

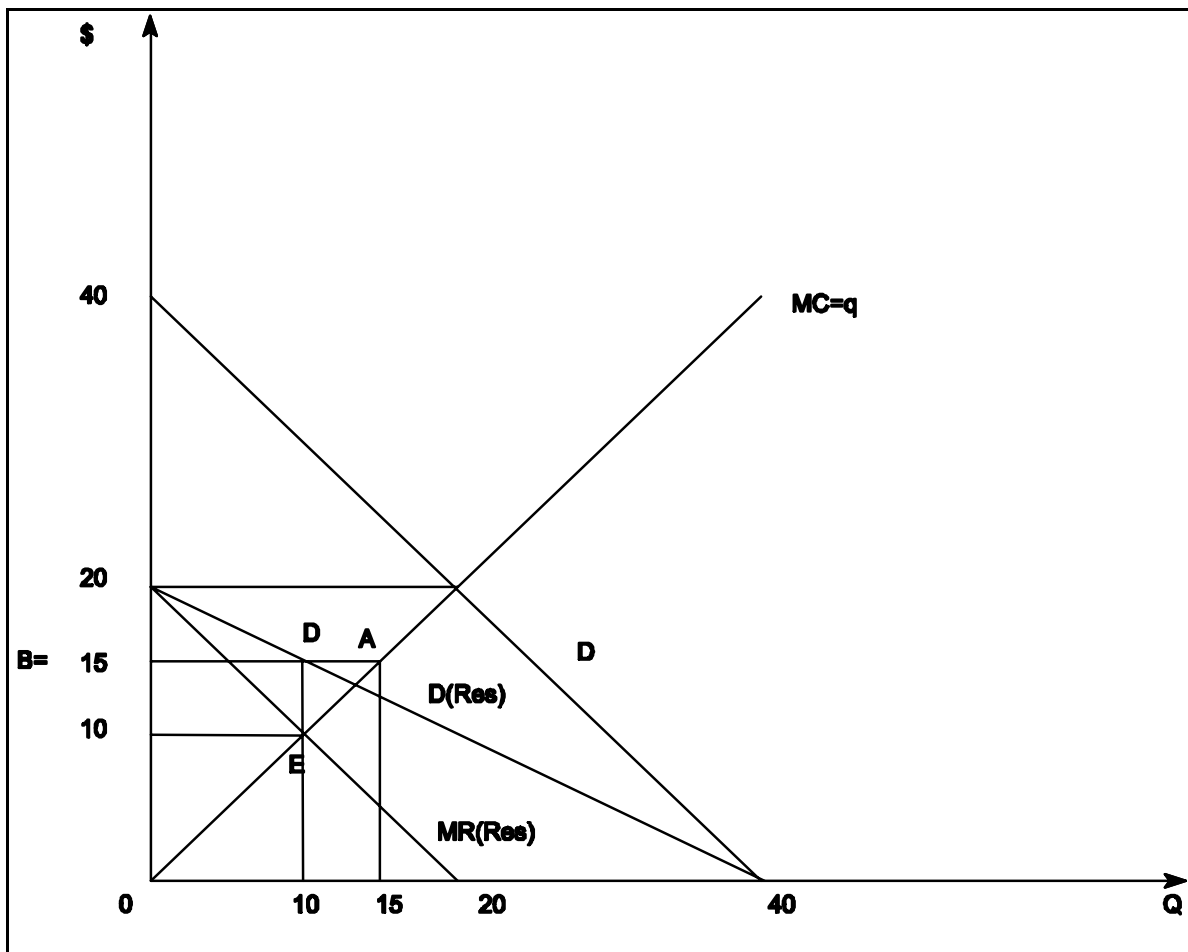
1.

(a) The world demand is $Q(p) = 40 - p$, while the supply of the followers is $q_f(p) = p$. Therefore the residual demand is $Q_R(p) = Q(p) - q_f(p) = 40 - 2p$.

(b) $Q_R = 40 - 2p \Leftrightarrow p = 20 - Q_R/2$. Therefore $MR_R = 20 - Q_R$. $MC_R = MR_R \Leftrightarrow 20 - Q_R = Q_R \Leftrightarrow Q_R = 10$. It follows that $p = 15$.

(c) At price $p = 15$, the follower produces at $MC = p$, i.e., at $q_f = 15$. Profit of the follower is the triangle $AB0 = (15)(15)/2 = 225/2 = 112.5$. Profits of the leader are $B0DE = AB0 - ADE = 112.5 - (5)(5)/2 = 112.5 - 12.5 = 100$.

(d) Each of the two firms has supply $q = p$. Therefore the world supply is $Q = 2p$. The equilibrium is at the intersection of this line and the demand. $40 - p = 2p \Leftrightarrow p = 40/3 = 13.33$, $q = 26.66$.



2. (a) $w = 10$, $U(C, R) = CR^2$. Budget Constraint:

$$C = wL = w(24 - R), \text{ i.e. } C = 10(24 - R).$$

Need to maximize $U(C, R) = CR^2 = 10(24 - R)R^2$.

$$dU/dR = 10[2R(24 - R) - R^2] = 10(48R - 3R^2) = 30R(16 - R) = 0.$$

Solving this we have $R = 16$, $L = 8$, $C = \$80$.

- (b) $w = 8$, $C = 8(24 - R)$. Again, we need to maximize

$$U(C, R) = CR^2 = 8(24 - R)R^2.$$

$$dU/dR = 8[2R(24 - R) - R^2] = 8(48R - 3R^2) = 0$$

This implies $R = 16$, $L = 8$, $C = \$64$.

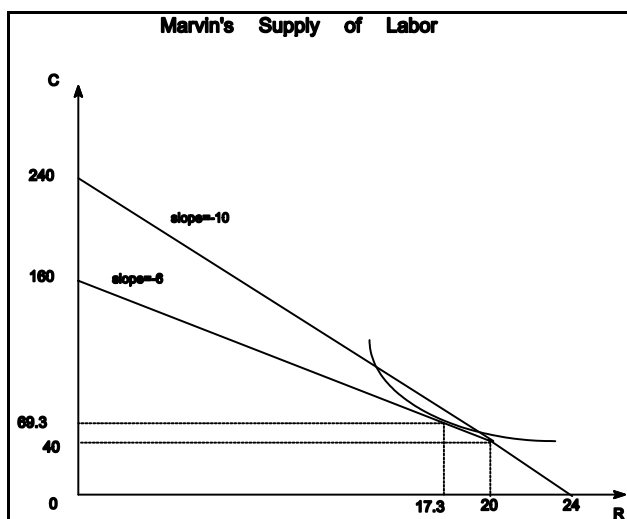
- (c) \$6 support payment, $C = 16 + 8(24 - R)$.

$$U = R^2(16 + 8(24 - R))$$

$$dU/dR = 2R(16 + 8(24 - R)) + R^2(-8) = 8R(4 + 48 - 3R) = 0.$$

This implies $R = 52/3 = 17.3$, $L = 6.7$.

- (d) $w = 10$. Tax rate = 40% for $wL > 40$, i.e. for $L > 4$.



(e) Kink is at $R = 20, C = 40$. For $R > 20$ the budget constraint is

$$C = 10(24-R). \quad (1)$$

For $R < 20$ the budget constraint is

$$C = 40 + 6(20-R) \quad (2)$$

(f) If $R < 20$, $MU_R/Mu_C = 2C/R = 6 \Rightarrow$

$$C = 3R. \quad (3)$$

From (3) and (2) we have $3R = 160 - 6R \Leftrightarrow R^* = 17.8, L = 6.2$. We check that $R^* = 17.8 < 20$. Since the budget set is concave and the indifference curves are strictly convex we know that there is a unique optimum and need not check further.