# Bundling and collusion in communications markets

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**Abstract:** This paper deals with competition in communications markets between an incumbent and a competitor. We analyse the effect of bundling strategy by a firm that enters an incumbent market. This market dimension has deep implications on the sustainability of collusion. In an infinitely repeated game framework, we show that the bundling strategy of the entrant might hinder collusion. Furthermore, we consider a setting in which the competitor uses a one-way access that the incumbent possesses. In such a situation, we show that when the competitor bundles its products, a low access charge might increase the feasibility of collusion. This result has an important policy implication.

**Keywords:** networks; bundling; access charge; unbundling local loop; collusion; telecommunications; competition; regulation; differentiation; communications; management.

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## 1 Introduction

Technological convergence appears to be well underway in the telecommunications industry. Several recent studies indicate that convergence facilitates the comparison of service offerings and intensifies competition between companies. Convergence is also changing the practices adopted by firms in terms of the pricing and structure of their service offerings. To reduce the intensity of competition, firms are pursuing strategies of price discrimination between consumers. As a result, companies are multiplying their bundles or tied offers that incorporate complementary or substitutable goods. Competitive pressure and changing consumption habits are encouraging firms to market bundles of services that include telephony, internet access and television. There are several goals behind this strategy, which vary depending on the type of player offering the bundles. For instance, bundling strategies can allow entrants to win market share and incumbents to offset losses in revenues.

The implications of convergence not only shape competition and pricing systems, but also lead to organisational convergence. Insofar as firms offering bundles of services do not historically come from the same markets, they do not have the same skills or core competences, and therefore do not have access to facilities enabling them to offer these services under the same conditions. The positioning of different firms in terms of the offering of service bundles effectively depends heavily on their core competence. Strategies of extending offerings consequently do not share the same dynamic: telecom operators have expanded their offerings to television, whereas cable operators are adopting strategies of extending their offers to telephony and high-speed internet access services.

Changes in the sector are raising interesting questions regarding aspects of competition. Major issues are the impacts of bundling offers both on the competitive behaviour of firms and access regulation. From this point of view, the entrance of cable operators into the telecommunications markets is one of the interesting examples. During the last years, cable operators have upgraded their cable network infrastructure to facilitate two-way data and voice transport for cable internet services. However, given the costs of new network deployments, cable operators could choose to extend their coverage via local loop unbundling[1] rather than by building new cable. Hence, even if cable operators have a strong market power on TV market, they might buy essential facilities for broadband internet access from telecom firms. In addition, the development of Voice over Internet Protocol (VoIP) allows cable operators to enter into telephony markets and to compete hardly the incumbent who offers the telephony over PSTN. Moreover, with VoIP, incumbents have to deal with competition with new upstart firms[2] offering VoIP services. Hence, anyone with a broadband connection (DSL or cable) can subscribe to a VoIP provider and make phone calls at a low rate.

The convergence raises interesting questions for the role of bundling on competition in telecommunications markets. For example, to what extent does competition in bundles require us to rethink the question of regulating access? Does the entrant have an incentive to use bundling to extend its market power? How does the easiness of collusion in such industries change with bundling? In this context, what is the role of access charge?

The recent literature on telecommunications competition and access regulation has been focused in situations of two-way access (Armstrong, 1998; Laffont et al., 1998a, 1998b; Valletti and Cambini, 2005). De Bijl and Peitz (2006) build a model on that literature and analyse the emergence of VoIP networks in a PSTN environment. They focus on the effect of access regulation of PSTN networks on the adoption of VoIP. In particular, they show that higher prices for terminating access to the PSTN network make VoIP less likely to succeed. More recently, Baranes and Poudou (2010) analyse how access regulation affects collusion in a differentiated duopoly. They show that the conventional doctrine "it is easier to collude among equals" is not always at work when there are asymmetries in price-sensitivity of demand in the industry. In particular, it does not apply when the competitor's demand is more sensible to price than the incumbent's one. Hence in this situation, cost symmetry may hinder collusion.

Our paper focuses on the relationship between bundling and the feasibility of collusion when a telecom firm competes with a newcomer who has a strong market power on a tying market. The new entrant is either a firm with a full-coverage network or a provider who uses local loop unbundling to reach end-users.

During the last two decades, bundling has become an intensive research topic for Industrial Organisation. Whinston (1990) clarifies the various aspects of bundling strategies and their antitrust issues. Papers initiated by Whinston (1990) have shown that the profitability of bundling results from economies of scale in the tied market. Other papers (Carbajo et al., 1990; Seidmann, 1991) have shown that bundling may mitigate competition by inducing more differentiation. More recently, Stole (2007), Armstrong and Vickers (2010), and Thanassoulis (2007) give an interesting overview on bundling. This literature developed with legal actions against Microsoft because many economists consider that bundling has been the main driver for the development of Microsoft (Nalebuff, 2000; Economides, 2001). This theoretical literature looks primarily at two cases. The first case corresponds to that of a monopolist who is threatened by an entrant and uses bundling or tying as a substitute to discrimination and to capture more consumer surplus (for instance, see Bakos and Brynjolfsson, 1999). The second case corresponds to that of an incumbent threatened by an outsider for whom bundling (or tying) is used as a means to foreclose entry (Rey and Tirole, 2005). A more recent literature, in line with Matutes and Regibeau (1992), analyses competition between firms that offer bundles. In particular, Reisinger (2004) shows that the consequences of bundling are less predictable in the duopoly than in the monopoly because the traditional sorting effect is in balance with a business-stealing effect.

Although a lot of economic literature exists on bundling, this has not given rise to many papers on the relationship between bundling and collusion. Yet, the existing relationship to bundling would seem to lie in the ability of firms to sustain collusion. Our framework aims to clarify that relationship and identify the lessons to be learnt in terms of antitrust policy. It will subsequently offer relevant economic arguments regarding the justification of regulating firms' content offerings and access regulation. In an infinitely repeated game, Spector (2007) shows that the anticompetitive use of bundling is possible even in the absence of economies of scale or scope in the tied market. The mechanism from which the bundling can mitigate competition is that bundling is a tool allowing firms to shift from non-cooperation to collusion. Spector (2007) claims that if collusion is feasible in the tied market, bundling may be a profitable strategy because it may facilitate collusion.

In this paper, we consider a model of competition with horizontal differentiation in the tied market between an incumbent and an entrant.[3] The incumbent possesses a complete local access network and offers PSTN telephony. The entrant offers the internet services (VoIP and TV services) with a full-coverage broadband network or using local loop unbundling to reach end-users. In this context, we examine how the feasibility of collusion in the tied market depends on the new upstart firm's offer strategy (bundling or not bundling). To focus on the impact of both bundling and regulation of access, we abstract from problems associated with the pricing of call termination. When the entrant offers a bundle, we show that differentiation might reduce the ability of firms to sustain the collusive equilibrium. In this setting, bundling might hinder collusion if it sufficiently rises the degree of product differentiation. When the bundling firm uses a one-way access that incumbent possesses, access charge might reduce the feasibility of collusion.

The remainder of the paper is organised as follows. Section 2 describes the model and examines the sustainability of collusion in the benchmark case where firms choose independent pricing. In Section 3, we consider pure bundling and we compare the condition of sustainability with the benchmark case. In Section 4, we introduce a one-way access model and we examine the impact of access charge on collusion. Conclusion appears in Section 5.

## 2 The framework

### 2.1 The model

We consider two independent markets, market $X$ and market $Y$, and we assume that the two products can be consumed independently. There are two firms, a competitor (firm $A$) and an incumbent (firm $B$). The incumbent owns a complete local access network. In this section, we suppose that the competitor has its own local network and thus can operate with a full-coverage broadband network.

Consumers have a unit demand for each product. The market for product $X$ is monopolised by firm $A$. Consumers have valuation of $\alpha$ for product $X$ and the unit cost of production for firm $A$ is $c_x$. We normalise $c_x$ to 0 and we suppose that $\alpha > 0$.

The market for product $Y$ is served by firm $A$ and firm $B$. The two firms are engaged in price competition. Suppose that their unit cost for product $Y$ is the same and given by $c_y$. We normalise $c_y$ to 0. We assume that product $Y$ is differentiated *à la* Hotelling. We denote the location of a consumer on a unit interval by $y$, $y \in [0, 1]$, in which consumers are uniformly distributed. The reservation value for product $Y$ is normalised to 1. The two firms are located at the end points of the unit interval and we assume that firm $A$ is located at $y_A = 0$ and firm $B$ at $y_B = 1$.

Let us assume that firm $i$ charges consumers with price $p_i$ for product $Y$. The utility of a consumer located at $y$ who would subscribe to firm $i$ is represented as:

$$U_i = 1 - p_i - t|y - y_i| \quad \text{with } i = A, B. \tag{1}$$

For simplicity, we only consider the full market coverage case. In addition, we assume a non-negative market share condition for firms $A$ and $B$ to further specify the range of the model parameters.

The analysis follows the standard repeated-game treatment of collusion. We assume that firms use the 'trigger strategy' of Friedman (1971) where $\delta$ is the rate of time preference of both firms, $0 \leq \delta \leq 1$. This strategy is described as follows. Firms charge collusive prices if neither firm has deviated in a previous stage. However, if either firm deviates, then both firms revert forever to the Nash equilibrium. Hence, firm $i$ sticks to the collusive price if:

$$\frac{\pi_i^c}{1-\delta} \geq \pi_i^d + \frac{\pi_i^*}{1-\delta} \qquad \text{(IC)}$$

where $\pi_i^*$, $\pi_i^c$ and $\pi_i^d$ are the one-shot Nash, collusive and deviation profits of firm *i,* respectively.

The incentive compatibility constraint (IC) faced by firm $i$ gives a condition on the rate of time preference:

$$\delta \geq \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^*}. \tag{2}$$

Each firm is then willing to stick to the collusive price if this rate is sufficiently large. The collusive prices constitute a subgame perfect equilibrium of the infinitely repeated game if and only if $\delta \geq \max_{i=A,B} \delta_i$.

In this framework, we solve the collusion game given separate goods in one case and bundled goods in the other case. We consider that the bundling decision is irreversible (Spector, 2007).[4] If she decides to bundle, firm *A* will offer a pair combining one unit of product *X* and one unit of product *Y*. If she decides not to bundle, she will offer the two products *X* and *Y* alone. Finally, firms interact in an infinitely repeated game framework by setting simultaneously prices for the products they sell. On the basis of the results, we conduct comparative statistics for the two cases.

## 2.2 The benchmark: independent pricing

As a benchmark, suppose that no bundled agreements have been made. Consumers will make their choice over the two products independently. We consider the possibility of sustaining collusion in the repeated-game using Nash reversion trigger strategies.

First, consider the non-cooperative Nash equilibrium. Firm *A* can extract the whole consumer surplus in market *X* with a price equal to $\alpha$, and have a profit $\alpha$.

The marginal consumer for product *Y* will be located at $\hat{y}$ such that:

$$\begin{aligned} &1 - t\hat{y} - p_A = 1 - t(1-\hat{y}) - p_B \\ &\Leftrightarrow \hat{y} = \frac{1}{2} + \frac{1}{2t}(p_B - p_A). \end{aligned} \tag{3}$$

Thus, all consumers located at $y \leq \hat{y}$ purchase *Y* from firm *A* whereas all other consumers purchase from firm *B*.

Each firm, $i = A, B$, simultaneously and independently sets prices to maximise profits:

$$\max_{p_i} p_i \left( \frac{1}{2} + \frac{1}{2t}(p_j - p_i) \right).$$

In the unique Nash equilibrium, prices are given by $p_A^* = p_B^* = t$ with one half of consumers buying product *Y* from firm *A* and the rest buying this product from firm *B*. It should be noted that at equilibrium, both firms serve the market[5] and the full market coverage condition[6] requires $t \leq 2/3$. Given this, the profits of firms for product *Y* are $\pi_A^* = \pi_B^* = t/2$ and the total profit for firm *A* is $\Pi_A^* = \alpha + t/2$.

Let us now calculate the prices and the profits under collusion and deviation, respectively.

When firms collude, they set prices to maximise the joint profit. The collusive prices for market *Y* are given by $p_A^c = p_B^c = 1 - \frac{1}{2}t$ and the joint profit is $\pi^c = 1 - \frac{1}{2}t$. If the firms adhere to the collusive agreement, each firm earns the collusive payoff:

$$\pi_A^c = \pi_B^c = \frac{1}{2}\left(1 - \frac{1}{2}t\right). \tag{4}$$

It should be noted that when firms collude the full market coverage condition is satisfied.

The price set by firm $i$ that deviates from the collusive equilibrium when the other firm sticks to it can be obtained as the solution of the following programme:

$$\max_{p_A} \pi_i(p_i, p_j^c).$$

From the first-order condition, we obtain the deviation price for firm $i$:

$$p_i^d = \frac{1}{4}(2+t)$$

and the deviation profit for firm $i$ is:

$$\pi_i^d = \frac{1}{32t}(2+t)^2.$$

When a firm deviates from the collusive equilibrium, the non-negative market share conditions require[7] that $t \geq 2/7$.

**Lemma 1:** *(i) The critical discount factor that makes collusion feasible in market Y is:*

$$\delta \geq \delta^* = \frac{2-3t}{2+5t}$$

*(ii) The feasibility of collusion is increasing with product differentiation.*

*Proof*: Substituting the equilibrium values of profits into equation (2), we obtain the result.

When product differentiation increases, collusion becomes easier to sustain. In the limit, if $t = 2/3$, $\delta^* = 0$ and collusion is always feasible. This is a traditional result in the models of collusion in an infinitely repeated framework with horizontally differentiated products (Chang, 1991).[8] The intuition of this classical result is simple. The discount factor above which collusion is sustainable depends on two effects: a *deviation effect*, $(\pi_i^d - \pi_i^c)$, and a *punishment effect*, $(\pi_i^d - \pi_i^*)$. Both the benefit from deviating and the loss from punishment are decreasing with differentiation. It is clear that differentiation decreases the benefit from deviating because firms have more market power. Moreover, differentiation decreases the benefit from collusion because the punishment profit (Nash profit) increases whereas the collusion profit is dragged down. However, when differentiation increases, the benefit from deviating decreases faster than the loss from punishment. In consequence, an increase in product differentiation makes a deviation from the collusive path less likely.

## 3 Pure bundling and collusion

In the previous section, we were concerned with the feasibility of collusion when firms choose independent pricing. In this section, we consider the case where firm $A$ offers a bundle combining in a fixed proportion product $X$ and product $Y$.

In the case of pure bundling, the consumer has only two choices: he can either buy the product $Y$ from firm $B$ or buy the bundle (one unit of product $X$ and one unit of product $Y$) from firm $A$. We assume that when a consumer buys a bundle, he cannot consume one product (product $A$) and buy the other product ($B$) from the competing firm. This assumption can be related to compatibility. Here, as in Matutes and Regibeau (1992), bundling is a way to make its products incompatible with the product of firm $B$.

Let $\hat{p}_A$ denote the price of the bundle offered by firm $A$ and $\hat{p}_B$ be the price of product $Y$ from firm $B$. We suppose that the bundling strategy of firm $A$ modifies the degree of differentiation between the two products offered by firms. We note $\tau$ the transportation cost and we assume that $\tau = \beta t$, where $\beta > 1$. This assumption means that bundling reduces the degree of substitutability between the two competing products.

Consumer has valuation of $\alpha + 1$ if he buys the bundled product. For the convenience of analysis, we put restrictions on parameters to ensure both full market coverage and non-negative market shares under collusion, deviation and non-cooperative equilibriums (see Appendix 1). Notice that these restrictions require $\alpha \leq 2$ and $\beta \in [\underline{\beta}, \overline{\beta}]$, where $\underline{\beta} = \frac{5\alpha+4}{14t}$ and $\overline{\beta} = \frac{\alpha+4}{6t}$. Following the first condition, we consider that the difference between consumers' valuation of the two independent goods ($X$ and $Y$) is relatively limited. If $\alpha > 1$ (respectively, $< 1$), consumers' valuation for product $X$ is higher (respectively, lower) than for product $Y$. It should be noted that if $\alpha = 0$, consumers value in the same manner the bundle offered by firm $A$ and the product $Y$ offered by firm $B$. The difference with the benchmark case is then only about the degree of substitution between the two offers. The condition on parameter $\beta$ ensures that differentiation between the bundle offered by firm $A$ and the product $Y$ offered by firm $B$ always allows a duopoly equilibrium with full market coverage.

For the bundled product to be chosen by the consumer located at $b$, $\hat{p}_A$ and $\hat{p}_B$ should satisfy the following condition:

$$\alpha + 1 - \hat{p}_A - \tau b \geq 1 - \hat{p}_B - \tau(1-b).$$

We can derive the demand function for each firm as:

$$b_A = \frac{\alpha - \hat{p}_A + \hat{p}_B + \beta t}{2\beta t} \quad \text{and} \quad b_B = 1 - b_A \tag{5}$$

and the profit for firm $i$ ($i = A, B$) is then given by $\pi_i = \hat{p}_i b_i$.

As in the previous section, we determine the non-cooperative Nash equilibrium and the prices and profits under collusion and deviation, respectively.

In the non-cooperative game, firms set prices to maximise their profits. The equilibrium prices are $\hat{p}^*_A = \beta t + \alpha/3$ and $\hat{p}^*_B = \beta t - \alpha/3$.

Thus, the equilibrium profits are:

$$\hat{\pi}^*_A = \frac{(3\beta t + \alpha)^2}{18\beta t} \quad \text{and} \quad \hat{\pi}^*_B = \frac{(3\beta t - \alpha)^2}{18\beta t}. \tag{6}$$

We now calculate the prices and the profits under collusion and deviation, respectively.

When firms collude, they set prices, which capture entirely the surplus of consumers. These prices are given by $\hat{p}_A = \alpha + 1 - \tau b$ and $\hat{p}_B = 1 - \tau(1-b)$. Firms determine market shares to maximise the joint profit with respect to $b$:

$$\max_b \widehat{\Pi}^c = (\alpha + 1 - \beta t b)b + (1 - \beta t + \beta t b)(1-b).$$

Finally, under collusion the market share for firm *A* and firm *B* are:

$$\widehat{b}_A^c = \frac{1}{2} + \frac{\alpha}{4\beta t} \quad \text{and} \quad \widehat{b}_B^c = \frac{1}{2} - \frac{\alpha}{4\beta t}$$

and the collusive prices are:

$$\widehat{p}_A^c = 1 + \frac{3}{4}\alpha - \frac{\beta t}{2} \quad \text{and} \quad \widehat{p}_B^c = 1 + \frac{\alpha}{4} - \frac{\beta t}{2}.$$

We can thus deduce easily the profits of firms when they collude:

$$\widehat{\pi}_A^c = \frac{(3\alpha + 4 - 2\beta t)(2\beta t + \alpha)}{16\beta t} \quad \text{and} \quad \widehat{\pi}_B^c = \frac{(\alpha + 4 - 2\beta t)(2\beta t - \alpha)}{16\beta t}. \tag{7}$$

When firm *i* deviates from the collusive equilibrium, she takes as given the collusive price of its rival and sets price to maximise its profit. The deviation prices for firm *A* and *B*, respectively, are then given by:

$$\widehat{p}_A^d = \frac{1}{2} + \frac{5\alpha}{8} - \frac{\beta t}{4} \quad \text{and} \quad \widehat{p}_B = \frac{1}{2} - \frac{\alpha}{8} + \frac{\beta t}{4}.$$

and the profits after deviation are:

$$\widehat{\pi}_A^d = \frac{(4 + 5\alpha + 2\beta t)^2}{128\beta t} \quad \text{and} \quad \widehat{\pi}_B^d = \frac{(4 - \alpha + 2\beta t)^2}{128\beta t}. \tag{8}$$

Using equation (2) together with equations (6)–(8), we can conclude that collusive prices are sustainable under bundling if $\delta \geq \max(\widehat{\delta}_A, \widehat{\delta}_B)$, where:

$$\widehat{\delta}_A = \frac{9(4 + \alpha - 6\beta t)^2}{(12 + 30\beta t + 23\alpha)(12 - 18\beta t + 7\alpha)}$$

$$\widehat{\delta}_B = \frac{9(4 + 3\alpha - 6\beta t)^2}{(12 + 5\alpha - 18\beta t)(12 - 11\alpha + 30\beta t)}.$$

We now turn to the impact of a change in the differentiation parameter $\beta$ on the threshold values $\widehat{\delta}_A$ and $\widehat{\delta}$. The standard result shows that differentiation increases the feasibility of collusion. We show here that bundling matters for the relationship between differentiation and the feasibility of collusion.

The following lemma summarises the impact of a change in the differentiation parameter $\beta$ on the critical discount factors.

**Lemma 2:** *Differentiation between the two offers:*

*(i) Decreases the critical discount factor for firm A*

*(ii) Increases the critical discount factor for firm B if* $\beta \geq \beta_1$

*Proof*: The comparative statistics on $\widehat{\delta}_A$ and $\widehat{\delta}_B$ give:

$$\frac{\partial \widehat{\delta}_A(\alpha,\beta)}{\partial \beta} = 1728\frac{\beta(\alpha+4-6\beta t)(-9\alpha^2-19\alpha+12\alpha\beta t-12+18\beta t)}{(-7\alpha-12+18\beta t)^2(23\alpha+12+30\beta t)^2}$$

$$\frac{\partial \widehat{\delta}_B(\alpha,\beta)}{\partial \beta} = 1728\frac{\beta(-6\beta t+3\alpha+4)(6\alpha\beta t+18\beta t-2\alpha^2-5\alpha-12)}{(-5\alpha-12+18\beta t)^2(-11\alpha+12+30\beta t)^2}$$

Since $\beta \leq \overline{\beta}$, it is easy to show that $\dfrac{\partial \widehat{\delta}_A(\alpha,t)}{\partial \beta} \leq 0.$

The impact of differentiation on the critical discount factor for firm *B* is quite ambiguous. Notice that the denominator of $\partial \delta_B(\alpha,\beta)/\partial \beta$ is positive. Since $\beta \leq \overline{\beta}$, it is easy to show that $(-6\beta t+3\alpha+4)>0.$ Let put $\beta_1 = 2\alpha^2+5\alpha+12/6t(\alpha+3).$ Then for any $\beta \in [\beta_1,\overline{\beta}]$, $(6\alpha\beta t+18\beta t-2\alpha^2-5\alpha-12)$ is positive.

As noted earlier, differentiation has two effects on the critical discount factors: an effect on the loss from punishment and an effect on the benefit from deviating. For the firm that offers the bundle, differentiation increases its incentive to sustain a collusive equilibrium. Then, for firm *A,* we obtain the classical effect of differentiation on the critical discount factor. In contrast, the incentive of firm *B* to sustain the collusive equilibrium depends on the level of differentiation. First, it should be noted that when firm *A* chooses to bundle its products, the firms' market shares under collusion do not evolve in the same direction when the degree of differentiation changes. We find that differentiation reduces the market served by firm *A* whereas it increases the market share of firm *B*. Thus, the collusive profit for firm *B* is increasing with differentiation. This effect could relax the incentive for firm *B* to deviate from the collusive path if the Nash profit (punishment) is low enough. This occurs when differentiation is not too large ($\beta < \beta_1$). When differentiation is high ($\beta \geq \beta_1$), the punishment profit for firm *B* is sufficiently high and increases faster than the collusive profit. In such a case, the loss from punishment decreases faster than the benefit from deviating. This makes a deviation from the collusive path more likely.

Now, we study the ranking of the critical discount factors $\widehat{\delta}_A$ and $\widehat{\delta}_B$. The following proposition gives the condition that makes collusion sustainable under bundling.

**Proposition 1:** *The critical discount factor that makes collusion sustainable under bundling is given by* $\delta \geq \delta_B$.

*Proof*: See Appendix 2.

This result indicates that it will be more difficult to discipline firm *B*. The reason is that firm *B* has a greater incentive to undercut its rival (deviation effect). Moreover, firm *B* has less to fear from a possible retaliation from firm *A* (punishment effect). Hence, the critical discount factor for firm *B* is higher than the critical discount factor for firm *A*.

Let us now determine how bundling affects the sustainability of collusion. As noted earlier, we only consider the case where both full market coverage condition and non-negative market shares condition are verified. To study the relationship between bundling and collusion, we have to compare the critical discount factor $\delta^*$, when firm *A* offers independently product *X* and product *Y*, with the critical discount factor $\delta_B$, under bundling.

Remember that Lemma 2 shows how the critical discount factor that makes collusion sustainable under bundling ( $\widehat{\delta}_B$ ) moves with the degree of differentiation. The following lemma considers the impact of consumers' valuation for product *X* on the threshold value $\delta_B$.

**Lemma 3:** *The threshold value* $\widehat{\delta}_B$ *rises with* $\alpha$.

*Proof*: Using restrictions on parameters, the sign of derivative can be obtained by direct computation.

This comparative statistic shows that the sustainability of collusion under bundling is decreasing with the valuation of consumers for product *X*. The valuation $\alpha$ modifies both the loss from punishment and the benefit from deviating. The intuition is as follows. When $\alpha$ increases, the benefit from deviating increases whereas the loss from punishment decreases. It is clear indeed that the valuation of the monopolised product decreases the non-cooperative market share of firm *B* and then reduces its profit. In the collusive equilibrium, $\alpha$ reduces the collusive profit of firm *B* too. However, the impact on the non-cooperative profit is lower than the impact on the collusive profit.

The following proposition establishes the relationship between bundling and the feasibility of collusion.

**Proposition 2:** *The comparison between the two critical discount factors,* $\widehat{\delta}_B$ *and* $\delta^*$*, gives:*

*(i)* $\widehat{\delta}_B \geq \delta^*$ *if* $\alpha \geq \overline{\alpha}$ *or* $\beta \geq \widehat{\beta}_2$

*(ii)* $\widehat{\delta}_B \geq \delta^*$ *if* $\alpha \leq \overline{\alpha}$ *and* $\beta \leq \widehat{\beta}_1$

*(iii)* $\widehat{\delta}_B < \delta^*$ *if* $\alpha < \overline{\alpha}$ *and* $\widehat{\beta}_1 < \beta < \min(\widehat{\beta}_2, \overline{\beta})$

*Proof*: Appendix 3.

This proposition establishes that the relationship between bundling and collusion sustainability depends both on the level of the consumers' valuation for product *X* and on the degree of product differentiation.

The intuition behind the results is as follows. Lemma 2 gives the condition under which differentiation may increase the critical discount factor for firm *B* and Lemma 3 shows that this critical factor rises with the valuation for the monopolised product. This implies that bundling reduces the likelihood of collusion ($\widehat{\delta}_B \geq \delta^*$) when consumers have a high valuation for the monopolised product ($\alpha \geq \overline{\alpha}$) or when differentiation is high enough ($\beta \geq \widehat{\beta}_2$). In other cases, there are two opposing effects at play that determine the impact of bundling on the sustainability of collusion. Indeed, when $\beta$ is lower than $\widehat{\beta}_2$ the impact of differentiation on $\widehat{\delta}_B$ depends on the level of the threshold value $\widehat{\beta}_1$ (see Lemma 2).[9] It is easy to show that the value of $\widehat{\beta}_1$ increases with $\alpha$. It turns out that when the valuation is low ($\alpha < \overline{\alpha}$), the threshold value $\widehat{\beta}_1$ is low and $\widehat{\delta}_B$ is more likely to increase with differentiation. On the other hand, it is clear that a lower valuation makes collusion less sustainable. Thus, the threshold value $\widehat{\beta}_1$ trades off the differentiation effect with the valuation effect on $\widehat{\delta}_B$ and finally on the impact of bundling on collusion $\widehat{(\delta_B - \delta^*)}$. Notice therefore that the interval $[\hat{\beta}_1, \hat{\beta}_2]$ within which bundling makes a deviation more profitable decreases with $\alpha$.

The following proposition examines how bundling affects collusion when there is no difference between valuations for products *X* and *Y*.

**Proposition 3:** *Suppose* $\alpha = 1$. *Bundling reduces the feasibility of collusion.*

*Proof*: Note that $\alpha = 1 > \bar{\alpha}$.

In words, with bundling there is less scope for collusion if consumers have the same valuation for both products. The intuition about why collusion becomes more difficult as $\alpha = 1$ is simple. Indeed, $\alpha = 1$ is sufficiently high with respect to the threshold value $\bar{\alpha}$ to make a deviation more profitable. In fact, by linking the monopolised market (market for product *X*) with a more competitive market (market for product *Y*), bundling may strengthen the gain from deviation for firm *B*. That is why collusion is more difficult to sustain. It should be noted that when $\alpha = 1$ the threshold value $\beta_1$ under which differentiation may reduce $\delta_B$ is inside the interval $[\underline{\beta}, \bar{\beta}]$ and closer to $\bar{\beta}$. Therefore, since the valuation for product *X* is high enough, the valuation effect offsets the potential negative effect of differentiation on $\delta_B$.

This result may have an important implication for competition policy and *ex-ante* regulation in network industries. The antitrust traditional view indeed regards bundling as an anticompetitive strategy used by dominant firms. This result shows that bundling may benefit to competition by lowering the feasibility of collusion.

## 4 Bundling with one-way access

In this section, we extend the paper. We consider now that firm *A* enters the market *Y* without network access. Since the local loop is an essential facility, the competitor must get access from the incumbent (firm *B*).

For simplicity, we assume that $\alpha$ and $\beta$ are normalised to 1, so that the valuations of both products are equal and bundling does not rise differentiation. Let *a* denote the unit access charge, which firm *A* pays to firm *B* for each unit of product *Y* she sells.

First, we consider the case where firm *A* offers the two products independently. Second, we consider the bundling case.

### *4.1 Independent pricing*

As before, we restrict our analysis in the case where both full market and duopoly conditions are verified. This requires that $\underline{a}_1 < a \leq \underline{a}_2$, where[10] $\underline{a}_1 = 1 - 7t/2$ and $\underline{a}_2 = 1 - 3t/2$.

The market shares are unchanged and given by (*y*). The non-cooperative profits on market *Y* are:

$$\pi_A = (p_A - a)\hat{y} \quad \text{and} \quad \pi_B = p_B(1 - \hat{y}) + a\hat{y}.$$

The profits maximisation gives the following equilibrium prices:

$$p_A^{**} = p_B^{**} = t + a$$

and profits are:

$$\pi_A^{**} = \frac{t}{2} \quad \text{and} \quad \pi_B^{**} = \frac{t}{2} + a.$$

We proceed as before to determine collusion and deviation profits. Collusion profits are given by:

$$\pi_A^c = \frac{2-t-2a}{4} \quad \text{and} \quad \pi_B^c = \frac{2-t+2a}{4}$$

and deviation profits are:

$$\pi_A^d = \frac{(2+t-2a)^2}{32t} \quad \text{and} \quad \pi_B^d = \frac{(2+t)^2 + 4a(7t-2+a)}{32t}.$$

Using IC, we determine easily the threshold value $\delta^{**}$ above which collusion is feasible:

$$\delta^{**} = \frac{2-3t-2a}{2+5t-2a}. \tag{9}$$

It is easy to check that the incremental payoff from deviating in a collusive path and the loss from punishment are the same for both firms. Thus, the critical discount factors for firm *A* and firm *B* are symmetric. Hence, the threshold value depends on two parameters, *t* and *a*. As in the benchmark, it is decreasing in the degree of differentiation, $\partial\delta^{**}/\partial t < 0$. Finally, it is easy to show that the threshold value $\delta^{**}$ is decreasing with access charge.[11] The following lemma summarises the results.

**Lemma 4:** *(i) Collusion is feasible in market Y with independent pricing if* $\delta \geq \delta^{**}$.

*(ii) Access charge is a tool for collusion:* $\dfrac{\partial\delta^{**}}{\partial a} < 0.$

If the discount factor is greater than $\delta^{**}$, collusion will be sustained From equation (9), we observe that $\delta^{**}$ is a function of access charge (*a*). The result shows that access charge decreases the discount factor, making collusion more sustainable. This highlights the collusive power of access charge. This result was discussed in a recent literature in different frameworks, which have examined the possible anticompetitive use of access charges between interconnected networks (Armstrong, 1998; Laffont et al., 1998a, 1998b; Dessein, 2004; Valletti and Cambini, 2005). In this paper, the collusive power of access charge appears in an infinitely repeated game framework when firm *A* offers product *X* and product *Y* separately. The intuition behind this result is that an increase in access charge induces a change in $(\pi^d - \pi^*)$, which exactly offsets the change in $(\pi^d - \pi^*)$. Thus, when access charge increases, the benefit from deviating decreases faster than loss from punishment, making access charge a tool for collusion.

## *4.2 Bundling*

We now consider the case where firm *A* offers a bundle combining product *X* and product *Y*. We assume again full market coverage and non-negative market shares conditions for all equilibriums with bundling. Thus, restrictions on parameters are given by $\overline{a}_1 \le a \le \overline{a}_2$, where:[12]

$$\overline{a}_1 = \frac{9-14t}{4} \quad \text{and} \quad \overline{a}_2 = \frac{5-6t}{4}.$$

The profit functions are:

$$\widehat{\pi}_A = (p_A - a)b_A \quad \text{and} \quad \widehat{\pi}_B = p_B(1-b_A) + ab_A.$$

By following the logic that we outlined in the previous section, we can show that the non-cooperative profits are:

$$\widehat{\pi}_A^* = \frac{(1+3t)^2}{18t} \quad \text{and} \quad \widehat{\pi}_B^* = \frac{9t^2+1+18at-6t}{18t}$$

and the collusive profits are given by:

$$\widehat{\pi}_A^c = \frac{(1+2t)(7-2t-4a)}{16t} \quad \text{and} \quad \widehat{\pi}_B^c = \frac{12t-4t^2+4a+8at-5}{16t}.$$

The best deviations for firm *A* and firm *B* give, respectively, the profits:

$$\widehat{\pi}_A^d = \frac{(9+2t-4a)^2}{128t} \quad \text{and} \quad \widehat{\pi}_B^d = \frac{4t^2+12t+9+16a^2+112at-24a}{128t}.$$

Using IC, we can now determine the critical discount factors both for firm *A* and for firm *B*:

$$\widehat{\delta}_A = \frac{9(6t+4a-5)^2}{(19-18t-12a)(35+30t-12a)}$$

$$\widehat{\delta}_B = \frac{9(6t+4a-7)^2}{(17-18t-12a)(1+30t-12a)}.$$

The following lemma gives the critical discount value above which collusion is feasible with bundling.

**Lemma 5:** *With bundling, collusion is feasible if* $\delta \ge \widehat{\delta}_B$.

*Proof*: See Appendix 4.

The above-mentioned lemma implies that with bundling the incentive to deviate is the greatest for firm *B*. Thus, the threshold value under which collusion is not feasible is $\widehat{\delta}_B$. This result is similar to the result obtained in the previous section (*Proposition 1*). However, the sustainability of collusion depends here on the level of access charge.

The following proposition gives the result on collusion sustainability and examines the impact of a change in access charge.

**Proposition 4:** *(i) Bundling hinders collusion* $(\widehat{\delta}_B > \delta^{**})$

*(ii) Access charge reduces the feasibility of collusion* $\left(\dfrac{\partial\widehat{\delta}_B}{\partial a} > 0\right)$.

*Proof*: See Appendix 4.

This result shows that when the bundling firm (firm *A*) is using a one-way access that an incumbent possesses (firm *B*), bundling always increases the ability for firm *B* to deviate making less probable collusion (i). The intuition is as follows. When firm *A* sells the two products separately, the collusive and the deviation profits for firm *B* are higher than those with bundling. First, the effect regarding the collusive profit for firm *B* is clear because when firm *A* chooses a bundle its collusive market share increases. Second, under bundling firm *B* has a low short-term gain from deviating because it is more difficult to undercut its rival. Therefore, with bundling, the collusive profit for firm *B* decreases faster than its deviation profit. Hence, the deviation effect offsets the punishment effect. This makes collusion more difficult to sustain.

Finally, *Proposition 4* shows that the lower the access charge is, the more sustainable the collusion (ii) is. The reason is simple. When access charge is reduced, the punishment profit decreases faster than the collusive profit. On the other hand, a lower access charge reduces the ability of firm *B* to undercut its rival. Thus, the loss from punishment rises faster than the benefit from deviating. Hence, in a context of tacit collusion with bundling, the incentive of the firm, which possesses the broadband access to sustain collusion, is rising when access charge is low. This result may have an important implication on access regulation: access charge regulation based on simple cost recovery rules risks encouraging collusive behaviours.

## 5 Conclusion

Our analysis has shed light on the effects of bundling on the sustainability of collusion.

We looked at two cases. In the first case, we assume that the competitor owns its local access network and then can self-supply broadband access to offer internet services. We show how product differentiation and the relative valuation for the monopolised product matter for the feasibility of collusion.

In the second case, we consider that the competitor cannot self-supply local access and then utilises a one-way access that the incumbent owns. We focus on the impact of access charge on the feasibility of collusion both with independent pricing and bundling. With independent pricing, access charge appears as a tool to increase the sustainability of collusion. In contrast, with bundling the sustainability of collusion is decreasing with the level of access charge. This main result has an important policy implication. This implies that regulatory authority should be careful when she regulates access price. In other words, a low access charge could not be desirable insofar as it could increase the feasibility of collusion and lead to high prices for consumers.

This research is perfectly in line with current regulatory debates on the bundling strategies in communications industries. It enables us to analyse the problems raised by convergence and competition between wired and wireless networks. In this case, regulators are looking at the effects of the bundled offerings marketed by an operator that is dominant in the mobile market and offers fixed services in a more competitive market.

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## Notes

[1]For a recent empirical framework about the benefits of entry into the local phone services see for example Economides *et al.* (2008).

[2]Alternative operators (Yahoo!BB, Time Warner, Free, Fastweb...) or pure VoIP service providers (eBay-Skype, Google, Yahoo!...)

[3]In this framework we do not consider entry decision. We suppose that entry occurred and that the two firms compete each other.

[4]Spector (2006) examines the robustness of the result when bundling decision is reversible. Other things equal, reversibility should not contradict the result.

[5]The non-negative market share conditions for firms $A$ and $B$ is $p_B - t \leq p_A \leq p_B + t$ which is verified at equilibrium.

[6]We obtain the full market coverage with: $1 - ty - p_A^* = 1 - t(1-y) - p_B^* \geq 0.$

[7]This condition is given by $\frac{2-3t}{2} \leq p_i^d \leq \frac{2+t}{2}.$

[8]See, among others, Chang (1991).

[9]Direct computation shows that $\widehat{\beta}_2 > \beta_1$.

[10]We assume $t \leq 2/3$ to have $\underline{a}_2 \geq 0$.

[11]The derivative of $\widehat{\delta}^{**}$ is $\frac{-16t}{(2+5t-2a)^2} < 0.$

[12]We assume $t \leq 5/6$ to have $\overline{a}_2 \geq 0$ and $t \geq 1/2$ to have non-negative market shares.

## Appendix 1

For the benchmark case (independent pricing), it is easy to show that full market coverage and non-negative market shares under non-cooperation, collusion and deviation equilibrium require $2/7 \leq t \leq 2/3$ .

Let us now consider the pure bundling case.

### Non-negative market shares conditions

*Non-cooperative equilibrium*

The market shares are given by:

$$\widehat{b}_A^* = \frac{3\beta t + \alpha}{6\beta t} \quad \text{and} \quad \widehat{b}_B = 1 - \widehat{b}_A.$$

Each firm has a positive market share if $0 \leq \widehat{b}_A \leq 1$. This requires $\beta \geq \beta_1 = \alpha / 3t$.

*Collusion*

The market shares are $\widehat{b}_A^c = \alpha + 2\beta t / 4\beta t$ and $\widehat{b}_B^c = 1 - \widehat{b}_A^c$. The condition is $\beta \geq \beta_2 = \alpha / 2t$.

Notice that $\beta_2 > \beta_1$.

*Deviation*

When firm *A* deviates, its market share is $\widehat{b}_A^d = \frac{5\alpha+4+2\beta t}{16\beta t}$. The non-negative market share condition is then $\beta \geq \underline{\beta} = \frac{5\alpha+4}{14t}$.

When firm *B* deviates, each firm has a positive market share if $\beta \geq \max(\beta_3, \beta_4)$, where $\beta_3 = \frac{4-\alpha}{14t}$ and $\beta_4 = \frac{\alpha-4}{2t}$. Notice that $\beta_3 \geq \beta_4$ if $\alpha \leq 4$.

Finally, to ensure non-negative market shares in all equilibriums, we must put restrictions on parameters such that:

$$\beta \geq \underline{\beta} \quad \text{if} \quad \alpha \leq 2 \quad \text{and} \quad \beta \geq \beta_2 \quad \text{if} \quad \alpha \leq 2.$$

**Full market coverage conditions**

We have to determine in each equilibrium the restrictions on parameters, which ensure $U_A(y) = U_B(y) \geq 0$.

*Non-cooperative equilibrium*

$$U_A(\widehat{b}_A^*) = U_B(\widehat{b}_A^*) \geq 0$$

$$\Leftrightarrow \beta \leq \beta_5 = \frac{2+\alpha}{3t}.$$

*Collusion*

At the collusion equilibrium, the market is always fully covered: $U_A(\widehat{b}_A^c) = U_B(\widehat{b}_A^c) = 0$.

*Deviation*

When firm *A* deviates, the condition is given by

$$\beta \leq \overline{\beta} = \frac{\alpha + 4}{6t}.$$

When firm *B* deviates, the condition is given by

$$\beta \leq \beta_6 = \frac{3\alpha + 4}{6t}.$$

Notice that $\overline{\beta} < \beta_6$, $\overline{\beta} < \beta_5$ and $\beta_6 > \beta_5$.

We conclude that to ensure full market coverage in all equilibriums, we must restrict parameters on $\beta \leq \overline{\beta}$.

Compatibility between full market condition and non-negative market shares. To have full market coverage and non-negative market shares in all equilibriums, we must put restrictions on parameters such that $\underline{\beta} \leq \beta \leq \overline{\beta}$ and $\alpha \leq 2$.

Conditions for full market coverage and duopoly equilibrium.

## Appendix 2

*Proof of Proposition 1*

To examine the effect of bundling on the sustainability of collusive pricing among firms, we have to compare $\max\{\widehat{\delta}_A, \widehat{\delta}_B\}$. Remember that we restrict our attention to $\alpha \leq 2$ and $\underline{\beta} \leq \beta \leq \overline{\beta}$.

Considering the difference between the two critical factors, we have

$$\widehat{\delta}_A - \widehat{\delta}_B = 288\tfrac{H(\beta)}{D}$$

where

$$D = (7\alpha + 12 - 18\beta t)(23\alpha + 12 + 30\beta t)(5\alpha + 12 - 18\beta t)(11\alpha - 12 - 30\beta t) < 0$$

and

$$H(\beta) = -216\beta^3 t^3 + 468t^2\beta^2(\alpha + 2) - 30t\beta(32 + 9\alpha^2 + 32\alpha) \\ + (\alpha + 2)(47\alpha^2 + 144\alpha + 144)$$

As $D < 0$, the sign of $\widehat{\delta}_A - \widehat{\delta}_B$ is given by $H(\beta)$. The derivation of $H(\beta)$ with respect to $\beta$ is:

$H'(\beta) = -648\beta^2 t^3 + 936t^2\beta(\alpha + 2) - 30t(32 + 9\alpha^2 + 32\alpha)$ admits two solutions:

$$\beta_1' = \frac{26 + 13\alpha - \sqrt{196 + 196\alpha + 34\alpha^2}}{18t}$$

$$\beta_1'' = \frac{26 + 13\alpha + \sqrt{196 + 196\alpha + 34\alpha^2}}{18t}$$

and we have: $H'(\beta) \geq 0$ if $\beta \in [\beta_1', \beta_1'']$ and $H'(\beta) < 0$ otherwise.

Moreover, we show that $\beta_1' \geq \overline{\beta}, H(0) > 0$ and $H(\overline{\beta}) = 2\alpha^2(7\alpha + 8) > 0$. This implies $H(\beta) > 0$ and finally $\widehat{\delta}_A - \widehat{\delta}_B < 0$.

## Appendix 3

*Proof of Proposition 2*

We have to compare the critical discount factors both under independent pricing, $\delta^*$, and under bundling, $\widehat{\delta}_B$. We show that:

$$\widehat{\delta}_B - \delta^* = 16\frac{NB(\beta)}{DB}$$

where

$$DB = -(5\alpha + 12 - 18\beta t)(11\alpha - 12 - 30\beta t)(5t + 2) > 0$$

and

$$NB(\beta) = 108\beta^2 t^2 - 12t\beta(3\alpha t + 7\alpha + 9t + 6) + 15\alpha^2 t + 17\alpha^2 + 54\alpha t + 36\alpha + 72t.$$

We note $g(\alpha) = (3\alpha^2 t + 27t + 18\alpha t + \alpha^2 - 18 + 12\alpha)$. A direct computation shows that the sign of $g(\alpha)$ is given by:

$$g(\alpha) \le 0 \quad \text{if} \quad \alpha \le \bar{\alpha} = \frac{-18t - 12 + 6\sqrt{15t+6}}{2(3t+1)} \quad \text{and} \quad g(\alpha) > 0 \quad \text{otherwise.}$$

Moreover, we have $g(0) = 9(3t - 2) \le 0$ because we assume that $t \le 2/3$ to ensure full market coverage in independent pricing.

The analysis of $NB(\beta)$ with respect to $\beta$ gives:

*Case 1*: if $\alpha \ge \bar{\alpha}$ then $NB(\beta) \ge 0$

*Case 2*: if $\alpha \le \bar{\alpha}$ then $g(\alpha) < 0$.

$NB(\beta) = 0$ has two roots given by $\widehat{\beta}_1$ and $\widehat{\beta}_2$ :

$$\widehat{\beta}_1 = \frac{3\alpha t + 7\alpha + 9t + 6 - \sqrt{(3t-2)(3\alpha^2 t + 27t + 18\alpha t + \alpha^2 - 18 + 12\alpha)}}{18t}$$

$$\widehat{\beta}_2 = \frac{3\alpha t + 7\alpha + 9t + 6 + \sqrt{(3t-2)(3\alpha^2 t + 27t + 18\alpha t + \alpha^2 - 18 + 12\alpha)}}{18t}$$

with $\widehat{\beta}_1 < \widehat{\beta}_2$.

In this case, we show that $NB(0) > 0$.

We evaluate now the values of $NB(\beta)$ at $\beta = \overline{\beta}$ and $\beta = \underline{\beta}$ and derivatives at these points.

i First, we consider $\beta = \underline{\beta}$.

We have:

$$NB(\underline{\beta}) = \frac{15}{7}\alpha^2 t + \frac{38}{49}\alpha^2 + \frac{36}{7}\alpha t + \frac{408}{49}\alpha + \frac{288}{7}t - \frac{576}{49}$$

and we show that $NB(\underline{\beta})$ has two roots given by:

$$\underline{\alpha}_1 < 0 \text{ and } \underline{\alpha}_2 = \frac{-252t - 408 + 84\sqrt{-111t + 20t + 36}}{2(38 + 105t)}.$$

Using $2/7 \le t \le 2/3$, we show that $\underline{\alpha}_2 < 0$ and then $NB(\underline{\beta}) > 0$.

The derivative of $NB(\beta)$ shows that $\dfrac{\partial NB(\underline{\beta})}{\partial \beta} < 0$.

ii Second, we consider $\beta = \overline{\beta}$.

We have:

$$NB(\overline{\beta}) = \alpha(9\alpha t + 6\alpha + 12 - 8) \geq 0 \text{ if } \alpha \geq \widehat{\alpha} = \frac{4}{3}\frac{2-3t}{3t+2}.$$

It is easy to show that $0 < \widehat{\alpha} < \overline{\alpha}$ and:

if $\alpha \leq \widehat{\alpha}$ then $NB(\overline{\beta}) \leq 0$ and $NB(\overline{\beta}) > 0$ otherwise.

The derivative of $NB(\beta)$ gives:

$$\frac{\partial NB(\overline{\beta})}{\partial \beta} < 0 \text{ for any } \alpha \leq \overline{\alpha}.$$

Finally:

- when $\alpha < \widehat{\alpha}$ we have:

  $NB(\beta) \leq 0$ if $\beta \in [\widehat{\beta}_1, \overline{\beta}]$ and $NB(\beta) > 0$ if $\beta \in [\underline{\beta}, \widehat{\beta}_1]$.

- when $\widehat{\alpha} \leq \alpha \leq \overline{\alpha}$, we have:

  $NB(\beta) \leq 0$ if $\beta \in [\widehat{\beta}_1, \widehat{\beta}_2]$ and $NB(\beta) > 0$ otherwise.

## Appendix 4

*Proof of Lemma 4:*

We have to compare the critical discount factors for both firms:

$$\widehat{\delta}_A - \widehat{\delta}_B = 288\frac{\widehat{H}(a)}{\widehat{D}(a)}$$

where:

$$\widehat{D}(a) = (18t + 12a - 19)(30t - 12a + 35)(18t - 17 + 12a)(30t + 1 - 12a)$$

since $a < \overline{a}_2$, $\widehat{D}(a) > 0$.

and

$$\widehat{H}(a) = 288a^3 + (960t - 1296)a^2 + (1966 - 2880t + 936t^2)a - 1005 + 2190t + 216t^3 - 1404t^2.$$

The derivative of $\widehat{H}(a)$ is:

$$\widehat{H}'(a) = 864a^2 + (1920t - 2592)a + 936t^2 + 1966 - 2880t.$$

$\widehat{H}'(a) = 0$ has two roots given by:

$$\widehat{a}_{1,2} = \frac{-10t}{9} + \frac{3}{2} \mp \frac{1}{36}\sqrt{196t^2 33}.$$

It is easy to show that $\widehat{a}_1 < \widehat{a}_2$ and $\widehat{a}_1 > \overline{a}_2$. We can deduce that $\widehat{H}'(a) > 0$. Note that $\widehat{H}(\overline{a}_2) < 0$ and $\widehat{H}(\overline{a}_1) < 0$. Then, $\widehat{H}(a) < 0$. Finally, $\widehat{\delta}_B > \widehat{\delta}_A$.

**Proof of Proposition 4:**

i We compare $\widehat{\delta}_B$ and $\delta^{**}$:

$$\widehat{\delta}_B - \delta^{**} = 16\frac{\widehat{NB}(a)}{\widehat{DB}(a)}$$

where:

$$\widehat{DB}(a) = (18t + 12a - 17)(30t + 1 - 12a) < 0 \quad \text{since} \quad a < \overline{a}_2.$$

and

$$\widehat{NB}(a) = 36t^2 + 15t - 30ta - 53 + 89a - 36a^2.$$

We show that $\widehat{NB}(a) = 0$ has two roots given by:

$$\widehat{a}_{1,2} = \frac{-5}{12} + \frac{89}{72} \mp \frac{1}{72}\sqrt{6084t^2 - 3180t + 289}.$$

It is easy to remark that $\widehat{a}_1 < \widehat{a}_2$ and $\widehat{a}_1 > \overline{a}_2$. We deduce that $\widehat{NB}(a) < 0$ and then $\widehat{\delta}_B > \delta^{**}$.

ii The derivative of $\widehat{\delta}_B$ with respect to $a$ is:

$$\frac{\partial\widehat{\delta}_B}{\partial a} = 288\frac{(7 - 6t - 4a)(108t^2 - 132t + 72ta + 43 - 36a)}{(18t - 17 + 12a)^2(30t + 1 - 12a)^2}.$$

Note that since $a < \overline{a}_2$ then $(7 - 6t - 4a) > 0$ and it is easy to show that $(108t^2 + 72at - 132t - 36a + 43) > 0$. Hence, $\frac{\partial\widehat{\delta}_B}{\partial a} > 0$.