by the following equation:

$$g = -\left[\ln\frac{\delta}{c} + \frac{\tau_f\bar{f}+\frac{1}{2}}{\tau_f+\tau_l+\tau_p} - \frac{\tau_f\bar{f}+\frac{1}{2}}{\tau_f+\tau_s+\tau_c} + \frac{\left(\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p\right)}{\tau_f+\tau_l+\tau_p}\left(\frac{\tau_f\bar{f}}{\tau_f+\tau_s+\tau_c}\right)\right.$$
$$+\left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2\frac{1}{2(\tau_f+\tau_s+\tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f+\tau_l+\tau_p}\right)^2\sigma_l^2$$
$$\left.+\frac{1}{2}\left(\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2\left(\frac{1}{1+k}\right)^2\sigma_\xi^2 - \frac{1}{2}\sigma_\xi^2\right] \Big/ \left(1+\frac{\tau_s}{\tau_f+\tau_s+\tau_c}\frac{\left(\tau_l+\left(\frac{1}{1+k}\right)\tau_p\right)}{\tau_f+\tau_l+\tau_p}\right) \quad (28)$$

Finally, we need to establish that a speculator observing a private signal below $g - k\tilde{s}_c$ prefers to short sell and a speculator observing a signal above $g - k\tilde{s}_c$ prefers not to short sell. Note that the derivative of a speculator's payoff from short selling in (15) with respect to $\tilde{s}_i$ is $\tau_s/(\tau_f+\tau_s+\tau_c)$ times

$$\frac{1}{\delta}\exp\left(\frac{\tau_f\bar{f}+\tau_s\tilde{s}_i+\tau_c\tilde{s}_c+\frac{1}{2}}{\tau_f+\tau_s+\tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2\right)$$
$$-\frac{1}{c}\left(\frac{\left(\tau_f+2\tau_l+\tau_p\left(1+\frac{1}{1+k}\right)\right)}{\tau_f+\tau_l+\tau_p}\right)\exp\left(\begin{array}{c}\frac{\tau_f\bar{f}+\tau_p\frac{k}{1+k}\tilde{s}_c+\frac{1}{2}}{\tau_f+\tau_l+\tau_p} + \frac{\left(\tau_f+2\tau_l+\tau_p\left(1+\frac{1}{1+k}\right)\right)}{\tau_f+\tau_l+\tau_p}\left(\frac{\tau_f\bar{f}+\tau_s\tilde{s}_i+\tau_c\tilde{s}_c}{\tau_f+\tau_s+\tau_c}\right)\\ +\left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2\frac{1}{2(\tau_f+\tau_s+\tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f+\tau_l+\tau_p}\right)^2\sigma_l^2\\ +\frac{1}{2}\left(\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2\left(\frac{1}{1+k}\right)^2\sigma_\xi^2\end{array}\right)$$

Note that the above is strictly negative whenever the speculator's payoff is zero for a given $\tilde{s}_i$ and $\tilde{s}_c$. This implies that for a given $\tilde{s}_c$ there is a unique $\tilde{s}_i$ at which the speculator is indifferent between buying the asset or shorting it and that the speculator wants to buy for $\tilde{s}_i$ above this level and short below it. QED.

Proof of Proposition 2: We plug k_{BM} in the right side of Equation (18) to obtain:

$$-\left(\frac{\tau_s\tau_l}{\tau_f+\tau_s+\tau_c} + (\tau_f+\tau_l+\tau_c)\right)\left(\frac{\tau_l\tau_c}{(\tau_f+\tau_s+\tau_c)(\tau_f+\tau_l)+\tau_l\tau_s}\right)^3$$
$$+\left(\frac{\tau_c\tau_l-\tau_s\tau_c}{\tau_f+\tau_s+\tau_c} - \tau_c\right)\left(\frac{\tau_l\tau_c}{(\tau_f+\tau_s+\tau_c)(\tau_f+\tau_l)+\tau_l\tau_s}\right)^2$$
$$+\tau_c\left(\frac{\tau_c-\tau_s}{\tau_f+\tau_s+\tau_c}\right)\left(\frac{\tau_l\tau_c}{(\tau_f+\tau_s+\tau_c)(\tau_f+\tau_l)+\tau_l\tau_s}\right) + \frac{\tau_c^2}{\tau_f+\tau_s+\tau_c}$$
$$\frac{\tau_l\tau_c^2}{\tau_\xi(\tau_f+\tau_s+\tau_c)}\left(\frac{-(\tau_s\tau_l+(\tau_f+\tau_l)(\tau_f+\tau_s+\tau_c))}{(\tau_f+\tau_s+\tau_c)(\tau_f+\tau_l)+\tau_l\tau_s}+1\right)$$

which (after some tedious algebra) can be shown to be strictly positive. Therefore, k_{BM} is strictly less than the equilibrium weight that the speculators put on the common signal when the capital provider learns from price. The statement $k_{BM} < \tau_c/\tau_s$ is immediate from Equation (23). Similarly, to show that $k^* < \tau_c/\tau_s$, we plug τ_c/τ_s in the right side of Equation (18) and find that it is strictly negative. QED.

Proof of Proposition 3: Consider the last two terms in Equation (18):

$$-\frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l + (\tau_f+\tau_l)(\tau_f+\tau_s+\tau_c)\right)k + \frac{\tau_c^2\tau_l}{\tau_\xi}.$$

Denote by $k^*(\tau_\xi)$ the equilibrium k for a given τ_ξ. We want to show that $k^*(\tau_\xi)$ is decreasing in $1/\tau_\xi$. Take a fixed $\hat{\tau}_\xi > 0$. Note that the above sum is negative at $k = k^*(\hat{\tau}_\xi)$ (since $k^*(\hat{\tau}_\xi) > \tau_l\tau_c/\left((\tau_f+\tau_s+\tau_c)(\tau_f+\tau_l)+\tau_l\tau_s\right)$ by Proposition 2.) As $1/\tau_\xi$ increases this sum becomes more negative at $k = k^*(\hat{\tau}_\xi)$. This means that for $\tau_\xi > \hat{\tau}_\xi$ the value of Equation (18) is strictly negative at $k = k^*(\hat{\tau}_\xi)$ and thus $k^*(\tau_\xi) < k^*(\hat{\tau}_\xi)$ .QED.

Proof of Proposition 4

We start with a lemma:

Lemma 1: The expression

$$\begin{aligned} D(k) &= -3k^2\left((\tau_l+\tau_c+\tau_f)(\tau_c+\tau_f+\tau_s)+\tau_l\tau_s\right) \\ &+2\tau_c k(\tau_l-2\tau_s-\tau_f-\tau_c)+\tau_c(\tau_c-\tau_s)-\frac{\tau_c}{\tau_\xi}\left(\tau_l\tau_s+(\tau_l+\tau_f)(\tau_c+\tau_f+\tau_s)\right) \end{aligned}$$

is negative at $k = k^*$.

Proof of Lemma 1 We know that Equation (18) crosses zero once and from above so its derivative with respect to k is negative at k^*. QED

Now we proceed with the proof of Proposition 4. To see $\partial k^*/\partial\tau_f < 0$ we take the total derivative of Equation (18) with respect to τ_f to obtain:

$$\frac{\partial k^*}{\partial\tau_f} = \frac{k^3(2\tau_f+2\tau_c+\tau_l+\tau_s)+\tau_c k^2+\tau_c\frac{k}{\tau_\xi}(2\tau_f+\tau_c+\tau_s+\tau_l)}{D(k)} < 0.$$

Taking total derivative of Equation (18) with respect to τ_s and using Lemma 1 establishes that $\partial k^*/\partial\tau_s < 0$.

Next we show $\partial k^*/\partial\tau_l < 0$ for small enough τ_f. Taking total derivative of Equation (18) with respect to τ_l we see that the derivative is given by:

$$(\tau_s+(\tau_c+\tau_f+\tau_s))k^3-\tau_c k^2+\frac{\tau_c}{\tau_\xi}(\tau_c+\tau_f+2\tau_s)k-\frac{\tau_c^2}{\tau_\xi}$$

divided by $D(k)$. The numerator is negative if and only if

$$k > \frac{\tau_c}{2\tau_s + \tau_f + \tau_c}.$$

We directly verify that the value of Equation (18) at $\tau_c / (2\tau_s + \tau_f + \tau_c)$ is positive if τ_f is small enough. The last result then again follows from Lemma 1.

Finally, taking total derivative of Equation (18) with respect to τ_c we obtain $\partial k^* / \partial \tau_c$ equals

$$\left[\begin{array}{c} k^3 (2\tau_c + 2\tau_f + \tau_s + \tau_l) + k^2 (2\tau_c + \tau_f - \tau_l + 2\tau_s) - k (2\tau_c - \tau_s) - 2\tau_c \\ + \frac{k}{\tau_\xi} (\tau_l \tau_s + (\tau_f + \tau_l)(2\tau_c + \tau_f + \tau_s)) - 2\tau_c \frac{\tau_l}{\tau_\xi} \end{array} \right] / D(k).$$

Using Equation (18) we can write the numerator as:

$$\frac{1}{\tau_c} \left[- \left(2\tau_s \tau_l + \tau_l \tau_f + \tau_s \tau_f + \tau_f^2\right) k^3 + \tau_c^2 \left(k^3 + k^2 - k - 1 + \frac{k}{\tau_\xi} (\tau_f + \tau_l) - \frac{\tau_l}{\tau_\xi} \right) \right].$$

Equation (18) evaluated at $k = 1$ is strictly negative thus in equilibrium $k^* < 1$. Moreover the expression

$$k^3 + k^2 - k - 1 + \frac{\tau_l}{\tau_\xi} (k - 1) < 0$$

for $k \in (0, 1)$. Therefore the numerator of $\partial k^* / \partial \tau_c$ is negative for small enough τ_f. Using Lemma 1 establishes that $\partial k^* / \partial \tau_c > 0$ for small enough τ_f. QED

Proof of Proposition 5:

We substitute optimal I into Equation (24) and compute the expectations:

$$
\begin{aligned}
& \frac{1}{c}E\left[\exp\left(\tilde{f}\right)\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\left(P\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\left(P\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
= & \frac{1}{c}E\left[\exp\left(2\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z\left(P\right)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z\left(P\right)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
= & E\frac{1}{c}\left[\exp\left(\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\tilde{f}+\frac{\tau_f\bar{f}}{\tau_f+\tau_l+\tau_p}+\frac{\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z\left(P\right)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z\left(P\right)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
= & \frac{1}{c}\exp\left(2\bar{f}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}+\frac{1}{2}\frac{\left(\tau_f+2\tau_l+2\tau_p\right)^2\frac{1}{\tau_f}}{\left(\tau_f+\tau_l+\tau_p\right)^2}+\frac{1}{2}\frac{\tau_l+\tau_p}{\left(\tau_f+\tau_l+\tau_p\right)^2}\right) \\
& -\frac{1}{c}\frac{1}{2}\exp\left(2\bar{f}+\frac{1}{(\tau_f+\tau_l+\tau_p)}+2\left(\frac{\left(\tau_l+\tau_p\right)^2\frac{1}{\tau_f}}{\left(\tau_f+\tau_l+\tau_p\right)^2}+\frac{\tau_l+\tau_p}{\left(\tau_f+\tau_l+\tau_p\right)^2}\right)\right) \\
= & \frac{1}{c}\exp\left(2\bar{f}+\frac{1}{\tau_f}\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right)-\frac{1}{c}\frac{1}{2}\exp\left(2\bar{f}+\frac{1}{\tau_f}\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right) \\
= & \frac{1}{c}\frac{1}{2}\exp\left(2\bar{f}+\frac{1}{\tau_f}\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right).
\end{aligned}
$$

Therefore the maximization problem can be viewed as maximizing the following expression in k :

$$\exp\left(\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right),$$

and this is equivalent to maximizing τ_p. Moreover τ_p is increasing for $k<\tau_c/\tau_\xi$ and decreasing for $k<\tau_c/\tau_\xi$ which proves the last statement. QED.

Proof of Proposition 6: Since $\tau_p = \left((1+k)^2\tau_c\tau_\xi\right)/\left(k^2\tau_\xi+\tau_c\right)$, its maximum is

achieved when $k = \tau_c/\tau_\xi$. We plug $k = \tau_c/\tau_\xi$ in the right side of Equation (18) to obtain:

$$\begin{aligned}
& -\left(\tau_s\tau_l + (\tau_f + \tau_s + \tau_c)(\tau_f + \tau_l + \tau_c)\right)\left(\frac{\tau_c}{\tau_\xi}\right)^3 + \tau_c\left(\tau_l - 2\tau_s - \tau_f - \tau_c\right)\left(\frac{\tau_c}{\tau_\xi}\right)^2 \\
& +\tau_c\left(\tau_c - \tau_s\right)\left(\frac{\tau_c}{\tau_\xi}\right) + \tau_c^2 - \frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l + (\tau_f + \tau_l)(\tau_f + \tau_s + \tau_c)\right)\left(\frac{\tau_c}{\tau_\xi}\right) + \frac{\tau_c^2\tau_l}{\tau_\xi} \\
= \; & -\frac{\tau_c^2}{\tau_\xi^3}\left(\tau_c + \tau_\xi\right)\left(\tau_c^2 + \tau_f^2 - \tau_\xi^2 + 2\tau_c\tau_f + \tau_c\tau_l + \tau_f\tau_l + \tau_c\tau_s + \tau_f\tau_s + 2\tau_l\tau_s - \tau_l\tau_\xi + \tau_s\tau_\xi\right).
\end{aligned}$$

There exists $\bar{\tau}_\xi$ such that the above expression is negative for $\tau_\xi < \bar{\tau}_\xi$ and positive for $\tau_\xi > \bar{\tau}_\xi$. To see this note that the sign of the above expression is the negative of the sign of the last part in brackets. It is easy to see that the last part is positive at $\tau_\xi = 0$, may increase as τ_ξ increases at first but will eventually decrease in τ_ξ and cross once and for all to the negative region. Using the logic in the proof of Proposition 1, this establishes the statement in the proposition. QED.

Proof of Proposition 7: (a) The market clearing price is

$$P = \frac{1}{\delta}\exp\left((1+k)\,\tilde{f} + k\sigma_c\tilde{\epsilon}_c - g + \tilde{\xi}\right),$$

and its excess volatility is defined as non-fundamental volatility which can be written as the volatility of the following:

$$z(P) - \tilde{f} = \frac{g + \ln(\delta P)}{1+k} - \tilde{f} = \frac{k}{1+k}\sigma_c\tilde{\epsilon}_c + \frac{1}{1+k}\tilde{\xi}.$$

It is straightforward to show that when $k = k_{OP} = \tau_c/\tau_\xi$, its excess volatility is the lowest and is

$$\text{Excess Volatility}\,(\text{Asset Price}) = \frac{1}{\tau_c + \tau_\xi}.$$

The rest of the statement follows immediately. QED.

(b) When $k = k_{OP} = \tau_c/\tau_\xi$, $\tau_p = \tau_c + \tau_\xi$. We know that:

$$I = \frac{1}{c}\exp\left(\frac{\tau_f\bar{f} + \tau_l s_l + \tau_p\left(\tilde{f} + \frac{k}{1+k}\sigma_c\tilde{\epsilon}_c + \frac{1}{1+k}\tilde{\xi}\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right).$$

Take logs on both sides, we obtain:

$$\ln I = \ln\left(\frac{1}{c}\right) + \left(\frac{\tau_f\bar{f} + \tau_l s_l + \tau_p\left(\tilde{f} + \frac{k}{1+k}\sigma_c\tilde{\epsilon}_c + \frac{1}{1+k}\tilde{\xi}\right)}{\tau_f + \tau_l + \tau_p} + \frac{1}{2(\tau_f + \tau_l + \tau_p)}\right).$$

We can define the excess volatility of the real investment as the volatility of the following:

$$\frac{(\tau_f+\tau_l+\tau_p)\left(\ln I-\ln\left(\frac{1}{c}\right)\right)-\frac{1}{2}-\tau_f\bar{f}}{\tau_l+\tau_p}-\tilde{f}=\frac{\tau_l\sigma_l\epsilon_l+\tau_p\left(\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{1}{1+k}\tilde{\xi}\right)}{\tau_l+\tau_p}$$

It is straightforward to show that when $k=k_{OP}=\tau_c/\tau_\xi$, $\tau_p=\tau_c+\tau_\xi$,and the excess volatility of the real investment is the lowest which is

$$\text{Excess Volatility}\,(\text{Real Investment})=\frac{1}{\tau_l+\tau_c+\tau_\xi}.$$

The rest of the statement follows immediately. QED.

Proof of Proposition 8:

Given the adjusted cost of investment solution to capital provider's problem is given by:

$$I=\frac{E[\tilde{F}|\mathcal{F}_l]}{cE[\tilde{F}^\beta|\mathcal{F}_l]}=\frac{1}{c}\exp\left((1-\beta)\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\,(P)}{\tau_f+\tau_l+\tau_p}\right)+\frac{(1-\beta^2)}{2(\tau_f+\tau_l+\tau_p)}\right).$$

Given the investment policy and the price in (8), we can now write speculator i's expected profit from buying the asset given the information that is available to her:

$$\begin{aligned}&E\left[\tilde{F}I-P|\tilde{s}_i,\tilde{s}_c\right] \qquad (29)\\ =\;&\frac{1}{c}E\left[\exp\left((1-\beta)\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z\,(P)}{\tau_f+\tau_l+\tau_p}\right)+\frac{(1-\beta^2)}{2}\frac{1}{\tau_f+\tau_l+\tau_p}+\tilde{f}\right)|\tilde{s}_i,\tilde{s}_c\right]\\ &-E\left[\frac{1}{\delta}\exp\left(\tilde{f}+k\tilde{s}_c-g+\tilde{\xi}\right)|\tilde{s}_i,\tilde{s}_c\right].\end{aligned}$$

As before, conditional on observing $\tilde{s}_i$ and $\tilde{s}_c$ speculator i believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f}{\tau_f+\tau_s+\tau_c}\bar{f}+\frac{\tau_s}{\tau_f+\tau_s+\tau_c}\tilde{s}_i+\frac{\tau_c}{\tau_f+\tau_s+\tau_c}\tilde{s}_c$$

and variance $1/\left(\tau_f+\tau_s+\tau_c\right)$. Hence, substituting for $z\,(P)$ (from (9)) and taking expectations, Equation (29) can be rewritten as:

$$\begin{aligned}&E\left[\tilde{F}I-P|\tilde{s}_i,\tilde{s}_c\right] \qquad (30)\\ =\;&\frac{1}{c}\exp\left(\begin{array}{c}\frac{(1-\beta)\tau_f\bar{f}}{\tau_f+\tau_l+\tau_p}+\frac{\left(\left(\frac{1-\beta}{1+k}+1\right)\right)\tau_p+\tau_f+(2-\beta)\tau_l}{\tau_f+\tau_l+\tau_p}\left(\frac{\tau_f\bar{f}+\tau_s\tilde{s}_i+\tau_c\tilde{s}_c}{\tau_f+\tau_s+\tau_c}\right)+\frac{1}{2}\left(\frac{\left(\left(\frac{1-\beta}{1+k}+1\right)\right)\tau_p+\tau_f+(2-\beta)\tau_l}{\tau_f+\tau_l+\tau_p}\right)^2\frac{1}{\tau_f+\tau_s+\tau_c}\\ +\frac{(1-\beta)^2}{2}\frac{\tau_l+\tau_p^2\left(\frac{1}{1+k}\right)^2\sigma_\xi^2}{\left(\tau_f+\tau_l+\tau_p\right)^2}+\frac{\tau_p\frac{k}{1+k}\tilde{s}_c}{\tau_f+\tau_l+\tau_p}+\frac{\left(1-\beta^2\right)}{2}\frac{1}{\tau_f+\tau_l+\tau_p}\end{array}\right)\\ &-\frac{1}{\delta}\exp\left(\frac{\tau_f\bar{f}+\tau_s\tilde{s}_i+\tau_c\tilde{s}_c+\frac{1}{2}}{\tau_f+\tau_s+\tau_c}+k\tilde{s}_c-g+\frac{1}{2}\sigma_\xi^2\right).\end{aligned}$$

In equilibrium, a speculator who receives a private signal $\tilde{s}_i = g - k\tilde{s}_c$ must be indifferent between shorting and buying the asset. That is,

$$E\left[\tilde{F}I - P|\tilde{s}_i = g - k\tilde{s}_c, \tilde{s}_c\right] = 0. \tag{31}$$

Substituting $\tilde{s}_i = g - k\tilde{s}_c$ into (30), and taking logs, the indifference condition for the marginal investor becomes:

$$\begin{aligned}
& \ln\frac{1}{\delta} + \frac{\tau_f\bar{f} + \tau_s\left(g - k\tilde{s}_c\right) + \tau_c\tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2 \\
= \; & \ln\frac{1}{c} + \frac{\left(1-\beta\right)\tau_f\bar{f}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\left(\frac{1-\beta}{1+k} + 1\right)\right)\tau_p + \tau_f + \left(2-\beta\right)\tau_l}{\tau_f + \tau_l + \tau_p}\left(\frac{\tau_f\bar{f} + \tau_s\left(g - k\tilde{s}_c\right) + \tau_c\tilde{s}_c}{\tau_f + \tau_s + \tau_c}\right) \\
& + \frac{1}{2}\left(\frac{\left(\left(\frac{1-\beta}{1+k} + 1\right)\right)\tau_p + \tau_f + \left(2-\beta\right)\tau_l}{\tau_f + \tau_l + \tau_p}\right)^2 \frac{1}{\tau_f + \tau_s + \tau_c} \\
& + \frac{\left(1-\beta\right)^2}{2}\frac{\tau_l + \tau_p^2\left(\frac{1}{1+k}\right)^2\sigma_\xi^2}{\left(\tau_f + \tau_l + \tau_p\right)^2} + \frac{\tau_p\frac{k}{1+k}\tilde{s}_c}{\tau_f + \tau_l + \tau_p} + \frac{\left(1-\beta^2\right)}{2}\frac{1}{\tau_f + \tau_l + \tau_p}.
\end{aligned}$$

In a linear equilibrium the above equality must hold for all $\tilde{s}_c$. Therefore, the coefficient of $\tilde{s}_c$ must be zero. That is, the equilibrium k in this case satisfies the following equation:

$$\frac{\tau_p}{\tau_f + \tau_l + \tau_p}\frac{k}{1+k} + \frac{\tau_f + \left(2-\beta\right)\tau_l + \left(\left(\frac{1-\beta}{1+k} + 1\right)\right)\tau_p}{\tau_f + \tau_l + \tau_p}\left(\frac{-\tau_s k + \tau_c}{\tau_f + \tau_s + \tau_c}\right) - \frac{-\tau_s k + \tau_c}{\tau_f + \tau_s + \tau_c} - k = 0.$$

Rearranging we obtain the following equation for the equilibrium k:

$$\begin{aligned}
& -\left(\tau_s\tau_l\left(1-\beta\right) + \left(\tau_f + \tau_l + \tau_c\right)\left(\tau_c + \tau_f + \tau_s\right)\right)k^3 + \tau_c\left(\tau_l\left(1-\beta\right) - \tau_s\left(2-\beta\right) - \tau_f - \tau_c\right)k^2 \\
& + \left(1-\beta\right)\tau_c\left(\tau_c - \tau_s\right)k + \tau_c^2\left(1-\beta\right) \\
& - \frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l\left(1-\beta\right) + \left(\tau_f + \tau_l\right)\left(\tau_s + \tau_c + \tau_f\right)\right)k + \left(\frac{\tau_c^2\tau_l}{\tau_\xi}\left(1-\beta\right)\right) = 0.
\end{aligned}$$

Let

$$\begin{aligned}
J\left(k\right) \; = \; & -\left(\tau_s\tau_l + \left(\tau_f + \tau_l + \tau_c\right)\left(\tau_c + \tau_f + \tau_s\right)\right)k^3 + \tau_c\left(\tau_l - 2\tau_s - \tau_f - \tau_c\right)k^2 + \tau_c\left(\tau_c - \tau_s\right)k + \tau_c^2 \\
& - \frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l + \left(\tau_f + \tau_l\right)\left(\tau_s + \tau_c + \tau_f\right)\right)k + \left(\frac{\tau_c^2\tau_l}{\tau_\xi}\right)
\end{aligned}$$

and

$$G\left(k\right) = -\tau_s\tau_l k^3 + \tau_c\left(\tau_l - \tau_s\right)k^2 + \tau_c\left(\tau_c - \tau_s\right)k + \tau_c^2 - \frac{\tau_c}{\tau_\xi}\tau_s\tau_l k + \frac{\tau_c^2\tau_l}{\tau_\xi}.$$

Thus the equilibrium condition is:

$$J\left(k\left(\beta\right)\right) - \beta G\left(k\left(\beta\right)\right) = 0. \tag{32}$$

From the proof of Proposition 1 we know that $J(k)$ has a unique strictly positive root. Thus for small enough β (32) has a unique strictly positive root as well. and the equilibrium without policy intervention is given by $k(0)$ that solves $H(k(0)) = 0$. It is easy to see that

$$H(k) = G(k) - ((\tau_f + \tau_l + \tau_c)(\tau_c + \tau_f + \tau_s))k^3 + \tau_c(-\tau_s - \tau_f - \tau_c)k^2 - \frac{\tau_c}{\tau_\xi}((\tau_f + \tau_l)(\tau_s + \tau_c + \tau_f))k.$$

Therefore, $G(k(0)) > 0$.

Since

$$\frac{\partial k(\beta)}{\partial \beta} = \frac{G(k(\beta))}{H'(k(\beta)) - \beta G'(k(\beta))}$$

and $H'(k(0)) < 0$ (from the derivation of equilibrium $k(0)$ without policy intervention) we see that

$$\frac{\partial k(0)}{\partial \beta} = \frac{G(k(0))}{H'(k(0))} < 0.$$

Therefore, for β close to zero, a policy with cost of funding positively correlated with the fundamental ($\beta > 0$) leads to less coordination among speculators and a policy with the cost of funding negatively correlated with the fundamental ($\beta < 0$) leads to more coordination among speculators. QED.