NOTES ON

ELEMENTARY GAME THEORY

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1. **Game theory**. **Games** describe situations where there is potential for conflict and for cooperation. Many business situations, as well as many other social interactions have both of these such features.

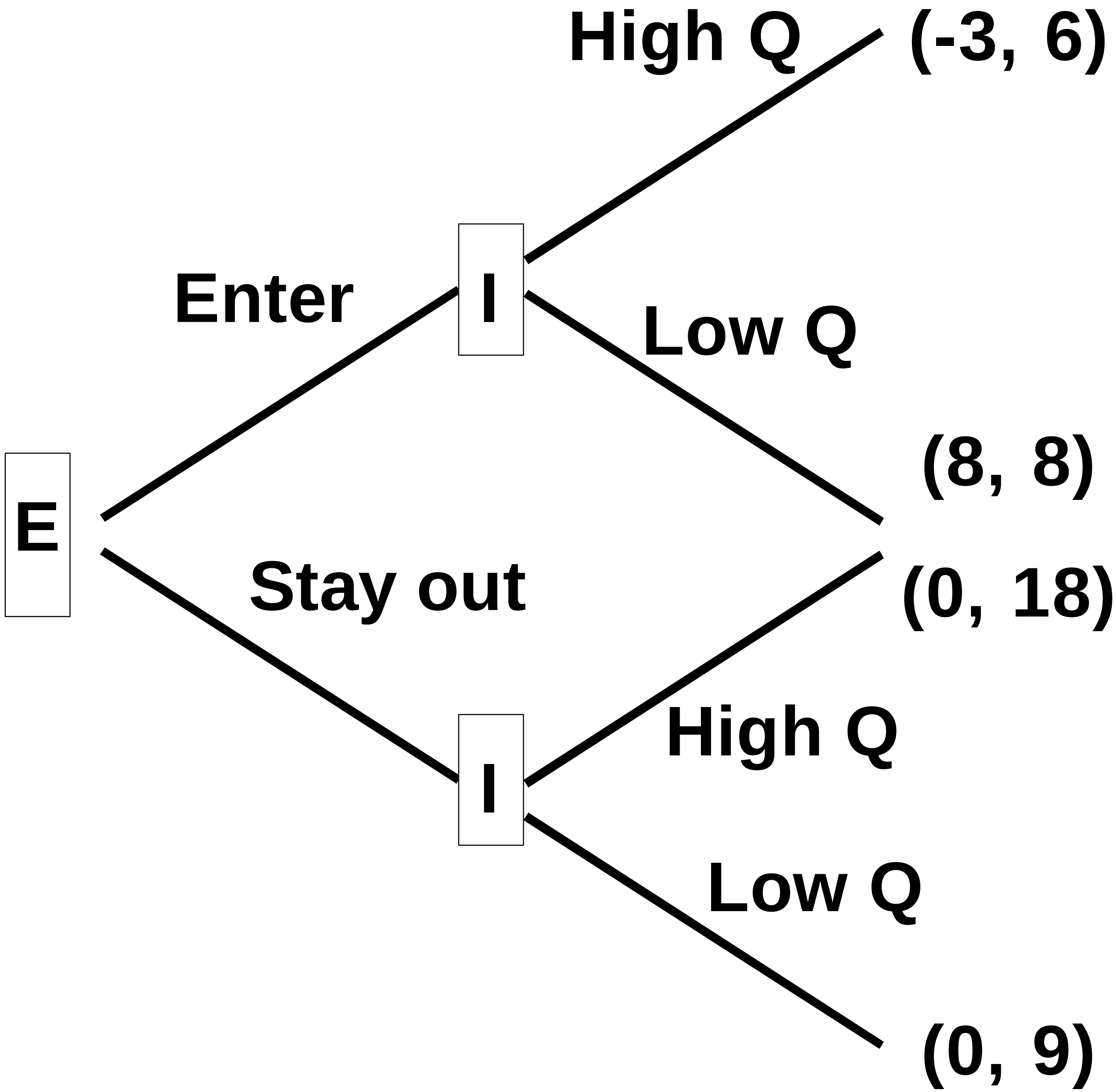
Example 1: Company X would like to be the only seller of a product (a monopolist). The existence of competing firm Y hurts the profits of firm X. Firms X and Y could cooperate, reduce total production, and increase profits. Or they could compete, produce a high quantity and realize small profits. What will they do?

Example 2: Bank 1 competes with bank 2 for customers. Many of their customers use Automated Teller Machines (ATMs). Suppose that each bank has a network of its own ATM machines which are currently available only to its customers. Should bank 1 allow the customers of the other bank to use its ATMs? Should bank 1 ask for reciprocity?

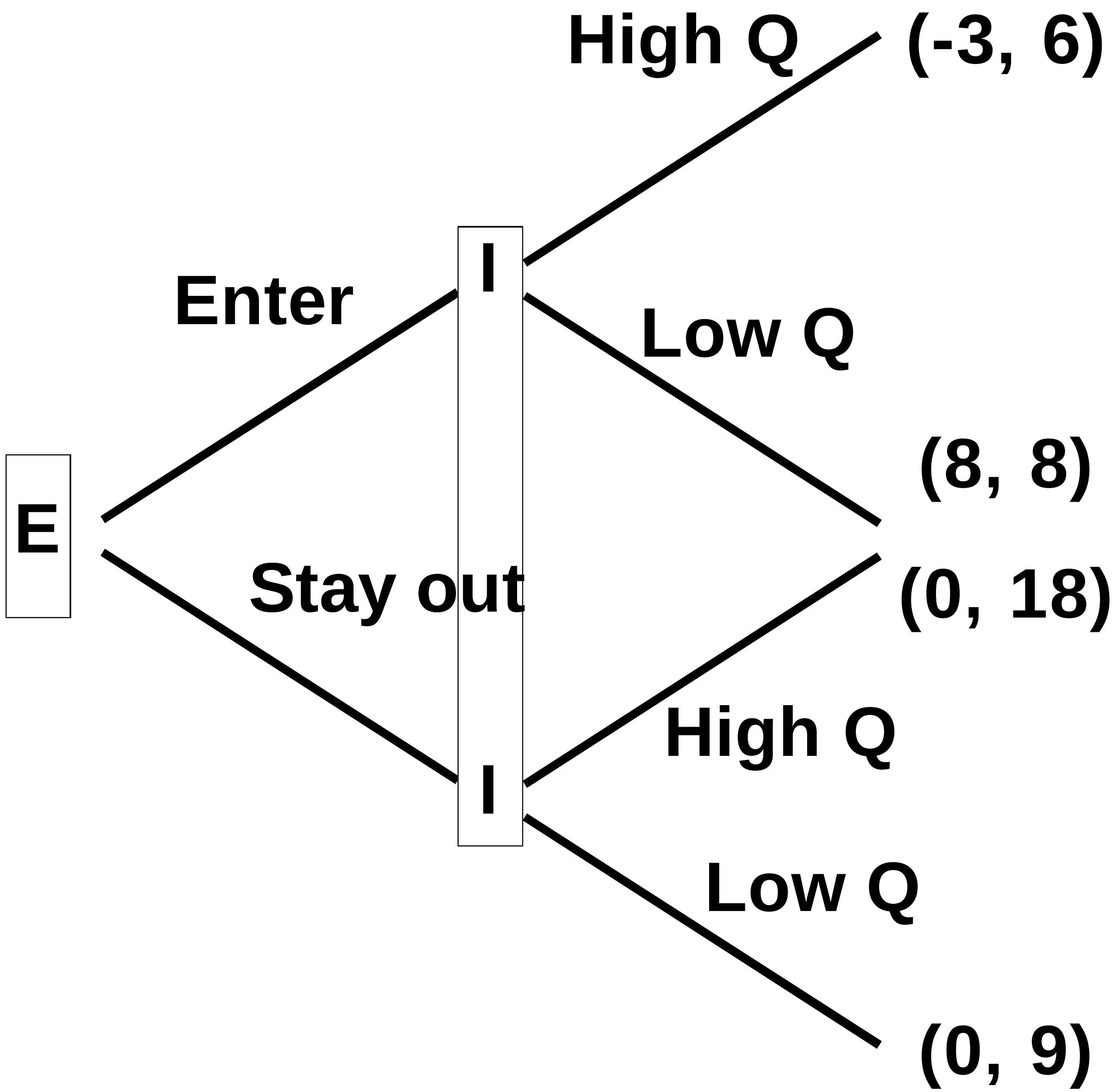
Example 3: Computer manufacturer 1 has a cost advantage in the production of network "cards" (interfaces) of type 1. Similarly manufacturer 2 has an advantage in network "cards" of type 2. If they end up producing cards of different types, their profits will be low. However, each firm makes higher profits when it produces the "card" on which it has a cost advantage. Will they produce "cards" of different types? Of the same type? Which type?

A **game in extensive form** is defined by a set of players, i = 1, ..., n, a **game tree**, **information sets**, **outcomes**, and **payoffs**. The game tree defines the sequence and availability of **moves** in every **decision node**. Each **decision node** is identified with the player that decides at that point. We assume there is only a finite number of possible moves at every node. Each branch of the tree ends at an event that we call an **outcome**. The utility associated with the outcome for every player we call his **payoff**. **Information sets** contain one or more nodes. They show the extent of knowledge of a player about his position in the tree. A player only knows that he is in an information set, which may contain more than one nodes. Information sets allow a game of simultaneous moves to be described by a game tree, despite the sequential nature of game trees. A game where each information set contains only one point is called a game of **perfect information**. (Otherwise it is of **imperfect information**.) For example, in the “Incumbent-Entrant” game (Figure 1), at every point, each player knows all the moves that have happened up to that point. All the information sets contain only a single decision node, and the game is of perfect information. In the “Simultaneous Incumbent-Entrant” game (Figure 2), player I is not sure of player E’s decision. He only knows that he is at one of the two positions included in his information set. It is as if players I and E move simultaneously. This is a game of imperfect information. Note that this small change in the information sets of player I makes a huge difference in what the game represents -- a simultaneous or a sequential decision process.

Incumbent-Entrant game



Simultaneous Incumbent-Entrant game



When the utility functions associated with the outcomes are known to both players, the game is of **complete information**. Otherwise it is of **incomplete information**. You may not know the opponent's utility function. For example, in a price war game, you may not know the value to the opponent firm (or the opponent manager) of a certain loss that you can inflict on them.

2. A **game in normal form** is a summary of the game in extensive form. This is facilitated by the use of **strategies**. *A strategy for player i defines a move for this player for every situation where player i might have to make a move in the game*. A strategy of player i is denoted by s_i, and it belongs in the set of available strategies of player i, S_i. By its nature, a strategy can be very complicated and long. For example, a strategy for white in chess would have to specify the opening move, the second move conditional on the 20 alternative first moves of the black, the third move conditional on the many (at least 20) alternative second moves of the black, and so on. The advantage of using strategies is that,

once each player has chosen a strategy, the outcome (and the corresponding payoffs) are immediately specified. Thus, the analysis of the game becomes quicker.

3. Example 1: **"Simultaneous Incumbent-Entrant"**

		Player 2 (Incumbent)	
		High Q	Low Q
Player 1 (Entrant)	Enter	(-3, 6)	(8, 8)
	Stay out	(0, 18)	(0, 9)

Strategies for Player 1: Enter, Stay out

Strategies for Player 2: High Q, Low Q

Example 2: **Prisoners' Dilemma**

		Player 2	
		silence	talk
Player 1	Silence	(5, 5)	(0, 6)
	Talk	(6, 0)	(2, 2)

Strategies for player 1: Silence, Talk.

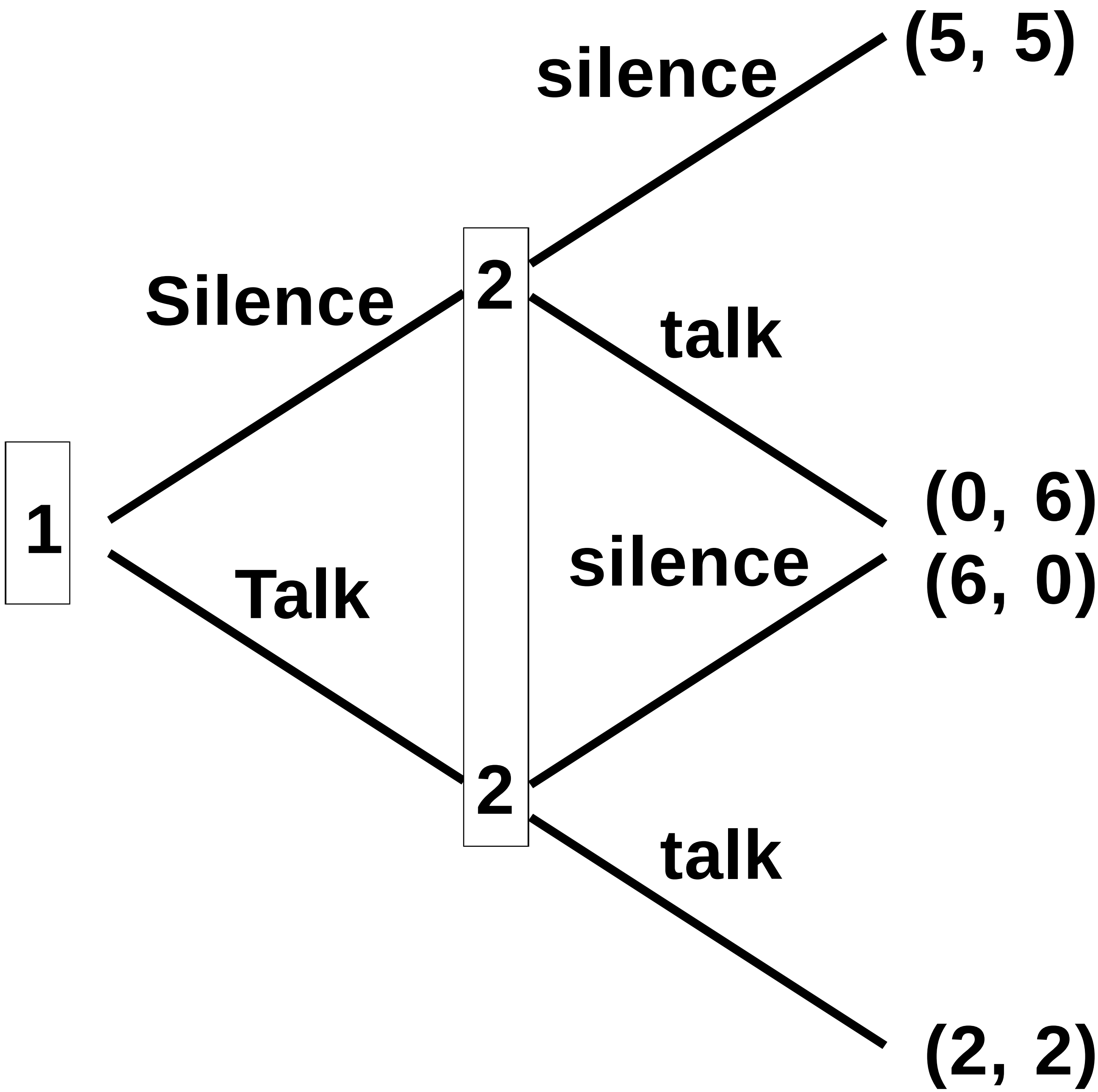
Strategies for player 2: silence, talk.

Figure 3 shows the Prisoners' Dilemma game in extensive form.

4. **Non-cooperative equilibrium**. A pair of strategies (s, s) is a **non-cooperative equilibrium** if and only if each player has no incentive to change his strategy provided that the opponent does not change his strategy. No player has an incentive to unilaterally deviate from an equilibrium position.

Looking at the Prisoners' Dilemma, we see that if player 2 plays "silence" and is expected to continue playing this strategy, player 1 prefers to play "Talk," since he makes 6 instead of 5 in the payoff. Since player 1 wants to deviate from it, (Silence, silence) is **not** a non-cooperative equilibrium. If player 2 plays "talk" and is expected to continue playing it, player 1 prefers to play "Talk," since he makes 2 instead of 0 in the payoff. Therefore (Talking, silence) is **not** a non-cooperative equilibrium. Finally, given that player 1

Prisoners' Dilemma



plays “Talk,” player 2 prefers to play “talk” since he gets 2 instead of 0. Since both players prefer not to deviate from the strategy they play in (Talk, talk) if the opponent does not deviate from his strategy in (Talk, talk), this **is** a non-cooperative equilibrium.

In the “Simultaneous Incumbent-Entrant” game, if E enters, the incumbent prefers to play L because his payoff is 8 rather than 6. If the incumbent plays L, the entrant chooses to play Enter, because he prefers 8 to 0. Therefore **no player has an incentive to deviate from (Enter, Low Q), and it is a non-cooperative equilibrium**. In the same game, if the entrant chooses to stay out, the incumbent replies by producing a high quantity (H) because 18 is better than 9. And, if the incumbent produces a high Q, the entrant prefers to stay out because 0 is better than -3. Therefore **no player has an incentive to deviate from (Stay out, High Q), and it is also a non-cooperative equilibrium**. Note that there are two equilibria in this game.

To find the equilibrium in the original "Sequential Incumbent-Entrant" game of Figure 1, note that if player E enters, player I prefers Low Q, and ends up at (8, 8). If E stays out, I prefers High Q, and ends up at (0, 18). Seeing this, E chooses to enter because in this way he realizes a profit of 8 rather than of 0. **Therefore the non-cooperative equilibrium is at (Enter, Low Q)**, and both firms realize a profit of 8. Note that *the other equilibrium of the simultaneous game was eliminated*.

5. **Dominant strategies**. In some games, no matter what strategy player 1 plays, there is a single strategy that maximizes the payoff of player 2. For example, in the Prisoners' Dilemma if player 1 plays "Silence," it is better for player 2 to play "talk"; and, if player 1 plays "Talk," it is better for player 2 to play "talk" again. Then "talk" is a **dominant strategy** for player 2. In the same game, note that "Talk" is a dominant strategy for player 1, because he prefers it no matter what player 2 plays. In a game such as this, where both players have a dominant strategy, there is an **equilibrium in**

dominant strategies, where each player plays his dominant strategy. An equilibrium in dominant strategies is necessarily a non-cooperative equilibrium. (Why? Make sure you understand that no player wants to unilaterally deviate from a dominant strategy equilibrium.)

There are games with no equilibrium in dominant strategies. For example, in the simultaneous incumbent-entrant game, the entrant prefers to stay out if the incumbent plays "H." However, the entrant prefers to enter if the incumbent plays "L." Since the entrant would choose a different strategy depending on what the incumbent does, the entrant does not have a dominant strategy. (Similarly, check that the incumbent does not have a dominant strategy.) Therefore in the simultaneous incumbent-entrant game there is no equilibrium in dominant strategies.

6. **Best replies**. Player 1's **best reply** to strategy s_2 of player 2 is defined as the best strategy that player 1 can play against strategy

s_2 of player 2. For example, in the simultaneous incumbent-entrant game, the best reply of the entrant to the incumbent playing "High Q" is "Stay out." Similarly, the best reply of the entrant to the incumbent playing "Low Q" is "Enter." From the point of view of the incumbent, his best reply to the entrant's choice of "Enter" is "Low Q," and his best reply to the entrant's choice of "Stay out" is "High Q." Notice that at a non-cooperative equilibrium both players play their best replies to the strategy of the opponent. For example, at (Stay out, High Q), as we saw, "Stay out" is a best reply to "High Q," and "High Q" is a best reply to "Stay out." This is no coincidence. At the non-cooperative equilibrium no player has an incentive to deviate from the strategy he plays. This means that his strategy is the best among all the available ones, as a reply to the choice of the opponent. This is just another way of saying that the player chooses the best reply strategy. Therefore **at equilibrium each player plays a best reply strategy**. This suggests that we can find equilibria by finding first the first reply strategies. We do this in the oligopoly games that we discuss next.

7. **Strategic Trade Policy**. The following examples show how industrial policy or trade policy of a country can have very significant impact on the entry of firms and the existence of an industry in a country.

Consider an entry game between Boeing and Airbus with the following profit matrix.

Entry in Airplane Manufacturing

		Player 2 (Airbus)	
		enter	don't enter
Player 1 (Boeing)	Enter	(-100, -100)	(500, 0)
	Don't Enter	(0, 500)	(0, 0)

This game has two non-cooperative equilibria. In each one of them, one firm enters and the other does not. However, if Airbus received a subsidy of 200 from France the profit matrix gets transformed into:

Entry in Airplane Manufacturing with Airbus Subsidy

		Player 2 (Airbus)	
		enter	don't enter
	Enter	(-100, 100)	(500, 0)
Player 1 (Boeing)	Don't Enter	(0, 700)	(0, 0)

Now the "enter" strategy becomes dominant for Airbus, and the only non-cooperative equilibrium is when Airbus enters and Boeing stays out. If the US were to subsidize Boeing by 200 in the absence of an Airbus subsidy, in the new matrix the "enter" strategy would now be dominant for Boeing and the only non-cooperative equilibrium would have Boeing entering and Airbus staying out. Of course, each country could each subsidize its company.

Entry in Airplane Manufacturing with Two Subsidies

		Player 2 (Airbus)	
		enter	don't enter
Player 1 (Boeing)	Enter	(100, 100)	(700, 0)
	Don't enter	(0, 700)	(0, 0)

Then both firms have a dominant strategy to enter. The public is the loser in this case, because the market is forced to have more firms than is efficient.