Pricing of Complements and Network Effects*

October 2007

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Abstract

We discuss the case of a monopolist of a base good in the presence of a complementary good provided either by it or by another firm. We assess and calibrate the extent of the influence on the profits from the base good that is created by the existence of the complementary good. We establish an equivalence between a model of a base and a complementary good and a reduced-form model of the base good in which network effects are assumed in the consumers' utility functions as a surrogate for the presence of direct or indirect network effects, such as complementary goods produced by other firms. We also assess and calibrate the influence on profits of the intensity of network effects and quality improvements in both goods. We evaluate the incentive that a monopolist of the base good has to improve its quality rather than that of the complementary good under different market structures. Finally, based on our results, we discuss a possible explanation of the fact that Microsoft Office has a significantly higher price than Microsoft Windows although both products have comparable market shares.

Key words: calibration; monopoly; network effects; complementary goods; software; Microsoft

JEL Classification Codes: L12; L13; C63; D42; D43

* We thank Ravi Mantena and Andy Skrzypacz for useful comments. Viard acknowledges financial support from the Fletcher Jones Foundation.

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1. Introduction

We examine a monopolist of a base good who benefits from a complementary good provided either by it or another firm. Use of the complementary good requires the base good, but not the reverse. We assess and calibrate the extent of the positive influence on the base good profits that is created by the existence of the two sources (internal or external) of the complementary good. We establish an equivalence between a model of a base and complementary good and a reduced form model of the base good where network effects are assumed in the utility function as a surrogate for the presence of direct network effects (*i.e.,* a consumer's utility directly increases in the number of other users) or indirect network effects (*i.e.,* arising from increased variety of complementary goods produced by other firms). This allows us to examine the pricing of the complementary good under different market structures and in the context of the effect of other complementary goods via the network effects. We assess and calibrate the influence of the intensity of network effects and quality improvements in the complementary good on profits from the base good. We also evaluate the incentive that a monopolist has to improve the quality of the base good rather than that of a complementary good that it produces.

Our model has implications for the base good monopolist's tradeoff in improving the quality of its own complementary good versus subsidizing increases in other network effects. The monopolist could subsidize increases in other network effects by, for example, taking actions to increase sales of the base good thereby increasing consumers' utility directly ("direct network effects") or facilitating or subsidizing increased variety of other complementary goods available ("indirect network effects"). The base good monopolist prefers that an independent firm offer an additional complementary good rather than improve the quality of a pre-existing complementary good by the same amount as the quality offered by the new good, assuming the costs of the two are the same. This results from the complementary goods firm's incentive to restrict output more at higher quality levels, limiting the increase in base good sales via sales of the complementary good. We also find that the base good monopolist gains more from adding a complementary good to its portfolio of products than increasing the quality of an existing portfolio product by the same quality as that of the new good if the costs of doing so are the same. The effect is stronger than if an independent firm produces the complementary good. This is because adding a complementary good increases sales of the base good because of the complementarities, but an increase in the quality of the complementary good does not affect sales of the base good because the monopolist can fully adjust the price of the complementary good to capture profits from its increased quality.

The model of this paper also has implications for the base good monopolist's incentives to invest in improving the base and complementary goods under different market structures and in making them compatible. An independent base good monopolist has a greater incentive to invest in improving the quality of the base good (at the margin) than a joint monopolist who produces both the base and complementary good. Improvements in the base good increase its price and therefore the effective price to use the complementary good. A producer of both internalizes this and has less incentive to improve the base good, while an independent monopolist does not. The flip side of this is that a monopolist who produces both the base and complementary goods has a greater incentive to improve the complementary good (on the margin) than an independent firm would have to improve it. Improvements in the complementary good's quality increase sales of the base good, which the joint monopolist internalizes but the independent firm does not. Finally, if a single firm owns both the base and complementary goods it has a greater incentive to make them compatible than if separate firms offered the two products because increasing compatibility improves sales of both. The base good benefits directly from a more compatible product and the complementary good benefits indirectly because it requires purchase of the base good. A joint monopolist internalizes this feedback while independent firms do not.

Based on our results, we discuss a possible explanation of the fact that Microsoft Office is significantly more expensive than Microsoft Windows. Microsoft has approximately the same market share (over 90%) in the market for operating systems for personal computers as in the market for "office applications" (a bundle of word processing, spreadsheet, presentation and database software). However, Microsoft charges a price for its Windows operating system that is significantly lower than the price of its office suite. Although our model does not address the level of prices for Windows and Office, it can explain this difference in relative prices for Windows and Office. A joint monopolist, such as Microsoft, has two price instruments, the base good and complementary good price. It is optimal to keep the operating system price low even if Office is quite valuable because some users buy Windows for use with other complementary goods. Raising the price of Office but keeping the price of Windows low allows the joint monopolist to capture some of the value provided by Office while not pricing users of other goods complementary to Windows out of the market.

Our model of a base and a complementary good is similar to models of "mix and match," where consumers assemble systems in fixed proportions, but differs from typical assumptions in these models because the base good in our model is valuable without use of the complementary good. In "mix and match" models typically neither good is valuable without the other. "Mix and match" models originated from Matutes and Regibeau (1988) and Economides (1989).[1, 2]

Our reduced form model of the base good in which network effects are summarized by a term that influences utility positively and is increasing in sales, derives from Katz and Shapiro (1985) and Farrell and Saloner (1986). The network effects summarized in the utility function can result from either direct or indirect network effects. In the former case, a consumer benefits directly from the number of individuals adopting the base good (for example, because there is a

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[1] Our model is an example of a "micro" model as defined by Economides (1996a) or the "components approach" as defined by Shy (2001).

[2] We do not consider the more distantly related effect of changes in "software variety" considered in Church and Gandal (1992) or Chou and Shy (1990). The former evaluate how the compatibility decisions of software firms affect the degree of standardization in the hardware market, while the latter demonstrate that increasing returns in the production of complementary goods can substitute for the assumption of network effects. These are different than our objective, which is to take network effects and a complementary good as given and evaluate their equivalence.

larger network to exchange files with) while in the latter case a consumer benefits indirectly from the number of individuals adopting the base good through the increased availability of software variety. Both of these effects are summarized by a term in consumers' utility functions, which is increasing in total sales of the base good.[3]

There has been little attempt to calibrate the size of the network effect used in the reduced form models. Two exceptions are Economides (1996b) and Clements (2004). Economides (1996b) calibrates the size of the network effect in the context of measuring the incentive of a patent-holding monopolist who also sells a complementary good to invite competitors in the complementary goods market so as to maximize the network effects. Clements (2004) evaluates the effect of the strength of network effects, degree of compatibility and the density of consumers in the market on standardization under oligopolistic competition. The objective of these papers is different than our objective of providing an equivalence of the two modeling approaches and evaluating the incentives of the base good provider to innovate, promote other complementary products and set compatibility standards.

Section 2 sets up the basic framework of our research. Section 3 develops and discusses the five models we use in this paper, which differ in the way that network effects and inherent product quality are modeled. Section 4 compares the equilibria of the five models. Section 5 discusses the incentives to invest in quality in either the base good or the complementary good in different ownership structures and under different intensities of network effects and also examines compatibility decisions made by the base good monopolist. Section 6 discusses the explanation of Microsoft's relative pricing provided by our analysis. Section 7 compares our results with the empirical literature on network effects. Section 8 has concluding remarks.

2. Basic Framework

We assume that consumers are differentiated in terms of their preferences for quality of the base good ("B") and quality of the complementary good ("C"). The second good requires the first good to provide positive utility.[4] For example, we can think of the Windows operating system as the base good, and an office suite (such as Microsoft Office) as the complementary good, not necessarily produced by the same company. Let the marginal utility of quality of the base good be θ and the marginal utility of quality of the complementary good be φ. The pair (θ,φ) defines a consumer type. We assume that both θ and φ are distributed independently and uniformly on [0, 1].

We assume that there are potentially network effects for the base good. These network effects could be direct effects that result from a consumer's utility directly increasing in the number of other users of the base good or indirect effects that result from other complementary

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3 This is an example of a "macro" model as defined by Economides (1996a) or the "network externalities approach" as defined by Shy (2001).

4 Since the complementary good requires the presence of the base good but not conversely, we expect that the equilibria in terms of prices and quantities will be asymmetric across firms.

goods whose existence positively influences consumers' willingness to pay for the base good via economies of scale in production. We assume in the latter case that the positive consumption effects between the base good and the complementary goods reinforce each other. We summarize these effects by adding a term proportional to sales of the base good in the utility function of a typical consumer.

When consuming one unit of the base good and possibly one unit of the complementary good, consumer (θ,φ) receives utility

$$U = \theta q_B - p_B + \alpha x_B + \delta V ,$$

where q_B is the quality of the base good, p_B is the price of the base good, V is the utility from the consumption of the complementary good, x_B is the sales of the base good, α measures the intensity of the network effects and δ is an indicator variable taking the value one if the complementary good is bought and zero otherwise. Thus, network effects arising out of direct or indirect network effects are summarized by an additive term in the utility function proportional to sales.[5] Consumers not purchasing receive zero utility. The utility from the consumption of the complementary good is

$$V = \varphi q_C - p_C ,$$

where q_C is the quality of the complementary good and p_C is the price of the complementary good.

We will consider five alternative models. The first model has a base good monopolist in a market where network effects are summarized in the utility function of consumers as proportional to sales. The second model has two monopolists (independent firms), one for the base good and one for the complementary good, and assumes no network effects. The third model adds network effects to the independent firms in Model 2. The fourth model has a single monopolist (joint monopolist) producing both the base and the complementary good. The fifth model adds network effects to the joint monopolist considered in Model 4.

3. Models

3.1 Model 1: Single Good Monopolist in a Market with Network Effects

We first consider a model of a single good monopolist selling the base good with network effects arising from direct or indirect effects due to the presence of other complementary goods produced with increasing returns to scale. In this case, $\delta = 0$ and consumer θ who buys one unit of the base good of quality q_B at price p_B receives utility of

[5] We assume that that the influence of positive consumption (network) effects on the willingness to pay for the base good can be summarized by an additive term which is proportional to sales of the base good. This assumes that higher sales of the base good are reflected in higher sales of other complementary goods and vice versa.

$$U = \theta q_B - p_B + \alpha x_B \qquad (1)$$

where $\alpha > 0$ measures the intensity of the network effect (marginal utility of network expansion). All consumers of type $\theta > \theta_B$ buy the good, where the marginal consumer is

$$\theta_B = (p_B - \alpha x_B)/q_B. \qquad (2)$$

Sales are

$$x_B = (1 - \theta_B) = 1 - (p_B - \alpha x_B)/q_B. \qquad (3)$$

Inverting the demand we have

$$x_B = (q_B - p_B)/(q_B - \alpha),\ \ \Pi_B = p_B x_B = p_B(q_B - p_B)/(q_B - \alpha).^{6} \qquad (4)$$

Assuming zero costs, maximizing profits implies:[7]

$$p_B^* = q_B/2,\ \ x_B^* = q_B/(2(q_B - \alpha)) \text{ and } \Pi_B^* = q_B^2/(4(q_B - \alpha)).^{8} \qquad (5)$$

In the case of no network effects, *i.e.*, when $\alpha = 0$, the demand without network effects is a pivot of the demand with network effects through the point $(0, q_B)$. It is well known that such pivots of linear demands lead to the same monopoly price. Thus, the equilibrium price is unaffected by network effects, while sales and profits are higher with them. Using the subscript 0 for the variables with no networks effects ($\alpha = 0$), we have

$$x_B^* = x_{B0}[q_B/(q_B - \alpha)],\ \ p_B^* = p_{B0} \text{ and } \Pi_B^* = \Pi_{B0}[q_B/(q_B - \alpha)]. \qquad (6)$$

3.2 Model 2: Independent Firms Without Network Effects

In Model 2, we consider two independent monopolists, one for the base good and another for the complementary good, and we assume no network effects. By comparing the equilibrium of this model to that of Model 1, we can calibrate the intensity of network effects necessary to generate base good profits equivalent to those generated by sales of a complementary good.

There are two groups of purchasers to consider (see Figure 1). First, consumers of type $\theta > \theta_B$, $\varphi < \varphi_{B,BU}$ buy the base good only, where θ_B is the marginal consumer indifferent between buying the base good and buying nothing, *i.e.*

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[6] We require $\alpha < q_B$ so that the demand is download sloping.

[7] We present the model with zero costs, but positive costs could easily be added. We have $d\Pi_B/dp_B = (q_B - 2p_B)/(q_B - \alpha) = 0$ and $d^2\Pi_B/dp_B^2 = -2/(q_B - \alpha) < 0$ since $q_B > \alpha$.

[8] We also require that everyone does not buy the good which implies $x_B^* < 1$ or $2(q_B - \alpha) > q_B$, i.e., $\alpha < q_B/2$.

$$\theta_B = p_B / q_B \,, \tag{7}$$

and $\varphi_{B,BU}$ is the marginal consumer indifferent between buying only the base good and buying both the base and complementary goods, *i.e.*

$$\varphi_{B,BU} = p_C / q_C \,. \tag{8}$$

Second, consumers of types $\varphi > \varphi_{B,BU}$, $\theta > \theta_B$, as well as of types $\varphi > \varphi_{BU}(\theta)$, $\theta < \theta_B$, buy both, where $\varphi_{BU}(\theta)$ is the marginal consumer of type θ indifferent between buying both goods and buying nothing, *i.e.*

$$\varphi_{BU}(\theta) = (p_B + p_C - \theta q_B)/q_C \,. \tag{9}$$

The profits for the base good monopolist are

$$\Pi_B = \left[(1-\theta_B)\varphi_{B,BU} + \left(1 - \varphi_{B,BU} - \left(\overline{\varphi_{BU}} - \varphi_{B,BU}\right)/2\right)\right] p_B \,, \tag{10}$$

where $\overline{\varphi_{BU}} = (p_B + p_C)/q_C$ is the consumer of type $\theta = 0$ who is indifferent between buying both goods and nothing. The profits for the complementary good monopolist are

$$\Pi_C = \left[(1 - \varphi_{B,BU}) - \left(\overline{\varphi_{BU}} - \varphi_{B,BU}\right)\theta_B / 2 \right] p_C \,. \tag{11}$$

At a Nash equilibrium in a price-setting game, the first-order conditions for the two monopolists are

$$p_B(3p_B + 4p_C) - 2q_B q_C = 0 \text{ and } p_B^2 - 2q_B(q_C - 2p_C) = 0 \,. \tag{12}$$

Since the first-order conditions are nonlinear we solve them numerically to find the equilibrium.[9] In our analysis we will restrict $q_C \geq q_B$ since we wish to consider interior solutions only and for $q_C > q_B$, $\overline{\varphi_{BU}} > 1$.[10]

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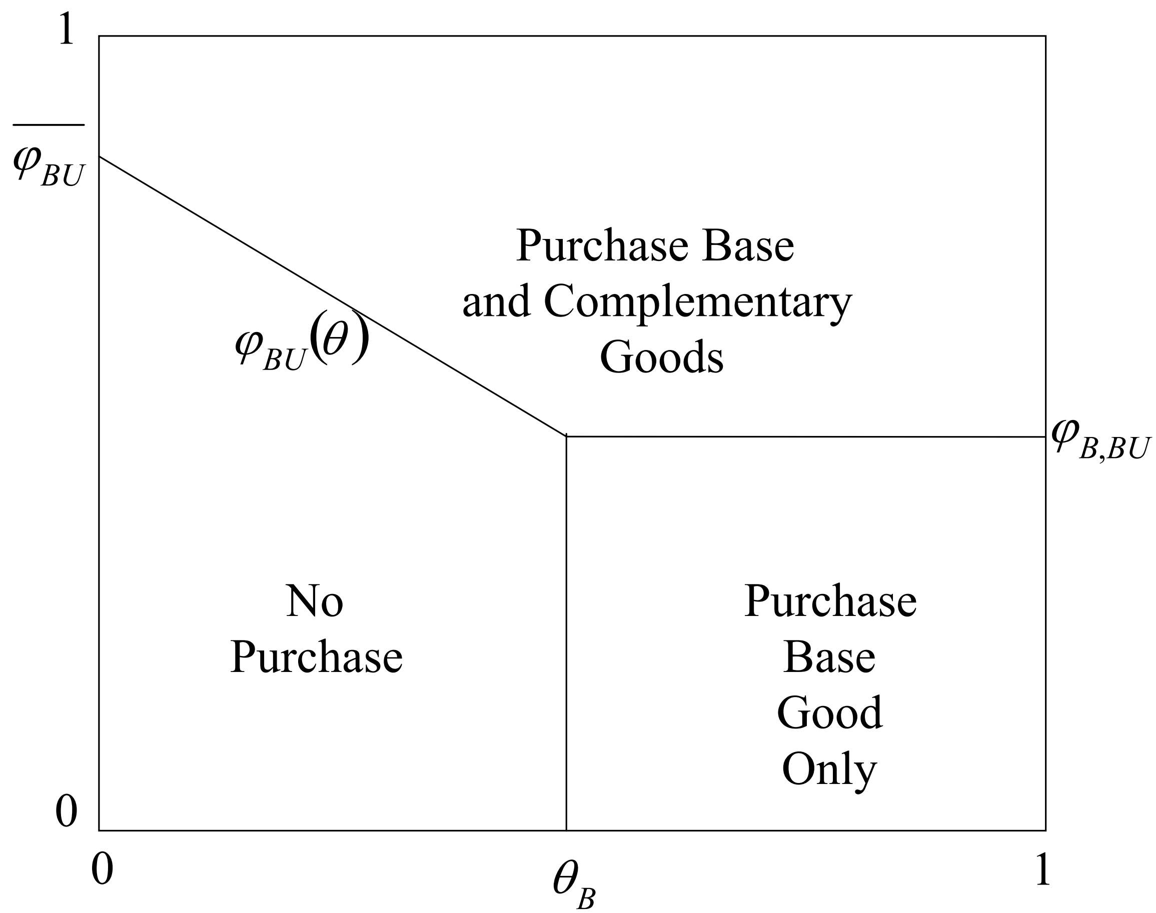
[9] The second order conditions are $-(3p_B + 2p_C)/q_B q_C < 0$ and $-2/q_C < 0$ respectively, both of which are met for all parameter values.

[10] Note that the first-order conditions themselves place no restrictions on the relative qualities. The positive root of the first first-order condition is $1/3\left(-2p_C + \sqrt{4p_C^2 + 6q_B q_C}\right)$ which is always positive. Solving the second first-order condition for p_C we get $q_C/2 + p_B^2/4q_B$ which is always positive as well.

Figure 1



3.3 Model 3: Independent Firms With Network Effects

In Model 3, we incorporate network effects arising from direct consumption externalities or other complementary goods into Model 2. The utility function of consumers (equation (1)) now has a positive α capturing the network effects. The same regions of consumer types buy as in Model 2, but some margins now depend on α . We use superscript n to denote the presence of network effects

$$\theta_B^n = (p_B - \alpha x_B)/q_B \ , \ \varphi_{B,BU}^n = \varphi_{B,BU} \ , \tag{13}$$

$$\varphi_{BU}^n = (p_B + p_C - \theta q_B - \alpha x_B)/q_C \ , \ \overline{\varphi_{BU}^n} = (p_B + p_C - \alpha x_B)/q_C \ . \tag{14}$$

Demand for the base good is given by solving for x_B in

$$x_B = 1 - \theta_B^n(x_B) + \left(1 - \left(\overline{\varphi_{BU}^n}(x_B) - \varphi_{B,BU}^n(x_B)\right)\theta_B^n / 2\right). \tag{15}$$

Since θ_B^n and φ_{BU}^n are both linear functions of x_B, this is a quadratic equation. Using the positive root, x_B^*, that solves this equation, the profit function for the base good monopolist is

$$\Pi_B^n = \left[\left(1-\theta_B^n\left(x_B^*\right)\right)\varphi_{B,BU}^n + \left(1-\varphi_{B,BU}^n - 1/2\left(\overline{\varphi_{BU}^n}\left(x_B^*\right)-\varphi_{B,BU}^n\right)\theta_B^n\left(x_B^*\right)\right)\right]p_B \tag{16}$$

and for the complementary good monopolist is

$$\Pi_C^n = \left[\left(1-\varphi_{B.BU}^n\right) - 1/2\left(\overline{\varphi_{BU}^n}\left(x_B^*\right)-\varphi_{B,BU}^n\right)\theta_B^n\left(x_B^*\right)\right]p_C . \tag{17}$$

The first-order conditions for the two firms are nonlinear functions of the prices so we solve them numerically.[11]

3.4 Model 4: Joint Monopolist Without Network Effects

In Model 4, the joint monopolist sells both the base and complementary goods. The marginal consumers are defined in the same manner as in Model 2, and the profit function for the joint monopolist is

$$\Pi_B + \Pi_C = \left(1-\theta_B\right)\varphi_{B,BU}\,p_B + \left(1-\varphi_{B,BU} - 1/2\left(\overline{\varphi_{BU}} - \varphi_{B,BU}\right)\theta_B\right)\left(p_B + p_C\right). \tag{18}$$

The joint monopolist chooses both prices to maximize its profits. The first-order conditions are

$$3p_B\left(p_B + 2p_C\right) - 2q_B q_C = 0 \text{ and } 3p_B^2 - 2q_B\left(q_C - 2p_C\right) = 0 . \tag{19}$$

These can be solved to get the equilibrium prices, quantities, and profits:

$$p_B = 2q_B/3 ,\ p_C = q_C/2 - q_B/3 , \tag{20}$$

$$x_B = 1 - p_B\left(p_B + 2p_C\right)/\left(2q_B q_C\right) \text{ and } x_C = 1 - \left(p_B^2 + 2p_C q_B\right)/\left(2q_B q_C\right).^{12} \tag{21}$$

Notice that the price of the base good is independent of the quality of the complementary good. This is true for general demand functions, since the marginal revenue of the joint monopolist from sales of the base good is independent of the quality and price of the complementary good, at the optimal complementary good price. To see this, consider general demand functions for the base and complementary goods, $D_B(p_B)$ and $D_C(p_B + p_C)$ respectively. Then profits are:

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[11] We also verify numerically that the nonlinear second-order conditions hold and that $\Pi_B^n\left(p_B;p_C^*\right)$ is quasiconcave in p_B and $\Pi_C^n\left(p_C;p_B^*\right)$ is quasiconcave in p_C.

[12] The second-order condition is met as the Hessian is negative definite for all parameter values.

$$\Pi_B = p_B[D_B(p_B) + D_C(p_B + p_C)] \text{ and} \tag{22}$$

$$\Pi_C = p_C D_C(p_B + p_C),$$

and joint profits are $\Pi = \Pi_B + \Pi_C$ so that the first order conditions are (where primes denote derivatives):

$$D_B + p_B D_B' + D_C + (p_B + p_C) D_C' = 0, \tag{23}$$

$$D_C + (p_B + p_C) D_C' = 0.$$

These imply $D_B(p_B) + p_B D_B'(p_B) = 0$. Therefore for the joint monopolist the choice of price for the base good is independent of the choice of price and quality of the complementary good.

The joint monopolist completely internalizes in the complementary good price any changes in the quality of the complementary good, and therefore the price of the basic good remains unaffected by such quality changes.[13] In our analysis we will only consider positive prices for the complementary good and therefore restrict $q_C > 2q_B/3$.[14]

3.5 Model 5: Joint Monopolist With Network Effects

In Model 5, we incorporate network effects for the base good into Model 4. The marginal consumers are defined in the same manner as in Model 3 and the profit function for the joint monopolist is

$$\Pi_B^n + \Pi_C^n = \left(1 - \theta_B^n\left(x_B^*\right)\right)\varphi_{B,BU}^n p_B + \left(1 - \varphi_{B,BU}^n - 1/2\left(\overline{\varphi_{BU}^n}\left(x_B^*\right) - \varphi_{B,BU}^n\right)\theta_B^n\left(x_B^*\right)\right)\left(p_B + p_C\right). \tag{24}$$

The first-order conditions for the firm are nonlinear functions of the prices so we solve them numerically.[15]

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[13] Also notice that, for independent firms, the first order conditions cannot be decomposed as in joint monopoly, and therefore the equilibrium prices of both the base and complementary good do depend on the quality levels of both goods. For independent firms, the first order conditions are:
(A) $D_B + p_B D_B' + D_C + p_B D_C' = 0$,
(B) $D_C + p_C D_C' = 0$.
Substitution from (B) into (A) cannot accomplish decomposition as in joint monopoly. Of course, comparison of (B) with (22) confirms Cournot's result that the total price $p_B + p_C$ is lower under joint monopoly.

[14] Although in principle the joint monopolist could choose to sell the complementary good below cost, such action could raise serious antitrust concerns.

[15] We also verify that the second-order conditions are met. We solve over a grid of possible prices to ensure that we obtain the global maximum.

4. Equivalence Results

In this section, we calibrate the size of network effects necessary to achieve the same base good profits as those arising from sales of the complementary good. This is possible since we have models that explicitly allow for positive effects of the complementary good sales as well as models that allow for network effects that are summarized in the utility function. Thus, we establish an equivalence between the network effects (defined as added profits to a base good monopolist) created by the presence of a complementary good and those summarized in the utility function. This is done in sections 4.1 to 4.4 for the various industry structures and for different quality levels. We use this equivalence in base good profits to analyze the incentive of the base good monopolist to offer its own complementary good, improve the quality of a complementary good that it offers, and subsidize an independent firm so that it offers or increases the quality of a complementary good it provides.

We focus on a particular type of equivalence, in base good profits, because we are primarily interested in the incentives of the base good monopolist. This equivalence, of course, does not ensure that consumer welfare is equated. Doing so would require determining how to weight the utility of consumers with high versus low valuations of each of the base and complementary goods since purchasing patterns will vary across different equilibria. In addition, profits could not be equilibrated at the same time as consumer welfare. One could also calibrate equivalence in total profits across both firms. However this would be an inappropriate comparison to make when comparing models with and without network effects, as we do. Similarly we could calibrate the equivalence in total (base and complementary good) profits for the base good monopolist. However this would also be an inappropriate comparison to make when comparing models in which the base good firm controls the complementary good to those in which it does not. We focus on equivalence in base good profits because our goal is to evaluate the incentives of the base good monopolist.[16]

An important property of all our models is scalability. It is easy to check that the equilibrium sales x_B and x_C are unaffected by a common scaling up or down of q_B, q_C, and α by the same positive coefficient, say $\lambda > 0$. Additionally, the equilibrium prices p_B and p_C are proportional to the common scaling factor λ of q_B, q_C, and α, and therefore their ratio $\left(p_C/p_B\right)$ is unaffected by scaling. It follows that equilibrium profits are also proportional in the scaling factor λ. Thus, we scale (normalize) all our variables in terms of the quality of the base good q_B, defining the "normalized quality" of the complementary good as $\tilde{q}_C = q_C/q_B$, the "normalized α" or "normalized network effects" as $\tilde{\alpha} = \alpha/q_B$, the normalized prices of the two goods as $\tilde{p}_B = p_B / q_B$, $\tilde{p}_C = p_C / q_B$, the "normalized relative price of the complementary good" in relation to the base good as $\tilde{R}_{CB} = \tilde{p}_C / \tilde{p}_B = p_C / p_B$, the "normalized base good profits" as $\tilde{\Pi}_B = \Pi_B/q_B$, and the "normalized complementary good profits" as $\tilde{\Pi}_C = \Pi_C/q_B$. All the

[16] We do not discuss the possibility of anti-competitive "leveraging" of monopoly power from the base to the complementary good.

normalized variables remain unaffected by the common scaling up or down of q_B, q_C, and α. Below, we will report results for all models in terms of these normalized variables.

4.1 Equivalence Between Network Effects And The Effects Of A Complementary Good Produced By An Independent Firm (Model 1 Versus Model 2)

We start with a model of two independent monopolists, one producing the base good and another producing a complementary good (Model 2). We compare this with a model of a single base good monopolist with network effects summarized in their utility function (Model 1). We establish an equivalence between the two models by equating the normalized base good equilibrium profits. An independent firm selling the complementary good results in increased sales of the base good. The equivalence finds the network effects (αx) in the utility of individual consumers, where $\tilde{\alpha}$ measures the intensity of the network effect necessary to equate base good profits.

Table 1 Independent Firms: Equivalence of Quality and Network Effects*

Normalized Complementary Good Quality $\left(\tilde{q}_C = \frac{q_C}{q_B}\right)$	Normalized Relative Price of Complementary Good $\left(\tilde{R}_{CB} = \frac{p_C}{p_B}\right)$**	Normalized Base Good Profits $\left(\tilde{\Pi}_B = \frac{\Pi_B}{q_B}\right)$	Normalized Equivalent α $\left(\tilde{\alpha} = \frac{\alpha}{q_B}\right)$
1	0.7071	0.3431	0.2714
2	1.2291	0.4003	0.3755
3	1.7375	0.4273	0.4149
5	2.7441	0.4532	0.4484
10	5.2481	0.4755	0.4743

* In this and all subsequent tables, we round results to four decimal places unless otherwise noted.
** These are equilibrium prices under presence of the complementary good but no network effects.

Table 1 shows the normalized network effects, $\tilde{\alpha}$, required to obtain equivalent normalized base good profits in the absence of the complementary good. For example, line three of the table indicates that a base good monopolist in the absence of a complementary good but with an $\tilde{\alpha}$ of 0.4149 earns the same normalized base good profits as a base good monopolist with an $\tilde{\alpha}$ of zero in the presence of an independent monopolist producing a complementary good of normalized quality $\tilde{q}_C = 3$. In this and all following analyses we choose $q_C \geq q_B$ to ensure an interior solution for the independent firms market structure and to ensure a positive price for the complementary good in the joint monopolist market structure as described earlier.