Risk Everywhere: Modeling and Managing Volatility

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Introduction

Realized Volatility across global asset classes

We study Realized Volatility (RV) for a wide set of global assets

- Literature has focused on a few cash equities and currencies.
- We expand to global equities, fixed income, commodities, and currencies
- We provide a standardized way to implement across assets

What do you do with RV? Run lagged regressions

- We like the simplicity of HAR. Make it better
  - "Smooth" decay in lag structure, impose regularity, and avoid discontinuities
  - "Anchor" to long term estimate
- Pool assets together

Accrue tangible benefits from volatility targeting

- Framework for volatility model evaluation
- Optimal risk-based allocation
- Investor willing to pay 55bps/year for dynamic, constant volatility strategy

RV With Global Asset Classes

A brief overview

Implementing Realized Volatility Everywhere

A DIY guide to calculating *RV*

RV "works" for 58 global assets

- 21 developed and emerging equity futures
- 20 commodity futures
- 8 developed fixed income futures
- 9 developed spot currencies

Practical implementation – the RV cookbook

- Which contracts?
  - Build a roll plan
- How often to sample?
  - Signature plots (ABDL 2000)
- When are markets open?
  - "Liquidity" plots and published market hours

Unconditional distribution very similar across assets

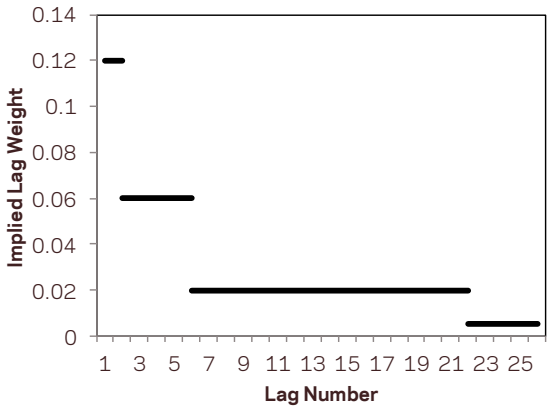
- Adjusted for level of volatility

Source: "Risk Everywhere: Modeling and Managing Volatility", Bollerslev, Hood, Huss, and Pedersen (2016). Please see the Appendix for more information on the global assets used throughout the presentation.

Regression Modeling

This Paper Is Not Necessarily About The Models

But we still need to define them



Baseline Model: HAR-RV (Corsi 2009)

- Flat day, week, and month factors, plus intercept

Adding Fixed Effects: HAR-FE

- Only matters for pooled regressions

More Flexibility: HAR-Free

- Individual factors for the first 6 days, plus monthly, annual, and long run (expanding) factors
- Adds long-run "anchoring", removing intercept and forcing coefficients to sum to one

Eliminate Discontinuities: HAR-Slope

- Sloped (declining) day, week, month, and annual factors, plus long run (expanding)
- Also "anchored" to the long run factor

Use EWMAs: HExp model ("Heterogeneous Exponential")

- EWMA factors with 1, 5, 25, and 125 day centers of mass, plus long run (expanding)
- Anchored to the long run factor

Example: HExp Model

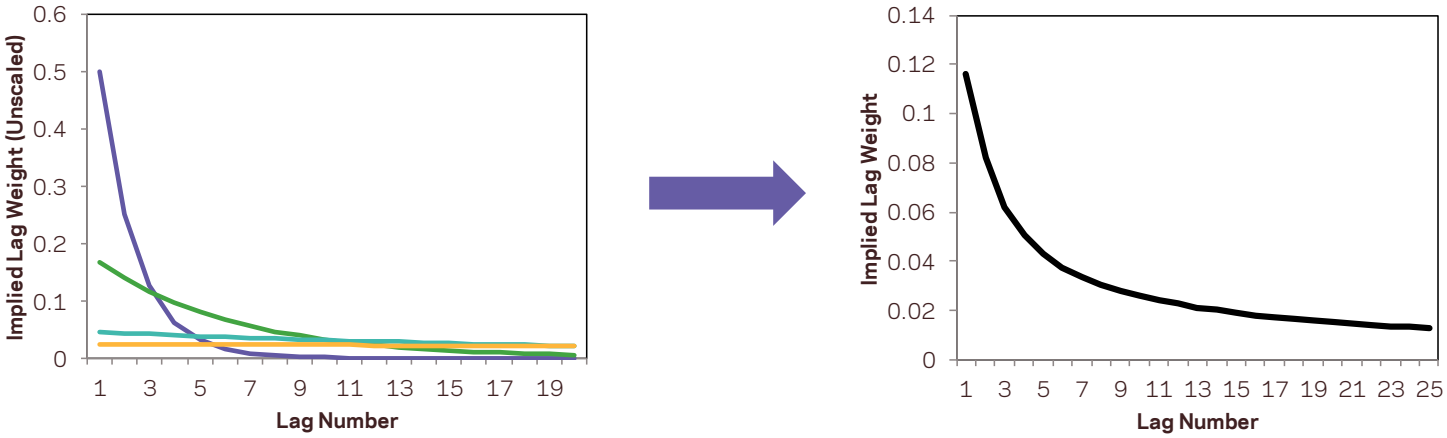
Heterogeneous Exponential: A simple combination of EWMAs

What if we use a spanning set of exponentially-weighted moving averages of RV?

- Heterogeneous Exponential RV (HExp-RV)
- RV lag space well-spanned using 1, 5, 25, and 125 day centers of mass plus long-term anchor
- Note: a combination of EWMAs is not itself an EWMA

Some benefits

- Smooth
- Very easy to implement
- Uses all historical data



Model Estimation: Pooling Assets Together

We run regressions for single assets, asset classes, and across all assets

We constrain model betas in three ways:

"Individual Assets" regression

- Each asset has its own set of model betas

"Panel" regression

- Common set of betas within asset class, different across asset classes

"Mega" panel regression

- Common betas for all assets

Pooling imposes more regularity, at the expense of flexibility

- Flexibility good in sample by definition
- What about out of sample?

Smoother Models Outperform

HExp gives highest R^2 both in- and out-of-sample

In-sample regressions show best fit for HExp model

- Improvement in R^2 of new models over HAR is very small

Monthly Predictive R^2s for All Assets

		HAR	HAR-FE	HAR-Free	Slope	HExp
In Sample	Individual Asset	44.5%	44.5%	44.8%	44.7%	44.9%
	Panel					
	Mega					
Out of Sample	Individual Asset					
	Panel					
	Mega					

Smoother Models Outperform

HExp gives highest R^2 both in- and out-of-sample

In-sample regressions show best fit for HExp model

- Improvement in R^2 of new models over HAR is very small

Out-of-sample improvement is bigger (still modest). We see:

- Improvement due to smoothness: HAR-Free > HAR
- Improvement due to increased regularity: Slope and HExp > any other model

If regularity of model improves R^2, what happens when we run pooled regressions?

Monthly Predictive R^2s for All Assets

		HAR	HAR-FE	HAR-Free	Slope	HExp
In Sample	Individual Asset	44.5%	44.5%	44.8%	44.7%	44.9%
	Panel					
	Mega					
Out of Sample	Individual Asset	42.2%	42.2%	44.0%	45.3%	45.4%
	Panel					
	Mega					

Small Benefit to Increasing Regularity

Pooled Regressions Are Better Too

Restricting coefficients to be same across all assets helps out of sample

Though pooling hurts in sample (by construction), it actually helps out of sample!

- This result holds for all models (one exception is HAR "Mega")
- Improvement in R^2 of HExp/Mega over HAR/Individual is ~5%

Intuition:

- RV lag structures more similar than not, even across asset classes
- The model for Asset A can "learn" how responsive to be from Asset B before experiencing its own shock
- Small fit penalty for being too responsive before shock, large benefit if/when it comes!

Monthly Predictive R^2s for All Assets

		HAR	HAR-FE	HAR-Free	Slope	HExp
In Sample	**Individual Asset**	44.5%	44.5%	44.8%	44.7%	44.9%
	Panel	40.7%	42.8%	42.9%	43.0%	43.1%
	Mega	39.0%	42.6%	42.8%	42.8%	42.9%
Out of Sample	**Individual Asset**	42.2%	42.2%	44.0%	45.3%	45.4%
	Panel	44.5%	44.5%	46.6%	46.8%	46.8%
	Mega	43.6%	44.9%	46.9%	47.2%	47.3%

Small Benefit to Increasing Regularity

In Sample: More **Fitting** is Better…

Out of Sample: More **Consistency** is Better!

The Utility of a Risk Model

Mean-Variance Utility for Investor

Assume independent Sharpe to remove isolate the volatility model

Standard mean-variance utility:

$$U(x_t) = W_t \left(x_t E_t(r^e_{t+1}) - \frac{\gamma}{2} x_t^2 E_t(RV_{t+1}) \right)$$

Realized utility depends on realized returns

- Assume constant/independent Sharpe, isolate effect of realized volatility

$$SR \equiv E_t(r^e_{t+1}) / \sqrt{E_t(RV_{t+1})}$$

Constant Sharpe mean-variance utility

$$U(x_t) = W_t \left(x_t SR \sqrt{E_t(RV_{t+1})} - \frac{\gamma}{2} x_t^2 E_t(RV_{t+1}) \right)$$

Optimal Solution

$$x_t^* = \frac{SR/\gamma}{\sqrt{E_t(RV_{t+1})}} \qquad U(x_t^*) = \frac{SR^2}{2\gamma} W_t$$

Intuition

Investor targets constant volatility

Investor targets a constant volatility of $^{SR}/_{\gamma}$

- If predicted vol is above (below) $^{SR}/_{\gamma}$, then $x_t{}^*$ is less (greater) than 1.

Suppose $SR = 0.3$ and $\gamma = 3$. Then

$$x_t^* = \frac{10\%}{\sqrt{E_t(RV_{t+1})}} \quad \textbf{and} \quad U(x_t^*) = 1.5\%\, W_t$$

Intuition

- Investor expects to receive excess return of 3%
- Half of return is "lost" to dis-utility of risk
- Vol of vol further reduces utility

Investor willing to pay 1.5% of wealth for access to perfectly vol-targeted risky asset

Dynamic Volatility Targeting Is Beneficial

Having a dynamic risk model matters

Compare utility of simple models to RV-based models

- Volatility targeting with HExp gains 0.55% over fixed-leverage strategy
- Simpler rolling windows almost as beneficial as HExp

Based on costless trading

- More dynamic strategy → more costly trading

	DAILY RETURNS			RV		FUTURE RV	
	In Sample Mean	**Expanding Mean**	**21 Day Window**	**21 Day RV**	**HExp**	**Future 20 Day RV**	**Future 1 Day RV**
COMMODITIES	0.89%	0.71%	1.26%	1.30%	1.30%	1.35%	1.50%
EQUITIES	0.57%	0.63%	1.26%	1.28%	1.29%	1.34%	1.50%
FIXED INCOME	0.90%	0.95%	1.25%	1.29%	1.30%	1.34%	1.50%
FX	0.74%	0.59%	1.24%	1.29%	1.30%	1.34%	1.50%
ALL ASSETS	0.75%	0.70%	1.26%	1.29%	1.30%	1.34%	1.50%

RV Matters Given High Transactions Costs

Improvement for better models survives transactions costs

Transactions cost assumption: Median bid-ask spread in 2014

- Utility given full spreads, as well as half spreads and 5x half spreads
- Choose your own cost level

RV **matters a lot more given transactions costs**

		DAILY RETURNS			RV		FUTURE RV	
	Transactions Cost	**In Sample Mean**	**Expanding Mean**	**21 Day Window**	**21 Day RV**	**HExp**	**Future 20 Day RV**	**Future 1 Day RV**
ALL ASSETS	Half-Spread	0.75%	0.69%	1.16%	1.24%	1.25%	1.29%	0.61%
	2x Half-Spread	0.75%	0.68%	1.07%	1.20%	1.20%	1.25%	-0.28%
	5x Half-Spread	0.75%	0.66%	0.78%	1.06%	1.04%	1.10%	-2.94%

Even if you could perfectly predict RV, you wouldn't want to!

Increase Your Gains With Slower Trading

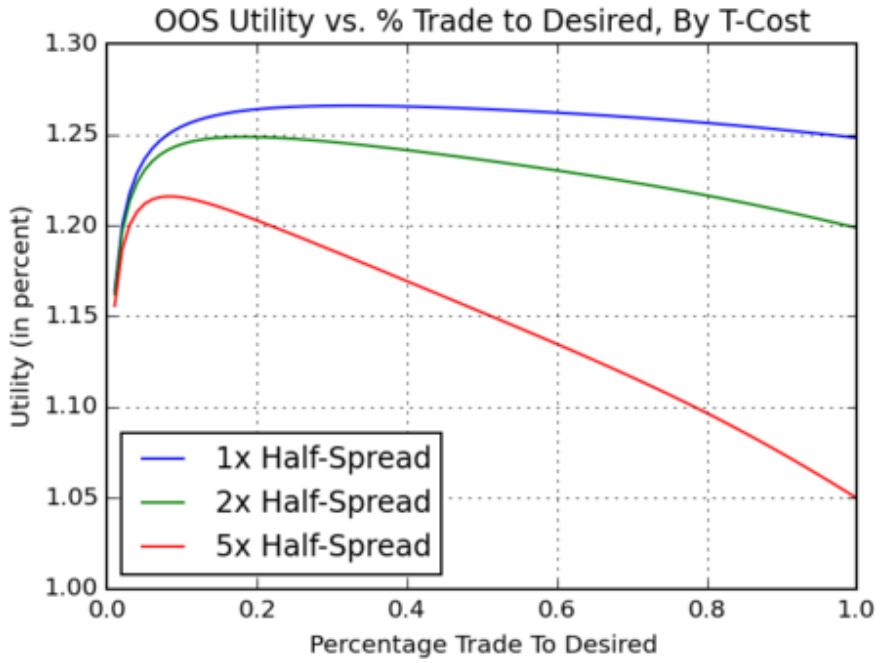
Constrained trading captures more of improved model benefits

High transactions costs eat into strategy utility

Simple solution: Slow down your trading

- Garleanu, Pedersen (2013)

Trade fixed percentage of the way to the target each day



		DAILY RETURNS			RV		FUTURE RV	
		In Sample Mean	**Expanding Mean**	**21 Day Window**	**21 Day RV**	**HExp**	**Future 20 Day RV**	**Future 1 Day RV**
5x Half Spread	100% to Target	0.75%	0.66%	0.78%	1.06%	1.04%	1.10%	-2.94%
	50% to Target	0.75%	0.67%	0.95%	1.12%	1.15%	1.18%	-0.38%
	25% to Target	0.75%	0.65%	1.02%	1.14%	1.19%	1.22%	0.49%
	12.5% to Target	0.75%	0.62%	1.07%	1.15%	1.21%	1.26%	0.85%

Another Way To Constrain Trading

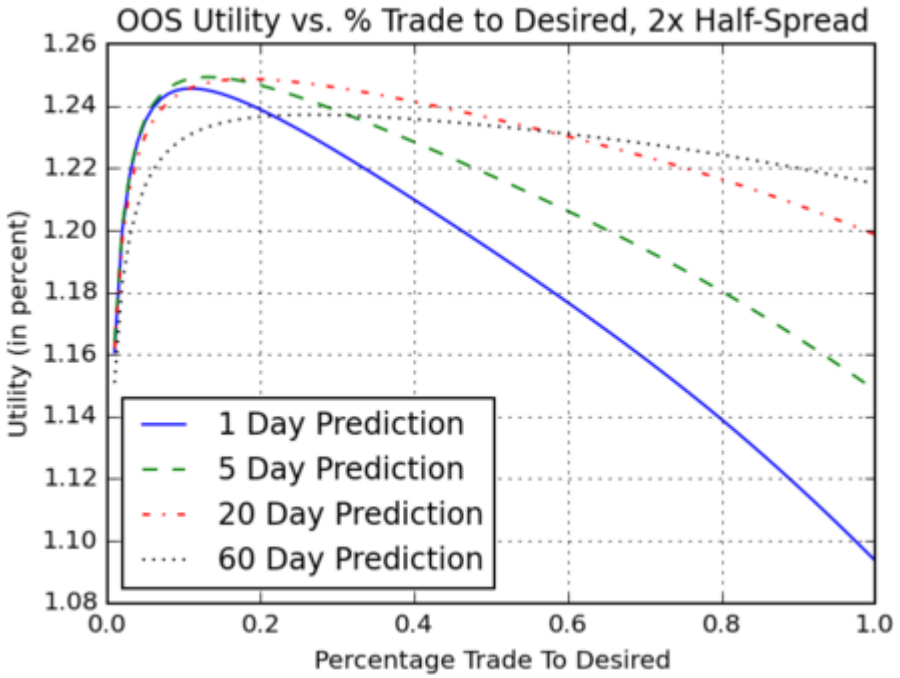
Longer horizon regressions

Why not just make the model slower?

- Use 20 day prediction horizon + trade rule
- Would it be better to use longer horizon regression?

We find utility is indifferent to model horizon

- We get roughly the same utility regardless of horizon
- Very long horizon can be slightly worse



Conclusion

Risk Everywhere

RV "works" for many global assets in different asset classes

- Simple guide to implementing globally

Use simple autoregressive models

- More regularity is slightly better

Pool assets when possible

- Single set of coefficients for all assets

Having a risk model matters

- But it doesn't matter which one (within reason)

References

Andersen, T. G., Bollerslev, T., Diebold, F. X., Labys, P., 2000. Great realizations. Risk, 105-108.

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Corsi, F., 2009. A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics 7 (2), 174-196.

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Appendix

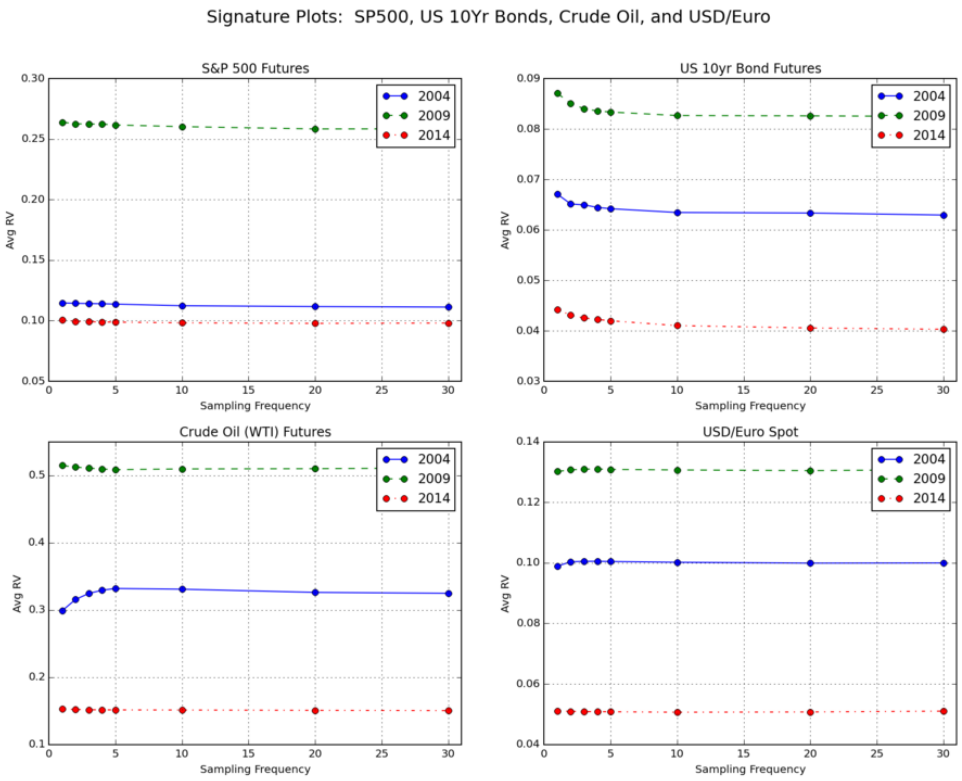
Signature Plots – A Diagnostic Tool

RV Estimation Parameters: How frequently can we sample?

"Volatility Signature Plots" show the average annualized RV value as a function of sampling frequency

- Andersen, Bollerslev, Diebold, and Labys (2000)
- Looking for clear bias due to market microstructure at higher sampling frequencies

Choose sampling frequency coarse enough to mitigate bias relative to longer horizon estimates



Liquidity Plots – Another Diagnostic Tool

RV Estimation Parameters: When is market open, and how liquid is it?

"Liquidity Plots" show average number of days with at least one trade in each minute

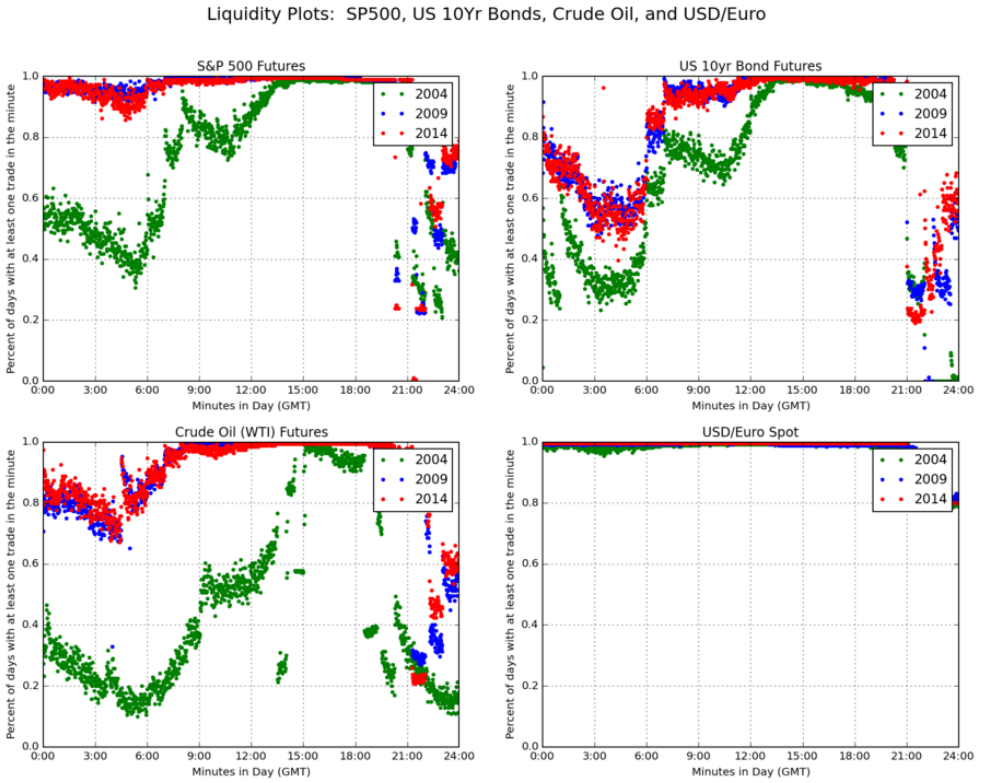
- High number → high liquidity time of day
- Low number → low liquidity time of day

These change a lot over time!

- Anecdote: Brazil shifts market hours twice a year to match NYC daylight savings

Diagnostic tool used to select appropriate market session

- Still recommend using official market hours
- Proper session may change over time



Summary Statistics and Average RV Pairwise Correlations

RV levels differ, but are correlated across assets and through time

Daily Realized Volatility Summary Statistics

	Commodities	Equities	Fixed Income	FX
Average	25.4%	20.6%	3.1%	10.3%
Standard Deviation	12.6%	13.7%	1.5%	5.7%
Skewness	2.6	3.4	2.3	3.1
Excess Kurtosis	16.9	22.9	11.6	18.5
Maximum	185.6%	186.6%	19.4%	74.1%
95th Percentile	47.8%	44.8%	5.8%	20.4%
50th Percentile	22.7%	17.0%	2.8%	9.0%
5th Percentile	11.6%	8.2%	1.5%	4.6%
Minimum	4.9%	3.0%	0.6%	1.2%
1 Day Autocorrelation	0.52	0.71	0.48	0.52
20 Day Autocorrelation	0.36	0.48	0.35	0.41
100 Day Autocorrelation	0.19	0.23	0.20	0.22
250 Day Autocorrelation	0.12	0.11	0.07	0.10

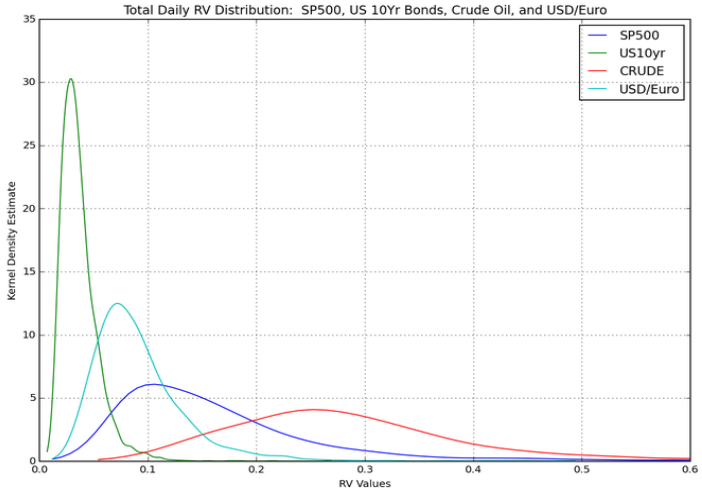
Average Correlations of Daily Realized Volatilities

	Commodities	Equities	Fixed Income	FX
Commodities	0.28	0.30	0.22	0.36
Equities		0.67	0.41	0.55
Fixed Income			0.47	0.43
FX				0.71

Unconditional Daily Volatility Distributions

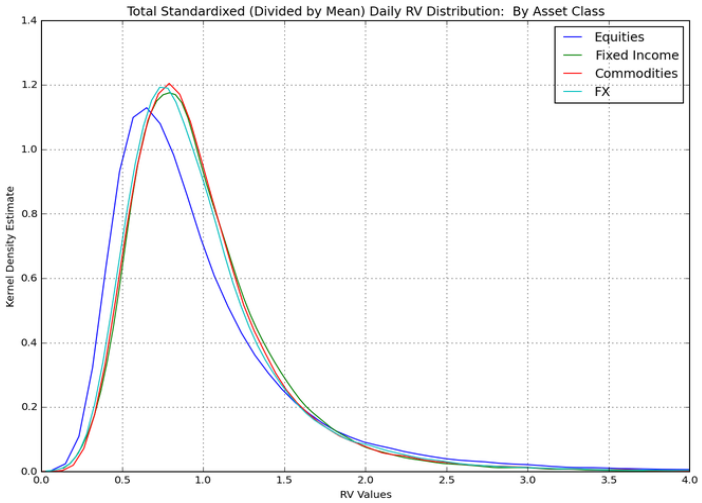
Standardizing by mean makes distributions similar

Daily realized volatility distributions vary greatly across asset classes



But after adjusting for only level, distributions are very similar

- We do not adjust for standard deviation
- Only removes one degree of freedom



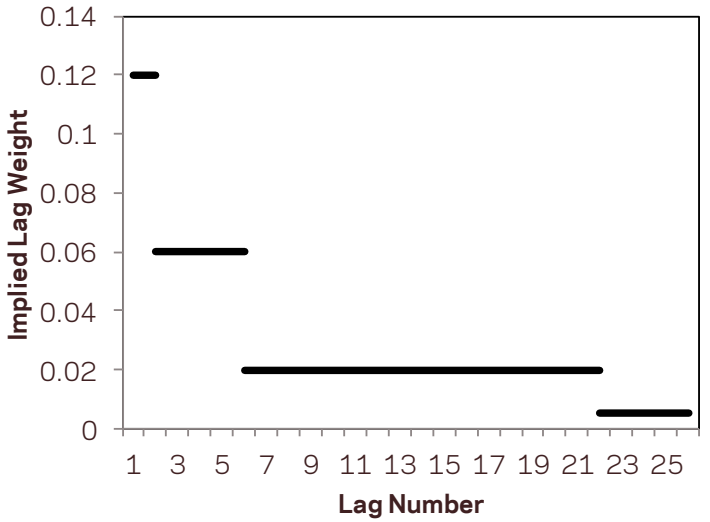
Model Definitions

HAR and its relatives

Baseline model: Heterogeneous Autoregressive (HAR) with multi-period forecasting

- Corsi (2009)
- h is forecasting horizon
- Coefficients depend on choice of h
- Results in "flat" factors

$$RV_{t+h}^h = \beta_0^h + \beta_D^h RV_t + \beta_W^h RV_t^W + \beta_M^h RV_t^M + \epsilon_t^h$$



Pooling with common intercept is too restrictive

- Solution: HAR with fixed effects (HAR-FE)
- Better solution: Long term "anchor"

Free up first 6 days and "anchor" to long run average (HAR-Free)

$$\begin{aligned} RV_{t+h}^h - RV_t^{LR} = {} & \beta_1(RV_t - RV_t^{LR}) + \beta_2(RV_{t-1} - RV_t^{LR}) + \beta_3(RV_{t-2} - RV_t^{LR}) \\ & + \beta_4(RV_{t-3} - RV_t^{LR}) + \beta_5(RV_{t-4} - RV_t^{LR}) + \beta_6(RV_{t-5} - RV_t^{LR}) \\ & + \beta_M(RV_t^M - RV_t^{LR}) + \beta_A(RV_t^A - RV_t^{LR}) + \epsilon_t. \end{aligned}$$

More Model Definitions

Better approximating ACF with sloped factors

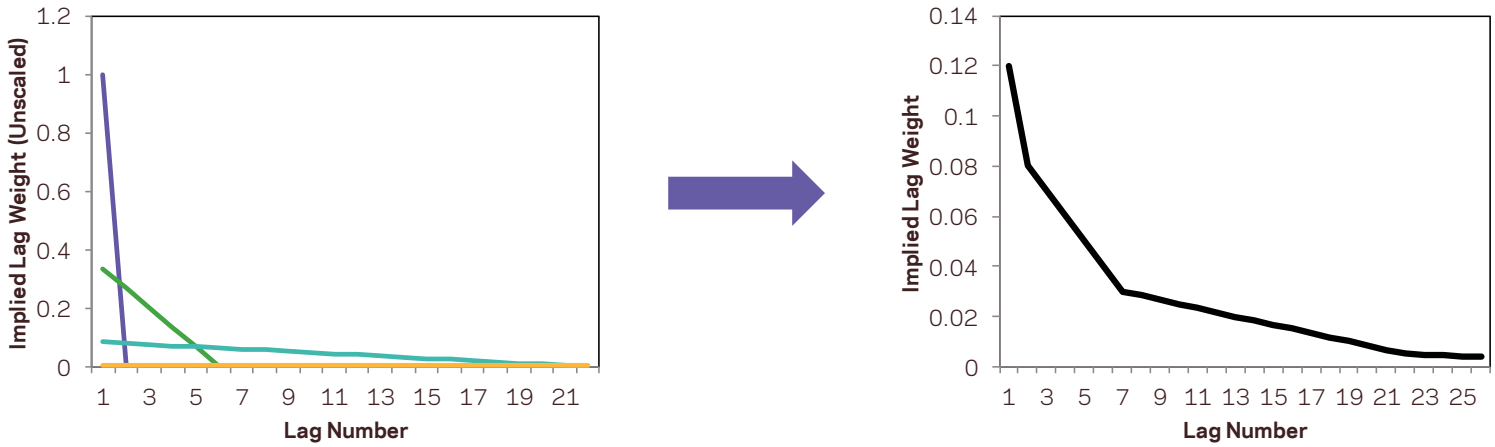
Can we get a "smoother" lag weighting structure?

- The HAR's flat factors impose unnatural drops in weights at deterministic lags
- Logical fix: tilt the flat factors!

$$SlopeRV_t^k \equiv \sum_{i=1}^{k} \left(\frac{k-i+1}{k+(k-1)+\ldots+1} \right) RV_{t+1-i}$$

We call this the Slope-RV model

- Based on Daily, Weekly, Monthly, and Annual slope factors
- Also anchors to the long-run average RV factor



Yet More Model Definitions

Heterogeneous Exponential: A simple combination of EWMAs

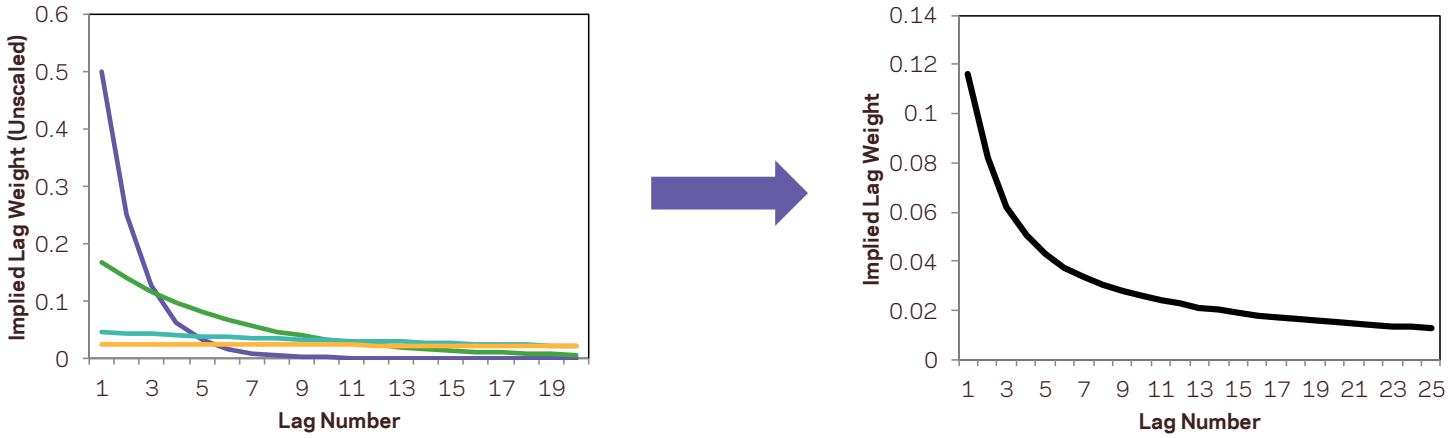
What if we use a spanning set of exponentially-weighted moving averages of RV?

- Heterogeneous Exponential RV (HExp-RV)
- RV lag space well-spanned using 1, 5, 25, and 125 day centers of mass plus long-term anchor
- Note: a combination of EWMAs is not itself an EWMA

Some benefits

- Smooth
- Very easy to implement
- Uses all historical data

$$RV_{t+h}^{h} - RV_t^{LR} = \beta_1(ExpRV_t^{1} - RV_t^{LR}) + \beta_5(ExpRV_t^{5} - RV_t^{LR}) + \beta_{25}(ExpRV_t^{25} - RV_t^{LR}) + \beta_{125}(ExpRV_t^{125} - RV_t^{LR}) + \epsilon_t$$



Model Evaluation Basics

We compute pooled R^2s, both in- and out-of-sample

We evaluate goodness-of-fit using adjusted R^2, with all assets pooled together

- This is always true, regardless of pooling for model estimation

Residual Sum of Squares: panel of $\hat{Y}_{i,t}$ versus $Y_{i,t}$ for each asset i and date t

- Imagine a panel with time as the index and assets on the columns
- Sum squared differences in both dimensions

Total Sum of Squares

- In sample: panel of $Y_{i,t}$ versus $\overline{Y_i}$, the full period average future RV for asset i
- Out of sample: panel of $Y_{i,t}$ versus $\overline{Y_{i,s<t}}$, the expanding window average future RV up to date t
- Interpretation: Out of sample R^2 is relative to a naïve forecast

Out of sample regression methodology

- Estimate betas at end of each month, use to get fitted values for subsequent month
- Hold out assets for 12 months before including in the sample

Asset Class Regressions

Benefits to more restrictive models not just in equities

Monthly Predictive R^2s Across Asset Classes

			HAR	HAR-FE	HAR-Free	Slope	HExp
COMMODITIES	In Sample	Individual Asset	45.3%	45.3%	45.8%	45.4%	45.9%
		Panel	40.1%	43.3%	43.7%	43.7%	43.8%
		Mega	39.3%	43.2%	43.6%	43.5%	43.7%
	Out of Sample	Individual Asset	44.4%	44.3%	44.8%	45.8%	46.2%
		Panel	42.9%	44.9%	46.1%	46.3%	46.5%
		Mega	42.9%	45.2%	46.5%	46.9%	47.1%
EQUITIES	In Sample	Individual Asset	43.6%	43.6%	43.7%	43.7%	43.8%
		Panel	41.2%	42.1%	42.0%	42.2%	42.2%
		Mega	40.2%	41.8%	41.8%	42.0%	42.1%
	Out of Sample	Individual Asset	40.2%	40.2%	43.2%	44.6%	44.3%
		Panel	46.5%	44.0%	47.2%	47.4%	47.0%
		Mega	47.4%	44.4%	47.2%	47.5%	47.4%
FIXED INCOME	In Sample	Individual Asset	43.2%	43.1%	45.0%	46.7%	46.8%
		Panel	40.1%	42.5%	44.2%	45.8%	46.0%
		Mega	<0	41.6%	42.9%	43.9%	44.1%
	Out of Sample	Individual Asset	43.5%	43.3%	46.2%	48.4%	48.2%
		Panel	43.0%	43.9%	46.3%	48.3%	48.3%
		Mega	<0	43.6%	46.9%	47.6%	47.7%
FX	In Sample	Individual Asset	53.7%	53.6%	53.7%	54.0%	54.4%
		Panel	52.4%	52.9%	52.9%	53.3%	53.5%
		Mega	7.9%	52.3%	51.9%	52.4%	52.7%
	Out of Sample	Individual Asset	11.1%	10.9%	45.8%	47.6%	49.4%
		Panel	43.3%	43.4%	48.6%	47.5%	47.4%
		Mega	<0	50.8%	51.1%	51.8%	51.8%

Regressions at Different Horizons

Model choice matters more at long horizons, pooling matters less

Predictive R^2s for All Assets at Different Prediction Horizons

			HAR	HAR-FE	HAR-Free	Slope	HExp
1 Day Prediction Horizon	In Sample	individual Asset	29.1%	29.1%	29.8%	29.4%	29.3%
		Panel	28.2%	28.4%	28.7%	28.7%	28.7%
		Mega	27.5%	27.8%	28.1%	28.2%	28.2%
	Out of Sample	individual Asset	30.6%	30.6%	30.6%	31.3%	31.8%
		Panel	31.3%	31.2%	31.4%	31.8%	32.0%
		Mega	31.2%	31.2%	31.7%	31.9%	31.8%
5 Day Prediction Horizon	In Sample	individual Asset	45.0%	45.0%	45.6%	45.6%	45.7%
		Panel	43.4%	44.0%	44.4%	44.6%	44.7%
		Mega	42.5%	43.5%	43.9%	44.2%	44.3%
	Out of Sample	individual Asset	45.7%	45.7%	46.1%	46.9%	47.5%
		Panel	46.7%	46.6%	47.4%	47.8%	47.9%
		Mega	46.5%	46.6%	47.5%	48.0%	48.0%
60 Day Prediction Horizon	In Sample	individual Asset	33.5%	33.5%	33.2%	32.8%	32.9%
		Panel	23.7%	30.4%	29.5%	29.7%	29.7%
		Mega	18.8%	30.3%	29.4%	29.6%	29.6%
	Out of Sample	individual Asset	28.6%	28.6%	34.4%	35.1%	35.4%
		Panel	28.9%	30.7%	34.7%	35.2%	35.0%
		Mega	25.3%	31.2%	35.2%	35.6%	35.6%

RV Predicts Both RV and Daily Better

Large benefit from reduction in noise for left-hand side variable

Daily Return versus RV-based Monthly Predictive R^2s

		HAR	HAR-FE	HAR-Free	Slope	HExp
In Sample	Daily pred Daily	26.6%	31.5%	31.6%	31.8%	32.0%
	Daily pred RV	33.7%	39.1%	39.5%	39.9%	40.2%
	RV pred Daily	31.9%	35.3%	34.8%	34.8%	34.9%
	RV pred RV	39.0%	42.6%	42.8%	42.8%	42.9%
Out of Sample	Daily pred Daily	30.4%	32.7%	34.9%	35.3%	35.5%
	Daily pred RV	40.5%	43.3%	43.2%	43.7%	44.0%
	RV pred Daily	35.9%	36.8%	38.5%	38.7%	38.7%
	RV pred RV	43.6%	44.9%	46.9%	47.2%	47.3%

Intraday Versus Overnight Regressions

Practically no improvement from fitting intraday/overnight separately

Intraday versus Overnight Monthly Predictive R^2s

			HAR	HAR-FE	HAR-Free	Slope	HExp
Intraday Only	In Sample	Individual Asset	53.7%	53.6%	54.1%	54.0%	54.1%
		Panel	50.9%	52.5%	52.6%	52.6%	52.8%
		Mega	49.1%	52.2%	52.3%	52.4%	52.6%
	Out of Sample	Individual Asset	51.7%	51.7%	54.0%	54.5%	54.6%
		Panel	53.8%	54.0%	55.3%	55.4%	55.3%
		Mega	52.8%	54.3%	55.4%	55.6%	55.7%
Overnight Only	In Sample	Individual Asset	29.1%	29.1%	29.1%	29.4%	29.7%
		Panel	24.4%	26.9%	27.0%	27.1%	27.1%
		Mega	23.1%	26.8%	26.9%	27.0%	27.0%
	Out of Sample	Individual Asset	23.7%	23.6%	26.1%	29.1%	30.4%
		Panel	28.6%	28.0%	31.2%	31.5%	31.3%
		Mega	28.1%	28.4%	31.5%	31.9%	31.8%
Intraday Plus Overnight	In Sample	Individual Asset	45.8%	45.8%	45.8%	45.8%	46.0%
		Panel	42.4%	44.1%	44.1%	44.1%	44.1%
		Mega	40.9%	43.8%	43.8%	43.8%	43.9%
	Out of Sample	Individual Asset	42.8%	42.8%	45.1%	46.3%	46.8%
		Panel	45.9%	44.9%	47.2%	47.4%	47.3%
		Mega	45.1%	45.2%	47.3%	47.6%	47.6%

Variance Space or Volatility Space?

Regression fit is similar in either case

Monthly Predictive R^2s Based on Volatility Space Regressions

		HAR	HAR-FE	HAR-Free	Slope	HExp
In Sample	individual Asset	45.4%	45.4%	46.4%	45.1%	45.1%
	Panel	42.3%	44.1%	45.3%	44.0%	43.7%
	Mega	39.9%	43.9%	45.3%	43.9%	43.7%
Out of Sample	individual Asset	47.4%	47.4%	48.8%	48.0%	48.0%
	Panel	47.2%	48.7%	49.8%	49.2%	49.0%
	Mega	45.9%	48.8%	49.8%	49.3%	49.2%

Effect of Assumptions on Utility

Higher Sharpe/risk aversion imply higher benefits to volatility targeting

		Risk Target				
		5%	**10%**	**15%**	**20%**	**25%**
Assumed Sharpe Ratio	**0.1**	0.09%	0.18%	0.27%	0.36%	0.45%
	0.2	0.18%	0.36%	0.55%	0.73%	0.91%
	0.3	0.27%	0.55%	0.82%	1.09%	1.36%
	0.4	0.36%	0.73%	1.09%	1.46%	1.82%
	0.5	0.45%	0.91%	1.36%	1.82%	2.27%
	0.6	0.55%	1.09%	1.64%	2.18%	2.73%
	0.7	0.64%	1.27%	1.91%	2.55%	3.18%
	0.8	0.73%	1.46%	2.18%	2.91%	3.64%

Utility By Asset Class

Assuming non-zero transactions costs

		DAILY RETURNS			RV		FUTURE RV	
	Transactions Cost	**In Sample Mean**	**Expanding Mean**	**21 Day Window**	**21 Day RV**	**HExp**	**Future 20 Day RV**	**Future 1 Day RV**
COMMODITIES	Half-Spread	0.89%	0.70%	1.19%	1.26%	1.27%	1.31%	0.83%
	2x Half-Spread	0.89%	0.70%	1.11%	1.23%	1.23%	1.27%	0.16%
	5x Half-Spread	0.89%	0.68%	0.88%	1.12%	1.12%	1.16%	-1.85%
EQUITIES	Half-Spread	0.57%	0.63%	1.22%	1.25%	1.27%	1.31%	1.08%
	2x Half-Spread	0.57%	0.63%	1.17%	1.23%	1.24%	1.28%	0.65%
	5x Half-Spread	0.57%	0.62%	1.02%	1.15%	1.16%	1.21%	-0.61%
FIXED INCOME	Half-Spread	0.90%	0.94%	1.06%	1.19%	1.24%	1.24%	-0.46%
	2x Half-Spread	0.90%	0.93%	0.86%	1.10%	1.17%	1.14%	-2.41%
	5x Half-Spread	0.90%	0.90%	0.28%	0.81%	0.98%	0.84%	-8.28%
FX	Half-Spread	0.74%	0.58%	1.09%	1.21%	1.21%	1.26%	-0.01%
	2x Half-Spread	0.74%	0.56%	0.93%	1.14%	1.12%	1.18%	-1.53%
	5x Half-Spread	0.74%	0.51%	0.47%	0.92%	0.85%	0.96%	-6.06%
ALL ASSETS	Half-Spread	0.75%	0.69%	1.16%	1.24%	1.25%	1.29%	0.61%
	2x Half-Spread	0.75%	0.68%	1.07%	1.20%	1.20%	1.25%	-0.28%
	5x Half-Spread	0.75%	0.66%	0.78%	1.06%	1.04%	1.10%	-2.94%

Utility By Trading Rule

Trading rule matters more given higher transactions costs

		DAILY RETURNS			RV		FUTURE RV	
		In Sample Mean	**Expanding Mean**	**21 Day Window**	**21 Day RV**	**HExp**	**Future 20 Day RV**	**Future 1 Day RV**
Half Spread	100% to Target	0.75%	0.69%	1.16%	1.24%	1.25%	1.29%	0.61%
	50% to Target	0.75%	0.68%	1.19%	1.25%	1.26%	1.31%	1.07%
	25% to Target	0.75%	0.67%	1.19%	1.24%	1.27%	1.33%	1.17%
	12.5% to Target	0.75%	0.64%	1.19%	1.23%	1.26%	1.34%	1.19%
2x Half Spread	100% to Target	0.75%	0.68%	1.07%	1.20%	1.20%	1.25%	-0.28%
	50% to Target	0.75%	0.68%	1.13%	1.21%	1.24%	1.28%	0.71%
	25% to Target	0.75%	0.66%	1.15%	1.22%	1.25%	1.30%	1.00%
	12.5% to Target	0.75%	0.63%	1.16%	1.21%	1.25%	1.32%	1.11%
5x Half Spread	100% to Target	0.75%	0.66%	0.78%	1.06%	1.04%	1.10%	-2.94%
	50% to Target	0.75%	0.67%	0.95%	1.12%	1.15%	1.18%	-0.38%
	25% to Target	0.75%	0.65%	1.02%	1.14%	1.19%	1.22%	0.49%
	12.5% to Target	0.75%	0.62%	1.07%	1.15%	1.21%	1.26%	0.85%

Equity Assets

Asset Class	Asset	Number of Assets	Total Days in Analysis	Primary Data Source	Used From	Secondary Data Source	Used From	Assumed T-Costs (in bps)
EQUITIES		21	80042	TRTH		NONE		
	Australia (SPI 200)	1	3472	TRTH	12/18/2000	NA	NA	1.9
	Germany (DAX 30)	1	4732	TRTH	1/3/1996	NA	NA	1.0
	Brazil (BOVESPA)	1	4577	TRTH	2/27/1996	NA	NA	2.8
	China (Hang Seng CEI)	1	2667	TRTH	12/9/2003	NA	NA	2.0
	Canada (S&P/TSX 60)	1	3773	TRTH	9/14/1999	NA	NA	1.3
	Spain (IBEX 35)	1	4698	TRTH	1/4/1996	NA	NA	2.0
	Eurostoxx	1	4130	TRTH	6/23/1998	NA	NA	3.2
	France (CAC 40)	1	4007	TRTH	1/7/1999	NA	NA	1.1
	Hong Kong (Hang Seng)	1	4591	TRTH	1/3/1996	NA	NA	1.2
	India (SGX NIFTY)	1	2213	TRTH	10/11/2005	NA	NA	1.7
	Italy (FTSE MIB)	1	2617	TRTH	6/15/2004	NA	NA	2.4
	Japan (TOPIX)	1	4570	TRTH	1/5/1996	NA	NA	4.1
	South Korea (KOSPI 200)	1	4466	TRTH	5/6/1996	NA	NA	1.9
	Netherlands (AEX)	1	4499	TRTH	1/9/1997	NA	NA	1.3
	South Africa (ALSI)	1	2308	TRTH	7/7/2005	NA	NA	1.7
	Switzerland (SMI)	1	4027	TRTH	9/15/1998	NA	NA	1.2
	Taiwan (SGX-MSCI Taiwan)	1	4295	TRTH	2/24/1997	NA	NA	3.1
	UK (FTSE 100)	1	4706	TRTH	1/3/1996	NA	NA	0.8
	US (S&P 500 E-Mini)	1	4274	TRTH	9/10/1997	NA	NA	1.3
	US (Russell 2000 E-Mini)	1	2234	TRTH	12/13/2005	NA	NA	0.9
	US (S&P 400 Mid Cap E-Mini)	1	3186	TRTH	1/29/2002	NA	NA	1.5

Commodities Assets

Asset Class	Asset	Number of Assets	Total Days in Analysis	Primary Data Source	Used From	Secondary Data Source	Used From	Assumed T-Costs (in bps)
COMMODITIES		20	108149	TRTH		TDC		
	Brent Oil	1	4754	TRTH	1/3/1996	TDC	1/3/1996	1.0
	Cattle	1	5483	TRTH	12/20/2004	TDC	11/30/1992	3.2
	Cocoa	1	5471	TRTH	4/1/2008	TDC	11/11/1992	3.4
	Coffee	1	5469	TRTH	4/1/2008	TDC	11/17/1992	8.0
	Corn	1	5502	TRTH	8/1/2006	TDC	11/19/1992	5.7
	Cotton	1	5453	TRTH	4/1/2008	TDC	11/12/1992	4.6
	Crude (WTI) Oil	1	5480	TRTH	9/5/2006	TDC	11/10/1992	1.0
	Feeder Cattle	1	5513	TRTH	8/1/2007	TDC	10/29/1992	4.5
	Gas Oil	1	4754	TRTH	1/3/1996	TDC	1/3/1996	2.8
	Gold	1	5471	TRTH	12/4/2006	TDC	12/2/1992	0.8
	Heating Oil	1	5480	TRTH	9/5/2006	TDC	11/16/1992	1.7
	Lean Hogs	1	5486	TRTH	2/15/2005	TDC	11/30/1992	4.5
	Natural Gass	1	5442	TRTH	8/23/2006	TDC	1/5/1993	4.0
	Silver	1	5412	TRTH	12/4/2006	TDC	1/5/1993	2.6
	Soybeans	1	5522	TRTH	8/1/2006	TDC	10/22/1992	2.1
	Soymeal	1	5502	TRTH	8/1/2006	TDC	11/19/1992	4.1
	Soyoil	1	5501	TRTH	8/1/2006	TDC	11/19/1992	3.0
	Sugar	1	5481	TRTH	4/1/2008	TDC	11/3/1992	5.9
	Unleaded (RBOB)	1	5475	TRTH	8/22/2006	TDC	11/16/1992	2.0
	Wheat	1	5498	TRTH	8/1/2006	TDC	11/19/1992	4.4

Fixed Income and Foreign Exchange Assets

Asset Class	Asset	Number of Assets	Total Days in Analysis	Primary Data Source	Used From	Secondary Data Source	Used From	Assumed T-Costs (in bps)
FIXED INCOME		**8**	**32333**	**TRTH**		**TDC**		
	Australia 10y	1	4734	TRTH	1/3/1996	TDC	1/3/1996	3.9
	Germany 10y	1	4499	TRTH	1/5/1999	TDC	1/3/1997	0.7
	Germany 5y	1	4493	TRTH	2/1/1999	TDC	1/3/1997	0.8
	Canada 10y	1	2771	TRTH	9/26/2000	TDC	9/2/2003	0.8
	Japan 10y	1	3605	TRTH	1/5/1996	TDC	1/4/2000	0.7
	UK 10y	1	4711	TRTH	1/3/1996	TDC	1/3/1996	0.9
	US 10y	1	3993	TRTH	1/1/2001	TDC	10/20/1998	1.3
	US 5y	1	3527	TRTH	7/1/2001	TDC	9/5/2000	0.7
FOREIGN EXCHANGE		**9**	**30161**	**Olsen Data**		**NONE**		
	Australia (AUD-USD)	1	2802	OlsenData	1/1/2004	NA	NA	2.2
	Eurozone (EUR-USD)	1	4103	OlsenData	1/1/1999	NA	NA	0.7
	Canada (USD-CAD)	1	3061	OlsenData	1/1/2003	NA	NA	2.5
	Japan (USD-JPY)	1	3841	OlsenData	1/1/2000	NA	NA	1.0
	Norway (USD-NOK)	1	2801	OlsenData	1/1/2004	NA	NA	8.1
	New Zealand (NZD-USD)	1	2803	OlsenData	1/1/2004	NA	NA	4.7
	Sweden (USD-SEK)	1	3062	OlsenData	1/1/2003	NA	NA	7.7
	Switzerland (USD-CHF)	1	3844	OlsenData	1/1/2000	NA	NA	1.7
	UK (GBP-USD)	1	3844	OlsenData	1/1/2000	NA	NA	1.5