Compatibility and Market Structure*

by

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Abstract

This paper studies the choice of compatibility in a model of two types of complementary products. It is assumed that the industry for the first type of the complementary components is a duopoly where firms realize positive profits, while the industry for the second type of complementary components is monopolistically competitive. Two regimes are compared. In the regime of compatibility, all components are readily combinable. In the regime of incompatibility, a type "B" producer has to provide two different versions of his product, each compatible with a specific type "A" component. It is shown that, at the long run free entry equilibrium, the number of active "B"-type producers is smaller in the regime of incompatibility. When there is no significant increase in industry demand as a result of an increase in the number of varieties of type "B", profits for "A"-type firms are higher in the regime of incompatibility, and these firms will prefer this regime. Conversely, when the addition of a variety results in a significant increase in industry demand, an "A"-type firm has higher profits in the regime of compatibility.

Key words: Compatibility, Standardization, Components, Variety.

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1. Introduction

The issue of technical compatibility has received significant attention by academic economists during the last few years. This is not surprising given the increasing complexity of products in the marketplace. Increasingly, there are more products available that are composed of two or more components, with each component made by a different manufacturer. The question of compatibility arises immediately, since many manufacturers have a choice between making their components compatible with components of other manufacturers or producing stand-alone systems, whose components are incompatible with those of other manufacturers.

The issue of compatibility inevitably prompts an analysis of the basic substitutability and complementarity relationships among components and systems. The following basic structure is useful in analyzing these relationships.\(^1\) There are two types of complementary components, "A" and "B". There are \(m\) producers of components of type "A", and there are \(n\) producers of type "B" goods. Under a regime of full compatibility, any component of type "A" can be readily combined with any component of type "B" to form system \(A_iB_j\).\(^2\) In a regime of incompatibility, there are two or more "standards," and only two components that conform to the same "standard" can be combined into a working system.

In the existing literature, there is a maintained assumption that all differentiated components of the same type are produced by different firms. Further, most of the analyses that use this structure have focused on parallelly vertically integrated firms, where each firm produces

\(^1\) Carmen Matutes and Pierre Regibeau (1988) introduced this "mix-and-match" model in a locational framework for \(n = m = 2\). This framework was generalized within the context of a locational setting by Nicholas Economides (1989a). A more general non-locational setting was introduced in Economides (1988), (1991). This framework has been recently used by Economides and Steve Salop (1992) to analyze pricing in alternative market structures in a regime of full compatibility.

\(^2\) Systems are assembled by consumers.
every type of the complementary products. A common result has emerged: profits are higher in a regime of full compatibility.  

This paper departs from both of these two common assumptions when analyzing the regime of incompatibility. First, it is assumed that each firm produces components of one type only. There are two firms of type "A", each with a different technical standard. Each type-"B" firm produces two versions of its product, each conforming to the different specifications of the two type-A producers. Note that this is common practice for firms that produce software for incompatible hardware platforms. For example, Microsoft Word has different versions for the MS-DOS and the Macintosh operating systems.

This paper also differs from most of the mix-and-match literature by assuming that the number of differentiated products is not fixed. It is assumed that the number of "B"-type firms is variable, endogenously determined through free entry. The market structure of the "B" side of the market is assumed to be monopolistic competition.

In contrast with the results in the existing literature, it is shown that prices and profits can be higher under incompatibility. The analysis is surprisingly simple. It is first shown that for every fixed number of firms producing "B"-type products, the equilibrium prices and profits for the "A"-type firms are the same across regimes. As the number of "B"-type firms increases parametrically, equilibrium prices and profits decrease for all firms. Free entry in the "B" market leads to fewer "B"-type firms under incompatibility than under compatibility. This result occurs because in this regime each firm incurs a fixed cost for the production of two different versions of its product and this cost is higher than the fixed cost of producing a single version under compatibility. Under incompatibility, there are fewer firms of type "B" at the free entry equilibrium. When the addition of an extra variety leads to little or no increase in industry demand, the profits of a firm of type "A" decrease in the number of varieties of "B"-type. Since

See for example, Chien-fu Chou and Oz Shy (1990), Nicholas Economides (1989a, 1991), and Carmen Matutes and Pierre Regibeau (1988).
the number of varieties at the free entry equilibrium is lower under incompatibility, the profits of a type-"A" firm will be higher under incompatibility, and type-"A" firms will choose this regime.

Conversely, when the addition of a new variety increases industry demand considerably, the profits of an "A"-type firm are increasing in the number of varieties. Then, at the free-entry equilibrium profits are higher under compatibility.

Section 2 sets up the model. Sections 2.1 and 2.2 analyze the short run equilibria of the incompatibility and compatibility regimes respectively. Section 2.3 compares the short run equilibria. Section 3 analyzes the equilibrium in the long run. Section 4 contains concluding remarks.

2. The Model and Short Run Analysis

There are two types of goods, type "A" and type "B". A good of type "A" is complementary with a good of type "B". There are two goods of type "A", A_1 and A_2, and n goods of type "B", B_1, ..., B_n. Consumers demand systems A_iB_j. Two regimes that differ in the compatibility between the components are analyzed. In the first regime, the specifications of goods A_1 and A_2 are incompatible; i.e., no component of type "B" that connects with A_1 can also connect with A_2. Thus, the producer of differentiated product B_i creates two versions of this product, one that is compatible with component A_1 and another one, B'_i, that is compatible with component A_2. In Figure 1, each product is denoted by a double arrow, and a box encloses the goods produced by the same firm.

<< Insert Figure 1 >>

In the second regime, there is full compatibility and all systems A_iB_j, i = 1, 2, j = 1, ..., n, are available. The available products and the ownership relationships are seen in Figure 2.

<< Insert Figure 2 >>
A three-stage game is analyzed. In stage 1, each firm of type "A" decides if it wants to sell components that are compatible with the components produced by the opponent. There are two possible outcomes of the first stage, full compatibility, and total incompatibility. It will be assumed that either type-"A" firm can force incompatibility in the industry. This stage is interpreted as involving a commitment for "the very long run". With the regime (compatibility or incompatibility) determined in stage 1, two stages remain. In stage 2, firms of type "B" enter in the market. This stage is interpreted as involving a commitment for "the long run". In stage 3, all firms choose prices. This stage is interpreted as involving "the short run". We seek to establish subgame-perfect equilibria.

2.1 Incompatibility

Let the price of good $A_i$ be $p_i$, the price of good $B_i$ be $q_i$, and the price of good $B'_i$ be $q'_i$. Systems $A_iB_i$, $i = 1, ..., n$, are available at prices $s_{1i} = p_i + q_i$. Systems $A_2B'_i$, $i = 1, ..., n$, are available at prices $s_{2i}' = p_2 + q'_i$. In the world of incompatibility, product $A_1B_1$ has $n-1$ substitutes that differ from it in the second component, i.e., $A_iB_i$, $i = 2, ..., n$. System $A_1B_1$ also has $n$ substitutes that differ from it in both components, $A_2B'_i$, $i = 1, ..., n$. It will be assumed (see Assumption 1, below) that there is equal substitutability between pairs of systems that differ in the same component. Denoting by $D^{ij}$ the demand for product $A_1B_j$, this means that $\partial D^{ij}/\partial s_{1i} = \partial D^{11}/\partial s_{1j} = c$, for $i \neq 1$, $j \neq 1$, and $\partial D^{11}/\partial s_{2i}' = \partial D^{11}/\partial s_{2j}' = d$, for all $i$, $j$.

Formally, the following assumptions are made and maintained throughout the paper.

**Assumption 1:** The demand structure is linear and symmetric, with equal substitutability between pairs of systems that differ in the same component.

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4 This is because there are many different dimensions in which incompatibilities can be introduced, and it seems unlikely that each incompatibility can be anticipated and counteracted by the opponent.
**Assumption 2:** Variable costs are zero.

**Assumption 3:** A firm of type "B" incurs a fixed cost $F$ (to produce product $B_i$) in the regime of compatibility, and a fixed cost $F'$ (to produce both products $B_i$ and $B_i'$) in the regime of incompatibility, with $F' > F$.

**Assumption 4:** An equal increase in the price of all systems decreases the demand for every system. This is incorporated below in setting $b(2n-1) > c(n-1) + nd$.

Assumption 1, allows for a workable, but not trivial, setting for the analysis. Different degrees of substitutability between systems could easily be incorporated in the analysis without any significant qualitative changes, but would have resulted in significantly more complicated mathematical expressions without additional insights. Assumption 2 is formally equivalent to an assumption of a constant marginal cost. Assumption 3 notes that the fixed cost of producing two versions of a product is higher than the fixed cost of producing a single version. Note that there is no restriction on the size of fixed cost $F'$ to be twice that of $F$, so that economies of scope in the production of "B"-type goods are allowed but are not necessary. Assumption 4 imposes a very weak restriction that most demand structures obey.

With $n$ active firms, the demand function for system $A_1B_j$ is

$$D_{1j} = a - b(2n-1)(p_1 + q_j) + c \sum_{k \neq j} (p_1 + q_k) + d \sum_k (p_2 + q_k').$$

(1)

with $a > 0$, $b$, $c$, $d > 0$, and $b(2n-1) > c(n-1) + nd$ so that Assumption 4 is fulfilled.

The profits of firm $A_1$ are

$$\Pi_{A_1} = p_1 \sum_{j=1}^n D_{1j}.$$

The demand function for system $A_2B_j'$ is similarly defined. The profits of firm $A_2$ are
\[ \Pi_{A_2} = p_2 \sum_{j=1}^{n} D^{2j}. \]

A typical firm of type "B", say firm \( j \), produces components \( B_j \) and \( B_j' \). Its profit function is

\[ \Pi_{B_j} = q_j D^{1j} + q_j' D^{2j} - F. \]

Next we calculate the equilibrium prices in the last stage of the game. Firms choose prices simultaneously. Profit maximization by firm \( A_i \), \( i = 1, 2 \), implies

\[ \frac{\partial \Pi_{A_i}}{\partial p_i} = \sum_{j=1}^{n} D^{ij} + p_i[-b(2n-1) + (n-1)c]n = 0. \] (2)

Profit maximization by firm \( B_j \), \( j = 1, ..., n \), implies

\[ \frac{\partial \Pi_{B_j}}{\partial q_j} = D^{1j} - b(2n-1)q_j + d q_j' = 0. \] (3)

and

\[ \frac{\partial \Pi_{B_j}}{\partial q_j'} = D^{2j} - b(2n-1)q_j' + d q_j = 0. \] (4)

The equilibrium prices in the regime of incompatibility are found as the solution of the system of equations (2)-(4),

\[ p_1^I = p_2^I = a[b(2n-1) - d]/\text{Den}, \quad q_1^I = q_2^I = a[b(2n-1) - (n - 1)c]/\text{Den}, \] (5)

where

\[ \text{Den} = [b(2n-1) - d][b(2n-1) - (n-1)c] + [2b(2n-1) - (n-1)c - d][b(2n-1) - (n-1)c - nd]. \] (6)

The implied short run equilibrium profits are

\[ \Pi_{A_1}^I = \Pi_{A_2}^I = n[b(2n - 1) - c(n - 1)](p_1^I)^2, \] (7)
\[ \Pi^I_B = 2[b(2n - 1) - d](q^I)^2 - F. \] (8)

### 2.2 Compatibility

In the regime of full compatibility (see Figure 2), let the price of good \( A_i \) be \( p_i \), and the price of good \( B_j \) be \( q_j \). All systems \( A_iB_j \), \( i = 1, 2, j = 1, ..., n \), are available at prices \( s_{ij} = p_i + q_j \). The demand function for system \( A_1B_j \) is

\[ D^1_j = [a - b(2n-1)(p_1 + q_j) + c \sum_{k \neq j} (p_1 + q_k) + c(p_2 + q_j) + d \sum_{k \neq j} (p_2 + q_k)]. \]

The profits of firm \( A_i \), \( i = 1, 2 \), are

\[ \Pi_{A_i} = p_i \sum_{k=1}^{n} D^{ik}, \]

Profit maximization by firm \( A_i \) implies

\[ \frac{\partial \Pi_{A_i}}{\partial p_i} = \sum_{k=1}^{n} D^{ik} + p_i[-b(2n-1) + (n-1)c]n = 0. \] (9)

The profits of firm \( B_j \), \( j = 1, ..., n \), are

\[ \Pi_{B_j} = q_j \sum_{k=1}^{2} D^{kj} - F, \]

and are maximized at the solution of

\[ \frac{\partial \Pi_{B_j}}{\partial q_j} = \sum_{k=1}^{2} D^{kj} + q_j[-b(2n-1) + d] = 0. \] (10)

Equilibrium prices in the regime of compatibility are found as the solution of the system of equations (9)-(10),

\[ p_1^C = p_2^C = a[b(2n-1) - d]/\text{Den}, \quad q^C = a[b(2n-1) - (n-1)c]/\text{Den}, \] (11)
where, as before,

\[ \text{Den} = [b(2n-1) - d][b(2n-1) - (n-1)c] + [2b(2n-1) - (n-1)c - d][b(2n-1) - (n-1)c - nd]. \]

The short run equilibrium profits are

\[ \Pi_{A_i}^C = \Pi_{A_2}^C = n(b(2n-1) - (n-1)c)(p_i^C)^2, \quad (12) \]
\[ \Pi_{B_j}^C = 2(b(2n-1) - d)(q_j^C)^2 - F. \quad (13) \]

2.3 Comparisons of the Short Run Equilibria

Comparing equations (5)-(8) with (11)-(13), note that (for a given n) the equilibrium prices and the profits for firms A_1 and A_2 are the same across regimes, i.e.,

\[ p^I = p^C, \quad q^I = q^C, \quad \Pi^I = \Pi^C. \]

But, since \( F' > F \), the realized profits for a "B" type firm are lower under incompatibility,

\[ \Pi_{B_j}^C > \Pi_{B_j}^I. \]

**Proposition 1**: In the short run, with the number of firms fixed, the equilibrium prices and the profits for A_1 and A_2 are equal under compatibility and under incompatibility; but "B" type firms have lower profits under incompatibility, \( \Pi_{B_j}^C > \Pi_{B_j}^I \).

It is noteworthy that there is no difference in the prices across the two regimes. The main reason for this is that, in the present model, no firm produces two complementary components, so it does not have to take into account vertical pricing effects. In the earlier literature, Matutes and Regibeau (1988) and Economides (1988, 1989a, 1991) have noted lower equilibrium prices under incompatibility. In those papers, firms were vertically integrated in a parallel fashion, so that each firm produced two complementary components. This drove a wedge between the
equilibrium price in the two regimes, even when the number of products was the same. As explained in detail in these papers, a firm faces a more elastic demand under incompatibility, and therefore it chooses a lower price in the incompatibility regime.

The second difference with the existing literature is in the range of products offered under incompatibility. In the present model, under incompatibility, all brands of type "B" can be combined with each of the incompatible components of type "A". Thus, given the same number of components-producers, the number of substitute systems is the same under compatibility and incompatibility. Therefore, the basic demand and profit conditions are very similar under compatibility and incompatibility. Then equal equilibrium prices are not surprising.

3. The Long Run Free-Entry Equilibria

In the long run, the number of firms of type B is determined through free entry. For a fixed number of firms, it has been shown that equilibrium prices are the same under compatibility and incompatibility, profits for an A-type firm are the same, and profits for a B-type firm are smaller under incompatibility because of the required extra fixed cost. The equilibrium profits of a B-type firm are expected to be decreasing in the number of competitors, n. Because of the extra fixed cost, the free entry equilibrium number of B-type varieties will be smaller under incompatibility. The interesting question is how does an increase in the number of B-type firms affect the profits of an A-type firm. Of course, the answer to this question will determine the choice of regime by an A-type firm.

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5 Specifically, because of the existence of two versions software j, B_j and B_j', each system has n-1 substitutes that differ from it in both components as well as the n-1 substitute systems that differ from it in the second component.

6 This result will also be true with asymmetric substitutabilities in the demand structure.
The crucial properties are the signs of the rates of change of the equilibrium prices and profits with the number of B-type varieties. Before we go into specific examples, let us define the desired features of the long run variations of prices and profits with \( n \). They are:

1. The price of a B-type good should be decreasing in the number of varieties.
2. The demand for a B-type variety should not be increasing in \( n \).
3. The total demand for all B-type goods should not be decreasing in \( n \).
4. The profits of a B-type firm should be decreasing in \( n \).

We see below that there are reasonable values of the parameters of the demand functions such that these properties are fulfilled, but they can lead to quite different variations of the profits of an A-type firm with \( n \).

A critical determining factor of the variation of profits with the number of varieties \( n \) is the extent to which demand for an individual B-type variety is affected by the addition of a new variety. Of course, this is determined by the extent to which a new variety is sold to consumers who did not buy any differentiated product before, as opposed to sales to old customers of other firms. The effect of the number of B-type varieties, \( n \), on demand for each individual variety can be measured by the variation in \( n \) of the intercept "a" of the demand function \( D_{ij} \) of equation (1). At the one extreme lie the models of differentiated models a-la-Hotelling with inelastic demand, where each B-type firm has a market area of the order of \( 1/n \), i.e., \( a = O(n^{-1}) \). At the other extreme, as for example in a market with considerable network externalities, the addition of an extra variety may leave the intercept of the demand for an individual variety unaffected -- "a" will be constant, \( a = O(1) \).

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7 See, for example, Salop (1979), Economides (1989b).
It is also worth noting that irrespective of the dependence of "a" on "n", the prices \( p \) and \( q \) are of the same order of \( n \), i.e., \( p/q = [b(2n-1) - d]/[b(2n-1) - c(n-1)] = O(1) \), and the profits of a type-A firm are one order higher than the profits of a B-type firm, \( \Pi_A/\Pi_B = n[b(2n-1) - d]/[b(2n-1) - c(n-1)] = O(n) \). This last fact allows for the profits of an A-type firm to be either increasing or decreasing in \( n \), at the same time as the profits of a B-type firm are decreasing in \( n \).

In a model with relative inelastic industry demand, such as the circular model of Salop (1979), the market area of each firm (that would be realized when all firms have zero prices) is of the order of the distance between consecutive firms, i.e. \( O(n^{-1}) \). By substitution in equations (11)-(13), we have that

\[
\text{Den} = O(n^2), \quad q^I = q^C = O(n^2), \quad p^I = p^C = O(n^2),
\]

and

\[
\Pi_A^I = \Pi_A^C = O(n^{-2}), \quad \Pi_B^C = O(n^3), \quad \Pi_B^I = O(n^3).
\]

Thus, equilibrium prices are of order \( n^2 \), and profits are of order \( n^3 \). They all tend to zero as the number of varieties goes to infinity.

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8 This is a rather surprising fact, and it begs a clarification of the effective meaning of "order of \( n^x \)". One may expect that an A-type firm would take advantage of the small number of firms of its type to receive a considerably higher price than a B-type firm. Indeed, it is true that \( p > q \), but \( p \) and \( q \) are \textit{of the same order of} \( n \), and both of these facts would be true even if the "A" market were a monopoly. Our results suggest that the monopoly power of A-type firms is limited by the fact that they have to sell their product together with a product of type "B".

9 \( \text{Den} = O(n)O(n) = O(n^2), \quad q^I = q^C = O(n^1)O(n)/O(n^2) = O(n^2), \quad p^I = p^C = O(n^1)O(n)/O(n^2) = O(n^2). \)

10 \( \Pi_A^I = \Pi_A^C = nO(n)O(n^2)^2 = O(n^2), \quad \Pi_B^C = O(n)O(n^2)^2 = O(n^3), \) and similarly for \( \Pi_B^I \).
On the other hand, in some markets, industry demand could expand significantly with the addition of an extra variety. In the extreme case, the demand for each of the other varieties remains unaffected at constant prices, i.e., the intercept is constant, \( a = O(1) \). Then

\[
\text{Den} = O(n^2), \quad q^1 = q^c = O(n^{-1}), \quad p^1 = p^c = O(n^{-1}),
\]

and

\[
\Pi^A_n = \Pi^C_n = O(1), \quad \Pi^C_{ Bj} = O(n^{-1}), \quad \Pi^B_{ Bj} = O(n^{-1}).
\]

Thus both prices as well as the profits of a B-type firm go to zero as \( n \to \infty \), while the profits of an A-type firm tend to a constant as \( n \to \infty \). It can be checked that although \( \Pi^A_n \) is of the order of a constant, it is an increasing function of \( n \).

The equilibrium profits of a B-type firm are decreasing in \( n \). From Proposition 1, for the same number of "B"-type competitors, \( n \), the equilibrium profits of a firm of type "B" are lower under incompatibility than under compatibility. It follows that, at the long run free-entry equilibrium, the number of active "B"-type producers will be lower under incompatibility, \( n^I < n^C \).

When industry demand does not expand significantly when a new variety is added, the profits of an "A"-type producer are also decreasing in the number of firms of type "B". Therefore the long-run equilibrium profits of a type-"A" producer are higher in the regime of incompatibility,

\[
\Pi_A(n^I) > \Pi_A(n^C).
\]

See Figure 3a.

**Proposition 2:** When industry demand for the complementary B-type good does not increase significantly as a result of the addition of a new variety of that good, the profits of an
"A"-type firm decrease in the number of varieties of the B-type good. An "A"-type firm realizes higher profits in the regime of incompatibility.

Since the short run equilibrium prices are decreasing in the number of active firms, and \( n^c > n^l \), the long run equilibrium prices are going to be lower under compatibility,

\[
p^c(n^c) < p^l(n^l), \quad q^c(n^c) < q^l(n^l),
\]

**Corollary 1**: At the long-run free-entry equilibrium, the prices for both components are lower in the regime of full compatibility.

It is immediate from the comparison of the equilibrium profits in the two regimes that in the first stage of the game each firm of type "A" will choose incompatibility.

**Proposition 3**: When industry demand for the complementary B-type good does not increase significantly as a result of the addition of a new variety of that good, at the subgame-perfect equilibrium both type-"A" firms choose to produce incompatible components. This implies a smaller number of type-"B" firms, and higher prices for all components than in a regime of full compatibility.

Of course, in a market where the addition of an extra variety results in a very significant increase in industry demand (for example when it does decrease the sales of other varieties at constant prices), these results (Propositions 2 and 3) are reversed. Then profits increase in the number of complementary varieties and since more varieties are produced under compatibility, an A-type firm will prefer compatibility. See Figure 3b.

<< Insert Figure 3b >>
4. Concluding Remarks

This paper shows that one of the most common results in the compatibility literature, namely, that prices and profits are higher in full compatibility that in a regime of incompatible components, can be reversed in an industry of intense competition in varieties. Two features of the present model were essentially different from previous models. First, that each of the monopolistically competitive "B"-type firms is able to offer its variety in both specifications when these specifications are incompatible. Second, no firms were vertically integrated. It is the first feature that drives the results of this paper. I have argued elsewhere (Economides (1991)), under the assumption of a fixed number of varieties, that the worst of all worlds for compatibility is a world of vertically integrated firms in a parallel fashion.\footnote{This is because a vertically integrated firm has to balance the increase of demand that accompanies compatibility with the increase in competition that it brings in both of its vertically related markets.} This applies here as well. A vertical merger of a firm of type "A" with a firm of type "B" would increase its tendency to prefer incompatibility. Therefore the lack of vertical integration in the model presented here does not bias the results towards incompatibility.
References


Church, Jeffrey, and Gandal, Neil, (1990), "Network Effects and Software Provision," mimeo.


Figure 1: Systems and ownership in the regime of incompatibility.
Figure 2: Systems and ownership in the regime of full compatibility.
Profits and prices for industry demand not very sensitive to changes in \( n \)

**Figure 3a**
Profits and prices for sensitive industry demand to changes in $n$

Figure 3b