COMPATIBILITY AND COMPETITION IN AIRLINES : DEMAND SIDE NETWORK EFFECT

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Abstract

Network effects arising in air transport markets are usually viewed as resulting from jointness on the cost side. In this paper, we focus on network effects originating on the demand side. We consider a network that links three cities involving two direct and one indirect connection. Two carriers, with asymmetric traffic rights on this network compete sequentially : at the first stage, they choose their departure times ; at the second, they set prices. We show that in order to compensate its network disadvantage the "weak" carrier chooses its departure time as close as possible to its competitor's. This is in contrast to the usual maximum differentiation principle. The network effect intensifies price competition. Depending on the size of the market and on consumers valuation of waiting time, various subgame perfect equilibrium configurations are exhibited.

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1 Introduction

Much of the literature on economics of air transportation relies essentially on two assumptions. The first is that competition between airlines occurs mainly between individual city pairs, each pair being considered as a distinct market by travellers. Under this assumption, demand functions between different city pairs are independent. The second assumption by contrast, emphasizes that air transport involves joint production. In this perspective, "network economies" in air transport result from the cost structure, i.e., from the supply side. The existing literature deals mainly with these supply side aspects (see, e.g., Reiss and Spiller (1989), Brueckner and Spiller (1991), Lederer (1993)). This is also the case of the vast literature devoted to deregulation (see for instance Berechman and Shy (1993), Bittlingmayer (1990), Morrisson and Winston (1986)). Airline liberalization changes drastically the way carriers operate[1]. Indeed, airline industry costs are characterized by substantial economies of density. These give rise to network externalities, which affect the network design. The latter, in turn, has a strong influence on the behavior of the market. The relationship between the cost structure and the network configuration has been explored among others by Spiller (1989), Reiss and Spiller (1989), and Morrisson and Winston (1990), whereas Borenstein ((1989) and (1990)) study the network restructuring in connection with airport dominance.

Both assumptions make sense within point to point networks. However, additional problems arise in more complex networks. For instance, if the traffic between two cities is low and does not allow direct service, or if in a regulatory context a route is served by one carrier only, passengers may face an arbitrage between direct and indirect flights. With an indirect flight, passengers generally incur a disutility due to connection. This disutility induces an additional cost that has to be compensated through a lower price. In other words, consumers have to choose between low quality and low prices associated to indirect flights on one hand, and higher quality and higher prices in the case of direct flights on the other hand (see Morisson and Winston (1985)). The functioning of "hub and spokes" networks (see for instance Dobson and Lederer (1993) for an analysis of the supply side)) reflects such consumer's choices. A consequence of these complex network structures is that demand functions are no longer independent, since they involve arbitrage between different routes.

The present paper focuses on these demand side aspects. These are closely related to the "compatibility" issues of the network literature (see Katz and Shapiro (1985), (1986-a and -b), Farrell and Saloner (1985), (1986), de Palma and Leruth (1989), Perrot (1993)). Consider the case of system goods, for instance, as in Matutes and Regibeau (1988). A system good is composed of two (or more) complementary components, none of which may be used alone. If different producers offer complementary goods, they may either choose to make their goods compatible or to make them incompatible. Compatibility

[1]For an analysis of the deregulation in the US, see Borenstein [1989], Morrisson and Winston [1986], Levine [1987].

increases the variety of systems available to consumers, but generally it also relaxes price competition because a price cut on the part of a given competitor benefits to competing carriers , through an increase in the demand for mixed systems. The analogy with air transport is quite clear. Consider a one stop flight between a city pair. The two legs of the journey are two elementary components of the complete flight. Full compatibility is achieved when the waiting time at the connection is zero, which results from strategic compatible scheduled flights of both carriers. However, there are some differences between the general case and the air transport situation. First, many system goods involve discrete compatibility choices in the sense that goods are either compatible or incompatible [2]. Here, if the degree of compatibility is understood as the length of waiting time at the connection, compatibility appears as a continuous characteristic of the system good. Second, in transportation each elementary component of a one stop flight can be considered either as a final good, or as a complementary good. By contrast, this dimension is generally absent from usual system goods. Third, in air transportation markets, a higher degree of compatibility between flights due to closer departure times, may increase price competition. This is in sharp contrast with some of the results established in the case of network externalities (see, for instance, Katz and Shapiro (1985)).

In this paper, we analyze both price and departure time competition between two carriers on a two-leg network that links three cities[3]. The paper is organized as follows. Section 2 presents the model. In section 3, we restrict ourselves to the case where elementary travels are only considered as parts of a one stop flight, but not as final goods. This allows to isolate the network effect. The complete model is studied in section 4. Conclusions are developed in section 5.

2 The model

We consider the following three-node network linking cities A, B and C.. Suppose for instance that B and C are two cities located within the same country, while A is a city located in another country [4]. There are three markets corresponding to travel demand between each of the city pairs. The pairs AB and BC are connected directly. A and C, by contrast, are linked indirectly : all journeys must pass through B. Two carriers fly the route AB while the route BC is flown only by carrier 2 [5].

Passengers travelling between A and C have two options : they can go with carrier 2 on both legs of the journey on or they can fly with carrier 1 between A and B and switch to carrier 2 for the remainder of the trip. In the latter case, their travel time

[2]Another issue is that compatibility may result either from individual decisions or from joint adoption. See Encaoua, Michel and Moreaux (1992).

[3]An earlier version of this model is Encaoua, Moreaux and Perrot (1992).

[4]For example, one can think of two European cities (B and C) and a non european one (A).

[5]This situation corresponds, for example, to the case where B and C belong to the same country in which domestic routes are regulated and served by a regulated monopoly.

is increased since they must wait until the departure time of carrier 2 from B. In the former, passengers make a connection at B without incurring any waiting time at the connection. The size of the market AB is normalized to 1, the sizes of markets AC and BC respectively are α and β.

It is assumed that travellers -indexed by m- differ in terms of their preferred departure times. The distribution of the parameter m is assumed uniform on $[0, 1]$. The time-distance metric used to measure customer disutility is a convex function, which reflects increasing returns in lost time disutility. For simplicity, we adopt a quadratic disutility function. Note however that if the interval $[0, 1]$ would represent scheduling daily flights, the adopted metric would no longer be appropriate in the sense that pathological behaviors would appear. For example, if flights from A to B were scheduled at 0.00 (carrier 1) and 18.00 (carrier 2), our metric would report that passengers with ideal departure time 23.00 would prefer to use carrier 2, when the daily scheduling model indicates the opposite preference. To avoid this contradiction, the unitary time period must be interpreted as the interval of opening hours of airports. In many countries, opening hours vary between 6.00 a.m. and 10.00 p.m. Our representation, where $[0, 1]$ is equivalent to [6.00 a.m., 10.00 p.m.], amounts thus to assume that time 1 on day 0 is very far from time 0 on day 1.

The departure times of both carriers from A to B are denoted m_1 and m_2 respectively, where m_1 and m_2 belong to $[0, 1]$. We assume that carriers 1 and 2 offer only one flight per unitary time period $[0, 1]$ on each route of their network.

Flying time between A and B is t for both carriers. It is assumed that the departure time of the flight between B and C offered by carrier 2 is $m_2 + t$, i.e., immediately upon landing of the flight of carrier 2 arriving from A. If $m_2 > m_1$, this implies that passengers going from A to C and arriving at B with carrier 1 must wait a time period $(m_2 - m_1)$ before boarding the connecting flight to C. It is also assumed that travellers between B and C differ in terms of preferred take-off time. It should be noted that although the model does not specify preferences for arrival times, these are indirectly taken into consideration on the market AB. On market AC, however, there exists no direct flight. Therefore, all passengers going from A to C arrive at destination at the same time, irrespective of the company they have chosen on the leg AB. The distribution of their preferences is also assumed uniform on $[0, 1]$[6].

Let p_1^{AB} and p_2^{AB} respectively denote the prices set by carriers 1 and 2 for a flight between A and B. Let p_2^{AC} and p_2^{BC} denote the prices which carrier 2 establishes on AC and BC respectively. We make two important simplifying assumptions. First, the price p_2^{BC} is fixed and exogeneous[7]. Second, we assume that carrier 2 cannot discriminate between the markets its serves. That is, the fare on market AC is the sum of the fares for AB and $BC : p_2^{AC} = p_2^{AB} + p_2^{BC}$. This assumption may be considered as unrealistic since airlines moft often offer fares with subadditive price structures. However, in a context

[6]Departure times from B should belong to the interval $[t, t + 1]$, which is renormalized to $[0, 1]$.

[7]This situation may result, for instance, from the price regulation of the route BC.

where carrier 1 belongs to the country of city A and carrier 2 to the country of cities B and C, and where the traffic rights are negotiated between both countries, carrier 1 may claim that price discrimination by carrier 2 would eliminate carrier 1 from market AC. Thus this pricing constraint may result from the bilateral agreements concerning international traffic rights.

Carriers compete sequentially. They first choose their times of departure m_1 and m_2 and second, they compete in prices.This order is based on the fact that scheduling is less flexible than prices. Schedules must be determined in advance because slots are generally scarce; by contrast, prices are more easily adjusted in the short run. Concerning the choices of departure times, two scenarii can be considered. In the first one, both carriers choose simultaneously their departure times m_1 and m_2. This leads to a two stage game. In the second scenario, carrier 2 acts as a Stackelberg leader and chooses its time of departure m_2 before carrier 1. This leads to a three-stage game. The motivation to analyze these two scenarii comes from the fact that the asymmetry of the network configurations between both carriers leads to some possible entry deterrence strategy by an appropriate choice by carrier 2 of its departure time. In each game, the equilibrium concept is the subgame perfect equilibrium.

Passengers select the routing and carriers that minimize the generalized prices associated to the various possibilities they face. These prices are obtained as the sum of the price of the ticket, the monetary value of the schedule delay and the monetary value of transfer time. The monetary value of the schedule delay is supposed to be quadratic and takes the form $a(m - m_i)^2$ for passenger m where m_i is the time departure offered by carrier i. The monetary value of the transfer time at the connection for passengers of the market AC who travel with carrier 1 on AB, and with carrier 2 on BC (this choice is denoted $1-2$) is also supposed to be quadratic and equal to $b(m_1 - m_2)^2$. The parameter b measures the valuation of connection time and is supposed to be greater than a. Indeed, according to some empirical estimations (see, for instance, Morrison and Winston (1985 and 1989)), the value of transfer time is more than ten times the value of schedule delay. This means that the disutility to air travellers of a time spent walking through terminals and waiting at their departure gates to make a connection is largely higher than the disutility due to the difference between the traveller's desired departure time and the available departure time. By an appropriate choice of units, we can measure the value of the tranfer time in terms of the value of the schedule delay. This allows to choose $a = 1$ and $b > 1$. Passengers may also choose carrier 2 on both segments AB and BC : this choice is denoted by $2 - 2$. In this case, they incur no transfer cost.

Let s, u, and v denote respectively the reservation values of passengers on markets AB, AC and BC. We assume that these values are large enough to insure that markets are covered when both carriers are active. However, the monopoly prices are constrained by these reservation values.

It is worth noting that this model involves both vertical and horizontal differentiation aspects. The horizontal dimension of the model comes from the heterogeneity of

consumers preferences with regard to departure times. The vertical dimension of the problem comes from the fact that if prices p_1^{AB} and p_2^{AB} were equal, all consumers of market AC would choose the option $2-2$, because it involves no waiting time at the connection[8]. However, by its choice of m_1, carrier 1 may reduce the waiting time and make its flight more compatible with carrier 2's flight from B. This effect is similar to that of networks models (see Katz and Shapiro (1986, 1988)) where the network externality plays the role of a quality parameter. Here, since a higher quality is obtained through a reduction in departure time variety, it may also hurt consumers through a horizontal differentiation effect, especially on markets of final goods AB and BC, where compatibility does not matter.

First we examine the simple case where AC is the only market being served. It can be considered as the limit case of a configuration where the market AC is very large relative to AB and BC. This allows to distinguish the quality -or compatibility- effect from the pure horizontal differentiation effect. Ignoring markets AB and BC amounts to consider a case similar to that studied by Matutes and Regibeau (1988), where two elementary goods are combined into a system but are not valuable by themselves. In other words, the two elementary goods AB and BC are not considered as final goods. In the present model, however, the degree of compatibility between the two components AB and BC varies continuously according to the extent of horizontal differentiation, a dimension which is absent in Matutes and Regibeau. The complete model will be analyzed in section 4.

3 Subgame perfect equilibria on the market AC

Consider the case of market AC alone. This simple case allows to capture the main aspects of the network effects that arise in the complete model.

We examine successively the equilibria of the three and two-stage games, depending on whether m_1 and m_2 are sequentially or simultaneously determined.

3.1 Sequential determination of departure times : the three-stage game

Note first that acting as a follower, carrier 1 would never choose m_1 higher than m_2 : indeed, it would be excluded from the market AC. If $m_1 = m_2$, Bertrand price competition prevails and profits are both equal to zero. It results that we need only to consider the situation where $m_1 < m_2$.

In the case where $m_1 < m_2$, the consumer of market AC who is indifferent between

[8]If the parameter b were equal to zero, the model would only involve this horizontal dimension. Similarly, if the parameter a were equal to zero, the model would only involve vertical differentiation.

the choice $1-2$ and the choice $2-2$ is located at $m^{AC} = \frac{p_2^{AB}-p_1^{AB}}{2(m_2-m_1)} + \frac{m_1+m_2}{2} - \frac{b}{2}(m_2-m_1)$[9]. This leads to three possible configurations of demand, according to the value of m^{AC} with respect to 0 and 1. Whatever the number of consumers who choose $1-2$, all travellers fly with carrier 2 between B and C. Thus, carrier 2 obtains a fixed profit on route BC equal to $\alpha\ p_2^{BC}$, and its total profit is equal to the sum of this fixed profit and of a variable part π_2, which depends on the configuration of demand. We only need to deal with this variable part. The three possible configurations of demand and profit functions are :

Region (i) : $m^{AC} \leq 0$: carrier 2 serves all travellers and profits are $p_1 = 0$ and $\pi_2 = \alpha p_2^{AB}$.

Region (ii) : $0 < m^{AC} < 1$: both carriers 1 and 2 are active and we have : $\pi_1 = \alpha m^{AC} p_1^{AB}$ and $\pi_2 = \alpha(1-m^{AC})p_2^{AB}$.

Region (iii) : $m^{AC} \geq 1$: all travellers prefer the combination $1-2$. Profits are therefore given by : $\pi_1 = \alpha p_1^{AB}$, and $\pi_2 = 0$.

3.1.1 Price competition on the market AC

For given values of m_1 and m_2, such that $m_2 > m_1$, the following lemma characterizes the price equilibria.

Lemma 1 : For any values of (m_1, m_2) such that $m_2 > m_1$, there exists a unique equilibrium which leads to an outcome in regions (i) or (ii).

If $b \geq \underline{b} \equiv \frac{2+m_1+m_2}{m_2-m_1}$, the equilibrium is at the frontier between regions (i) and (ii) and is given by :

$p_1^{AB*}(m_1, m_2) = 0;\ p_2^{AB*}(m_1, m_2) = (m_2 - m_1)[b(m_2 - m_1) - (m_1 + m_2)]$.

If $b < \underline{b}$, the price equilibrium is interior to the region (ii) and is given by :

(1.a) $p_1^{AB*}(m_1, m_2) = \frac{m_2-m_1}{3}[2 + m_1 + m_2 - b(m_2 - m_1)]$

(1.b) $p_2^{AB*}(m_1, m_2) = \frac{m_2-m_1}{3}[4 - (m_1 + m_2) + b(m_2 - m_1)]$

Proof : All the proofs are in Appendix.

This result asserts that for high values of the waiting cost disutility parameter b ($b > \underline{b}$), carrier 1 is forced to push its price down to zero in order to capture some passengers of the market AC. The threshold level $\underline{b}$ depends on the values m_1 and m_2 of the departure times and is greater than 1 for all values of m_2 and m_1. For b in the interval $[1, \underline{b}]$, the horizontal dimension of the price competition outweighs the vertical dimension of the model. When $b > \underline{b}$, the vertical differentiation effect is predominant. Note that $b < \underline{b}$ is equivalent to $m_1 > \frac{(b-1)m_2-2}{b+1}$ (or $m_2 > \frac{2+m_1(1+b)}{b-1}$).

Therefore, the equilibrium values of prices lead to the following expressions of the profit functions :

[9]Note that $m^{AC} = 0$ is equivalent to $p_1^{AB} = p_2^{AB} + (m_2 - m_1)[m_2 + m_1 - b(m_2 - m_1)]$ and $m^{AC} = 1$ is equivalent to $p_1^{AB} = p_2^{AB} + (m_2 - m_1)[m_2 + m_1 - b(m_2 - m_1) - 2]$.

(2.a) $\pi_1(m_1, m_2) = \begin{cases} 0 & \text{if } m_1 \leq \frac{(b-1)m_2-2}{b+1} \\ \alpha\frac{m_2-m_1}{18}[2+(m_1+m_2)-b(m_2-m_1)]^2 & \text{if } m_1 > \frac{(b-1)m_2-2}{b+1} \end{cases}$

and :

(2.b) $\pi_2(m_1, m_2) = \begin{cases} \alpha(m_2-m_1)[b(m_2-m_1)-(m_1+m_2)] & \text{if } m_2 \geq \frac{2+m_1(1+b)}{b-1} \\ \alpha\frac{m_2-m_1}{18}[4+(m_1+m_2)+b(m_2-m_1)]^2 & \text{if } m_2 < \frac{2+m_1(1+b)}{b-1} \end{cases}$

It is worth noting that in region (ii), we have $\pi_1(m_1, m_2) = \alpha\frac{(p_1^{AB*})^2}{2(m_2-m_1)}$ and $\pi_2(m_1, m_2) = \alpha\frac{(p_2^{AB*})^2}{2(m_2-m_1)}$. These profit functions allow us to derive the equilibrium in departure times. We first consider the situation corresponding to a sequential determination of these values, carriers 1 and 2 being respectively the follower and the leader.

3.1.2 Stackelberg equilibrium in departure times

At the second stage of the game, m_2 is given and carrier 1 must choose $m_1 < m_2$ in order to be active on market AC. The optimal choice of m_1 is given by the following lemma.

Lemma 2 : The best reply reply function of carrier 1 verifies $m_1 * (m_2) \geq m_2$ and is given by :

$m_1{}^*(m_2) = \begin{cases} 0 & \text{if } 0 \leq m_2 \leq \frac{2}{1+3b} \\ m_2\frac{(1+3b)-2}{3(1+b)} & \text{if } \frac{2}{1+3b} < m_2 \leq 1 \end{cases}$

The profit function $\pi_1(m_1, m_2)$ for $m_2 > \frac{2}{1+3b}$ has three parts : it is first equal to zero, then it is convex, then concave.

When $m_2 > \frac{2}{1+3b}$, $m_1^*(m_2) = \text{Argmax } \pi_1(m_1, m_2)$ is obtained on the concave part of $\pi_1(m_1, m_2)$. Note also that if $m_2 = 0$, the only solution consists in choosing $m_1^*(m_2) = 0$. According to lemma 2, the function $m_1^*(m_2)$ is a non-decreasing function of m_2, which implies a less than maximum differentiation principle. For high values of b, that is, for a high disutility of connection time, carrier 1 has to increase its departure time in order to capture passengers of market AC.

We are now in position to derive the optimal location of carrier 2 (first stage of the game).

Lemma 3 : The optimal choice of m_2 for carrier 2 is $m_2^* = \frac{1}{2}$.

These three lemmas lead to the following proposition which characterizes completely the subgame perfect equilibrium of the three stage game.

Proposition 1 : For any value of the parameter b such that $b > 1$, the unique subgame perfect equilibrium of the three stage game is given by :

$m_1^* = \frac{b-1}{2(1+b)}, \quad m_2^* = \frac{1}{2}$

$p_1^{AB*}(m_1^*, m_2^*) = \frac{2}{3(1+b)}, p_2^{AB*}(m_1^*, m_2^*) = \frac{4}{3(1+b)}$

$\pi^{1*} = \frac{2\alpha}{9(1+b)}, \pi_2^* = \frac{8\alpha}{9(1+b)}$

These values are sustained by the price functions given by (1.a) and (1.b) in lemma

1 and the best response function relative to departure time of carrier 1 *given by equation (3) in lemma 2.*

According to this result, the difference between departure times is equal to $\frac{1}{1+b}$. It is always smaller than $\frac{1}{2}$ (because $b > 1$) and it tends to zero when b tends to infinity. This result thus illustrates the existence of a "demand side network effect" : at the perfect equilibrium of the three stage game, the location of the leader is $m_2^* = \frac{1}{2}$ and corresponds to the monopoly solution, whereas the location of the follower ($m_1^* = \frac{b-1}{2(1+b)}$) becomes closer to $\frac{1}{2}$ as the disutility of the waiting time at the connection increases.

In order to illustrate the way this demand side network effect is affected by the specified order of decisions, we now turn to the two-stage game where m_1 and m_2 are simultaneously chosen by carriers 1 and 2.

3.2 Simultaneous determination of departure times

In the game where m_1 and m_2 are simultaneously chosen, one has to consider three cases, according to the fact that m_1 is inferior, equal or superior to m_2. First, can a pair (m_1, m_2) with $m_1 > m_2$ be an equilibrium ? Note that in this case, carrier 1 cannot be active on market AC and thus its profit is zero, whereas for any given positive value of m_2, carrier 1 can make a positive profit by choosing $m_1 < m_2$. Thus, for any positive value of m_2, the best reply of carrier 1 is given by lemma 2 and leads to $m_1 < m_2$. When $m_2 = 0$, the best reply of carrier 1 is to choose $m_1 = 0$. But it is easy to check that the best reply of carrier 2 to $m_1 = 0$ is $m_2 = 1$. Indeed, in this case, the profit function of carrier 2 is given by $(2.b)$ for $m_2 \geq m_1$ and is equal to zero when $m_2 = m_1 = 0$. Carrier's 2 profit is increasing in m_2 and reaches its maximum for $m_2 = 1$. It results that $m_1 = m_2 = 0$ cannot be an equilibrium. We have thus eliminated the possibility that $m_1 > m_2$ occurs at equilibrium.

Two cases remain : $m_1 = m_2 > 0$ or $m_1 < m_2$. If $m_1 = m_2$, Bertrand competition prevails and both profits are zero. But this cannot be an equilibrium, since carrier 2 makes a positive profit by choosing $m_2 > m_1$. Thus the only possible equilibrium verifies $m_2 > m_1$. Such an equilibrium exists provided that carrier 2 has no incentive to lower its time departure m_2 under m_1 in order to obtain a monopoly position. The corresponding monopoly profit is bounded by a value which depends positively on the consumers reservation price (u) on market AC. It follows that a deviation towards a monopoly position is unprofitable if this reservation $((u)$ price is not too high. The following lemma characterizes in this case the duopoly equilibrium on market AC.

Lemma 4 : The only equilibrium in departure times of the simultaneous game is given by :

$m_1^* = \frac{3b-1}{3(1+b)}; m_2^* = 1$

It is important to remark that in the simultaneous game, both carriers are active on market AC. Indeed, the only possibility of carrier 2 to eliminate carrier 1 from market

AC is to choose $m_2 = 0$. But this choice is clearly dominated. The difference between departure times at equilibrium is $m_2^* - m_1^* = \frac{4}{3(1+b)}$. It is a decreasing function of b. This expresses the fact that in order to be present on market AC, carrier 1 has to come closer to carrier 2 as b grows.

The following proposition sums up the results relative to the two-stage game.

Proposition 2 : For any value of the parameter b such that $b > 1$, there exists a unique subgame perfect equilibrium given by $m_1^* = \frac{3b-1}{3(b+1)}$ and $m_2^* = 1$. This equilibrium is sustained by the price equilibrium functions given by (1.a) and (1.b). At the subgame perfect equilibrium, the prices are :

$p_1^{AB*}(m_1^*, m_2^*) = \frac{32}{27(1+b)}$; $p_2^{AB*}(m_1^*, m_2^*) = \frac{40}{27(1+b)}$

The corresponding values of profits are :

$\pi_1^* = [\frac{8}{3}]^2 \frac{2\alpha}{27(1+b)}$; $\pi_2^* = [\frac{10}{3}]^2 \frac{2\alpha}{27(1+b)}$

According to proposition 2, the network effect, which is only due to the demand side aspects, prevails again even when m_1 and m_2 are simultaneously determined. If the disadvantage of carrier 1 relative to carrier 2 were exogeneously given, that is, if the cost of the waiting time were independent of m_1, then carrier 2 could exclude carrier 1 from the market for high values of the parameter b. But in fact, in our model, this cost is endogeneous since it results from the location choices m_1 and m_2. Therefore, firm 1 can choose its departure time close to that of carrier 2 in order to reduce the waiting cost. Carrier 2 cannot exclude carrier 1 from the market. This constitutes the network effect arising from demand aspects in airlines. This phenomenon is quite close to that obtained in other models of network competition (see, for instance, Katz and Shapiro (1985, 1986)) : the firm with a network disadvantage is in favor of compatibility, whereas the stronger competitors are reluctant to produce compatible goods. This is in contrast with the usual maximum differentiation principle (see d'Aspremont, Gabszewicz and Thisse (1979)). In fact m_1^* is an increasing function of b, and tends to 1 when b tends to infinity. This property expresses a kind of "minimum differentiation principle". This result has been obtained in the case where competition is restricted to the market AC.

When the complete model is considered, firms also compete on the market AB. The intuition is that if the size of market AB is large relative to that of market AC, carrier 1 may find an advantage, for some values of b, in leaving the market AC. Some trade-off between the attractiveness of market AC measured by the size a of this market and the disadvantage due to the cost of waiting time b is expected to occur.

4 A taxonomy of equilibria in the complete network

We turn now to the complete model where AB and BC are considered as final goods as well as AC. The model thus involves both vertical and horizontal differentiation aspects.

4.1 Demand configurations

When the complete network is considered, one cannot exclude a priori the case where $m_1 > m_2$. Thus two cases must be distinguished according to whether $m_2 > m_1$ or $m_2 < m_1$.

4.1.1 First case : $m_2 > m_1$

The consumer indifferent between 1 and 2 on market AB is located at :

(4) $m^{AB} = \frac{(p_2^{AB} - p_1^{AB})}{2(m_2 - m_1)} + \frac{m_1 + m_2}{2}$

whereas on market AC, the consumer indifferent between $1-2$ and $2-2$ is located at

(5) $m^{AC} = \frac{p_2^{AB} - p_1^{AB}}{2(m_2 - m_1)} + \frac{m_1 + m_2}{2} - b\frac{m_2 - m_1}{2}$

According to the values of m^{AB} and m^{AC}, various configurations of demand may prevail. Whatever the demand, recall that all passengers fly with carrier 2 on the leg BC of the journey. Thus carrier 2 obtains a fixed profit $(\alpha + \beta)p_2^{BC}$ from its activity on BC. As in the previous section, we only need to consider the variable part of carrier 2's profit. The various configurations of demand lead to the following partition of the price space (p_1, p_2) :

Region (1) : $m^{AB} < 0$ and $m^{AC} < 0$: no demand is addressed to carrier 1 on any market. The profits are $\pi_1 = 0$ and $\pi_2 = p_2^{AB}(1 + \alpha)$.

Region (2) : $m^{AB} \in [0, 1]$ and $m^{AC} < 0$: both carriers share the demand on market AB and carrier 2 serves alone the market AC. Profits are $\pi_1 = p_1^{AB} m^{AB}$ and $\pi_2 = p_2^{AB}(1 + \alpha - m^{AB})$.

Region (3) : $m^{AB} \in [0, 1]$ and $m^{AC} \in [0, 1]$: both firms are active on both markets. Profits are $\pi_1 = p_1^{AB}(m^{AB} + \alpha m^{AC})$ and $\pi_2 = p_2^{AB}[1 - m^{AB} + \alpha(1 - m^{AC})]$.

Region (3') : $m^{AB} > 1$ and $m^{AC} < 0$: carrier 1 is alone on market AB and carrier 2 is alone on AC. Profits are $\pi_1 = p_1^{AB}$ and $\pi_2 = \alpha p_2^{AB}$.

Region (4) : $m^{AB} > 1$ and $m^{AC} \in [0, 1]$: carrier 1 is alone on market AB and shares AC with its rival. Profits are $\pi_1 = p_1^{AB}(1 + \alpha m^{AC})$ and $\pi_2 = p_2^{AB}\alpha(1 - m^{AC})$.

Region (5) : $m^{AB} > 1$ and $m^{AC} > 1$: carrier 1 has a monopoly position on AB and AC. In this case, $\pi_1 = p_1^{AB}(1 + \alpha)$ and $\pi_2 = 0$.

Suppose now that p_1^{AB} is fixed and let p_2^{AB} vary from 0 to ∞. Low values of p_2^{AB} correspond to region (1) until we have $m^{AB} = 0$. For higher values of p_2^{AB}, the pair (p_1^{AB}, p_2^{AB}) is in region (2). When p_2^{AB} continues to increase, two cases may prevail according to whether b is smaller or greater than $\frac{2}{m_2 - m_1}$: either one crosses first the line $m^{AC} = 0$ ($b < \frac{2}{m_2 - m_1}$) and the next region is region (3) ; or one crosses first the line $m^{AB} = 1$ ($b > \frac{2}{m_2 - m_1}$) : in this case, the next region is region (3'). Then finally, for higher values of p_2^{AB}, regions (4) and (5) are achieved. Thus, for a given p_1^{AB} and for increasing values of p_2^{AB}, the order of the five regions is (1), (2), (3), (4), (5) if $b < \frac{2}{m_2 - m_1}$.

This order is (1), (2), (3'), (4), (5) if $b > \frac{2}{m_2 - m_1}$. Note that if $b = \frac{2}{m_2 - m_1}$, regions (3) and (3') disappear.

4.1.2 Second case : $m_2 < m_1$.

In this case, the demand addressed to carrier 1 on market AC is zero and the consumer indifferent between carriers 1 and 2 on market AB is located at m^{AB}. Thus three configurations may prevail, according to the value of m^{AB}. If $m^{AB} < 0$, we have $D_1^{AB} = 1$ and $D_2^{AB} = 0$. Profits are $\pi_1 = p_1^{AB}$ and $\pi_2 = \alpha p_2^{AB}$. If $0 \leq m^{AB} \leq 1$, we have $D_1^{AB} = 1 - m^{AB}$ and $D_2^{AB} = m^{AB}$. Profits are $\pi_1 = p_1^{AB}(1 - m^{AB})$ and $\pi_2 = p_2^{AB}(m^{AB} + \alpha)$. If $m^{AB} > 1$, we have $D_1^{AB} = 0$ and $D_2^{AB} = 1$; profits are $\pi_1 = 0$ and $\pi_2 = p_2^{AB}(1 + \alpha)$.

4.2 Price competition

According to the previous discussion on demand configurations, we have to distinguish the cases $m_2 > m_1$ and $m_2 < m_1$.

4.2.1 Case : $m_2 > m_1$:

The following lemma reduces the set of possible equilibrium prices.

Lemma 5 : If $m_2 > m_1$, price equilibria cannot belong to regions (1), (3), (3') or (5). Moreover, if there exists a price equilibrium in region (2), it is given by :

(6.a) $p_1^{AB*} = \frac{(m_2 - m_1)}{3}[2(1 + \alpha) + m_1 + m_2]$;

(6.b) $p_2^{AB*} = \frac{(m_2 - m_1)}{3}[4(1 + \alpha) - (m_1 + m_2)]$.

If there exists a price equilibrium in region (4), it is given by :

(7.a) $\hat{p}_1^{AB} = \frac{m_2 - m_1}{3\alpha}[4 + 2\alpha + \alpha m_1(1 + b) + \alpha m_2(1 - b)]$

(7.b) $\hat{p}_2^{AB} = \frac{m_2 - m_1}{3\alpha}[2 + 4\alpha - \alpha m_1(1 + b) - \alpha m_2(1 - b)]$

The crucial result of this lemma is that both firms cannot simultaneously be active on both markets. This means that a price equilibrium cannot belong to region (3). By itself, this is an interesting result since it asserts that the trade off between the attractiveness of market AC resulting from its size α and its disadvantage for carrier 1 due to the waiting cost parameter b leads to the fact that carriers avoid to be simultaneously active on both markets.

When the price equilibrium belongs to region (4), carrier 1 is active on market AC because it offers a sufficiently low price to compensate its network disadvantage. But then, price competition is very intense on market AB. Since carrier 2 has no advantage on market AB, it prefers to give up serving consumers of this market by choosing a high price $\hat{p}_2^{AB}$ and to concentrate its activity on market AC.

When the price equilibrium belongs to region (2), entering market AC is too costly for carrier 1. Thus, no demand is addressed to this carrier on this market, which is then served by carrier 2 only. This means that passengers of market AC do not consider that the price p_1^{AB} proposed by carrier 1 on the leg AB of their journey is attractive enough to compensate the disadvantage of the waiting time.

Characterizing the parameters values for which an equilibrium exists either in region (2) or in region (4) is a difficult problem. However, the proof of lemma 6 gives an idea of the steps one has to follow in order to obtain such a characterization.

Lemma 6 : If a price equilibrium exists, there is a threshold $\alpha(b)$ such that the price equilibrium belongs to region (4) if the size α of market AC is greater than $\alpha(b)$, and to region (2) if α is lower than $\alpha(b)$.

The intuition behind lemma 6 is clear. Price competition on the complete network involves two opposite effects : a competitive effect and an attractiveness effect. According to the respective values of α and b, one of these two effects dominates and leads to an equilibrium in region (2) or in region (4). The first effect pushes carrier 1 out of market AC in order to concentrate its activity on market AB, avoiding thus a tough price competition on market AB. However, if the size α of market AC is high, the second effect is present, according to which carrier 1 has an incentive to lower its price in order to remain active on market AC. Lemma 6 shows that there exists a function $\alpha(b)$ such that, when the size of market AC is sufficiently high (that is, $\alpha > \alpha(b)$), this second effect, due to the relative market sizes, outweighs the first effect. Finding the analytical expression of $\alpha(b)$ is a boring task which does not add much to the economic intuition of the result.

4.2.2 Case : $m_2 < m_1$

In this case, our assumptions on the functionning of the flights during the unit time period imply that carrier 1 is absent from market AC. However, by contrast with the case examined in section 3, carrier 1 and 2 compete now on market AB. Since carrier 2 takes into account its global profit on both markets when it determines its price, both prices will depend on α at equilibrium. Straightforward computations lead to the following expressions for equilibrium prices :

(8.a) $\underline{p}_1^{AB} = \frac{m_1 - m_2}{3}[4 + 2\alpha - (m_1 + m_2)]$

(8.b) $\underline{p}_2^{AB} = \frac{m_1 - m_2}{3}[2 + 4\alpha + (m_1 + m_2)]$

The corresponding profits are :

(9.a) $\underline{\pi}_1(m_1, m_2) = \frac{m_1 - m_2}{18}[4 + 2\alpha - (m_1 + m_2)]^2$

(9.b) $\underline{\pi}_2(m_1, m_2) = \frac{m_1 - m_2}{18}[2 + 4\alpha + (m_1 + m_2)]^2$

Let us now sum up the results relative to the price competition stage of the game. When the departure time m_2 of carrier 2, which has a network advantage, is higher than the departure time m_1 of its competitor, two types of price equilibria can emerge.

In the first one (region (2)), passengers on market AC never choose carrier 1, whereas competition prevails on market AB. In the second one (region (4)), carrier 2 exploits its network advantage on market AC, which allows a high price p_2^{AB}, which in turn dicourages consumers of market AB from flying with this carrier. It is precisely in this case that we could obtain a "less than maximum differentiation" result concerning the choice of departure times. When $m_2 < m_1$, competition between carriers 1 and 2 prevails only on market AB, and one can expect a classical result of maximum differentiation for the choice of time locations.

4.3 Equilibria in departure times

At the first stage of the game, carriers choose simultaneously their departure times. The complete characterization of the subgame perfect equilibria is very complex. We limit ourselves to identify the three possible types of equilibria in departure times. The following proposition gives these three equilibria.

Proposition 3 : Three types of equilibria in departure times may occur :

(i) $m_1 = 1$ *and* $m_2 = 0$. *In this case, carrier* 1 *cannot be active on market* AC *because of its departure time. Both carriers compete on market* AB. *This case corresponds to an "eviction by departure times" (EDT).*

(ii) $m_1 = 0$ *and* $m_2 = 1$. *In this case, price equilibrium is in region (2) and carrier* 1 *is again evicted from market* AC, *due to the structure of the price equilibrium in this region. Both carriers serve again the market* AB. *This case corresponds to an "eviction by price competition" (EPC).*

(iii) $m_1 = \frac{b}{1+b} - \frac{\alpha+4}{3\alpha(1+b)}$ *and* $m_2 = 1$. *In this case, price equilibrium is in region (4). Both carriers are active on market* AC *whereas carrier* 2 *serves the market* AB *alone. This equilibrium corresponds to a "network equilibrium" (NE).*

This proposition allows some important conclusions. First, for any value of the parameters α and b, one possible equilibrium leads carrier 2, who has a network advantage, to choose its departure time at $m_2 = 0$, so that carrier 1 has no possibility to attract consumers of market AC. This, in turn, leads carrier 1 to choose $m_1 = 1$. At this EDT equilibrium, one can easily compute the equilibrium prices from (8.a) and (8.b) :

$\underline{p}_1^{AB} = \frac{1}{3}(3 + 2\alpha)$; $\underline{p}_2^{AB} = \frac{1}{3}(3 + 4\alpha)$

and the corresponding profits from (9-a) and (9-b) :

$\underline{\pi}_1 = \frac{1}{6}(3 + 2\alpha)^2$; $\underline{\pi}_2 = \frac{1}{6}(3 + 4\alpha)^2$

In this case, the eviction of carrier 1 from market AC is obtained by carrier 2 without incurring any cost in terms of price competition.

Second, for some values of the parameters α and b, carrier 2 can reach the same result by a tougher price competition, rather than by the previous choice of departure times. This corresponds to the EPC equilibrium where $m_1 = 0$ and $m_2 = 1$. The corresponding equilibrium prices are obtained from (6.a) and (6.b), and lead exactly to the same values

than the EDT equilibrium, namely : $p_1^{AB*} = \underline{p}_1^{AB}$ and $p_2^{AB*} = \underline{p}_2^{AB}$. Note however, that the corresponding profits π_1^* and π_2^* are smaller than the profits $\underline{\pi}_1$ and $\underline{\pi}_2$ obtained in the EDT equilibrium. Indeed, one can easily verify that $\pi_1^* = \frac{1}{3}\underline{\pi}_1$ and $\pi_2^* = \frac{1}{3}\underline{\pi}_2$. Thus the EPC equilibrium is Pareto dominated by the EDT equilibrium. In an intertemporal setting, a renegotiation argument would lead to the adoption of the EDT equilibrium rather than to the EPC equilibrium. But without such an argument, one cannot abandon the EPC equilibrium. Note that in order to obtain an EPC equilibrium, one has to check that the corresponding prices belong to region (2), which means that they satisfy both conditions : $0 < m^{AB} < 1$ and $m^{AC} < 0$. This gives two necessary conditions : $\alpha < \frac{3}{2}$ and $b > \frac{2\alpha+3}{3}$.

Third, the network equilibrium (NE) exhibits a less than maximum differentiation principle. The discrepancy between departure times is equal to : $m_2 - m_1 = \frac{4(1+\alpha)}{3a(1+b)}$. It tends to zero when b tends to infinity. Note that when α tends to infinity, we obtain the same value for the limit of $m_2 - m_1$ than in the case of market AC alone examined in section 3.

In order to ensure that the NE equilibrium indeed leads to prices in region (4) (that is $0 < m^{AC} < 1$ and $m^{AB} > 1$), some necessary conditions have to be satisfied. Substituting for the convenient values of m_1 and m_2 in equations (7.a) and (7.b), one obtains the following equilibrium prices and profits :

$\hat{p}_1^{AB} = \frac{32}{27}\frac{(1+\alpha)^2}{\alpha^2}(1+b) \quad \hat{p}_2^{AB} = \frac{40}{27}\frac{(1+\alpha)^2}{\alpha^2(1+b)}$

$\hat{\pi}_1^{AB} = \frac{128}{243}\frac{(1+\alpha)^3}{\alpha^2(1+b)} \quad \hat{\pi}_2^{AB} = \frac{200}{243}\frac{(1+\alpha)^3}{\alpha^2(1+b)}$

The necessary conditions for $\hat{p}_1^{AB}$ and $\hat{p}_2^{AB}$ belonging to region (4) are thus $b > 5$ and $\alpha > \frac{5}{4}$. This means that when carrier 2 has a big network advantage ($b > 5$) and when the size of market AC is large relative to that of market $AB(\alpha > \frac{5}{4})$, one possible equilibrium consists in carrier 2 concentrating its activity on market AC with a high price $\hat{p}_2^{AB}$. Moreover, in order to be active on market AC, carrier 1 has to become more compatible with carrier 2, and this is obtained by reducing the discrepancy between departure times, even if this leads to a more intense price competition. This constitutes the essence of the "demand side network effect" in airlines.

5 Conclusion

Some general principles emerge from the above analysis. First, as in usual network models, firms endowed with asymmetric networks or technologies receive different incentives to produce compatible products : the stronger firm generally dislikes compatibility, whereas the weaker one prefers it. However, this result is obtained here through a mechanism that differs from those emphasized in the literature. It is not the cost structure which here explains departure time choices by carriers, but the joint demand aspects inherent either to the hub and spoke organization of the domestic airline networks, or in an international context, to different regulatory settings. When consumers face high

connecting costs, the firm with the smallest network is either evicted or constrained to choose its departure time close to its rival. Second, the interactions between markets of elementary direct and indirect flights are complex. When the markets of elementary components are not considered by themselves, i. e., when these goods are not considered as final goods only, this network effect prevails. In other words, the carrier with a competitive disadvantage always benefits from being active on the market, which is possible if it chooses a departure time very close to that of its rival. However, this minimum differentiation principle intensifies the price competition. It implies that when the complete network is taken into account, this strategy may increase competition on some elementary parts of the network. In some cases, a firm might prefer to leave one market in order to concentrate on other markets and to benefit from a strategy of maximum differentiation on these markets.

These results show that interactions between airline markets of a complex network may lead to paradoxical results with regard to usual competitive behavior in a context of differentiated products. This may have important implications for competition policy in airlines (see, for instance, Morrisson and Winston (1986), Encaoua (1991) and Encaoua and Perrot (1992) for a discussion of policy problems in the European context). In particular, it highlights the fact that deregulating a route which belongs to a complex network could affect the equilibrium on all other routes of the network. This is because demand behavior and compatibility choices associated to this joint demand phenomenon induce network effects that link markets together. Further analysis is needed to characterize the magnitude of these network effects in more general network structure and under various regulatory assumptions. The welfare implications of pricing strategy in different types of networks will be investigated in a future paper.

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Appendix

Proof of lemma 1 :

The profit functions $\pi_1(p_1^{AB}, p_2^{AB})$ and $\pi_2(p_1^{AB}, p_2^{AB})$ are globally continuous and locally concave respectively with respect to p_1^{AB} and p_2^{AB} in each of the three regions. Consider a pair (p_1^{AB}, p_2^{AB}) such that, given m_1 and m_2, we have $m^{AC} > 1$. Then $\pi_2 = 0$, which implies that it would be profitable for carrier 2 to decrease p_2 in order to obtain an outcome of the form (ii). Therefore an outcome in which carrier 1 is the only carrier active on AC cannot be an equilibrium.

The same argument, however, cannot be applied to region (i) (where carrier 1 makes no profit) because of the asymmetry of the networks : firm 2 may in some cases evict firm 1 out of the market.

It results that only equilibrium configurations in regions (i) and (ii) need to be considered. Two possibilities remain : either there exists an equilibrium at the frontier between (i) and (ii), either there is an interior equilibrium in region (ii).

Consider now a pair of prices (p_1^{AB}, p_2^{AB}) on the frontier between (i) and (ii). If $p_1^{AB} > 0$, carrier 1 can lower its price in order to obtain a positive demand. Thus, an equilibrium can be on the frontier between regions (i) and (ii) only if carrier 2 can profitably exclude carrier 1 from the market even when carrier 1 sets $p_1^{AB} = 0$. A price equilibrium does indeed exist on this frontier if firm 2 cannot do better in region (ii) when $p_1^{AB} = 0$. This result emerges when the sign of the derivative of carrier 2's profit at the frontier between regions (i) and (ii) is non positive.

The profit of carrier 2 in region (ii) is given by : $\pi_2 = \alpha(1 - m^{AC})p_2^{AB}$ and its derivative has the same sign as $2 + m_1 + m_2 - b(m_2 - m_1)$. Thus, two types of equilibria may arise according to the value taken by b relative to m_1 and m_2.

- If $b \geq \frac{2+m_1+m_2}{m_2-m_1}$ (we will refer to this inequality as condition C), the derivative is negative and there is a price equilibrium at the frontier of regions (i) and (ii). Equilibrium prices are :

(A1-1) $\quad p_1^{AB*}(m_1, m_2) = 0$

(A1-2) $\quad p_2^{AB*}(m_1, m_2) = (m_2 - m_1)[b(m_2 - m_1) - (m_1 + m_2)]$.

- If $b < \frac{2+m_1+m_2}{m_2-m_1}$ (condition C'), the value of m^{AC} is $m^{AC}(p_1^{AB*}, p_2^{AB*}) = \frac{1}{6}[2 + m_1 + m_2 - b(m_2 - m_1)]$, which belongs to $]0, 1[$. Price equilibrium belongs to the interior of region (ii). Equilibrium prices, derived from the first order conditions $\partial p_i / \partial p_i^{AB} = 0$ for $i = 1, 2$, where profits are taken according to their expression in region (ii), are :

(A1-3) $p_1^{AB*}(m_1, m_2) = \frac{m2-m1}{3}[2 + m_1 + m_2 - b(m_2 - m_1)]$

(A1-4) $p_2^{AB*}(m_1, m_2) = \frac{m2-m1}{3}[4 - (m_1 + m_2) + b(m_2 - m_1)]$

which are (1.a) and (1.b) in the text. Q.E.D

Proof of lemma 2 :

Note first that, since the demand functions are continuous, the profit functions $\pi_1(m_1, m_2)$ and $\pi_2(m_1, m_2)$ are continuous on $[0, 1]$. We first study the shapes of these function in regions (C) and (C') (see proof of lemma 1 for the definition of C and C'). The derivative $\frac{\partial \pi_1}{\partial m_1}(m_1, m_2)$ in region (C') is given by :

(A2-1) $\frac{\partial \pi_1}{\partial m_1}(m_1, m_2) = \frac{\alpha}{18}[2 + m_1(1 + b) + m_2(1 - b)][m_2(1 + 3b) - 3m_1(1 + b) - 2]$.

The first bracket of (A2-1) is proportional to the price $p_1^{AB*}(m_1, m_2)$ given in (1-a); hence, it is positive. It follows that the sign of $\frac{\partial \pi_1}{\partial m_1}(m_1, m_2)$ is the same as the sign of the second bracket of (A2-1). The second order derivative $\frac{\partial^2 \pi_1}{\partial m_1^2}(m_1, m_2)$ is proportional to $m_2[3b - 1] - 3m_1(1 + b) - 4$. The sign of this expression is easily determined :

$m_1 > \frac{m_2(3b-1)-4}{3(1+b)} \Leftrightarrow \frac{\partial^2 p_1(m_1,m_2)}{\partial m_1^2} < 0$

Note that, for all values of m_2 in $[0, 1]$, we have :

(i) $\frac{m_2(3b-1)-4}{3(1+b))} < 1$

(ii) $\frac{m_2(3b-1)-4}{3(1+b))} > \frac{m_2(b-1)-2}{b+2)}$

The threshold $\frac{m_2(b-1)-2}{b+2}$ defines the frontier between regions (C) and (C') :

$m_1 < \frac{m_2(b-1)-2}{b+2} \Leftrightarrow (m_1, m_2) \in (C) \Leftrightarrow b > \frac{2+m_1+m_2}{m_2-m_1}$.

In region (C), $\pi_1(m_1, m_2)$ is equal to zero. Thus we have the following properties :

(A2.2) $m_1 \in [0, \frac{m_2(b-1)-2}{b+2}] \Rightarrow \pi_1(m_1, m_2) = 0$

(A2.3) $m_1 \in]\frac{m_2(b-1)-2}{b+2}, \frac{m_2(3b-1)-4}{3(1+b)}] \Rightarrow \pi_1(m_1, m_2)$ is convex in m_1.

(A2.4) $m_1 \in]\frac{m_2(3b-1)-4}{3(1+b)}, 1] \Rightarrow \pi_1(m_1, m_2)$ is concave in m_1.

Moreover, the first derivative $\frac{\partial \pi_1(m_1,m_2)}{\partial m_1}$ evaluated at $m_1 = \frac{m_2(3b-1)-4}{3(1+b)}$ is positive and this first derivative evaluated at $m_1 = 1$ is negative. For a given value of m_2 in $[0, 1]$, the maximum of $\pi_1(m_1, m_2)$ is achieved for $m_1(m_2) = \frac{m_2(1+3b)-2}{3(1+b)}$. Note that this value belongs to the subregion of (C') in which $\pi_1(m_1, m_2)$ is concave. Note also that this value $m_1(m_2)$ is smaller than m_2. Finally, note that $m_1(m_2) > 0$ is equivalent to $m_2 > \frac{2}{1+3b}$. Q.E.D.

Proof of lemma 3 :

In order to derive the profit function $\pi_2(m_1(m_2), m_2) \equiv \tilde{\pi}_2(m_2)$, we have to compare the two thresholds involved respectively in the best reply function $m_1(m_2)$, (that is, $\frac{2}{1+3b}$), and in the profit function of carrier 2, (namely, $\frac{2+m_1(1+b)}{b-1}$). It is easy to check that for all m_1, we have : $\frac{2+m_1(1+b)}{b-1} > \frac{2}{1+3b}$. It results that the function $\tilde{\pi}_2(m_2)$ has the following configuration :

$$\tilde{\pi}_2(m_2) = \begin{cases} \frac{\alpha}{18} m_2[4 + m_2(b-1)]^2 & \text{if } m_2 \leq \frac{2}{1+3b} \\ \\ \frac{4\alpha}{243(1+b)}(m_2+1)[7-2m_2]^2 & \text{if } m_2 > \frac{2}{1+3b} \end{cases}$$

Note that it is impossible to have simultaneously $m_2 > \frac{2+m_1(1+b)}{b-1}$ and $m_2 < \frac{2}{1+3b}$. It is easily verified that the global maximum of the function $\tilde{\pi}_2(m_2)$ is reached for $m_2 = \frac{1}{2}$. Q.E.D.

Proof of lemma 4 :

When $m_2 > m_1$, the derivative of carrier 2's profit is :

(A4.1) $\frac{\partial \pi_2}{\partial m_2}(m_1, m_2) = \frac{\alpha}{18}[4 - (m_1 + m_2) + b(m_2 - m_1)][3m_2(b-1) - m_1(3b-1) + 4]$

The first bracket of (A4.1) is proportionnal to π_2^{AB*} and is thus positive. Hence, $\frac{\partial \pi_2}{\partial m_2}(m_1, m_2)$ has the same sign as $[3m_2(b-1) - m_1(3b-1) + 4]$. This expression is positive for $m_2 > m_1$. The second order derivative $\frac{\partial 2\pi_2}{\partial m_2^2}(m_1, m_2)$ has the sign of the following expression $3(b-1)m_2 - m_1(1+3b) + 8$. As $m_2 > m_1$, this expression is always positive. Thus, $\pi_2(m_1, m_2)$ is an increasing convex function in region (C'). In region (C), the derivative of $\pi_2(m_1, m_2)$ is $2\alpha[(b-1)m_2 - bm_1]$. The second order derivative is $2\alpha(b-1)$ which is positive since $b > 1$. The function $\pi_2(m_1, m_2)$ is therefore also convex is region (C). Moreover, $\pi_2(m_1, m_2)$ is an increasing function of m_2 in this region. Using lemma 2, we thus obtain the equilibrium in departure times. It is given by :

(A4.2) $m_1^* = \frac{3b-1}{3(b+1)}, \quad m_2^* = 1.$ Q.E.D.

Proof of lemma 5 :

First, note that in region (1), $\pi_1 = 0$ and $\pi_2 = (1+\alpha)p_2^{AB}$. If $p_2 > 0$, firm 1 can lower its price in order to obtain a positive market share either on market AB or on market AC. The same argument applies to region (5) where $\pi_2 = 0$. Thus equilibria cannot be in regions (1) and (5).

In region (3'), which appears when $b > \frac{2}{m_2 - m_1}$, $\pi_1(p_1^{AB}, p_2^{AB})$ is increasing in p_1^{AB} and $\pi_2(p_1^{AB}, p_2^{AB})$ is increasing in π_2^{AB}. It results that if an equilibrium were in region (3'), carrier 1 would choose its price at the frontier between the regions (3') and (2), that is $p_1^{AB} = p_2^{AB} + (m_2 - m_1)[m_1 + m_2 - 2]$, and carrier 2 would choose its price at the frontier between (3') and (4), that is $p_2^{AB} = p_1^{AB} - (m_2 - m_1)[m_1 + m_2 - b(m_2 - m_1)]$. This is impossible unless $b(m_2 - m_1) = 2$.

Let us now eliminate the possibility of an equilibrium in region (3) when $b < \frac{2}{m_2 - m_1}$. First, the derivative of the profit function $\pi_2(p_1^{AB}, p_2^{AB}) \equiv p_2^{AB} D_2(p_1^{AB}, p_2^{AB}$ with respect to p_2^{AB} is negative at the frontier between regions (2) and (3) when p_1^{AB} is held at the value p_1^{AB*} which maximizes the profit function of carrier 1 in region 2. Second, the derivative of the profit function $\pi_2(p_1^{AB}, p_2^{AB})$ with respect to p_2^{AB} is lower in region (3) than in region (2). Indeed, this derivative is equal to $D_2 - \frac{(1+\alpha)p_2}{2(m_2 - m_1)}$, where D_2 is taken according to its value in region (3), whereas in region (2) it is equal to $D_2 - \frac{p_2}{2(m_2 - m_1)}$ where D_2 is taken according to its value in region (2). Since D_2 in region (3) is lower than D_2 in region (2), it results that the profit function of carrier 2 is decreasing in region (3).

Moreover, the maximum of $\pi_2(p_1^{AB}, p_2^{AB})$ in region (2) is interior to this region. Thus, if price equilibria exist, they necessarily belong to regions (2) or (4). In each of these regions, they are easily obtained by the first order conditions since the profit functions p_i, as defined in regions (2) and (4), are concave in $p_i (i = 1, 2)$. Q.E.D.

Proof of lemma 6 :

According to lemma 5, if a price equilibrium exists in region (2), it is given by equations (6.a) and (6.b). Corresponding profits are :

(7.a) $\pi_1^* = \frac{m_2 - m_1}{18}[2(1+\alpha) + m_1 + m_2]^2$

(7.b) $\pi_2^* = \frac{m_2 - m_1}{18}[4(1+\alpha) - (m_1 + m_2)]^2$

In order to have an equilibrium in region (2), we must eliminate the possibility of profitable deviations of carrier 1 and carrier 2 from region (2) towards regions (4), (5), (3') (when $b > \frac{2}{m_2 - m_1}$) and (3) (when $b < \frac{2}{m_2 - m_1}$). The possible deviations of carrier 1 necessarily involve a price decrease from p_1^{AB*} whereas deviations of carrier 2 involve either a price decrease in order to reach region (1) or a price increase in order to reach regions (3), (3') or (4). Consider first a deviation of carrier 1 from p_1^* to a price $\underline{p}_1^{AB}$ such that $D_1^{AB}(\underline{p}_1^{AB}, p_2^{AB*}) = 1$ and $D_1^{AC}(\underline{p}_1^{AB}, p_2^{AB*}) = \alpha$, that is, $m^{AB}(\underline{p}_1^{AB}, p_2^{AB*}) > 1$ and $m^{AC}(\underline{p}_1^{AB}, p_2^{AB*}) = 1$. Such a deviation leads to a pair of prices in region (5). It is possible if and only if $p_2^{AB*} > p_2^\circ$ where p_2° is given by $m^{AC} = 1$ and $p_1^{AB} = 0$. Note that the price $\underline{p}_1^{AB}$ is positive if and only if $p_2^{AB*} > p_2^\circ$. This deviation from region (2) to region (5) is profitable for carrier 1 if and only if : $\pi_1(\underline{p}_1^{AB}, p_2^{AB*}) > \pi_1^*$ which is equivalent to the following inequality (A) : $\frac{10}{9}(m_2 - m_1)(1+\alpha)^2 - \frac{(m_2^2 - m_1^2)}{18}(m_1 + m_2) - (1+\alpha)[\frac{5}{9}(m_2^2 - m_1^2) - (m_1 + m_2) + b(m_2 - m_1) + 2] > 0$.

The left hand side is a convex function of $(1+\alpha)$ which has two roots with opposirte signs. Denote by $A^5(b)$ the positive root. If $\alpha > A^5(b)$, the inequality (A) is satisfied. Moreover, it is easy to check that if $\alpha > A^5(b), p_2^{AB*} > p_2^\circ$. Thus if $\alpha > A^5(b), \underline{p}_1^{AB}$ is a possible and profitable deviation of carrier 1 from region (2) to region (5). Thus the pair (p_1^{AB*}, p_2^{AB*}) cannot be an equilibrium.

Similar analysis of deviations of carrier 1 from region (2) to respectively regions (3') and (4) show that there exist two thresholds for α, respectively $A^3(b)$ and $A^4(b)$ such that if $b > \frac{2}{m_2 - m_1}$ and $\alpha > A^3(b)$ (resp. $A^4(b)$), then carrier 1 can profitably deviate from region (2) to region (3') (resp. (4)).

It results that if $b > \frac{2}{m_2 - m_1}$ and $\alpha < A(b) = \text{Min}(A^5(b), A^4(b), A^3(b))$, carrier 1 has no profitable and possible deviations from region (2).

Deviations of carrier 2 to regions (1), (3') or (4) lead to similar results : there exists a threshold $B(b) = \text{Min}(B^1(b), B^3(b), B^4(b))$ such that if $\alpha < B(b)$, carrier 2 has no profitable deviation from region (2).

Together with lemma 5, this insures that if an equilibrium exists, it is necessarily in region (4) if $\alpha > \underline{\alpha}(b) \equiv \text{Min}(A(b), B(b))$, and in region (2) if $\alpha < \underline{\alpha}(b) \equiv \text{Min}(A(b), B(b))$. Q.E.D.

Proof of proposition 3 :

When $m_1 > m_2$, the equilibrium prices are $\underline{p}_1$ and $\underline{p}_2$ given by (8.a) and (8.b) and the corresponding profits are

$\underline{\pi}_1(m_1, m_2) = \frac{m_1 - m_2}{18}[4 + 2\alpha - (m_1 + m_2)]^2$

$\underline{\pi}_2(m_1, m_2) = \frac{m_1 - m_2}{18}[2 + 4\alpha + m_1 + m_2]^2$.

When $m_1 < m_2$ and the equilibrium is in region (2) the prices are p_1^* and p_2^* given by (6.a) and (6.b) and the corresponding profits are :

$\pi_1^*(m_1, m_2) = \frac{m_2 - m_1}{18}[2(1 + \alpha) + m_1 + m_2]^2$

$\pi_2^*(m_1, m_2) = \frac{m_2 - m_1}{18}[4(1 + \alpha) - (m_1 + m_2)]^2$

When $m_1 < m_2$ and the equilibrium is in region (4), the prices are $\hat{p}_1$ and $\hat{p}_2$ given by (7.a) and (7.b) and the corresponding profits are :

$\hat{\pi}_1(m_1, m_2) = \frac{m_2 - m_1}{18\alpha}[4 + 2\alpha + \alpha m_1(1 + b) + \alpha m_2(1 - b)]^2$

$\hat{\pi}_2(m_1, m_2) = \frac{m_2 - m_1}{18\alpha}[2 + 4\alpha - \alpha m_1(1 + b) - \alpha m_2(1 - b)]^2$

Both $\pi_2^*(m_1, m_2)$ and $\hat{\pi}_2(m_1, m_2)$ are increasing function of m_2 over $[m_1, 1]$ and reach their maximum at $m_2 = 1$. $\underline{\pi_2}(m_1, m_2)$ is decreasing in m_2 and is maximum when $m_2 = 0$. Thus if the equilibrium price when $m_1 < m_2$ is in region (2), the best reply function of carrier 2 to the location m_1 of carrier 1 is given by :

$$m_2(m_1) \begin{cases} = 0 & \text{if } \underline{\pi}_2(m_1, 0) > \pi_2^*(m_1, 1) \quad \text{(condition (H))} \\ \\ = 1 & \text{if } \underline{\pi}_2(m_1, 0) < \pi_2^*(m_1, 1) \end{cases}$$

If the equilibrium price when $m_1 < m_2$ is in region (4), the best reply function of carrier 2 to the location m_1 of carrier 1 is given by :

$$m_2(m_1) \begin{cases} = 0 & \text{if } \underline{\pi}_2(m_1, 0) > \hat{\pi}_2(m_1, 1) \quad \text{(condition (K))} \\ \\ = 1 & \text{if } \underline{\pi}_2(m_1, 0) < \hat{\pi}_2(m_1, 1) \end{cases}$$

Condition (H) can be written as :

$m_1[2 + 4\alpha + m_1]^2 > \frac{1 - m_1}{3}[4(1 + \alpha) - (m_1 + 1)]^2$

and condition (K) as :

$m_1[2(1 + 2\alpha) + m_1]^2 > \frac{1 - m_1}{3\alpha}[2 + 3\alpha + \alpha b - \alpha m_1(1 + b)]^2$

It follows in particular that $m_2(1) = 0$ and $m_2(0) = 1$.

A similar analysis of the profit function $\pi_1(m_1, m_2)$ shows that if the equilibrium prices when $m_1 < m_2$ belong to region (2), the best reply function of carrier 1 is :

$$m_1(m_2) \begin{cases} = 1 & \text{if } \underline{\pi}_1(1, m_2) > \pi_1^*(0, m_2) \quad \text{(condition (L))} \\ \\ = 0 & \text{if } \pi_1^*(0, m_2) > \underline{\pi}_1(1, m_2) \end{cases}$$

When the equilibrium prices belong to region (4), the best reply function of carrier 1 is :

$m_1(m_2) = \frac{\alpha m_2(1 + 3b) - 2(\alpha + 2)}{3\alpha(1 + b)}$

Condition L expresses as :

$(1 - m_2)[3 + 2\alpha - m_2]^2 > \frac{m_2}{3}[2(1 + \alpha) + m_2]^2$

It is clear in particular that the best response to $m_2 = 0$ is $m_1 = 1$ (that is, conditions H, K, L are simultaneously verified). The best response to $m_2 = 1$ is $m_1 = 0$ if equilibrium prices belong to region (2) and $m_1 = \frac{3\alpha b - \alpha - 4}{3\alpha(1+b)}$ if the equilibrium prices belong to region (4). These possibilities correspond to the three types of equilibria which appear in proposition 3. Q.E.D.