Interest Rate Uncertainty and Economic Fluctuations

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Monetary policy transmission

monetary policy → short/medium rate → long rate → macro economy

interest rate uncertainty
Question: interest rate uncertainty $\rightarrow$ macroeconomy
Literature

Uncertainty

- first moment
- second moment
  - SV in VAR: Cogley and Sargent (2001, 2005), and Primiceri (2005)
- This paper: first moment + second moment

Term structure models

  - does not fit yield volatility
  - restrict yield fitting
  - only 1 volatility factor
- This paper: fit both yields and volatility
Contribution: a new model for interest rate uncertainty

Contribution to the uncertainty literature:
- jointly model the first and second moments
  - first moment: conditional mean of macro variables
  - second moment: volatility of interest rates

Contribution to the term structure literature:
- introduce multiple volatility factors that fit the data
  - volatility factors and yield factors are distinct
Result highlight: two dimensions of uncertainty

We find

- 2 volatility factors capture the cross section of yield volatility
- We rotate to “short-term” uncertainty and “long-term” uncertainty
- Increases in either of them lead higher unemployment rates
- But they interact with inflation in opposite directions.
Outline

1. Model and estimation
2. Economic implication
3. Yield curve fitting
Factors

- $m_t : M \times 1$ Macro factors
- $g_t : G \times 1$ Gaussian yield factors
- $h_t : H \times 1$ yield volatility factors
Dynamics

\[
m_{t+1} = \mu_m + \Phi_m m_t + \Phi_m g_t + \Phi_m h_t + \sum_m \epsilon_{m,t+1}.
\]
\[
g_{t+1} = \mu_g + \Phi_g m_t + \Phi_g g_t + \Phi_g h_t + \sum_g \epsilon_{m,t+1} + \sum_g D_t \epsilon_{g,t+1},
\]
\[
h_{t+1} = \mu_h + \Phi_h h_t + \sum_h \epsilon_{m,t+1} + \sum_h D_t \epsilon_{g,t+1} + \sum_h \epsilon_{h,t+1}.
\]

where the diagonal time-varying volatility is a function of \( h_t \)

\[
D_t = \text{diag} \left( \exp \left( \frac{\Gamma_0 + \Gamma_1 h_t}{2} \right) \right).
\]

\( h_t \) enters the model through

- conditional mean: \( h_t \)
- conditional variance: \( D_t \)
**Bond prices**

**Short rate**

\[ r_t = \delta_0 + \delta_1 g_t. \]

**Pricing equation**

\[ P^n_t = E^Q_t [\exp (-r_t) P^{n-1}_{t+1}] \]

under risk neutral dynamics

\[ g_{t+1} = \mu_g^Q + \Phi_g^Q g_t + \sum_{g, t+1} \varepsilon_g^Q \]
Bond prices

Bond prices are exponentially affine

\[ P_t^n = \exp \left( \bar{a}_n + \bar{b}'_n g_t \right) \]

where

\[ \bar{a}_n = -\delta_0 + \bar{a}_{n-1} + \mu_g \bar{b}_{n-1} + \frac{1}{2} \bar{b}'_{n-1} \Sigma_g \Sigma_g' \bar{b}_{n-1}, \]

\[ \bar{b}_n = -\delta_1 + \Phi_g \bar{b}_{n-1}. \]

Yields \( y_t^n \equiv -\frac{1}{n} \log P_t^n \) are linear

\[ y_t^n = a_n + b'_n g_t \]

with \( a_n = -\frac{1}{n} \bar{a}_n, \ b_n = -\frac{1}{n} \bar{b}_n. \)

Novel approach

- bond prices identical to Gaussian ATSMs
Tension between fitting the yield curve and volatility

$$y^m_t = a_n + b'_n g_t + b'_{n,h} h_t$$

Spanned models ($b_{n,h} \neq 0$)
- dual role: volatility factors price bonds
- $h_t$ are forced to fit the conditional mean of yields.

Unspanned models/USV ($b_{n,h} = 0$)
- fit volatility better, but only allow one factor
- restrict yield fitting, see Creal and Wu (2015)

Our model ($b_{n,h} = 0$)
- no restriction on fitting yield curve
- multiple volatility factors
Bayesian estimation

**Model**
- non-Gaussian non-linear state space model
- likelihood not known in closed form

**MCMC**
- In each step, conditionally linear Gaussian state space model
- Kalman filter: draw parameters not conditioning on the state variables
- forward filtering and backward sampling: draw state variables jointly

*particle filter: compute likelihood*
Data and factors

Monthly from June 1953 to December 2013

Yields
- Fama-Bliss zero-coupon yields from CRSP
- maturities: 1m, 3m, 1y, 2y, 3y, 4y, 5y

Macro
- FRED

Factors
- $g_t$: 3m, 5y and 1y with errors
- $h_t$: volatility of 3m and 5y
- $m_t$: inflation and unemployment
Impulse responses

- Short unc -> short unc
- Short unc -> inflation
- Short unc -> unemployment
- Long unc -> long unc
- Long unc -> inflation
- Long unc -> unemployment
Time-varying impulse responses

- Short unc $\rightarrow$ Short unc
- Short unc $\rightarrow$ Inflation
- Short unc $\rightarrow$ Unemployment
- Long unc $\rightarrow$ Long unc
- Long unc $\rightarrow$ Inflation
- Long unc $\rightarrow$ Unemployment

Great Recession
Great Inflation
Volcker
Great Moderation
Greenspan

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Uncertainty and recession

\[ h_{jt} = \alpha + \beta \mathbb{1}_{\text{recession},t} + u_{jt} \]

- Coeff: 2.3 for short term; 0.6 for long term
- \( p \)-values: 0 for both
Model specification

- $M = 0, 2$
- $G = 3$
- $H = 0, 1, 2, 3$
Yield volatilities: how many factors?

BIC chooses \( H = 2 \) as well.
Yield volatilities: adding macro variables

- Term structure of yield volatility: $H = 2$
- Term structure of yield volatility: macro model with $H = 2$
Cross section of yields

**Table:** measurement errors

<table>
<thead>
<tr>
<th>model unites</th>
<th>( H_0 ) %</th>
<th>( H_1 )</th>
<th>( H_2 )</th>
<th>( H_3 )</th>
<th>macro</th>
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<td>1m</td>
<td>0.2524</td>
<td>0.9917</td>
<td>1.0170</td>
<td>1.0539</td>
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<td>3m</td>
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</table>
Conclusion

We propose a new model

- study the effect of interest rate uncertainty on macro variables
- uncertainty enters both the first and second moments
- the model has multiple volatility factors
- volatility factors evolve separately from yield factors

We find

- 2 volatility factors capture the cross section of yield volatility
- increases in either of them lead higher unemployment rates
- but they interact with inflation in opposite directions.
Literature

Volatility in mean with different applications

- **SV**: *Jo* (2013)

Bayesian

- *Chib and Ergashev* (2009) and *Bauer* (2014)
Stochastic discount factor

Pricing equation I

\[ P^n_t = \mathbb{E}^Q_t \left[ \exp (-r_t) P^{n-1}_{t+1} \right] \]

Pricing equation II

\[ P^n_t = \mathbb{E}_t \left[ M_{t+1} P^{n-1}_{t+1} \right] . \]

Pricing kernel for any process of \( h_t \) under \( Q \).

\[ M_{t+1} = \frac{\exp (-r_t) p^Q (g_{t+1}|I_t; \theta) p^Q (h_{t+1}|I_t; \theta)}{p (g_{t+1}|I_t; \theta) p (h_{t+1}|I_t; \theta)} \]

If we assume the process for \( h_t \) is the same under \( P \) and \( Q \)

\[ M_{t+1} = \frac{\exp (-r_t) p^Q (g_{t+1}|I_t; \theta)}{p (g_{t+1}|I_t; \theta)} \]
Observed yields

Stack

\[ y^n_t = a_n + b'_n g_t \]

for different maturities \( n_1, n_2, \ldots, n_N \) to

\[ Y_t = A + B g_t + \eta_t \]

where \( A = (a_{n_1}, \ldots, a_{n_N})' \), \( B = (b'_{n_1}, \ldots, b'_{n_N})' \).
State space form I conditional on $h_{0:T}$

Transition equation

$$g_{t+1} = \mu_g + \Phi_{gm} m_t + \Phi_g g_t + \Phi_{gh} h_t + \sum g m \varepsilon_{m,t+1} + \sum g D_t \varepsilon_{g,t+1}$$

Observation equations

$$m_{t+1} = \mu_m + \Phi_m m_t + \Phi_{mg} g_t + \Phi_{mh} h_t + \sum m \varepsilon_{m,t+1}$$

$$h_{t+1} = \mu_h + \Phi_h h_t + \sum h m \varepsilon_{m,t+1} + \sum h g D_t \varepsilon_{g,t+1} + \sum h \varepsilon_{h,t+1}$$

$$Y_{t+1} = A + B g_{t+1} + \eta_{t+1}$$

- The volatilities $h_{0:T}$ are known
- Gaussian factors $g_{1:T}$ are latent
State space form II conditional on $g_{1:T}$

Transition equation

$$h_{t+1} = \mu_h + \Phi_h h_t + \sum_{hm} \varepsilon_{m,t+1} + \sum_{hg} D_t \varepsilon_{g,t+1} + \sum_{h} \varepsilon_{h,t+1}$$

Observation equations

$$m_{t+1} = \mu_m + \Phi_m m_t + \Phi_{mg} g_t + \Phi_{mh} h_t + \sum_{m} \varepsilon_{m,t+1}$$

$$\hat{g}_{t+1} = \Gamma_0 + \Gamma_1 h_t + \hat{\varepsilon}_{t+1}$$

where we define $\tilde{g}_{t+1} = D_t \varepsilon_{g,t+1}$, $\hat{g}_{t+1} = \log (\tilde{g}_{t+1} \odot \tilde{g}_{t+1})$.

- Gaussian factors $g_{1:T}$ are observed.
- The volatilities $h_{0:T}$ are latent.
- Approximate the error with mixture of normals using Omori, Chib, Shephard, and Nakajima(2007).
Sketch of MCMC algorithm

- Conditional on \( h_{0:T} \), use state space form I
  - Draw \( \theta_g \) using Kalman filter without depending on \( g_{1:T} \)
  - Draw \( g_{1:T} \) using forward filtering and backward sampling

- Conditional on \( g_{1:T} \), use state space form II
  - Draw \( \theta_h \) using Kalman filter without depending on \( h_{0:T} \)
  - Draw \( h_{0:T-1} \) using forward filtering and backward sampling

- Draw the remaining parameters
Particle filter

- Calculate the likelihood of the model: $p(Y_{1:T}; \theta)$
- Calculate filtered estimates
- We use the mixture Kalman filter, see Chen and Liu (2000)
Interpretation of Gaussian factors

We rotate the state vector as

\[
\begin{pmatrix}
 y^3_t \\
 y^{60}_t \\
 y^{12}_t \\
 \vdots
\end{pmatrix}
 =
\begin{pmatrix}
 0 \\
 0 \\
 0 \\
 \vdots
\end{pmatrix}
 +
\begin{pmatrix}
 1 & 0 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 1 \\
 \vdots & \vdots & \vdots
\end{pmatrix}
 g_t + \eta_t
\]

The provides an interpretation of the state variables \( g_t = (g_{1t}, g_{2t}, g_{3t})' \).

- \( g_{1t} = y^3_t \) is the short-term maturity - m.e.
- \( g_{2t} = y^{60}_t \) is the long-term maturity - m.e.
- \( g_{3t} = y^{12}_t \) is the mid-term maturity - m.e.
Magnitude of uncertainty