Perpetual Versus Subscription Licensing
Under Quality Uncertainty and Network Externality Effects

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ABSTRACT: We discuss the optimal way for a software vendor to license software: a perpetual license at a posted price, a subscription contract that subscribers receive automatic updates for periodic payment, or a hybrid approach that involves both. By addressing specific issues in the software market such as network effects, quality uncertainty, upgrade compatibility, and the vendor’s ability to commit to future prices in a dynamic environment, we demonstrate how a software vendor can manage the trade-offs of perpetual licensing and subscription to optimize profit, as well as the corresponding welfare effect on consumers. Although the subscription model helps the vendor lock in consumers so as to increase profit when there is great uncertainty
associated with the next version software, it destroys the path dependence in creating network externalities. Therefore, when the network effect is sufficiently large, it is more profitable for a software vendor to provide both perpetual licensing and subscription.

**Key words and phrases:** compatibility, network externality, price discrimination, quality uncertainty, software licensing, upgrades.

Software products traditionally have been sold as a property via a perpetual ownership licensing model. Customers acquire the permanent right to use and own the version of the software by paying up front. Software publishers have adjusted their licensing models from the traditional perpetual licensing model in favor of software subscription models. They deliver application software and services (software maintenance, upgrades, staff training, etc.) over the Web on a lease or subscription basis. For example, Microsoft bundled its Enterprise Subscription Agreement for business users with Software Assurance at an annual rate on a three-year term; vendors such as Sun, Oracle, SAS Institute, Computer Associates, and BMC Software have also offered their own subscription licenses on their major products. Despite the various forms of licensing models, the vendors all aim at generating perpetual revenue streams by transforming software into a subscription-based service. Industry practitioners claim that the subscription model can lower the cost of ownership and grant users access to up-to-date software at a predictable cost without a large up-front investment. They have even claimed that “traditional software is already dead” [25]. International Data Corporation (IDC) has also predicted that the software industry will experience “a transition to subscription licensing” [24].

Software can be used for a period of time without replacement, though its value may depreciate. In this sense, it is a kind of durable good. Yet software as a commodity has some special characteristics that differentiate it from other durable goods: (1) in general, software cannot be resold or appropriated due to license restrictions, so a secondhand market such as that for used cars does not exist; therefore, we can ignore the resell problem in selling other durable goods; (2) the production of software that is an information good has cost-side economy of scale—it is costly to create the first copy but has negligible marginal production cost for additional copies; (3) with the development of information technology (IT), it is relatively easy to improve the value of installed software through upgrades; and (4) the use of software has a strong network externality; that is, the value of using any particular software increases with the number of its adopters. This network externality, therefore, is like the “chicken and egg” problem [15]. As a result, we cannot apply the established theories about selling or leasing durable goods directly to the licensing models of software.

This paper addresses these issues by studying the optimal licensing policy of a monopolist software vendor who provides packaged off-the-shelf products through a perpetual licensing or subscription model exclusively, or through a mixed strategy of both. In order to provide strategic suggestions to software vendors as well as recom-
recommendations to users, we address the following research questions by incorporating the special characteristics of software products:

**RQ1:** What are the benefits and costs of perpetual and subscription licensing models for a vendor?

**RQ2:** Trading off those benefits and costs, what is the optimal way for vendors to license software? Should they use subscription licensing to completely replace the traditional perpetual licensing?

**RQ3:** How are consumers’ purchasing behaviors influenced by different licensing models?

By addressing the specific issues related to licensing models, such as compatibility, network externalities, quality uncertainty, and commitment, our paper fills a gap in the literature by examining the impact of quality uncertainty and network externalities on a monopoly seller’s licensing models. With our more generalized new findings, our paper extends the previous results of selling and leasing durable goods [4, 7, 8, 16] to a broader scope, and it provides key insights to software vendors as well.

**Literature Review**

Many economics and marketing researchers have examined the ways of selling durable goods. Coase [7] conjectured that a monopolist seller of such goods could not sell them at the monopoly price because rational and patient consumers would anticipate a future price drop and delay their purchase. Coase suggested that leasing durable goods would solve this time-inconsistency problem. Bulow [4] formally proved the Coase conjecture and further affirmed that leasing can achieve the optimal profit for a monopolist seller of nondurable goods; other studies verifying the conjecture followed, including, for example, Gul et al. [12]. Some works, however, have challenged the assumptions of the Coase model and have proposed conditions under which leasing can coexist with selling as the optimal strategy for a monopolist seller of durable goods. Bucovetsky and Chilton [3], for example, suggested threat of entry as a reason for the monopolist to mix selling and leasing. Desai and Purohit [8] found that leasing does not dominate selling in all cases if depreciation of the durable goods is taken into account and that a mixed strategy is optimal when the depreciation rates differ between selling and leasing. We investigate a monopolist software vendor who sells or leases his or her product to the market, which is similar to this stream of research. In order to examine the impact of the network externality effect and uncertainty regarding quality on the vendor’s strategy, we exclude from consideration the potential effects of competition and product depreciation.

We deal here with intertemporal product quality improvement, which differs from vertical product differentiation in a static state, as studied by Bhargava and Choudhary [1], Moorthy [18], and Mussa and Rosen [19], among others. A related stream of literature concerns product upgrades in selling durable goods. Levinthal and Purohit [17] considered the pricing of two generations of a durable good but do
not allow for an upgrade price. Dhebar [9] discussed the existence of pure strategy subgame-perfect equilibria in two-period models of overlapping generations of products with and without upgrades. Padmanabhan et al. [20] show that firms may issue upgrades as a signal of potential product acceptance when consumers face uncertainty about the degree of network effects for the product. Fudenberg and Tirole [11] discussed how a monopolist prices the new generation of durable goods based on the information he or she has about past customers. Viard [26] studied a monopolist’s upgrade strategy in an infinite-period setting and empirically verifies the results with data from the PC software industry. Sankaranarayanan [22] found that a monopolist software vendor will have an incentive to release new updates without cannibalizing his or her existing customers when the fixed cost of developing software is relatively high.

Among the research on selling and leasing packaged software products, Choudhary et al. [6] suggested that renting software in the first period to those who otherwise would adopt later can increase the seller’s profit; Choudhary [5] considered the effect of the selling and leasing schemes on the publisher’s incentive to invest in software quality. Harmon et al. [13] provided a thorough review of the literature on IT services pricing and presented a value-based approach.

Our work also draws on the literature about markets that involve direct or indirect network effects. Katz and Shapiro [15] categorized such markets and identify the issues firms and consumers face in them. Brynjolfsson and Kemerer [2] constructed a hedonic model and empirically report that the network externalities significantly contribute to the success of spreadsheet products: a 1 percent increase in a product’s installed base was associated with a 0.75 percent increase in its price.

Network externalities endogenize a user’s decision on the other users’ utilities and decisions. Combined with compatibility, versioning, and upgrades, they significantly complicate a software vendor’s strategic decisions. The most current attempt to address these issues is by Ellison and Fudenberg [10], who examined software upgrades with network externalities under the pure selling policy. They found that the network effect causes upgrades beyond the socially optimal level. Our paper uses a similar setting, but we instead look at the seller’s licensing and pricing strategies.

Building on prior literature, we consider software upgrades, quality uncertainty, compatibility, the network effect, and the ability to commit in our model of a monopoly software vendor’s optimal licensing policy, taking into account users’ responses. We provide vendors with important insights on licensing policies and improve our understanding of the implications of licensing model selection on consumer surplus and social welfare.

Model

We model the intertemporal consumer behavior and the monopolist software vendor’s strategic licensing policy with a two-period model as in Ellison and Fudenberg [10] and Fudenberg and Tirole [11]. Following literature such as Ellison and Fudenberg [10], Mussa and Rosen [19], and Salant [21], we assume that a software user’s net utility of consuming the software product is
where $\theta$ is the intensity of the quality preference, $q$ represents its quality, $e$ measures the intensity of the network externality effect, $x$ is the mass of the adopters of the product, and $p$ is the price paid for the software. Consumers are heterogeneous in their quality preference $\theta$, which is assumed to be uniformly distributed on the support $[0, 1]$. Consumers are homogeneous in their sensitivity to network externality $e$. Software “quality” ($q$) includes dimensions such as features, speed, functionalities, user interface, ease of learning, and other characteristics that affect users’ valuation of the product. Potential buyers can assess the quality of existing software, yet the quality of the future (upgrade) version is unknown. Users who subscribe to the software with periodic upgrades are exposed to the risk of buying into future upgrades with uncertain quality, whereas those who purchase the software can choose to upgrade or not in a future period when the quality of the new version is realized.

The monopolist vendor continuously invests in R&D to improve product quality, offering version 1 of the software in period 1 and an improved version—version 2—in period 2. When a consumer makes the adoption decision in period 1, the quality of version 1, $q_1$, is already realized, whereas the quality of version 2, $q_2$, is uncertain due to risks in research and development (R&D) and market condition changes during period 1. Until $q_2$ is realized at the beginning of period 2, its distribution is public knowledge: with probability $\rho$, the quality is high ($q_2^h = q_2^h$) and with probability $1 – \rho$ the quality is low ($q_2^l = q_2^l$). Assume that the quality level cannot go down: $q_2^l > q_1$.

The quality uncertainty of version 2 software is measured by the variance of $q_2$, $\text{var}(q_2) = \rho(1 - \rho)(q_2^h - q_2^l)^2$. The vendor can control the quality improvements by controlling R&D costs [5]. However, in order to simplify the model and focus on our research problem of the optimal licensing strategy, we assume that the R&D investments are sunk and the quality of version 2 is exogenous.

Software is usually designed to be compatible with previous versions (backward compatibility) to take advantage of the existing network of users, but forward compatibility (compatibility with future versions) is not easy to achieve. For example, the new version of Microsoft Office software will allow users to open files created by previous versions natively, but older software may not open the files created in the new. We study the most common case of software production in terms of compatibility: the software is backward compatible but forward incompatible. That is, the later version of the software can successfully use interfaces and data formats from the earlier version, but the original software is not designed in such a way that it can seamlessly accommodate files produced with the planned future version. Hence, consumers who upgrade to or buy later versions can enjoy network externalities from the users of both versions, but those who continue using the older version only have a network of users of the same version.

Consumers discount their future utility gain by a factor of $\beta \in [0, 1]$. To reduce the number of cases under consideration, we assume $(1 + \beta)q_1 \geq \beta E(q_2)$ so that a consumer with a higher quality preference $\theta$ prefers buying version 1 in period 1 to waiting to buy version 2 in period 2. This assumption is similar to the one in Fudenberg and
Tirole [11], which only excludes very large improvements between the two versions, but considerably simplifies the analysis.

As suggested by Waldman [27], we assume that the monopolist vendor can commit to future prices through means such as best-price provisions or limited editions. Relaxing this assumption does not change our major conclusions, but significantly complicates the problem. Hence, we adopt the commitment assumption in order to separate other effects on the licensing models and focus on the effects of quality uncertainty and network effect. We will discuss relaxing this assumption in the fourth section.

The software vendor can offer the software to the market through one-time sales, sales of upgrades, and subscription contracts. Depending on the licensing model adopted, the software vendor sets the selling price $p_1$, upgrade price $p_u$, the second-period selling price $p_2$, or the per period subscription fee $p_r$ at the beginning of period 1. Because the vendor cannot tell whether a buyer in the second period owns version 1, the upgrade price $p_u$ has to be no higher than the sale price of version 2, $p_2$—that is, $p_u \leq p_2$. The penalty for deviating from the subscription contract is large enough that the user commits to paying for both periods. Thus, it is the total subscription payment over the two periods that matters to the vendor, but not how it is allocated over the two periods. Without any loss of generality, we assume the per period subscription fee $p_r$ stays the same over the two periods, as adopted in the subscription practices of the software industry, for example, Microsoft’s Enterprise Subscription Model, “Oracle On-Demand,” and “McAfee Subscription Plus.”

Figure 1 depicts all the available consumer choices. Those who purchase version 1 in period 1 have the option of upgrading to the new version in period 2 at a cost of $p_u$, but those who enter the subscription contract in period 1 will use the new version software without any additional charges besides the per period subscription fee. To those consumers who do not adopt in period 1, the software vendor is better off selling only version 2 in period 2 to achieve a higher profit and avoid market cannibalization. We combine the option of subscribing only in period 2 with that of purchasing in period 2 because (1) consumers receive the same level of utility from either of the two options in the two-period model and therefore will simply choose the option with a lower price, and (2) the subscription fee and sale price in period 2 are not correlated with other decision variables (i.e., other pricing schedules).

The expected value for a risk-neutral user who buys the software in period 1 is given by

$$V_B(\theta) = U(q_1, x_0, p_1; \theta) + \beta \max \{U(q_2, x_2, p_u; \theta), U(q_1, x_1, 0; \theta)\}$$

which is the sum of the net utility of a user with quality preference $\theta$ in period 1 and the discounted net utility from the second period, when the buyer can decide to upgrade to version 2 or keep using version 1. $x_0$ denotes the mass of adopters in period 1, and $x_1$ denotes the mass of first-period adopters who do not upgrade in period 2. By our backward-compatibility assumption, $x_2$ is the mass of total adopters of either version in period 2.
Under the subscription contract, the user receives continuous supply of software with updates for a per period payment $p_r$. The expected value is

$$V_L(\theta) = U(q_1, x_0, p_r; \theta) + \beta U(q_2, x_2, p_r; \theta)$$

$$= \theta q_1 + ex_0 - p_r + \beta E[\theta q_2 + ex_2 - p_r],$$

which is the sum of the subscriber’s net utility from using the latest version of the software over the two periods.

Finally, the consumer who is inactive in the first period can either buy the new version of the software in period 2 or remain inactive. His or her expected discounted value is

$$V_I(\theta) = \beta \max \left\{ U(q_2, x_2, p_r; \theta), 0 \right\}$$

$$= \beta \max \left\{ E[\theta q_2 + ex_2 - p_r], 0 \right\}.$$  

The consumer with quality preference $\theta$ will choose how to adopt the software by maximizing the expected total discounted value:

$$V(\theta) = \max \left\{ V_H(\theta), V_L(\theta), V_I(\theta) \right\}.$$  

Consumers’ adopting strategies determine the market segmentations. Taking that into account, the software vendor will decide the pricing schedule for each licensing model and choose the optimal one. Now we solve the market equilibrium under each of the three licensing models.

**Perpetual Licensing**

We consider the case in which the vendor offers the software through selling: the vendor sells version 1 to a mass of $x_0$ users at price $p_1$ in period 1, and in period 2 the

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*Figure 1. Consumer Strategies*
vendor sells version 2 to a mass of \((x_2 - x_0)\) users at the full price \(p_2\), and a mass of \((x_0 - x_1)\) early adopters at the upgrade price \(p_u\).

At the beginning of period 1, a consumer of type \(\theta\) evaluates his or her expected value of buying in period 1, \(V_b(\theta)\) (Equation (2)), with the expected value of delaying the decision until period 2, \(V_I(\theta)\) (Equation (4)). There exists a trade-off between buying and waiting: if buying in period 1, the user can start using the software from that period and has the option of upgrading in period 2; otherwise, he or she cannot use the software in period 1 but retains the option of buying the new version in period 2 when the quality of version 2 is realized. Thus, a user has six unique strategies over the two periods: buy and upgrade (BU): buy version 1 in period 1 and upgrade to version 2 in period 2; buy and contingently upgrade (BC): buy version 1 and upgrade only when the quality improvement is large in period 2; buy and hold (BH): buy version 1 in period 1 and keep using it in period 2; buy version 2 in period 2 (IB); buy version 2 in period 2 only when the quality of version 2 is high (IC); and remain inactive in both periods (II). By the assumption we made above about quality improvement, \((1 + \beta)q_3 \geq \beta Eq_2\), we have the potential market segmentation depicted in Figure 2.

\section*{Lemma 1: Consumers with quality preference \(\theta \in [0, \theta_0]\) are inactive (II); those with \(\theta \in (\theta_0, \theta_1]\) buy version 2 in period 2 only when the quality of version 2 is high (IC); those with \(\theta \in (\theta_1, \theta_2]\) wait to buy version 2 in the second period (IB); those with \(\theta \in (\theta_2, \theta_3]\) buy version 1 in period 1 but do not upgrade in period 2 (BH); those with \(\theta \in (\theta_3, \theta_4]\) buy version 1 in period 1 and upgrade contingent on the realized quality in period 2 (BC); and those with \(\theta \in (\theta_4, 1]\) always use the latest version of the software during the two periods (BU).}

\section*{Proof: See Appendix A.}

The cutoff values \(\theta_0, \theta_1, \theta_2, \theta_3, \text{ and } \theta_4\) make consumers indifferent to two adjacent choices:

\[\theta_0 = \frac{p_2 - e x_2}{d_2^h}, \theta_1 = \frac{p_2 - e x_2}{d_2^l}, \theta_2 = \frac{p_1 - \beta p_2 + e\big(\beta(x_2 - x_1) - x_0\big)}{(1 + \beta)q_1 - \beta Eq_2}, \theta_3 = \frac{p_u - e(x_2 - x_1)}{q_2^h - q_1}, \theta_4 = \frac{p_u - e(x_2 - x_1)}{q_2^l - q_1}.\]  

(6)

Because the population of potential users is uniformly distributed on the \([0, 1]\) support, we plug Equation (7) into (6).
\[ x_0 = 1 - \theta_2 \]
\[ x_1 = (1 - \rho)(\theta_4 - \theta_3) + \theta_3 - \theta_2 \]
\[ x_2 = 1 - \theta_1 + \rho(\theta_1 - \theta_0) \].

Solving them simultaneously, we get the market segmentation of the consumers in response to the seller’s price schedule \( p_1, p_2, \) and \( p_u \).

Taking into account the consumers’ self-selection behavior, the software vendor decides prices to maximize his or her discounted total profit over the two periods:

\[
\max_{p_1, p_u, p_2} \Pi(p_1, p_u) = p_1x_0 + \beta p_u (x_0 - x_1) + \beta p_2 (x_2 - x_0)
\]
\[
\text{s.t. } \begin{cases} 
0 \leq x_1 \leq x_0 \leq x_2 \\
p_2 \geq p_u
\end{cases}
\]  

Applying the Kuhn–Tucker theorem, we solve this constrained optimization problem by comparing the profits under different combinations of Kuhn–Tucker conditions. We find that consistent with Fudenberg and Tirole [11], the optimal solution to Equation (8) involves no users buying only in period 2 (leapfrogging). When the expected quality improvement of the software is reasonably large and the intensity of the network effect on utility is small relative to the quality effect, the resulting market segmentation is (II, BC, BU). We use the superscript \( P \) to represent the equilibrium under the perpetual licensing strategy. The optimal price schedule for the software vendor is \( (p_1^P, p_u^P, p_2^P) \). The software vendor gets profit \( \Pi^P \). Those equilibrium results are shown graphically in Figure 3.

The software vendor will be better off committing to a very high price of version 2 software to prevent consumers from balking to purchase it in period 2 (leapfrogging). Some users will upgrade contingent on the realized quality of the new version, whereas other users will upgrade to version 2 no matter what the realized quality is. The vendor will increase both \( p_1 \) and \( p_u \) with a higher expected quality of version 2, and thus the market share will be reduced due to a higher price margin (Figure 3a). With the increase of the network effect, the vendor will lower the price of version 1 to attract more users in period 1. With the enlarged installed base, the vendor can charge a higher price for the upgrades and still receive a higher profit (Figure 3b).

Subscription

Due to a confluence of economic, market, and technological factors, the software vendor and the users have gradually adopted subscription as a licensing option. We study the types of subscription models as a contract between the software vendor and the users about delivery of the software product together with the latest upgrades for a fixed periodic payment over a certain number of periods. If a user subscribes, he or she will pay a rent \( p_r \) at the beginning of each period and enjoy the latest version of the software without any additional charge.
The expected discounted value for a subscriber is given in Equation (3). Consumers with \( q \geq q_L \) will get positive utility from the subscription contract. The level of \( q_L \) can be obtained by solving \( V_L(q_L) = 0 \):

\[
\theta_L = \frac{(1 + \beta)(p_r - e)}{q_1 + \beta E(q_2) - e(1 + \beta)}. \tag{9}
\]

Given the consumers’ choice, the software vendor sets the optimal subscription fee in the contract by maximizing his or her expected discounted total profit:

\[
\max_{p_r} \Pi(p_r) = (1 + \beta) p_r (1 - \theta_L)
\]

s.t. \( \theta_L \leq 1. \tag{10} \]

Here we use the superscript \( L \) to represent the equilibrium under the subscription (a leasing type of) strategy. We have the optimal price schedule for the software vendor,

\[
p_L^r = \frac{q_1 + E(q_2)}{2(1 + \beta)}. \]
The software vendor gets profit
\[ \Pi^T = \frac{(q_1 + \beta E q_2)^2}{4(q_1 + \beta E q_2 - e(1 + \beta))}, \]
which is affected by the mean of quality of version 2 software but not its variance.

A Hybrid Model of Perpetual Licensing and Subscription

This section considers the optimal pricing policy for a hybrid strategy whereby the monopolist software vendor offers consumers a choice of either perpetual licensing or subscribing to the software. The consumers are faced with a decision of whether to pay an up-front fee to buy the software and have the choice of upgrading in the second period or to subscribe and automatically receive the upgrade with an uncertain quality.

Consumers with the highest level of quality preference would like to adopt the latest version of the software. Hence, they receive the same utility through subscription or buying plus upgrading. When both policies are available, those consumers will choose the one with a lower cost. Consequently, these two market segments will not coexist unless they cost the same, under which condition high-end consumers are indifferent to either strategy. For simplicity and without loss of generality, we only consider consumers’ pure strategy equilibrium and ignore a mixed strategy of either buying or subscription. Thus, for the hybrid case to exist, we need to have the total cost of subscription lower than that of buying and upgrading \((1 + \beta)p_r < p_1 + \beta p_u\). By the same reasoning as in Lemma 1, the potential market segmentation is as shown in Figure 4.

The cutoff values \(\theta_0, \theta_1, \theta_2, \theta_3,\) and \(\theta_4\) make consumers indifferent to two adjacent choices:
\[
\theta_0 = \frac{p_2 - e x_2}{q_1^2}, \quad \theta_1 = \frac{p_2 - e x_2}{q_2^2}, \quad \theta_2 = \frac{p_1 - \beta p_2 + e(\beta(x_2 - x_1) - x_0)}{(1 + \beta)q_1 - \beta E q_2}, \quad \theta_3 = \frac{p_u - e(x_2 - x_1)}{q_1^2 - q_1}, \quad \theta_4 = \frac{(1 + \beta)p_r - p_1 - \beta p_u - \beta e(1 - \rho)(x_2 - x_1)}{\beta(1 - \rho)(q_1^2 - q_1)}. \tag{11}
\]

Plugging Equation (7) into (11), we get the market segmentation of the consumers in response to the seller’s price schedule \(p_1, p_2,\) and \(p_u\).

Taking into account the consumers’ self-selection behavior, the software vendor decides prices to maximize his or her discounted total profit over the two periods:
\[
\max_{p_1, p_u, p_2, p_r} \Pi(p_1, p_u, p_2, p_r) = p_1(\theta_4 - \theta_2) + \beta p_u(\theta_4 - \theta_3) + \beta p_2(\theta_2 - \theta_1 + \rho(\theta_1 - \theta_0)) + (1 + \beta)p_r(1 - \theta_4)
\]
\[
\text{s.t. } \begin{cases} 0 \leq \theta_0 \leq \theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4 \leq 1 \\ p_2 \geq p_u. \end{cases} \tag{12}
\]
Applying the Kuhn–Tucker theorem, we solve this constrained optimization problem by comparing the profits under different combinations of Kuhn–Tucker conditions. As in the case for equilibrium under the perpetual licensing model, we find that the optimal solution to Equation (12) involves no users leapfrogging. When the expected quality improvement of the software is reasonable and the intensity of the network effect on utility is small relative to the quality effect, the resulting market segmentation is \((\text{II}, \text{BH}, \text{L})\). We use the superscript \(H\) to represent the equilibrium under the hybrid model. The optimal price schedule for the software vendor is \((p_1^H, p_u^H, p_2^H)\), which is given by Proposition 1.

**Proposition 1:** When the software vendor adopts the hybrid licensing model and can commit to future prices, there exists a feasible set of parameter values such that the profit-maximizing problem of the software vendor is joint concave of the prices. Hence, a unique equilibrium exists under which the software vendor charges for version 1 software

\[
\frac{1}{2}(1+\beta) \left( q_1 - \frac{e\beta \left( 2(q_1(Eq_2-q_1)-eEq_2)+eq_1 \right)}{4(1+\beta)(q_1(Eq_2-q_1)-eEq_2)+(4+3\beta)e^2} \right)
\]

in period 1, upgrade price

\[
\frac{1}{2}(q_2^h-q_1) + \frac{e \left[ 2(1+\beta)q_1(2Eq_2-q_2^h-q_1)-e(4(1+\beta)Eq_2-(4+3\beta)q_2^h+(2+\beta)q_1) \right]}{2 \left[ 4(1+\beta)(q_1(Eq_2-q_1)-eEq_2)+(4+3\beta)e^2 \right]}
\]

in period 2, and per period subscription fee

\[
\frac{1}{2}(1+\beta) \left( q_1 + \beta E(q_2) + \frac{e^2\beta \left( (2+\beta)Eq_2-(3+2\beta)q_1 \right)}{4(1+\beta)(q_1(Eq_2-q_1)-eEq_2)+(4+3\beta)e^2} \right).
\]

**Proof:** See Appendix A.

The optimal price for version 1 software is based on its quality, and the vendor will have incentive to lower the price of the first version to increase the installed base in period 1 due to the network effect. The vendor charges the subscription fee based on the quality of version 1 and the (discounted) expected quality of version 2. He or she also takes into account the network effect, trading off the conflict of profit margin and market share.

The price discount offered by the subscription contract to those consumers who always adopt the highest-quality product,
is increasing with $q^h_2 - q^l_2$, which is one of the measures of the variance of the quality of version 2. This is because with the increase in the uncertainty of the future-version software, users are more willing to purchase version 1 and keep the option of upgrading depending on the realized quality value, rather than enter the subscription contract. Hence, the vendor has to offer a greater price discount to those users to attract them to subscribe from period 1. Equation (13) suggests that price discount disappears when there is no uncertainty in quality of version 2: $q^h_2 = q^l_2$.

Figure 5 shows the equilibrium market segmentation under the hybrid licensing policy with the changes of probability of a high quality for version 2, $r$, and of the intensity of the network effect, $e$.

As in market segmentation under the perpetual licensing model, the software vendor will be better off committing to a very high price for version 2 software to prevent consumers from balking to purchase it in period 2 (leapfrogging) and providing enough incentive for those users with high quality preference to enter the subscription contract. Those with a medium quality preference will purchase version 1 in period 1 and keep using it without upgrading.

When there is no network effect—that is, $e = 0$—those users who buy version 1 but do not upgrade will not contribute to the vendor’s second-period profit. Therefore, it is optimal for the vendor to charge a high price for version 1 to convert those consumers to either subscribe or stay inactive in both periods. Thus, the equilibrium market share and vendor profit are the same as those under the pure subscription strategy in the Subscription subsection above.

When the network effect is significant, $e > 0$, a consumer’s utility increases with the installed base, so the vendor has a greater incentive to increase the market share in the early period. Those customers who do not upgrade will increase the installed base of

Figure 5. Market Segmentation in Equilibrium Under the Hybrid Licensing Policy
Notes: $\beta = 0.8$, $q_1 = 1$, $q^l_2 = 1.4$, $q^h_2 = 1.9$; (a) $e = 0.2$; (b) $\rho = 0.8$. 

$$
\frac{\beta}{2} (1-r) \left( q^h_2 - q^l_2 \right) \left( 1 - \frac{e(2(1+\beta)q_1 - (4+3\beta)e)}{4(1+\beta)(q_1(Eq_2 - q_1) - eEq_2) + (4+3\beta)e^2} \right),
$$

(13)
the software, and they thus help increase other users’ utility to the benefit of the vendor. With an increase in the intensity of the network effect, the software vendor will lower the first-period price to attract more early buyers. As a result, the vendor can charge more users a higher subscription and upgrade price. Summing up the discounted profits over the two periods, the vendor is better off with a greater network effect.

We next compare the three licensing strategies in terms of pricing, profit, consumer surplus, and social welfare.

Comparison of the Three Licensing Models

Comparing the market structure and the software vendor’s profit among the three license models—perpetual license, subscription, and a hybrid model—we have the following conclusions from our analytical results:

Proposition 2: When a software vendor can commit to future prices and there is no network effect,

(a) the monopolist software vendor receives less market share but a higher profit by adopting the subscription model rather than the perpetual licensing model; the hybrid licensing model has the same equilibrium as the subscription model: \( \Pi^p \leq \Pi^L = \Pi^H \);

(b) those consumers who buy and do not upgrade incur a lower total expense under the perpetual licensing model than under the subscription model \( (p_1 < (1 + \beta)p_r) \), whereas those who always use the latest version have to pay more under the subscription model than under perpetual licensing \( (p_1 + \beta p_u < (1 + \beta)p_r) \); and

(c) the consumer surplus and social welfare when the vendor chooses the perpetual licensing model are higher than or equal to those under the subscription model: \( CS^L = CS^H \leq CS^p \) and \( W^L = W^H \leq W^p \).

Proof: See Appendix A.

If there is no network externality effect \( (e = 0) \) and the vendor can commit to future pricing, it is optimal for the vendor to sell only to high-end users who always adopt the latest-version software but not to those users who buy and hold, those who selectively upgrade, or those who opportunistically leapfrog. The vendor prefers the subscription strategy, under which the vendor targets a smaller market share but extracts more surplus from those consumers. The difference of the profits under the subscription and the perpetual licensing models is proportional to \( (q^h_2 - q^l_1)^2 \). Thus, the higher the variance of the quality of version 2 software, the more likely the vendor chooses the subscription licensing model. The vendor’s choice, however, is inconsistent with the social optimum.

Software, however, has a strong network effect because there is value created from file sharing and knowledge exchange among adopters as well as from compatible products [2]. Users therefore will value more a software product with a greater population of adopters. When we consider the network externality effect and compare the market equilibria under the three models, we obtain Proposition 3:
Proposition 3: When a software vendor can commit to future decisions and there is no uncertainty regarding quality of version 2,

(a) the perpetual licensing or hybrid model weakly dominates the subscription model in terms of both market share and profit: \( \Pi^\ell \leq \Pi^p = \Pi^{hl} \);

(b) those consumers who buy and do not upgrade incur a lower total expense under the perpetual licensing model than under the subscription model \( (p_1 < (1 + \beta) p_r) \), whereas those who always use the latest version have to pay more under the perpetual licensing model than under the subscription model \( (p_1 + \beta p_u > (1 + \beta) p_r) \); and

(c) the consumer surplus and social welfare are no higher when the vendor chooses the subscription model: \( CS^\ell \leq CS^p = CS^{hl} \), \( WL^\ell \leq WL^p = WL^{hl} \).

Proof: See Appendix A.

When the committed vendor sells the software in a market with network effect and there is no uncertainty regarding the quality of version 2 software, the perpetual licensing model will generate both a greater market share (Figure 6) and a higher profit (Figure 7). The differences of the market share and profit between the perpetual or hybrid licensing model and the subscription model are increasing with the intensity of the network effect \( e \).

Consumers are also better off under the perpetual or hybrid licensing model. The perpetual licensing model allows the medium-value consumers to buy version 1 without the obligation of upgrading, whereas a subscription model forces them to upgrade by contractual agreement. The purchase option therefore significantly reduces those users’ cost of using the first version of the software (Figure 8). With the consequent increase in the population of users, the network externality effect becomes stronger, enabling the vendor to charge a higher upgrade price to the high-value consumers. Figure 8 indicates that consumers who always use the latest version of the software thus incur a higher total cost under the perpetual or hybrid licensing model than under the subscription model. Their loss, however, is outweighed by other consumers’ gains, so the total consumer surplus under the perpetual or hybrid licensing model is greater (Figure 9).

Proposition 3 has important implications for the recent subscription licensing activities in the software industry. Subscriptions with automatic upgrades deprive some medium-value consumers of the flexibility of buying but not upgrading. A subscription model thus forgoes some market share, which has great value for goods with a network effect. We show that when the software vendor can commit to future prices and there is no uncertainty regarding the quality of the future-version software, both the vendor and users prefer the perpetual or hybrid licensing model to a subscription model.

The foregoing results suggest that when the quality uncertainty or the network effect is considered alone, the software vendor prefers the subscription model when the uncertainty of the quality of version 2 is high and prefers the perpetual licensing model when the network effect is great. When there is both network effect associated with the product and uncertainty in the quality of version 2 software, it is unclear which licensing model will be a better choice for the vendor. Because the analytical
results are not very straightforward to derive comparison results, we use the following numerical example to compare the equilibria of the three licensing models. We pick the values of the parameters to make sure both the quality uncertainty of version 2 and the network externality effect are not too small and all the assumptions of the model are satisfied:

\[ \beta = 0.8, q_1 = 1, q_2^l = 1.5, q_2^h = 2.1, \text{ and } \rho = 0.7. \]

When the network effect is large enough, the software vendor receives the highest profit (Figure 10) and the greatest market share (Figure 11) through the hybrid licensing model. The vendor has a medium-level profit but the smallest market share under the subscription model. The market share and the gain in profit are increasing with the intensity of the network effect. The perpetual licensing model provides users
with the option of buying version 1 without the obligation of upgrading. Buy-and-hold consumers can increase the software’s user population to the benefit of the vendor. On the other hand, under the perpetual licensing model, the software vendor has to charge the same upgrade price $p_u$ to all the adopters of version 1 with different quality preferences. Adding subscription to the perpetual licensing model can better refine those users with the subscription fee $p_r$ and the upgrade price $p_u$. By narrowing the segment of conditional upgrading consumers, the vendor can reduce the downside risk of the quality uncertainty on his or her profit. Therefore, when the quality of new-version software is uncertain and the network effect is large enough, the vendor should choose the hybrid licensing model.

With the increase of the network effect, the vendor will have a greater incentive to cut the first-period prices to enlarge the installed base (Figure 12) so that he or she

---

**Figure 8.** Total Cost of Using the Latest-Version Software Under the Perpetual or the Hybrid Licensing Model (the long-dotted curve) Versus Total Rents Under Subscription (the solid curve) Versus Cost of Buy-and-Hold Under the Perpetual Licensing (the short-dotted curve) When $\beta = 0.7, q_1 = 0.9, q_2 = 2.18$

**Figure 9.** Consumer Surplus of the Perpetual or the Hybrid Licensing Model (the dotted curve) Versus Subscription (the solid curve) When $\beta = 0.7, q_1 = 0.9, q_2 = 2.18$
can charge a higher upgrade price in the second period (Figure 13) to extract part of the consumer surplus increased due to the network effect.

Figures 14 and 15 show that adding perpetual licensing to the subscription model will not only increase the vendor’s profit but will also increase consumer surplus and social welfare. Consumers benefit from the flexibility in updating under the perpetual licensing model, which has the highest consumer surplus. When the network effect is strong enough, the vendor’s licensing model selection (the hybrid model) is consistent with social optimum (Figure 15).

This section provides key theorems with ample analysis and evidence to show how the vendor should trade off the benefit and cost of the perpetual and subscription licensing models to choose the optimal one based on the levels of the network effect and of the uncertainty regarding the quality of future-version software: when the network effect is large, he or she will be more likely to choose the perpetual licensing model to increase the installed base in period 1; when the quality uncertainty is large, he or she will be more likely to adopt the subscription licensing model to lock in the users.
from period 1; and when both effects exist and are significant, it may be optimal for the vendor to adopt a mixed strategy of both licensing models.

Even though the foregoing results are based on the assumption that the vendor can commit to future prices, we now discuss that relaxing this assumption does not change our major conclusions. When the vendor cannot commit to the future prices, we will solve the problem as a two-stage game by backward induction. The vendor will announce the second-period selling and upgrading prices after the quality of version 2 is realized. Therefore, given a quality level of version 2, the vendor will price conditionally and receive a conditional second-period profit. Then the vendor will consider the uncertain profit of period 2 when he or she decides the first-period prices by maximizing the discounted total profit of the two periods. Consistent with our results under the committed scenario, a higher uncertainty in quality of version 2 will cause bigger losses in the vendor’s discounted total profit, and the vendor will have greater incentive to adopt the subscription licensing model to remove the variance in profit. Even when the vendor cannot commit to future prices, the path dependence
of the network effect still exists. Hence, those consumers who buy and hold who can only exist in the perpetual licensing model will still add value to the utilities of later adopters and indirectly contribute to the vendor’s profit. When the network effect is large enough, the vendor is better off choosing a hybrid licensing model. In this paper we focus on the effects of network effect and quality uncertainty on the vendor’s licensing model selection assuming the vendor can commit to future prices because (1) this assumption is supported by literature such as Waldman [27], (2) considering the uncommitted case will lose the tractability of the problem without adding enough new insights, and (3) the main insights of the paper will still hold in the uncommitted scenario.

Conclusions and Suggestions for Future Research

Software vendors now can choose to deliver software based on either a perpetual ownership licensing model, which offers customers the permanent right to use and own the version of the software they purchased, or a multiperiod subscription model, which includes a commitment to a periodic payment for continuous usage of the latest version of the software during the contract periods. Recent speculations of practitioners in the software industry predict that subscription is likely to become the dominant means of licensing software.

Our paper addresses this issue by examining the optimal licensing strategy for a monopoly software vendor with a two-period analytical model combining vertical differentiation and intertemporal price discrimination. We suggest that software vendors should consider the uncertainty of quality improvement and the intensity of the network effect in choosing their optimal licensing models. A high level of quality uncertainty will increase the likelihood of consumers’ delaying making the upgrade decisions until the quality is realized. In the presence of quality uncertainty, vendors should consider using subscription licensing to lock consumers in during the first period to prevent their opportunistic behaviors. For example, one of Microsoft’s strategies
to fight the slowing speed of introducing new versions of software products and the competition with itself is providing and actively promoting the subscription licensing policy to its enterprise users.

However, when there is a strong network effect, as is the case with most software, the subscription model prevents some earlier adopters from generating positive externalities to the later users, which reduces the value of the software to consumers and hurts the vendor’s bottom line. On the other hand, perpetual licensing provides users the right to keep using the same version of the software in later periods without further payment. Due to backward compatibility, those users will contribute to the network size and add value to later adopters. Therefore, a strong network effect will increase the vendor’s incentive to lower the price of the first version in order to increase the installed base, thereby increasing the network effect and values of future adopters, which will in turn allow the vendor to extract more surpluses from adopters of later periods. When this effect is pronounced, the vendor is better off choosing a hybrid model rather than the pure subscription model. Our results are validated by the actual licensing practices in the software market. The majority of the software vendors, such as Microsoft, Sun, Oracle, and SAS, are not completely using subscription to replace the perpetual licensing model but have added subscription as another option in their original licensing models.

Our findings should also help users realize the benefits and costs of each licensing model and help them make the best decision, taking into account their own characteristics, product upgrades, quality uncertainty, and network effect. A subscription model smooths out cash payments and replaces a single lump sum with a per period payment, which may be attractive when software budgets are limited. Consumers need to consider the total cost of ownership, however, when evaluating their options. Our model shows that under the subscription model, users can always enjoy the latest version of the software, but they are deprived of the option of not upgrading. The total cost of subscription is higher than buying but not upgrading. Thus, the claim that the subscription model lowers the cost of ownership is misleading and myopic.

Figure 15. Social Welfare Under the Hybrid (the solid curve), Perpetual (the short-dotted curve), and Subscription (the long-dotted curve) Licensing Policies with the Change of the Network Effect When $\beta = 0.8$, $q_1 = 1$, $q_2^f = 1.5$, $q_2^h = 2.1$, and $\rho = 0.7$
Our consideration of consumer surplus and social welfare likewise has implications for policymakers. The subscription model gives the software vendor more market power but deprives consumers of upgrade options. Consumer surplus is higher under the perpetual and hybrid licensing models. Moreover, social welfare is also lowest under the subscription model according to Propositions 2 and 3. Hence, policymakers should encourage a mix of the perpetual and subscription models rather than the pure subscription model in the software market.

Network externalities and quality uncertainty combined with compatibility issues endogenize the externality of a user’s adoption decision into the decision of the other users. Thus, network effects significantly complicate the study of software licensing models. In order to explore the effect of network externalities in particular, we have made several assumptions to simplify the problem. Relaxing those assumptions does not affect our major conclusions but does create interesting opportunities for future research.

First, like Fudenberg and Tirole [11], we only considered the general case, when quality improvement is moderate. When quality improvement is large, Dhebar [9] has shown that if the durable goods producer cannot credibly commit to future prices and quality, then no equilibrium strategy exists. Kornish [16] shows that an equilibrium pricing strategy exists if the durable goods monopolist does not offer upgrade pricing. Because some of the features of software—such as negligible marginal production costs, lack of residual value, and easy upgrading—can simplify the durable goods problem, future research can look into the effect of network externalities on software licensing models when there is significant technological advancement.

Second, we assume that the second-period quality is exogenous so as to remove the quality selection stage in the vendor’s decisions. Choudhary [5] studies the effect of a subscription policy on the software vendor’s investment in development. A more complete study is expected to address the interaction of licensing policy selections, investment in innovation, and pricing structures.

Third, our work assumes that the vendor only charges a periodic rent for subscriptions, which is a simplification of a more general fixed-fee plus usage-based pricing model. Fixed-fee pricing, which is studied in this paper, is common among large independent software vendors, but application service providers and cloud computing providers generally prefer to use the two-part tariff pricing model. Huang and Sundararajan [14] and Seidmann and Ma [23] examine pricing strategies in a pay-per-use, on-demand environment. We can generalize our results by applying a more complicated pricing structure in the subscription model.

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Note

1. Refer to the notation in Appendix B.
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Appendix A

Proof to Lemma 1

The values of a consumer under each of the six choices (II, IC, IB, BH, BC, BU) are as follows:

\[ V_{BU}(\theta) = \theta(q_1 + \beta E(q_2)) + e(x_0 + \beta x_2) - (p_1 + \beta p_u) \]
\[ V_{BC}(\theta) = \theta(q_1 + \beta p q_2^h + \beta (1 - \rho) q_1) + e(x_0 + \beta p x_2 + \beta (1 - \rho) x_1) - (p_1 + \beta p p_u) \]
\[ V_{BH}(\theta) = (1 + \beta) \theta q_1 + e(x_0 + \beta x_1) - p_1 \]
\[ V_{IB}(\theta) = \beta(\theta E q_2 + e x_2 - p_2) \]
\[ V_{IC}(\theta) = \beta p(\theta q_2^h + e x_2 - p_2) \]
\[ V_{IU}(\theta) = 0. \]

By the assumption that \((1 + \beta)q_1 \geq \beta E q_2\), the sensitivities of a consumer’s expected values of different adoption strategies with respect to his or her type \(\theta\) are in the order of

\[ 0 < V^*_IC(\theta) < V^*_IB(\theta) < V^*_BH(\theta) < V^*_BC(\theta) < V^*_BU(\theta). \]

Consumers with the highest quality preference \(\theta\) therefore will choose to buy and upgrade; those with a next highest \(\theta\) will buy version 1 in period 1 but only upgrade to version 2 when its quality is high; those with a relatively high \(\theta\) will choose to buy and hold; those with a moderate \(\theta\) will wait to buy in period 2; those with a low \(\theta\) will buy in period 2 only if the quality of version 2 is high; and those with the lowest \(\theta\) will not participate in either period. Q.E.D.

Proof to Proposition 1

In the hybrid licensing model, consumers have the option of subscribing or buying, and if buying, they can decide in which period to purchase and whether to upgrade (see Figure 4). Taking into account the consumers’ self-selection behavior, the software vendor has demand for his or her first-period sales \(d_1\), demand for the subscription contract \(d_s\), demand for the upgrade version in period 2 \(d_u\), and demand for purchasing version 2 in period 2 \(d_2\).
Taking into account the consumers’ self-selection behavior, the software vendor decides prices $p_1, p_r, p_2,$ and $p_u$ to maximize his or her discounted total profit over the two periods:

$$\max_{p_1, p_u, p_2, p_r} \Pi(p_1, p_u, p_2, p_r) = p_1d_1 + \beta pp_u d_u + \beta p_2 d_2 + (1+\beta) p_r d_r$$

s.t. \begin{align*}
0 \leq d_u \leq d_1 \\
0 \leq d_2 \leq 0 \\
0 \leq d_r \leq 0 \\
p_2 \geq p_u.
\end{align*}

Applying the Kuhn–Tucker theorem to solve this constrained optimization problem, we compare the profits under different combinations of Kuhn–Tucker conditions and find that the optimal solution to Equation (A2) involves no user leapfrogging. Assuming the value of the parameters are in the range that makes the profit function joint concave, we have the optimal price schedule for the software vendor (we use the superscript $H$ to represent the hybrid licensing policy):

\begin{align*}
\left\{ \begin{array}{l}
\left( \begin{array}{l}
p^H_{1} = \frac{1}{2}(1+\beta) \left[ q_1 - \frac{e\beta (2q_1(Eq_2-q_1) - eEq_2) + eq_1}{4(1+\beta)(q_1(Eq_2-q_1) - eEq_2) + (4+3\beta)e^2} \right]
\end{array} \right)
\left( \begin{array}{l}
p^H_{r} = \frac{1}{2(1+\beta)} \left[ \beta E(q_2) + q_1 + \frac{e^2\beta (2+\beta)Eq_2 - (3+2\beta)q_1}{4(1+\beta)(q_1(Eq_2-q_1) - eEq_2) + (4+3\beta)e^2} \right)
\end{array} \right)
\left( \begin{array}{l}
p^H_{u} = \frac{1}{2} \left[ q_2 - q_1 \right]
\end{array} \right)
\end{align*}

$$p^H_{2} = \frac{1}{2} \left[ 2(1+\beta)q_1 \left( 2Eq_2 - q_2^h - q_1 \right) - e \left( 4(1+\beta)Eq_2 - (4+3\beta)q_2^h + (2+\beta)q_1 \right) \right]$$

\begin{align*}
\left( \begin{array}{l}
2 \left[ 4(1+\beta)(q_1(Eq_2-q_1) - eEq_2) + (4+3\beta)e^2 \right]
\end{array} \right).
\end{align*}

Q.E.D.

**Proof to Proposition 2**

When $e = 0$, by the market share and optimal profits obtained from the equilibrium results of the three models, we have the following:
(a) Given $0 \leq \rho \leq 1$, $\beta > 0$, $q_i \leq q^i_2 \leq q^h_2$, we have

$$\theta^H_0 - \theta^L_0 = 0$$

$$\theta^P_0 - \theta^L_0 = \frac{-\beta(1-\rho)(q^h_2 - q^l_2)(q^h_2 - q_1)}{2\left(\beta q^h_2 ((1-\rho)q^h_2 + \rho q^l_2) + (q^l_2 + \beta(Eq^h_2 - 2q^h_2)) q_1 - q_1^2\right)} < 0$$

$$\Pi^L = \Pi^H = \frac{q_1 + \beta Eq^h_2}{4}$$

$$\Pi^H - \Pi^P = \frac{\beta(1-\rho)(q^h_2 - q^l_2)^2 \left(\beta p(q^h_2 - q_1) + (1+\beta)q_1\right)}{4\left(\beta q^h_2 ((1-\rho)q^h_2 + \rho q^l_2) + (q^l_2 + \beta(Eq^h_2 - 2q^h_2)) q_1 - q_1^2\right)} > 0.$$ Therefore, the perpetual licensing model has a greater market share but a lower profit than the subscription model.

(b) 

$$(1+\beta)p_e - (p_1 + \beta p_u) = \frac{\beta(1-\rho)(q^h_2 - q^l_2)(\beta p(q^h_2 - q_1) + q^h_2 q_1)}{2\left(\beta q^h_2 ((1-\rho)q^h_2 + \rho q^l_2) + (q^l_2 + \beta(Eq^h_2 - 2q^h_2)) q_1 - q_1^2\right)} \geq 0.$$ Thus, we have $(1+\beta)p_e \geq (p_1 + \beta p_u) > p_1$.

(c) 

$$CS^P - CS^L = \frac{3\beta(1-\rho)(q^h_2 - q^l_2)^2 \left(\beta p(q^h_2 - q_1) + (1+\beta)q_1\right)}{8\left(\beta q^h_2 ((1-\rho)q^h_2 + \rho q^l_2) + (q^l_2 + \beta(Eq^h_2 - 2q^h_2)) q_1 - q_1^2\right)} \geq 0$$

$$WP^P - W^L = \frac{\beta(1-\rho)(q^h_2 - q^l_2)^2 \left(\beta p(q^h_2 - q_1) + (1+\beta)q_1\right)}{8\left(\beta q^h_2 ((1-\rho)q^h_2 + \rho q^l_2) + (q^l_2 + \beta(Eq^h_2 - 2q^h_2)) q_1 - q_1^2\right)} \geq 0.$$ Therefore, we have $CS^L = CS^H \leq CS^P$, $W^L = W^H \leq W^P$. Q.E.D.

Proof to Proposition 3

When there is no uncertainty regarding $q_2$, and by the equilibria of the three models presented in the equilibrium results of the three models, we have the following:

(a) 

$$\Pi^P - \Pi^L = \frac{\beta e^2 ((2 + \beta)q_2 - (3 + 2\beta)q_1)^2}{4\left(q_1 + \beta q_2 - (1 + \beta)e\right)\left((4 + 3\beta)e^2 + 4(1+\beta)(q_1 (q_2 - q_1) - e q_2)\right)} \geq 0$$

$$\Pi^H - \Pi^P = 0.$$
In the range of values of parameters of this equilibrium, we have $\Pi^L \leq \Pi^P = \Pi^H$.

(b) 

$$p_1 + \beta p_u - (1 + \beta) p_r = \frac{\beta \left( (2 + \beta) q_2 - (3 + 2\beta) q_1 \right)^2}{2 \left( (4 + 3\beta) e^2 + 4 (1 + \beta) \left( q_1 (q_2 - q_1) - e q_2 \right) \right)} > 0.$$ 

Given the range of values of parameters of this equilibrium, we have $p_1 + \beta p_u > (1 + \beta) p_r$.

$$\frac{(1 + \beta) p_r - p_1}{\beta \left( (2 + \beta) q_2 - (3 + 2\beta) q_1 \right) e^2 - 2 (1 + \beta) (q_2 - q_1) (2 q_2 - q_1) e + 4 (1 + \beta) q_1 (q_2 - q_1)^2}{2 \left( (4 + 3\beta) e^2 + 4 (1 + \beta) \left( q_1 (q_2 - q_1) - e q_2 \right) \right)}$$

$$= \frac{(q_2 - q_1)^2 \left[ (2 + \beta) q_2 - (3 + 2\beta) q_1 \right]^2 + (4 + 10\beta + 5\beta^2) q_1^2}{2 \left( (2 + \beta) q_2 - (3 + 2\beta) q_1 \right) \left( (4 + 3\beta) e^2 + 4 (1 + \beta) \left( q_1 (q_2 - q_1) - e q_2 \right) \right)} > 0.$$ 

Given the range of values of parameters of this equilibrium, we have $(1 + \beta) p_r > p_1$.

(c) The vendor’s market share under the perpetual or hybrid licensing model is greater than that under the subscription model, and the medium- and low-value consumers pay less to use the software. The total gain of those consumers outweighs the loss of those high-value consumers who pay more to buy and upgrade instead of subscribing:

$$CS^P - CS^L = \frac{\beta e^2 \left( (3 + 2\beta) q_1 - (2 + \beta) q_2 \right)^2}{8 (q_1 + \beta q_2 - (1 + \beta) e) \left( (4 + 3\beta) e^2 + 4 (1 + \beta) q_1 (q_2 - q_1) - 4 (1 + \beta) e q_2 \right)^2}.$$ 

$$\left[ 12 (1 + \beta) q_1 (q_2 - q_1) (q_1 + \beta q_2) - 8 (1 + \beta) e \left( \beta q_2^2 + (2 + \beta) q_1 q_2 - (1 + \beta) q_1^2 \right) + e^2 \left( (4 + 3\beta) q_1^2 + (4 + 12\beta + 7\beta^2) q_2 \right) \right].$$

Although the network effect considered is small compared with the quality effect—that is, $e$ is small compared with $q_1$ and $q_2$—the total consumer surplus under the perpetual or hybrid licensing model is higher: $CS^P \geq CS^L$.

Because social welfare is defined as the sum of firm profit and consumer surplus, $W = \Pi + CS$ and $\Pi^P \geq \Pi^L$ and $CS^P \geq CS^L$, we have $W^P \geq W^L$. Because $CS^H = CS^P$ and $W^H = W^P$, the results can be summarized as $CS^L \leq CS^P = CS^H$ and $W^L \leq W^P = W^H$. Q.E.D.
Appendix B

List of Variables Used in the Analytical Model

- $\theta$: A consumer’s intensity of the quality preference, assumed to be uniformly distributed on the support $[0, 1]$.
- $e$: The intensity of the network externality effect.
- $q$: Quality of the software includes dimensions such as features, speed, functionalities, user interface, ease of learning, and other characteristics that affect users’ valuation of the product.
- $q_1$: The quality of version 1.
- $q_2$: The quality of version 2: can be either $q_2^h$ or $q_2^l$.
- $\rho$: Probability the quality of version 2 is high $q_2^h$.
- $\beta$: Discount factor in the range of $[0, 1]$.
- $p$: Price paid for the software.
- $p_1$: Selling price of version 1.
- $p_2$: Selling price of version 2.
- $p_r$: Per period subscription fee.
- $p_u$: Upgrade price.
- $x_0$: The mass of adopters in period 1.
- $x_1$: The mass of first-period adopters who do not upgrade in period 2.
- $x_2$: Total adopters of either version in period 2.
- $V_L(\theta)$: The total expected value of a consumer of type $\theta$ when he or she subscribes to the software in both periods.
- $V_p(\theta)$: The total expected value of a consumer of type $\theta$ when he or she buys the software in period 1 and conditionally upgrades in period 2.
- $V_i(\theta)$: The total expected value of a consumer of type $\theta$ when he or she waits to adopt the software in period 2.
- Superscript $P$: The equilibrium result under the perpetual licensing strategy.
- Superscript $L$: The equilibrium result under the subscription strategy.
- Superscript $H$: The equilibrium result under the hybrid strategy.
- $\Pi$: Vendor’s total discounted profit.
- $CS$: Consumer surplus.
- $W$: Social welfare.

Summary of Equilibria Results Under Different Licensing Strategies

Subscription Strategy

Segmentation:

$$\theta_L = \frac{(1+\beta)(p_r - e)}{q_1 + \beta E(q_2) - e(1+\beta)}.$$
Pricing:

\[ p^L_t = \frac{q_1 + E(q_2)}{2(1+\beta)}. \]

Total discounted profit:

\[ \Pi^L = \frac{(q_1 + \beta E q_2)^2}{4(q_1 + \beta E q_2 - e(1+\beta))}. \]

Perpetual Licensing Strategy

The equilibrium results when there exist positive network effects too massive to show. Therefore, we show the results when there is no network effect \((e = 0)\), which is a special case of the general result.

Segmentation:

\[ x^P_H = \frac{(q_1 + \beta q^h) (q'_2 - q_1)}{2q^h (1-\rho) q^h_2 + \rho q'_2 - q_2 (1-\beta(1-\rho)) - q_1^2}, \]

\[ x^P_{BC} = \frac{(q_1 + \beta q^h) (q'_2 - q_2)}{2q^h (1-\rho) q^h_2 + \rho q'_2 - q_2 (1-\beta(1-\rho)) - q_1^2}, \]

\[ x^P_{BV} = 1 - x^P_{BC} - x^P_H. \]

Pricing:

\[ p^P_t = \frac{(1+\beta)q_1 (q_1 + \beta q^h) (q'_2 - q_1)}{2q^h (1-\rho) q^h_2 + \rho q'_2 - q_2 (1-\beta(1-\rho)) - q_1^2} \]

\[ p^P_u = \frac{(q_1 + \beta q^h) (q'_2 - q_1) (q'_2 - q_1)}{2q^h (1-\rho) q^h_2 + \rho q'_2 - q_2 (1-\beta(1-\rho)) - q_1^2}. \]

Total discounted profit:

\[ \Pi^P = (p^P_t + \beta p^P_u) x^P_{BU} + (p^P_t + \beta p^P_u) x^P_{BC}. \]
Hybrid Strategy

Segmentation:

\[ x_H^H = \frac{(2(1+\beta)q_1 - (4 + 3\beta)e)(Eq_2 - q_1 - e)}{4(1 + \beta)(q_1(Eq_2 - q_1) - eE_q_2) + (4 + 3\beta)e^2} \]

\[ x_{BH}^H = \frac{e((2 + \beta)Eq_2 - (3 + 2\beta)q_1)}{4(1 + \beta)(q_1(Eq_2 - q_1) - eE_q_2) + (4 + 3\beta)e^2} \]

\[ x_{BU}^H = 1 - x_{BH}^H - x_H^H. \]

Pricing:

\[ p_1^H = \frac{1}{2}(1+\beta)\left\{ q_1 - \frac{\beta E(2q_1 - q_2 - q_1 - e)(E_q_2 - q_1 - eE_q_2) + e_1}{4(1 + \beta)(q_1(Eq_2 - q_1) - eE_q_2) + (4 + 3\beta)e^2} \right\} \]

\[ p_r^H = \frac{1}{2}(1+\beta)\left( \beta E(q_2) + q_1 + \frac{e^2\beta((2 + \beta)Eq_2 - (3 + 2\beta)q_1)}{4(1 + \beta)(q_1(Eq_2 - q_1) - eE_q_2) + (4 + 3\beta)e^2} \right) \]

\[ p_u^H = \frac{1}{2}(q_2^h - q_1) \]

\[ + \frac{e\left[ 2(1 + \beta)(2 E_q_2 - q_2^h - q_1) - e\left( (1 + \beta)Eq_2 - (4 + 3\beta)q_2^h + (2 + \beta)q_1 \right) \right]}{2\left( 4(1 + \beta)(q_1(Eq_2 - q_1) - eE_q_2) + (4 + 3\beta)e^2 \right)}. \]

Total discounted profit:

\[ \Pi^H = (p_1^H + \beta p_u^H)x_{BU}^H + p_1^H x_{BH}^H. \]