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ON NASH EQUILIBRIUM EXISTENCE AND OPTIMALITY IN OLIGOPOLISTIC COMPETITION IN PRICES AND VARIETIES

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I. *INTRODUCTION*

Recently attention has been focused on the problem of oligopolistic competition among firms selling differentiated products. The revival of the interest started with the re-examination of the exposition of Hotelling (1929).[1] Hotelling examined oligopolistic competition where firms choose prices and varieties in the framework of two games. In the short run game firms compete non-cooperatively in prices while varieties are fixed. The long run game is defined for those varieties which result in a (unique) Nash equilibrium in prices for the first game. In the long run game firms compete in varieties, with payoffs the Nash equilibrium payoffs of the first game.

The two stage framework is usually justified in two ways: First, as a temporal framework; firms are assumed to take decisions about product variety at a different point in time than decisions about prices. Second, as a sophisticated game that firms play to avoid cut-throat competition. The first reason is valid for a class of firms which take decisions in that sequential manner. In this paper we will show that the second reason lacks validity.

Here we allow for price and variety to be played *simultaneously*. Use of price and variety as simultaneous strategic variables allows for the possibility of more intense competition. This leads to the elimination of all equilibria which entailed some degree of competition. At any Nash equilibrium of the simultaneous price-variety game firms are local monopolists. Thus, allowing a high degree of competition leads to equilibria entailing the least competition. The simultaneous use of price and variety is justified when the technology is such that both variables are equally easy to vary in the same time framework.

We add a stage for entry to the game. The entry stage can be interpreted as the long run, while the stage of simultaneous choice of prices and varieties can

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1. Hotelling's (1929) model has been re-examined by d'Aspremont *et al.* (1979), Salop (1979), Novshek (1980) and Economides (1981), (1984) & (1986).

be interpreted as the short run. We note that here entry means the payment for the fixed capital which allows the firm to produce *any specification* of differentiated products. For example the fixed cost may represent the cost of a machine which is used to paint the objects which are differentiated by color. Then, variation of the color input can be as easy as a change of price, but entry entails the payment for the setup of the machine.

We show that in this framework, depending on the significance of fixed costs, product diversity at the long run equilibrium may be lower or higher than optimal. This is in contrast with the usual results of similar models where product diversity is higher than optimal.

The model is presented in section 2. In section 3 we show non-existence of equilibrium where firms are in direct interaction. In section 4 we characterise the existing local monopolistic equilibria. Section 5 is devoted to the analysis of the long run equilibrium and its comparison with the surplus maximizing product diversity.

II. *THE MODEL*

Briefly the model is specified as follows :[2]

(a) There are $j = 1, \ldots, n$ firms, $n \geqslant 2$, offering differentiated products $x_1, \ldots, x_n$ respectively. These products are ordered in [0, 1].

(b) Consumers have utility functions separable in money (Hicksian composite commodity) and one unit of a differentiated product. Consumers' choice is limited to buying a unit of one differentiated product or no differentiated products. Consumer "w" prefers most product w. His utility function is :

$$U_w(m, z, p_z) = m - p_z + V_w(z), \quad V_w(z) = k - f(||z - w||)$$

$f(d)$ measures the disutility of distance in the space of characteristics. Clearly, $f(0) = 0, f'(.) > 0$. It also assumed that $f''(.) \geqslant 0$.[3] k is the reservation price, for any differentiated product. Consumers are distributed in [0, 1] according to $G(w)$ which is absolutely continuous.

2. For a more detailed exposition of the model see Economides (1984).

3. (Weak) convexity of $f(.)$ guarantees that consumers who have close tastes buy the same product. A concave $f(.)$ can result in a firm serving consumers who are located far apart (in terms of their most preferred good), while not serving consumers located between those served. Although a concave $f(.)$ may seem appropriate in the locational interpretation of the model, it seems totally inappropriate in the differentiated products interpretation.

(c) Firms face the same cost functions $C(q) = E + cq$. The short run strategic variables for each firm j are its price p_j and the variety x_j it produces; $s_j \equiv (p_j, x_j)$. In the long run, firms decide on entry expecting to receive the short run equilibrium profits if they enter, and zero otherwise.

Given the prices and varieties of all firms, consumer w purchases the commodity which gives him the largest utility, or buys nothing if this last choice maximizes utility. Formally, consumer w picks the choice which is the minimum of $p_1 + f(||x_1 - w||)$, $p_2 + f(||x_2 - w||)$, ..., $p_n + f(||x_n - w||)$, k.

Let $\underset{\sim}{p} = (p_1 \ldots, p_n)$, $\underset{\sim}{x} = (x_1, \ldots, x_n)$ denote the prices and locations of all firms. The following notation is introduced:
$\underset{\sim}{p}_{-j} = (p_1, \ldots, p_{j-1}, p_{j+1}, \ldots, p_n)$, $\underset{\sim}{x}_{-j} = (x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n)$.
The demand and profits functions are: $D_j \equiv D_j(p_j, x_j; \underset{\sim}{p}_{-j}, \underset{\sim}{x}_{-j})$,
$\Pi_j \equiv \Pi_j(p_j, x_j; \underset{\sim}{p}_{-j}, \underset{\sim}{x}_{-j}) \equiv p_j D_j(p_j, x_j; \underset{\sim}{p}_{-j}, \underset{\sim}{x}_{-j}) - C(D_j(p_j, x_j; \underset{\sim}{p}_{-j}, \underset{\sim}{x}_{-j}))$.
For completeness we include the well known definition of a non-cooperative equilibrium: An n-tuple of strategies, $\underset{\sim}{s}^* \equiv (\underset{\sim}{p}^*, \underset{\sim}{x}^*)$ is a Nash equilibrium if and only if no agent has an incentive to depart from it unilaterally, i.e. if for $j = 1, \ldots, n$ and all (p_j, x_j) it is true that

$$\Pi_j(\underset{\sim}{p}^*, \underset{\sim}{x}^*) \geqslant \Pi_j(p_j, x_j; \underset{\sim}{p}^*_{-j}, \underset{\sim}{x}^*_{-j}).$$

III. *A NON-EXISTENCE RESULT*

We distinguish configurations of reversation price, spacing and prices at which neighboring firms are in direct competition with each other. We define firm j to be "strictly competitive to the right" if and only if

$$x_j - x_{j+1} < f^{-1}(k - p_j) + f^{-1}(k - p_{j+1}).$$

A firm is "weakly competitive to the right" when the above relation holds with equality. This relation says that the consumer who is marginal between buying x_j and x_{j+1} is strictly better off buying any of these two differentiated products rather than not buying any differentiated product. Firms j and $j + 1$ are in direct competition with each other if and only if firm j is competitive to to the right. Similarly, we define firm j to be "(strictly) competitive to the left" if firm $j - 1$ is "(strictly) competitive to the right."

We now show that an equilibrium cannot exist if there is even one firm in direct competition with its neighbors.

Theorem 1a: There is no equilibrium where a firm is strictly competitive either to the left or to the right.[4]

Proof: Say firm j is strictly competitive to the right so that all consumers between x_j and x_{j+1} buy a differentiated product. See Figure 1. I can assume without loss of generality that $\Pi_j(\underset{\sim}{p}, \underset{\sim}{x}) \leqslant \Pi_{j+1}(\underset{\sim}{p}, \underset{\sim}{x})$. In the n-tuple of strategies $(\underset{\sim}{p}, \underset{\sim}{x})$, firm j plays strategy (p_j, x_j). Now consider the following alternative strategy for firm j: Relocate to x_{j+1} and charge $p_{j+1} - \varepsilon$ when ε is a small positive number. Then firm j will receive the same profits as firm $j + 1$ from consumers to the right of x_{j+1} plus or minus an amount of order ε. Firm j will also receive, from consumers to the left of x_{j+1}, higher profits than firm

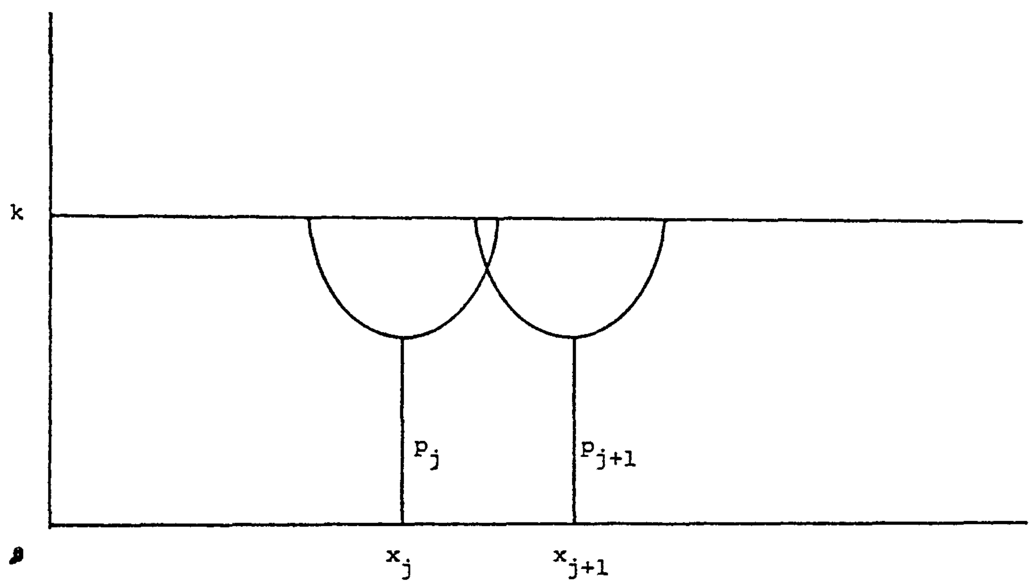

FIGURE 1

$j + 1$ used to make, as long as firm j had some positive demand at its old position. Then the profits resulting when firm j uses strategy $(p_{j+1} - \varepsilon, x_{j+1})$ are higher than the profits of firm $j + 1$ before the move, and these by assumption are higher than the old profits of firm j:

$$\Pi_j(p_{j+1} - \varepsilon,\ x_{j+1};\ \underset{\sim}{p}_{-j},\ \underset{\sim}{x}_{-j}) > \Pi_{j+1}(\underset{\sim}{p}, \underset{\sim}{x}) \geqslant \Pi_j(\underset{\sim}{p}, \underset{\sim}{x}).$$

4. Theorems 1a and 1b are also true for a concave "transportation cost function" $f(.)$.

Therefore for firm j, the strategy $(p_{j+1} - \varepsilon, x_{j+1})$ is superior to (p_j, x_j); hence $(\underset{\sim}{p}, \underset{\sim}{x})$ is *not* a non-cooperative equilibrium.[5] QED.

Next we show that equilibria where firms are weakly competitive but are not local monopolistic, in the sense that their behavior does not coincide with firm behavior if there were no other firms in the market, can also be shown not to exist; thus *only truly local monopolistic equilibria exist.* A firm is in a local monopolistic equilibrium if it does not face competition in its "neighborhood" and chooses a price which it would have chosen even if no other firms existed in the market.

Now we can precisely state :

Theorem 1b: There is no equilibrium where a firm is weakly competitive (to the left or to the right) and it is not a local monopolist.

Proof: Consider a proposed "equilibrium" configuration $(\underset{\sim}{x}^*, \underset{\sim}{p}^*)$ where firm $j + 1$ is weakly competitive to the left but does not charge the local monopolistic price (which it would charge if firm j did not exist). Then firm $j + 1$ is charging the price that corresponds to the kink of its (residual) demand function. If firm j is not there, firm $j + 1$ faces a higher (residual) demand and marginal revenue functions and will choose to produce more at a lower price. This is exactly the situation in which firm j finds itself when it moves from x_j to x_{j+1}. Thus $\Pi_j(x_{j+1}, p_{j+1} - \varepsilon; \underset{\sim}{p}^*_{-j}, \underset{\sim}{x}^*_{-j}) > \Pi_{j+1}(\underset{\sim}{x}^*, \underset{\sim}{p}^*) \geqslant \Pi_j(\underset{\sim}{x}^*, \underset{\sim}{p}^*)$. Hence it is profitable for firm j to deviate, and the proposed configuration does not constitute an equilibrium. QED.

This basic non-existence of a Nash equilibrium in a "competitive" structure in the game of simultaneous price and variety choice has little relation to the non-existence of non-cooperative equilibrium in Hotelling's (1929) *duopoly when prices are strategic variables and varieties are fixed.*

When only price is a strategic variable d'Aspremont *et al.* (1979) have shown that a Nash equilibrium does not exist for close locations and linear disutility of distance [linear $f(d)$]. However, the non-existence region shrinks when a low reservation price k is introduced [Economides (1984)], and when the exponent of the disutility of distance function increases, $f(d) = d^a$, $1 < a < 2$ [Economides (1986)]. The non-existence region disappears for $f(d) = d^2$

5. The same argument goes through if firm j relocates at $x_{j+1} - \delta$ and charges $p_{j+1} - \varepsilon'$. This is important when the strategy space of locations for firm j is restricted to the open set (x_{j-1}, x_{j+1}).

[d'Aspremont *et al.* (1979)]. In contrast, the non-existence in the model examined here holds for any increasing function $f(.)$.[6]

The basic non-existence of a non-cooperative equilibrium where firms compete directly cannot be cured easily. Novshek (1980) gets around the non-existence by changing the strategic rules of the game so that a firm reacts by cutting prices when its demand falls to zero as a result of an aggressive undercutting act of an opponent. Although the "fix" works well in his model where the disutility of distance function $f(.)$ is linear, it cannot work for a general disutility of distance function $f(.)$ because a firm $j+1$ may not be completely driven out of business at a proposed deviation from equilibrium which is preferred by firm j to the candidate equilibrium configuration.[7] It may well be optimal for firm j not to undercut its opponent. Hence, a much more complex change of the strategic rules is required to get around this non-existence. A strategic rule which will guarantee existence is that firms have expectations of reactions of other firms prices which depend on the distance between firms, $(dp_i/dp_j)^c = R(||x_j - x_i||)$ with $R(.) \gg 0$ and decreasing in distance.[8] This is clearly outside the non-cooperative equilibrium framework.[9]

IV. *AN EXISTENCE RESULT*

Now consider a configuration of prices and varieties such that no firm is "competitive" to the left or to the right. There are consumers between any firms that prefer not to buy any differentiated product. Each firm is a "local monopolist" in a part of the market. Assume that the distribution of consumers is uniform.

Let $(\underset{\sim}{p}, \underset{\sim}{x})$ define the local monopolistic structure. Firm j serves consumers in $(x_j - D_j/2, x_j + D_j/2)$ where $D_j = 2f^{-1}(k - p_j)$ and $\Pi_j = p_jD_j -$

6. D'Aspremont *et al.* (1979) noted in their footnote 2 that equilibrium would fail to exist in their model of linear disutility of distance if price and location were to be chosen simultaneously.

7. See footnote 5. A relocation of firm j at $x_{j+1} - \delta$ where it charges $p_{j+1} - \varepsilon'$ leaves a positive market share to firm $j+1$ and is preferred by firm j to the candidate equilibrium configuration.

8. Novshek's (1980) "fix" can be thought of as a special case of this rule. In our terms, he required $R(0) = 1$ if $dp_j < 0$.

9. Salop (1979) overcomes the non-existence problem for asymmetric locations by assuming that increasing marginal costs do not allow the firm to cover costs when expanding production to undercut its opponents. In a similar context Prescott and Visscher (1977) overcome the problem of non-existence by assuming irreversible locational decisions by firms.

$C(D_j)$. It is easily verified that Π_j is concave in p_j and therefore the optimal p_j (given x_j) is unique.[10] Let p_j^* maximize Π_j by solving:

$$\partial \Pi_j / \partial p_j = 2[f^{-1}(k - p_j) - (p_j - C'(.))/f'(f^{-1}(k - p_j))] = 0. \qquad (1)$$

It is clear that marginal changes of x_j which do not change the local monopolistic configuration make no difference in profits. Strategies which make firm j competitive to the left (or the right) can only yield lower profits than those of the local monopolistic configuration, since in the competitive configuration firm j has to gain consumers that can buy from $j + 1$ at a utility cost of less than k.[11] Locating to the left of x_{j+1} and charging less than p_{j+1} (so that some consumers to the left of x_{j+1} are gained) is inferior to (p_j^*, x_j) by the same argument; and by considering the fact that marginal costs are not falling. Finally, playing $(p_{j+1} - \varepsilon, x_{j+1})$ gives order of ε less profits than $\Pi_j(p_{j+1}^*, x_{j+1}) = \Pi_j(p_j^*, x_j)$. Therefore, it is optimal to play (p_j^*, x_j).

By the above argument, a Nash equilibrium $(\underset{\sim}{p}^*, \underset{\sim}{x}^*)$ where $p_j^* = p^*$, the solution of (1), and $x_{j+1} - x_j \geqslant 2f^{-1}(k - p^*)$, will exist if it is feasible to have n local monopolists in the market, i.e. if $1 \geqslant \sum_{j=1}^{n} D_j = 2nf^{-1}(k - p^*)$, or, equivalently, if

$$p^* + f(1/(2n)) \geqslant k. \qquad (2)$$

From (1), implicitly differentiating, we have :

$$\frac{dp_j^*}{dk} = -\frac{\dfrac{\partial^2 \Pi_j}{\partial p_j \, \partial k}}{\dfrac{\partial^2 \Pi_j}{\partial p_j^2}} = \frac{2C'' + f' + f^{-1}f''}{2C'' + 2f' + f^{-1}f''} < 1. \qquad (3)$$

Therefore, for large k equation (2) does not hold. Define $\bar{k}$ as the solution of (2) as equality:

$$p^*(\bar{k}) + f(1/(2n)) = \bar{k}. \qquad (4)$$

10. $\dfrac{\partial^2 \Pi_j}{\partial p_j^2} = \dfrac{-2}{(f')^2}(2f' + f''(p_j - C')/f' + 2C'') < 0$, since $f'' > 0$, $p_j \geqslant C'$, $C'' \geqslant 0$.

11. After firm j has taken over consumer $x_{j+1} - D_j/2$ (who is indifferent between buying from firm $j + 1$ and not buying a differentiated product) it has to decrease its price below k following $f^{-1}(x_{j+1} - z)$ to be able to gain consumer z.

We have shown:[12]

Theorem 2: In Hotelling's model, wherein strategies are both prices and varieties, n-firm Nash equilibria, $(\underset{\sim}{p}^*, \underset{\sim}{x}^*)$, exist only if the reservation price is relatively low, $k \leqslant \bar{k}$, where $\bar{k}$ solves (4). For $k = \bar{k}$ a unique equilibrium $(\underset{\sim}{p}^*, \underset{\sim}{x}^*)$ exists at varieties $x_j^* = (2j - 1)f^{-1}(k - p^*), j = 1, \ldots, n$, and prices $p_j^* = p^*(k)$ defined as the solution of (1). For every $k < \bar{k}$ there is a continuum of equilibria such that all firms charge $p^*(k)$ and their locations are constrained by $x_1 \geqslant f^{-1}(k - p^*)$, $x_{j+1} - x_j \geqslant 2f^{-1}(k - p^*), j = 1, \ldots, n - 1$, and $x_n \leqslant 1 - f^{-1}(k - p^*)$.

V. *LONG RUN EQUILIBRIUM AND OPTIMAL PRODUCT DIVERSITY*

In the long run firms have the option of entry and exit. We model the entry decision as a move played in an earlier stage than the one of simultaneous price-variety choice. We consider equilibria of the entry stage which are subgame-perfect in the price-variety subgame.

Let $\Pi^*(n)$ denote the profits of a local monopolist when there are n firms in the market. Profits are in general dependent on n as the entry of firms may bid up input prices. In general $d\Pi^*/dn < 0$.[13] For a fixed reservation price k, let $L(n) = 2nf^{-1}(k - p^*(n))$ be the minimal length of the market required to fit n (profit maximizing) local monopolistic firms. When the addition of an extra firm has relatively small influence on marginal costs ($dC'/dn \geqslant 0$, small), the total length of the market served increases with n, $dL/dn > 0$.[14] Of

12. A similar result may be derived when consumers located at each point z have a general downward sloping demand.

13. From the first order condition (equation 1).,
$p^* - C' = f^{-1}(k - p^*)\, f'(f^{-1}(k - p^*))$. Implicitly differentiating,
$\dfrac{dp^*}{dC'} = \dfrac{f'}{2f' + f''f^{-1}} > 0$, so that $dp^*/dn = (dp^*/dc)(dc/dn) \geqslant 0$. Equilibrium profits are,
$\Pi^* = \Pi(p^*) = 2(f^{-1}(k - p^*))^2 f'(f^{-1}(k - p^*)) - E$, and since
$\dfrac{d\Pi^*}{dp^*} = -2(2f^{-1} + \dfrac{f''(f^{-1})^2}{f'}) < 0$, it follows that
$d\Pi^*/dn = (d\Pi^*/dp^*)\,(dp^*/dn) \leqslant 0$.

14. $\dfrac{dL^*}{dn} = 2[f^{-1}(k - p^*) - \dfrac{n}{f'}\dfrac{dp^*}{dC'}\dfrac{dC'}{dn}] = 2[f^{-1}(.) - \dfrac{n}{2f' + f''f^{-1}}\dfrac{dC'}{dn}]$,

$\dfrac{dL^*}{dn} > 0 <=> \dfrac{dC'}{dn} < f^{-1}(.)\,(2f' + f''f^{-1}(.))/n$. Therefore if $\dfrac{dC'}{dn}$ is relatively small, equilibrium prices increase slowly with n and, although the size of the market of course, this case includes the "standard" case in the literature where the addition of firms has no influence on costs.

Let n_1 be the integer part of the solution of $L(n) = 1$ and n_2 the integer part of the solution of $\Pi^*(n) = 0$. (If $\Pi^*(n) > 0$ for all $n > 0$, set $n_2 = \infty$). n_1 is the maximal number of firms which "fit" in [0, 1]. No short run equilibrium with more than n_1 active firms exists by theorem 1. No long run equilibrium with more than n_2 firms exists because firms have the option of not entering a market where they will make negative profits. Further, for any short run equilibrium with less than $\min(n_1, n_2)$ firms there exists a feasible short run equilibrium with $1 + \min(n_1, n_2)$ firms where all firms make nonnegative profits. Thus, at a subgame perfect equilibrium there will be $n_e = \min(n_1, n_2)$ active firms.

Theorem 3: At a subgame perfect free entry equilibrium there will be $n_e = \min(n_1, n_2)$ active firms.

An important question in models of differentiated products concerns product diversity. Does the market solution entail too few or too many differentiated products compared to the number that maximizes economic welfare? In models where consumers have single-peaked preferences in the space of characteristics the result has generally been that there is higher than optimal product diversity at the zero profit long run equilibrium.[15]

In this model, the long run equilibrium does not necessarily entail zero profits. When $n_e = n_1 = \min(n_1, n_2)$, at the long run equilibrium there are n_1 firms completely filling the market [0, 1], all at positive profits. Thus, although the welfare maximizing number of differentiated products n_0 is, in general, smaller than n_2, it is not necessarily smaller than the equilibrium number n_e. When $n_e = n_1 < n_2$, it may well be that $n_e < n_0$, so that there is *lower* than optimal diversity at the long run equilibrium.

To see this concretely, consider the simple case that has been extensively

each firm drops, the total size of the market served by all n firms increases. In the (uninteresting) case of strong external diseconomies when $\frac{dC'}{dn}$ is large, equilibrium prices increase quickly in n and the decrease in individual firm market size is not offset by the increase in n, so that the total market size decreases in n.

15. See for example Lancaster (1979), Salop (1979) and Economides (1981), (1989). This contrasts with the lower than optimal product diversity reported by Dixit and Stiglitz (1977) in a model where consumers have a taste for variety. Spence (1976) reports the possibility of lower than optimal equilibrium diversity when the demand for each differentiated product is highly elastic. For a general comparison of the optimal and equilibrium numbers of firms see Mankiw and Whinston (1986).

discussed in the literature when entry has no influence on marginal costs ($dC'/dn = 0$, $C'' = 0$) and the disutility of distance is linear, $f(d) = d$. The number of products which maximizes welfare is $n_0 = 1/(2\sqrt{E})$ if $E < (k - c)^2$ and $n_0 = 0$ otherwise.[16] The maximal number of products which "fit" in [0, 1] is $n_1 = 1/(k-c)$.[17] The number of firms which would be in the market on profit considerations alone is $n_2 = \infty$ if $E < (k - c)^2/2$ and $n_2 = 0$ otherwise.[18] Thus for relatively small fixed costs, $E < (k - c)^2/4$, it is true that $1/(k - c) < 1/(2\sqrt{E}) < \infty$,[19] i.e. $n_1 < n_0 < n_2$, and thus $n_0 > n_e = \min(n_1, n_2)$ i.e., optimal diversity is higher than the long run equilibrium product diversity.

Theorem 4: Equilibrium product diversity is higher (lower) than optimal if and only if the fixed cost is high (low) : $E - (k - c)^2/4 > 0$ (<0).

VI. *CONCLUSION*

Hotelling's model of oligopolistic competition was modified so that price and variety are simultaneous strategic variables. Although the possibilities of more intense competition were enhanced, at equilibrium firms are local monopolists. At all Nash equilibria there are consumers (located between neighboring firms) who are (at least weakly) better off by not buying any differentiated product at the prevailing prices. The result was established for products differentiated by one of their characteristics, but it readily generalizes to products differentiated by any number of characteristics. Existence of equilibria where firms are in direct competition with each other can be restored if a conjectured reaction $(dp_i/dp_j)^c = R(\|x_i - x_j\|) > 0$ is assumed, but this is outside the Nash equilibrium framework.

We modelled entry as a move in the game happening before the price-variety move. At the long run free entry equilibrium which is subgame perfect in the price-variety truncation there may be more or less active firms than the surplus maximizing optimum number, depending on how low fixed costs are. This contrasts with the usual result of more than optimal product diversity.

An important avenue of further research is the examination of the exi-

16. Welfare is equal to consumers plus producers' surplus, $W = CS + PS$, and $CS = n(k - p)^2$, $PS = n[2(k - p)(p - c) - E]$, so that using $k - p = 1/(2n)$ we have $W(n) = k - c - 1/(4n) - nE$. Then $\partial W/\partial n = 0 <=> n_0 = 1/(2\sqrt{E})$ and $W(n_0) = k - c - \sqrt{E}$. Thus the optimal number of firms is $n_0 = 1/(2\sqrt{E})$ if $E < (k - c)^2$ and $n_0 = 0$ otherwise.

17. $L(n) = 1 <=> n = 1/[2(k - p^*)]$ and here $p^* = k(+ c)/2$ so that $n_1 = 1/(k - c)$.

18. Maximized profits are $\Pi^* = (b - c)^2/2 - E$.

19. $1/(k - c) < 1/(2\sqrt{E}) <=> E < (k - c)^2/4$.

stence problem in mixed strategies. Recent results by Dasgupta and Maskin (1986) established existence of equilibria in mixed strategies in the symmetric two-stage game of Hotelling. However, when prices and varieties are simultaneous strategic variables the task is much more formitable. Possibilities of undercutting are so abundant that it is very doubtful if a similar existence result can be established for the "competitive region" of the game of this paper.

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