Affordable Housing and City Welfare

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Abstract

Housing affordability has become the main policy challenge for most large cities in the world. Key policy levers are zoning, rent control, housing vouchers, and tax credits. We build a new dynamic stochastic spatial equilibrium model to evaluate the effect of these policies on house prices, rents, residential construction, labor supply, output, income and wealth inequality, as well as the location decision of households within the city. The analysis incorporates risk, wealth effects, and dynamic spatial equilibrium. We calibrate the model to the New York MSA, incorporating current zoning and rent control policies. Our model suggests sizable welfare gains from relaxing zoning regulations in the city center, as well as from expanding rent control and housing voucher programs. Housing affordability policies have a hitherto under-appreciated insurance value which needs to be traded off against potential efficiency losses. The calibrated model implies gains in social welfare from reducing housing inequality.

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1 Introduction

The increasing appeal of major urban centers has brought on an unprecedented housing affordability crisis. Ever more urban households are burdened by rents or mortgage payments that take up a large fraction of their paycheck and/or by long commutes. The share of cost-burdened renters in the United States has risen from 23.8% in the 1960s to 47.5% in 2016. Over this period, median home value rose 112%, far outpacing the 50% increase in the median owner income (Joint Center for Housing Studies of Harvard University, 2018). Housing affordability has macro-economic consequences. Hsieh and Moretti (2019) have argued that our most productive cities are smaller than they should be because of lack of affordable housing options.

Policy makers are under increasing pressure to improve affordability. They have four broad categories of tools to address housing affordability: rent control (RC), zoning policies, housing vouchers, and tax credits for developers. We conceive RC broadly to include all government-provided or regulated housing, which rents at below-market rates. It has multiple policy levers: the size of the RC program, the discount to the market rent, the income threshold for qualifying, the depth of the affordability, and the spatial distribution of the housing stock. Zoning governs land use and can be changed to increase the supply of housing, and all else equal, reduce its cost. Zoning changes are often associated with requirements on developers to set aside affordable housing units (inclusionary zoning). Housing vouchers are subsidies to households to be used to lower their housing expenditures, leaving them free to choose where to locate. Finally, tax credits directly affect developers’ incentives to build. Each policy affects the quantity and price of owned and rented housing and its spatial distribution. It affects incentives to work, wages, commuting patterns, and ultimately output. Each policy affects wealth inequality in the city and in each of its neighborhoods.

While there is much work, both empirical and theoretical, on housing affordability, what is missing is a general equilibrium model that can quantify the impact of such policies on prices and quantities, on the spatial distribution of households, on income inequality within and across neighborhoods, and ultimately on individual and city-wide welfare. This paper sets up and solves a model that is able to address these questions.

We model a metropolitan area (city), which consists of two zones, the city center or central business district (zone 1) and the rest of the metropolitan area (zone 2). Working-age households who live in zone 2 commute to zone 1 for work. The model is an overlapping generations model with risk averse households that face labor income risk during

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1 Fifteen cities in California have rent control. A November 2018 California state ballot initiative proposed to overturn the 1995 Costa-Hawkins Act, clearing the way for more rent control. Bill de Blasio, the mayor of New York City, was elected on a platform to preserve or add 200,000 affordable housing units.
their life-cycle as well as mortality risk, and make dynamic decisions on non-housing and housing consumption, savings in bonds and investment property, labor supply, tenure status (own or rent), and location. Since households cannot perfectly hedge labor income and longevity risk, markets are incomplete. This incompleteness opens up the possibility for housing affordability policies to provide insurance. We model a progressive tax system to capture the existing insurance provided by other government tax and transfer programs beside housing policies. The model generates a rich cross-sectional distribution over age, labor income, tenure status, housing wealth, and financial wealth. We show that this richness is paramount to understanding the distributional effects from housing affordability policies.

On the firm side, the city produces tradable goods and residential housing in each zone, subject to decreasing returns to scale. Construction is subject to zoning regulation which limits the maximum amount of housing that can be built, lowering the housing supply elasticity. The city has a rent control system in place. Qualifying households enter in a housing lottery that randomly allocates housing units at discounted rents for those households that qualify. Once in the RC system, a tenant has strong priority to stay put. Rent control affects rents earned by landlords, which lowers the average price of rental buildings, and thereby the incentives for residential development. Wages, house prices, and market rents are determined in the city’s equilibrium.

We calibrate the model to the New York metropolitan area and designate Manhattan as the urban core, or zone 1, and the rest of the metropolitan area (MSA) as zone 2. Our calibration matches key features of the data, including the relative size of Manhattan versus the rest of the MSA, the New York MSA income distribution, commuting times and costs, the housing supply elasticity, current zoning laws, the current size and scope of the rent control system, and the current federal, state, and local tax and transfer system.

We use this model as a laboratory to explore a range of housing affordability policies. In the first experiment, we find that expanding the quantity of rent controlled housing is welfare-increasing. A 50% increase in RC square footage increases welfare by 0.15% in consumption equivalent units. Rent control acts as an insurance device that particularly benefits households in the bottom of the income distribution. The gain from improved access outweighs the aggregate cost from RC which results from a spatial and sectoral misallocation of labor and housing. For the parameters that best fit the New York MSA data, we find that the reduction in inequality more than compensates for the reduction in efficiency. In contrast with received wisdom, increasing the share of housing devoted to RC does not lead to an overall decline in the quantity of housing. The housing stock

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2In a 1992 survey of professional economists by Alston, Kearl, and Vaughn (1992), 92.9% agreed that “A ceiling on rents reduces the quantity and quality of housing available.” Micheli and Schmidt (2015)
declines in the urban core, consistent with partial equilibrium logic (Autor, Palmer, and Pathak (2014), Diamond, McQuade, and Qian (2017)). With endogenous location choice, the population share of the urban core falls and demand for housing in zone 2, where housing is cheaper, rises. The expansion of the housing stock in zone 2 more than offsets the fall in the housing stock in zone 1. Another interesting finding, in common with several other experiments, is that standard housing affordability metrics such as rent-to-income ratios and the fraction of severely rent-burdened households, do not capture the improved availability of affordable housing. The rent-to-income ratio increases in zone 1, reflecting not only the higher rents resulting from a smaller housing stock but also the lower average income of the new demographic make-up (de-gentrification). Indeed, a policy that symmetrically expands rent recontrol in both zones ends up reducing inequality between zones in equilibrium. The welfare effects are not symmetric. We show that an equal-sized reduction in rent control leads to a much larger welfare loss than the welfare gain from expanding it.

To explore the spatial aspect of our model, we study a policy that concentrates all affordable housing units in zone 2 while keeping the overall size of the program unchanged. In equilibrium, fewer households end up in larger RC units. The reduced access to the insurance provided by RC hurts lower-income households who suffer an adverse income shock and ultimately reduces aggregate welfare (-0.45%). Despite the improved incentives for residential development in zone 1, the housing stock shrinks. The removal of RC units from zone 1 frees up space for high-income households, who are more likely to own, but it reduces the overall demand for space in zone 1. The population of zone 2 grows. Despite the worsened incentives for residential development in zone 2, the housing stock expands to accommodate the extra population. Partial equilibrium logic is deceptive.

The third main policy experiment concerns zoning. The policy makes it easier to build in zone 1 by increasing the allowed maximum residential buildable area. Specifically, we study a policy that ends up increasing the population share of Manhattan by 10%. There are substantial welfare gains from this policy (0.36%). The policy is a Pareto improvement, benefitting all age, productivity, income, and wealth groups. Because of endogenous population changes, affordability metrics such as rent-to-income and the share of rent burdened households deteriorate. They fail to capture the benefit of living closer to work for many household. The housing boom in Manhattan ends up increasing metro-wide wages and reducing the competitiveness of the area as a whole. Output declines slightly.

The fourth main policy experiment studies an expansion of the housing voucher system. Vouchers are cash subsidies provided through the tax and transfer system to low income households to be used for housing expenditures. An expansion of the voucher system is welfare increasing (0.90%). The policy completes markets, in that marginal util-
ity growth becomes less volatile. Because the vouchers are targeted to high marginal utility households, they increase aggregate welfare. There are substantial costs from this policy since we assume, consistent with reality, that a voucher expansion would be financed with a more progressive tax system. Since labor income taxation is distortionary, high- and middle-productivity households reduce labor supply which results in a decline in output. The voucher expansion, whose direct effects are independent of location, triggers an interesting spatial response. Some high- and middle-productivity households decide to move out of Manhattan after the tax increase. In equilibrium, the vouchers do not allow poor households to “move to opportunity” but rather seem to “remove from opportunity” some high-productivity households.

Our final main experiment is a tax credit program. Developers are subsidized for constructing affordable housing. The tax credits are paid for by higher taxes. In our calibration, the distortionary tax effects depress housing demand, resulting in a lower rather than higher equilibrium housing stock. The policy fails to bring significant indirect benefits from lower rents, so that the distortionary tax effects dominate. Aggregate welfare falls (-0.52%).

The appendix studies five more policies that change the levers of the rent control system. The most welfare beneficial one is a policy that tightens the income qualification requirements for RC. By more explicitly targeting low-income households, the policy improves access to RC insurance for those who need it most without creating more macroeconomic distortions.

**Related Literature** Our work is at the intersection of the macro-finance and urban economics literatures. On the one hand, a large literature in finance solves partial-equilibrium models of portfolio choice between housing (extensive and intensive margin), financial assets, and mortgages. Examples are Campbell and Cocco (2003), Cocco (2005), Yao and Zhang (2004), and Berger, Guerrieri, Lorenzoni, and Vavra (2017). Davis and Van Nieuwerburgh (2015) summarize this literature. More recent work in macro-finance has solved such models in general equilibrium, adding aggregate risk, endogenizing house prices and sometimes also interest rates. E.g., Favilukis, Ludvigson, and Van Nieuwerburgh (2017) and Kaplan, Mitman, and Violante (2017). Imrohoroglu, Matoba, and Tuzel (2016) study the effect of the 1978 passage of Proposition 13 which lowered property taxes in California. They find quantitatively meaningful effects on house prices, moving rates, and welfare. Like the former literature, our model features a life-cycle and a rich portfolio choice problem. It aims to capture key quantitative features of observed wealth accumulation and home ownership over the life-cycle. Like the latter literature, house prices, rents, and wages are determined in equilibrium. Because we model one city, interest rates and
tradeable goods prices are naturally taken as given. Like the macro-finance literature, we aim to capture key features of house prices, income inequality, and wealth inequality. However, we abstract from aggregate risk which is not central to our analysis. Our key contribution to the macro-finance literature is to add a spatial dimension by introducing a cost of commuting, differing housing supply elasticity, and local amenities.

On the other hand, a voluminous literature in urban economics studies the spatial location of households and firms in urban areas. Brueckner (1987) summarizes the Muth-Mills monocentric city model. Rosen (1979) and Roback (1982) introduce spatial equilibrium. This literature studies the trade-off between the commuting costs and housing expenditures. These models tend to be static and households tend to be risk neutral or have quasi-linear preferences. Recent work on spatial sorting across cities includes Van Nieuwerburgh and Weill (2010), Behrens, Duranton, and Robert-Nicoud (2014) and Eeckhout, Pinheiro, and Schmidheiny (2014). Rappaport (2014) introduces leisure as a source of utility and argues that the monocentric model remains empirically relevant. Guerrieri, Hartley, and Hurst (2013) study house price dynamics in a city and focus on neighborhood consumption externalities, in part based on empirical evidence in Rossi-Hansberg, Sarte, and Owens (2010). The lack of risk, investment demand for housing by local residents, and wealth effects makes it hard to connect these spatial models to the macro-finance literature.

Hizmo (2015) and Ortalo-Magné and Prat (2016) bridge some of this gap when they study a portfolio choice problem where households make a once-and-for-all location choice between cities. Conditional on the location choice, they are exposed to local labor income risk and make an optimal portfolio choice. They have constant absolute risk aversion preferences and consume at the end of life. The model features absentee landlords. Their models are complementary to ours in that they solve a richer portfolio choice problem in closed-form, and have a location choice across cities. We solve a within-city location choice, but allow for preferences that admit wealth effects, and allow for consumption and location choice each period. We close the housing market in that local, risk averse landlords who own more housing than they consume rent to other locals. Studying the welfare effects of housing affordability policies requires incorporating such wealth effects.

Because it is a heterogeneous-agent, incomplete-markets model, agents choices and equilibrium prices depend on the entire wealth distribution. Because of the spatial dimension, households’ location is an additional state variable that needs to be kept track of. We use state-of-the-art methods to solve the model. We extend the approach of Favilukis et al. (2017), which itself extends Gomes and Michaelides (2008) and Krusell and Smith (1998) before that. The resulting model is a new laboratory that can be used to study how place-
based policies affect the spatial distribution of people, labor supply, house prices, and inequality. Favilukis and Van Nieuwerburgh (2018) use a related framework to study the effect of out-of-town investors on residential property prices. A key difference is that the current paper adds a spatial dimension, which substantially enriches the model and adds cross-equation restrictions that can be used to discipline the fit to the data and to generate new predictions related to spatial inequality, is calibrated to New York, and studies housing affordability policies.

Finally, our model connects to a growing empirical literature that studies the effect of rent control and zoning policies on rents, house prices, and housing supply. Autor et al. (2014); Autor, Palmer, and Pathak (2017) find that the elimination of the rent control mandate on prices in Cambridge increased the value of decontrolled units and neighboring properties in the following decade, by allowing constrained owners to raise rents and increasing the amenity value of those neighborhoods through housing market externalities. The price increase spurred new construction, increasing the rental stock. Diamond et al. (2017) show that the expansion of the rent control mandate in San Francisco led to a reduction in the supply of available housing, by decreasing owners' incentives to rent below market prices, paradoxically contributing to rising rents and the gentrification of the area. While beneficial to tenants in rent control, it resulted in an aggregate welfare loss. Diamond and McQuade (forthcoming) find that the use of the Low Income Housing Tax Credit, a financial incentive for landlords to rent their properties below market prices to low-income tenants, leads to house price appreciation and decreasing segregation in low-income neighborhoods, thereby increasing welfare. As in our paper, the nature of the rent control policy and its distributional consequences are essential. In contemporaneous work, Sieg and Yoon (2017) also study the welfare gains from affordable housing policies in New York. Different from our set up, they consider public housing and rent-stabilized housing as two distinct housing options aside from market rentals. They do not allow for home ownership. Since they only focus on Manhattan, their model does not consider the equilibrium spillover effects from changes in Manhattan housing policies on the rest of the metropolitan area.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 describes the calibration to the New York metropolitan area. Section 4 discusses its main results and implications for quantities and prices, the distribution of households and affordability. Section 5 studies counterfactual policy experiments in which we vary the nature and strength of the rent control and zoning tools. Section 6 concludes.

2 Model

The model consists of two geographies, the “urban core” and the “periphery”, whose union forms the “metropolitan area” or “city.” The urban core, which we refer to as zone 1, is the central business district where all employment takes place. The periphery, or zone 2, contains the outer boroughs of the city as well as the suburban areas that belong to the metropolitan area. In the context of the New York calibration in Section 3, zone 1 will be Manhattan while zone 2 contains the other 24 counties that make up the metro area. While clearly an abstraction of the more complex production and commuting patterns in large cities, the monocentric city assumption captures the essence of commuting patterns (Rappaport, 2014) and is the simplest way to introduce a spatial aspect in the model. The two zones are allowed to differ in size. The city has a fixed population.\(^4\)

2.1 Households

Preferences The economy consists of overlapping generations of risk averse households. There is a continuum of households of a given age \(a\). Each household maximizes a utility function \(u\) over consumption goods \(c\), housing \(h\), and labor supply \(n\). Utility depends on location \(\ell\) and age \(a\), allowing the model to capture commuting time and amenity differences across locations.

The period utility function is a CES aggregator of \(c\) and \(h\) and leisure \(l\):

\[
U(c_t, h_t, n_t, \ell_t, a) = \frac{\chi^T_{\ell, a} C(c_t, h_t, l_t)}{1 - \gamma},
\]

\[
C(c_t, h_t, l_t) = \left(1 - \alpha_n\right) \left((1 - \alpha_h)c_t^\ell + \alpha_hh_t^\ell\right)^{\eta} + \alpha_n n_t^\eta \right)^{\frac{1}{\eta}},
\]

\[
h_t \geq \frac{h}{(1 - \alpha_n)(1 - \alpha_h)c_t^\ell + \alpha_hh_t^\ell + \alpha_n n_t^\eta}
\]

\[
n_t^a = \begin{cases} 
1 - \phi_T - l_t \geq n & \text{if } a < 65 \\
0 & \text{if } a \geq 65
\end{cases}
\]

\(^4\)Future work could study interactions between affordability policies and net migration patterns in an open-city model. Such a model would need to take a stance on a reservation utility of moving and on the moving costs. These reservation utilities would naturally differ by age, productivity, and wealth, leading to a proliferation of free parameters. The lack of guidance from the literature would pose a substantial challenge to calibration. Furthermore, the empirical evidence for the New York metropolitan area, discussed in Appendix B.6, suggests that the zero net migration assumption fits the data well. These two considerations motivate the closed-city model assumption.
\begin{equation}
\chi_{t}^{\ell,a} = \begin{cases} 
\chi^{1} & \text{if } \ell = 1 \text{ and } c_t < \zeta \\
\chi^{1} \chi^{W} & \text{if } \ell = 1 \text{ and } a < 65 \text{ and } c_t \geq \zeta \\
\chi^{1} \chi^{R} & \text{if } \ell = 1 \text{ and } a \geq 65 \text{ and } c_t \geq \zeta \\
1 & \text{if } \ell = 2
\end{cases}
\end{equation}

(4)

The coefficient of relative risk aversion is \( \gamma \). The Frisch elasticity of labor supply is affected by all utility parameters, but mostly governed by \( \eta \). The parameter \( \epsilon \) controls the intra-temporal elasticity of substitution between housing and non-housing consumption.

Equation (2) imposes a minimum house size requirement \((h_0)\), capturing the notion that a minimum amount of shelter is necessary for a household.

Total non-sleeping time in equation (3) is normalized to 1 and allocated to work \((n_t)\), leisure \((l_t)\), and commuting time \(\phi_{T}^\ell\). We normalize commuting time for zone 1 residents to zero: \(\phi_{T}^2 > \phi_{T}^1 = 0\). Since we will match income data that exclude the unemployed, we impose a minimum constraint on the number of hours worked \((n)\) for working-age households. This restriction will also help us match the correlation between income and wealth. There is an exogenous retirement age of 65. Retirees supply no labor.

The age- and location-specific taste-shifter \(\chi_{t}^{\ell,a}(c_t)\) is normalized to one for all zone 2 residents. The shifter \(\chi^{1} \geq 1\) increase the utility of all zone 1 residents. The shifter \(\chi^{W} (\chi^{R})\) increases the utility for working-age (retired) households that live in zone 1 and consume above a threshold \(\zeta\). This creates a complementarity between living in zone 1 and high consumption levels. This modeling device captures that luxury amenities such as high-end entertainment, restaurants, museums, or art galleries are concentrated in the urban core. By assuming that the benefit from such luxury amenities only accrue above a certain consumption threshold, this provides an extra pull for rich households to live in the city center, beyond the pull provided by the opportunity cost of commuting. Guerrieri et al. (2013) achieve a similar outcome through a neighborhood consumption externality. A special case of the model arises for \(\chi^{1} = \chi^{R} = \chi^{W} = 1\); location choice is solely determined by commuting costs. Another special case is \(\zeta = 0\), which gives the same amenity value of the city center to all households, rich or poor.

There are two types of households in terms of the time discount factor. One group of households have a high degree of patience \(\beta^H\) while the rest have a low degree of patience \(\beta^L\). This preference heterogeneity helps the model match observed patterns of wealth inequality and wealth accumulation over the life cycle. A special case of the model sets \(\beta^H = \beta^L\).

**Endowments** A household’s labor income \(y_{t}^{lab}\) depends on the number of hours worked \(n\), the wage per hour worked \(W\), a deterministic component \(G(a)\) which captures the hump-shaped pattern in average labor income over the life-cycle, and an idiosyncratic

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labor productivity $z$, which is stochastic and persistent.

After retirement, households earn a retirement income which is the product of an aggregate component $\Psi$ and an idiosyncratic component $\psi^a z$. The idiosyncratic component has cross-sectional mean of one, and is determined by productivity during the last year of work. Labor income is taxed linearly at rate $\tau^{SS}$ to finance retirement income. All other taxes and transfers are captured by the function $T(\cdot)$ which maps total pre-tax income into a net tax (negative if transfer). Net tax revenue goes to finance a public good which does not enter in household utility.

Households face mortality risk which depends on age, $p^a$. Although there is no intentional bequest motive, households who die leave accidental bequests. We assume that the number of agents who die with positive wealth leave a bequest to the same number of agents alive of ages 21 to 65. These recipient agents are randomly chosen, with one restriction. Patient agents ($\beta^H$) only leave bequests to other patient agents and impatient agents ($\beta^L$) only leave bequests to other impatient agents. One interpretation is that attitudes towards saving are passed on from parents to children. Conditional on receiving a bequest, the size of the bequest $\hat{b}_{t+1}$ is a draw from the relevant distribution, which differs for $\beta^H$ and $\beta^L$ types. Because housing wealth is part of the bequest and the house price depends on the aggregate state of the economy, the size of the bequest is stochastic. Agents know the distribution of bequests, conditional on $\beta$ type. This structure captures several features of real-world bequests: many households receive no bequest, bequests typically arrive later in life and at different points in time for different households, and there is substantial heterogeneity among bequest sizes for those who receive a bequest.

**Rent Control**  We model the rent-controlled (RC) housing sector as a set of housing units that rent at a fraction $\kappa_1 < 1$ of the free-market rent.\(^5\) Every household in the model enters the affordable housing lottery every period. A household that wins the lottery has the option to move into a RC unit in a zone assigned by the lottery, provided it qualifies.\(^6\) A winning household can choose to turn down the RC unit, and rent or own the unit of its choice in the location of its choice on the free market. If the household accepts the RC lottery win, it must abide by two conditions: (i) its income must be below a cutoff, expressed as a fraction $\kappa_2$ of area median income (AMI), when it first moves into the unit – not in subsequent periods – and (ii), the rent paid on the RC unit must be below a

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\(^5\)Since our model is stationary, this is equivalent to assuming that RC rents grow at the same rate as private market rents. We think this is a reasonable approximation to capture a mixture of units that have a nominal rent cap and units that are rent stabilized in that rents grow at a low and stable rate. We discuss how we map our model to the NYC affordable housing data in the calibration section below.

\(^6\)There is a single lottery for all RC units. A certain lottery number range gives access to RC housing in zone 1, while a second range gives access to housing in zone 2. Households with lottery numbers outside these ranges lose the housing lottery.
fraction $\kappa_3$ of AMI. The latter condition effectively restricts the maximum size of the RC unit. Both of these conditions are consistent with typical rent regulation and affordable housing specifications.

Importantly, we assume that rent control is persistent. Households that lived in a RC unit in a given zone in the previous period have an exogenously set, high probability of winning the lottery in that same zone.\(^7\) This parameter value determines the persistence of the RC system. For households that were not previously in RC, the probability of winning the lottery for each zone is endogenously determined to equate the residual demand (once accounting for persistent RC renters) and the supply of RC units in each zone. Households form beliefs about this probability. This belief must be consistent with rational expectations, and is updated as part of the equilibrium determination.

Rent control engenders four distortions. Since labor supply is endogenous, a household which wins the RC lottery may choose to reduce hours worked in order to qualify for a RC unit. This has adverse implications for the level of output. Second, a household may choose to consume a larger amount of housing, conditional on being awarded a RC unit, than it would on the free market. This may lead to misallocation of housing in the cross-section of households. Third, a household who would otherwise live in zone 2 but wins the housing lottery in zone 1 may choose to live in zone 1 and vice versa. Those three distortions are worsened by the persistence of RC. Fourth, RC blunts developers’ incentives to construct housing, as explained below.

**Location and Tenure Choice** Denote by $p^{RC,\ell}$ the probability of winning the RC lottery and being offered a unit in zone $\ell$. The household chooses whether to accept the RC option with value $V_{RC,\ell}$, or to turn it down and go to the private housing market with value $V_{free}$. The value function $V$ is:

$$V = p^{RC,1} \max \{ V_{RC,1}, V_{free} \} + p^{RC,2} \max \{ V_{RC,2}, V_{free} \} + \left( 1 - p^{RC,1} - p^{RC,2} \right) V_{free}.$$  

A household that loses the lottery or wins it but turns it down, freely chooses in which location $\ell \in \{1, 2\}$ to live and whether to be an owner ($O$) or a renter ($R$).

$$V_{free} = \max \{ V_{O,1}, V_{R,1}, V_{O,2}, V_{R,2} \}.$$  

The Bellman equations for $V_{RC,\ell}$, $V_{R,\ell}$ and $V_{O,\ell}$ are defined below.

Let $S_t$ be the aggregate state of the world, which includes the wage $W_t$, as well as the housing price $P_{t,\ell}$, the market rent $R_{t,\ell}$ and previous housing stock $H_{t-1,\ell}$ for each location $\ell$. The household’s individual state variables are: its net worth at the start of the period $x_t$,

\(^7\)For these households, the probability of winning the RC lottery in the other zone is set to zero.
its idiosyncratic productivity level $z_t$, its age $a$, and its rent control status in the previous period $d$ (equal to 0 if the household was not in RC, to 1 if it was in RC in zone 1, and to 2 if it was in RC in zone 2). The latter affects the probabilities of winning the lottery, and whether the income constraint applies in the current period conditional on choosing RC. We suppress the dependence on $\beta$-type in the problem formulation below, but note here that there is one set of Bellman equations for each $\beta$-type.

**Market Renter Problem** A renter household on the free rental market in location $\ell$ chooses non-durable consumption $c_t$, housing consumption $h_t$, and working hours $n_t$ to solve:

$$V_{R,\ell}(x_t, z_t, a, d_t, S_t) = \max_{c_t, h_t, n_t, b_{t+1}} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta\mathbb{E}_t[V(x_{t+1}, z_{t+1}, a + 1, 0, S_{t+1})]$$

s.t.

$$c_t + R_{t} h_t + Qb_{t+1} + \phi_{F}^\ell = (1 - \tau^{SS}) y_{t}^{lab} + \Psi_{t} + x_t - T(y_{t}^{lot}),$$

$$y_{t}^{lab} = W_{t} n_{t} G^{a} z_{t},$$

$$y_{t}^{lot} = y_{t}^{lab} + \left(1 - \frac{1}{Q}\right) x_t,$$

$$x_{t+1} = b_{t+1} + \hat{b}_{t+1} \geq 0,$$

and equations (1), (2), (3), (4).

The renter’s savings in the risk-free bond, $Qb_{t+1}$, are obtained from the budget constraint. Pre-tax labor income $y_{t}^{lab}$ is the product of wages $W$ per efficiency unit of labor, the number of hours $n_t$ and the productivity per hour $G^{a} z_{t}$. Total pre-tax income, $y_{t}^{lot}$, is comprised of labor income and financial income. The latter is the interest income on bonds. Next period’s financial wealth $x_{t+1}$ consists of savings $b_{t+1}$ plus any accidental bequests $\hat{b}_{t+1}$. Housing and labor choices are subject to minimum constraints discussed above. In addition to a time cost, residents of zone 2 face a financial cost of commuting $\phi_{F}^2$. Like we did for the time cost, we normalize the financial cost of commuting in zone 1 to zero: $\phi_{F}^1 = 0.$
RC Renter Problem  A renter household in the RC system in location $\ell$ chooses non-durable consumption $c_t$, housing consumption $h_t$, and working hours $n_t$ to solve:

$$V_{RC,\ell}(x_t, z_t, a, d_t, S_t) = \max_{c_t, h_t, n_t, b_t, \ell} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta E_t[V(x_{t+1}, z_{t+1}, a + 1, \ell, S_{t+1})]$$

s.t.

$$c_t + \kappa_1 R^\ell_t h_t + \hat{Q} b_{t+1} + \phi F^\ell_t = (1 - \tau^{SS}) y^l_{t+1} + \Psi_t h_t + x_t - T (y^l_{t})$$

$$x_{t+1} = b_{t+1} + b_{t+1} \geq 0,$$

$$y^l_{t} \leq \kappa_2 \hat{Y}_t \text{ if } d_t = 0,$$

$$h_t \leq \frac{\kappa_3 \hat{Y}_t}{\kappa_1 R^\ell_t},$$

and equations (1), (2), (3), (4).

The per square foot rent of a RC unit is a fraction $\kappa_1$ of the market rent $R^\ell_t$. For households who were not previously in the RC system ($d_t = 0$) to qualify for RC, labor income must not exceed a fraction $\kappa_2$ of area median income (AMI), $\hat{Y}_t = \text{Median}[y^l_{t}],$ the median across all residents in the metro area. The last inequality says that expenditures on rent ($\kappa_1 R^\ell_t h_t$) must not exceed a fraction $\kappa_3$ of AMI.\(^8\) We impose the same minimum house size constraint in the RC system. We note that a household in RC in the current period carries over her priority status for RC into the next period; the value function next period has RC flag $d_{t+1} = \ell$.

Owner’s Problem  An owner in location $\ell$ chooses non-durable consumption $c_t$, housing consumption $h_t$, working hours $n_t$, and investment property $\hat{h}_t$ to solve:

$$V_{O,\ell}(x_t, z_t, a, d_t, S_t) = \max_{c_t, h_t, n_t, \hat{h}_t, \ell} U(c_t, h_t, n_t, \ell_t) + (1 - p^a)\beta E_t[V(x_{t+1}, z_{t+1}, a + 1, 0, S_{t+1})]$$

s.t.

$$c_t + P^\ell_t h_t + Q b_{t+1} + \kappa_4 P^\ell_t \hat{h}_t + \phi F^\ell_t = (1 - \tau^{SS}) y^l_{t+1} + \Psi_t \psi^z + x_t + \kappa_4 R^\ell_t h_t - T (y^l_{t})$$

$$x_{t+1} = b_{t+1} + \tilde{b}_{t+1} + P^\ell_t h_t (1 - \delta^{\ell} - \tau^{P,\ell}) + \kappa_4 P^\ell_t \hat{h}_t (1 - \delta^{\ell} - \tau^{P,\ell}),$$

$$-Q_t b_{t+1} \leq P^\ell_t \left( \theta_{res} \hat{h}_t + \theta_{inv} \kappa_4 \hat{h}_t \right) - \kappa_4 R^\ell_t \hat{h}_t - (y^l_{t} - c_t),$$

$$\hat{h}_t \geq 0,$$

$$\kappa_4 = 1 - \eta^{\ell} + \eta^{\ell} \kappa_1,$$

and equations (1), (2), (3), (4).

Local home owners are the landlords to the local renters. This is a departure from the typical assumption of absentee landlords. Our landlords are risk-averse households.

---

\(^8\)In the implementation, we assume that the income and size qualification cutoffs for RC are constants. We then compute what fractions $\kappa_2$ and $\kappa_3$ of AMI they represent. This allows us to sidestep the issue that the AMI may change with RC policies.
inside the model. For simplicity, we assume that renters cannot buy investment property and that owners can only buy investment property in the zone of their primary residence. Landlords earn rental income on their investment units.

Landlords in the model are required to buy $\eta^\ell$ square feet of RC property for every $1 - \eta^\ell$ square feet of free-market rental property. This captures the institutional reality of affordable housing programs in NYC and elsewhere. The effective rent earned per square foot of investment property is $\kappa^\ell R^\ell_t$. Since the blended rent is a multiple $\kappa^\ell P^\ell_t$. Because prices and rents scale by the same constant, the return on investing in owner-occupied and rental property is the same. As a result, landlords are unaffected by rent regulation. However, the lower average price for rental property ($\kappa^\ell < 1$) has important effects on housing supply/development, as discussed below.

The physical rate of depreciation for housing units is $\delta^\ell$. The term $Ph^\ell\delta^\ell$ is a financial costs, i.e., a maintenance cost. As shown in equation (10) below, the physical depreciation can be offset by residential investment undertaken by the construction sector.

Property taxes on the housing owned in period $t$ are paid in year $t + 1$; the tax rate is $\tau_{P,\ell}$. Property tax revenue finances local government spending which does not confer utility to the households. Housing serves as a collateral asset for debt. For simplicity, mortgages are negative short-term safe assets. Households can borrow a fraction $\theta_{res}$ of the market value of their primary residence and a potentially different fraction $\theta_{inv}$ against investment property. The empirically relevant case is $\theta_{res} \geq \theta_{inv}$. We exclude current-period rental income and savings from the pledgable collateral. In light of the fact that one period is four years in the calibration, we do not want to include four years worth of (future) rental income and savings for fear of making the borrowing constraint too loose.

In the appendix we show that for renters, the choices of $h_t$ and $n_t$ are analytic functions of $c_t$, therefore the renter’s problem can be rewritten with just two choices: consumption $c_t$ and location $\ell$. For owners, the choices of $h_t$ and $n_t$ are analytic functions of $c_t$ and $\hat{h}_t$, therefore the owner’s problem can be rewritten with just three choices: consumption $c_t$.

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Footnotes:

9. Examples of incentives provided for the development of affordable housing in NYC are (i) the 80/20 new construction housing program, a state program that gives low-cost financing to developers who set aside at least 20% of the units in a property for lower-income families; (ii) the 421a program, which gives tax breaks (up to 25 years) for the development of under-utilized or vacant sites often conditional on providing at least 20% affordable units (i.e., used in conjunction with the 80-20 program), (iii) the Federal Low Income Housing Tax Credits (LITC) program, which gives tax credits to developers directly linked to the number of low-income households served, and (iv) Mandatory Inclusionary Housing, a New York City program that lets developers build bigger buildings and gives them tax breaks if they reserve some of the units for (permanently) affordable housing.

10. Alternatively, one could solve a model where the public good enters the utility function. That would require taking a stance on the elasticity of substitution between private and public consumption.
investment property size \( \hat{h}_t \), and location \( \ell \).

### 2.2 Firms

**Goods Producers** There are a large number \( n_f \) of identical, competitive firms located in the urban core (zone 1), all of which produce the numéraire consumption good.\(^{11}\) This good is traded nationally; its price is unaffected by events in the city and normalized to 1. The firms have decreasing returns to scale and choose efficiency units of labor to maximize profit each period:

\[
\Pi_{c,t} = \max_{N_{c,t}} N_{c,t}^\theta c - W_t N_{c,t} \tag{5}
\]

We assume that these firms are owned by equity owners outside the city. In robustness analysis, we consider a model where half of the profits accrue to locals.

**Developers and Affordable Housing Mandate** In each location \( \ell \) there is a large number \( n_f \) of identical, competitive construction firms (developers) which produce new housing units and sell them locally. All developers are headquartered in the urban core, regardless of where their construction activity takes place. Like goods firms, construction firms are owned by equity owners outside the city. Again, we relax this assumption in robustness.

The cost of the affordable housing mandate is born by developers. Affordable housing regulation stipulates that for every \( 1 - \eta^\ell \) square foot of market rental units built in zone \( \ell \), \( \eta^\ell \) square feet of RC units must be built. Developers receive an average price per foot for rental property of \( \kappa^\ell P_t^\ell \), while they receive a price per foot of \( P_t^\ell \) for owner-occupied units. Given a home ownership rate in zone \( \ell \) of \( ho^\ell \), developers receive an average price per foot \( P_t^\ell \):

\[
P_t^\ell = \left( ho^\ell + (1 - ho^\ell) \kappa^\ell \right) P_t^\ell. \tag{6}
\]

The cost of construction of owner-occupied and rental property in a given location is the same. After completion of construction but prior to sale, some of the newly constructed housing units are designated as rental units and the remainder as ownership units. The renter-occupancy designation triggers affordable housing regulation. It results in a lower rent and price than for owner-occupied units. Developers would like to sell ownership

\(^{11}\) We assume that the number of firms is proportional to the number of households in the city when solving the model. With this assumption, our numerical solution is invariant to the number of households. Due to decreasing return to scale, the numerical solution would depend on the number of households otherwise.
units rather than rental units, but the home ownership rate is determined in equilibrium. Developers are price takers in the market for space, and face an average sale price of $\overline{P}_t^\ell$.

A special case of the model is the case without rent control: $\kappa_4^\ell = 1$ either because $\eta^\ell = 0$ or $\kappa_1 = 1$. In that case, $\overline{P}_t^\ell = P_t^\ell$. Without rent control, the higher sale price for housing increases incentives to develop more housing.

**Zoning** Given the existing housing stock in location $\ell$, $H_t^{\ell-1}$, and average sale price of $\overline{P}_t^\ell$, construction firms have decreasing returns to scale and choose labor to maximize profit each period:

$$\Pi_{h,t}^\ell = \max_{N_{c,t}} P_t^\ell \left( 1 - \frac{H_t^{\ell-1}}{H^\ell} \right) N_{c,t}^{\rho_h} - W_t N_{c,t}$$

(7)

The production function of housing has two nonlinearities. First, as for consumption good firms, there are decreasing returns to scale because $\rho_h < 1$.

Second, construction is limited by zoning laws. The maximal amount of square footage zoned for residential use in zone $\ell$ is given by $\overline{H}^\ell$. We interpret $\overline{H}^\ell$ as the total land area zoned for residential use multiplied by the maximum permitted number of floors that could be built on this land, the floor area ratio (FAR). This term captures the idea that, the more housing is already built in a zone, the more expensive it is to build additional housing. For example, additional construction may have to take the form of taller structures, buildings on less suitable terrain, or irregular infill lots. Therefore, producing twice as much housing requires more than twice as much labor. Laxer zoning policy, modeled as a larger $\overline{H}^\ell$, makes development cheaper, and all else equal, will expand the supply of housing.

When $\overline{H}^\ell$ is sufficiently high, the model’s solution becomes independent of $\overline{H}^\ell$, and the supply of housing is governed solely by $\rho_h$. When $\overline{H}^\ell$ is sufficiently low, the housing supply elasticity depends on both $\overline{H}^\ell$ and $\rho_h$.\(^{12}\)

### 2.3 Equilibrium

Given parameters, a competitive equilibrium is a price vector $(W_t, P_t^\ell, R_t^\ell)$ and an allocation, namely aggregate residential demand by market renters $H_t^{R,\ell}$, RC renters $H_t^{RC,\ell}$, and owners $H_t^{O,\ell}$, aggregate investment demand by owners $\hat{H}_t^\ell$, aggregate housing supply, aggregate labor demand by goods and housing producing firms $(N_{c,t}, N_{c,t})$, and aggregate labor supply $N_t$ such that households and firms optimize and markets clear.

\(^{12}\)In this sense, the model captures that construction firms must pay more for land when land is scarce or difficult to build on due to regulatory constraints. This scarcity is reflected in equilibrium house prices.
The following conditions characterize the equilibrium. First, given wages and average prices given by (6), firms optimize their labor demand, resulting in the first-order conditions:

\[ N_{c,t} = \left( \frac{\rho_c}{W_t} \right)^{\frac{1}{1-\rho_c}} \quad \text{and} \quad N_{\ell,t} = \left( \frac{1 - \frac{H^\ell_{t-1}}{H^\ell_t}}{\rho_h} \frac{\bar{P}^\ell_t \rho_h}{W_t} \right)^{\frac{1}{1-\rho_h}}. \]  

(8)

Second, labor demand equals labor supply:

\[ n_f \left( N_{c,t} + \sum_\ell N_{\ell,t} \right) = N_t. \]  

(9)

Third, the housing market clears in each location \( \ell \):

\[ (1 - \delta^\ell) H^\ell_{t-1} + n_f \left( 1 - \frac{H^\ell_{t-1}}{H^\ell_t} \right) N_{\ell,t}^{\rho_h} = H_t^{O,\ell} + \hat{H}^\ell_t. \]  

(10)

The left-hand-side is the supply of housing which consists of the non-depreciated housing stock and new residential construction. The right-hand-side is the demand for those housing units by owner-occupiers and landlords. Fourth, the supply of rental units in each location \( \ell \) must equal the demand from market tenants and RC tenants:

\[ \hat{H}^\ell_t = H_t^{R,\ell} + H_t^{RC,\ell}. \]  

(11)

Fifth, total pension payments equal to total Social Security taxes collected:

\[ \bar{\Psi}_t N_{ret} = \tau^{SS} N_t W_t, \]  

(12)

where \( N_{ret} \) is the total number of retirees, which is a constant, and \( N_t \) are total efficiency units of labor. Sixth, the aggregate state \( S_t \) evolves according to rational expectations. Seventh, the value of all bequests received is equal to the wealth of all agents who die.

2.4 Welfare Effects of Affordability Policies

We compute the welfare effect of an affordability policy using the following procedure. Denote agent \( i \)'s value function under benchmark policy \( \theta_b \) as \( V_i(x, z, a, S; \theta_b) \). Consider an alternative policy \( \theta_a \) with value function \( V_i(x, z, a, S; \theta_a) \). The alternative policy implies a change in value functions, which we express in consumption equivalent units. We
compare steady state welfare for agents belonging to a group \( g \) with cardinality \( G \):

\[
\mathcal{W}_g = \left( \frac{1}{G} \sum_{i \in g} V_i(\theta_a) \right)^{\frac{1}{1-\gamma(1-\alpha_n)}} - 1. \tag{13}
\]

We focus on the following groups: the entire population, age groups, productivity groups, income quartiles, and wealth quartiles. The groups always have the same cardinality under the benchmark and alternative policies. One caveat is that income and wealth are endogenously determined and non policy-invariant. For this reason, we do not aggregate by tenure status. Tenure status groups do not have fixed cardinality and are strongly policy-dependent. For example, a policy may increase the number of households who receive a RC unit, but RC units may be smaller in the alternative economy. Comparing the welfare of RC households in the benchmark to that of RC households in the alternative economy misses the compositional change, and it may show a welfare reduction because of the smaller unit size. The welfare gain may concentrated on households who transition from, say, being a market renter to a RC renter, which is missed by a comparison of tenure status groups.

3 Calibration

We calibrate the model to the New York metropolitan area. Data sources and details are described in Appendix B. Table 1 summarizes the chosen model parameters.

**Geography**  The New York metro consists of 25 counties located in New York (12), New Jersey (12), and Pennsylvania (1). We assume that Manhattan (New York County) represents zone 1 and the other 24 counties make up zone 2.\(^\text{13}\) The zones differ in the maximum buildable residential square footage, captured by \( \bar{H} \). Appendix B describes detailed calculations on the relative size of Manhattan and the rest of the metro area, which imply that \( \bar{H}^1 = 0.0238 \times \bar{H}^2 \). We then choose \( \bar{H}^2 \) such that the fraction of households living in zone 1 equals 10.5% of the total, the fraction observed in in the data. Since the model has no vacancies, we equate the number of households in the model with the number of occupied housing units in the data.

\(^{13}\)Alternative choices are to designate (i) New York City (five counties coinciding with the five boroughs of NYC) as zone 1 and the rest of the metro as zone 2, or (ii) Manhattan as zone 1 and the other four counties in New York City as zone 2. Both choices ignore that the dominant commuting pattern is from the rest of the metro area to Manhattan.
Demographics  The model is calibrated so that one model period is equivalent to 4 years. Households enter the model at age 21, work until age 64, and retire with a pension at age 65. Survival probabilities $p(a)$ are calibrated to mortality data from the Census Bureau. People age 65 and over comprise 19.1% of the population age 21 and over in the data. In the model, we get 21.8%. The average New York metro resident is 47.6 years old in the data and 47.4 years old in the model.

Labor Income  Recall that pre-tax labor income for household $i$ of age $a$ is $y_{i}^{ab} = W_{i}n_{i}^{G}a^{z_{i}^{a}}$, where the household takes wages as given and chooses labor supply $n_{i}^{G}$. The choice of hours is subject to a minimum hours constraint, which is set to 0.5 times average hours worked. This constraint rules out a choice of a positive but very small number of hours, which we do not see in the data given the indivisibility of jobs. It also rules out unemployment since our earnings data are for the (part-time and full-time) employed. This constraint binds for only 0.01% of workers in equilibrium.

Efficiency units of labor $G^{a}z_{i}^{a}$ consist of a deterministic component that depends on age ($G^{a}$) and a stochastic component $z_{i}^{a}$ that captures idiosyncratic income risk. The $G^{a}$ function is chosen to enable the model to match the mean of labor earnings by age. We use data from ten waves of the Survey of Consumer Finance (1983-2010) to estimate $G^{a}$.

The idiosyncratic productivity process $z$ is chosen to both match earnings inequality in the NY metro data and to generate realistic persistence in earnings. We discretize $z$ as a 4-state Markov chain. The four states vary by age to capture the rising variance of earnings with age. We use the same SCF data to discipline this increase in variance by age. The four grid points at the average age are chosen to match the NY metro pre-tax earnings distribution from the Census Bureau. We choose annual household-level earnings cutoffs in the data of $41,000, 82,000, and 164,000$. This results in four earnings groups with average earnings of $28,125, 60,951, 116,738, and 309,016$. Average New York MSA household earnings are $124,091$. The four point grid for productivity $z$ is chosen to match the average earnings in each group. This is an iterative process since labor supply is endogenous and depends on all other parameters and features of the model.

The $4 \times 4$ transition probability matrix for $z_{i}^{a}$ is age-invariant, but is allowed to depend on $\beta$ type. Specifically, the expected duration of the highest productivity state is higher for the more patient agents. There are five unique parameters governing transition probabilities which are pinned down by five moments in the data. The four income groups have population shares in the data of 16.1%, 29.8%, 34.2%, and 19.9%, respectively. Since the shares sum to 1, that delivers three restrictions on the transition matrix. Matching the persistence of labor income to 0.9 delivers a fourth restriction. Finally, the dependence on
\[ \beta \] is calibrated to deliver the observed correlation between income and wealth in the SCF data. Appendix C contains the parameter values and further details.

**Taxation**  Since our model is an incomplete markets model in which housing affordability policies can act as an insurance device and help to “complete the market,” it is important to realistically calibrate the redistribution provided through the tax code. We follow Heathcote, Storesletten, and Violante (2017) and choose an income tax schedule that captures the observed progressivity of the U.S. tax code in a parsimonious way. Net taxes are given by the function \( T(\cdot) \):

\[
T(y_{tot}) = y_{tot} - \lambda (y_{tot})^{1-\tau}
\]

The parameter \( \tau \) governs the progressivity of the tax and transfer system. We set \( \tau = 0.17 \) to match the average income-weighted marginal tax rate of 34% for the U.S. It is close to the value of 0.18 estimated by Heathcote et al. (2017). We set \( \lambda \) to match state and local government spending to aggregate income in the NY metro area, equal to 15-20%.\(^{14}\) This delivers \( \lambda = 0.75 \). Appendix D shows the resulting tax and after-tax income along the before-tax income distribution.

**Retirement Income**  Social Security taxes and receipts are treated separately from the tax and transfer system. Social security taxes are proportional to labor earnings and set to \( \tau_{SS} = 0.10 \). Retirement income is increasing in the household’s last productivity level prior to retirement, but is capped for higher income levels. We use actual Social Security rules to estimate each productivity group’s pension relative to the average pension. The resulting pension income states are \( \psi_z = [0.50, 1.03, 1.13, 1.13] \), where \( z \) reflects the last productivity level prior to retirement. They are multiplied by average retirement income \( \Psi \), which is endogenously determined in equation (12) to balance the social security budget. Average retirement income \( \Psi \) is $44357, which corresponds to 36% of average earnings.

**Commuting Cost**  We choose the time cost to match the time spent commuting for the average New York metro area resident. This time cost is the average of all commutes, including those within Manhattan. Since the model normalizes the commuting time within zone 1 to zero, we target the additional commuting time of zone 2 residents. The addi-

\(^{14}\)Depending on what share of NY State spending goes to the NY metro area, we get a different number in this range.
Table 1: Calibration

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<tr>
<td>Rental share threshold for RC</td>
<td>( k_{3} )</td>
<td>0.35</td>
<td>Fraction of income Q3/Q4 hhs in RC</td>
</tr>
</tbody>
</table>
tional commuting time amounts to 25 minutes per trip for 10 commuting trips per week.\textsuperscript{15} The 4.2 hours represent 3.7\% of the 112 hours of weekly non-sleeping time. Hence, we set $\phi_T^2 = 0.037$.

The financial cost of commuting $\phi^2_T$ is set to 1.8\% of average labor earnings, or $2274 per household per year. This is a reasonable estimate for the commuting cost in excess of the commuting cost within Manhattan, which is normalized to zero.\textsuperscript{16}

We assume that retirees have time and financial commuting costs that are 1/3 of those of workers. This captures that retirees make fewer trips, travel at off-peak hours, and receive transportation discounts.

Preferences The functional form for the utility function is given in equation (1). We set risk aversion $\gamma = 5$, a standard value in the asset pricing literature. Since risk aversion governs the value of insurance against risk, it is a key parameter and we do robustness to its value.

The observed average workweek for New York metro residents is 42.8 hours or 38.2\% of available non-sleeping time. Since there are 1.66 workers on average per household, household time spent working is $38.2\% \times 1.66/2 = 31.7\%$. We set $\alpha_n$ to match household time spent working. The model generates 31.1\% of time worked.

We set $\alpha_h$ in order to match the ratio of average market rent to metro-wide average income. Income data discussed above and rental data from Zillow, detailed in Appendix B, indicate that this ratio is 23\% for the New York MSA in 2015. The model generates 23.5\%.

We set $\beta^H = 1.28$ (1.06 per year) and $\beta^L = 0.98$ (1.00 per year). A 25\% share of agents has $\beta^H$, the rest has $\beta^L$. This delivers an average $\beta$ of 1.06, chosen to match the average wealth-income ratio which is 5.69 in the 1998-2010 SCF data. The model generates 6.26. The dispersion in betas delivers a wealth Gini coefficient of 0.79, matching the observed wealth Gini coefficient of 0.80 for the U.S.\textsuperscript{17} Note that because of mortality, the effective time discount rate is $(1 - p(a))\beta$.

\textsuperscript{15}The 25 minute additional commute results from a 15 minute commute within Manhattan and a 40 minute commute from zone 2 to zone 1. With 10.5\% of the population living in Manhattan, the average commuting time is 37.4 minutes per trip or 6.2 hours a week. This is exactly the observed average for the New York metro from Census data.

\textsuperscript{16}In NYC, an unlimited subway pass costs around $1,400 per year per person. Rail passes from the suburbs cost around $2400 per year per person, depending on the railway station of departure. If zone 1 residents need a subway pass while zone 2 residents need a rail pass, the cost difference is about $1000 per person. With 1.66 workers per household, the cost difference is $1660 per household. The cost of commuting by car is at least as high as the cost of rail once the costs of owning, insuring, parking, and fueling the car and tolls for roads, bridges, and tunnels are factored in.

\textsuperscript{17}No wealth data is available for the NY metro. We believe it is likely that wealth inequality is at least as high in the NY metro than in the rest of the U.S.
The taste-shifter for zone 1 is parameterized as: $\chi^1 = 1.080$, $\chi^W = 1.004$, $\chi^R = 1.038$, $\zeta = 0.45$. Living in Manhattan gives a substantial utility boost, equivalent to a 8% higher consumption bundle. About 26% of the Manhattan population consumes above the threshold $\zeta$. This group derives extra utility from living in Manhattan, especially the retirees in this group ($\chi^1 \chi^R = 1.121$). We chose these four parameters to get our model to better match the following four ratios of zone 1 relative to zone 2 variables, given all other parameters: the relative fraction of retirees of 0.91, a relative household income ratio of 1.66, the relative ratio of market rents per square foot of 2.78, and the relative home ownership rate of 0.42. In the model, these ratios are 0.98, 1.69, 2.78, and 0.79, respectively.

**Housing**  
The price for the one-period (4-year) bond is set to $Q = 0.89$ to match the average house price to rent ratio for the New York metro, which is 17.79. The model delivers 17.36. Under the logic of the user cost model, the price-to-rent ratio depends on the interest rate, the depreciation rate, and the property tax rate.

The property tax rate in Manhattan is $\tau^{P,1} = 0.029$ or 0.73% per year, and that in zone 2 is $\tau^{P,2} = 0.053$ or 1.33% per year. These match the observed tax rates averaged over 2007-2011 according to the Brookings Institution.$^{18}$

The housing depreciation rate in Manhattan is $\delta^1 = 0.058$ or 1.45% per year, and that in zone 2 is $\delta^2 = 0.096$ or 2.41% per year. This delivers a metro-wide average depreciation rate of 2.39% per year, equal to the average depreciation rate for privately-held residential property in the BEA Fixed Asset tables for the period 1972-2016. The annual depreciation wedge of 0.96% between zones 1 and 2 is chosen to match the relative fraction of buildings that were built before 1939.$^{19}$

We set the maximum loan-to-value ratio (LTV) for the primary residence at 80% ($\theta_{res} = 0.8$), implying a 20% down payment requirement. This is the median downpayment in the U.S. data on purchase mortgages. The LTV for investment property is also set at 80% ($\theta_{inv} = 0.8$).

Finally, we impose a minimum housing size of 554 square feet, a realistic value for New York. This represents 38% of the average housing unit size of 1445 square feet.

**Production and Construction**  
We assume that the return to scale $\rho_c = 0.66$. This value implies a labor share of 66% of output, consistent with the data.$^{18}$

---

$^{18}$The zone-2 property tax rate is computed as the weighted average across the 24 counties, weighted by the number of housing units.

$^{19}$Data from the 5-year American Community Survey from 2017 give the distribution of housing units by year built for each of the 25 counties in the New York MSA. In Manhattan, 42.8% of units are built before 1939. The housing-weighted average among the 24 counties of zone 2 is 26.6%. Assuming geometric depreciation, matching this fraction requires a 0.96% per year depreciation wedge.
For the housing sector, we set $\rho_h = 0.66$ in order to match the housing supply elasticity, given the other parameters. The long-run housing supply elasticity in the model is derived in Appendix E. Saiz (2010) reports a housing supply elasticity for the New York metro area of 0.76. The model delivers 0.70. The housing supply elasticity is much lower in zone 1 (0.08) than in zone 2 (0.73), because in zone 1 the housing stock is much closer to $\overline{H}$ (11% from the constraint) than in zone 2 (71% from the constraint). Since the housing stock of the metro area is concentrated in zone 2, the city-wide elasticity is dominated by that in zone 2.

Rent Control Rent regulation plays a major role in the New York housing market. We define RC housing as all housing units that are (i) rent controlled, (ii) public housing, (iii) Mitchell Lama housing, (iv) all other government-assisted or regulated housing. Appendix B.5 contains a detailed description of our rent regulation data and definitions.\(^ {20}\) Using data from the New York City Housing and Vacancy Survey and county-level data on affordable housing for the New York metro area counties outside of New York City, we find that 13.0% of zone 1 households and 4.7% of zone 2 households live in RC units. The metro-wide average is 5.57%, the ratio of zone 1 to zone 2 is 2.77. We set the share of square feet of housing devoted to RC units, $\eta^1 = 23.85\%$ and $\eta^2 = 15.95\%$, to match the fraction of households that are in RC units in each zone. This fraction is endogenous since housing size is a choice variable.

According to the same definition and data sources, the average rent in RC units is 50% below that in all other rentals. We set $\kappa_1 = 50\%$ to the observed rent discount. It follows that $\kappa_4^1 = 0.88$ and $\kappa_4^2 = 0.92$, so that landlords earn 12% lower rents in zone 1 and 8% lower rents in zone 2 than they would in an unregulated market.

We set the income qualification threshold to a fraction $\kappa_2 = 60\%$ of AMI. This is a standard income threshold for affordable housing. We set $\kappa_3 = 35\%$ so that households in affordable housing spend no more than 35% of their income on rent, again a standard value. These two parameters affect the composition of households who live in RC housing. The lower $\kappa_2$, the larger the share of RC tenants that come from the bottom of the income distribution. Similarly, $\kappa_3$ affects who lives in RC. High-income agents in the model want to live in a house that is larger than the maximum allowed size under RC and would turn down a RC lottery win.

Finally, we need to calibrate the persistence of the RC system. We assume that households who were in RC in the previous period have a probability of 76.4% to qualify for RC in the same zone this period. The value is chosen to match the fraction of RC tenants

\(^ {20}\)Note that we chose to exclude rent stabilized units from this definition, of which there are many in NYC.
who have lived in a RC unit for 20 years or more. That number in the data is 23.1%. It is 20.9% in the model.

4 Baseline Model Results

We start by discussing the implications of the baseline model for the spatial distribution of population, housing, income, and wealth. We also discuss house prices and rents for the city as a whole and for the two zones. Then we look at the model’s implications for income, wealth, and home ownership over the life-cycle.

4.1 Demographics, Income, and Wealth

Demographics The first three rows of Table 2 show that the model matches basic demographic moments. One moment not targeted by the calibration is the relative age of zone 1 to zone 2 households. The average Manhattan resident is younger (41) than the average resident in the rest of the metro (48). The model generates the same age gap as the balance of two forces. On the one hand, younger households in the working phase of life value proximity to work. They also tend to have lower income, which increases the importance of not having to shoulder the financial cost of commuting. On the other hand, they may not be able to afford the high rents in Manhattan until later in their life-cycle when earnings are higher and they have been able to save more.

In the data, retirees represent 17.6% of Manhattan residents, and the model matches this share. On the one hand, retirees have lower time and financial costs of commuting, giving them a comparative advantage to living in zone 2. On the other hand, retirees tend to be wealthier making living in Manhattan financially feasible. Absent a taste for Manhattan, the commuting cost effect would allocate most retirees in zone 2. A fairly strong preference for living in Manhattan is needed to offset the commuting effect ($\chi^1 = 1.080, \chi_R = 1.038$) and match the relative share of retirees in zone 1 to zone 2.

Housing Units In the data, the typical housing unit is much smaller in Manhattan than in the rest of the metro area. We back out the typical house size (in square feet) in each county as the ratio of the median house value and the median house value per square foot, using 2015 year-end values from Zillow. We obtain an average housing unit size of 1,445 sqft in the MSA and 826 sqft in Manhattan. The ratio of zone 1 to zone 2 is 0.59. In

\[21\text{See Table H of the NYU Furman Institutes’ 2014 “Profile of Rent-Stabilized Units and Tenants in New York City.”}\]
Table 2: New York Metro Data Targets and Model Fit

<table>
<thead>
<tr>
<th></th>
<th>Data ratio zone 1/zone 2</th>
<th>Model ratio zone 1/zone 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Households (thousands)</td>
<td>7124.9 0.12</td>
<td>7124.9 0.12</td>
</tr>
<tr>
<td>2 Avg. hh age, cond over 20</td>
<td>47.6 0.95</td>
<td>47.4 0.87</td>
</tr>
<tr>
<td>3 People over 65 as % over 20</td>
<td>19.1 0.91</td>
<td>21.8 0.98</td>
</tr>
<tr>
<td>4 Avg. house size (sqft)</td>
<td>1445 0.59</td>
<td>1448 0.63</td>
</tr>
<tr>
<td>5 Avg. pre-tax lab income ($)</td>
<td>124091 1.66</td>
<td>124325 1.69</td>
</tr>
<tr>
<td>6 Home ownership rate (%)</td>
<td>51.5 0.42</td>
<td>57.4 0.79</td>
</tr>
<tr>
<td>7 Median mkt price per unit ($)</td>
<td>510051 3.11</td>
<td>506592 2.34</td>
</tr>
<tr>
<td>8 Median mkt price per sqft ($)</td>
<td>353 5.24</td>
<td>348 3.58</td>
</tr>
<tr>
<td>9 Median mkt rent per unit (monthly $)</td>
<td>2390 1.65</td>
<td>2432 1.82</td>
</tr>
<tr>
<td>10 Median mkt rent per sqft (monthly $)</td>
<td>1.65 2.78</td>
<td>1.67 2.78</td>
</tr>
<tr>
<td>11 Median mkt price/median mkt rent (annual)</td>
<td>17.79 1.89</td>
<td>17.36 1.29</td>
</tr>
<tr>
<td>12 Mkt price/avg. income (annual)</td>
<td>3.99 1.71</td>
<td>4.08 1.38</td>
</tr>
<tr>
<td>13 Avg. rent/avg. income (%)</td>
<td>23.0 1.00</td>
<td>23.5 1.07</td>
</tr>
<tr>
<td>14 Avg. rent/income ratio for renters (%)</td>
<td>42.3* 0.93*</td>
<td>28.8 0.9</td>
</tr>
<tr>
<td>15 Rent burdened (%)</td>
<td>23.5* 0.91*</td>
<td>36.3 0.18</td>
</tr>
<tr>
<td>16 Severely rent burdened (%)</td>
<td>31.7* 0.87*</td>
<td>9.2 2.21</td>
</tr>
<tr>
<td>17 % Rent regulated of all housing units</td>
<td>5.57 2.77</td>
<td>5.25 2.87</td>
</tr>
</tbody>
</table>

Notes: Columns 2-3 report the values for the data of the variables listed in the first column. Data sources and construction are described in detail in Appendix B. Column 3 reports the ratio of the zone 1 value to the zone 2 value in the data. Column 5 reports the same ratio in the model. Data indicated with * are only available for New York City, not for the 20 counties in the NY metro outside of NYC.

The model, households freely choose their housing size subject to a minimum house size constraint. The model generates a similar ratio of house size in zone 1 to zone 2 of 0.63.

Figure 1 shows the distribution of house sizes. The model (left panel) matches the data (right panel) quite well, even though these moments are not targeted by the calibration. The size distribution of owner-occupied housing is shifted to the right from the size distribution of renter-occupied housing units in both model and data.

**Mobility** The model implies realistic moving rates from zone 1 to zone 2 and vice versa, despite the absence of moving costs. Mobility rates are not targeted by the calibration. Figure 7 in Appendix F shows that mobility is highest for the youngest agents (21-25) and for middle-aged households (35-40). For these two groups, the annual mobility rate is 4% annually. The overall mobility rate across neighborhoods in the model is about 2% annually. These data are consistent with the facts for the NY MSA, where 2.1% of households move between counties within the MSA annually.\(^\text{22}\)

**Income** Average income in the metro area matches the data (row 5 of Table 2) by virtue of the calibration. The ratio of average income in zone 1 to zone 2 is 1.69 in the model and 1.66 in the data. The productivity distribution is substantially different in the two zones.

\(^22\)Data from the U.S. Census Bureau on annual average county-to-county migration rates for 2012-2016.
Figure 1: House size distribution in Model (L) and Data (R)


Zone 1 contains workers that are on average 74% more productive than in zone 2. Productive working-age households have a high opportunity cost of time and prefer to live close to work given the time cost of commuting. Mitigating the high opportunity cost of time is the high cost of living in Manhattan. Indeed, some high-productivity workers may still be early in the life-cycle when earnings are lower and accumulated wealth smaller. Also contributing to the income gap between zone 1 and zone 2 is the luxury amenity value of living in Manhattan for working-age households, $\chi^W$. Our calibration shows that its value is close to 1, implying that the commuting cost is a strong enough force to generate the observed income gap.

The top panel of Figure 2 shows household labor income over the life-cycle, measured as pre-tax earnings during the working phase and as social security income in retirement. We plot average income for the bottom 25% of the income distribution, for the middle of the income distribution (25-50%), and for the top 25% of the distribution, as well as the overall average income. Labor income has the familiar hump-shaped profile over the life-cycle inherited from the deterministic productivity process $G^a$.

The model generates a large amount of income inequality at every age. The model’s
earnings Gini of 0.45 matches the 0.47 value in the 2015 NY metro data.\textsuperscript{23} Earnings inequality in the model is lower within zone 1 (Gini of 0.37) than within zone 2 (Gini of 0.45).

**Wealth** The model makes predictions for average wealth, the distribution of wealth across households, as well as how that wealth is spatially distributed. Average wealth to average total income ($y^{tot}$) in the metro area is 6.26. Wealth inequality is high, with a wealth Gini coefficient of 0.79. Both are close to the data by virtue of the calibration.

The probability of receiving a bequest equals the number of households between ages 21 and 65 divided by the number of dead households. It is equal to 10\% over each 4-year period, and identical for $\beta^H$ and $\beta^L$ household types. Under our calibration, about 1.2\% of wealth is bequeathed each year. This number matches the data. Some agents with low productivity who receive a bequest decide not to work and choose to live in zone 2.

The model predicts a ratio of average wealth in zone 1 to zone 2 of 1.17. While this number is not observable in the data, it seems reasonable to expect a geographic wealth inequality pattern that, at least directionally, mirrors that in income inequality.

The middle panel of Figure 2 shows household wealth over the life cycle at the same income percentiles as in the top panel. It shows that the model endogenously generates substantial wealth accumulation for the average New York resident as well as a large amount of wealth inequality between income groups. This wealth inequality grows with age.

### 4.2 Home Ownership, House Prices, and Rents

Next, we explore the model’s predictions for home ownership, house prices, and rents. The model manages to drive a large wedge between house prices, rents, and home ownership rates between zones 1 and 2 for realistic commuting costs.

**Home Ownership** The model generates a home ownership rate of 57.4\%, fairly close to the 51.5\% in the New York metro. The bottom panel of Figure 2 plots the home ownership rate over the life-cycle. It displays a hump-shape over the life-cycle with interesting variation across income groups. High-income households become home owners at a younger age than low-income households, achieve a higher ownership rate, and remain home owners for longer during retirement. These patterns are broadly consistent with the data.

Row 6 of Table 2 shows that the observed home ownership rate in Manhattan, at 23.1\%, is far below that in the rest of the metro area, at 54.9\%. The ratio of these two numbers

\textsuperscript{23}The Gini in the data is calculated by fitting a log-normal distribution to the mean and median of earnings.
Figure 2: Income, Wealth, and Home Ownership over the Life-Cycle

is 0.42. The model generates a home ownership rate of 45.9% for Manhattan, which is substantially lower than the predicted 58.4% for zone 2. The model’s prediction for home ownership in zone 2, where 89.5% of the population lives, is close to the data, but the model fails to generate the very low home ownership rate in Manhattan. The model’s failure to generate a large enough wedge in home ownership rates between zones is con-
nected to its inability to generate a large enough wedge in the price-rent ratios across zones. Intuitively, since owning is only somewhat more expensive than renting in zone 1 relative to zone 2, the home ownership rate in zone 1 is only somewhat lower than in zone 2. We discuss the price-rent ratio wedge below.

**Market Prices and Rents** Turning to rents and house prices, row 7 of Table 2 shows the median price per housing unit, row 8 the median price per square foot (the ratio of rows 8 and 2), row 9 the median rent per unit, and row 10 the median rent per foot. In the data, we use the Zillow home value index (ZHVI) to measure the median price of owner-occupied units, the Zillow median home value per square foot, and the Zillow rental index (ZRI) for the median rent. These indices are available for each county in the New York metro, and we use the year-end 2015 values. Zillow excludes non-arms’ length transactions and rent-controlled rentals. To aggregate across the 24 counties in zone 2, we calculate the median price as the weighted average of the median prices in each county, where the weights are the shares of housing units. Similarly, for the median rent of zone 2, we average median rents of the 24 counties using housing unit shares as weights. Zillow uses a machine-learning algorithm that ensures that the ZHVI and ZRI pertain to the same, typical, constant-quality unit, in a particular geography. The ratio of the ZHVI to the ZRI in a county, is the price-rent ratio, reported in row 11 of the table.

To ensure consistency with the empirical procedure, we calculate the median house size in each zone including both owner- and renter-occupied units (but excluding RC units) in the model. Call these $\bar{h}_\ell$. We form the median price per unit as the product of the market price times the typical unit size $P_\ell \bar{h}_\ell$. The market rent is $R_\ell \bar{h}_\ell$. The price-rent ratio is simply $P_\ell / R_\ell$. To form metro-wide averages, we use the number housing units in each zone as weights, just like in the data. The median house value in the NY metro area is $510,051 in the data compared to $506,592 in the model. The median is $1.3 million in Manhattan and $417,000 outside Manhattan in the data, a ratio of 3.11. This 3.11 house value ratio is the product of a house size ratio of 0.59 and a price per sqft ratio of 5.24. The model generates a ratio of prices of 2.34, the product of a house size ratio of 0.63 and a price per sqft ratio of 3.58.

The Zillow data indicate a monthly rent on a typical market-rate unit of $2,390 per month in the metro area. The model predicts $2,432. The ratio of rents per square foot in zone 1 to zone 2 is 2.78 in the data and closely matched by the 2.78 in the model. The ratio of rents per unit in zone 1 to zone 2 is 1.65 in the data, with a somewhat higher value of 1.82 obtained by the model.

The proximity to jobs and to amenities are the reasons why the model generates higher housing demand in Manhattan. Because of the highly inelastic housing supply in Man-
hattan, this translates into higher house prices and higher rents.

The model also comes close to matching the metro-wide price/rent ratio level of 17.79 (row 11). A simple user cost model would imply a steady state 4-year price-rent ratio of \((1 - Q \times (1 - \delta - \tau))^{-1}\). Plugging in for Q and the zone-specific property tax and depreciation parameters, the user cost formula generates price-rent ratios of 20.8 for zone 1 and 16.2 for zone 2. The model has borrowing constraints so that housing has an additional collateral value component which increases its price. There also is a small housing risk premium, which lowers the price. These two additional effects are quantitatively small and offsetting, so that the price-rent ratios are essentially driven by the user cost formula. In the data, the price-rent ratio in Manhattan is 26.65, or 1.72 times the 15.52 value in zone 2. In the model that ratio is 1.29. In other words, the property tax and depreciation wedges generate too little spatial variation in price-to-rent ratios. Outside of the model, houses in Manhattan may be less risky than in zone 2 which would increase the price-rent ratio wedge. Another feature of the real world that is absent from the model is that owner-occupied housing in Manhattan may be of higher quality than in zone 2. The lower depreciation rate in zone 1 than in zone 2 may not fully capture such quality differences.

### 4.3 Housing Affordability and Rent Control

**Price-Income and Rent-Income** Row 12 of Table 2 reports the ratio of the median value of owner-occupied housing to average earnings in each zone. Average earnings refer to all working-age residents in a zone, both owners and renters. The median home price to the average income is an often-used metric of housing affordability. In the NY metro data, the median owner-occupied house costs 3.99 times average income. Price-income is 6.08 in Manhattan compared to 3.55 outside Manhattan, a ratio of 1.71. We use the same definition as in the data and compute average pre-tax labor earnings among the working-age population. The model generates a metro-wide price-income ratio of 4.08, very close to the data. It generates a ratio across zones of 1.38. While generating a substantial spatial wedge in this housing affordability metric, the model’s understate ment is a direct consequence of not generating enough spatial variation in median house values, as noted above.

Row 13 reports average rent paid by market renters divided by average income of all

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24 The model has no aggregate risk and risk aversion is only 5. To generate meaningful variation in housing risk premia would require, for example, changes in mortgage lending standards as in Favilukis et al. (2017). If homeowners in Manhattan are exposed less to changes in mortgage lending standards, housing in Manhattan would carry a lower risk premium. This would increase the price-rent ratio in Manhattan, and increase the wedge with the price-rent ratio in zone 2.
residents in a zone. This moment was the target for the housing preference parameter $a^h$. The model matches the 23% target.

To get at the household-level rent burden, we compute three additional moments reported in rows 14-16 of Table 2. A caveat is that the household-level data come from the NYHVS, which covers the five counties in New York City. We have no household-level data for the 20 counties in zone 2 counties outside of NYC. Also, average rent for NYC from the NYHVS is substantially lower than average rent for NYC from Zillow, possibly resulting in overstated rent burden numbers. The first statistic computes household-level rent to income ratios for renters with positive income, caps the ratios at 101%, and takes the average across households. For this calculation, income is earnings for working-age households and social security income for retirees. The observed average share of income spent on rent by renters is 40% in Manhattan and 43% in NYC ex-Manhattan. The second moment computes the fraction of renters whose rent is between 30% and 50% of income. These households are known as rent-burdened. The third measure computes the fraction of renters whose rent exceeds 50% of income and includes the renters with zero income. This group is known as severely rent-burdened. In the Manhattan data, 21.4% of renters are rent burdened and another 29% are severely rent burdened. In NYC ex-Manhattan, 23.5% of renters are rent-burdened and an additional 33% severely rent-burdened.

The model generates a rent burden for renters of 28.8% of income, much higher than the city-wide average among all residents of 23% but below the (possibly overstated) number in the data of 42%. The fraction of rent burdened households is 36.3% in the model with another 9.2% of households severely rent burdened. The model numbers include the rest of the metro outside of NYC, which makes them difficult to compare to the data. Still, the model generates a large “housing affordability crisis,” with nearly half of renters spending more than 30% of their income on rent, like what we see in the data.

Rent Control By virtue of the calibration, the model generates the right share of rent-controlled households in the population in each zone (row 17 of Table 2).

Figure 3 plots the model’s fraction of households that are in RC for the bottom 25%, middle 50% and top 25% of the income distribution against age in the left panel, and for the wealth distribution in the right panel. At younger ages (before age 30), some households in the middle of the income distribution (conditional on age) obtain get RC, and even 4% of top-income young households are in RC. At later ages, those middle- and high-income households find staying in RC housing increasingly unattractive as they would like to consume more housing. This is not possible because of the constraint on RC housing size. Or they would have to reduce labor supply which is too unappealing given their high productivity. Also middle- and high-income who were not yet in RC housing
no longer satisfy the income requirement as their income grows with age. The share of RC that goes to the bottom-25% of the income and wealth distribution rises. Still, because of persistence in the RC system, a large fraction of middle-income households remains in the system for a very long time.

Rent control acts as an insurance device in our incomplete markets model. If it is difficult for a low-income household to get into the RC system, then the value of that insurance is low. The model predicts that the probability of winning the affordable housing lottery and accepting the assignment for a low-income household (bottom 25%) that was not yet in the RC system is 3.6% in the metro area. This breaks down into 0.9% for zone 1 and 2.7% for zone 2 RC housing. Including the low-income households that already were in RC, the likelihood of getting RC housing is 9.7%. We investigate below how removing the right to stay in RC without income test affects the allocation of RC across the income distribution, i.e. its insurance value, and ultimately welfare.

Figure 3: Distribution of rent-controlled agents by age, and income and net worth quartiles.

5 Affordability Policies

Having developed a quantitatively plausible dynamic stochastic spatial equilibrium model of the New York housing market, we now turn to policy counter-factuals. We run ten policy experiments. Five main ones are discussed in this section; the remaining five are discussed in Appendix H and briefly summarized at the end of this section. Table 3 summarizes how key moments of the model change under the five main policy experiments.
The first column reports the benchmark model, while the other columns report the percentage change in moments relative to the benchmark for each of the policy experiments. Figure 4 plots the associated welfare changes. Welfare changes are changes in value functions, expressed in consumption equivalent units; see equation (13). The five policies have in common that they all increase the generosity of affordable housing system and none of them leads to higher government outlays.

Table 3: Main moments of the model under affordability policies that modify features of the RC system and the spatial allocation of housing.

<table>
<thead>
<tr>
<th>Moment Description</th>
<th>Benchmark</th>
<th>RC share</th>
<th>All RC in Z2</th>
<th>Zoning Z1</th>
<th>Vouchers</th>
<th>LIHTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg (Rent/income) among renters, Z1 (%)</td>
<td>26.6</td>
<td>6.16%</td>
<td>-12.02%</td>
<td>0.80%</td>
<td>0.86%</td>
<td>0.43%</td>
</tr>
<tr>
<td>Avg (Rent/income) among renters, Z2 (%)</td>
<td>29.1</td>
<td>-0.82%</td>
<td>1.04%</td>
<td>0.25%</td>
<td>1.03%</td>
<td>-0.45%</td>
</tr>
<tr>
<td>Fraction of households in RC (%)</td>
<td>5.25</td>
<td>47.24%</td>
<td>-19.46%</td>
<td>0.51%</td>
<td>-8.32%</td>
<td>1.75%</td>
</tr>
<tr>
<td>Fraction of those in income Q1 (%)</td>
<td>9.70</td>
<td>45.40%</td>
<td>-18.28%</td>
<td>0.08%</td>
<td>-24.53%</td>
<td>1.92%</td>
</tr>
<tr>
<td>Frac. severely rent-burdened (%)</td>
<td>3.9</td>
<td>30.31%</td>
<td>-22.29%</td>
<td>1.34%</td>
<td>4.05%</td>
<td>0.24%</td>
</tr>
<tr>
<td>Avg (Rent/income) among renters, Z2 (%)</td>
<td>29.1</td>
<td>-0.82%</td>
<td>1.04%</td>
<td>0.25%</td>
<td>1.03%</td>
<td>-0.45%</td>
</tr>
<tr>
<td>Avg. size of RC unit in Z1 (sqft)</td>
<td>716</td>
<td>0.64%</td>
<td>-8.00%</td>
<td>3.33%</td>
<td>4.96%</td>
<td>0.24%</td>
</tr>
<tr>
<td>Avg. size of RC unit in Z2 (sqft)</td>
<td>1419</td>
<td>-0.57%</td>
<td>-0.63%</td>
<td>0.49%</td>
<td>10.50%</td>
<td>-0.13%</td>
</tr>
<tr>
<td>Avg. size of a Z1 mkt unit (sqft)</td>
<td>985</td>
<td>2.53%</td>
<td>-3.21%</td>
<td>-1.20%</td>
<td>0.73%</td>
<td>-2.01%</td>
</tr>
<tr>
<td>Avg. size of a Z2 mkt unit (sqft)</td>
<td>1507</td>
<td>0.28%</td>
<td>-0.64%</td>
<td>0.35%</td>
<td>-1.96%</td>
<td>-0.14%</td>
</tr>
<tr>
<td>Frac. of population living in Z1 (%)</td>
<td>10.6</td>
<td>-0.54%</td>
<td>-1.79%</td>
<td>9.94%</td>
<td>-1.68%</td>
<td>1.87%</td>
</tr>
<tr>
<td>Frac. of retirees living in Z1 (%)</td>
<td>21.2</td>
<td>6.01%</td>
<td>-6.12%</td>
<td>-1.41%</td>
<td>13.48%</td>
<td>-4.89%</td>
</tr>
<tr>
<td>Housing stock in Z1</td>
<td>-</td>
<td>-0.37%</td>
<td>-1.49%</td>
<td>8.90%</td>
<td>-0.37%</td>
<td>-0.10%</td>
</tr>
<tr>
<td>Housing stock in Z2</td>
<td>-</td>
<td>0.14%</td>
<td>-0.53%</td>
<td>-0.88%</td>
<td>-1.24%</td>
<td>-0.36%</td>
</tr>
<tr>
<td>Rent/sqft Z1 ($)</td>
<td>4.16</td>
<td>1.04%</td>
<td>-0.13%</td>
<td>-0.32%</td>
<td>-0.65%</td>
<td>-0.34%</td>
</tr>
<tr>
<td>Rent/sqft Z2 ($)</td>
<td>1.49</td>
<td>1.32%</td>
<td>-0.14%</td>
<td>-0.37%</td>
<td>-0.68%</td>
<td>-0.48%</td>
</tr>
<tr>
<td>Price/sqft Z1 ($)</td>
<td>1064</td>
<td>1.05%</td>
<td>-0.35%</td>
<td>-0.71%</td>
<td>-0.99%</td>
<td>-0.36%</td>
</tr>
<tr>
<td>Price/sqft Z2 ($)</td>
<td>297</td>
<td>1.47%</td>
<td>-0.28%</td>
<td>-0.98%</td>
<td>-1.06%</td>
<td>-0.53%</td>
</tr>
<tr>
<td>Home ownership rate in Z1 (%)</td>
<td>45.9</td>
<td>-5.01%</td>
<td>5.07%</td>
<td>-0.81%</td>
<td>1.48%</td>
<td>-6.78%</td>
</tr>
<tr>
<td>Home ownership rate in Z2 (%)</td>
<td>58.4</td>
<td>-2.18%</td>
<td>0.59%</td>
<td>0.34%</td>
<td>0.13%</td>
<td>-0.01%</td>
</tr>
<tr>
<td>Avg. income Z1 among working-age hhs ($)</td>
<td>164491</td>
<td>-1.71%</td>
<td>3.41%</td>
<td>-4.35%</td>
<td>-6.76%</td>
<td>-1.04%</td>
</tr>
<tr>
<td>Avg. income Z2 among working-age hhs ($)</td>
<td>100135</td>
<td>0.27%</td>
<td>-1.32%</td>
<td>0.07%</td>
<td>0.42%</td>
<td>0.07%</td>
</tr>
<tr>
<td>Frac. of high productivity hhs living in Z1 (%)</td>
<td>27.7</td>
<td>-0.56%</td>
<td>-0.84%</td>
<td>0.15%</td>
<td>-5.40%</td>
<td>-0.68%</td>
</tr>
<tr>
<td>Total hours worked in economy</td>
<td>-</td>
<td>-0.18%</td>
<td>-0.03%</td>
<td>-0.08%</td>
<td>-1.12%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Total hours worked in efficiency units</td>
<td>-</td>
<td>-0.11%</td>
<td>-0.08%</td>
<td>-0.12%</td>
<td>-1.18%</td>
<td>-0.02%</td>
</tr>
<tr>
<td>Total output</td>
<td>-</td>
<td>-0.08%</td>
<td>-0.09%</td>
<td>-0.10%</td>
<td>-0.76%</td>
<td>-0.01%</td>
</tr>
<tr>
<td>Total commuting time across all households</td>
<td>-</td>
<td>0.18%</td>
<td>0.05%</td>
<td>-1.28%</td>
<td>0.47%</td>
<td>-0.32%</td>
</tr>
<tr>
<td>Prob. of first-time access to RC if negative shock (%)</td>
<td>3.9</td>
<td>45.65%</td>
<td>-30.97%</td>
<td>1.54%</td>
<td>-21.76%</td>
<td>1.18%</td>
</tr>
<tr>
<td>Prob. of staying in RC when income Q1 (%)</td>
<td>69.1</td>
<td>1.57%</td>
<td>1.58%</td>
<td>-0.13%</td>
<td>-1.07%</td>
<td>-0.18%</td>
</tr>
<tr>
<td>Std. MU growth, nondurables</td>
<td>0.45</td>
<td>0.05%</td>
<td>0.11%</td>
<td>-0.15%</td>
<td>-1.86%</td>
<td>32.03%</td>
</tr>
<tr>
<td>Std. MU growth, housing</td>
<td>0.46</td>
<td>-1.23%</td>
<td>5.29%</td>
<td>3.48%</td>
<td>-2.55%</td>
<td>43.88%</td>
</tr>
</tbody>
</table>

Notes: Column “Benchmark” reports values of the moments for the benchmark model. Columns “RC share” to “Vouchers” report percentage changes of the moments in the policy experiments relative to the benchmark. Rows 1-9 report housing affordability moments, rows 10-26 aggregate moments across the two zones, rows 27-30 capture insurance provision, and row 31 is the percentage aggregate welfare change (in CEV units) between the alternative and the benchmark economy. Z1 stands for zone 1 (Manhattan), Z2 for the rest of the metro area.
5.1 Increasing the Amount of Rent Controlled Housing

In the first experiment, we increase the square foot amount of RC housing in each zone by half; the new $\eta^\ell$ values are 50% larger than their benchmark values. The affordable housing requirements on developers increase commensurately.

Column “RC share” of Table 3 shows that the fraction of households in RC increases by 47.24%. The distribution of RC units across the income distribution becomes worse, in that the fraction of bottom-25% income households only grows by 45.4%.

The increase in RC further blunts developers’ incentives to build since it lowers the average sale price developers earn. Housing supply in zone 1 decreases by -0.37%. This is consistent with the empirical literature, which finds that increased incentives of landlords to renovate their properties and of developers to invest in new construction generate a modest housing boom in decontrolled areas (Autor et al. (2014), Diamond et al. (2017)). Rents increase by 1.04% in zone 1. The population in Manhattan decreases (-0.54%) because of the combination of a lower housing stock and larger housing units (+0.15%).

Developer incentives are also blunted in zone 2. Nevertheless, the housing supply in zone 2 increases 0.14%. The reason is the increased demand for housing in zone 2 due to the increased population share of zone 2. This illustrates the importance of spatial equilibrium. Rents in zone increase by 1.32% in zone 2.

Standard housing affordability metrics deteriorate, even though many more households are helped by cheap RC housing. Average rent-income ratios increase in both zones. The fraction of severely rent-burdened renters increases by 30.31% metro-wide. House prices also rise in both zones. The large rise in the price-rent ratio in zone 1 is consistent with the large decline in home ownership.

The policy results not only in a smaller but also in a different population in Manhattan. With the 47.24% growth in RC households, Manhattan becomes a more “mixed” neighborhood. Average income falls (-1.71%) and there are fewer top-productivity working-age households in the urban core. Row 20 reports that the fraction of households of working age in the highest productivity state that live in zone 1 falls by -0.56%, an indicator of a less efficient spatial allocation of labor. The reallocation of the housing stock towards affordable housing units pushes more middle- and upper-middle-income households out of the urban core. This de-gentrification is consistent with the empirical evidence in Autor et al. (2014), who show that richer households moved into units previously occupied by poorer RC tenants after a reduction in control in Cambridge, MA.

The average effect of increasing the size of the RC program is a welfare gain of 0.15% in CEV units. This welfare gain is the net result of reduced efficiency but improved equality. Several metrics point to reduced efficiency. More time is spent commuting because fewer households live in zone 1. This reduces leisure and uses resources that could otherwise
be consumed. Time spent working also falls (-0.18%), which further reduces leisure and utility. Total efficiency units of labor fall (-0.11%). In other words, rent control engenders a worsened spatial allocation of labor, as well as distortions on the intensive labor margin, which combined generate a drop in output (-0.08%). These are the distortions emphasized by economists; we find that they are quantitatively modest, even though rent control is highly persistent.

The key source of the welfare gain is that the policy reduces inequality. The extension of affordable housing enhances an important source of insurance for households facing a bad income shock. Rows 27-30 of Table 3 provide several metrics that capture insurance provision. Row 27 reports the probability of being able to getting into a RC unit for households who were not yet in a RC unit and who suffer a negative productivity shock from the second to the first or from the third to the second productivity level. It measures access to the insurance that RC provides for middle- and low-income households who fall on hard times. Expanding RC by 50% greatly improves access to RC insurance (+45.65%). Row 28 reports the likelihood of staying in a RC unit for a household that was in a RC unit in the previous period and that currently is in the bottom quartile of the income distribution. This metric captures the stability of the insurance. It improve slightly (+1.57%) from the benchmark likelihood of 69.1%. Rows 29 and 30 provide two more generic measures of risk sharing. They are the time-series standard deviation of marginal utility growth for a household, averaged across households. The first one refers to the marginal utility of non-housing consumption, the second one to housing consumption. In a complete market, households can perfectly smooth consumption and marginal utility ratios are constant; their standard deviation is zero. Our benchmark model displays severe incompleteness with volatilities of 0.45 and 0.46. Increasing the RC housing stock improves the volatility of the marginal utility growth of housing (-1.23%).

Figure 4 shows that households in the bottom half of the productivity and income distribution win from the policy while households in the top half loose. The distributional implications from the policy and how they affect the poor, high marginal utility households are a key driver of the aggregate welfare effect.

5.2 Spatial Allocation of RC Housing

In the second experiment, we explore the spatial dimension of our model and study the effects of a policy that shifts all RC units from Manhattan to zone 2 but holds fixed the overall amount of RC housing. That is \( \eta^1 \) is now set to zero and \( \eta^2 \) is increased from 0.0953 to 0.1133 to make up for the loss of RC housing in zone 1. This policy leads to a sizeable decrease (-19.46%) in the fraction of households in RC. The average RC unit is
Figure 4: Welfare effects of affordability policies by age, income and net worth deciles, and productivity quartiles.

Notes: The baseline model has the following rent control parameters: $\eta^1 = 23.85$, $\eta^2 = 15.95$, $\kappa_1 = 0.50$, $\kappa_2 = 0.60$, $\kappa_3 = 0.35$. Policy experiments, each panel: decrease in the mandated fraction of RC sqft by 50%, shift of all RC sqft from zone 1 to zone 2, zoning relaxation in zone 1, increase in amount of housing vouchers given to subsidized households, implementing LIHTC. Top left panel: by age. Top right panel: by productivity level. Bottom left panel: by income quartile. Bottom right panel: by net worth quartile. The welfare changes are measured as consumption equivalent variations for an average household in each group.

12.52% larger, a composition effect because RC units in zone 1 were (endogenously) much smaller than in zone 2 in the benchmark. RC units in zone 2 become smaller than they were in the benchmark (-0.61%).

Despite the removal of developer distortions in Manhattan, the housing stock shrinks (-1.49%) along with the population share (-1.79%). The space formerly occupied by RC units is now available for market renters and owners, and reduces the demand for new development. Manhattan sees a small reduction in rents (-0.13%). With the departure of RC tenants, the population of Manhattan becomes more affluent. Average income in zone 1 is 3.41% higher and the average dwelling size increases (0.23%). The home ownership rate increases 5.07%. Because of lower rents but especially because of higher average income, rent-income in zone 1 falls -12.02%.

The expansion of RC in zone 2 reduces incentives to build in zone 2. But demand for housing increases because of the households who move from zone 1 to zone 2. On net, the housing stock decreases modestly (-0.53%) along with average unit size (-0.74%) and rents (-0.14%). Average rent to income increases 1.04% in zone 2. As measured by the fraction of severely rent-burdened households metro-wide (-22.29%), housing affordabil-
ity improves with this spatial reallocation of RC. With more working-age households in zone 2, total commuting time increases by 0.05%.

This policy generates a fairly substantial welfare loss (-0.45%). The main reason for the loss is that quite a few low-income households lose access to RC housing. The probability of first-time access for low-income households with a negative income shock drops substantially (-30.97%). Risk-sharing opportunities worsen more broadly with both marginal utility growth rates becoming more volatile relative to the benchmark model. The top left panel of Figure 4 summarizes the welfare effects. Low-income and low-wealth households lose. The welfare losses would be larger still households had an explicit preference for socio-economic diversity in every neighborhood.

5.3 Relaxing Zoning Rules in Zone 1

The third experiment studies a favorite policy among economists: allowing for more housing in the city center. We think of this policy as relaxing zoning laws, often referred to as upzoning. In the model, more construction is permitted in zone 1 when we increase $H^1$. As in the benchmark model, for every square foot of rental housing built, a fraction $\eta^1$ must be affordable. This policy is akin to NYC’s Mandatory Inclusionary Zoning policy whereby zoning variances are given in exchange for a fraction of affordable units built. This policy has no direct fiscal outlay. However, its affects all equilibrium prices and quantities indirectly, as shown in the column “Zoning Z1” of Table 3.

For our chosen increase in $H^1$, the equilibrium housing stock in Manhattan increases by 8.90% and rents fall by -0.32%. The average unit size in zone 1 decreases by -1.03% so that the population share of Manhattan rises by 9.94%. Because of the population reallocation, average income in Manhattan decreases by -4.35%. More middle-income households can now afford Manhattan and the additional affordable housing that is associated with the new construction also brings in lower-income households. Surprisingly, the fraction of top-productivity households that lives in Manhattan falls by 0.15%. House prices fall by -0.71%, more than rents do, yet home ownership decreases in Manhattan (-0.81%) due to the change in socio-economic makeup. With more working-age households in Manhattan, aggregate commuting time falls by -1.28%.

The housing stock in zone 2 falls by -0.88% as developers shift their resources towards zone 1 where the population has swelled. Rents in zone 2 fall by -0.37%, despite the reduced supply, as demand for housing weakens due to the population pivot to Manhattan.

Housing affordability metrics deteriorate slightly, underscoring their limited usefulness. Average rent-to-income among renters increases by 0.80% in zone 1 and by 0.25% in zone 2. The fraction of severely rent-burdened households increases by 1.34%. These
metrics fail to capture is that more households can now afford to live close to work.

This experiment leads to a -0.10% decrease in output. Output in the Manhattan construction sector increases substantially (10.33%). But there is an output loss in the much larger construction sector of zone 2 (-0.90%), and a small loss in the output of the very large tradable sector (-0.06%). The latter is due to the slightly higher wages (0.03%) triggered by the Manhattan construction boom, signaling a reduction in the overall competitiveness of the city. The aggregate time saved commuting goes towards leisure, since aggregate time spent working falls slightly.

Increasing housing supply in zone 1 generates an aggregate welfare gain of 0.36%. As can be seen in Figure 4, this policy brings benefits to all productivity groups, income, and wealth quartiles, and to nearly all age groups. The gains tend to be similar across groups. The zoning change leaves the RC system largely unaffected in equilibrium.

5.4 Vouchers

One important pillar of housing policy is the Section 8 voucher program, housing assistance provided by the federal government to low-income households. Since this policy is part of the tax and transfer system, it is already captured by the function $T(\cdot)$. We study the effects of increasing the size of the voucher program. We engineer the increase by increasing the tax progressivity parameter $\tau$ from 0.17 to 0.18 while changing the overall size of the tax and transfer system from $\lambda$ of 0.75 to 0.7422. This dual parameter change leaves the total net tax collected unchanged, so that there is no direct fiscal cost associated with the voucher expansion, consistent with the other policies. Because of the increased progressivity, the system generates both higher tax revenues and higher transfers. The policy experiment translates into a $780 million increase in spending on the Section 8 voucher program in the New York metropolitan area.\textsuperscript{25} We enforce that all households who receive an additional transfer spend at least the amount of the additional transfer on housing, to capture the institutional reality that vouchers must be used for housing expenses.

The fourth column of Table 3 shows the results. The policy leads to a reduction in the population share of zone 1 (-1.66%), a decline in its housing stock (-0.37%), and an

\textsuperscript{25}Data compiled from the Housing and Urban Development department show that the housing authorities responsible for the 25 counties in the New York MSA disbursed $2.06 billion in 246,000 Section 8 vouchers in the year 2013 (latest available). This amounts to an average of $8,300 per year per voucher. The policy exercise we consider raises tax revenues from $5.434 billion to $6.215 billion, an increase of $738 million. At the cost of $8,300 per voucher, this translates into 94,100 additional vouchers. In the model, the tax change increases the transfers of those who already received transfers before, and the transfer is higher the lower is household income. It also creates new beneficiaries who now receive a transfer while they were paying a tax in the benchmark. Thus, the policy both increases the amount of the existing vouchers and disburses additional smaller vouchers.
increase in commuting (0.47%). It also substantially reduces the average income of zone 1 (-6.76%) and the fraction of top-productivity households who live in zone 1 (-5.40%). Total output falls by a sizeable -0.76% along with total hours worked (-1.12% and -1.18% in efficiency units).

Economists typically favor given cash transfers to low-income households over RC under the presumption that the former are less distortionary than rent control because they give households a free choice of where to live and how much housing to get. Vouchers are such cash transfers, but must typically be paid for with distortionary labor income taxes, like in our model. The increase in taxes to pay for the voucher expansion, which fall disproportionately on the high earners, drives some high-productivity, high-income households out of zone 1. This unintended consequence which arises in spatial equilibrium reduces the efficiency of the labor allocation. While vouchers are often conceived of as “moving” lower-income households “to opportunity,” in equilibrium they crowd out higher productivity households instead, “removing” them “from opportunity.”

Despite the labor tax distortions, reduced aggregate output, and increased commuting times, aggregate welfare increases when the housing voucher program is expanded (0.90%). In fact, this is the largest welfare gain among all of our experiments. As Figure 4 shows, young, low-productivity, low-income, and low-wealth households gain substantially from the policy, and the reduced inequality is welfare improving. These results are directionally consistent with the RC expansion experiment in Section 5.1, but the redistribution in the voucher experiment is starker. The substantial reduction in the volatility of marginal utility growth is consistent with the improved overall insurance in society.

Vouchers crowd out rent control to some extent. Fewer households live in larger RC units. the reduction in RC households is particularly strong among low-income households who enjoy the largest increase in vouchers.

5.5 Low-Income Housing Tax Credit (LIHTC)

The final main policy experiment studies a common policy tool: tax credits for developers. The idea is to directly incentivize the development of affordable housing units by giving developers essentially cash subsidies.\(^{26}\) Some programs, like Tax Incremental Financing, are ran by municipal governments. The main program, the federal Low Income Housing Tax Credit (LIHTC) subsidizes 30% of the construction cost associated with the affordable units.\(^{27}\) We model a LIHTC policy the same way in the model; Appendix G contains the

\(^{26}\)Developers can sell the tax credits to other profitable firms; they fetch prices above 90 cents on the dollar. Recent reductions in corporate tax rates have reduced the value of the tax credits somewhat.

\(^{27}\)This is known as the 4% program. A 4% subsidy of construction costs is given over a 10-year period, and is worth 30% in present-value terms. There is a second program, the 9% subsidy for 10 years, which is
details of the calculation. The subsidy ends up increasing the average price earned by developers, $P^\ell$, by 3.2% in zone 1 and 2.4% in zone 2. We lower the parameter $\lambda$ in the tax-and-transfer function $T(\cdot)$ from 0.75 to 0.746 to raise the necessary additional tax revenue to exactly pay for the tax credits that are distributed in equilibrium. The policy is budget neutral, like all previous experiments.

This policy has the most adverse effect among the five main affordability policies. Aggregate welfare falls substantially (-0.52%). All households lose, across age, productivity, income, and wealth groups. Most surprisingly, the policy fails its intended purpose, to increase the equilibrium housing supply (-0.10% in zone 1 and -0.36% in zone 2). While developer incentives to build are strengthened, equilibrium housing demand becomes weaker because of the higher tax burden to pay for the tax credits. The reduction in equilibrium rents and house prices is consistent with this downward shift in the housing demand curve. The reduction in house prices we find is consistent with Diamond and McQuade (forthcoming), who find negative spillovers on house prices in higher-income neighborhoods from LIHTC properties.\footnote{They find the opposite effect in low-income neighborhoods, and attribute the positive spillovers to neighborhood revitalization and crime reductions which attract a different population. We are holding the amenity value of a neighborhood constant. Also consistent with their findings, a symmetric expansion of LIHTC results in less inequality between neighborhoods.}

Higher taxes reduce labor supply, but a wealth effect from lower house prices counters the effect, so that hours and output are unaffected. More importantly, since the policy does not lead to a meaningful expansion of affordable housing in equilibrium, it fails to help the poor, high-marginal utility households. The higher taxes result in a negative level effect on consumption as well as an increase in the volatility of the marginal utility of consumption (+32.03%) and housing (+43.88%). A model where (some) developer profits remained within the city may mitigate the negative welfare effect. But the attenuation would depend on how developer profits are redistributed across the household distribution.

\section{5.6 Other RC Policies}

Appendix H discusses five more policies that change the various policy levers of the RC system. We start by discussing the opposite policy of that in section 5.1: a reduction in the size of the rent control system by half, allowing to assess the symmetry of the policies. We find welfare losses that are twice as large than the welfare gains from expanding the RC system by the same percentage. This asymmetry reflects the high marginal utility of low-income households most affected by the policy change. Second, we lower the rent worth 70% in present-value terms, which is aimed at more deeply affordable housing units. We focus on the 4\% program. Total spending on LIHTC is $9 billion annually nationwide; about $50 million in the New York MSA.
subsidy, governed by $\kappa_1$. The policy lowers welfare because it reduces the value of the insurance provided by the RC system. Third, lowering the income qualification threshold, governed by $\kappa_2$, is strongly beneficial. This policy directs the affordable housing units towards the most needy without creating any more distortions. Fourth, reducing the size of the RC units, governed by $\kappa_3$, has similar distributional effects but the welfare gains are much smaller. We finish with an experiment that forces households to re-qualify for the RC system, passing an income test each period. This reduces the persistence in the RC system. This policy also improves the targeting of the RC system but not as much as the income threshold policy. The welfare gain is positive but smaller than expected.

6 Conclusion

In a world with rising urbanization rates, the high cost of urban housing has surfaced as a daunting policy challenge. Existing policy tools affect the supply of housing, how the housing stock is used (owned, rented, rent controlled), and how it is distributed in space. Households of different tenure status, age, income, and wealth are differentially affected by changes in policy. This paper develops a novel dynamic stochastic spatial equilibrium model with wealth effects and rich household heterogeneity that allows, for the first time, to quantitatively assess the welfare implications of the various housing affordability policies.

The model is calibrated to the New York metropolitan area. It matches patterns of average earnings, wealth accumulation, and home ownership over the life-cycle, delivers realistic house prices, rents, and wages, and matches key facts on the spatial differences in income and rents between the urban core and the periphery. The model’s rent control sector matches the key features of the New York rent control system as well as restrictions on residential land use (zoning).

We use the model to evaluate various policy changes to the rent control system as well as to zoning policy and the size of the housing voucher system. We trace out their aggregate, distributional, and spatial implications. These policies have quantitatively important general equilibrium effects that are often at odds with partial equilibrium logic.

Contrary to conventional wisdom that highlights the housing and labor market distortions generated by various housing policies, our work finds important insurance benefits from housing affordability policies. Consistent with the redistributive role of housing policy, increasing the generosity of the rent control or housing voucher systems is welfare increasing. Increasing the housing safety net for the poorest households creates welfare gains that exceed the efficiency losses that accompany the policy design.

The results underscore the need for rich models of household heterogeneity to un-
derstand both the aggregate and the distributional implications of housing affordability policies.

References


SIEG, H. AND C. YOON (2017): “Waiting for Affordable Housing,”.


A  Appendix

A.1 Analytical solution for housing and labor supply choices

Preferences are a CES aggregator over leisure and a CES aggregator of nondurable consumption and housing. We will solve only the worker’s problem here. A retiree’s problem is analogous, but simpler because there is one fewer choice as \( n_t = 0 \).

The utility function is \( U(c, h, n) = \frac{C(c, h, n)^{1-\gamma}}{1-\gamma} \), where

\[
C(c, h, n) = \left[ (1 - \alpha_N) \left( (1 - \alpha_H)c^\epsilon + \alpha_H h^\epsilon \right)^{\frac{\eta}{\gamma}} + \alpha_N \left[ 1 - \Phi_T - n \right]^\chi \right]^{\frac{1}{\gamma}}
\]

when \(|\eta|, |\epsilon| > 0\) (case (i)). \( \chi_0 \) makes leisure nonlinear in hours (we set it to 1), and we impose a lower bound on hours, \( n \geq n_{\text{min}} \).

When \(|\eta| > 0, |\epsilon| = 0\), \( u(c, h, n) = \left[ (1 - \alpha_N) \left[ c^{1-aH} h^{\alpha H} \right]^\eta + \alpha_N \left[ 1 - \Phi_T - n \right]^\chi \right]^{\frac{1}{\gamma}} \) (case (ii)).

When \(|\eta| = 0, |\epsilon| > 0\), \( u(c, h, n) = \left[ (1 - \alpha_H)c^\epsilon + \alpha_H h^\epsilon \right]^{\frac{1-n_H}{\epsilon}} \left[ 1 - \Phi_T - n \right]^\chi \) (case (iii)).

When \(|\eta| = |\epsilon| = 0\), \( u(c, h, n) = \left[ c^{1-aH} h^{\alpha H} \right]^{1-\alpha_N} \left[ 1 - \Phi_T - n \right]^\chi \) (case (iv)).

Renter  First, consider the renter’s problem and let \( \lambda_t \) be the Lagrange multiplier on the budget constraint, \( \nu_t \) be the Lagrange multiplier on the borrowing constraint, and \( \xi_t \) be the Lagrange multiplier on the non-negativity labor constraint. The numerical strategy is to choose \( c_t \) in order to maximize the household’s utility, and \( l_t \) to solve the non-linear equation for labor supply. Here we will show that the other choices \( (h_t, b_{t+1}) \) can be written as analytic functions of \( c_t \) and \( n_t \). Denote \( C_t = C(c_t, h_t) \). We ignore the taste shifter (which is multiplicative and raised to the power \( 1 - \gamma \) in the equations involving \( C_t \)), and assume \( \bar{b}_{t+1} = 0 \). The budget constraint simplifies to:

\[
c_t + R_t^h h_t + Q_t b_{t+1} + \phi^\ell_{t+1} = V_t \psi^\ell + \left( 2 - \frac{1}{Q_t} \right) x_t + \lambda \left( W_t G^a z_t n_t + \left( 1 - 1 \right) x_t \right)^{1-\gamma} - \tau S_t W_t G^a z_t n_t
\]

The first order conditions for \( c_t, l_t, h_t, b_{t+1} \) are respectively:

\[
\begin{align*}
C_t^{1-\gamma} (1 - \alpha_N)(1 - \alpha_h) \left( (1 - \alpha_H)c^\epsilon + \alpha_H h^\epsilon \right)^{\frac{\eta}{\gamma}} c^{\gamma-1} & = \lambda_t \\
C_t^{1-\gamma} \alpha_n (1 - \Phi_T - n_t)^{\gamma-1} & = \lambda_t W_t G^a z_t \left[ (1 - \gamma) \left( W_t G^a z_t (1 - \phi^\ell - l_t) \right) \right]^{-\gamma} - \tau S_t w_t^n + \xi_t \\
C_t^{1-\gamma} (1 - \alpha_h) \left( 1 - \alpha_Hc^\epsilon + \alpha_H h^\epsilon \right)^{\frac{\eta}{\gamma}} h^{\gamma-1} & = \lambda_t R_t^f \\
\lambda_t Q_t = (1 - p^\ell) \beta \Psi_t \left[ \frac{\partial}{\partial x_t} V (x_{t+1}, z_{t+1}, a + 1, S_{t+1}) \right] + v_t
\end{align*}
\]

where

\[
\frac{\partial}{\partial x_t} V (x_t, z_t, a, S_t) = \lambda_t \left( 2 - \frac{1}{Q_t} + \lambda (1 - \gamma) \left( \frac{1}{Q_t} - 1 \right) \left( W_t G^a z_t n_t + \left( 1 - 1 \right) x_t \right)^{-\gamma} \right),
\]

since the marginal value of net worth is the same in the problem of renters, RC renters and owners, and the probabilities to receive rent control sum to 1.

Case 1: \( v_t = 0 \) and \( \xi_t = 0 \). In this case the household is unconstrained. Residential housing is obtained by combining the conditions for \( c_t \) and \( h_t \): \( h_t = \left( \frac{a_t}{(1 - \alpha_h) h_t} \right)^{\gamma} c_t \).

Case (i) The nonlinear equation for hours \( n_t \) is obtained by combining the first order conditions for
\( c_t \) and \( n_t \), and substituting for \( h_t \) as a function of \( c_t \) in the CES aggregator:

\[
\chi_0 a_N (1 - \Phi_T - n)^{\lambda \eta - 1} - (1 - a_N)(1 - a_H) \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right]^{\frac{\eta - \epsilon}{\tau}} LG_a z
\]

(16)

The Jacobian is:

\[
-\alpha_N \chi_0 (\lambda - 1)(1 - \Phi_T - n)^{\lambda \eta - 2} + (1 - a_N)(1 - a_H) \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right]^{\frac{\eta - \epsilon}{\tau}} LG_a z^2
\]

Absence HSV taxes, the analytic solution for hours is:

\[
n = 1 - \Phi_T - \left[ \frac{(1 - a_N)(1 - a_H)}{\lambda \alpha^H_N} \right] \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right] \frac{\eta - \epsilon}{\tau} LG_a z (1 - \tau^{SS}) \left[ \frac{1}{\chi_0^{\frac{\eta - \epsilon}{\tau}}} \right] \left[ \frac{1}{\chi_0^{\frac{\eta - \epsilon}{\tau}}} \right]
\]

(17)

Case (ii) The nonlinear equation for hours is:

\[
\chi_0 a_N (1 - \Phi_T - n)^{\lambda \eta - 1} - (1 - a_N)(1 - a_H) \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right]^{\frac{\eta - \epsilon}{\tau}} LG_a z
\]

(19)

The Jacobian is:

\[
-\chi_0 a_N (\lambda - 1)(1 - \Phi_T - n)^{\lambda \eta - 2} + (1 - a_N)(1 - a_H) \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right]^{\frac{\eta - \epsilon}{\tau}} LG_a z^2
\]

Absence HSV taxes, the analytic solution for hours is:

\[
n = 1 - \Phi_T - \left[ \frac{(1 - a_N)(1 - a_H)}{\lambda \alpha^H_N} \right] \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right] \frac{\eta - \epsilon}{\tau} LG_a z (1 - \tau^{SS}) \left[ \frac{1}{\chi_0^{\frac{\eta - \epsilon}{\tau}}} \right] \left[ \frac{1}{\chi_0^{\frac{\eta - \epsilon}{\tau}}} \right]
\]

(21)

Case (iii) The nonlinear equation for hours is:

\[
\chi_0 a_N \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right] \frac{\eta - \epsilon}{\tau} c
\]

(22)

\[-(1 - a_N)(1 - a_H)(1 - \Phi_T - n) LG_a z \left\{ \lambda (1 - \tau) (WG_a zn + rx)^{-\tau - \tau^{SS}} \right\} = 0
\]

The Jacobian is:

\[
(1 - a_N)(1 - a_H) LG_a z \left\{ \lambda (1 - \tau) (WG_a zn + rx)^{-\tau - \tau^{SS}} \right\} + (1 - \Phi_T - n) \lambda (1 - \tau) (WG_a zn + rx + \Pi)^{-\tau - 1}
\]

(23)

Absence HSV taxes, the analytic solution for hours is:

\[
n = 1 - \Phi_T - \left[ (1 - a_H) + a_H \left( \frac{\alpha_H}{1 - \alpha_H} \right)^{\frac{\eta - \epsilon}{\tau}} \right] \frac{\eta - \epsilon}{\tau} \frac{\chi_0 a_N}{(1 - a_N)(1 - a_H) LG_a z (1 - \tau^{SS}) c}
\]

(24)
The nonlinear equation for hours is:
\[
\chi_0 a_N c - (1 - a_N) (1 - a_H) (1 - \Phi_T - n) W G_a z \left\{ \lambda (1 - \tau) (W G_a z n + r x)^{-\tau} - \tau^{SS} \right\} = 0
\] (25)

The Jacobian is:
\[
(1 - a_N) (1 - a_H) W G_a z \left\{ \lambda (1 - \tau) (W G_a z n + r x + \Pi)^{-\tau} - \tau^{SS} \right\} +
(1 - \Phi_T - n) \lambda (1 - \tau) (W G_a z n + r x)^{-\tau - 1}
\] (26)

Absent HSV taxes, the analytic solution for hours is:
\[
n = 1 - \Phi_T - \frac{\chi_0 a_N}{(1 - a_N) (1 + a_H) W G_a z (1 - \tau^{SS})} c
\] (27)

Given \(c_t\), we obtain \(l_t\) (hence \(n_t\)) by numerically solving the labor supply equation. Given \(c_t, n_t, l_t\), we obtain \(b_{t+1}\) from the budget constraint:
\[
b_{t+1} = \frac{1}{\ell_t} \left[ \Psi_t \psi_t^z - \phi_{F,t} + 2 \left( 1 - \frac{1}{\ell_t} \right) x_t + \lambda \left( W_t G^a z_t n_t + \left( 1 - \frac{1}{\ell_t} \right) x_t \right) 1^{1-\tau} - \tau^{SS} W_t G^a z_t n_t - c_t - R_{f,t} l_t \right]
\] (28)

**Case 2:** \(v_t > 0\) and \(\xi_t = 0\). In this case the borrowing constraint binds and \(b_{t+1} = 0\) but the labor constraint does not. The first order conditions in the first three lines of equation 15 are still correct. It is still the case that conditional on choosing a location \(\ell_t, h_t = \left( \frac{a_h}{(1 - a_h) R_f} \right)^{1/\tau} c_t\) and \(l_t\) is the solution to the nonlinear equation in Case 1. Given those, \(c_t\) can be obtained from the budget constraint:
\[
c_t = \frac{\Psi_t \psi_t^z - \phi_{F,t} + 2 \left( 1 - \frac{1}{\ell_t} \right) x_t + \lambda \left( W_t G^a z_t n_t + \left( 1 - \frac{1}{\ell_t} \right) x_t \right) 1^{1-\tau} - \tau^{SS} W_t G^a z_t n_t}{1 + R_{f,t} \left( \frac{a_h}{(1 - a_h) R_f} \right)^{1/\tau}}
\] (29)

**Case 3:** \(v_t = 0\) and \(\xi_t > 0\). In this case the borrowing constraint does not bind, but the labor constraint does, implying \(l_t = 1 - \phi_{F,t}^\ell\), hence \(n_t = 0\). The first order conditions in the first, third, and fourth lines of equation 15 are still correct. As in Case 1, conditional on choosing a location \(\ell\), \(h_t = \left( \frac{a_h}{(1 - a_h) R_f} \right)^{1/\tau} c_t\). We obtain \(b_{t+1}\) from the budget constraint.

**Case 4:** \(v_t > 0\) and \(\xi_t > 0\). In this case both constraints bind, implying \(n_t = 0\) and \(b_{t+1} = 0\). The first order conditions in the first and third lines of equation 15 are still correct, so conditional on choosing a location \(\ell\), \(h_t = \left( \frac{a_h}{(1 - a_h) R_f} \right)^{1/\tau} c_t\). By plugging this into the budget constraint, we can explicitly solve for \(c_t = \frac{\Psi_t \psi_t^z - \phi_{F,t} + 2 \left( 1 - \frac{1}{\ell_t} \right) x_t + \lambda \left( 1 - \frac{1}{\ell_t} \right) x_t 1^{1-\tau}}{1 + R_{f,t} \left( \frac{a_h}{(1 - a_h) R_f} \right)^{1/\tau}}\).

In the case with \(\lambda = 1, \tau = 0\), we simply have \(c_t = \frac{\Psi_t \psi_t^z - \phi_{F,t} + x_t}{1 + R_{f,t} \left( \frac{a_h}{(1 - a_h) R_f} \right)^{1/\tau}}\).

**RC renter** Next, the RC renter’s problem is the same as the renter’s problem, with additional Lagrange multipliers on the income and the rent restriction constraints, \(\xi_t\) and \(\xi_t\), and \(k_1\) the rent
control discount multiplying $R^\ell_t$ wherever it appears. Note that we cannot have both $\xi_t, \xi^v_t > 0$ (we rule out $\frac{\kappa Y_t}{W_t G^2 z_t} = 0$). Because households are atomistic we ignore the derivative $\frac{\partial Y_t}{\partial n_t}$.

**Case 1:** $\xi_t = 0$ and $\xi^v_t = 0$. There is an interior solution $n_{min} < n_t < \frac{\kappa Y_t}{W_t G^2 z_t}$, which solves the same labor supply equation as the market renter (with $\kappa_1 R^\ell_t$ instead of $R^\ell_t$). If $\xi^v_t > 0$, then the residential housing choice is constrained: $h_t = \frac{\kappa Y_t}{\kappa_1 R_t}$. If $\xi^v_t = 0$, then combining the conditions for $c_t$ and for $h_t$, we obtain $h_t = \left(\frac{\alpha H}{1-\alpha H}\right) \frac{1}{\kappa_1 R_t} c_t$. If $\nu_t > 0$, then savings are constrained and $b_{t+1} = 0$. If $\nu_t = 0$, then given $c_t, n_t, h_t$, we obtain $b_{t+1}$ from the budget constraint.

**Case 2:** $\xi_t > 0$ and $\xi^v_t = 0$. The leisure choice is constrained at its upper bound, and $n_t = 0$. Choices for $h_t$ and $b_{t+1}$ as functions of $c_t$ are identical to Case 1.

**Case 3:** $\xi_t = 0$ and $\xi^v_t > 0$. The labor choice is constrained at the upper bound implied by RC, and $n_t = \frac{\kappa Y_t}{W_t G^2 z_t}$. Choices for $h_t$ and $b_{t+1}$ as functions of $c_t$ are identical to Case 1.

**Owner** Finally, consider the owner’s problem and let $\lambda_t$ be the Lagrange multiplier on the budget constraint, $\nu_t$ be the Lagrange multiplier on the borrowing constraint, and $\xi_t$ be the Lagrange multiplier on the labor constraint. The numerical strategy is to choose $c_t$ and $h_t$ in order to maximize the household’s utility (therefore, we ignore the multiplier on the non-negativity constraint $\tilde{h}_t \geq 0$), and $n_t$ to solve the nonlinear equation for labor supply. Here we will show that the other choices ($h_t$ and $b_{t+1}$) can be written as analytic functions of $c_t$ and $\tilde{h}_t$. The budget constraint simplifies to:

$$c_t + P_t h_t + Q_t b_{t+1} + \kappa^\ell_t P_t \xi_t h_t + \phi^\ell_{F,t}$$

$$= \psi_t \psi - \left(2 - \frac{1}{\xi_t}\right) x_t + \kappa_d R^\ell_t h_t + \lambda \left(W_t G^2 z_t n_t + \left(\frac{1}{\xi_t} - 1\right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^2 z_t n_t$$

The first-order conditions for $c_t, l_t$ are identical to the market renter. The first-order conditions for $h_t, h_t, b_{t+1}$ are respectively:

$$c_t^{1-\gamma-\eta} (1-\alpha_H) a_h ((1-\alpha_H)c^c + a_H h^c) \frac{\partial c^c}{\partial c^c} + (1-p^\eta) \beta (1-\delta - \tau^\ell) E_t \left[P^\ell_{t+1} \frac{\partial}{\partial x_{t+1}} V (x_{t+1}, z_{t+1}, a + 1, S_{t+1})\right] + \nu_t \theta_{res} P^\ell_t = \lambda_t P^\ell_t$$

$$= (1-p^\eta) \beta (1-\delta - \tau^\ell) E_t \left(P^\ell_{t+1} \frac{\partial}{\partial x_{t+1}} V (x_{t+1}, z_{t+1}, a + 1, S_{t+1})\right) + \nu_t \theta_{int} P^\ell_t = \lambda_t (P^\ell_t - R_t)$$

$$\lambda_t Q_t = (1-p^\eta) \beta E_t \left[\frac{\partial}{\partial x_{t+1}} V (x_{t+1}, z_{t+1}, a + 1, S_{t+1})\right] + \nu_t Q_t$$

**Case 1:** $\nu_t = 0$ and $\xi_t = 0$. In this case the household is unconstrained. Combining the conditions for $h_t$ and $\tilde{h}_t$, and combining the result with the condition for $c_t$, we can solve analytically for $h_t = \left(\frac{\alpha H}{1-\alpha H}\right) \frac{1}{\kappa_1 R_t} c_t$, as in the market renter’s case. Then, the nonlinear labor supply equation for $n_t$ is the same.
Given \( c_t, \hat{h}_t, h_t, n_t \), we obtain \( b_{t+1} \) from the budget constraint:

\[
\begin{align*}
b_{t+1} &= \frac{1}{Q_t} \left[ \Psi_t \psi^z - \phi^e_{F,t} + \left( 2 - \frac{1}{Q_t} \right) x_t + \kappa_4^e \left( R^e_t - P^e_t \right) \hat{h}_t \\
&+ \lambda \left( W_t G^a z_t n_t + \left( \frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} - \tau^{SS} W_t G^a z_t n_t - c_t - P^e_t h_t \right]
\end{align*}
\]

(32)

**Case 2:** \( v_t > 0 \) and \( \zeta_t = 0 \). In this case the borrowing constraint binds implying \( b_{t+1} = -\frac{\theta_{exc} P^e_t h_t + \theta_{inv} \kappa_4^e P^e_t \hat{h}_t}{Q_t} \), but the leisure constraint does not bind. We solve for \( l_t \) as in Case 1, as the nonlinear equation for hours is unaffected. Given \( c_t, \hat{h}_t, n_t \), we use the budget constraint to solve for \( h_t \) analytically:

\[
\begin{align*}
h_t &= \frac{1}{P^e_t (1 - \theta_{res})} \left[ \Psi_t \psi^z + \left( 2 - \frac{1}{Q_t} \right) x_t + \kappa_4^e R^e_t \hat{h}_t + \lambda \left( W_t G^a z_t n_t + \left( \frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} \\
&- \phi^e_{F,t} - c_t - (1 - \theta_{inv}) \kappa_4^e P^e_t \hat{h}_t - \tau^{SS} W_t G^a z_t n_t \right]
\end{align*}
\]

(33)

In the case with \( \lambda = 1, \tau = 0 \), we have

\[
h_t = \frac{1}{P^e_t (1 - \theta_{res})} \left[ \Psi_t \psi^z + x_t + (1 - \tau^{SS}) W_t G^a z_t n_t + \kappa_4^e R^e_t \hat{h}_t - \phi^e_{F,t} - c_t - (1 - \theta_{inv}) \kappa_4^e P^e_t \hat{h}_t \right]
\]

**Case 3:** \( v_t = 0 \) and \( \zeta_t > 0 \). In this case the borrowing constraint does not bind, but the leisure constraint does, implying \( n_t = 0 \). Conditional on choosing a location \( \ell, h_t \) is identical to Case 1. From the budget constraint, we deduce:

\[
\begin{align*}
b_{t+1} &= \frac{1}{Q_t} \left[ \Psi_t \psi^z - \phi^e_{F,t} + x_t \left( 2 - \frac{1}{Q_t} \right) + \kappa_4 (R^e_t - P^e_t) \hat{h}_t + \lambda \left( \left( \frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} - \left( 1 + \frac{\alpha_b}{(1 - \alpha_b) R^e_t} \right)^{\frac{1}{\tau}} P^e_t \right] c_t
\end{align*}
\]

(34)

**Case 4:** \( v_t > 0 \) and \( \zeta_t > 0 \). In this case both constraints bind, implying \( n_t = 0 \) and \( b_{t+1} = -\frac{\theta_{exc} P^e_t h_t + \theta_{inv} \kappa_4^e P^e_t \hat{h}_t}{Q_t} \). Eliminating \( b_{t+1} \) and \( n_t \) from the budget constraint, we can solve analytically for \( h_t \) as a function of \( c_t \) and \( \hat{h}_t \) just as in case 2:

\[
\begin{align*}
h_t &= \frac{1}{P^e_t (1 - \theta_{res})} \left[ \Psi_t \psi^z + x_t + \kappa_4^e R^e_t \hat{h}_t + \lambda \left( \left( \frac{1}{Q_t} - 1 \right) x_t \right)^{1-\tau} - \phi^e_{F,t} - c_t - (1 - \theta_{inv}) \kappa_4^e P^e_t \hat{h}_t \right]
\end{align*}
\]

(35)

In the case with \( \lambda = 1, \tau = 0 \), we have

\[
h_t = \frac{1}{P^e_t (1 - \theta_{res})} \left[ \Psi_t \psi^z + x_t + \kappa_4^e R^e_t \hat{h}_t - \phi^e_{F,t} - c_t - (1 - \theta_{inv}) \kappa_4^e P^e_t \hat{h}_t \right]
\]

**A.2 Special case which can be solved analytically**

Here we use Cobb-Douglas preferences as a special case of the CES aggregator described earlier. Consider a perpetual renter who is facing a constant wage \( W \) and a constant rent \( R \), who is not choosing location, who is not constrained, who faces no idiosyncratic shocks \((A = 1)\), and whose productivity and utility are not age dependent \((G^a = 1, \alpha_{c,a} = \alpha_c, \text{ and } \alpha_{h,a} = \alpha_h \forall a)\). His problem
can be written as:

\[
v(x_{st}, a) = \max_{c_{st}, h_{st}} \frac{1}{1-\gamma} \left( c_{st}^{\alpha_c} h_{st}^{\alpha_h} (1 - n_t)^{a_n} \right) ^{1-\gamma} + \beta E_t[v(x_{st+1}, a + 1)] \text{ s.t.}
\]

\[
x_{st+1} = \frac{1}{Q} (x_{st} + n_t W - c_{st} - h_{st} R)
\]

As shown earlier, the optimal housing and labor choices satisfy: \( h_{st} = \frac{a_n}{\alpha_c} \frac{1}{R} c_{st} \) and \( n_t = 1 - \frac{a_n}{\alpha_c} \frac{1}{R} c_{st} \). Redefining \( \hat{c}_s \) as \( \frac{1}{a_c} c_{st} \) and plugging these into the maximization problem, the problem is rewritten as:

\[
v(x_{st}, a) = \max_{\hat{c}_s} \frac{1}{1-\gamma} \hat{c}_s^{1-\gamma} + \beta E_t[v(x_{st+1}, a + 1)] \text{ s.t.}
\]

\[
x_{st+1} = \frac{1}{Q} (x_{st} + \hat{c}_s W - \hat{c}_s)
\]

where \( \bar{U} = (a_c a_h a_n R^{-a_n} W^{-a_n})^{1-\gamma} \). Next we can guess and verify that the value function has the form \( v(x_{st}, a) = \frac{\hat{v}_a}{\gamma} * \left( x_{st} + \frac{1}{Q_a} W \right)^{\frac{\gamma}{1-\gamma}} \) where \( \hat{v}_a \) and \( Q_a \) are constants that depend on age \( a \). Suppose this is true for \( a + 1 \). Then the problem is:

\[
v(x_{st}, a) = \max_{\hat{c}_s} \frac{1}{1-\gamma} \hat{c}_s^{1-\gamma} + \frac{\hat{v}_a}{\gamma} * \left( x_{st} + \frac{1}{Q_a} W \right)^{\frac{\gamma}{1-\gamma}} \text{ s.t.}
\]

\[
x_{st+1} = \frac{1}{Q} (x_{st} + \hat{c}_s W - \hat{c}_s)
\]

Define \( X_{a+1} = \hat{v}_a + Q^{-1}(1-\gamma) \beta \). Then the first order condition is: \( \bar{U} * \hat{c}_s^{1-\gamma} = X_{a+1} * (x_{st} - \hat{c}_s + W^{1-Q_{a+1} Q^{-1}})^{-\gamma} \). Rearranging, we can solve for optimal consumption:

\[
\hat{c}_s = \frac{\left( x_{a+1} \bar{U} \right)^{-1/\gamma}}{1 + \left( x_{a+1} \bar{U} \right)^{-1/\gamma}} \left( x_{st} + W^{1-Q_{a+1} Q^{-1}} \right)
\]

\[
x_{st+1} = \frac{1}{1 + \left( x_{a+1} \bar{U} \right)^{-1/\gamma}} \left( x_{st} + W^{1-Q_{a+1} Q^{-1}} \right)
\]

Plugging this back into the original problem:

\[
v(x_{st}, a) = \left( \bar{U} \left( \left( x_{a+1} \bar{U} \right)^{-1/\gamma} \right)^{-\gamma} \right) \left( x_{st} + W^{1-Q_{a+1} Q^{-1}} \right)^{1-\gamma}
\]

\[
= \bar{U} \left( 1 + \left( x_{a+1} \bar{U} \right)^{-1/\gamma} \left( x_{a+1} \bar{U} \right)^{-1/\gamma} \right)^{-\gamma} \left( x_{st} + W^{1-Q_{a+1} Q^{-1}} \right)^{1-\gamma}
\]

\[
= X_{a+1} \left( 1 + \left( x_{a+1} \bar{U} \right)^{-1/\gamma} \right)^{-\gamma} \left( x_{st} + W^{1-Q_{a+1} Q^{-1}} \right)^{1-\gamma}
\]

This verifies the conjecture. The age dependent constants take the following form:

\[
v_a = X_{a+1} \left( 1 + \left( x_{a+1} \bar{U} \right)^{-1/\gamma} \right)^{-\gamma}
\]

\[
Q_a = \frac{1}{1 + Q^{-1} Q_{a+1}}
\]
Note that $Q_\infty = Q$ and $v_\infty = U \left( 1 - \beta \frac{1}{\gamma} Q^{-\frac{(1-\gamma)}{\gamma}} \right)^{-\gamma}$.

### A.3 Commuting costs and composition of Zone 1

From the household’s FOC, we know that $\frac{\partial U}{\partial C} = \frac{\partial U}{\partial N} \times \frac{1}{w}$ where $C$ is the numeraire, $N$ is hours worked, and $w$ is the wage. Suppose that moving one unit of distance towards center decreases the hourly commuting cost by $\phi_T$ and the financial commuting cost by $\phi_F$. Also, suppose that the price is a function of distance from center $P(x)$.

First, consider time costs only ($\phi_F = 0$). The cost of decreasing the commute by $d$ is $d \times H \times P'(x) \times \frac{\partial U}{\partial C}$, this is the amount of housing consumed $H$, multiplied by the price increase at the current location $P'(x) \times d$, multiplied by the marginal utility of the numeraire good. The benefit of decreasing the commute by $d$ is $d \times \phi_T \times \frac{\partial U}{\partial W} = d \times \phi_T \times w \times \frac{\partial U}{\partial C}$, this is the marginal utility of leisure, multiplied by the extra leisure $d \times \phi_T$. Equating the cost to the benefit and rearranging: $P'(x) = \phi_T \frac{w}{H}$. The left hand side represents one’s willingness to pay per square foot implying that agents with high $\frac{w}{H}$ are willing to pay a higher price. For a fixed amount of wealth, high income agents have higher $\frac{w}{H}$ because individual productivity is stationary, therefore high income agents tend to save relatively more and consume relatively less of their wealth ($\frac{w}{H}$ would be constant if individual productivity had permanent shocks). For a fixed income, high wealth agents have higher $\frac{w}{H}$ because, consistent with the Permanent Income Hypothesis, for a fixed $\frac{w}{H}$, this is the amount of housing consumed $H$ multiplied by the marginal utility of the numeraire good. The benefit of decreasing the commute by $d \times \phi_T \times \frac{\partial U}{\partial W}$, multiplied by the marginal utility of the numeraire. Equating the cost to the benefit and rearranging:

$P'(x) = \phi_T \frac{w}{H}$. The benefit of decreasing the commute is the same as before $d \times H \times P'(x) \times \frac{\partial U}{\partial C}$. The benefit of decreasing the commute by $d \times \phi_F \times \frac{\partial U}{\partial W}$ multiplied by the marginal utility of the numeraire. Equating the cost to the benefit: $P'(x) = \phi_T \frac{w}{H}$. Low $H$ agents are willing to pay a higher price. Agents who have low wealth or low income tend to have lower housing demand $H$ and are willing to pay more per square foot to reduce their commute. The intuition is that the financial cost is fixed, thus agents with low housing demand are willing to pay a much higher price per square foot to ‘ammortize’ the benefit of not paying the fixed cost.

### A.4 One-period case which can be solved analytically

There are $m$ agents, $m^c$ consumption producing firms, $m^l$ construction firms in zone 1, and $m^2$ construction firms in zone 2. There are two zones with sizes $mH^1$ and $mH^2$. Agents have initial wealth $W = 0$ and earn a wage $w$. They live for one period only, and there is no resale value for the housing that they buy.

Conditional on a zone, a household maximizes $U = c^{a_c} h^{a_h} (1 - \lambda - x)^{a_n} \text{subject to} c + P * h = W + w * x$ where $\lambda$ is a zone specific time cost and $P$ is a zone specific housing price ($\lambda = 0$ in zone 1). This can be rewritten as:

$$U = \max_{h, x}(W + w * x - P * h)^{a_c} h^{a_h} (1 - \lambda - x)^{a_n}$$

(42)

The first order conditions imply the following solution:

$$c = a_c((1 - \lambda)w + W)$$
$$h = a_h((1 - \lambda)w + W)$$
$$x = (a_c + a_h)(1 - \lambda) - a_n \frac{W}{w}$$

$$U = \left( \frac{1}{P} \right)^{a_h} \left( \frac{1}{w} \right)^{a_n} a_c^{a_c} a_h^{a_h} a_n^{a_n} (1 - \lambda)w + W$$

(43)
Here we used \( \alpha_c + \alpha_h + \alpha_n = 1 \).

Each consumption producing firm chooses hours \( x_c \) to maximize \( \pi_c = x_c^{\rho_c} - wx_c \) which implies that \( w = \rho_c x_c^{\rho_c - 1} \). Each construction firm in zone 1 maximizes \( \pi_1 = \left( 1 - \frac{H^1}{mh} \right) P_1 x_1^{\rho_h} - w x_1 \) which implies that \( w = \left( 1 - \frac{H^1}{mh} \right) P_1 \rho_h x_1^{\rho_h - 1} \). Each construction firm in zone 2 maximizes \( \pi_2 = \left( 1 - \frac{H^2}{mh} \right) P_2 x_2^{\rho_h} - w x_2 \) which implies that \( w = \left( 1 - \frac{H^2}{mh} \right) P_2 \rho_h x_2^{\rho_h - 1} \). Here \( H^1 \) and \( H^2 \) are the total amount of housing built in each zone.

Equilibrium implies that the following equations must be satisfied.

\[
P_2 = P_1 (1 - \lambda)^{1/\alpha_h} \tag{44}
\]

Equation 44 says that for households to be indifferent between the two zones, their utility of living in each zone must be the same.

\[
n_1 = \frac{H^1 p_1}{\alpha_h w} \tag{45}
\]

\[
n_2 = \frac{H^2 p_2}{\alpha_h w (1 - \lambda)} \tag{46}
\]

\[
n_1 + n_2 = m \tag{47}
\]

Equations 45 and 46 say that the total number of households in each zone (\( N_1 \) and \( N_2 \)) must equal to the total housing in each zone, divided by the housing size an agent in that zone would demand. The housing size comes from the solution of the agent’s problem. Equation 47 says that the sum of agents living in zones 1 and 2 must equal to the total number of agents.

\[
w = \rho_c x_c^{\rho_c - 1} \tag{48}
\]

\[
w = \left( 1 - \frac{H^1}{mh} \right) P_1 \rho_h x_1^{\rho_h - 1} \tag{49}
\]

\[
w = \left( 1 - \frac{H^2}{mh} \right) P_2 \rho_h x_2^{\rho_h - 1} \tag{50}
\]

Equations 48, 49, and 50 relate each firm’s optimal behavior to the wage.

\[
H^1 = \left( 1 - \frac{H^1}{mh^1} \right) m_1 x_1^{\rho_h} \tag{51}
\]

\[
H^2 = \left( 1 - \frac{H^2}{mh^2} \right) m_1 x_2^{\rho_h} \tag{52}
\]

Equations 51 and 52 relate each firm’s output to the total output of housing in each zone. They can be rewritten as \( H^1 = \frac{mh^1 m_1 x_1^{\rho_h}}{mh^1 + m_1 x_1^{\rho_h}} \) and \( H^2 = \frac{mh^2 m_2 x_2^{\rho_h}}{mh^2 + m_2 x_2^{\rho_h}} \).

\[
(\alpha_c + \alpha_h)(n_1 + n_2(1 - \lambda)) = m_c x_c + m_1 x_1 + m_2 x_2 \tag{53}
\]

Equation 53 relates labor supply, on the left side, to labor demand, on the right side.

This is 10 equations and 10 unknowns: prices \( P_1, P_2 \); labor demand for each firm type \( x_1, x_2, x_c \); number of households living in each zone \( n_1, n_2 \); total housing in each zone \( H^1, H^2 \); and the wage \( w \). This can be reduced to a single equation.
First, plug $H$ and $P$ into equations (49) and (50): \[ w = P_1 \rho_h \frac{m h \lambda}{m h + m_1 x} + P_2 \rho_h \frac{m h \lambda}{m h + m_2 x} \]

Second, plug the wage into equations (45) and (46): \[ n_1 = \frac{m_1 x}{m_1 \rho_h} \quad \text{and} \quad n_2 = \frac{m_2 x}{m_2 \rho_h} \]

Third, plug $n_1$ and $n_2$ into equation (47) to solve for $x_2$ in terms of $x_1$: \[ x_2 = \frac{1 - \lambda}{m_2} (m x \rho_h - m_1 x) = A_0 + A_1 x_1 \text{ where } A_0 = \frac{1 - \lambda}{m_2} m x \rho_h \quad \text{and} \quad A_1 = -m_1 \frac{1 - \lambda}{m_2}. \]

Fourth, plug $x_2 = A_0 + A_1 x_1$ into the equality between zone 1 and zone 2 firms’ wages derived earlier and use equation (44) to get rid of prices: \[ \frac{m h \lambda}{m h + m_1 x} = (1 - \lambda)^{1/\alpha} \frac{m h (A_0 + A_1 x_1) x}{m h^2 + m_2 (A_0 + A_1 x_1) x}. \] This is now one equation with one unknown and can be solved numerically.

Fifth, once we have $x_1$ we can immediately calculate $x_2$, $n_1$, $n_2$, $H^1$, $H^2$ but we still need to solve for $w$ and $P_1$. We can solve for $w$ as a function of $P_1$ using equation (49). We can then solve for $x_c$ as a function of $P_1$ using equation (48). We can then plug this into equation (53) to solve for $P_1$.

## B Data Appendix: New York

### B.1 The New York Metro Area

U.S. Office of Management and Budget publishes the list and delineations of Metropolitan Statistical Areas (MSAs) on the Census website (https://www.census.gov/population/metro/data/metrodef.html). The current delineation is as of July 2015. New York-Newark-Jersey City, NY-NJ-PA MSA (NYC MSA) is the most populous MSA among the 382 MSAs in the nation.

NYC MSA consists of 4 metropolitan divisions and 25 counties, spanning three states around New York City. The complete list of counties with state and zone information is presented in Table 4. As previously defined, only New York County (Manhattan borough) is categorized as zone 1 and the rest 24 counties are categorized as zone 2. For informational purposes, the five counties of New York City are appended with parenthesized borough names used in New York City.

### B.2 Population, Housing Stock, and Land Area

The main source for population, housing stock and land area is US Census Bureau American FactFinder (http://factfinder.census.gov). American FactFinder provides comprehensive survey data on a wide range of demographic and housing topics. Using the Advanced Search option on the webpage, topics such as population and housing can be queried alongside geographic filters. We select the DP02 table (selected social characteristics) for population estimates, the DP04 table (selected housing characteristics) for housing estimates, and the GCT-PH1 table (population, housing units, area and density) for land area information. Adding 25 counties separately in the geographic filter, all queried information is retrieved at the county level. We then aggregate the 24 columns as a single zone 2 column.

Since the ACS (American Community Survey) surveys are conducted regularly, the survey year must be additionally specified. We use the 2015 1-year ACS dataset as it contains the most up-to-date numbers available. For Pike County, PA, the 2015 ACS data is not available and we use the 2014 5-year ACS number instead. Given that Pike County accounts only for 0.3% of zone 2 population, the effect of using lagged numbers for Pike County is minimal.

The ratio of the land mass of zone 1 (Manhattan) to the land mass of zone 2 (the other 24 counties of the NY MSA) is 0.0028. However, that ratio is not the appropriate measure of the
Table 4: Counties in the New York MSA

<table>
<thead>
<tr>
<th>County</th>
<th>State</th>
<th>Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York (Manhattan)</td>
<td>NY</td>
<td>Zone 1</td>
</tr>
<tr>
<td>Bergen</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Bronx (Bronx)</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Dutchess</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Essex</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Hudson</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Hunterdon</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Kings (Brooklyn)</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Middlesex</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Monmouth</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Morris</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Nassau</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Ocean</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Orange</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Passaic</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Pike</td>
<td>PA</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Putnam</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Queens (Queens)</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Richmond (Staten Island)</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Rockland</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Somerset</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Suffolk</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Sussex</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Union</td>
<td>NJ</td>
<td>Zone 2</td>
</tr>
<tr>
<td>Westchester</td>
<td>NY</td>
<td>Zone 2</td>
</tr>
</tbody>
</table>

relative maximum availability of housing in each of the zones since Manhattan zoning allows for taller buildings, smaller lot sizes, etc.

Data on the maximum buildable residential area are graciously computed and shared by Chamna Yoon from Baruch College. He combines the maximum allowed floor area ratio (FAR) to each parcel to construct the maximum residential area for each of the five counties (boroughs) that make up New York City. Manhattan has a maximum residential area of 1,812,692,477 square feet. This is our measure for $\hat{H}_1$. The other four boroughs of NYC combine for a maximum buildable residential area of 4,870,924,726 square feet. Using the land area of each of the boroughs (expressed in square feet), we can calculate the ratio of maximum buildable residential area (sqft) to the land area (sqft). For Manhattan, this number is 2.85. For the other four boroughs of NYC it is 0.62. For Staten Island, the most suburban of the boroughs, it is 0.32. We assume that the Staten Island ratio is representative of the 20 counties in the New York MSA that lie outside NYC since these are more suburban. Applying this ratio to their land area of 222,808,633,344 square feet, this delivers a maximum buildable residential square feet for those 20 counties of 71,305,449,967 square feet. Combining that with the four NYC counties in zone 2, we get a maximum buildable residential area for zone 2 of 76,176,377,693 square feet. This is $\hat{H}_2$. The ratio $\hat{H}_1 / \hat{H}_2$ is 0.0238. We argue that this ratio better reflects the relative scarcity of space in Manhattan than the corresponding land mass ratio.
B.3 Income

The main source for the income distribution data is again US Census Bureau American FactFinder. From table DP03 (selected economic characteristics), we retrieve the number of households in each of 10 income brackets, ranging from “less than $10,000” for the lowest to “$200,000 or more” for the highest bracket. The distribution suffers from top-coding problem, so we additionally estimate the conditional means for the households in each income bracket. For the eight income brackets except for the lowest and the highest, we simply assume the midpoint of the interval as the conditional mean. For example, for the households in $50,000 to $74,999 bracket, the conditional mean income is assumed to be $62,500. For the lowest bracket, (less than $10,000) we assume the conditional mean is $7,500. Then we can calculate the conditional mean of the highest income bracket, using the average household income and conditional means of the other brackets, since the reported unconditional mean is based on all data.

Our concept of income is household income before taxes. It includes income from (i) wages, salaries, commissions, bonuses, or tips, (ii) farm or non-farm business, proprietorship, or partnership, (iii) social security and railroad retirement payments, (iv) retirement, survivor, or disability pensions, (v) SSI, TANF, family assistance, safety net, other public assistance, or public welfare, (vi) interest, dividends, royalties, estates and trusts.

We aggregate the county-level income distribution into a zone 2 income distribution in two steps. First, the aggregate number of households included in each income bracket is the simple sum of county-level household numbers in the bracket. Second, we calculate the zone 2 conditional mean of the income brackets using the weighted average methods. For the lower nine income brackets, the conditional means are assumed to be constant across counties, so zone 2 conditional means are also the same. For the highest income bracket, we use the county-specific conditional mean of the highest bracket, and calculate its weighted average over the 24 counties. Using these conditional means, and the household distribution over 10 income brackets, the zone 2 average household income can be calculated.

B.4 House Prices, Rental Prices, and Home Ownership

Housing prices and rental prices data come from Zillow (http://www.zillow.com/research/data) indices. Zillow publishes Zillow Home Value Index (ZHVI) and Zillow Rent Index (ZRI) monthly. The main advantage of using Zillow indices compared to other indices is that it overcomes sales-composition bias by constantly estimating hypothetical market prices, controlling for hedonics such as house size. We use 2015 year-end data to be consistent with the ACS dataset. There are a few missing counties in ZHVI and ZRI. For the five counties with missing ZHVI index price, we search those counties from Zillow (http://www.zillow.com) website, and use the median listing prices instead. For the two counties with missing ZRI index price, we estimate the rents using the price/rent ratio of comparable counties.

Home ownership data is directly from American FactFinder. In table DP04 (selected housing characteristics), the Total housing units number is divided by Occupied housing units and Vacant housing units. Occupied housing units are further classified into Owner-occupied and Renter-occupied housing units, which enables us to calculate the home ownership ratio.

B.5 Rent Regulation

The main source for rent regulation data is US Census Bureau New York City Housing and Vacancy Survey (NYCHVS; http://www.census.gov/housing/nychvs). NYCHVS is conducted ev-
Every three years to comply with New York state and New York City’s rent regulation laws. We use the 2014 survey data table, which is the most recent survey data. In Series IA table 14, the number of housing units under various rent-control regulations are available for each of the five NYC boroughs. We define rent-regulated units as those units that are (i) rent controlled, (ii) public housing, (iii) Mitchell Lama housing, (iv) all other government-assisted or regulated housing. We exclude rent-stabilized units from our definition. Rent stabilized units are restricted in terms of their annual rent increases. The vast majority of units built after 1947 that are rent stabilized are so voluntarily. They receive tax abatement in lieu of subjecting their property to rent stabilization for a defined period of time. Both rent levels and income levels of tenants in rent-stabilized units are in between those of rent-regulated and unregulated units.

We calculate the proportion of rent-regulated units among all the renter-occupied units. The proportion is 16.9% for Manhattan and 13.2% for the other four NYC boroughs. We use a different data source for the other 20 counties outside of New York City. Affordable Housing Online (http://affordablehousingonline.com) provides various rent-related statistics at the county level. For each of the 20 counties outside NYC, we calculate the fraction of rent-regulated units by dividing Federally Assisted Units number by Renter Households number reported on each county’s webpage. We then multiply these %-numbers with the renter-occupied units in ACS data set to calculate the rent-regulated units for the 20 counties. Along with the NYCHVS numbers for the four NYC boroughs, we can aggregate all the 24 counties in zone 2 to calculate the fraction of rent-regulated units. The four NYC boroughs have 1.53 million renter-occupied housing units while the rest of zone 2 has 1.30 million. The resulting fraction of rent-regulated units in zone 2 is 10.4%.

From the NYCHVS, we also calculate the percentage difference in average rent in New York City between our definition of regulated rentals and the others (unregulated plus rent-stabilized). That percentage difference is 49.9%. We apply the same percentage difference to all of the MSA in our model.

Finally, we calculate the percentage difference in average household income (Series IA - Table 9) in New York City between our definition of regulated rentals and the others (unregulated plus rent-stabilized). That percentage difference is 54.2%. This is a moment we can compute in the model and compare to the data.

B.6 Migration

We use county-to-county migration data for 2006-2010 and 2010-2014 from the 5-year American Community Survey for the 25 counties in the New York metropolitan area. For each county and survey wave, we compute net migration rates (inflow minus outflow divided by population). When one person enters the New York labor market and another one leaves, the model is unchanged, so net migration is the relevant concept for the model. We aggregate net migration for the 24 counties in zone 2. The net migration rate over the 5-year period between 2010-2014 for the entire MSA is -0.15%, or -0.03% per year. First, this net migration rate is minuscule: only about 30,000 people moved in over a 5-year period on a MSA population of 20 million. Of course, this masks much larger gross flows: about 824,000 came into the MSA and 854,000 left. Second, Manhattan (zone 1) saw a net inflow of 30,000 people coming from outside the MSA while the rest of the MSA (zone 2) saw a net outflow to the rest of the country/world of 60,000. This is the opposite pattern than what we would expect if the out-of-town (OOT) purchases prompted migration of residents, since OOT purchases were much stronger in Manhattan than in the rest of the MSA (twice as large). Third, comparing the net migration in the 2010-2014 period to that in the 2006-
2010 period, we find that the net migration rate rose, from -73,000 to -30,000. The net migration rate rises from -0.38% in 2006-2010 to -0.15% in the 2010-2014 period. The rise in OOT purchases over time did not coincide with a decline in net migration, but with an increase. In other words, not only are the relevant net migration rates tiny, they also have the wrong-sign cross-sectional correlation with the spatial OOT pattern, and with the time-series of OOT purchases. We conclude that there is little evidence in the New York data of substantial net migration responses to OOT purchases.

C  Earnings Calibration

Before-tax earnings for household \(i\) of age \(a\) is given by:

\[
y_{i,a}^{lab} = W_i n_i^a G_a z_i
\]

where \(G^a\) is a function of age and \(z^i\) is the idiosyncratic component of productivity. Since endogenous labor supply decisions depend on all other parameters and state variables of the model, exactly matching earnings in model and data is a non-trivial task.

We determine \(G^a\) as follows. For each wave of the Survey of Consumer Finance (SCF, every 3 years form 1983-2010), we compute average earnings in each 4-year age bucket (above age 21), and divide it by the average income of all households (above age 21). This gives us an average relative income at each age. We then average this relative age-income across all 10 SCF waves.

We also use SCF data to determine how the dispersion of income changes with age. We choose four grid points for income, corresponding to fixed percentiles (0-25, 25-75, 75-90, 90-100). To compute the idiosyncratic income \(z_{a,j}\) of each group \(i \in \{1, 2, 3, 4\}\) at a particular age \(a\), relative to the average income of all households of that age we do the following:

Step 1: For each positive-earnings household, we compute which earnings group it belongs to among the households of the same age.

Step 2: For each 4-year age bucket, we compute average earnings of all earners in a group.

Step 3: We normalize each group’s income by the average income in each age group, to get each group’s relative income.

Step 4: Steps 1-3 above are done separately for each wave of the SCF. We compute an equal-weighted average across all 10 waves to get an average relative income for each age and income group. This gives us four 11x1 vectors \(z_{a,j}\) since there are 11 4-year age groups between ages 21 (entry into job market) and 65 (retirement). Note that the average \(z\) across all households of a particular age group is always one: \(E[z_{a,j}|a] = 1\).

Step 5: We regress each vector, on a linear trend to get a linearly fitted value for each group’s relative income at each age. The reason we perform Step 5, rather stopping at Step 4 is that the relative income at age 4 exhibits some small non-monotonicities that are likely caused by statistical noise (sampling and measurement error). Step 5 smoothes this out.

We set the productivity states to \(z^i \in Z = [0.255, 0.753, 1.453, 3.522]\) to match the observed mean NY household income levels, scaled by the NY metro area average, in the income groups below $41,000, between $41,000 and $82,000, between $82,000 and $164,000, and above $164,000. Those bins respectively correspond to bins for individual earnings below $25,000, between $25,000
and $50,000, between $50,000 and $100,000, and above $100,000, adjusted for the number of working adults in the average New York household (1.64). The NY income data is top-coded. For each county in the NY metro area, we observe the number of individuals whose earnings exceed $100,000. Because we also observe average earnings (without top-coding), we can infer the average income of those in the top coded group.

The transition probability matrix for \( z \) is \( P \) for \( \beta^L \) agents. We impose the following restrictions:

\[
P = \begin{bmatrix}
p_{11} & 1 - p_{11} & 0 & 0 \\
(1 - p_{22})/2 & p_{22} & (1 - p_{22})/2 & 0 \\
0 & (1 - p_{33})/2 & p_{33} & (1 - p_{33})/2 \\
0 & 0 & 1 - p_{44} & p_{44}
\end{bmatrix}
\]

For \( \beta^H \) types, the transition probability matrix is the same, except for the last two entries which are \( 1 - p_{44} - p^H \) and \( p_{44} + p^H \), where \( p^H < 1 - p_{44} \). We pin down the five parameters

\[
(p_{11}, p_{22}, p_{33}, p_{44}, p^H) = (0.93, 0.92, 0.92, 0.82, 0.02)
\]

to match the following five moments. We match the population shares in each of the four income groups defined above: 16.1%, 29.8%, 34.2%, and 19.9%, respectively (taken from the individual earnings data). Given that population shares sum to one, that delivers three moments. We match the persistence of individual labor income to a value of 0.9, based on evidence form the PSID in Storesletten, Telmer, and Yaron (2006). Finally, we choose \( p^H \) to match the fraction of high-wealth households in the top 10% of the income distribution.

Table 5 summarizes the results we obtain in the model. Average earnings are reported annually. Earnings autocorrelation and volatility are reported for 4 years.

<table>
<thead>
<tr>
<th>Group</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg earnings (Data)</td>
<td>25109</td>
<td>65603</td>
<td>123356</td>
<td>343908</td>
<td></td>
</tr>
<tr>
<td>Pop shares (Data)</td>
<td>20.5%</td>
<td>29.2%</td>
<td>33.3%</td>
<td>17.1%</td>
<td></td>
</tr>
<tr>
<td>Earnings autocorr.</td>
<td>0.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earnings vol.</td>
<td>0.125</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr. (income,wealth)</td>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Averages by bins in the data are obtained by multiplying average labor earnings ($124,091 annually) by the ratio of average earnings in each bin to the overall average (0.23, 0.50, 0.96, 2.42). Average household earnings and population shares in the data are denoted in parentheses and obtained from Census Bureau data for the year 2016.
D Progressive Taxation

Households with income $y^{tot} < y_0^{tot} = \lambda T$ receive transfers $T(y^{tot}) < 0$, and those with $y^{tot} \geq y_0^{tot}$ pay taxes $T(y^{tot}) \geq 0$. As a result of our calibration, 35% of households are subsidized by the progressive tax system, and 28% receive a subsidy after subtracting Social Security taxes. Figure 5 describes the progressive taxation system. At low total income values, some households receive a subsidy, which progressively decreases. At higher incomes, taxes increase faster than income. This is reflected in households’ after-tax income, shown in Figure 6.

Figure 5: Progressive Taxes

Notes: Horizontal axis: total income (in dollars, annual), measured as the sum of labor earnings, pensions, and financial income. Vertical axis: taxes minus transfers excluding Social Security taxes (in dollars, annual; left panel), total taxes minus transfers including Social Security taxes (in dollars, annual; left panel). The dashed line plots the zero-tax case.

Figure 6: After-tax Total Income

Notes: Horizontal axis: total income (in dollars, annual). Vertical axis: post-tax income excluding Social Security taxes (in dollars, annual; left panel), post-tax income including Social Security taxes (in dollars, annual; left panel). The dashed line is the 45 degree line.
E Housing Supply Elasticity Calibration

We compute the long-run housing supply elasticity. It measures what happens to the housing quantity and housing investment in response to a 1% permanent increase in house prices. Define housing investment for a given zone, dropping the location superscript since the treatment is parallel for both zones, as:

\[ Y^h_t = \left(1 - \frac{H_{t-1}}{H}ight) N^p_h. \]

Note that \( H_{t+1} = (1 - \delta)H_t + Y^h_t \), so that in steady state, \( Y^h = \delta H \). Rewriting the steady state housing investment equation in terms of equilibrium quantities using (8) delivers:

\[ H = \frac{1}{\delta} \left(1 - \frac{H}{H} \right) \frac{\rho^p_h}{\rho^p_h} \frac{\rho^p_h}{\rho^p_h} \frac{\rho^p_h}{\rho^p_h} W \frac{\rho^p_h}{\rho^p_h} \frac{\rho^p_h}{\rho^p_h} \]

Rewrite in logs, using lowercase letters to denote logs:

\[ h = -\log(\delta) + \frac{1}{1 - \rho_h} \log(1 - \exp(h - \tilde{h})) + \frac{\rho_h}{1 - \rho_h} \tilde{p} - \frac{\rho_h}{1 - \rho_h} w \]

Rearrange and substitute for \( \tilde{p} \) in terms of the market price \( \tilde{p} = \log(ho + (1 - ho)\kappa_4) + p \):

\[ p = \frac{1 - \rho_h}{\rho_h} h - \frac{1}{\rho_h} \log(1 - \exp(h - \tilde{h})) + k \]

where

\[ k \equiv \frac{1 - \rho_h}{\rho_h} \log(\delta) + w - \log(ho + (1 - ho)\kappa_4) \]

Now take the partial derivative of \( p \) w.r.t. \( h \):

\[ \frac{\partial p}{\partial h} = \frac{1 - \rho_h}{\rho_h} + \frac{1}{\rho_h 1 - \exp(h - \tilde{h})} + \frac{\partial k}{\partial h} \]

Invert this expression delivers the housing supply elasticity:

\[ \frac{\partial h}{\partial p} = \frac{\rho_h}{1 - \rho_h + \frac{\exp(h - \tilde{h})}{1 - \exp(h - \tilde{h})} + \rho_h \frac{\partial \delta h}{\partial h} - \rho_h \frac{(1 - \kappa_4) \partial ho}{ho(1 - ho) \kappa_4 \partial h}} \]

If (i) the elasticity of wages to housing supply is small (\( \frac{\partial \delta h}{\partial h} \approx 0 \)) and either the rent control distortions are small (\( \kappa_4 \approx 1 \)) or the home ownership rate is inelastic to the housing supply (\( \frac{\partial ho}{\partial h} \approx 0 \)), or (ii) if the two terms in square brackets are positive but approximately cancel each other out, then the last two terms are small. In that case, the housing supply elasticity simplifies to:

\[ \frac{\partial h}{\partial p} \approx \frac{\rho_h}{1 - \rho_h + \frac{\exp(h - \tilde{h})}{1 - \exp(h - \tilde{h})}} \]

Since, in equilibrium, \( Y^h = \delta H \), \( \partial y^h / \partial p = \partial h / \partial p \).

Note that \( h - \tilde{h} \) measures how far the housing stock is from the constraint, in percentage terms.
As $H$ approaches $\bar{H}$, the term $\frac{\exp(h-H)}{1-\exp(h-H)}$ approaches $+\infty$ and the elasticity approaches zero. This is approximately the case in zone 1 for our calibration. If $H$ is far below $\bar{H}$, that term is close to zero and the housing supply elasticity is close to $\frac{\rho_h}{1-\rho_h}$. That is approximately the case for zone 2 in our calibration. Since zone 2 is by far the largest component of the New York metro housing stock, zone 2 dominates the overall housing supply elasticity we calibrate to.

In the calibration, we use equation (54) to measure the housing supply elasticity and set $\frac{\partial w}{\partial h} = 0.25$ based on evidence from Favilukis and Van Nieuwerburgh (2018), who study a model with aggregate shocks to housing demand driven by out-of-town home buyers. We also set $\kappa_4 = 1$.

F Mobility Rates

Figure 7: Moving Rates

Notes: Mobility rates by age are measured as the annual probability to move across zones.
G Modeling Tax Credits

In our model, developers in a given zone earn a price per sqft built equal to the market price times a discount; recall equation (6) repeated here for convenience:

$$\bar{p}_t^\ell = \left( h\alpha_t^\ell + (1 - h\alpha_t^\ell)\kappa^\ell_4 \right) P_t^\ell.$$

The discount depend on the fraction of units that are owned ($h\alpha_t^\ell$) and the rent discount due to RC housing $\kappa^\ell_4$. We now assume that developers receive a subsidy to offset the rent discount due to RC:

$$\kappa^\ell_4 = 1 - \eta^\ell + \eta^\ell \kappa_1 (1 + LIHTC).$$

In the benchmark model, the parameter $LIHTC = 0$. In the LIHTC experiment, we choose $LIHTC$ such that the total value of LIHTC subsidies, aggregated first across all firms within each zone and then across zones, is equal to 30% of the construction costs associated with the construction of RC sqft of housing (also aggregating across firms and zones). We compute the construction costs of RC housing as follows. Since the only input is labor, we take the total wage bill in each zone (aggregating across firms) and multiply it by the share of RC sqft to compute the construction costs associated with RC housing in that zone; then we sum across zones and multiply by 30%. That gives us the total value of LIHTC subsidies. Matching the construction costs of RC housing requires a value for the parameter $LIHTC$ of 0.06 or 6%. We assume it is identical across zones.

When accounting for ownership rates and shares of RC sqft in each zone, the tax credits increases the average price $\bar{p}_t^\ell$ earned by developers in zone 1 by 3.2%, and in zone 2 by 2.4%. As described in the main text, we then change the value of $\lambda$ in the tax-and-transfer function to generate enough additional tax revenue to exactly pay for the aggregate tax credit outlays.
Additional Affordability Policies

In this appendix, we study five additional housing affordability experiments. All of them make the RC program less generous. We (i) reduce the size of the system (governed by $\eta^\ell$), (ii) reduce the rent subsidy (governed by $\kappa_1$), (iii) increase the income qualification threshold (governed by $\kappa_2$), (iv) lower the size of the RC units (governed by $\kappa_3$), and (v) force households to re-apply to the affordable housing lottery and meet the income qualification test every period (governed by $p^{RC,exog}$). Table 6 and Figure 8 summarize the results.

H.1 Reducing the Amount of RC Housing

This policy is the opposite policy of the one discussed in section 5.1. We reduce $\eta^\ell$ in each zone by 50%. This allows us to study the symmetry of the welfare results. Given the steep curvature of the value function at low wealth levels, poor households may be much more adversely affected by a discrete reduction in the RC system than they are helped by a same-sized expansion. Indeed, the 50% reduction leads to a larger welfare loss of -0.27% than the 0.15% gain from the 50% expansion.

The 50% reduction in the square footage of RC leads to a -76.07% drop in the fraction of households in RC, due to a decrease in the average size of a RC unit. The fraction of low-income households in RC falls by nearly as much. This leads to a large number of low-income losers from the policy, explaining the substantial aggregate welfare loss. Both access to (-71.90%) and stability of (-4.12%) RC insurance deteriorate. A slight improvement in output (0.03%), a slightly better spatial allocation of labor (0.12% more top-productivity households live in zone 1), and a reduction in commuting times (-0.52%) cannot make up for the concentrated losses on the poor.

H.2 Reducing the Rent Subsidy for RC Housing

In the second additional experiment, labeled “RC discount,” we reduce the rental discount that RC households enjoy. Specifically, we lower the rent discount parameter $\kappa_1$ from 50% to 25%. The total square footage that goes to RC housing in each zone ($\eta^\ell$) remains at the benchmark values. Surprisingly, the reduced generosity of the RC system leads to a 24.10% increase in the fraction of households in RC, due to a decrease in the average size of a RC unit. The fraction of low-income households in RC falls by nearly as much. This leads to a large number of low-income losers from the policy, explaining the substantial aggregate welfare loss. Both access to (-71.90%) and stability of (-4.12%) RC insurance deteriorate. A slight improvement in output (0.03%), a slightly better spatial allocation of labor (0.12% more top-productivity households live in zone 1), and a reduction in commuting times (-0.52%) cannot make up for the concentrated losses on the poor.

This policy reduces welfare by -0.21%, similar to the policy that reduces the amount of rent control by half in the first column. The winners and losers from this policy are also similar to the previous experiment, as can be seen in Figure 8. Many other features such as the increasing population share of Manhattan, the changes in the housing stock and in rents, and the change in commuting time are similar. Housing affordability metrics improve similarly, underscoring that maximizing welfare and improving housing affordability metrics can be conflicting objectives.

29Such a policy advocated by several policy institutes. For instance the NYU Furman Center (2018). New York City historically discouraged the development of small apartment units due to a variety of rules and regulations.
The aggregate welfare loss occurs despite the reduced frictions on housing construction. The aggregate housing stock ends up falling, illustrating the pitfalls of partial equilibrium logic. Output falls modestly as hours worked fall. In sum, the main source of the welfare loss is the reduced value of the insurance provided by the RC system which disproportionately hurts low-income households. While access to insurance improves, the extent of insurance provided deteriorates.

Table 6: Main moments of the model under affordability policies that modify features of the RC system and the spatial allocation of housing.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>RC share</th>
<th>RC discount</th>
<th>Inc. cutoff</th>
<th>RC housing size</th>
<th>Re-Qualify</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Avg (Rent/income) among renters, Z1 (%)</td>
<td>26.6</td>
<td>-6.39%</td>
<td>-11.13%</td>
<td>5.03%</td>
<td>-0.72%</td>
<td>3.91%</td>
</tr>
<tr>
<td>2 Avg (Rent/income) among renters, Z2 (%)</td>
<td>29.1</td>
<td>0.40%</td>
<td>-3.87%</td>
<td>-0.82%</td>
<td>-0.78%</td>
<td>1.10%</td>
</tr>
<tr>
<td>3 Frac. RC (%)</td>
<td>5.25</td>
<td>-76.07%</td>
<td>24.10%</td>
<td>15.37%</td>
<td>10.51%</td>
<td>1.10%</td>
</tr>
<tr>
<td>4 Frac. RC of those in income Q1 (%)</td>
<td>9.70</td>
<td>-73.81%</td>
<td>13.18%</td>
<td>41.86%</td>
<td>10.52%</td>
<td>-11.25%</td>
</tr>
<tr>
<td>5 Frac. rent-burdened (%)</td>
<td>3.9</td>
<td>-39.49%</td>
<td>-26.26%</td>
<td>2.29%</td>
<td>4.29%</td>
<td>17.33%</td>
</tr>
<tr>
<td>6 Avg. size RC unit in Z1 (sqft)</td>
<td>716</td>
<td>-1.39%</td>
<td>-15.20%</td>
<td>-6.65%</td>
<td>-15.40%</td>
<td>-5.59%</td>
</tr>
<tr>
<td>7 Avg. size RC unit in Z2 (sqft)</td>
<td>1419</td>
<td>-0.29%</td>
<td>-28.76%</td>
<td>-19.24%</td>
<td>-9.60%</td>
<td>-4.33%</td>
</tr>
<tr>
<td>8 Avg. size Z1 mkt unit (sqft)</td>
<td>985</td>
<td>-4.32%</td>
<td>-2.31%</td>
<td>-0.21%</td>
<td>0.13%</td>
<td>-1.97%</td>
</tr>
<tr>
<td>9 Avg. size Z2 mkt unit (sqft)</td>
<td>1507</td>
<td>0.04%</td>
<td>1.84%</td>
<td>1.09%</td>
<td>0.50%</td>
<td>0.06%</td>
</tr>
<tr>
<td>10 Frac. pop. Z1 (%)</td>
<td>10.6</td>
<td>2.61%</td>
<td>4.08%</td>
<td>0.83%</td>
<td>1.93%</td>
<td>2.86%</td>
</tr>
<tr>
<td>11 Frac. retirees Z1 (%)</td>
<td>21.2</td>
<td>-10.75%</td>
<td>-9.63%</td>
<td>-5.11%</td>
<td>0.51%</td>
<td>0.65%</td>
</tr>
<tr>
<td>12 Housing stock Z1</td>
<td>-</td>
<td>0.30%</td>
<td>0.27%</td>
<td>-0.15%</td>
<td>0.11%</td>
<td>0.21%</td>
</tr>
<tr>
<td>13 Housing stock Z2</td>
<td>-</td>
<td>-0.14%</td>
<td>-0.20%</td>
<td>0.01%</td>
<td>-0.19%</td>
<td>-0.47%</td>
</tr>
<tr>
<td>14 Rent/sqft Z1 ($)</td>
<td>4.16</td>
<td>-0.86%</td>
<td>-0.93%</td>
<td>-0.04%</td>
<td>-0.07%</td>
<td>-0.16%</td>
</tr>
<tr>
<td>15 Rent/sqft Z2 ($)</td>
<td>1.49</td>
<td>-1.07%</td>
<td>-1.19%</td>
<td>-0.05%</td>
<td>-0.08%</td>
<td>-0.22%</td>
</tr>
<tr>
<td>16 Price/sqft Z1 ($)</td>
<td>1064</td>
<td>-1.03%</td>
<td>-1.01%</td>
<td>-0.07%</td>
<td>-0.19%</td>
<td>-0.32%</td>
</tr>
<tr>
<td>17 Price/sqft Z2 ($)</td>
<td>297</td>
<td>-1.25%</td>
<td>-1.25%</td>
<td>-0.05%</td>
<td>-0.18%</td>
<td>-0.36%</td>
</tr>
<tr>
<td>18 Home ownership Z1 (%)</td>
<td>45.9</td>
<td>10.31%</td>
<td>4.64%</td>
<td>-0.50%</td>
<td>-0.68%</td>
<td>-3.42%</td>
</tr>
<tr>
<td>19 Home ownership Z2 (%)</td>
<td>58.4</td>
<td>1.38%</td>
<td>-1.84%</td>
<td>0.62%</td>
<td>-0.24%</td>
<td>0.13%</td>
</tr>
<tr>
<td>20 Avg. income Z1 ($)</td>
<td>164491</td>
<td>-0.06%</td>
<td>-2.43%</td>
<td>-1.03%</td>
<td>-1.75%</td>
<td>-2.65%</td>
</tr>
<tr>
<td>21 Avg. income Z2 ($)</td>
<td>100135</td>
<td>-0.11%</td>
<td>0.11%</td>
<td>0.14%</td>
<td>0.10%</td>
<td>0.01%</td>
</tr>
<tr>
<td>22 Frac. high prod. Z1 (%)</td>
<td>27.7</td>
<td>0.12%</td>
<td>-1.21%</td>
<td>0.81%</td>
<td>-0.37%</td>
<td>-1.15%</td>
</tr>
<tr>
<td>23 Total hours worked</td>
<td>-</td>
<td>0.16%</td>
<td>-0.12%</td>
<td>-0.14%</td>
<td>-0.13%</td>
<td>-0.54%</td>
</tr>
<tr>
<td>24 Total hours worked (efficiency)</td>
<td>-</td>
<td>0.07%</td>
<td>-0.05%</td>
<td>0.00%</td>
<td>-0.12%</td>
<td>-0.37%</td>
</tr>
<tr>
<td>25 Total output</td>
<td>-</td>
<td>0.03%</td>
<td>-0.05%</td>
<td>0.00%</td>
<td>-0.08%</td>
<td>-0.25%</td>
</tr>
<tr>
<td>26 Total commuting time</td>
<td>-</td>
<td>-0.52%</td>
<td>-0.68%</td>
<td>-0.19%</td>
<td>-0.22%</td>
<td>-0.33%</td>
</tr>
<tr>
<td>27 Prob. of first-time access to RC if negative shock (%)</td>
<td>3.9</td>
<td>-71.90%</td>
<td>25.23%</td>
<td>44.08%</td>
<td>11.92%</td>
<td>85.90%</td>
</tr>
<tr>
<td>28 Prob. of staying in RC when income Q1 (%)</td>
<td>69.1</td>
<td>-4.12%</td>
<td>-0.98%</td>
<td>2.73%</td>
<td>-0.09%</td>
<td>-204.80%</td>
</tr>
<tr>
<td>29 Std. MU growth, nondurables</td>
<td>0.45</td>
<td>-0.28%</td>
<td>-0.03%</td>
<td>0.20%</td>
<td>0.02%</td>
<td>-0.67%</td>
</tr>
<tr>
<td>30 Std. MU growth, housing</td>
<td>0.46</td>
<td>2.02%</td>
<td>-0.52%</td>
<td>-0.58%</td>
<td>0.00%</td>
<td>5.56%</td>
</tr>
<tr>
<td>31 Aggregate welfare change</td>
<td>-</td>
<td>-0.27%</td>
<td>-0.21%</td>
<td>0.69%</td>
<td>0.16%</td>
<td>0.04%</td>
</tr>
</tbody>
</table>

Notes: Columns “Benchmark” reports values of the moments for the benchmark model. Columns “RC discount” to “RC housing size” report percentage changes of the moments in the policy experiments relative to the benchmark. Rows 1-8 report housing affordability moments, rows 9-24 aggregate moments across the two zones, and rows 25-26 welfare moments. Z1 stands for zone 1 (Manhattan), Z2 for the rest of the metro area. Row 20 reports what fraction of working age top-productivity households live in zone 1.

H.3 Lowering the Income Qualification Threshold for RC Housing

In the third additional experiment, “Inc. cutoff,” we tighten the income requirements to qualify for rent control, governed by the parameter $\kappa_2$. Households must make less than 40% of AMI to qualify, compared to 60% in the benchmark economy. AMI in the benchmark is $82,731. Having only very low-income households qualify, rather than all low-income households reduces the generosity of the RC system.
This policy results in a substantial welfare gain of 0.69%. The main source of this welfare gain is that the RC system becomes much better targeted. The fraction of households that receives RC grows (15.37%) but the fraction of low-income (first quartile) households who receive RC grows much more strongly (41.86%). Because households receiving RC must now have a lower income to qualify, they live in smaller units (-6.65% in zone 1, -19.24% in zone 2). Access to RC insurance for outsiders with a negative income shock improves substantially (44.08%), and the stability of the insurance also goes up (2.73%). Figure 8 shows that the welfare gains are equally distributed across ages but heavily concentrated on the lowest-productivity, lowest-income, and lowest-wealth groups. Ironically, by making the RC system less generous in some sense, it provides much better insurance to the poor. The policy does not affect the population share much, nor does it affect output or the housing stock.

Figure 8: Welfare effects of tightening RC

H.4 Reducing the Maximum Size of RC Housing

The fourth additional experiment changes how “deeply affordable” RC units must be, governed by the parameter $\kappa_3$. In the benchmark, affordable housing units have a rent expenditure cap of
35% of AMI on rent, or about $2400 per month. Here we change this cap to 20% of AMI or about $1400 per month. This effectively tightens the constraint on the maximum size of a RC unit.

As shown in the column “RC housing size,” the policy change indeed lowers the average RC unit size by -15.40% in zone 1 and -9.60% in zone 2. Because the total amount of square footage is unaffected by the policy change, the fraction of households in RC increases by nearly the same amount (10.51%). Unlike tighter income requirements, tighter RC size requirements are not particularly effective at improving the targeting of the RC system. The fraction of low-income households in RC grows at the same rate (10.52%) as the aggregate. This suggests that the size constraint has only limited bite, at least given the income limit already in place. As a result of this less effective redistribution and insurance, the policy has smaller welfare gains than the previous one. The overall welfare change is modestly positive (0.16%). Figure 8 confirms that the welfare effects in the cross-section are directionally similar but much smaller in magnitude than in the previous experiment.

H.5 Re-Qualifying for RC Each Period

The final experiment reduces the persistence in the RC system. Rather than give existing tenants a high, exogenously given probability of being able to stay in the RC system without having to satisfy the income qualification, it forces them to re-apply every period (four years). There are no more insiders or outsiders, everyone has equal probability of winning the housing lottery. By removing the preference for insiders, i.e., by setting the parameter \( p^{RC,exog} = 0 \), the (endogenously determined) probability of winning the PC lottery for outsiders increases substantially. Outsiders with falling income enjoy a large gain in access to RC (85.90%). By the same token, the likelihood that existing RC tenants can stay in RC falls substantially. For low income RC tenants, the drop is large (-204.80%). The policy change causes more volatility in the marginal utility growth of housing because it triggers more instability in the housing choices of households (+5.56%). The last column of Table 6, labeled “Re-Qualify,” shows that the policy change results in a small aggregate welfare gain of 0.04%.

This policy, like the previous two, is aimed at improving the targeting of RC to the most needy. Figure 9 shows that the policy is successful at getting more RC housing to low-income households, relative to the benchmark model that is plotted Figure 3. Figure 9 also shows that retirees are now more likely to get RC because even “middle-income” retirees have low incomes relative to the population at large to qualify. The statistic on the fraction of bottom-quartile households that are in RC (-11.25%) fails to capture this improved targeting. The reason is that many retirees are in the second income quartile. When considering the bottom half of the income distribution, the targeting indeed improves substantially. The income panel of Figure 8 confirms that second-income quartile households gain the most from the policy change. That said, the targeting of RC to low-income households improve much more for the experiment where we lower the income threshold (Section H.3). While the young stand to gain as much as in the income threshold experiment, there are welfare losses rather than welfare gains at all other ages.

This policy leads to a fairly substantial output loss (-0.25%) driven by a reduction in hours and efficiency units of labor (-0.37%). Closer inspection reveal that productivity type-2 reduce hours the most (-1.30%) versus the lowest productivity types (-0.10%) or the top two productivity types 3 and 4 (-0.30% each). The reduction in hours for type-2 households is concentrated in zone 1. Type two households have a strong financial incentive to live in zone 1, given that the fixed cost of commuting represents a large share of their modest income. Put differently, if they live in zone 2 and commute, they work harder to make up for the cost of commuting. If they live in zone 1, they
reduce labor supply. This reduction is larger in the re-qualify than in the benchmark economy. The reason is that type-2 household snow have a better chance of getting into RC housing in zone 1 and rents are lower even if they do not get in. In addition, we have more type-2 households living in zone 1 in the re-qualify experiment. In sum, the improved insurance value of RC combined with the spatial reallocation it causes, leads to a reduction in labor supply and output.

Figure 9: Distribution of rent-controlled agents by age, and income and net worth quartiles.
I Graphing the Household Distribution Across Space, Productivity, and Tenure Status

Below, we present various graphs that graph the household distribution, always comparing the benchmark model (left panel) to an alternative model that undergoes one of the main policy experiments discussed in the main text (right panel). The vertical axes measures the total square footage devoted to the various types of housing in each zone (model units for readability, equal to the measure in sqft divided by 1,976): owner-occupied, rental, rent controlled housing in zone 1 and in zone 2. Values reported on the top of the bars correspond to the percentage of households in each category. These percentages add up to 100% across the six housing categories. Colors correspond to productivity levels: increasing from yellow (low, $z = 1$) to red (high, $z = 4$) for working-age households, and green for retirees.

Figure 10: Benchmark (left panel) versus increasing the fraction of rent controlled housing by 50% (right panel).
Figure 11: Benchmark (left panel) versus spatial allocation of RC housing (right panel).

Figure 12: Benchmark (left panel) versus relaxing zoning laws in zone (right panel).
Figure 13: Benchmark (left panel) versus more housing vouchers (right panel).

![Benchmark Vouchers](image1)

Figure 14: Benchmark (left panel) versus LIHTC (right panel).

![Benchmark LIHTC](image2)
Robustness

J.1 Lower Risk Aversion

In the benchmark calibration, risk aversion $\gamma = 5$. We study an alternative economy where $\gamma = 2$ and compare it to the benchmark economy. We recalibrate the average level of the time discount factor $\beta$ to match average net worth/income to the level in the data and in the benchmark model. All other parameters are kept at their benchmark values. Wealth inequality in this model is lower than in the benchmark; the wealth Gini is 0.67 versus 0.79 in the benchmark and in the data. There are too few households in RC units in zone 2 (3.5% vs 4.7% in data). Otherwise, this calibration matches the targets about equally well as the benchmark calibration.

Lowering risk aversion in heterogeneous agent models with incomplete markets in which the overall amount of risk is not too high results in an increase in savings. The stronger intertemporal substitution motive dominates the weakened precautionary savings motive. These effects are stronger for lower- and middle-income households. We find a strong rise in the metro-wide home ownership rate of 12% points relative to the benchmark, driven by lower- and middle-income households who save enough to make the downpayment earlier. The increase in home ownership takes place entirely in zone 2; as a result, the ownership rate wedge between zones grows to 0.61. The home ownership rate decreases more slowly with age for the low- and middle-income households. A lower value for $\gamma$ lowers the value of insurance provided by affordable housing programs. The acceptance rate of RC units, conditional on winning the lottery falls by about 15% points. Relative to the benchmark, there are fewer households in RC (-14.87%), but there are more low-income households in RC (10.38%). The higher savings rates of the middle-income households reduce their willingness to live in RC units, resulting in a better targeted RC system. RC also skews more towards the young than in the benchmark economy. The volatilities of marginal utility growth of non-housing (-62.61%) and housing consumption (-31.32%) fall substantially, as a result of the reduced curvature of the utility function.

J.2 No Luxury Taste Shifter for Manhattan

We solve a special case of the model where $\chi^R = \chi^W = 1$ so that there is no extra taste shifter for zone 1 for wealthy households ($c_i > c_i^W$). We recall that in the benchmark calibration, the luxury taste shifter for retirees $\chi^R = 1.038$ was substantially above 1 while that for workers $\chi^W = 1.004$ was not. We keep the overall taste shifter for zone 1: $\chi^1 > 1$ in place. All other parameters are the same as in the benchmark economy.

The main change is that there are now very few retirees (10%) who choose to live in zone 1. Because they do not work, they do not value the time and financial savings of living close to work. Also, they face lower time and financial commuting costs. A larger overall fraction of households live in zone 1 in equilibrium (14.3% compared to 10.5% in the benchmark), and relative dwelling size falls from a ratio of 0.66 to 0.44. Manhattan contains more young households and also more top-productivity households. Average income in Manhattan falls. Income inequality between the two zones falls substantially: from a ratio of 1.66 to a ratio of 1.37. Price per unit and rent per unit fall with the reduction in unit size, but the price and rent per square foot as well as their ratio across zones are not affected. We also observe lower mobility rates after initial location choices have been made.
J.3 Profit Redistribution

In the benchmark model, all profits from tradeable and construction firms left the city. In an alternative calibration, we assume that 50% of profits are redistributed to local households. Importantly, we assume that this redistribution occurs uniformly to all households. The results may differ if the redistribution is done, say, proportionately to household productivity level. All other parameters are kept at the benchmark level.

We find that zone 1 is smaller than in the benchmark (7% of population vs. 10.5%) and contains many more retirees than in the benchmark (the ratio of retirees in zone 1 relative zone 2 is 3 vs. 1). This increases wealth inequality between zones. Income between inequality is also greater. Essentially, there are fewer but higher-income households in Manhattan, living in larger apartments. Zone 1 is less affordable as measured by price-income and rent-income ratios, and there are substantially more rent-burdened households. More households live in RC units that are endogenously smaller. The volatility of marginal utility growth decreases because the extra profits provide a buffer for households, making it easier to smooth consumption. For this reason, aggregate welfare is higher.