Discussion of “Decentralized Trading with Private Information,” by Michael Golosov, Guido Lorenzoni, Aleh Tsyvinski

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Learning in Markets

Informational role of prices: Hayek (1945).

- Centralized Markets:
  - Grossman (1976), Grossman and Stiglitz (1981).
  - Wilson (1977), Townsend (1978), Milgrom (1981), Vives (1993), Pesendorfer and Swinkels (1997), and Reny and Perry (2006).
- Decentralized Markets: What if learning happens through local interactions/private negotiations?

Other Applications

- centralized exchanges, decentralized information transmission
- bank runs
- viral marketing
- knowledge spillovers
- technology diffusion

Summary of Golosov, Lorenzoni, Tsyvinski (2010)

- Informed investors.
- Random matching, private negotiations.
- Convergence to REE
- Value of Information
- Numerical Computation

Related to Information Percolation Work

- Duffie and Manso (2007)
- Duffie, Giroux, and Manso (2010)
- Duffie, Malamud, and Manso (2010a,2010b)

Some Differences

1. Discrete-time, full-matching model
2. Take-it-or-leave-it offer
3. Value of Information

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Continuous-time, partial matching

- For each agent, meetings according to a Poisson arrival process with intensity λ.
- Boltzmann Equation

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t \circ \mu_t.$$

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Auctions

- One-Sided Seller Auction
  - continuum of risk-neutral informed sellers.
  - each seller meets an uninformed buyer and another informed sellers with intensity λ.
  - auction to sell to the uninformed buyer an asset that pays 1 if $X = H$ and 0 otherwise.
- Double Auction
  - Different groups of agents with different valuations for an asset.
  - Meeting with intensity λ.
  - Participate in a k-double auction.

Equilibrium in strictly increasing strategies.

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Log-likelihood Ratio

After observing signals signals $S = \{s_1, \ldots, s_n\}$, the logarithm of the likelihood ratio between states $Y = 0$ and $Y = 1$ is by Bayes' rule:

$$\log \frac{\mathrm{P}(Y = 0 \,|\, s_1, \ldots, s_n)}{\mathrm{P}(Y = 1 \,|\, s_1, \ldots, s_n)} = \log \frac{\mathrm{P}(Y = 0)}{\mathrm{P}(Y = 1)} + \sum_{i=1}^{n} \log \frac{\mathrm{P}(s_i \,|\, Y = 0)}{\mathrm{P}(s_i \,|\, Y = 1)}.$$

Solution for Cross-Sectional Distribution of Information

The Boltzmann equation for the cross-sectional distribution μ_t of types is

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t * \mu_t.$$

where $*$ is the convolution operator and μ_0 is the initial distribution of types.

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Proposition: The unique solution of (12) is the Wild sum

$$\mu_t = \sum_{n\geq 1} e^{-\lambda t}(1-e^{-\lambda t})^{n-1}\mu_0^{*n}.$$

Proof of Wild Summation

Taking the Fourier transform μ_t of the Boltzmann equation

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t * \mu_t.$$

we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^2.$$

whose solution is

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{\lambda t}(1-\hat{\mu}_0)+\hat{\mu}_0}.$$

This solution can be expanded as

$$\hat{\mu}_t = \sum_{n\geq 1} e^{-\lambda t}(1-e^{-\lambda t})^{n-1}\hat{\mu}_0^n,$$

which is the Fourier transform of the Wild sum (12).

Multi-Agent Meetings

The Boltzmann equation for the cross-sectional distribution μ_t of types is

$$\frac{d}{dt}\mu_t = -\lambda\,\mu_t + \lambda\,\mu_t^{*m}.$$

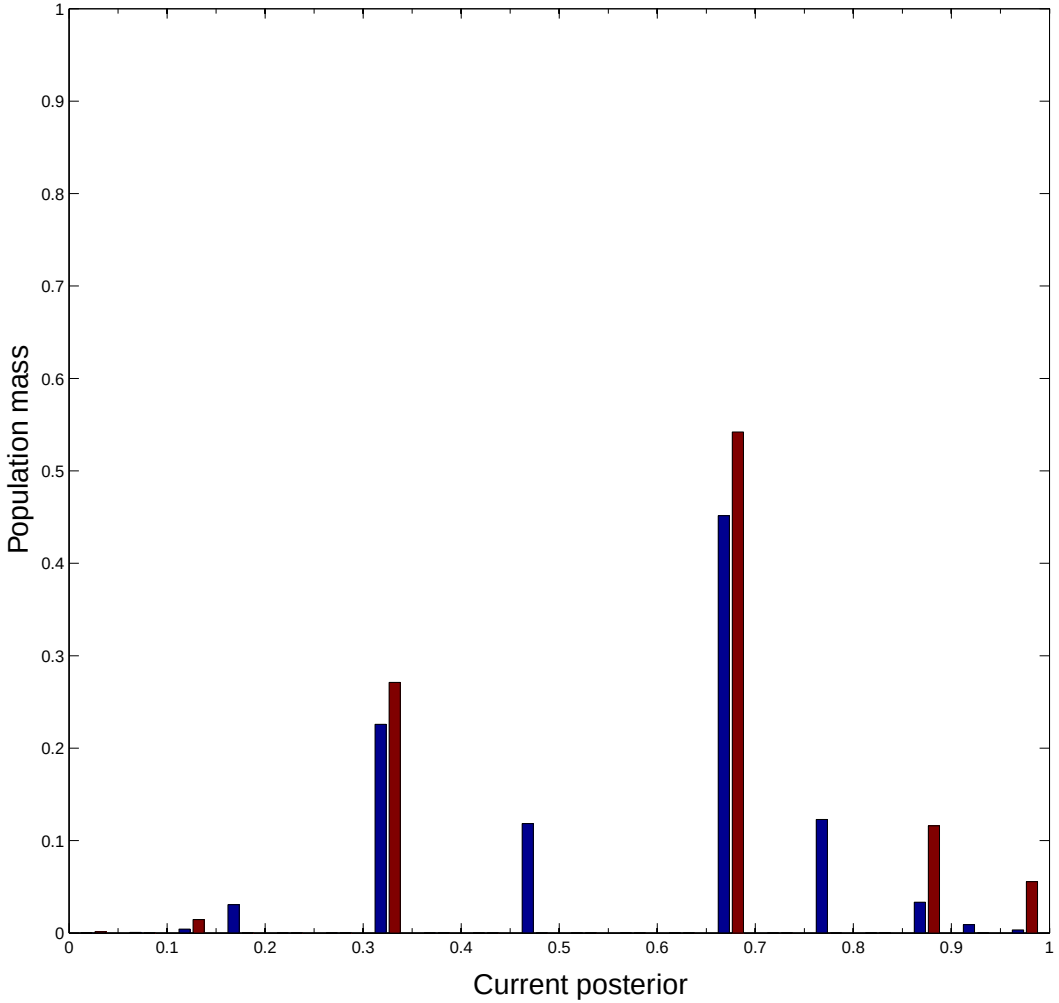
Taking the Fourier transform, we obtain the ODE,

$$\frac{d}{dt}\hat{\mu}_t = -\lambda\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^{m}.$$

whose solution satisfies

$$\hat{\mu}_t^{m-1} = \frac{\hat{\mu}_0^{m-1}}{e^{(m-1)\lambda t}(1-\hat{\mu}_0^{m-1}) + \hat{\mu}_0^{m-1}}. \tag{1}$$

Groups of 2 (blue) versus Groups of 3 (red)



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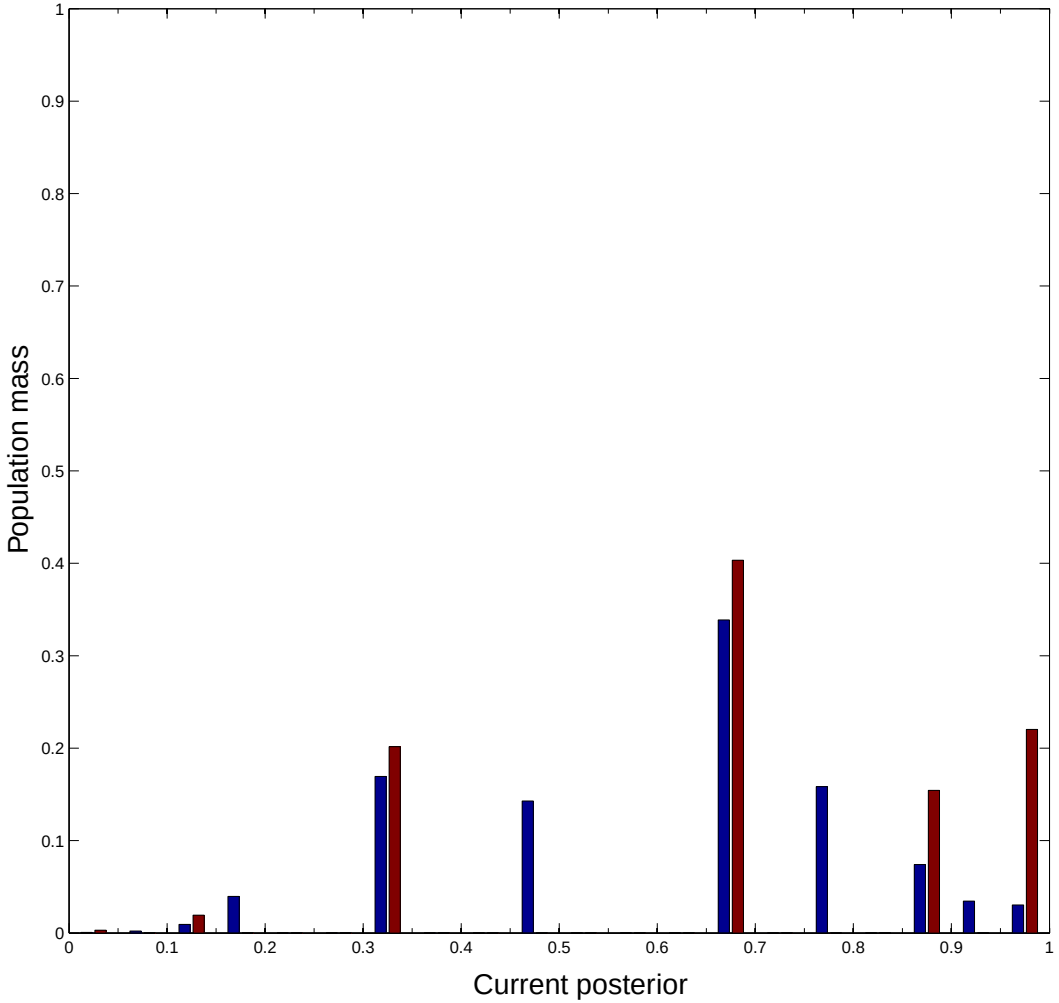

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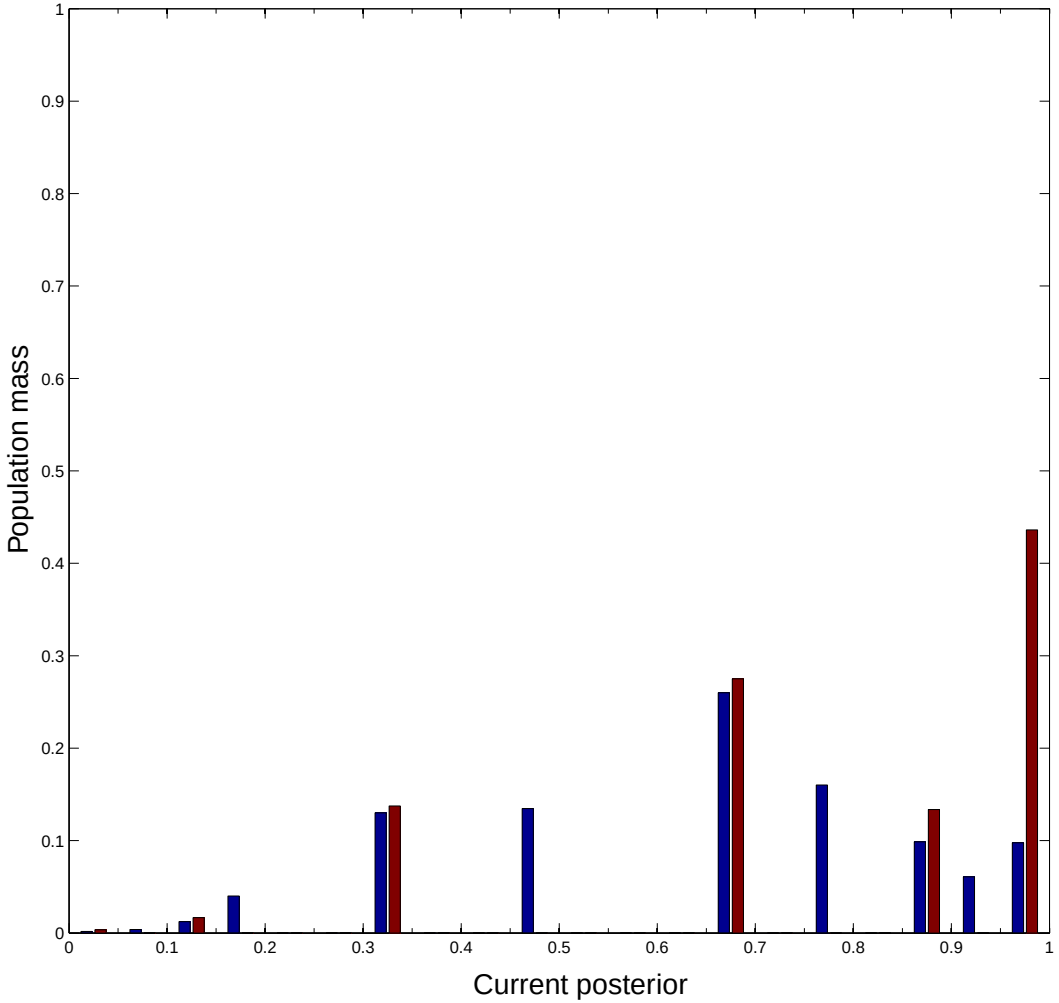

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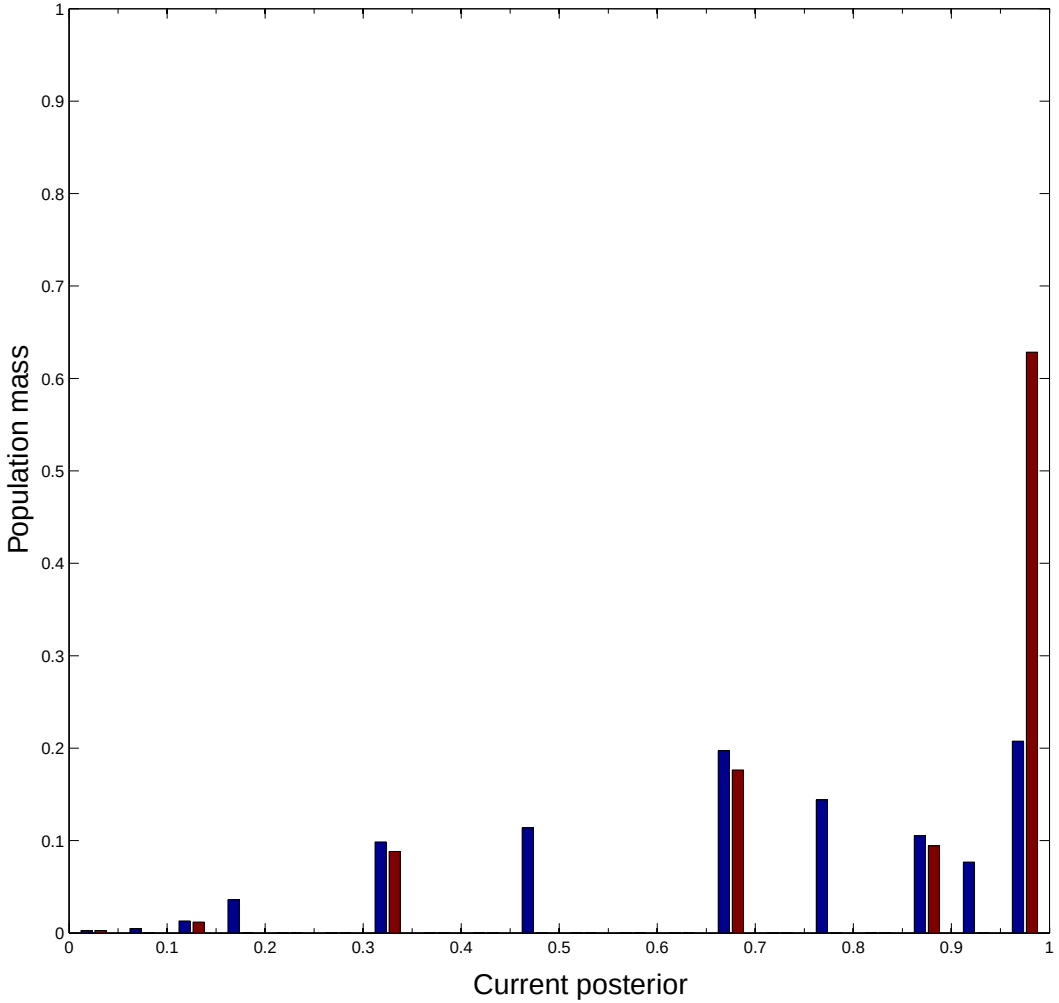

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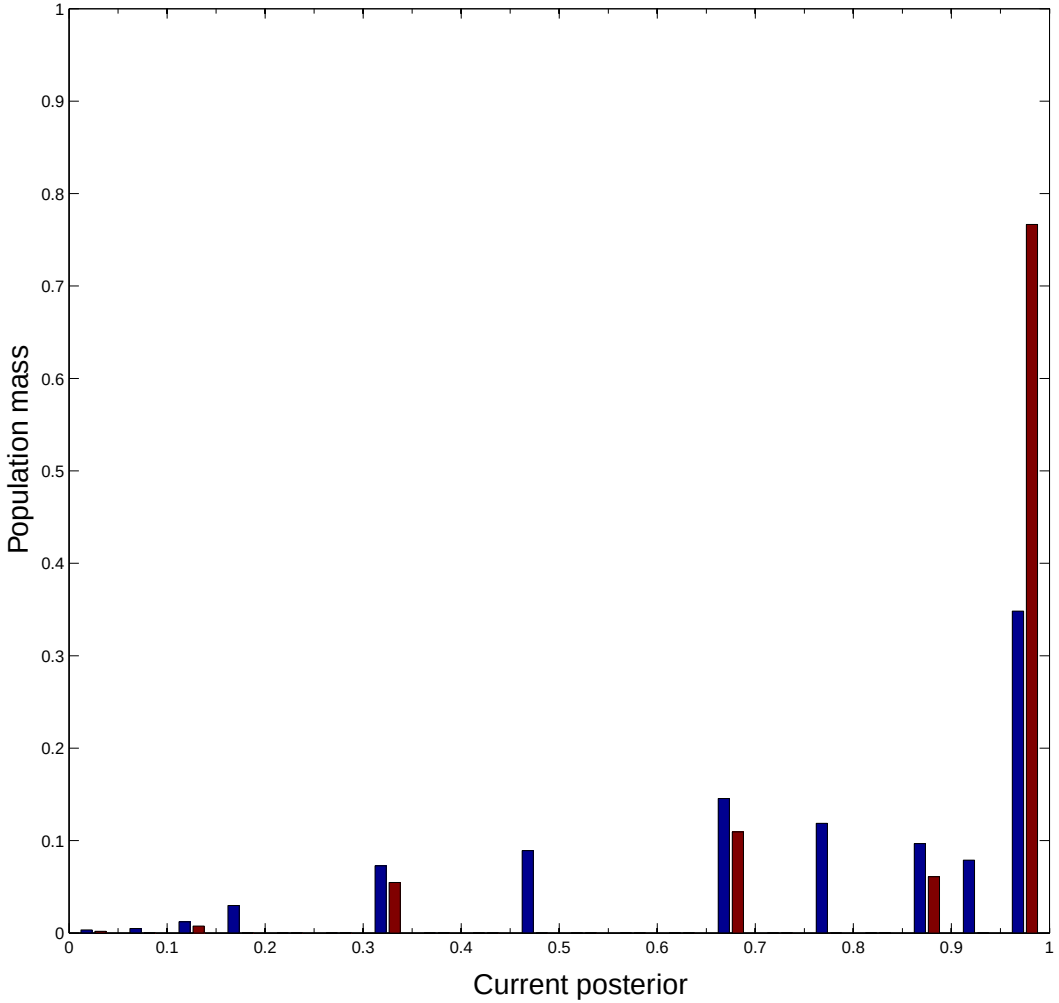

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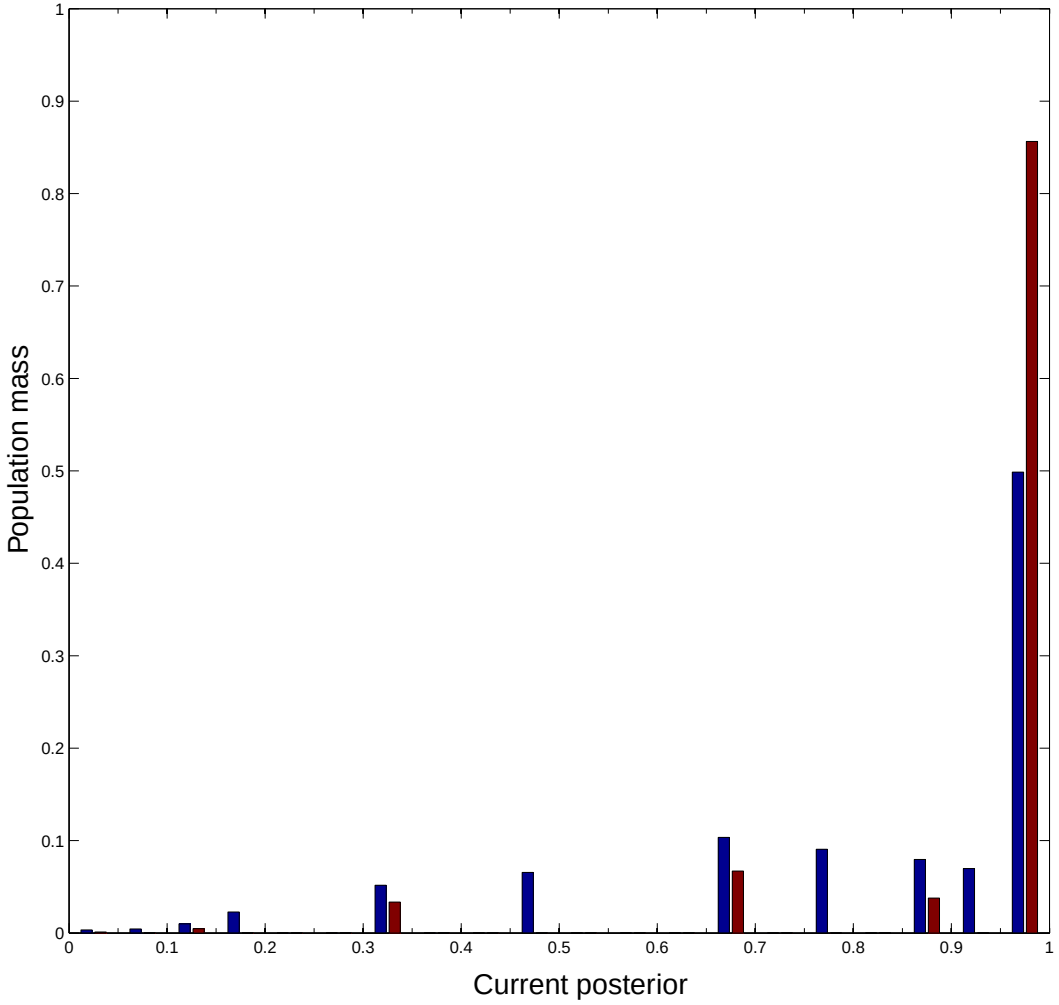

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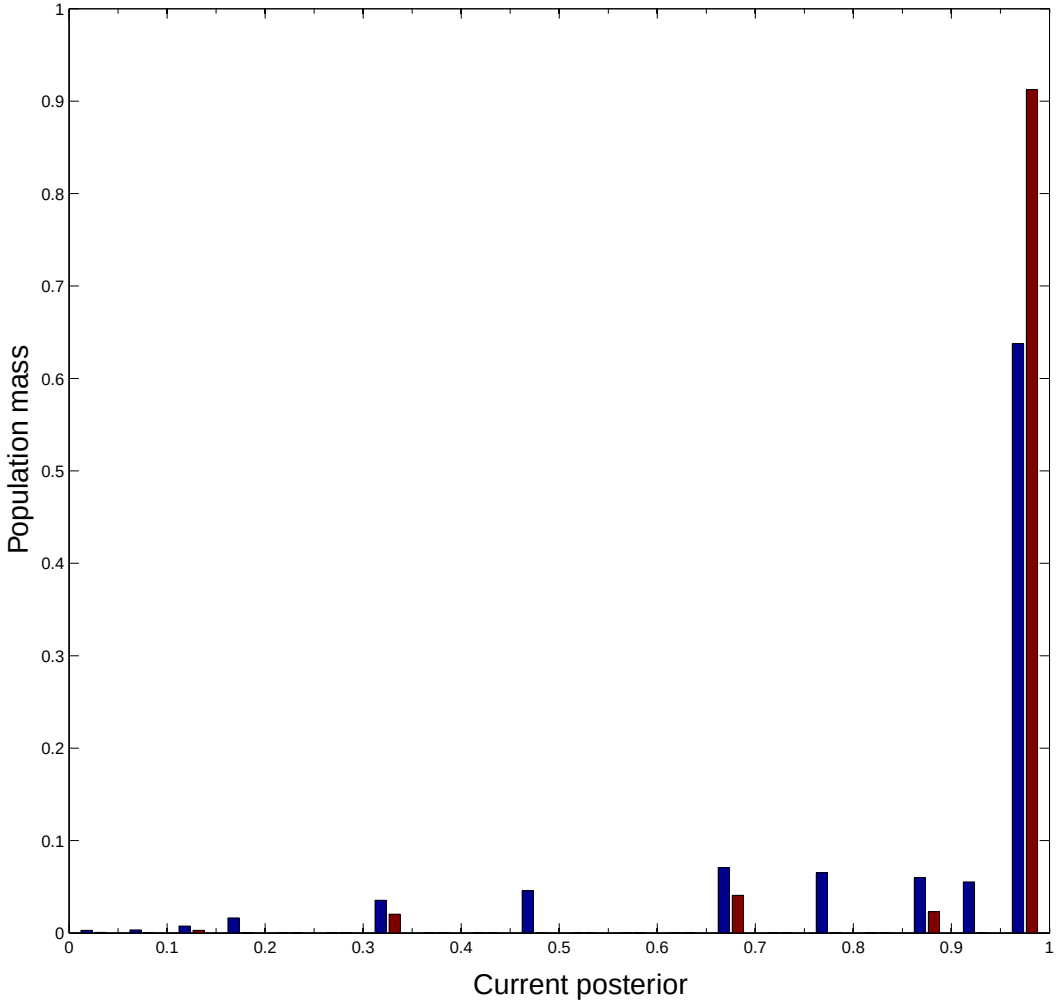

New Private Information

Suppose that, independently across agents as above, each agent receives, at Poisson mean arrival rate ρ, a new private set of signals whose type outcome y is distributed according to a probability measure ν. Then the evolution equation is extended to

$$\frac{d}{dt}\mu_t = -(\lambda+\rho)\,\mu_t + \lambda\,\mu_t * \mu_t + \rho\,\mu_t * \nu.$$

Taking Fourier transforms, we obtain the following ODE

$$\frac{d}{dt}\hat{\mu}_t = -(\lambda+\rho)\,\hat{\mu}_t + \lambda\,\hat{\mu}_t^2 + \rho\,\hat{\mu}_t\,\hat{\nu}.$$

whose solution satisfies

$$\hat{\mu}_t = \frac{\hat{\mu}_0}{e^{(\lambda+\rho(1-\hat{\nu}))t}(1-\hat{\mu}_0)+\hat{\mu}_0}$$

Segmented Markets

Same as the previous model except that:

- N classes of investors.
- Agent of class i has matching intensity λ_i.
- Upon meeting, the probability that a class-j agent is selected as a counterparty is κ_{ij}.

Evolution of Type Distribution

The evolution equation is given by:

$$\frac{d}{dt}\psi_{it} \;=\; -\lambda_i\,\psi_{it} \;+\; \lambda_i\,\psi_{it} * \sum_{j=1}^{N} \kappa_{ij}\,\psi_{jt}, \quad i \in \{1,\ldots,N\},$$

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General Case: Wild Sum Representation

Theorem: There is a unique solution of the evolution equation, given by

$$\psi_{it} \;=\; \sum_{k\in\mathbb{Z}_+^N} a_{it}(k)\,\psi_{10}^{*k_1} * \cdots * \psi_{N0}^{*k_N},$$

where ψ_{i0}^{*n} denotes n-fold convolution,

$$a_{it}' \;=\; -\lambda_i\, a_{it} \;+\; \lambda_i\, a_{it} * \sum_{j=1}^{N} \kappa_{ij}\, a_{jt}, \quad a_{i0} = \delta_{e_i},$$

$$(a_{it} * a_{jt})(k_1,\ldots,k_N) \;=\; \sum_{l=(l_1,\ldots,l_N)\in\mathbb{Z}_+^N,\, l<k} a_{it}(l)\, a_{jt}(k-l),$$

and

$$a_{it}(e_i) \;=\; e^{-\lambda_i t}\, a_{i0}(e_i).$$

Other Extensions

- Public information releases
  - Duffie, Malamud, and Manso (2010).
- Endogenous search intensity
  - Duffie, Malamud, and Manso (2009).

A Few Results

1. Prices converge exponentially to the rational expectations price.
2. The rate of convergence does not depend on the number of agents in each auction.
3. Public and private channels contribute equally to rate of convergence. Discontinuity at $\lambda = 0$.

Some Differences

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Value of Information

Effects of information and connectivity on profits:

- more informed/connected investors attain higher expected profits than less informed/connected investors if they can disguise trades.
- more informed/connected investors may not attain higher expected profits than less informed/ connected investors if characteristics are commonly observed.
- if investors have access to a cheap source of additional information, in equilibrium, they always choose to acquire information, even when this means lower expected profits.

Important Questions

- Endogenous information acquisition and convergence.
- Market design