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6 Differentiated Public Goods: Privatization and Optimality

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6.1 INTRODUCTION

Many commentators criticize centralized government production of public goods for not providing sufficient variety. Citizens with widely varying tastes all consume the same type and level of services. With fixed costs of production and variations in tastes and incomes, an efficient solution, familiar in the public finance literature, can be generated by a single government producer that supplies the public good and charges citizens different tax prices to take account of the variations in their tastes and incomes.[1] This solution, while efficient, is not feasible. Citizens have, in general, no incentive to reveal their willingness to pay, and recent advances in demand-revealing processes have not produced entirely satisfactory solutions.[2] In addition, if citizens' tastes diverge sharply enough and if production costs are low enough, efficient solutions may also exist with several suppliers each producing a different variety of output for a subset of citizens.[3,4]

But how should the number of suppliers be determined and how should they be financed given the public-good character of the output? This chapter analyzes a possible solution in which private suppliers provide 'local' public goods financed by tax dollars. Individual tax-payers, however, decide how their own tax money will be spent. The government requires each individual to pay a one-dollar fee for public services but individuals can decide which provider should receive his or her dollar. The central government taxes everyone $1 but permits a 100 per cent deduction from taxes for gifts made to organizations that produce differentiated public goods.

Producers are private organizations that compete for donations by providing both a particular variety of service and a quality (or quantity) of output that is a public good to all contributors. Suppliers seek

to maximize profits, and entry is free so long as suppliers can cover their fixed and variable costs of production. We assume that the public goods provided by private firms are 'local' in the sense that donors obtain utility only from the services produced by the firm to which they donate. The donor, by his gift, 'buys in' to the supplier's entire output. Thus, each donor finds the provider whose service mix provides him or her with the highest level of utility, and donates his entire dollar to that producer.[5]

Our modelling effort begins after the decision to publicly finance a type of service has been made. Thus we do not analyze the broader question of whether a particular type of activity deserves public subsidy in the first place. Instead, we suppose that the public policy question before us is the choice between direct public production of a uniform service versus a variety of types produced by private firms but financed by tax dollars. Can the efficiency benefits of competitive private markets be captured by letting citizens choose how to allocate their tax dollars among private providers?

The services that most closely match the model's assumptions are recreational and cultural activities where exclusion is inexpensive, but where benefits are enjoyed in common by those who gain entry. Examples are swimming pools, parks, museums, theatrical and musical performances, and sporting events. These services fit the model both because they are excludable public goods and because people are likely to have widely varying tastes for these services. Of course, because it is possible to charge admission, many of these services can be provided without a tax or subsidy. But the resulting pattern of consumption will not, in general, be efficient, and, in addition, public subsidy may also be justified on a variety of distributive grounds.

Also close to our theoretical formulation are advocacy organizations that promote various causes through political lobbying or campaigns of public education, organizations that support research, and those that provide charitable services to the needy. With a totally private system, free-riding behaviour would be a serious problem for all of these activities. Such behaviour would be overcome by the tax credit scheme outlined here. The major divergence between our model and these activities is our assumption that people gain only from the provider who receives their contribution. Nevertheless, since these are services about which people have sharply divergent preferences, our model has something to contribute to an analysis of these services as well.

Even schooling, the most important local government function, fits roughly within our model. While we ignore many issues of educational policy, such as the benefits of mixing children of different racial, ethnic and socioeconomic backgrounds and the role of schools in producing good future citizens, we provide a new kind of theoretical critique of voucher systems – a critique based solely on the economic efficiency rationales that are seen by others as the major justifications for such systems. While the degree to which our results can be generalized has not been tested, we believe that the conclusions obtained with our very abstract model will carry over to more realistic situations. Private production of publicly funded excludable public goods with citizens choosing which varieties to support is likely to require considerable regulation of entry and service levels, and may be inferior to direct public production.

In our model, most of the features that distinguish non-profit firms from for-profit producers are assumed away.[6] Firms face no trust or credibility problems.[7] When a producer announces the variety of service it will produce and its quantity (or quality), donors believe (correctly) that the producer will actually provide the promised service. Donors have no difficulty evaluating output. Furthermore, producers have no ideological commitment to particular varieties or levels of output.[8] Therefore, the providers can either be thought of as profit-maximizing suppliers, or as non-profits maximizing revenue or level of service provision. In the formal model below, they are characterized as for-profit firms, but, as we shall show, the generalization to non-profits, even ones with ideologically committed managers, is straightforward.

The work in public finance that is closest to this effort is Charles Tiebout's classic work on local public goods which concludes that intergovernmental competition could produce optimality.[9] His work has been criticized for excluding both geography and politics, and more recent work has tried to remedy this lack in ways that demonstrate the fragility of his speculations about optimal public-good production.[10] Our critique, however, takes a different approach. Like Tiebout we exclude politics. Unlike Tiebout, we are writing with the benefit of recent work on the inefficiency of markets with monopolistic competition for differentiated products.[11] In the present model, products differ in their locations in geographic space (or in a space of characteristics) as well as in their quality level.[12] Thus, while we share Tiebout's interest in the optimal variety of public goods, we do not

begin our modelling effort with his optimistic belief in the benefits of imitating such markets in the public sphere.

Our model departs from the traditional models of variety differentiation in the public-good nature of the goods as well as the way in which firms receive revenue. Our model combines variety and quality differentiation as in the recent work of Economides (1993) and Neven and Thisse (1990). However, in the present model *there are no prices*. Donors contribute directly to the organization that fits with their most desired position in the spectrum of characteristics. In the absence of prices to be used as strategic variables, firms compete more aggressively in the level of quality, and this leads to quite different results than in the traditional models of variety differentiation.

One purpose of this chapter is to determine whether a private market in public goods needs to be regulated. Will the free entry of profit-maximizing suppliers produce optimality when citizens have a wide range of tastes for public services and are given a 100 per cent tax deduction for donations? We demonstrate below that at equilibrium the level of quality (or quantity) of output per producer is, in general, above the optimum, and the difference between the equilibrium and optimal varieties increases as fixed costs decreases. Furthermore, we show that in most realistic situations, the optimal number of suppliers is below the number of suppliers at the free entry equilibrium. Thus, in a wide range of cases, contrary to the usual results for public goods, the free market produces too many varieties and too much output. This result occurs because the overall pool of financial resources is fixed and therefore firms, unable to compete in prices, compete instead by supplying ever higher levels of quality (or quantity). We also show that, in the absence of policies restricting entry, relying on a single public producer is generally superior to an unregulated free market equilibrium.

The rest of the chapter is organized as follows. Section 6.2 presents the model and derives the market equilibrium. Section 6.3 presents comparative statics on the features of the equilibrium. Section 6.4 discusses the optimal market structure. In Section 6.5 we compare the equilibrium and optimal solutions. In Section 6.6 we compare a single supplier with unregulated private production. In Section 6.7 we extend our results to non-profit firms. In Section 6.8 we present concluding remarks.

6.2 SUBGAME-PERFECT EQUILIBRIUM

In this section we describe the basic structure of the model. More detailed mathematical underpinnings are provided in the appendix. We first point out the distinction between quality and variety, developed in the study of monopolistic competition. All consumers desire more of a quality feature. Examples might be student/teacher ratios in schools, nurse/patient ratios in hospitals, the number of knots per square inch in an oriental rug, and the freshness of produce. In contrast, product varieties cannot be unambiguously ranked without knowledge of the individual consumer's utility function. Different consumers rank variety features differently. Examples are a school's educational philosophy, the pattern and colour of a carpet, the dryness of wine. While, in practice, variety is a multidimensional concept, its essential aspects can be captured by assuming that 'local' public goods vary by type and are indexed by a continuous one-dimensional variable x. We represent each variety of public good as a point on a circumference of length one. Every individual has a most preferred type of public service. For expositional convenience suppose that individual z's most preferred type is z, a location on the circumference.

Individuals, however, care about the quality of the service as well as its type. Quality is measured by the variable a independently of type. Because the good is a public good to donors, quality and quantity can be analyzed in the same way. More of either one is always desirable, and donors to a particular supplier benefit from that firm's entire production. Output is not assigned to particular donors. In the discussion to follow we refer to a as quantity, but the term quality can always be substituted.

Given the incentives for donations assumed above, and assuming that everyone possesses at least a dollar, the donor's problem is to choose the supplier which gives her the most utility. Recall that we assume a 'buying-in' mentality where the donor does not consider the benefits from services of producers to which she does not donate. Assuming that the utility of the public good is separable from other aspects of the utility function, let the utility of donor z when he donates to charity j (which produces variety x_j) be

$$U_z(a_j, x_j) = a_j - \beta(x_j - z)^2$$

where $\beta > 0$. The parameter β measures the intensity of an individual's preference for donating to a supplier that offers a variety x

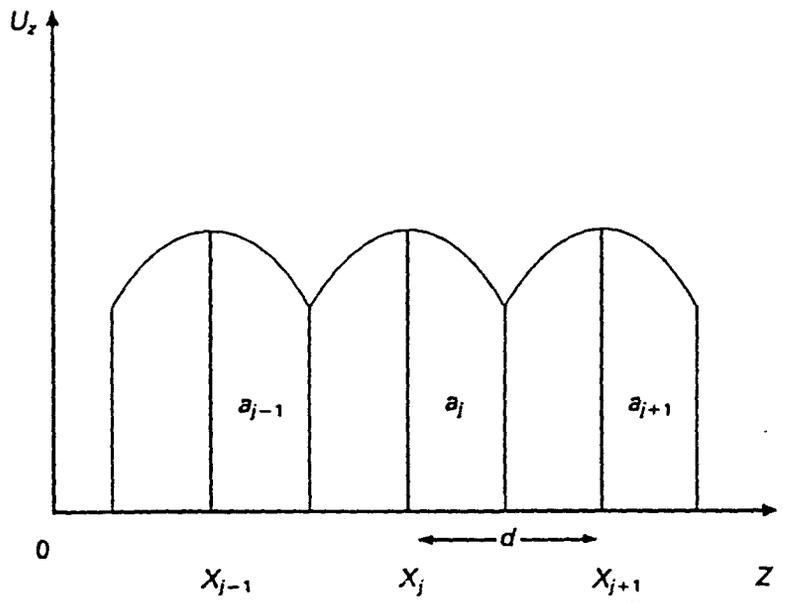

Figure 6.1 Utility to donor 'z'

close to the individual's most preferred variety z. Given n varieties located at $x = (x_1, \ldots, x_n)$, the donor contributes one dollar to the particular charity that realizes the highest utility $U_z(a_j, x_j)$ for her. See Figure 6.1.

Suppose that the consumers are uniformly distributed in variety space, that is, each of the possible types has the same number of people who believe that it is the most desirable sort of public good. Firms play a three-stage game. In the first stage, they decide whether or not to enter, in the second stage they decide what kind of service to provide (location on the circumference), and in the third stage they choose quantity (or quality). Firms never cooperate with each other, but each firm has correct information about the subsequent equilibrium and takes account of the way its own choices will affect the future choices of all firms.[13]

The model is solved by starting from the last stage and working backwards. Thus, suppose that n firms are in the market and have chosen locations $x = (x_1, \ldots, x_n)$ in earlier stages. In the last stage each firm j chooses quantity a_j to maximize its profits. We assume that the number of firms is large enough so that they are in direct competition. Thus the marginal donor to supplier j is indifferent between giving to j and giving to a 'neighbouring' firm $j - 1$.[14]

Assuming that firms collect revenues only from contributions, the profit function of firm j can be represented as

$$\Pi_j(x, a) = D_j - ca_j^2/2 - F,$$

where D_j is donations to firm j, F is fixed cost, and $ca_j^2/2$ is the variable cost of producing a_j. Notice that the fixed cost, F, can be thought of as the marginal cost of variety since it is the cost of adding a new firm to the market. The marginal cost of quantity (or quality) is, of course, ca_j. Let $a^*(x) = (a_1^*(x), \ldots, a_n^*(x))$ be the vector of equilibrium outputs (or quality levels) chosen by the n firms.[15]

In the second stage, each of the n firms decides what type of service to provide anticipating that the equilibrium described above will prevail in the third stage. Thus, firms take into account both the direct effects of their choice of service type on profits, as well as the indirect effect of service type on profits through the equilibrium output levels. The objective function for a firm j in the second stage is thus

$$\Pi_j^v(x) \equiv \Pi_j(x, a^*(x)).$$

We show in the appendix that, at the equilibrium of the second stage, the firms are equidistant from each other along the circumference which indicates the possible types of public services. Let d be the 'distance' between firms. Since the circle has a circumference of one, $d = 1/n$. Let μ be the density of consumers at each point in variety space, that is, the measure of the number of donors at each point on the circle. Then the donations obtained by any firm j are μd or μ/n. It is shown in the appendix that the equilibrium quantity (or quality) per firm is

$$a_j^m(n) = \mu n/(\beta c) \tag{6.1}$$

The resulting equilibrium profits per firm are

$$\Pi(n) = \mu/n - \mu^2 n^2/(2\beta^2 c) - F \tag{6.2}$$

6.3 COMPARATIVE STATICS

Output (or quality) is lower the larger are the cost parameter, c, and the utility function parameter which measures donors' concern with

variety, β. Notice that variety and quantity (or quality) are *positively* related in equilibrium. The more varieties of local public services there are in the market, the higher the level of output each produces.

Now consider the impact of the cost and utility parameters on profits. Notice that both have a positive impact on profits. Thus *a firm's profits are higher the greater the slope of the marginal cost curve.* At first this result may seem paradoxical, but it can be explained by recalling that c is assumed to be the same for all firms and by observing that the positive effect of costs on profits is larger the more firms there are in the market. An increase in c raises the costs of a firm's competitors and hence reduces competitive pressures. With a fixed number of firms, when the cost parameter is high, each firm produces less output (or lower-quality services) and since the volume of donations is fixed, this increases profits. This result would not necessarily follow if total donations were sensitive to quantity and quality levels, that is, if citizens were deciding not only which firms to donate to but also how much to give.

Next consider the effect on profits of the intensity of preference for one's preferred type of local public service. Notice that the higher is β, the more unwilling are donors to accept services located away from their preferred variety. A high β means that products located at a given 'distance' on the circle from the most preferred variety, z, are weaker substitutes for z than when β is low. Therefore, with a fixed number of firms, the local monopoly power of each firm increases with the intensity of preference, β, and so do profits per firm.

Now consider how the number of competitors in the market affects output per firm and profits. As the number of competitors increases, the size of the market for a firm shrinks. The firm responds by increasing the quantity (or quality) of output.[16] Profits are squeezed by the decline in the size of the market and by the cost of increasing output. Thus profits fall as the number of firms increases and are negative for large n.

Finally, consider the first stage of the game in which firms enter until profits are negative for a potential entrant. Thus the equilibrium number of firms, n^*, is the largest integer such that overall profits are greater or less than zero and profits with $n^* + 1$ firms are negative. Since overall profits are decreasing in n and become negative for large n, n^* is a finite, positive number. In the discussion below we will assume that the number of firms is large enough so that n can be treated as a continuous variable with little loss of generality.

We show in the appendix that the equilibrium number of firms is increasing in population density, μ, in the slope of the marginal cost curve, c, and in the intensity of preference for one's preferred type, β. Both the equilibrium level of production per firm and the equilibrium number of firms decrease as fixed costs, F, rise. These results demonstrate the basic trade-off in the model between fixed costs and diverse preferences. The greater the population density at each point and the stronger the preference for being close to one's most preferred type, the more varieties that can be profitably produced. Conversely, the greater the cost of producing a new variety, the fewer the types of local public services that will be produced. Notice, however, that the greater the slope of the marginal cost of quantity curve, the greater the equilibrium number of firms. This result holds because as the cost of additional units of quantity (or quality) increases, the quantity produced per firm falls, and thus, for a fixed number of firms facing a fixed revenue pool, profits increase. This increase in profits induces entry. Profits fall as the number of firms increases until the marginal firm is earning zero profits.

6.4 OPTIMALITY

Suppose that a social planner can place suppliers optimally around the 'variety circle' in a way that maximizes surplus. Recall that, under the conditions of our model, citizens only benefit from the services provided by the supplier that receives their donation. Then the shaded area in Figure 6.1 illustrates the utility obtained by contributors gross of their one dollar contribution. A little manipulation (see the appendix) yields the result that the optimal output per firm is

$$a^s(n) = \mu/(nc). \tag{6.3}$$

Note that the optimum quantity (or quality) per firm, a^s, is inversely related to the number of firms, while in the competitive equilibrium described above, quantity is positively related to n. Furthermore, the optimum output is independent of the utility function parameter, β, measuring the value of being close to one's preferred variety. In contrast, the equilibrium level is inversely related to β.

Given the optimal output (quality) level for each firm, the optimal number of suppliers, n^o, is found by maximizing total surplus, $S(n)$,

with respect to n (see the appendix). Let the optimal number of firms be n^o, and let the corresponding optimal level of quality be $a^o = a^s(n^o)$.

6.5 COMPARISON OF THE SOCIAL OPTIMUM WITH THE SUBGAME-PERFECT EQUILIBRIUM

We now compare the optimal results (n^o, a^o) with the equilibrium ones (n^*, a^*).[17] There are two basic features of the dependence of n and a on F. First, both n^* and n^o are decreasing in F. This is natural. The higher the fixed cost, the fewer firms will operate at the free entry equilibrium, and it is optimal to have fewer firms in operation. Second, the optimal level of quality decreases in n but the equilibrium level of quality *increases* in n. The relationship of the optimal quality with n is natural. The relationship of equilibrium quality with the number of firms is remarkable and surprising at first glance. But remember that a firm does not have the opportunity to compete in prices. Thus, when squeezed by its nearby competitors as their number increases, it competes by increasing its quality level. Because n^o and n^* vary in the same direction with F, but a^o and a^* vary in opposite directions with n, a^o and a^* vary in opposite directions with F. Indeed, a^o increases in F while a^* decreases in F. Thus, for all $F < \bar{F}$, $a^o < a^*$, and for all $F > \bar{F}$, $a^o > a^*$. See Figure 6.2. The relationship between a^o and a^* can be seen as follows. Higher levels of fixed cost are optimally matched with higher levels of quality. But a high level of fixed cost results in a small number of firms at the free entry equilibrium. This implies a small incentive to compete in quality, and therefore a low level of quality at equilibrium. It is shown in the appendix that $\bar{F}$ corresponds to less than two active firms. Thus, we can safely say that for all relevant costs $F \in [0, \bar{F}]$, the optimal quality exceeds the equilibrium one, $a^o < a^*$.

The comparison between the equilibrium and the optimal number of firms is more complex. They are both decreasing functions of the fixed cost, and they have two intersections (F_1, n_1), (F_2, n_2) as seen in Figures 6.3(a) and (b). It is shown in the appendix that F_2 is exceptionally small in comparison with $\bar{F}$. In the case of weak preference for variety (small β), $\bar{F} > F_1$, so that the equilibrium number of varieties is smaller than the optimal one, $n^* < n^o$, for all fixed costs except exceptionally small ones. See Figure 6.3(b). When consumers have strong preferences for particular brands, that is, for large β,

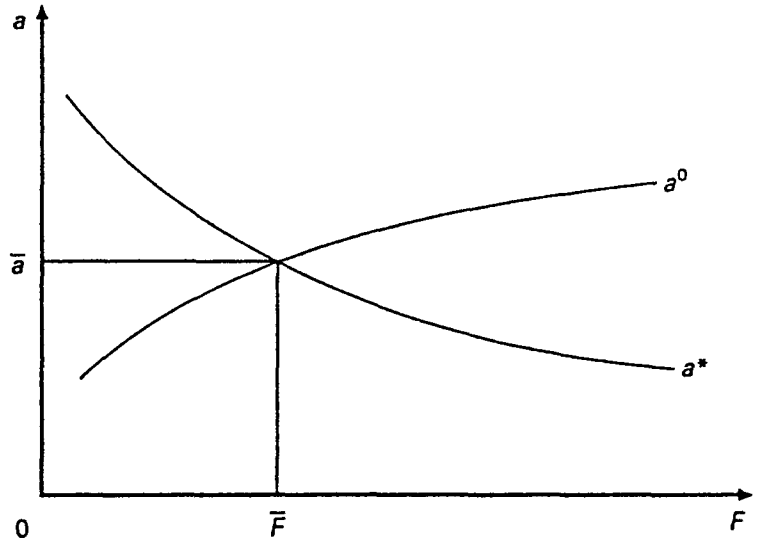

Figure 6.2 Optimal versus equilibrium quality

$F_2 < \bar{F} < F_1$, the equilibrium number of varieties exceeds the optimal number except for very small and very large fixed costs. Both of the latter cases are unlikely. Our basic conclusion is then a simple one. The market equilibrium is unlikely to be optimal and will generally producc a situation in which too much of both quantity (or quality) and variety are produced.

Table 6.1 summarizes the results of the comparison of the equilibrium and the optimal number of firms and levels of quality for strong or weak preference for particular brands (large or small β).

6.6 SECOND-BEST CONSIDERATIONS: COMPARISON OF A SINGLE SUPPLIER WITH UNREGULATED PRIVATE PRODUCTION

A society's regulatory options may be limited so that the social optimum cannot be obtained. Suppose, in particular, that the alternative to unregulated private production is governmental production of a single variety. Assume, however, that, subject to that constraint, the public authority selects the optimal quantity (or quality). Obviously, with the population distributed evenly over variety space all varieties are equally desirable. We show in the appendix that unless donors have extremely strong preferences for their most pre-

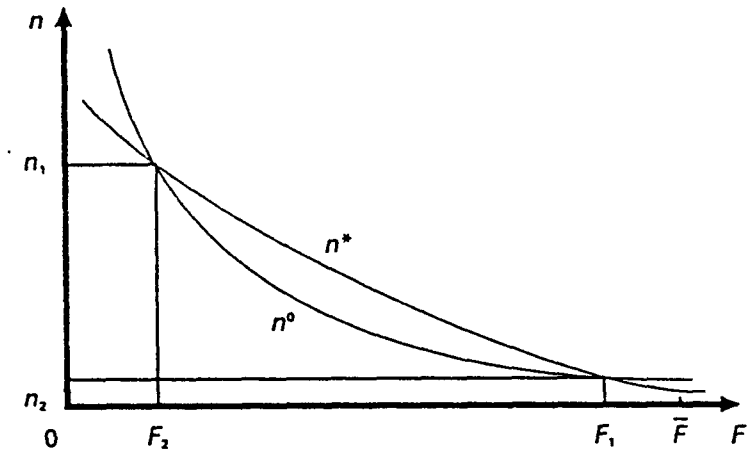

Figure 6.3(a) Optimal versus equilibrium, β large

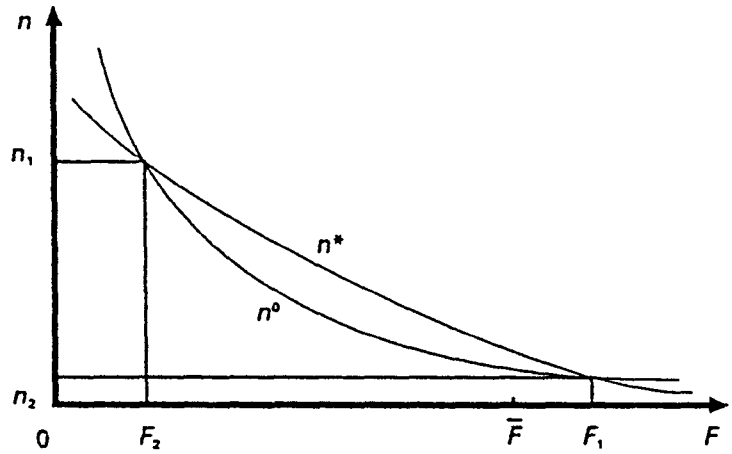

Figure 6.3(b) Optimal versus equilibrium, β small

ferred type or unless fixed costs are very small, surplus is larger with undifferentiated public production because of the savings of $(n^* - 1)F$ in fixed costs compared to the free entry equilibrium. Social welfare is higher in a wide range of cases with a government that imperfectly reflects the preferences of the population by producing a single type of service and taxing everyone equally than it is under a regime of unregulated private suppliers financed by gifts that are 100 per cent tax-deductible. The benefits of diversity require regulation of private production.

Table 6.1 Strong versus weak preferences for variety

Strong preference for variety, β large. Pictured in Figure 6.3(a).

Fixed cost	$F < F_2$		$F_2 < F < \bar{F}$
Number of firms	$n^o > n^*$		$n^o < n^*$
Quality levels		$a^o < a^*$	

Weak preference for variety, β small. Pictured in Figure 6.3(b).

Fixed cost	$F < F_2$	$F_2 < F < F_1$	$F_1 < F < \bar{F}$
Number of firms	$n^o > n^*$	$n^o < n^*$	$n^o < n^*$
Quality levels		$a^o < a^*$	

6.7 EXTENSION TO NON-PROFIT FIRMS

Suppose that the private providers of public goods funded by tax credits are non-profit corporations instead of for-profit firms. If we suppose that managers of non-profits seek to maximize quantity (or quality) subject to a break-even constraint, there will be no change in our basic results. For any number of firms less than or equal to the number in competitive equilibrium, n^*, profits will be zero. However, for $n < n^*$, entry will still occur because potential entrants do not care about profits *per se* but instead consider whether they can enter and break even by producing a positive quantity of some variety x. So long as we assume that there is no 'scarcity' of non-profit entrepreneurs and that suppliers have no personal preferences for particular varieties,[18] entry will occur until $\Pi(n^*) = 0$, that is, until new entrants cannot break even. An equilibrium will exist with producers evenly spaced in variety space, each producing the same level of output.[19] Therefore, all the results developed above carry over to this case. Thus, even if one believes that private non-profit suppliers are to be preferred to for-profits on quality control grounds,[20] the same problems of overproduction and excessive variety remain in a free entry equilibrium.

6.8 CONCLUDING REMARKS

Our model demonstrates that those who advocate the widespread privatization of public functions should proceed with caution.[21] We have shown that, while the provision of differentiated public services can improve consumer welfare, these benefits will generally require regulation of both entry and service levels to reap the full benefits of multiple providers. The basic trade-off between the costs and benefits of adding varieties will not be optimally resolved by private suppliers. Furthermore, private firms will overproduce. In contrast to the usual results in models with public goods, in our model when producers of 'local' public goods compete for a fixed quantity of financial resources, private firms will generally produce too wide a variety and too much total output relative to the social optimum. Even an inefficient government may be preferable to the production of public services by competitive private firms, be they non-profit charities or for-profit firms.

We have, however, only analyzed one aspect of the issue: the range of service types and quantities (or quality levels) produced by private firms in unregulated competitive equilibrium. In order to focus on this problem we have assumed that all suppliers face equivalent production and cost functions and are equally productive. We have not addressed the issue of shirking in either the for-profit, non-profit or governmental sectors. One should recognize, however, that the inclusion of this factor would not necessarily change our results. Many of the publicly provided goods and services that are candidates for privatization, such as the provision of social services to the needy, basic research, primary and secondary education, are ones that are difficult for tax-payers to monitor. In fact, this is one reason why the services are included in the public sector in the first place. Thus for-profits will not, in this context, necessarily be superior on productive efficiency grounds, but this is an issue that requires analysis in particular cases.

Appendix

Donor z contributes one dollar to the charity that maximizes $U_z = \max_j (a_j - \beta(x_j - z)^2)$. Let consumers be distributed uniformly with density μ on the circumference of length one. Suppose that n firms have entered in stage 1 and they have chosen locations $x = (x_1, \ldots, x_n)$ in stage 2. In the third stage

firms choose quantities $a = (a_1, \ldots, a_n)$. Let the marginal donor between firms j and $j + 1$ be z_j. z_j solves

$$a_j - \beta(x_j - z_j)^2 = a_{j+1} - \beta(x_{j+1} - z_j)^2 \Rightarrow$$
$$z_j = (x_j + x_{j+1})/2 + (a_j - a_{j+1})/[2\beta(x_{j+1} - x_j)]$$

Then donations to firm j are

$$D_j = \mu(z_j - z_{j-1}) = \mu\{x_{j+1} - x_{j-1} + (a_j - a_{j+1})/[\beta(x_{j+1} - x_j)] + (a_j - a_{j-1})/[\beta(x_j - x_{j-1})]\}/2 \tag{A6.1}$$

The profit function of firm j,

$$\Pi_j(x, a) = D_j - ca_j^2/2 - F \tag{A6.2}$$

is concave in a_j, and it is maximized at

$$a_j^* = \mu[1/(x_{j+1} - x_j) + 1/(x_j - x_{j-1})]/(2\beta c) \tag{A6.3}$$

The vector $a^*(x) = (a_1^*(x), \ldots, a_n^*(x))$ defines the level of output at the unique non-cooperative equilibrium of the quantities' subgame played for locations x.

The objective function of firm j in a subgame that starts in the second stage is

$$\Pi_j^v(x) \equiv \Pi_j(x, a^*(x))$$

We are interested in the existence of a symmetric subgame-perfect equilibrium, where the firms are equidistant from each other along the circumference. Suppose that all firms except firm j are located symmetrically $x_i = x_{i+1} - d$, $(i \neq j, i \neq j - 1)$, and $x_{j+1} - x_{j-1} = 2d$, where d is a constant. A symmetric perfect equilibrium exists if and only if Π_j^v is maximized at $x_j = (x_{j+1} + x_{j-1})/2$. This is guaranteed by Lemma 6.1.

LEMMA 6.1 *Π_j^v is quasiconcave for $x_j \in [x_{j-1}, x_{j+1}]$ and is maximized at $x_j = (x_{j+1} + x_{j-1})/2$.*

PROOF We show that $\Pi_j^v(x) \equiv \Pi_j(x, a^*(x))$ is maximized with respect to x_j at $(x_{j+1} + x_{j-1})/2$ when $x_i = x_{i+1} - d$ for all $i \neq j$, $i \neq j - 1$, and $x_{j+1} - x_{j-1} = 2d$. Substituting (A6.1) and (A6.3) in (A6.2) yields

$$\begin{aligned}\Pi_j^v &= \mu(x_{j+1} - x_{j-1})/2 + \mu^2[1/(x_{j+1} - x_j) + 1/(x_j - x_{j-1})]^2/(8c\beta^2) \\ &\quad - \mu^2[1/(x_{j+2} - x_{j+1}) + 1/(x_{j+1} - x_j)]/[4c\beta^2(x_{j+1} - x_j)] \\ &\quad - \mu^2[1/(x_j - x_{j-1}) + 1/(x_{j-1} - x_{j-2})]/[4c\beta^2(x_j - x_{j-1})]\end{aligned}$$

Let $x_j = (x_{j+1} - x_{j-1})/2 + \epsilon$, so that $x_{j+1} - x_j = d - \epsilon$ and $x_j - x_{j-1} = d + \epsilon$, where $\epsilon \in (-d, d)$. After substitution and a few steps we derive

$$\Pi_j^v = \mu d - \mu^2 d^2/2c\beta^2[(d^2 - \epsilon^2)]^2 - F$$

It follows that

$$\partial\Pi_j^v/\partial x_j = \partial\Pi_j^v/\partial\epsilon = -2\epsilon d^2\mu^2/[c\beta^2(d^2 - \epsilon^2)^3]$$

Therefore $\partial\Pi_j^v/\partial\epsilon = 0$ at $\epsilon = 0$, and $\partial\Pi_j^v/\partial\epsilon < 0$ for $\epsilon > 0$, while $\partial\Pi_j^v/\partial\epsilon > 0$ for $\epsilon < 0$. Thus, Π_j^v is maximized at $x_j^* = (x_{j+1} + x_{j-1})/2$.

For symmetric locations of firms d apart, $\epsilon = 0$, and profits per firm are

$$\Pi(d) = \mu d - \mu^2/(2c\beta^2 d^2) - F$$

Since $d = 1/n$, the realized profits per firm at an n-firm symmetric perfect equilibrium of the subgame that starts with the choice of locations are

$$\Pi(n) = \mu/n - \mu^2 n^2/(2\beta^2 c) - F \tag{A6.4}$$

Quantity per firm is

$$a_j^m(n) = \mu n/(\beta c) \tag{A6.5}$$

In the first stage, firms enter until the profits at the symmetric equilibrium of the subgame become negative for a potential entrant. The equilibrium number of firms is n^* such that $\Pi(n^*) \geq 0$ and $\Pi(n^* + 1) < 0$. Thus, n^* is the integer part of n^z that solves

$$\Pi(n^z) = 0$$

Since $\Pi(n)$ is decreasing in n and becomes negative for large n, there exists a finite and positive equilibrium number of firms n^*, provided that $\Pi(1) \geq 0$.

PROPOSITION 6.1 *The three-stage game (entry, location, production) has a symmetric perfect equilibrium with n^* equispaced firms producing $a^* = a_j^m(n^*)$ each.*

COROLLARY 6.1 *The equilibrium level of production per firm is increasing with population density, μ, and decreasing with the slope of the marginal cost curve, c, with the intensity of preference on variety, β, and with fixed cost, F.*

PROOF Eliminating n^* between (A6.5) and (A6.4) we can express profits per firm as a function of their level of production a^* at the free entry equilibrium as,

$$\Pi(a^*) = \mu^2/(a^*\beta c) - (a^*)^2 c/2 - F = 0 \tag{A6.6}$$

We use the implicit function theorem on $\Pi(a^*)$ defined in (A6.6). $da^*/dy = -(\partial\Pi/\partial y)/(\partial\Pi/\partial a^*)$, where the variable y is substituted for μ, c, and F. $\Pi(a^*)$ is decreasing in a^*: $\partial\Pi(a^*)/\partial a^* = -\mu^2(a^{*2}c\beta) - a^*c < 0$. Thus,

$\partial\Pi(a^*)/\partial\mu = 2\mu/(a^*c\beta) > 0$ implies $da^*/d\mu > 0$. $\partial\Pi(a^*)/\partial c = -\mu^2/a^*\beta c^2 - a^{*2}/2 < 0$ implies $da^*/dc < 0$. $\partial\Pi(a^*)/\partial\beta = -\mu^2/(a^*\beta^2 c) < 0$ implies $da^*/d\beta < 0$. $\partial\Pi(a^*)/\partial F = -1 < 0$ implies $da^*/dF < 0$.

COROLLARY 6.2 *The perfect equilibrium number of firms is increasing in population density, μ, in the slope of the marginal cost curve, c, and in the intensity of preference on variety, β. The equilibrium number of firms is decreasing in fixed cost, F.*

PROOF We use the implicit function theorem on $\Pi(n^*)$: $dn^*/dy = -(\partial\Pi/\partial y)/(\partial\Pi/\partial n^*)$, where the variable y is substituted for μ, c, and F. $\Pi(n^*)$ is decreasing in n^*: $\partial\Pi(n)/\partial n = -\mu/n^2 - \mu^2 n/(c\beta^2) < 0$. Thus, $\partial\Pi(n^*)/\partial\mu = 1/n^* - \mu n^{*2}/(c\beta^2) > 0$ implies $dn^*/d\mu > 0$. $\partial\Pi(n^*)/\partial c = \mu^2 n^{*2}(2c^2\beta^2) < 0$ implies $dn^*/dc > 0$. $\partial\Pi(n^*)/\partial\beta = \mu^2 n^{*2}/(\beta^3 c) > 0$ implies $dn^*/d\beta > 0$. $\partial\Pi(n^*)/\partial F = -1 < 0$ implies $dn^*/dF = 0$.

Consumers' plus producers' surplus per firm for a symmetric arrangement of firms d apart producing a each can then be expressed as:

$$\begin{aligned} s(a, d) &= \mu[ad - 2\beta \int_0^{d/2} x^2 dx] - ca^2/2 - F \\ &= \mu ad - \mu\beta d^3/12 - ca^2/2 - F \end{aligned}$$

Total surplus for $n = 1/d$ firms is

$$S(n, a) \equiv ns(a, 1/n) = \mu a - \mu\beta/(12n^2) - nca^2/2 - nF \tag{A6.7}$$

Maximization with respect to a yields

$$a^s(n) = \mu/(nc) \tag{A6.8}$$

Substituting (A6.8) in (A6.7) yields

$$S(n) \equiv S(n, a^s(n)) = -\mu\beta/(12n^2) + \mu^2/(2nc) - nF$$

Maximization with respect to n yields n^o that must solve:

$$S'(n) = \mu\beta/(6n^3) - \mu^2/(2n^2c) - F = 0 \tag{A6.9}$$

The social optimum is characterized by n^o firms, with $a^o = a^s(n^o)$ production per firm.

We first compare the equilibrium and the optimal levels of production per firm. As shown earlier, equilibrium production a^* must solve

$$\Pi(a^*) = \mu^2/(a^*\beta c) - (a^*)^2 c/2 - F = 0 \tag{A6.6}$$

Define $\tilde{\Pi}(a) = \Pi(a) + F$. Substituting n^o as a function of a^o from (A6.8) into (A6.9) yields that a^o must solve

$$S'(a) = \beta a^3c^3/(6\mu^2) - a^2c/2 - F = 0$$

Define $\widetilde{S}'(a) = S'(a) + F$. $\widetilde{\Pi}(a)$ and $\widetilde{S}'(a)$ have a unique intersection, $\bar{a}$, since

$$\widetilde{\Pi}(a) = \widetilde{S}'(a) \Leftrightarrow \mu^2/(a\beta c) = \beta a^3c^3/(6\mu^2) \Leftrightarrow \bar{a} = 6^{1/4}\mu/(c\sqrt{\beta})$$

At fixed cost $\bar{F} \equiv \widetilde{\Pi}(\bar{a})$, $a^* = a^o = \bar{a}$. See Figure 6.2. For $F < \bar{F}$, firms overproduce at the perfect equilibrium, $a^o < a^*$. The equilibrium number of firms corresponding to $\bar{F}$ is $n^*(\bar{a}) = 6^{1/4} \simeq 1.565 < 3$. Since our model assumes that each firm has two neighbouring firms, we are only concerned with cases in which there are at least three active firms. Therefore fixed costs above $\bar{F}$ are irrelevant and for all relevant fixed costs $a^o < a^*$.

To compare optimal with equilibrium diversity we define $\widetilde{\Pi}(n)$ as the profits per firm at an n-firm equilibrium net of fixed costs, $\widetilde{\Pi}(n) = \Pi(n) + F$. Similarly we define $\widetilde{S}'(n) = S'(n) + F$. The functions $\widetilde{\Pi}(n)$ and $\widetilde{S}'(n)$ are both decreasing in the region where they are positive, and they have two intersections at F_1 and F_2. The zero of $S'(n)$ is at $n_s = \beta c/(3\mu)$ while the zero of $\widetilde{\Pi}(n)$ is at $n_\pi = (2c\beta^2/\mu)^{1/3}$. $n_\pi < n_s \Leftrightarrow c\sqrt{\beta}/\mu > \sqrt{54} \simeq 7.348$. For high or low fixed costs, where $F > F_1$ or $F < F_2$, optimal diversity exceeds the equilibrium one, $n^o > n^*$; for $F_2 < F < F_1$, optimal diversity falls short of the equilibrium one, $n^o < n^*$. To find out if $F > F_1$ is a plausible case we need to place $\bar{F}$ relative to F_1 and F_2. Thus, we now compare $n^o(\bar{a})$ and $n^*(\bar{a})$, the optimal and equilibrium numbers of firms corresponding to $\bar{F}$. If $n^*(\bar{a}) > n^o(\bar{a})$, then $\bar{F} < F_1$, and fixed costs above F_1 are irrelevant. Conversely, if $n^*(\bar{a}) < n^o(\bar{a})$, then $\bar{F} > F_1$, and fixed costs in the range $(F_1, \bar{F})$ are worth considering.

It can be shown that

$$n^*(\bar{a}) > n^o(\bar{a}) \Leftrightarrow \beta < 6 \equiv \hat{\beta}$$

Consequently, for $\beta < \hat{\beta}$ we have $F_2 < \bar{F} < F_1$. Figure 6.3(b) shows n_o and n_e under these conditions. Similarly, for $\beta > \hat{\beta}$ we have $F_2 < F_1 < \bar{F}$. Figure 6.3(a) shows n_o and n_e for this case. Since $F > \bar{F}$ implies $n^* < 3$, in both cases the relevant range of fixed costs is $(0, \bar{F})$.

Now consider the intersection of n_o and n_e at (n_2, F_2). n_2 solves

$$6n^2\beta^2 - 3n^5\mu/c = \beta^3 - 3n\mu\beta^2/c$$

Assuming that μ/c and $n\mu/c$ are small, while n is large, this equation is approximated by $6n^2\beta^2 - 3n^5\mu/c = 0$ which is solved by $n_2 \simeq (2c\beta^2/\mu)^{1/3} = n_\pi$, the zero of $\widetilde{\Pi}(n)$. Thus, for small μ/c, F_2 is approximately zero. Compared with $\bar{F}$, F_2 is very small. $F_2 < 0.003\bar{F}$.

Thus we have two cases, illustrated in Figures 6.3(a) and 6.3(b); for small $\beta < \hat{\beta}$, the equilibrium level of diversity exceeds the optimum, except for very small level of fixed cost. For large $\beta > \hat{\beta}$, equilibrium diversity also exceeds the optimal one except for very small and very large fixed costs. Thus, in general, a competitive system produces too much of too many different kinds of public services. However, when fixed costs approach zero, it is optimal to have a very large number of service types each producing very

little output. Then competition produces too little diversity. If citizens care a great deal about donating to a producer whose service type is close to their own most preferred variety, then diversity is socially very valuable. Competition, however, still results in too much diversity in this case unless the fixed costs are either very high or very low. Therefore, we have:

PROPOSITION 6.2 *Equilibrium production per firm always exceeds the optimal one. When consumers have a relatively weak preference for variety, for all fixed costs, except those that are extremely small, equilibrium product diversity exceeds the optimal diversity. When consumers have a relatively strong preference for variety, equilibrium diversity exceeds the optimal one except for very large and very small fixed costs.*

Suppose that the alternative to unregulated private production is governmental production of a single variety. In such a situation where no regulation of private producers is possible, the relevant comparison is between, $S(1) \equiv S(1, a^s(1))$ and $B(n^*) \equiv S(n^*, a^m(n^*))$, where $n^* = n^*(\mu, c, F)$ is the perfect equilibrium number of firms. $B(n)$ is the realized total surplus at an n-firm equilibrium:

$$B(n) \equiv S(n, a^m(n)) = \mu^2[n/c\beta) - n^3/(2c\beta^2)] - \beta\mu/(12n^2) - nF$$

and

$$B(n) - S(1) = \mu^2(n/\beta - n^3/2\beta^2 - 1/2)/c + \mu\beta(1 - 1/n^2)/12 - F(n - 1)$$

Substituting from (A6.4) for the fixed cost, F, that makes $\Pi(n^*) = 0$ yields

$$\begin{aligned} B(n^*) - S(1) &= \mu\{(n - 1)[\beta(n^* + 1) - 12n^*]/(12n^{*2}) \\ &\quad - \mu(n^* - \beta)^2/[2\beta^2 c(n^* - 1)]\} \end{aligned} \tag{A6.10}$$

This expression is negative when the first brackets are negative, for which it is sufficient that $\beta < \hat{\beta} \equiv 6$. When donors do not have extremely strong preferences on variety, surplus is larger with undifferentiated public production because of the savings of $(n^* - 1)F$ start-up costs compared to the perfect equilibrium. Thus we have:

PROPOSITION 6.3 *Except when donors have extremely intense preferences for variety, total surplus with a single surplus-maximizing supplier exceeds total surplus at the perfect equilibrium of competitive firms.*

Notes

1. See Lindahl, 1919; Samuelson, 1954. Feldman (1980, pp. 106–22) provides a clear exposition.
2. See Groves and Ledyard, 1977; Tideman and Tullock, 1976. See Feld-

man (1980, pp. 122–34) for an exposition and critique of demand-revealing schemes.

3. McGuire, 1974.
4. These solutions may also be feasible since individuals reveal their demands through their choice of a supplier.
5. Ties are broken by lot.
6. For overviews of recent economic work on non-profits, see James and Rose-Ackerman, 1986; Rose-Ackerman, 1986.
7. For work which stresses this approach, see Hansmann, 1980; Easley and O'Hara, 1983.
8. For work which takes this perspective, see James, 1987; Rose-Ackerman, 1981; Young, 1983.
9. Tiebout, 1956.
10. For a summary and critique of this work, see Rose-Ackerman, 1979, 1983.
11. Economides, 1989a, 1989b, 1992; Salop, 1979. This work builds on Hotelling's (1929) seminal paper.
12. In the tradition of Hotelling (1929), we shall be referring to a geographic space, but the equivalence with a space of differentiated goods is well established in the literature.
13. Formally, we are seeking subgame-perfect equilibria in this three-stage game. An n-tuple of strategies is a subgame-perfect equilibrium if, when restricted to any subgame, it forms a non-cooperative equilibrium for that subgame. See Selten, 1975.
14. Similarly, there is a marginal consumer between firms j and $j + 1$.
15. It can be shown that the quantity produced by a firm will be higher the lower is the variable cost parameter, c, and the *closer* the firm is to its neighbours. A firm with close neighbours is in a difficult competitive position and can only compete by offering high level of quantity (or quality). It has very little market power over its customers.
16. This result is opposite to the standard result for similar models with private goods. In those models firms act less aggressively when their 'natural' markets shrink. See Economides, 1993. The difference arises because in the present model firms cannot use prices as strategic instruments.
17. This is done by comparing the point where the profits of the last firm are equal zero with the point where surplus is maximized, that is, where $S'(n) = 0$ (see the appendix for their explicit forms).
18. For models that take a contrasting view, see Rose-Ackerman, 1981, 1987.
19. If, in contrast, non-profit entrepreneurs are ideological and in short supply, they are unlikely to be spread out evenly in variety space in equilibrium, but the qualitative nature of the results would not be fundamentally changed.
20. Hansmann, 1980; Easley and O'Hara, 1983.
21. For a manifesto from an advocate of privatization, see Savas, 1982.

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