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A NOTE ON EQUILIBRIUM IN PRICE-QUALITY COMPETITION

By Nicholas S. Economides

*I. INTRODUCTION**

In markets of differentiated products the questions of existence of (non-cooperative) equilibria and of the pattern of production at equilibrium are of paramount importance. Here we consider a market for quality differentiated products. Let there be a spectrum $[q_l , q_h]$ of quality differentiated goods of the same generic type. Let consumers have the same direction of preferences (they all prefer a higher quality to a lower quality) but let them have differing valuations of the attribute q[1]. Mussa and Rosen (1978) compared the equilibrium patterns of production in a market for quality differentiated products under monopoly to those under perfect competition[2]. Existence of equilibrium and pricing in oligopoly remained open questions.

Any attempt to describe the pattern of pricing and production in this market in an oligopolistic setting requires the formulation of a strategic framework in which competition takes place. The specification of the strategy space used by competing firms can have significant implications on the nature of the resulting equilibria[3]. When the product mix is not a-priori specified, the strategic specification can also have important implications on the product mix at equilibrium. The existing models of oligopolistic competition

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1. The unanimity in the direction of preference (while there are differences in the intensity of preference) defines *quality* differentiation. When consumers differ in the direction of preferences (among differentiated goods) we have *variety* differentiation.

2. Among other results they prove: 1) that the monopolist reduces the quality sold to any consumer (compared with perfect competition), 2) that the range of qualities produced by the monopolist is always larger than under perfect competition, 3) prices are higher and quantities lower than under perfect competition and consumers of low intensity preference for q may be priced out of the market. All these result from the best possible attempt of the monopolist to discriminate between consumers and to segment the market.

3. The differences of the properties of equilibria of Cournot competition in quantities to those of Bertrand competition in prices and to those of competition in supply functions (Grossman (1981)) are well known in the case of homogeneous goods.

in differentiated products usually assume that each firm is allowed to produce only one product[4]. We believe that to the extent possible such a property should be the result of a model rather than an assumption[5].

In this paper we allow each firm $j = 1, \ldots, n$ to use as a strategy a schedule (function) $p_j(q)$ of prices at which it is willing to offer various qualities q in $[q_l, q_h]$. This is competition 'a la Bertrand' applied to differentiated products. We characterize pricing at the non-cooperative equilibria and thereby we are able to determine patterns of production at equilibrium.

Section 2 characterizes equilibria with production technology of no fixed costs. Section 3 analyses the case of fixed costs of production. In Section 4 we conclude.

2. *EQUILIBRIA WITH NO FIXED COSTS OF PRODUCTION*

Let there be a spectrum $[q_l, q_h]$ of products differentiated by quality q in $[q_2, q_h]$. All consumers prefer a higher quality good to a lower quality good (at equal prices) but have different valuations (willingness to pay) of the attribute q, i.e. their utility functions $U(q, \theta)$ have the property $\partial U/\partial q > 0$. Consumers are indexed by θ, the intensity of their preference for quality. We assume that $\partial^2 U/\partial\theta\partial q > 0$ so that the marginal utility of quality is increasing in the index θ. Consumers are distributed on θ according to the absolutely continuous distribution function $G(\theta)$.

Assume that there are no cost savings from production of more than one quality, and that the marginal cost $c(q)$ of production of quality q is constant for any amount of production. The cost of producing x units of quality q is $C(x(q)) = x(q)c(q)$.

Assumption 1 : No fixed costs.

Assumption 2 : There are $j = 1, \ldots, n$, $n \geqslant 2$, firms in the market each using as a strategy a schedule $p_j(q)$ of prices at which it is willing to sell qualities q.

We first characterize pricing at the non-cooperative equilibria.

Proposition 1 : At a non-cooperative equilibrium prices are equal to marginal cost.

To prove this consider a proposed equilibrium configuration $p(q) = (p_1(q), \ldots, p_n(q))$ where some product $\bar{q}$ is produced by firm j and sold at

4. See for example Shaked and Sutton (1982).

5. Of course, the fact that two firms do not produce the same product is usually a result. Here we make a statement about the ability of a firm to produce more than one product.

$p_j(\bar{q}) > c(\bar{q})$. Let $\Pi_j^* > 0$ indicate the profits of firm j from sales of product $\bar{q}$. Then firm $k \neq j$ can "undercut" j by offering product $\bar{q}$ at $p_j(\bar{q}) - \varepsilon$ (where $\varepsilon > 0$) thus realizing profits $\Pi_j^* - O_1(\varepsilon)$ from sales of product $\bar{q}$, where $O_1(\varepsilon)$ indicates an amount of order ε. The maximal loss on the profits of firm k from its sales of other qualities (resulting from the "undercutting" action) is $O_2(\varepsilon)$, also of order ε. The total effect is $\Pi_j^* - O_1(\varepsilon) - O_2(\varepsilon) > 0$ (for some ε), since Π_j^* is positive. Therefore "undercutting" is profitable and the proposed configuration is not an equilibrium *QED*.

Now we examine under what production configurations a marginal cost pricing non-cooperative equilibrium is possible.

Clearly there is no non-cooperative equilibrium where a finite number of products is produced with at least one product produced exclusively by one firm and all prices are equal to marginal cost. The reason is clear: when produced qualities are separated by non-infinitessimal distances clearly the firm which has exclusive production of a quality can realize positive profits by charging a price above marginal cost. Even when all products are produced, a marginal cost pricing Nash equilibrium may fail to exist as the following proposition shows.

Proposition 2: There is no non-cooperative equilibrium where a firm produces exclusively all products in an interval (q_1, q_2) in $[q_l, q_h]$.

To show this observe that if at equilibrium a product arbitrarily close to q_1 is produced (product $q_1 - \varepsilon$) at price p^1 then firm j will have to price $p_j(q_1)$ arbitrarily close to p^1. Similarly if at equilibrium a product arbitrarily close to q_2 is produced $(q_2 + \varepsilon)$ at price p^2 then the equilibrium price $p_j(q_2)$ will be arbitrarily close to p^2. Firm j can increase all prices of interior products by amounts of order ε retaining all its customers product by product and fulfilling both end point conditions[6] *QED*.

Proposition 2 shows that not all marginal cost pricing equilibria are non-cooperative equilibria in this game. On the other hand, there is a lot of non-cooperative equilibria at prices equal to marginal cost: If each product in $[q_l, q_h]$ is produced by two or more firms then all prices will be equal to marginal cost and this production schedule and the corresponding marginal cost prices will be a non-cooperative equilibrium. This equilibrium requires only a finite number of firms in production, each producing an infinity of products. A non-cooperative equilibrium with a finite number of firms in production necessarily involves overlapping production sets (among firms).

There can also be non-cooperative, marginal cost pricing equilibria

6. The exact function of order ε to be added to marginal cost will depend on the particular utility functions of the consumers.

where each product is produced exclusively by one firm. Suppose that there is an infinity of firms and each firm produces a distinct product q in $[q_l, q_h]$ so that the whole interval is covered. Although no two firms produce the same product, prices are driven down to marginal cost as there are arbitrarily close subsitutes.

Under a stronger assumption we can show that nearly all products in $[q_l, q_h]$ are produced, although no "bunch" of products is produced exclusively by one firm.

Assumption 4 : Let consumer of type θ have utility function $U_\theta(m, q) = m + \theta V(q)$, where q represents one unit of quality q and m is the consumption of the homogeneous good. Let the intensity of preference θ be distributed as $G(\theta)$ and assume that $V'(.) > 0$, $c'(.) > 0$, and $c''/c' > V''/V'$.

Now we can establish the following proposition.

Proposition 3 : There is no non-cooperative equilibrium where all prices are equal to marginal cost and there is an interval of non-produced qualities lying between produced qualities[7,8].

To show this consider any two produced qualities q_1, q_2, $q_1 < q_2$ such that there is no quality between them which is produced. We will show that there exists a quality $\bar{q}$ between q_1 and q_2 such that if sold at $c(\bar{q}) + \varepsilon$ it can attract some consumers who used to buy q_1 and q_2 at marginal cost. Let $q_L < q_1$ be the closest quality to q_1 lower than q_1. Let $A = (c(q_1) - c(q_L))/(V(q_1) - V(q_L))$. [If there is no closest produced quality to q_1, let $A = \lim_{q_L \to q_1} (c(q_1) - c(q_L))/(V(q_1) - V(q_L))$]. Let $q_H > q_2$ be the closest produced quality to q_2, higher than q_2. Let $C = (c(q_H) - c(q_2))/(V(q_H) - V(q_2))$. [If there is no closest produced quality to q_2, let $C = \lim_{q_H \to q_2} (c(q_H) - c(q_2))/(V(q_H) - V(q_2))$]. Further, let $B = (c(q_2) - c(q_1))/(V(q_2) - V(q_1))$. Then $(c'/V')' > 0$ implies $A < B < C$. When q_1 and q_2 are offered at marginal cost, all consumers with θ in (A, B) buy q_1 and all consumers with θ in (B, C) buy q_2. Let $\bar{q}$ in (q_1, q_2) be offered at marginal cost $c(\bar{q})$. Let $K = (c(\bar{q}) - c(q_1))/(V(\bar{q}) - V(q_1))$. Using $(c'/V')' > 0$ we deduce $A < K < B$ which implies that all consumers in (K, B) prefer $\bar{q}$ to q_1 if both are sold at marginal cost. Let $M = (c(q_2) - c(\bar{q}))/(V(q_2) - V(\bar{q}))$. Again using $(c'/V')' > 0$ we deduce $B < M < C$, which implies that all consumers in (B, M) prefer $\bar{q}$ to q_2 if both are sold at marginal cost. By charging

7. There can be equilibria where all products in $[q_1, q_h]$ are produced and none of those in (q_l, q_1) is produced because all consumers prefer to buy q_1 at $c(q_1)$ rather than any lower quality at marginal cost.

8. A special case of this result is when $V(\theta) = \theta$ and $c'' > 0$. This utility specification has been used by Mussa and Rosen (1978).

$c(\overline{q}) + \varepsilon$ a firm can make positive profits. Therefore the proposed equilibrium does not exist because it pays for one firm to deviate from it *QED*.

A common feature of the existing non-cooperative equilibria is that they involve production of an infinity of qualities (almost all products in $[q_l, q_h]$ are produced) and that no firm has exclusive production of a "bunch" of adjacent qualities. As we have seen there may be only a finite number of firms in production. Production of an infinity of qualities is a direct result of the no fixed cost technology. In the next section we change this assumption.

3. *EQUILIBRIUM WITH FIXED COSTS OF PRODUCTION*

We now discuss the case of production technologies with a fixed setup cost. We substitute assumption 1 by:

Assumption 1′: There is a fixed cost $F(q) > 0$ for production of quality q.

The fixed cost $F(q)$ is added to the constant marginal cost $c(q)$ for every quality produced, so that the cost of production of x units of quality q are $C(x(q)) = F(q) + x(q)c(q)$. Clearly a finite number of products will be produced and marginal cost pricing is unfeasible. The following lemma holds with exactly the same reasoning as proposition 1 if "marginal cost" is substituted by "average cost".

Lemma 1: When there are positive setup costs, prices of produced goods at a non-cooperative equilibrium are equal to average cost.

No equilibrium can exist with two or more firms producing the same product because price competition will derive their prices to marginal costs; i.e. below average costs. Thus:

Lemma 2: When there are positive setup costs a non-cooperative equilibrium will involve production of a finite number of products, with no product produced by more than one firm.

An argument similar to the argument of proposition 1 rules out non-cooperative equilibria with two or more active firms in the market: Consider an equilibrium configuration with two or more firms producing one product each. Let profits of firm j (which produces quality q_j at price p_j) be Π_j^* and profits of firms $j-1$ be Π_{j-1}^* (both equal to zero by lemma 1). Consider a move by firm $j-1$ (which produces a neighboring product to firm j at the proposed configuration) to production of quality q_j at price $p_j - \varepsilon$. Then profits for firm $j-1$ are $\Pi_j^* - 0(\varepsilon)$ from sales to old costumers of firm j, where $0(\varepsilon)$ signifies an amount of order ε. At its new position firm $j-1$ will also satisfy a non-infinitessimal demand by consumers who used to buy

from it at its old position. Let profits generated from these "old customers" be denoted by Π_0. These profits are positive and non-infinitessimal, being generated by the non-infinitessimal demand from old customers who switch to product q_j when product q_{j-1} is not sold anymore. Then the total profits for firm j after the move are $\Pi_j^* + \Pi_0 - 0(\varepsilon) > \Pi_{j-1}^* = \Pi_j^*$ because $\Pi_0 > 0(\varepsilon)$. Therefore it pays for a firm to undercut its neighbor at the proposed configuration. Therefore the proposed configuration is not a non-cooperative equilibrium[9]. We have established :

Lemma 3 : At a non-cooperative equilibrium there can be only one active firm.

But clearly, by a similar argument to the one of lemma 3 the zero profit making monopolist has incentives to eliminate some of his own products and thus increase profits, assuming that the rest of the industry will stay out. This plainly says that the zero profit making monopolist will not be at a non-cooperative equilibrium. Since all other equilibrium possibilities have been eliminated, we conclude that :

Proposition 4 : With a fixed set-up cost technology there is no non-cooperative equilibrium in price-quality competition.

It is important to note that if active and non-active firms are not treated symmetrically then other possibilities arise. To see this suppose that active firms (which produce positive amounts) are Stackelberg leaders with respect to inactive firms, while active firms play a Nash game among themselves. In this case active firms take into account the reaction of the non-active players. The non-cooperative game among active firms results in the elimination of all but one active firm as in lemma 3. Anticipation of (entry) reactions by inactive firms keeps the only active firm from eliminating any of its own product lines. At the same time the existence of potential entrants keeps the profits of the active firm at zero.

Proposition 5 : When active firms act as Stackelberg leaders with respect to inactive firms (potential entrants) and there is a fixed cost technology there exists a non-cooperative equilibrium with one active firm producing a number of qualities and realizing zero profits.

9. This result is true even if $\Pi_j^* \neq \Pi_{j-1}^*$. Then we select the firm with the lower profits. (at the proposed equilibrium configuration) to undertake the "undercutting" strategy.

4. *CONCLUDING REMARKS*

We analysed oligopolistic competition where firms use price-quality schedules as strategies. When the technology involves a constant marginal cost $c(q)$ for the production of quality q we showed that at all non-cooperative equilibria firms have to price at marginal cost. Otherwise there would be "undercutting" strategies and the proposed configuration would not be a non-cooperative equilibrium. At an equilibrium no firm has exclusive production of any interval of qualities and, under reasonable cost assumptions, almost all qualities are produced.

When a fixed cost per quality $F(q)$ is added to the constant marginal cost technology, we showed that in a symmetric non-cooperative game no equilibrium exists. In a game where firms have Stackelberg conjectures on non-active firms the equilibrium involve a monopolist selling a finite number of products at average cost. It is well worth noting that in this case we have a monopolist charging average cost, although the number of competitors in the market may be as low as two.

The "negative" flavor of these results is a consequence of the intense competition which we postulated. Slightly more encouraging results can be obtained when firms play a two-stage quality-price game, with qualities chosen first and prices subsequently. In such a duopoly game Shaked and Sutton (1982) have shown the existence of a (subgame perfect) equilibrium. The question of existence of subgame perfect equilibria in a multifirm oligopoly game is still open.

Columbia University, New York.

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