Transportation Models

• large variety of models due to the many modes of transportation
  – roads
  – railroad
  – shipping
  – airlines

• as a consequence different type of equipment and resources with different characteristics are involved
  – cars, trucks, roads
  – trains, tracks and stations
  – ships and ports
  – planes and airports

• consider two specific problems
Tanker Scheduling

Basic Characteristics

- consider the problem from the view of a company
- the planning process normally is done in a 'rolling horizon' fashion
- company operates a fleet of ships consisting of
  - own ships \( \{1, \ldots, T\} \)
  - chartered ships
- the operating costs of these two types are different
- only the own ships are scheduled
- using chartered ships only leads to costs which are given by the spot market
Tanker Scheduling

Basic Characteristics (cont.)

- each own ship $i$ is characterized by its
  - capacity $cap_i$
  - draught $dr_i$
  - range of possible speeds
  - location $l_i$ and time $r_i$ at which it is ready to start next trip
  - . . .


Tanker Scheduling

Basic Characteristics (cont.)

- the company has $n$ cargos to be transported
- cargo $j$ is characterized by
  - type $t_j$ (e.g. crude type)
  - quantity $p_j$
  - load port $port^l_j$ and delivery port $port^d_j$
  - time windows $[r^l_j, d^l_j]$ and $[r^d_j, d^d_j]$ for loading and delivery
  - load and unload times $t^l_j$ and $t^d_j$
  - costs $c^*_j$ denoting the price which has to be paid on the spot market to transport cargo $j$
Tanker Scheduling

Basic Characteristics (cont.)

- there are $p$ different ports
- port $k$ is characterized by
  - its location
  - limitations on the physical characteristics (e.g. length, draught, deadweight, ...) of the ships which may enter the port
  - local government rules (e.g. in Nigeria a ship has to be loaded above 90% to be allowed to sail)
  - 
  - 
  - ...
Tanker Scheduling

Basic Characteristics (cont.)

- the objective is to minimize the total cost of transporting all cargos
- hereby a cargo can be assigned to a ship of the company or ’sold’ on the spot market and thus be transported by a chartered ship
- costs consist of
  - operating costs for own ships
  - spot charter rates
  - fuel costs
  - port charges, which depend on the deadweight of the ship
Tanker Scheduling

ILP modeling

- straightforward choice of variables would be to use $0-1$-variables for assigning cargos to ships
- problem: these assignment variables do not define the schedule/route for the ship and thus feasibility and costs of the assignment can not be determined
- alternative approach: generate a set of possible schedules/routes for each ship and afterwards use assignment variables to assign schedules/routes to ships
- problem splits up into two subproblems:
  - generate schedules for ships
  - assign schedules to ships
Tanker Scheduling

**ILP modeling - generate schedules**

- a schedule for a ship consist of an assignment of cargos to the ship and a sequence in which the corresponding ports are visited
- generation of schedules can be done by ad-hoc heuristics which consider
  - ship constraints like capacity, speed, availability, . . .
  - port constraints
  - time windows of cargos
- each schedule leads to a certain cost
- for each ship enough potential schedules should be generated in order to get feasible and good solutions for the second subproblem
Tanker Scheduling

ILP modeling - generate schedules (cont.)

- the output of the first subproblem is
  - a set $S_i$ of possible schedules for ship $i$
  - each schedule $l \in S_i$ is characterized by
    * a vector $(a_{i1}^l, \ldots, a_{in}^l)$ where $a_{ij}^l = 1$ if cargo $j$ is transported by ship $i$ in schedule $l$ and 0 otherwise
    * costs $c_i^l$ denoting the incremental costs of operating ship $i$ under schedule $l$ versus keeping it idle over the planning horizon
    * profit $\pi_i^l = \sum_{j=1}^n a_{ij}^l c_j^* - c_i^l$ by using schedule $l$ for ship $i$ instead of paying the spot market
Remarks:

- All the feasibility constraints of the ports and ships are now within the schedule.
- All cost aspects are summarized in the values $c^l_i$ resp. $\pi^l_i$.
- The sequences belonging to the schedules determine feasibility and the costs $c^l_i$ but are not part of the output since they are not needed in the second subproblem.
Tanker Scheduling

ILP modeling - assign schedules to ships

- variables $x^l_i = \begin{cases} 1 & \text{if ship } i \text{ follows schedule } l \\ 0 & \text{else} \end{cases}$

- objective: $\max \sum_{i=1}^{T} \sum_{l \in S_i} \pi^l_i x^l_i$

- constraint:
  
  $- \sum_{i=1}^{T} \sum_{l \in S_i} a_{ij}^l x^l_i \leq 1; \quad j = 1, \ldots, n$ (each cargo at most once)

  $- \sum_{l \in S_i} x^l_i \leq 1; \quad i = 1, \ldots, T$ (each ship at most one schedule)
Tanker Scheduling

ILP modeling - assign schedules to ships (cont.)

- the ILP model is a set-packing problem and well studied in the literature

- can be solved by branch and bound procedures

- possible branchings:
  - chose a variable $x^l_i$ and branch on the two possibilities $x^l_i = 0$ and $x^l_i = 1$
    select $x^l_i$ on base of the solution of the LP-relaxation: choose a variable with value close to 0.5
  - chose a ship $i$ and branch on the possible schedules $l \in S_i$
    selection of ship $i$ is e.g. be done using the LP-relaxation: choose a ship with a highly fractional solution
**Tanker Scheduling**

ILP modeling - assign schedules to ships (cont.)

- lower bounds can be achieved by generating feasible solutions via clever heuristics (feasible solution = lower bound since we have a maximization problem)

- upper bounds can be obtained via relaxing the integrality constraints and solving the resulting LP (note, that this LP-solution is also used for branching!)

- for a small example, the behavior of the branch and bound method is given in the handouts
Remarks Two Phase Approach

- in general the solution after solving the two subproblems is only a heuristic solution of the overall problem.

- if in the first subproblem all possible schedules/routes for each ship are generated (i.e. $S_i$ is equal to the set $S_{i}^{all}$ of all feasible schedules for ship $i$), the optimal solution of the second subproblem is an optimal solution for the overall problem.

- for real life instances the cardinalities of the sets $S_{i}^{all}$ are too large to allow a complete generation (i.e. $S_i$ is always a (small) subset of $S_{i}^{all}$).

- column generation can be used to improve the overall quality of the resulting solution.
Train Timetabling

General Remarks

- in the railway world lots of scheduling problems are of importance
  - scheduling trains in a timetable
  - routing of material
  - staff planning
  - ...
- currently lots of subproblems are investigated
- the goal is to achieve an overall decision support system for the whole planning process
- we consider one important subproblem
Train Timetabling

Decomposition of the Train Timetabling

- mostly the overall railway network consists of some major stations and ‘lines/corridors’ connecting them

- a corridor normally consists of two independent one-way tracks

- having good timetables for the trains in the corridors makes it often easy to find timetables for the trains on the other lines
Train Timetabling

Scheduling Train on a Track

- consider a track between two mayor stations
- in between the two mayor stations several smaller stations exists

R  RN  RA  CS  NI  G  GG  GC  W  U
R  Rotterdam Centraal  CS  Capelle Schollevaar  GG  Gouda Goverwelle
RN  Rotterdam Noord  NI  Nieuwerkerk ad IJssel  W  Woerden
RA  Rotterdam Alexander  G  Gouda  U  Utrecht Centraal

- trains may or may not stop at these stations
- trains can only overtake each other at stations
Train Timetabling

Problem Definition Track Scheduling

- time period 1, \ldots, q, where q is the length of the planning period (typically measured in minutes; e.g. \( q = 1440 \))
- \( L + 1 \) stations 0, \ldots, \( L \)
- \( L \) consecutive links;
- link \( j \) connects station \( j - 1 \) and \( j \)
- trains travel in the direction from station 0 to \( L \)
- \( T \): set of trains that are candidates to run during planning period
- for link \( j \), \( T_j \subseteq T \) denotes the trains passing the link
Train Timetabling

Problem Definition Track Scheduling (cont.)

- Train schedules are usually depicted in so-called time-space diagrams.

- Diagrams enable user to see conflicts.
Train Timetabling

Problem Definition Track Scheduling (cont.)

- train schedules are usually depicted in so-called time-space diagrams

- diagrams enable user to see conflicts
Train Timetabling

Problem Definition Track Scheduling (cont.)

- each train has an most desirable timetable (arrivals, departures, travel time on links, stopping time at stations), achieved e.g. via marketing department
- putting all these most desirable timetables together, surely will lead to conflicts on the track
- possibilities to change a timetable:
  - slow down train on link
  - increase stopping time at a station
  - modify departure time at first station
  - cancel the train
Train Timetabling

Problem Definition Track Scheduling (cont.)

- cost of deviating from a given time $\hat{t}$:
  - specifies the revenue loss due to a deviation from $\hat{t}$
  - the cost function has its minimum in $\hat{t}$, is convex, and often modeled by a piecewise linear function

- piecewise linear helps in ILP models!
Train Timetabling

Variables for Track Scheduling

- variables represent departure and arrival times from stations
  - $y_{ij}$: time train $i$ enters link $j$
    - $= \text{time train } i \text{ departs from station } j - 1$
    - (defined if $i \in T_j$)
  - $z_{ij}$: time train $i$ leaves link $j$
    - $= \text{time train } i \text{ arrives at station } j$
    - (defined if $i \in T_j$)

- $c_{ij}^d(y_{ij})$ ($c_{ij}^a(z_{ij})$) denotes the cost resulting from the deviation of the departure time $y_{ij}$ (arrival time $z_{ij}$) from its most desirable value
Train Timetabling

Variables for Track Scheduling (cont.)

- variables resulting from the departures and arrivals times:
  \[ \tau_{ij} = z_{ij} - y_{ij} \]: travel time of train \( i \) on link \( j \)

  \[ \delta_{ij} = y_{i,j+1} - z_{ij} \]: stopping time of train \( i \) at station \( j \)

- \( c^\tau_{ij}(\tau_{ij}) \) \( (c^\delta_{ij}(\delta_{ij})) \) denotes the cost resulting from the deviation of the travel time \( \tau_{ij} \) (stopping time \( \delta_{ij} \)) from its most desirable value

- all cost functions \( c^d_{ij}, c^a_{ij}, c^\tau_{ij}, c^\delta_{ij} \) have the mentioned structure
Train Timetabling

Objective function

- minimize

\[
\sum_{j=1}^{L} \sum_{i \in T_j} (c_{ij}^d(y_{ij}) + c_{ij}^a(z_{ij}) + c_{ij}^T(z_{ij} - y_{ij})) + \sum_{j=1}^{L-1} \sum_{i \in T_j} c_{ij}^\delta(y_{i,j+1} - z_{ij})
\]
Train Timetabling

Constraints

- minimum travel times for train $i$ over link $j$: $\tau_{ij}^{min}$
- minimum stopping times for train $i$ at station $j$: $\delta_{ij}^{min}$
- safety distance:
  - minimum headway between departure times of train $h$ and train $i$ from station $j$: $H_{hij}^d$
  - minimum headway between arrival times of train $h$ and train $i$ from station $j$: $H_{hij}^a$
- lower and upper bounds on departure and arrival times:
  $y_{ij}^{min}, y_{ij}^{max}, z_{ij}^{min}, z_{ij}^{max}$
Train Timetabling

Constraints (cont.)

- to be able to model the minimum headway constraints, variables have to be introduced which control the order of the trains on the links
- \( x_{hi,j} = \begin{cases} 
1 & \text{if train } h \text{ immediately preceeds train } i \text{ on link } j \\
0 & \text{else} 
\end{cases} \)
- using the variables \( x_{hi,j} \), the minimum headway constraints can be formulated via ’big M’-constraints:

\[
\begin{align*}
    y_{i,j+1} - y_{h,j+1} + (1 - x_{hi,j})M & \geq H_{hi,j}^d \\
    z_{ij} - z_{hj} + (1 - x_{hi,j})M & \geq H_{hi,j}^a
\end{align*}
\]
Train Timetabling

Constraints (cont.)

- two dummy trains 0 and * are added, representing the start and end of the planning period (fix departure and arrival times appropriate ensuring that 0 is sequenced before all other trains and * after all other trains)
Train Timetabling

Constraints (cont.)

\[ y_{ij} \geq y_{ij}^{\text{min}} \quad j = 1, \ldots, L; \ i \in T_j \]
\[ y_{ij} \leq y_{ij}^{\text{max}} \quad j = 1, \ldots, L; \ i \in T_j \]
\[ z_{ij} \geq z_{ij}^{\text{min}} \quad j = 1, \ldots, L; \ i \in T_j \]
\[ z_{ij} \leq z_{ij}^{\text{max}} \quad j = 1, \ldots, L; \ i \in T_j \]
\[ z_{ij} - y_{ij} \geq \tau_{ij}^{\text{min}} \quad j = 1, \ldots, L; \ i \in T_j \]
\[ y_{i,j+1} - z_{ij} \geq \delta_{ij}^{\text{min}} \quad j = 1, \ldots, L - 1; \ i \in T_j \]
\[ y_{i,j+1} - y_{h,j+1} + (1 - x_{hij})M \geq H_{hij}^d \quad j = 0, \ldots, L - 1; \ i, h \in T_j \]
\[ z_{ij} - z_{hj} + (1 - x_{hij})M \geq H_{hij}^a \quad j = 1, \ldots, L; \ i, h \in T_j \]
\[ \sum_{h \in T_j \setminus \{i\}} x_{hij} = 1 \quad j = 1, \ldots, L; \ i \in T_j \]
\[ x_{hij} \in \{0, 1\} \quad j = 1, \ldots, L; \ i, h \in T_j \]
Train Timetabling

Remarks on ILP Model

- the number of 0-1 variables gets already for moderate instances quite large
- the single track problem is only a subproblem in the whole timetabling process and needs therefore to be solved often
- as a consequence, the computational time for solving the single track problem must be small
- this asks for heuristic approaches to solve the single track problem
Train Timetabling

Decomposition Approach: General Idea

- schedule the trains iteratively one by one
- initially, the two dummy trains 0 and * are scheduled
- the selection of the next train to be scheduled is done on base of priorities
- possible priorities are
  - earliest desired departure time
  - decreasing order of importance (importance may be e.g. measured by train type, speed, expected revenue, ...)
  - smallest flexibility in departure and arrival
  - combinations of the above
Train Timetabling

Decomposition Approach: Realization

- $T_0$: set of already scheduled trains
- initially $T_0 = \{0,*\}$
- after each iteration a schedule of the trains from $T_0$ is given
- however, for the next iteration only the sequence in which the trains from $T_0$ traverse the links is taken into account
- $S_j = (0 = j_0, j_1, \ldots, j_n, j_{n+1} = *):$ sequence of trains from $T_0$ on link $j$
- if train $k$ is chosen to be scheduled in an iteration, we have to insert $k$ in all sequences $S_j$ where $k \in T_j$
- this problem is called $Insert(k, T_0)$
Train Timetabling

ILP Formulation of $\text{Insert}(k, T_0)$

Adapt the 'standard' constraints and the objective to $T_0$:

$$\min \sum_{j=1}^{L} \sum_{i \in T_j} (c_{ij}^d(y_{ij}) + c_{ij}^a(z_{ij}) + c_{ij}^\tau(z_{ij} - y_{ij}))$$

$$+ \sum_{j=1}^{L_1} \sum_{i \in T_j} c_{ij}^\delta(y_{i,j+1} - z_{ij})$$

subject to

$$y_{ij} \geq y_{ij}^{\min} \quad j = 1, \ldots, L; \ i \in T_0 \cap T_j$$

$$y_{ij} \leq y_{ij}^{\max} \quad j = 1, \ldots, L; \ i \in T_0 \cap T_j$$

$$z_{ij} \geq z_{ij}^{\min} \quad j = 1, \ldots, L; \ i \in T_0 \cap T_j$$

$$z_{ij} \leq z_{ij}^{\max} \quad j = 1, \ldots, L; \ i \in T_0 \cap T_j$$

$$z_{ij} - y_{ij} \geq \tau_{ij}^{\min} \quad j = 1, \ldots, L; \ i \in T_0 \cap T_j$$

$$y_{i,j+1} - z_{ij} \geq \delta ij^{\min} \quad j = 1, \ldots, L - 1; \ i \in T_0 \cap T_j$$
Train Timetabling

ILP Formulation of Insert($k, T_0$) (cont.)

- adapt $y_{i,j+1} - y_{h,j+1} + (1 - x_{hij})M \geq H^d_{hij}$ for trains from $T_0$

$$y_{ji+1,j} - y_{ji,j} \geq H^d_{ji,ji+1,j-1} \quad \text{for } j = 1, \ldots, L, i = 0, \ldots, n_j$$

- adapt $z_{ij} - z_{hj} + (1 - x_{hij})M \geq H^a_{hij}$ for trains from $T_0$

$$z_{ji+1,j} - z_{ji,j} \geq H^a_{ji,ji+1,j} \quad \text{for } j = 1, \ldots, L, i = 0, \ldots, n_j$$
Train Timetabling

ILP Formulation of \( \text{Insert}(k, T_0) \) (cont.)

- insert \( k \) on link \( j \) via variables
  \[
  x_{ij} = \begin{cases} 
  1 & \text{if train } k \text{ immediately precedes train } j_i \text{ on link } j \\
  0 & \text{else}
  \end{cases}
  \]

- new constraints for \( j = 1, \ldots, L, i = 0, \ldots, n_j \):
  
  \[
  \begin{align*}
  - y_{k,j} - y_{j_i,j} + (1 - x_{ij})M & \geq H_{jikj}^d \\
  - y_{j_{i+1},j} - y_{k,j} + (1 - x_{ij})M & \geq H_{kj_{i+1}j}^d \\
  - z_{k,j} - z_{j_i,j} + (1 - x_{ij})M & \geq H_{jikj}^a \\
  - z_{j_{i+1},j} - z_{k,j} + (1 - x_{ij})M & \geq H_{kj_{i+1}j}^a
  \end{align*}
  \]

- 0-1 constraints and sum constraint on \( x_{ij} \) values
Train Timetabling

Remarks on ILP Formulation of $Insert(k, T_0)$

- the ILP Formulation of $Insert(k, T_0)$ has the same order of continuous constraints $(y_{ij}, z_{ij})$ but far fewer 0-1 variables than the original MIP.

- a preprocessing may help to fix $x_{ij}$ variables since on base of the lower and upper bound on the departure and arrival times of train $k$ many options may be impossible.

- solving $Insert(k, T_0)$ may be done by branch and bound.
Train Timetabling

Solving the overall problem

- an heuristic for the overall problem may follow the ideas of the shifting bottleneck heuristic
  - select a new train \( k \) (machine) which is most ’urgent’
  - solve for this new train \( k \) the problem \( \text{Insert}(k, T_0) \)
  - reoptimize the resulting schedule by rescheduling the trains from \( T_0 \)

- rescheduling of a train \( l \in T_0 \) can be done by solving the problem \( \text{Insert}(l, T_0 \cup \{k\} \setminus \{l\}) \) using the schedule which results from deleting train \( l \) from the schedule achieved by \( \text{Insert}(k, T_0) \)