Take home problems; Series 1; Scheduling

1. Give an instance of a scheduling problem with the following property: if the processing time of one job is reduced by 1, the makespan of the best nondelay schedule increases.

2. Give an instance of a parallel machine scheduling problem with the following property: if one additional machine is added, the makespan of the best nondelay schedule increases.

3. Proof that the WSPT-rule is optimal for $1|| \sum w_j C_j$

4. Show that problem $1|prec|L_{max}$ can be solved to optimality without knowledge of the processing times $p_j$. Give an example that this is not true for the problem $1|| \max w_j L_j$.

5. Show that the preemptive EDD-rule solves problem $1|r_j|L_{max}$ to optimality if the release and due dates are similarly ordered, i.e. $d_j \leq d_k$ whenever $r_j < r_k$.

6. Consider problem $1|| \sum w_j T_j$. Prove or disprove:
   - if $w_j/p_j > w_k/p_k$, $p_j < p_k$ and $d_j < d_k$ then there exists an optimal schedule in which job $j$ appears before job $k$.

7. Show that if an optimal schedule for problem $P||C_{max}$ results in at most 2 jobs on any machine, then the LPT-rule is optimal

8. Show that the bound $(4/3 - 1/3m)$ on the approximation ratio of the LPT-rule for problem $P||C_{max}$ is tied; i.e. for each $m$ give an instance where the approximation ratio is $(4/3 - 1/3m)$

9. Proof that problem $P|| \sum w_j C_j$ is NP-hard in the strong sense by reducing 3-PARTITION to it.

10. Proof that for problem $F||C_{max}$ an optimal schedule exists with
    - the job sequence on the first two machine is the same
    - the job sequence on the last two machine is the same

11. Give an instance of problem $F||C_{max}$ for which no permutation schedule is optimal

Series 1 is now closed.