The Demographic Deficit

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Abstract

There has been a slowdown in growth in the world's strongest economies. One explanation for the slowdown is that it represents a permanent shift in potential output because the innovations available to us in the future will not lead to the productivity growth that we have experienced in the recent past. An alternative argument, the Secular Stagnation hypothesis, holds that higher propensities to save made the real interest rate required to equate savings and investment at full employment negative. In this paper we argue that changing demographics, in particular aging populations, combined with increased life expectancy can account for slower growth, falling interest rates and falling productivity. Using Japan and the U.S. as a case study we provide estimates of the growth deficit that arises from an aging cohort structure and increasing life expectancy. We also provide some quantitative evidence of the importance of demographics for the G7 set of countries.

JEL Classification Codes: F21, J21.

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1 Introduction

The recovery from the Great Recession in both the U.S. and Europe has been anemic in spite of aggressive monetary stimulus. Productivity, which typically improves after a recession, has also been slow to recover. The persistent pattern of slow growth has led to much speculation that something fundamental has changed in the economic environment that is causing growth rates to be slow. In this proposal we argue that an important persistent element in the growth slowdown is the change in demographic profile of the population.

There are two popular accounts of why current and expected future growth rates are low. One view, articulated by Gordon (2016), holds that aggregate supply may be impaired because the opportunities for technological change that exist in the future are not going to provide the dramatic increases in productivity that important innovations in the past have delivered. That is, the set of blueprints available to us now and in the future are not as transformative as those we have had in the past.[1] An alternative view, the secular stagnation hypothesis associated with former Treasury Secretary Larry Summers, is that future growth is likely to be constrained by insufficient investment demand. On this view our current slow growth is more than just a hangover of the financial crisis. It is a consequence of the fall in the real interest rate that prevails in equilibrium. If the real interest rate is well below zero, then monetary policy is going to have a hard time delivering a real rate that is consistent with long term growth as we have experienced it in the past.[2]

The common element in these accounts is that the slowdown will be persistent. We do have other examples of persistent stagnation in previously fast growing economies, the most notable being Japan. Japan has been stagnant since the early 1990's in spite of aggressive monetary and fiscal stimulus. The Japanese slowdown was initially attributed to the collapse of the asset price bubble in 1991 and subsequently to shortcomings in the response of monetary and fiscal policy to the Asian Financial Crisis. In the run up to the financial crisis of 2007-2009 and the Great Recession that followed, Europe and the U.S. also experienced asset inflation followed by a shock to the financial system that was global and severe. As in Japan, the growth rate after the financial crisis has been below that in previous recoveries from recession in both Europe and the U.S. and below the trend growth in the period leading up to the crisis.

The challenge is to identify factors that can account for the persistence of the slowdown. One low frequency factor is the demographic structure of an economy. It is widely recognized that demographic changes have important implications for economic growth. But the channels through which these changes work are less well understood. In this paper we make them precise in the context of a life cycle model with rich demographics.

[1] The more nuanced statement of this view argues that the period from 1970-1994 when total factor productivity grew at an average annual rate of 0.5% (compared to 1.89% from 1920-1970) is likely to be characteristic of the future largely because the potential for technological innovation is unlikely to offer the opportunities for the kind of big increases in productivity that we experienced in the past.

[2] Summers proposed solution to this problem is to compensate for the lower investment with large scale public investment.

There is increasing recognition that demographic changes are an important driver of many economic phenomena. In the years prior to the Financial Crisis there was a lot of concern about global current account imbalances and capital flows. A number of papers recongnized that changing demographics could account for a lot of the observed. decline in interest rates and cross border capital flows. Henriksen (2002) and later Backus, Cooley, and Henriksen (2014) showed the effect of demographic changes on capital flows and interest rates in the U.S. and Japan. Feroli (2003) explored the role of demographics for capital flows among the G7 nations. Krueger and Ludwig (2007) studied the consequences of demographic changes for rates of return on capital, wages, and wealth in the OECD Countries. More recently Gagnon, Johannson, and Lopez-Salido (2016), Carvalho, Ferrero, and Nechio (2016) and Ikeda and Saito (2014) have shown the impact of demographic changes on the real interest rate and investment in the U.S. and Japan as a consequence of the exogenous impact of these changes on the aggregate labor supply. All these papers have focused on how demographic change affects supply of and demand for capital.

Demographic change affects factor supply through changes in life expectancy and the age-cohort distribution of the population. Changes in life expectancy impact both individuals' life-cycle savings and their labor-supply decisions. Changes in cohort distributions affect how these decisions are aggregated. We analytically distinguish between these two features of demographic change.

Growth accounting shows that that growth differentials both across countries and over time are not only driven by TFP and capital accumulation, but labor supply on the extensive margin, labor supply on the intensive margin, and (obviously) population growth. One straightforward way in which demographics impact changes in aggregate economic activity is through their impact on aggregate factor supply. Data shows that households steadily decrease labor supply both on the intensive and extensive margin in the latter part of their working lives. This is in contrast to usual assumptions in overlapping-generations models, where household supply labor inelastically until retirement age. Changes in life expectancy and cohort distributions will therefore affect both labor market participation and average hours worked. Faced with increases in life expectancy individuals need to provide for more years in retirement during their working life. In addition, aging populations means more people will be in their highest savings years. This may lead to changes in aggregate capital supply. Lastly, demographic change affect the composition of the work force and its productivity. Changes in the average efficiency of the individuals working will manifest itself in changes in TFP.

Economists have struggled to reconcile labor supply elasticities estimated from microeconomic data and elasticities implied by macro-economic adjustments. But the key to reconciling these two is to distinguish all the margins of adjustment of labor supply.[3] . The assumption common in life-cycle models that labor is supplied inelastically from individuals enter the labor market in their early 20s until they exit the labor market at retirement

[3]Keane and Rogerson (2011) and (Prescott, Rogerson, and Wallenius, 2009) discuss the biases in the estimates of labor supply elastcity that result from ignoring the margins of adjustment

age is at odds with the evidence that labor supply on the extensive margin (labor-market participation) and on the intensive margin (hours worked conditional on being in the labor market) have a pronounced hump-shape over the life cycle (see eg. Bick, Fuchs-Schündeln, and Lagakos, 2016). [4]

Figures 10 and 11 show labor-market participation rates for Japan and the United States in 1990 and 2008. There has been a notable decline in participation of prime age workers in the U.S. during the last three decades. Much has been written about this including Aguiar and Hurst (2007) and Aguiar, Bils, Hurst, and Charles (2016) who have shown that less educated prime age workers are allocating more time to leisure. Changes in demographics affect how households make labor-supply decisions over the life cycle and how these decision are aggregated.

The general equilibrium effect of demographic changes is also important and is manifested through the impact on the capital stock, changes in the relative supply and demand of labor and capital, and through that the wage rate and the real rate of interest. The effects on aggregate factor prices may magnify or dampen the individual life-cycle decisions shaped by demographic change.

Measured aggregate productivity change is a measure of output change adjusted for changes in factor inputs. Due to measurement issues, labor is usually measured as number of hours worked. The efficiency of a worker, conditional on numbers of hours worked, will on average also depend on age. In the literature estimating life-cycle income uncertainty the predictable component of wage rates have been found to be hump-shaped (see eg. Huggett, Ventura, and Yaron, 2011). Demographic change, both through changes in labor-supply decisions and through the aggregation, will affect aggregate productivity and measured TFP. The asssumption of a hump-shaped eficiency-wage profile is not incontroversial. Rupert and Zanella (2015) and Casanova (2013) argue that the earnings profile is humped shaped, whereas the wage / productivity profile stays flat or is a step function with discrete changes associated with transition to part-time work. Kambourov and Manovski (2009) also document a flattenig of life-cycle earnings profiles since the late 1960's, but a steepening of earnings inequality profiles. As we show later, our preliminary results turn out to broadly consistent with this.

Understanding the magnitude of the contribution of demographics to long term growth and productivity is particularly important because the demographic forces are essentially baked in to the future path of the economy. They are not easily altered or easily influenced by economic policy other than perhaps by immigration policy. But understanding the behavioral response to these economic forces at the household level is important for understanding the connection to interest rates and the corresponding connection to policy. Aging populations present a number of challenges for the social organization of societies

[4] In the U.S. two thirds of the labor market adjustment over the business cycle occur on the intensive margin, meaning that changes in employment dominate changes in hours (Cho and Cooley, 1994). Llosa, Ohanian, Raffo, and Rogerson (2012) show in countries with employment protection laws and much large fraction of the adjustment takes place on the intensive margin.

and for health care, retirement insurance and the sustainability of public debt.[5] We don't address these issues in the paper but provide the most parsimonious framework to account for the effect of demographic change on long-run growth and a framework within which it is possible to analyze policy responses. For that reason we focus sharply in the paper on estimating the impact of the evolving cohort structure on growth.

2 The Growth and Productivity Slowdown

The growth and productivity slowdown is evident in trend GDP growth for a number of countries. Figure 1 shows the trend GDP for the U.S., France (as a representative of the other G7 countries), and Japan.

The downward shift in growth in Japan is evident beginning in the early 1990's while the U.S. and France show a noticeable downward shift beginning in about 2007. Figure 2 shows the 10 year Treasury Rate for Japan and the U.S.. In Japan there was a sharp decline beginning in the early 1990's and in the U.S. beginning in the 2007. This is consistent with the secular stagnation view and with an unanticipated increase in life expectancy. Figure 3 shows the pattern of life expectancy for a number of countries. It is interesting to note life expectancy in the U.S. in 2010 reached the approximate life expectancy in Japan in 1990.

In this paper we estimate the magnitude of the contribution of demographics to these trends. The two most significant demographic trends over the past several decades have been the increases in life expectancy and in the average age. The combination of increasing life expectancy and decreasing fertility means that the median age of the population is increasing, in some cases dramatically. This pattern is most evident in Japan where the combination of low fertility and increasing life expectancy is causing the population to shrink by a million people a year and the median age to increase from 37 years to 44 years between 1990 and 2008.

Here we are concerned with the impact of these trends on economic growth.[6] Growth accounting is a useful framework for addressing the question at hand because it makes clear the relationship between changes in factor inputs and measured total factor productivity. Consider the standard neoclassical production function:

$$Y_t = A_t K_t^{\alpha} \left(L_t h_t\right)^{1-\alpha} \tag{1}$$

[5]For example Attanasio, Kitao, and Violante (2007) and Kitao (2014), study the impact of demographics on the sustainability of social security programs.

[6] One approach is that taken by Favero and Galasso (2015) who treat this as an empirical question that can be answered by projecting mortality based trends on growth rates and interest rates. They suggest, based on their methodology, that demographic factors account for lower growth but not lower interest rates. We study the relationship between demographics and growth and interest rates through the lense of a structural general equilibrium model driven by rich demographics. This provides us a sharper understanding of the margins through which demographics are driving growth and a laboratory in which to analyze potentially welfare-improving policy options.

where Y is output, K is the capital stock, L is employment (the extensive margin of labor supply), and h is average hours conditional on being in the labor force (the intensive margin of labor supply). Dividing through by population, Pop, we can write this as

$$Y_t = A_t \cdot \left(\frac{K_t}{L_t}\right)^{\alpha} \cdot Pop_t \cdot \frac{L_t}{Pop_t} \cdot h_t^{1-\alpha} \tag{2}$$

and

$$A_t = \frac{Y_t}{\left(\frac{K_t}{L_t}\right)^{\alpha} \cdot L_t \cdot h_t^{1-\alpha}}$$

The average, continuously-compounded growth rate of aggregate output can be decomposed into the continuously-compounded growth rate of each of these components

$$\gamma_Y = \gamma_A + \alpha\gamma_{K/L} + \gamma_{Pop} + \gamma_{L/Pop} + (1-\alpha)\gamma_h, \tag{3}$$

where γ_Y is the continuously-compounded growth rate of aggregate output, γ_A is the growth rate of TFP, and so on.

All of the right-hand side terms are affected by demographic change. γ_{Pop} is (obviously) determined by demographics. Changes in labor supply on the extensive ($\gamma_{L/Pop}$) and intensive (γ_h) are partly determined by households' labor supply decisions over the life cycle and conditional on their life expectancy. Likewise, capital supply ($\gamma_{K/L}$) is partly determined by households' savings decisions over the life cycle and conditional on their life expectancy. Both changes in the age structure and selection effects will change the average productivity of each hour supplied. These demographic effects will be manifested in changes in *measured* TFP.

For illustration we compute this decomposition for the G7 group of countries for the period 1990-2007. The time period covers the decades when growth in many of the G7 countries was strong but growth in Japan was weak. This is also the period when Japan's demographics changed significantly as shown in the cohort distribution for Japan. When do not include later periods because the long run growth effects of the aging cohort distributions are swamped by the cyclical effects of the financial crisis and ensuing recession. For the purpose of this decomposition we assume that capital's share is 1/3.

It is clear that demographics are an important contributor to the growth experience of the G7 countries. One country that stands out is Japan. From the cohort distribution for Japan it is clear that Japan is undergoing an important demographic evolution. In the period from 1990 to 2007 declining labor force participation and declining hours of work account for 0.60% decline in output growth in Japan. In addition, total factor productivity (TFP) was low compared to previous decades and compared to the U.S. possible reflecting the lower productivity of older workers.

The combined effects of low fertility and increased life expectancy is shifting the cohort structure of the population. In some countries this is offset by immigration, but with more

Table 1: Growth Accounting G7

	γ_Y	γ_A	$\alpha \cdot \gamma_{K/L}$	γ_{Pop}	$\gamma_{L/Pop}$	$(1-\alpha) \cdot \gamma_h$
United States	2.76	1.28	0.52	1.10	-0.10	-0.05
Canada	2.61	0.45	0.80	0.99	0.52	-0.15
UK	2.48	1.53	0.72	0.40	0.09	-0.25
France	1.78	0.97	0.45	0.54	0.19	-0.36
Germany	1.67	1.03	0.55	0.18	0.28	-0.38
Italy	1.30	0.30	0.54	0.32	0.26	-0.13
Japan	1.11	0.75	0.83	0.17	-0.16	-0.47

Table 2: Growth Accounting United States and Japan

	γ_Y	γ_A	$\alpha \cdot \gamma_{K/L}$	γ_{Pop}	$\gamma_{L/Pop}$	$(1-\alpha) \cdot \gamma_h$
United States	2.76	1.28	0.52	1.10	-0.10	-0.05
Japan	1.11	0.75	0.83	0.17	-0.16	-0.47
Difference	1.65	0.53	-0.31	0.93	0.06	0.42

population concentrated in later cohorts this affects both labor force participation and hours worked since older people participate less and when they do they work fewer hours. [7]

As we noted earlier, the increase in life expectancy can lead to increased saving in working years and thus an increase in the capital stock. The Table below shows the changes in capital/output ratio across these countries from 1990 to 2007. The most striking are Japan and Italy.

	K/Y 1990	K/Y 2007
United States	3.16	3.06
Japan	3.27	4.19
Canada	2.12	2.67
Great Britain	2.26	2.33
France	3.25	3.41
Germany	2.78	3.01
Italy	3.53	4.16

[7] Fisher, Gorry, and vom Lehn (2016) show that changes in age demographics and fiscal policy may together account for roughly half of the decline in hours worked.

3 Model Economy

Growth accounting shows that a large part of both the levels of growth rates and the difference in growth rates between countries are due to changes in factor supplies. These changes and differences in factor supply may potentially be accounted for by changes in demographic structure. The remaining part – the level of and the differences in measured TFP growth – may also partially be accounted for by demographic change.

In order to study to what extent demographics may account for changes and differences in growth, both through changes in factor supplies and through changes in TFP, the model economy must not only (and trivially) account for population growth, but also allow for factor supply decisions with respect to capital, labor on the extensive margin, and labor on the intensive margin. The evolution of demographics is persistent and predictable so model that captures that connection between demographics and growth can be used to project the future path of growth.

General equilibrium identifies two dimensions through which demographics may be important for macroeconomic phenomena: life expectancy and cohort distributions. Life expectancy is crucial for individual decision making at different ages, and cohort distributions aggregate the decisions made by individuals of different ages. The evolution of the cohort distribution is a function of fertility rates, mortality rates (ie. life expectancy), and immigration rates. Fertility rates and mortality rates only indirectly affect macroeconomic outcomes.

In order to account for labor choice over the life cycle both on the intensive and extensive margin a couple of features of the model economy are crucial. First, individuals in the model economy must have the choice between labor supply on the intensive margin and labor supply on the extensive margin over the life cycle. Second, in order to account for labor supply both on the intensive and extensive margin, the model economy must account for labor-productivity over the life cycle. Individual labor productivity over the life cycle has two components: a deterministic, age-dependent, hump-shaped part and an idiosyncratic component, which is specified as a first-order autoregressive process. And third, disutility associated with the participation in the labor market, measured in terms of lost time for leisure, varies by age.

3.1 Demographics

Time is discrete and the model is populated by up to J overlapping generations. The population is stationary. Each household faces a positive probability of death at each age. Let s_j denote the conditional survival probability from age j to age $j+1$. There are no annuity markets and a fraction of households therefore leaves unintended bequests that are redistributed in a lump-sum manner across all individuals currently alive.

The economy consists of overlapping generations of ex ante identical agents who live up to J periods, with ages denoted by $i \in \mathscr{I} \equiv \{1, \ldots, J\}$. At every point in time, there are I different cohorts alive. Individuals remain children for J_0 periods. As children they neither consume, accumulate capital nor supply labor. After J_0 periods the agents enter the economy as autonomous decision makers.

The survival probability between age j and $j+1$ is denoted $s_{j,t}$ and varies with ages j and time. The unconditional probability of reaching age j is denoted s^j and is the product of conditional survival probability rates; $s^j = \prod_{i=1}^{j-1} s_i$.

Let $x_t \in \mathbb{R}^I$ be the vector of number of members in each cohort in period t. The demographic structure of the population changes through changes in fertility, mortality and immigration. According to time and age specific fertility rates $\varphi_{i,t}$, in each period these individuals give birth to a certain number of new individuals, and the number of newborns in period $t+1$, $x_{1,t+1}$, is the product of x_t and the vector of fertility rates φ_t. Then the law of motion of a population with survival rates as given above, but deterministic fertility, can be described by a simple $(J \times J)$ matrix[8]

$$\hat{\Gamma} = \begin{pmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \cdots & \varphi_I \\ s_1 & 0 & 0 & \cdots & 0 \\ 0 & s_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & s_{I-1} & 0 \end{pmatrix},$$

where the diagonal elements $(s_1, \ldots, s_{J-1})$ are the conditional survival probabilities.

Let $m_t \in \mathbb{R}^J$ be a vector with each element representing the cohort specific number of net immigrants at time t. Denoting $\hat{\Gamma}_t$ the matrix of deterministic fertility and mortality rates at time t, the law of motion for the population may be written

$$x_{t+1} = \hat{\Gamma}_t x_t + m_t.$$

As we will see in Subsection 4.1, only life expectancy s and the cohort distribution x are directly relevant for macroeconomic outcomes.

3.2 Endowments and preferences

Households are endowed with one unit of time in each period of their lives and enter the economy with no assets, expect their lump-sum share of accidental bequests. They spend their time supplying labor to a competitive market or consuming leisure.

[8] The largest eigenvalue of the matrix Γ is the rate of growth of the population in steady state regardless of the initial condition. The eigenvector corresponding to this eigenvalue describes the share of each age group in the steady state population.

Households are heterogeneous along three dimensions that affect their labor productivity and cost of supplying labor. First, they differ by age in their average labor productivity ψ, which governs the average productivity of an age cohort. Secondly, households of the same age face idiosyncratic risk with respect to their individual labor productivity. Let ν denote a generic realization of this idiosyncratic labor productivity uncertainty in the current period. The stochastic process for labor productivity status is identical and independent across households and follows a finite-state Markov process with stationary transition probabilities over time. Thirdly, cost of participation in the labor market is a monotonically increasing function in age.

At any given time households are characterized by (j, a, η), where j is age, a is current asset or accumulated savings, and η is idiosyncratic labor productivity status.

Households order the sequence of consumption and labor supply over the life-cycle according to a time-separable utility function

$$\max_{\{c_j, h_j\}} \mathrm{E}_{t_0} \left\{ \sum_{j=j_0}^{J} \beta^{j-j_0} s_j u(c_{j,t_0+j}, h_{j,t_0+j}) \right\} \tag{4}$$

where β is the subjective discount factor, s_j is the unconditional survival probability to age j, and the instantaneous utility function over consumption and hours worked at age j, denoted as c_j and h_j, is given by

$$u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} + \chi \frac{(1 - h - \theta_j \cdot i_p)^{1-\gamma}}{1-\gamma}. \tag{5}$$

The parameter χ represents the weight on utility from leisure relative to consumption, σ is the intertemporal rate of substitution, γ is the curvature of the utility function for leisure, and i_p is an indicator that takes a value 0 when $h = 0$ and 1 otherwise .

θ_j represents the disutility associated with the participation in the labor market. Following Kitao (2014) we assume that the fixed cost of participation is measured in terms of lost time for leisure and varies by age. The fixed cost of participation conditional on age j is given the following functional form

$$\theta_j = \kappa_1 + \kappa_2 j^{\kappa_3}. \tag{6}$$

Our results will point to that the labor supply elasticity and the trade-off between labor supply and leisure are key determinants of our findings, and more generally, key to understand the economic effects of the demographic transition.

3.3 Individual budgets and aggregate resource constraints

Individuals maximize expected utility subject to their period-by-period budget constraint

$$c_{j+1,t+1} + a_{j+1,t+1} = (1 + r_t) a_{j,t} + w_t \cdot h_{j,t} \cdot \psi_j \cdot \eta_{j,t} + b_t, \tag{7}$$

and the constraints following from the absence of an intentional bequest motive

$$a_{I_0,t} = a_{I+1,t} = 0, \tag{8}$$

where $a_{j,t}$ represents asset holdings, r_t is the rate of return on capital, w_t is the market price of one efficiency unit of labor, and accidental bequests, ψ_j is the individual age-specific, systematic productivity component, η_j is the idiosyncratic component of an individual's productivity, and b_t, is the fraction of total inheritance or bequests received by each individual alive at time t.

The process for the exogenous uninsurable idiosyncratic productivity shocks, η_j, is specified as a first-order autoregressive process. Empirical studies indicate that a persistent autoregressive component and a transitory component accurately describes the data (see eg. Meghir and Pistaferri, 2004; Heathcote, Storesletten, and Violante, 2010)

$$\log \eta_{j+1} = \rho \log \eta_j + \varepsilon_{j+1} \qquad \varepsilon \sim \mathcal{N}(0, \sigma^2). \tag{9}$$

3.4 Technology

We assume competitive firms demand labor and capital, supply consumption goods, and have access to a constant elasticity of substitution technology with the form:

$$Y_t = A[\alpha K^\gamma + (1-\alpha) L^\theta]^{\frac{1}{\theta}} \tag{10}$$

where K_t and L_t represent the aggregate capital stock, and aggregate labor input (measured in efficiency units) in period t. The aggregate law of motion for capital stock is

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{11}$$

where δ is depreciation rate and I_t is aggregate net savings.

3.5 Competitive equilibrium

An equilibrium for this economy is defined by:

1. Individuals optimize and choose quantities demanded and supplied given prices

    - Each individual's optimization problem

    $$v(j, a, \eta) = \max_{h, a'} \left\{ u(c, h) + \beta \cdot s \cdot \mathrm{E}_{\eta'|\eta} v(j+1, a', \eta') \right\}$$

- Individuals' quantity choices are aggregated by the number of individuals in each cohort

$$K_t^s = \sum_i x_i \int_j a_{j,i,t} d\eta_j$$

and

$$L_t^s = \sum_i x_i \int_j h_{j,i,t} \psi_i \eta_j d\eta_j$$

2. Firms optimize and choose quantities demanded and supplied given prices

$$\max_{K_t^d, L_t^d} \left\{ \left(K_t^d\right)^{\alpha} \left(L_t^d\right)^{1-\alpha} - r_t K_t^d - w_t L_t^d \right\}$$

3. Markets clear: prices are set such that demand equals supply

$$\{r_t, w_t\} \ : \ K_t^s = K_t^d \ \text{ and } \ \ L_t^s = L_t^d$$

As we see from the definition of competitive equilibrium, there are two features of demographic change that affect macroeconomic outcomes: individuals' conditional life expectancies, s, affect individual choices, and the cohort distribution, x, determines how these individual choices are aggregated.

4 Calibration

In order to carry out the numerical simulations and compute average growth rate we first choose a model parameterization.

4.1 Demographics

Households start making autonomous consumption-savings and labor-leisure choices at age 21. Retirement age is endogenous and individual. As shown in Section the two crucial demographic variables are individual life expectancy and cohort distribution.

Each year households face a mortality risk. The sequence of annual mortality rates is computed using the algorithm described by Henriksen (2015) to match the life expectancy reported by the United Nations Population Prospects (2015) for the given country and year.The sequence of mortality rates determines the theoretical maximum age.

Cohort distributions are based on the United Nations Population Prospects (2015). We linearly interpolate to compute one-year cohort bins based on the five-year cohorts reported by the UNPP.

The cohort distributions do not necessarily match the stationary distributions associated with the mortality rates. As shown in Section , the competitive equilibrium may still be computed.

4.2 Preferences

Households have preferences that are additively separable over time and additively separable over consumption and leisure. Households discount the future with the product of the factor β and the country, time and age specific survival probability s.

The age-invariant discount factor β is set to match a capital-output ratio of 3.0 in the initial steady state. The parameter χ is set so that the average work hours of working individuals equal to 38% of disposable time as in the PSID data. The risk-aversion parameter σ is set at 2 and γ at 4.0, which implies the intertemporal labor-supply elasticity of about 0.32 on average.

The three parameters in the fixed cost of participation κ_1, κ_2 and κ_3 are calibrated to match the following three targets: average participation rate at age 60 and at age 70, and average work years over the life-cycle. The calibrated cost function is plotted in Figure 7.

Because our results will point to the labor supply elasticity as a key determinant of our findings so further sensitivity analyses may be warranted.

4.3 Technology

In the benchmark calibration, the CES production function has unit elasticity of substitution ($\theta = 0$) so we have the standard Cobb-Douglas case.

The capital share parameter α is set to 0.33, a standard value in the literature. The depreciation rate is set to match a real interest rate of 3% in 1990, $\delta = 0.06$.

4.4 Labor Productivity Processes

A household's labor productivity depends on two components: a deterministic age-dependent component ψ_j, and a persistent, idiosyncratic shock η.

Following Hansen (1993), we have used a hump-shaped profile for the age-specific component of individual productivity ψ_j, see Figure 6. Several contributions have later questioned this parametrization, including Casanova (2013) and Rupert and Zanella (2015). As we will show and in line with the results reported in these papers, due to selection effects average earnings are flatter than average productivity profiles.

The persistence parameter of the idiosyncratic component η of a worker's wage is set to $\rho = 0.97$ and the variance of the white noise is set to $\sigma^2 = 0.02$, which lie in the range of estimates in the literature (see, for example, Meghir and Pistaferri, 2004; Heathcote et al., 2010).

The calibrated parameters are summarized in Table 3.

Table 3: Calibration

Demographics		
x	cohort sizes	UN
s	conditional survival probabilities	UN, Henriksen (2015)
Preferences		
β	subjective discount factor	0.9615
χ	weight on leisure	0.5123
γ	leisure utility curvature	4.0
$\kappa_1, \kappa_2, \kappa_3$	cost of labor force participation	0.0531, 0.000298, 2.780
Labor productivity process		
ρ	persistence parameter	0.97
σ^2	variance	0.02
ψ	age-dependent productivity	PSID
Technology and production		
α	capital share of output	1/3
δ	depreciation rate of capital	0.06

5 Computational experiment

In our first exercise, we use the evolution of the contributions to growth in Japan and the U.S. between 1990 and 2007 and ask whether the observed contributions of demographic factors in the growth accounting exercise reported in Section 2 is consistent with the general equilibrium economy described above. We do this for at least two reasons. First, Japan 1990 shared several demographic similarities with the United States at the onset of the Financial Crisis, in particular in terms of cohort distribution and life expectancy. Second, the growth experience in Japan in the 1990s shares similarities with the growth experience in the United States following the Financial Crisis. And third, this is part of the period which was referred to as the great moderation. It may therefore provide us with a benchmark to evaluate the potential contributions of demographics to economic growth.

We calibrate the structural parameters of the model economy to the United States in 1990 and solve for the steady state distributions of the economy with these structural parameters, but the demographic structure in the U.S. 1990, the U.S. 2007, Japan 1990, and Japan 2007, respectively. We then do the growth accounting on the implied data and compare it to the results of the previous exercise.

During this period there were both substantial increases in life expectancy and changes in average age and the cohort distributions The anchor for these computations are the life expectancies and cohort distributions for Japan and the U.S. in 1990 and 2007. The cohort distributions are shown in Figures 3 and 4 and the evolution of life expectancy at birth is shown in Figure 5.

The main reason for why we solve for steady-state distributions instead of transitions between steady states is to keep the exercise as accountable and parsimonious as possible. Computing transitions would be numerically and computationally feasible, but the economic results would necessarily hinge on choice of initial and terminal conditions and therefore the results substantially much harder to analyze.

6 Results

In order to assess the potential importance of demographic changes for economic growth over time and across nations, for reasons mentioned about we study the United States and Japan between 1990 and 2007.

6.1 Comparing Japan and the United States 1990 to 2007

The model economy is scaled by the size of the population so we demographics can trivially account for the population-growth part on average 1.10% and 0.17% annually for United States and Japan, respectivley.

Since we may trivially account for the aggregate growth contribution from population growth we are interested in growth ex population growth, or, equivalently, in growth per capita. Table 4 shows the growth accounting for United States and Japan from Table 2 net of population growth.

The question is to what extent the evolution of the contributions to growth observed in the growth accounting exercise reported in Section 2 are consistent with the general equilibrium economy described above.

Table 4: Data: Growth Accounting Net of Population Growth

	$\gamma_{Y/Pop}$	γ_A	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-\alpha) \cdot \gamma_h$
United States	1.66	1.28	0.52	-0.10	-0.05
Japan	0.94	0.75	0.83	-0.16	-0.47
Difference	0.72	0.53	-0.31	0.06	0.42

To understand how much of the changes in growth and in total factor productivity we compute the steady states of the model for the U.S. in 1990 and in 2007 and for Japan in 1990 and 2007. We then do the growth accounting on the implied data and compare it to the results of the previous exercise.

After computing stationary distributions, factor prices and levels of aggregate variables for the two economies with their given demographic structure at the two years we did a similar growth accounting exercise on the data from the model economy. The model implied total growth over the period is given in Table 5, and the the model-implied annual growth over the period is given in Table 6.

Table 5: Model: Total Growth Accounting Net of Population 1990-2007

	$\gamma_{Y/Pop}$	γ_A	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-\alpha) \cdot \gamma_h$
United States	6.39	3.89	6.39	-1.29	-2.60
Japan	4.26	2.51	10.10	-2.44	-5.92

Table 6: Model: Annual Growth Accounting Net of Population 1990-2007

	$\gamma_{Y/Pop}$	γ_A	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-\alpha) \cdot \gamma_h$
United States	0.35	0.22	0.35	-0.07	-0.14
Japan	0.24	0.14	0.56	-0.14	-0.33

The results show that about 1/4 of the level of growth for both United States and Japan, net of population growth, may be accounted for by changes in life expectancy and in the cohort distributions.

Most surprising may be the sizeable contribution from demographics to measured TFP growth. This result is partly due to a composition effect and partly due to selection effects. Almost mechanically, if a larger fraction of the population is in their most productive years that will be measured as an increase in aggregate TFP. In addition there is a selection effect interacting with increases in life expectancy. Individuals who receive a sequence of positive idiosyncratic productivity shocks tend to supply more labor on the intensive margin and stay longer in the labor force. This effect seems to be reinforced by changes in longevity.

Capital accumulation is an important contributor to growth. Figure 8 shows asset profiles over the life cycle. As we see, due to idiosyncractic risk, there is a wide dispersion in asset holdings over the life cycle.

There has been debate about how productivity on average varies with age. E.g. Rupert and Zanella (2015) argue that the wage profile does not decline with age, while the earnings profile does. Our results are broadly consistent with this finding. Figure 9 shows the average productivity of a unit of time worked, conditional on age, together with the deterministic profile for psi_i. As the figure shows, due to selection effects, there is little decline in the average productivity of, and hence wage for, a unit of time worked.

7 Projections for future growth

The model economy we described in the previous section can also be used to project the expected impact of demographic changes on the growth experience of countries going forward. Using tge projections for life expectancies and cohort distributions reported by the United Nations *World Population Pospects*, we can compute the projected impact on future growth.

Using exactly the same calibration as in our baseline model we project the impact on growth from 2015-2030. Over this period the predected life expectancy in Japan grows from 84.09 to 86.21 years. The predicted life expectancy for the U.S. grows from 79.57 years to 81.79 years.

The impact of these changes for average annual growth rates are shown in Table 7.

Table 7: Growth Projections -

	$\gamma_{Y/Pop}$	γ_A	$\alpha \cdot \gamma_{K/L}$	$\gamma_{L/Pop}$	$(1-\alpha) \cdot \gamma_h$
Japan	-0.02%	0.04%	0.50%	-0.35%	-0.21%
United States	-0.19%	0.02%	0.29%	-0.52%	-0.01%

These results are quite striking. They suggest that over time demographic changes are a significant drag on economic growth in the United States and more so for the U.S. than for Japan.

8 Concluding remarks

There has been a slowdown in growth in the world's most advanced economies. In this paper, we have documented that the slowdown in growth has been due in part to the combination of lower population growth, decrease in labor supply, both on the intensive and the extensive margin, and lower TFP growth. Increased capital accumulation has somewhat mitigated the decrease in growth rate coming from these other factors.

We have argued that changing demograpics affect factor supplies, factor accumulation, and TFP. Changing demographics, in particular aging populations and increased life expectancy, may account for slower growth, and falling productivity. With our calibrated model, we found that a parsimonious overlapping generations model in which households make labor supply and savings decisions based on (changing) conditional life expectancy can account for about a quarter of the growth rate experience of Japan and the United States between the start of Japan's "lost decades" and the start of the Financial Crisis.

The objective of the paper is to provide a transparent framework to analyze historical data and, potentially, make predictions for future growth rates. This framework shows that elasticities of labor supply, of intertemporal substitution, and of substitution between capital and labor are crucial for both these results and for modeling the effects of the demographic transition on future economic growth. Further refinement of the calibration may show that these demographic factors may account for an even larger part of economic growth.

This framework may also serve as a laboratory to address future challenges related to issues such as fiscal sustainability if an increase in the number of retirees coincides with lower economic growth. As we have indirectly shown, the incentive effects from capital and labor may not only affect labor supply on the intensive and extensive margin, but also aggregate productivity growth.

The current version has abstracted from crucial developments such as increasing female labor supply and decreasing prime-age male labor supply. These are important issues, which could be addressed within the current framework when appropriately extended.

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A Figures

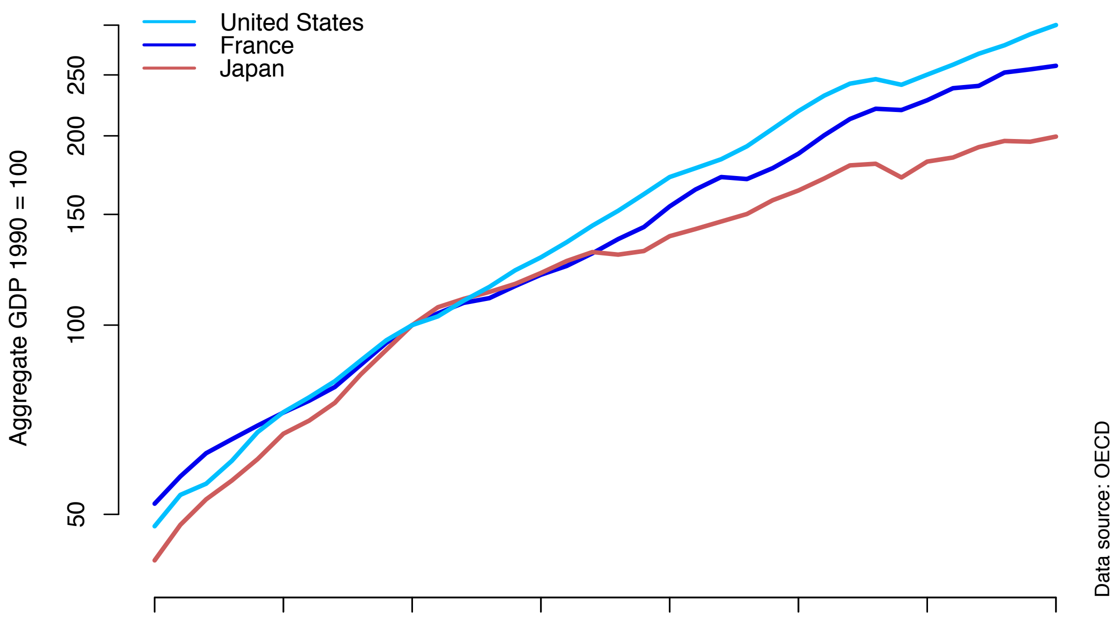

Figure 1: Aggregate Real GDP United States, Japan and France

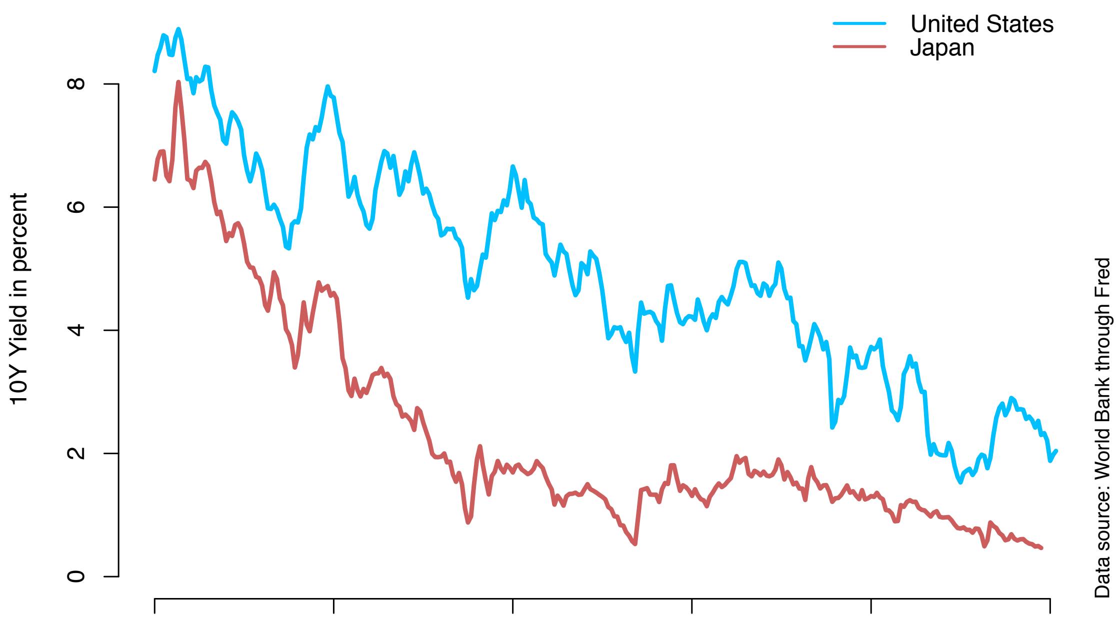

Figure 2: Yield 10Y Government Bonds United States and Japane

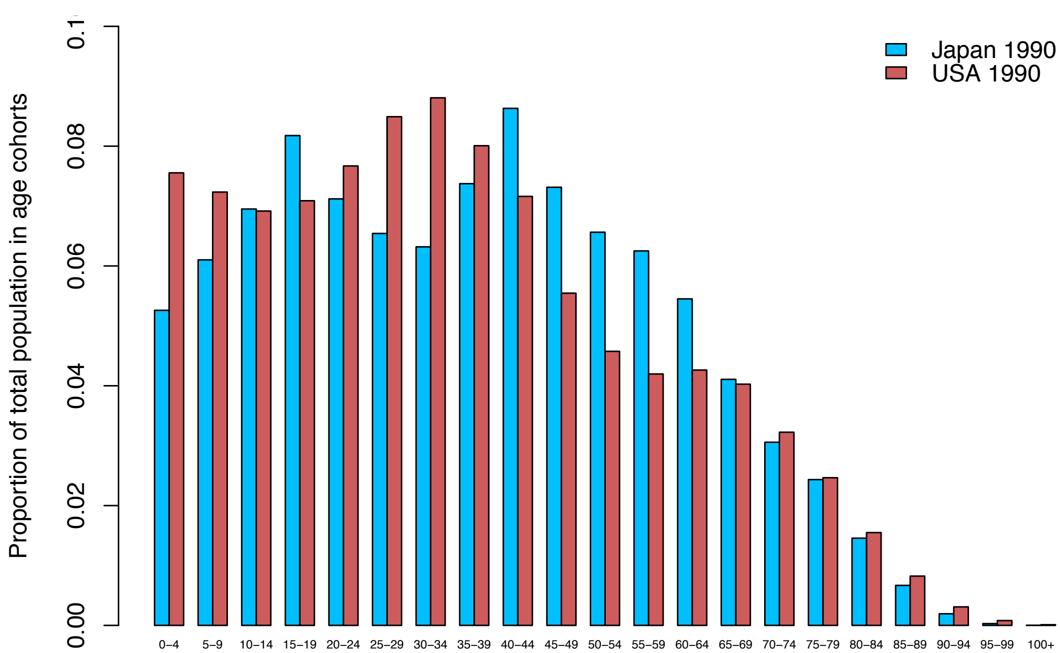

Figure 3: Cohort distribution United States and Japan 1990

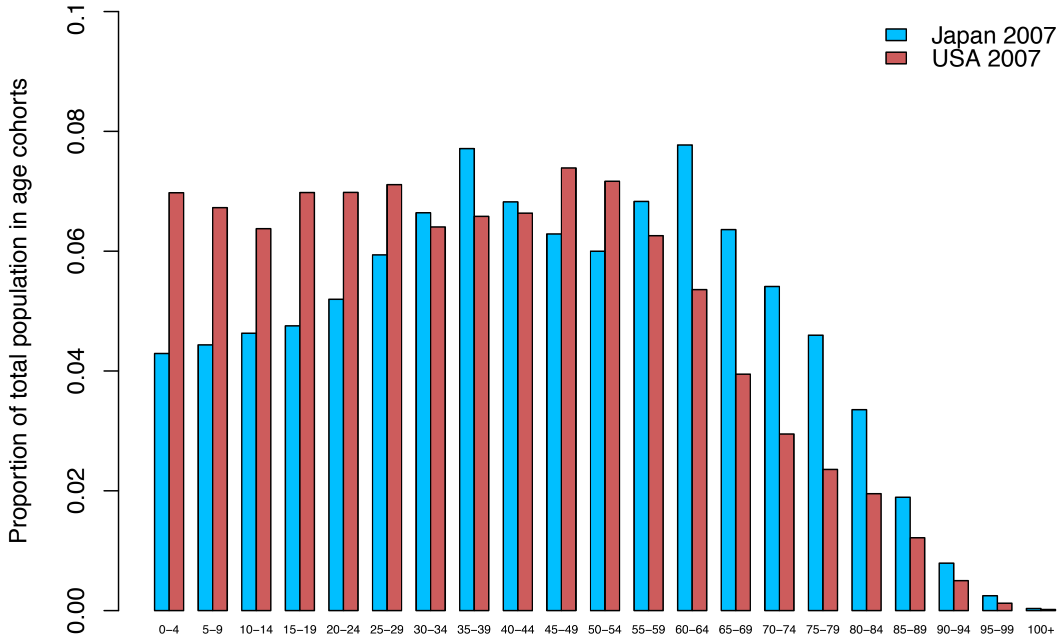

Figure 4: Cohort distribution United States and Japan 2007

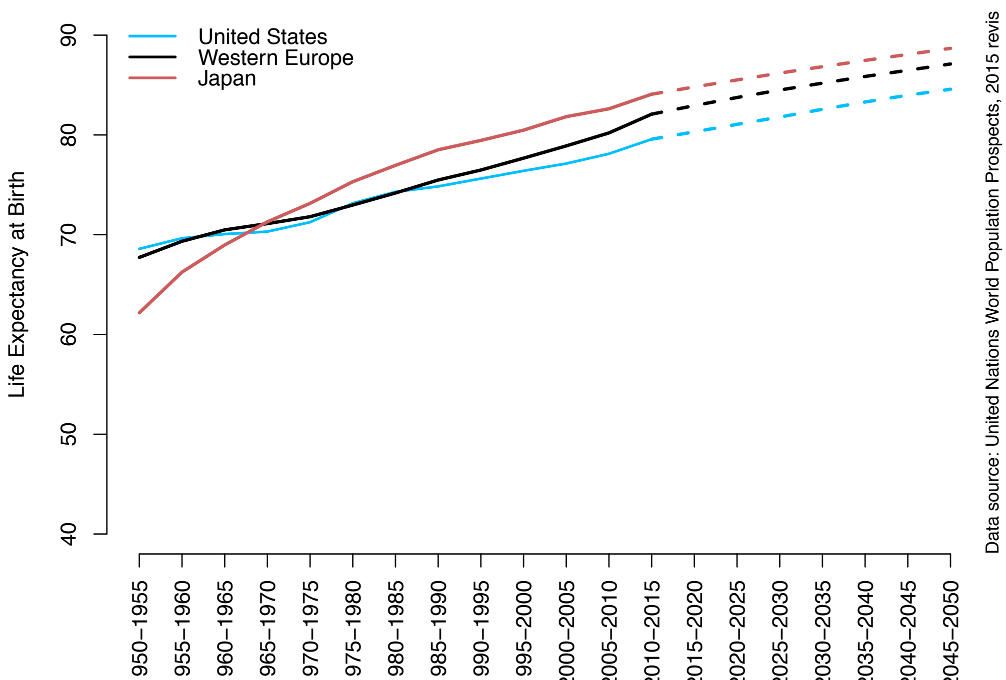

Figure 5: Life expectancy at birth, United States, Japan, and Western Europe

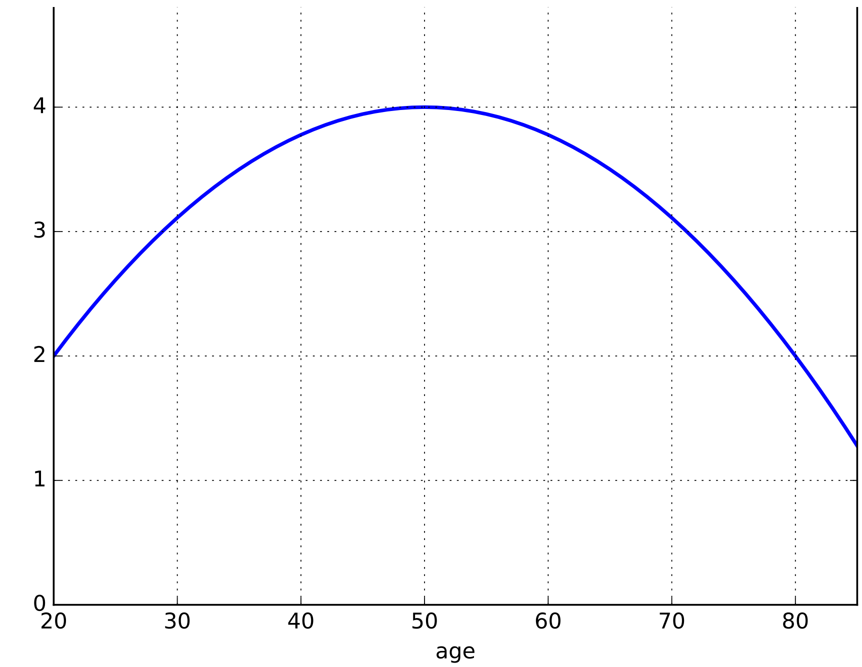

Figure 6: Life-cycle productivity profile

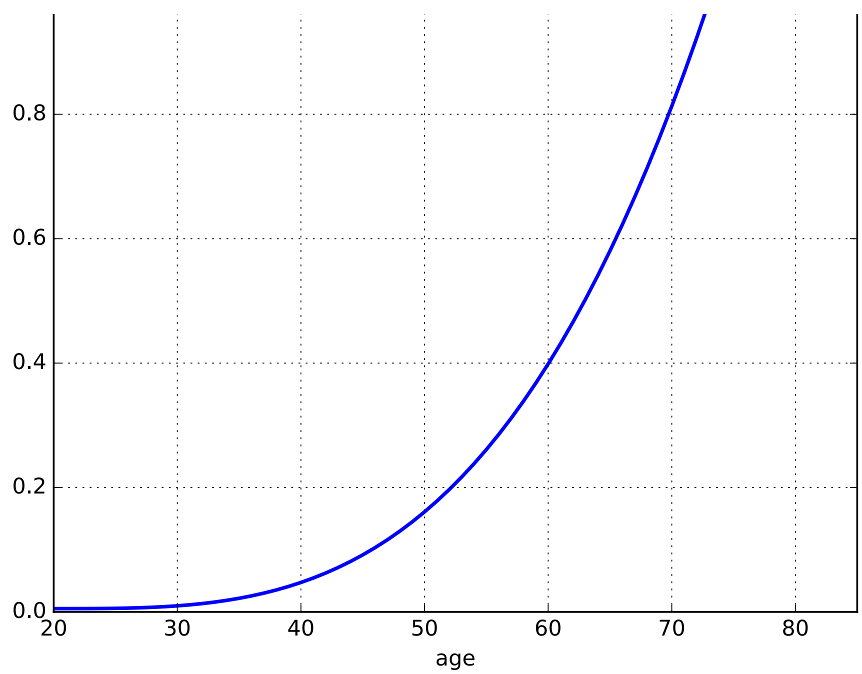

Figure 7: Participation cost at different ages

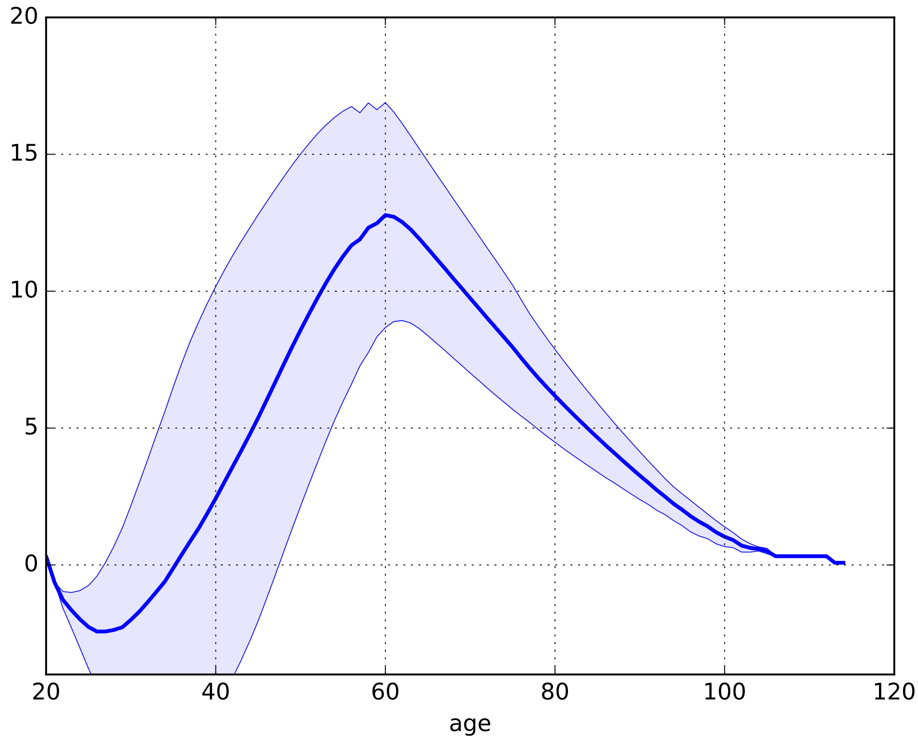

Figure 8: Model asset profile over the life cycle given US life expectancy and equilibrium factor prices in 2007

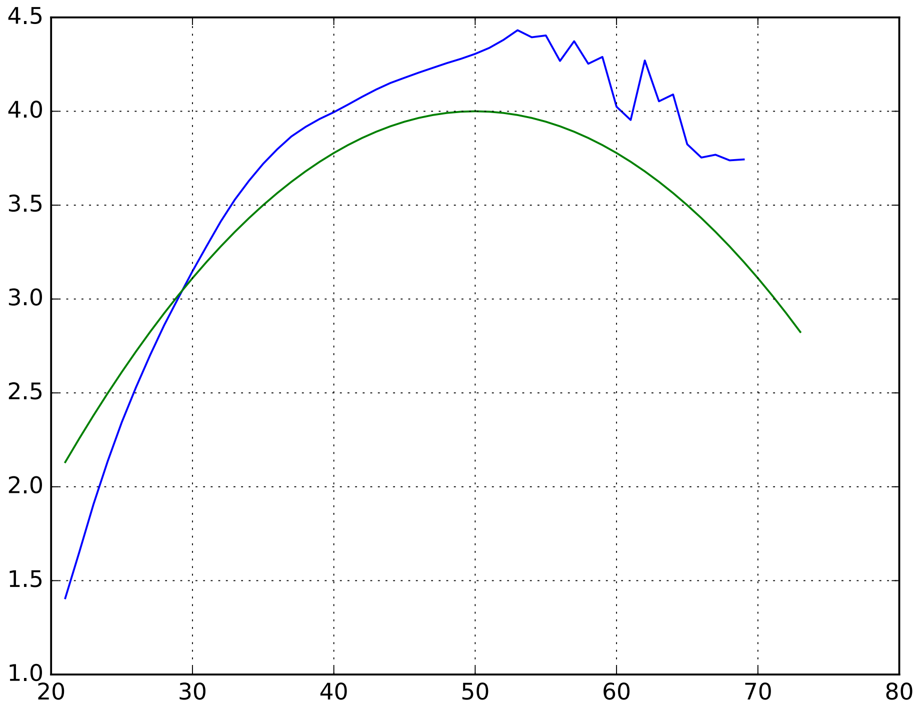

Figure 9: Average productivity of a unit of time worked compared to the systematic productivity profile

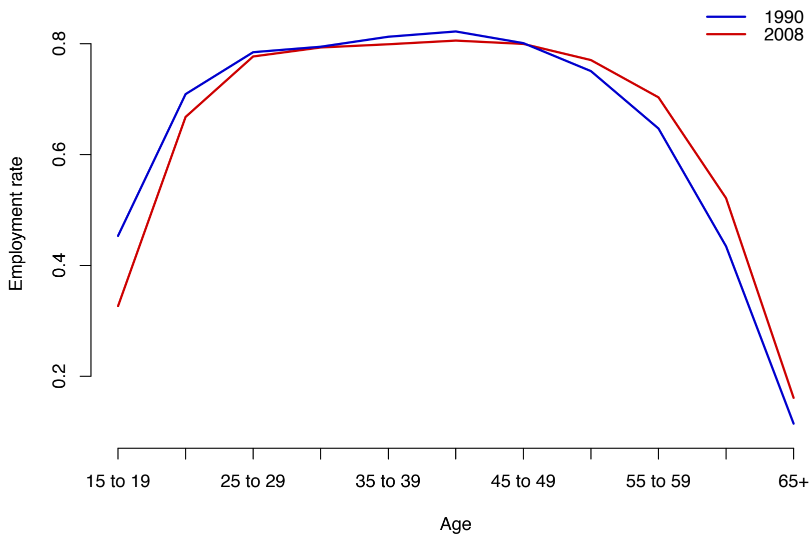

Figure 10: Employment rate: United States 1990 and 2008

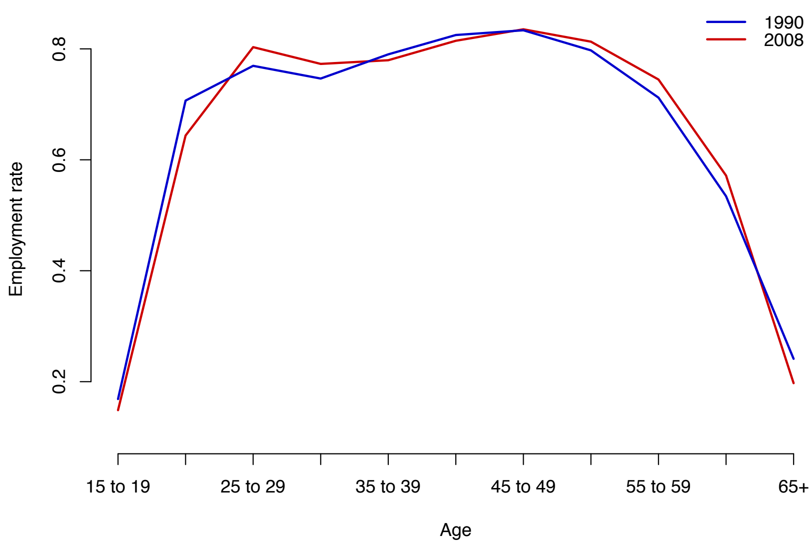

Figure 11: Employment rate: Japan 1990 and 2008

B Appendix: Computing Survival Probabilities

In our model the state variable driving individual decisions about labor supply and saving is the survival probability at every age. We estimate this following the procedure in Henriksen (2015). Given reported changes in life expectancies at birth, e_0, a model is necessary to compute how conditional survival probabilities will evolve.

It is known among demographers that death rates increase exponentially with age, or equivalently that the logarithm of death rates increases linearly with age for all ages after early adulthood. The near-log-linear relationship between age and conditional mortality suggests the following parametric form:

$$\log[m(x, e_0)] = \alpha + \beta^{e_0} x + \varepsilon_{x,t}, \tag{12}$$

where x is age, e_0 is life expectancy at birth, and β^{e_0} is a linear function of life expectancy

$$\beta^{e_0} = \gamma + \theta e_0 + \varepsilon_{e_0}. \tag{13}$$

This may be estimated using a two-stage procedure. The first stage is a weighted least squares estimation. The ranking criterion for the results is out-of-sample absolute deviation between life-expectancy at birth in the data and life-expectancy at birth life predicted by the model. In the second stage, a simulated method of moments procedure provides exact estimates of life expectancy at birth.

This approach shares similarities with, among others, the principal-components-based model of Lee and Carter (1992), which is referred to as the "leading statistical model of mortality [forecasting] in the demographic literature" (Deaton and Paxson, 2004). Lee and Carter developed their approach on historical U.S. mortality data, 1933-1987. However, the method is now being applied to all-cause and cause-specific mortality data from many countries and time periods (Girosi and King, 2008, p.34). It was used as a benchmark for the Census Bureau population forecasts (Hollmann, Mulder, and Kallan, 2000), two U.S. Social Security Technical Advisory Panels, recommended its use, or the use of a method consistent with it (Lee and Miller, 2001), and the United Nations Population Forecast used it (Li and Gerland, 2011).