

Prepayment Risk and Expected MBS Returns*

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Abstract

We present a simple, linear asset pricing model of the cross section of Mortgage-Backed Security (MBS) returns in which MBS earn risk premia as compensation for their exposure to prepayment risk. We measure prepayment risk and estimate security risk loadings using real data on prepayment forecasts vs. realizations. Estimated loadings are monotonic in securities' coupons relative to the par coupon, as predicted by the model. Prepayment risks appear to be priced by specialized MBS investors. In particular, we find convincing evidence that prepayment risk prices change sign over time with the sign of a representative MBS investor's exposure to prepayment risk.

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1 Introduction

We develop a model of the time series and cross section of treasury-hedged returns to Mortgage-Backed Securities (MBS), and find robust empirical support for our model's main implications. Our model is a simple and transparent linear asset pricing model. Its key feature is that prepayment risk which is not readily hedgeable using US treasuries is priced by specialized MBS investors, as in Gabaix, Krishnamurthy, and Vigneron (2007). These specialized investors require prepayment risk premia which not only vary across securities, but also over time depending on the share of MBS which is trading above par value. We find that a tradable strategy optimized according to our theory across securities and over time has a Sharpe ratio which is 2.7 times that of a passive, value-weighted MBS index.

While prepayments due to interest rate movements of government securities can be hedged, the requirement of a hedging model leads to model risk. Moreover, it is even more challenging, if not impossible, to hedge against prepayment risk driven by shocks to spreads between government and mortgage rates, changing credit conditions, house price appreciation, and regulatory changes. As result, MBS which load on the unhedgeable component of prepayment risk can exhibit risk premia even if returns are effectively duration hedged to US treasuries. We show that this is indeed the case by measuring MBS securities' loadings on prepayment risk shocks measured using actual forecast and realized prepayment data and showing that these risk loadings are priced. However, we also show that the factor risk prices change sign over time. Such time varying risk premia have important implications for the time series and cross section variation in MBS average returns. Indeed, failing to account for time varying risk premia leads to estimates for expected returns which are misleading because positive expected returns are biased towards zero.

We show that the composition of the market between discount and premium securities drives the sign of prepayment risk premia. This idea, first proposed by Gabaix et al. (2007), makes sense in the context of segmented markets and specialized MBS investors. Discount securities trade below face value and are prepaid at face value. Thus, for discount securities, or securities with prices below par, prepayment is value increasing. The opposite is true for premium securities, whose value declines

with early prepayment at a face value which is lower than their market value. As interest rates move, the moneyness of borrowers' options to prepay, and the prices of outstanding securities relative to the par security, vary considerably. As a result, the composition of the MBS universe changes substantially over time, and the market moves between being mainly comprised by discount securities to primarily premium. Thus, prepayments can be value increasing for the market overall when the majority of MBS, weighted by their remaining principal balance, are trading at a discount to their face value, and value decreasing when the market features more premium securities. We provide evidence that MBS markets are indeed segmented. In particular, when prepayment is wealth decreasing for a representative MBS investor who holds the MBS market, we find negative prepayment risk prices. On the other hand, when early prepayment increases the wealth of such an investor, we find a positive price of prepayment risk.

We provide a simple, linear asset pricing model for the cross section of hedged MBS returns which features two prepayment risk factors, and prices of risk for these two factors which vary with market composition. The first risk factor is a level factor, which shifts prepayments across all coupon levels up or down. The second factor is a rate-sensitivity factor. This factor determines how sensitive borrowers are to prepayment options, conditional on their option moneyness. Although active MBS investors duration hedge, they cannot hedge shocks to the level of prepayments, or shocks to borrowers' sensitivity to their rate incentive. These "level" and "rate-sensitivity" factors were termed "turnover" and "refi" risk by Levin and Davidson (2005). Chernov et al. (2015) greatly extend this idea and, using a structural estimation, derive implied time series for these two factors from their model and data on MBS prices.

Our study makes several contributions. First, we provide the first comprehensive study explaining a long time series and broad cross section of MBS expected returns. We study the pattern of expected returns in this important fixed income market using a linear factor model which is straightforward to estimate and easy to interpret. Second, we are the first to show that risk premia are earned on MBS investments which load on prepayment risk in a study which uses actual prepayment data to measure innovations to prepayment risk factors. We find substantial evidence that non-interest rate driven prepayment shocks drive MBS returns, and this has important

implications for prepayment modeling. Third, our study helps to explain why option adjusted spreads (OAS), which should be zero if MBS are only exposed to interest rate risk, are non-zero on average and exhibit a U-shaped pattern in pooled time series cross section data. This pattern is emphasized by Boyarchenko, Fuster, and Lucca (2014) in their closely related study of the OAS smile using data on interest only and principal only strips. Our findings suggest that OAS on MBS reflect premia for prepayment risk which change sign so that the pattern of expected returns in the cross section is downward sloping in discount markets, and upward sloping in premium markets, leading to a U-shape in the pooled time series cross section. Importantly, since OAS are model-specific, we use data on MBS returns to measure risk premia. We show that average monthly returns display precisely this time varying pattern in the cross section conditional on market type. Our analysis also explains why studies using different time samples find different rankings amongst MBS strategies which are long either discount, par, or premium securities. Duarte, Longstaff, and Yu (2006) finds a ranking of discount, par, premium for average strategy level returns using data from 1996 to 2004. Using our sample from 1994 to the present, we find the opposite ranking, consistent with the findings of positive prepayment risk premia in Gabaix, Krishnamurthy, and Vigneron (2007). The difference is due to variation in the composition of the MBS market over time. Discount securities were more prevalent in the period studied by Duarte, Longstaff, and Yu (2006), implying a negative prepayment risk premium. Finally, our study contributes to the mounting evidence that markets are segmented, and that risks may be priced by specialized investors. Our findings present evidence that MBS risk prices actually change sign over time with whether prepayment is wealth increasing or wealth decreasing for a representative MBS investor who is specialized in MBS and holds the MBS market.¹

An accurate understanding and measurement of risk premia in MBS markets is important of its own accord. The market for agency MBS pass-through securities represents over \$6.3 Trillion in market value.² MBS are a very important part of the fixed income portfolios of most banks, asset managers, pension funds, and insurance

¹That risk prices can change sign over time may be a more pervasive phenomenon. For example, see Campbell, Pflueger, and Viceira (2016) for evidence of changing stock bond correlations.

²See the Table describing US Mortgage-Related Issuance and Outstanding at www.sifma.org/research/statistics.aspx.

company portfolios. Such securities constitute about 23% of the Barclays Capital US Aggregate Bond Index, a key benchmark for fixed income portfolio allocations. Moreover, the risk premia on MBS are key inputs into the pass-through from monetary policy, which primarily operates through treasury pricing, and the mortgage rates faced by borrowers.

2 Literature Review

Our study is most closely related to Gabaix, Krishnamurthy, and Vigneron (2007). Their study provides convincing evidence that MBS returns are driven in large part by limits to arbitrage. Importantly, they show that although prepayment risk is partly common within a class of MBS securities, the risk in MBS investing is negatively correlated with the aggregate risks born by a representative consumer, as measured by consumption growth. The main differences between our study and theirs are that they use a shorter time period, in which prepayment risk carries a consistently positive risk premium, and they study Collateralized Mortgage Obligations (CMO's), rather than pass-through securities. We greatly extend their results on the cross section and time series of MBS returns by using a long time series and broad cross section on MBS pass-through returns. Pass-through securities constitute 90% of MBS outstanding, while CMO's comprise the remaining 10%. Finally, Gabaix, Krishnamurthy, and Vigneron (2007) measure prepayment risk as errors from a stylized prepayment model, rather than using actual data on prepayment forecasts and realizations as our study does.

We also build on recent contributions, including Chernov, Dunn, and Longstaff (2015). Chernov, Dunn, and Longstaff (2015) use a structural model to derive more accurate MBS prices, and back out the model implied level and rate-sensitivity levels of prepayments. They provide convincing evidence that there are systematic shocks to the level and rate-sensitivity of prepayments, and that these shocks drive the level of MBS prices.³ By contrast, we focus on understanding the cross section of returns, rather than the levels of prices. Another difference is that Chernov, Dunn, and Longstaff (2015) use their model, along with MBS pricing data, to back out the

³See also Levin and Davidson (2005), who develop and calibrate a model of MBS option adjusted spreads which includes turnover and refinancing risk factors.

properties of the priced turnover and rate-sensitivity risk factors. Implied risk factors can be sensitive to model specification. By contrast, we use real variables as factors, and we measure prepayment risk factors directly using prepayment data. Our factor measurement is thus in the spirit of the macroeconomic factors constructed in Chen, Roll, and Ross (1986). Chernov, Dunn, and Longstaff (2015) argue convincingly that MBS pricing is driven by exposure to turnover and rate-sensitivity risks, and our finding that exposures to these risks measured using actual prepayment data explain the cross section and time series of MBS returns builds heavily on, and supports, their findings.

Another closely related recent study is Boyarchenko, Fuster, and Lucca (2014), who also argue that unhedgeable prepayment risk is priced in their study of the OAS smile. The notion of systematic, priced, non-interest rate prepayment risk is also proposed by Boudoukh, Richardson, Stanton, and Whitelaw (1997). Boyarchenko, Fuster, and Lucca (2014) use interest only and principal only strips to show that more extreme coupons seem to have higher prepayment risk exposure, and thus have higher OAS. Again, our study focuses on actual MBS returns, and on risk loadings which we derive independently from our pricing model. In contrast to Boyarchenko, Fuster, and Lucca (2014), we emphasize that the U-shaped unconditional average return pattern is driven by conditional patterns of returns that are sometimes upward sloping, and other times downward sloping, depending on the MBS market composition.

Finally, our study follows many papers which study prepayment behavior and the effect of prepayment on MBS pricing. Important examples include Dunn and McConnell (1981a), Dunn and McConnell (1981b), Schwartz and Torous (1992), Stanton (1995), Longstaff (2005), Downing, Stanton, and Wallace (2005), and Agarwal, Driscoll, and Laibson (2013).

3 Model

We develop a linear pricing model in which risk premia and expected excess returns are earned for loading (β) on priced risks (λ). In particular, following Levin and Davidson (2005) and Chernov, Dunn, and Longstaff (2015), we posit a two-factor model, in which prepayment shocks arise from innovations to the level of prepayments,

x , and innovations to the sensitivity of prepayments to interest rate incentives, y . MBS investors price and hedge their portfolios using pricing models in which interest rates are the main stochastic state variable. Moreover, other variables which drive MBS cash flows, such as house price appreciation and credit conditions, do not have traded derivatives, making hedging changes in these systematic state variables costly, imperfect, or infeasible. Thus, although MBS investors duration hedge, the level and sensitivity of prepayments to rate incentives varies systematically, conditional on rate realizations. Our model is aimed at pricing prepayment risk in treasury hedged MBS.

Further, we assume a segmented market in which the stochastic discount factor (SDF) arises from a representative MBS investor who is undiversified and holds the universe of MBS. Such a stochastic discount factor can be motivated by specialized investors as in Gabaix, Krishnamurthy, and Vigneron (2007) and He and Krishnamurthy (2013). In particular, we assume the following SDF:

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_{x,M} dZ_t^x - \lambda_{y,M} dZ_t^y \quad (1)$$

where $\lambda_{x,M}$ is the price of risk for “turnover” risk, x_t , and $\lambda_{y,M}$ is the price of risk for “rate-sensitivity” risk, y_t , and $M \in \{DM, PM\}$ indicates that risk prices are conditional on market type; either discount (DM) or premium (PM). Market type is determined by which security type is predominant, either discount (price below par) or premium (price above par). The type of security which is predominant in terms of remaining principal balance determines whether prepayment is either value increasing or decreasing for the overall MBS market. We then derive our linear asset pricing model by computing the difference in drifts in expected MBS returns under the physical and risk-neutral measure as follows:

$$\overbrace{(\mu^i - r_f)}^{E_M[R^{ei}]} dt = \lambda_{x,M} \overbrace{\sigma_x \frac{\partial P^i}{\partial x} \frac{1}{P^i}}^{\beta_x^i} dt + \lambda_{y,M} \overbrace{\sigma_y \frac{\partial P^i}{\partial y} \frac{1}{P^i}}^{\beta_y^i} dt, \quad (2)$$

using the notation $E_M[R^{ei}]$ to denote expected returns conditional on market type $M \in \{DM, PM\}$, where e denotes the excess return after treasury hedging, and

i denotes the security.⁴ We define securities by the coupon of the MBS security relative to the par coupon, and note that discount securities have coupons lower than the par coupon, while premium securities have coupons higher than the par coupon. Simplifying notation, this leads to the following conditional linear model, familiar-looking from linear equity pricing models, for the cross section of treasury-hedged MBS returns:⁵

$$E_M[R^{ei}] = \lambda_{x,M}\beta_x^i + \lambda_{y,M}\beta_y^i. \quad (3)$$

Following, Gabaix, Krishnamurthy, and Vigneron (2007) and Boyarchenko, Fuster, and Lucca (2014), we develop the intuition for our model using a first order approximation of MBS prices around the no prepayment uncertainty case. There is a constant par coupon rate, r , which represents the opportunity cost of capital for the representative, specialized, MBS investor who can reinvest portfolio proceeds in par MBS securities, as in Fabozzi (2006). There is a securitized mortgage pool (MBS) i with prepayment rate ϕ^i and coupon c^i . We normalize the initial mortgage pool balance b_0^i to one and denote the remaining principal balance b_t^i , with

$$\frac{db_t^i}{dt} = -\phi^i b_t^i. \quad (4)$$

The first order linear approximation of the value of the MBS pass-through around the no prepayment uncertainty case is then given by:

$$P_0^i = \int_0^\infty e^{-rt} (b_t^i c^i - db_t^i) dt = b_0^i + (c^i - r) \int_0^\infty e^{-(r+\phi^i)t} dt = 1 + \frac{c^i - r}{r + \phi^i} \quad (5)$$

Thus, the value of the MBS is approximately its par value plus the value of the coupon strip. The value of the coupon strip increases in the difference between the coupon and current rates, and is negative for discount securities and positive for premium securities. Accordingly, the value of the coupon strip decreases with the speed of prepayment if $c^i - r$ is positive, and increases with the speed of prepayment if $c^i - r$ is negative. Using this first-order approximation, we can derive expressions for the

⁴Our model is a conditional asset pricing model, as in Jagannathan and Wang (1996), with one difference. In our model, loadings are constant, and only prices of risk vary.

⁵See Cochrane (2005) for a textbook description of the theory and econometrics of linear asset pricing models, including models with conditioning information for risk prices.

approximate factor loadings on turnover and rate-sensitivity shocks, β_x^i and β_y^i as follows:

$$\beta_x^i = \sigma_x \frac{\partial P^i}{\partial x} \frac{1}{P^i} = \sigma_x \frac{\partial P^i}{\partial \phi^i} \frac{\partial \phi^i}{\partial x} \frac{1}{P^i} = \sigma_x \frac{r - c^i}{(r + \phi^i)(\phi^i + c^i)} \frac{\partial \phi^i}{\partial x}, \quad (6)$$

and,

$$\beta_y^i = \sigma_y \frac{\partial P^i}{\partial y} \frac{1}{P^i} = \sigma_y \frac{\partial P^i}{\partial \phi^i} \frac{\partial \phi^i}{\partial y} \frac{1}{P^i} = \sigma_y \frac{r - c^i}{(r + \phi^i)(\phi^i + c^i)} \frac{\partial \phi^i}{\partial y}. \quad (7)$$

Given that prepayment a positive shock to either x or y implies an increase in prepayment, these expressions give us the first testable hypothesis of our model, which we state in Lemma 1:

Lemma 1. *If $c^i - r > 0$, then $\beta_x^i < 0$ and $\beta_y^i < 0$. If $c^i - r < 0$, then $\beta_x^i > 0$ and $\beta_y^i \geq 0$.*

In other words, premium securities, for which $c^i - r > 0$, will have negative loadings on turnover and rate-sensitivity risk. Intuitively, these securities have coupons that are above current mortgage rates, and so their value deteriorates with faster prepayment. On the other hand, discount securities, for which $c^i - r < 0$, load positively on prepayment risk. Discount securities have coupon rates that are below the opportunity cost of re-invested capital, and hence their value increases if prepayment speeds increase. This intuition is readily apparent from the right hand side of Equation (5). The prepayment rate ϕ^i acts like an additional discount rate of the cash flows in the numerator. When $c^i < r$, the numerator is negative and an increase in the prepayment rate essentially discounts that negative cash flow more, increasing the value of the discount MBS. When $c^i > r$, the numerator is positive and an increase in discounting in the denominator, reduces the value of the premium MBS.

We further specify the following stylized model for prepayment, where our notation now allows prepayment to vary over time in order make the connection with our empirical work clear:

$$\phi_t^i = x_t + y_t \max(0, m^i - m_t). \quad (8)$$

We use m^i to denote the borrowers' loan rates for the loans underlying the MBS with coupon i (i.e. the coupon i MBS's "Weighted Average Coupon" or WAC), and m_t to denote the current mortgage rate (measured by the Freddie Mac Primary Mortgage Market Survey rate, for example). We assume that $c^i - r = m^i - m_t$, so that

the moneyness of the borrowers' long prepayment options matches that of the MBS investors' short options. This assumption is not crucial but it helps facilitate exposition. Although we abstract from variation in the spread between the MBS coupons, c^i , and the underlying borrowers' loan rates, m^i , we will use separate data on each of these rates in our empirical work and so we use separate notation for clarity. The moneyness of borrowers' prepayment options ("borrower moneyness") is measured by $m^i - m_t$. The moneyness from investors' perspective ("investor moneyness"), $c^i - r$ captures how the security's value changes with prepayment, which sets the security's value to par value. We use borrower moneyness to estimate the prepayment risk factor, since the borrowers themselves make the prepayment decisions. Then, to define securities, and to study financial payoffs and returns to these securities, we use investor moneyness.

Figure 1 plots prepayment as a function of borrower moneyness and the realization of the x and y prepayment factors. Using this model, we have for discount securities:

$$\phi_t^{i,\text{disc}} = x_t, \quad (9)$$

and for premium securities

$$\phi_t^{i,\text{prem}} = x_t + y_t \max(0, m^i - m_t). \quad (10)$$

Superscripts denote securities by relative coupon, $i = c^i - c^{\text{par}}$ and prem indicates that the MBS is a premium security, i.e. $c^i - c^{\text{par}} > 0$. Further, we have that for discount securities,

$$\frac{\partial \phi^{i,\text{disc}}}{\partial x} = 1 \quad \text{and} \quad \frac{\partial \phi^{i,\text{disc}}}{\partial y} = 0. \quad (11)$$

For premium securities, we have

$$\frac{\partial \phi^{i,\text{prem}}}{\partial x} = 1 \quad \text{and} \quad \frac{\partial \phi^{i,\text{prem}}}{\partial y} = (m^i - m_t). \quad (12)$$

Using these expressions in Equations (6) and (7) for β_x^i and β_y^i , we have the following additional testable implications for the two prepayment risk factor loadings:

Lemma 2. *For discount securities, using i to denote the security defined by $c^i - r$*

where for discounts $c^i - r < 0$ we have:

(i) $\beta_x^{i, disc}$ is monotonically decreasing in c^i . That is, we expect securities which trade at a larger discount to par have larger positive loadings on the turnover prepayment risk factor.

(ii) $\beta_y^{i, disc} = 0$.

For premium securities, using i to denote the security defined by $c^i - r$ where for premiums $c^i - r > 0$ we have:

(i) $|\beta_x^{i, prem}|$ is monotonically increasing in c^i . That is, we expect securities which trade at a larger premium relative to par have more negative loadings on the turnover prepayment risk factor.

(ii) $|\beta_y^{i, prem}|$ is monotonically increasing in c^i . That is, we expect securities which trade at a larger premium relative to par to have more negative loadings on the rate-sensitivity prepayment risk factor.

See Table 1 for a tabular summary the model's main predictions for factor loadings from Lemmas 1 and 2 describing the signs and relative absolute magnitudes of the prepayment risk factor loadings across discount and premium securities defined by their coupon relative to the par coupon.

With these results in hand, we now turn to our model's predictions for the signs of the prices of risk $\lambda_{x,M}$ and $\lambda_{y,M}$. Because markets are segmented, and the signs of securities' changes in value in response to prepayment shocks vary across the coupon stack, the sensitivity of the representative MBS investor's wealth can change sign with the composition of the market, whether comprised predominantly by premium or discount securities.⁶ That is, we expect the prepayment risk prices to vary over time, and to change sign as the market moves from discount heavy to premium heavy. This is because, if the market is comprised mostly of discount securities, then the representative investor is averse to states of the world in which discount securities deteriorate in value, namely low prepayment states. On the other hand, if the market

⁶Note that we do not need strict market segmentation. It is sufficient that the sensitivity of the representative MBS investor's wealth can change sign with the composition of the market.

is comprised mostly of premium securities, then the representative investor demands compensation for securities which increase their downside exposure in states of the world in which prepayment is high, causing premium securities to lose value. In other words, the prices of risk are determined by the sign of the change in wealth for a representative, specialized MBS investor who invests in the universe of MBS securities. To fix ideas, consider that, in a strictly segmented market and under the standard assumptions necessary to guarantee the existence of a representative agent, we can write the wealth of the representative MBS investor that holds the MBS portfolio as:

$$W = \sum_i P^i \text{RPB}^i$$

where P^i is given in equation (5), and RPB^i denotes the remaining principal balance of security i . It is clear that $\frac{\partial W}{\partial x}$ and $\frac{\partial W}{\partial y}$ inherit the sign of the partial derivative of the price of the majority RPB security type with respect to the shock. As long as the representative investor dislikes states of the world in which their wealth declines, we have that investors will require compensating risk premia for holding securities whose returns are positively correlated with changes in their wealth.⁷ Namely, they will require positive risk premia for the predominant security type, either discount or premium. Thus, in a premium heavy market, we expect that

$$E_{\text{PM}}[R^{ei,\text{prem}}] = \lambda_{x,\text{PM}}\beta_x^{i,\text{prem}} + \lambda_{y,\text{PM}}\beta_y^{i,\text{prem}} > 0, \quad (13)$$

where we use PM to denote the expectation conditional on “premium market” dates, namely dates at which more than 50% of total MBS remaining principal balance trades at a premium. Again, superscripts denote securities by relative coupon, $i = c^i - r$ and prem indicates that the MBS is a premium security, i.e. $c^i - r > 0$. Since $\beta_x^{i,\text{prem}}$ and $\beta_y^{i,\text{prem}}$ are both negative, we expect that both $\lambda_{x,\text{PM}}$ and $\lambda_{y,\text{PM}}$ are negative.

By contrast, in a discount market, we expect that:

$$E_{\text{DM}}[R^{ei,\text{disc}}] = \lambda_{x,\text{DM}}\beta_x^{i,\text{disc}} > 0 \quad (14)$$

⁷High state prices when investor wealth declines can be motivated, for example, by short termism, value at risk constraints, or compensation concerns.

where we use E_{DM} to denote the expectation conditional on “discount market” dates, namely dates at which 50% or more of total MBS remaining principal balance trades at a discount. Superscripts denote securities by relative coupon, $i = c^i - r$ and disc indicates that the MBS is a discount security, i.e. $c^i - r$. Since $\beta_x^{i, disc} > 0$, implies that $\lambda_{x, DM} > 0$. The sign on λ_y , however, is less straightforward in discount heavy markets. The loading on y for discount securities is zero. If the realized rate-sensitivity shock is high, our model predicts that there is no effect on the valuation of discount securities. However, MBS investors should require compensation from premium securities from their exposure to the y shock, despite the fact that premium securities are a less important part of their portfolio in discount markets. That is, $\lambda_{y, M}$ should be negative in both premium and discount markets. This is because discount securities, which drive risk pricing of level risk in a discount market, do not load on the y shock and so these shocks should be priced by their (always negative) effect on the cash flows from premium securities. This implies that, in principal, in a discount market, expected returns on premium securities may be positive or negative:

$$E_{DM}[R^{ei, prem}] = \lambda_{x, DM}\beta_x^{i, prem} + \lambda_{y, DM}\beta_y^{i, prem} <> 0. \quad (15)$$

Then, we have the following hypothesis regarding the signs of the prices of prepayment risk:

Hypothesis 1. *In a segmented market, in which a representative MBS investor holds the aggregate MBS portfolio, we have the following signs for the prices of level and rate-sensitivity risk, depending on market type:*

- (i) **Premium Market:** *When the market is comprised mainly by premium securities, the representative investor requires compensation for bearing the risk that prepayment is higher than expected due to either factor. That is, we expect that $\lambda_{x, PM}$ and $\lambda_{y, PM}$ are both negative. Given the predictions for the signs of the risk loadings (β 's) from Lemma 1, this implies that $E_{PM}[R^{ei, prem}] > 0$ and $E_{PM}[R^{ei, disc}] < 0$.*
- (ii) **Discount Market:** *When the market is comprised mainly by discount securi-*

ties, the representative investor requires compensation for bearing the risk that prepayment is lower than expected. That is, we expect that $\lambda_{x,DM} > 0$. Because discount securities should not load on rate-sensitivity risk ($\beta_y^{i,disc} = 0$), we expect that $\lambda_{y,DM} < 0$, the same as in premium heavy markets. Given the predictions for the signs of the risk loadings (β 's) from Lemma 1, this implies that $E_{DM}[R^{ei,prem}] < 0$ and $E_{DM}[R^{ei,disc}] > 0$.

Table 2 summarizes these predictions for the prices of risk by market type. Figure 2 graphs the model's predictions for relative coupon expected returns by market type.

4 Data

The Appendix contains a detailed description of the data and its construction. The following is a brief introduction to our data sources and methodology.

Our return data come from Bloomberg Barclays Hedged MBS Return indices. Index returns are available at a monthly frequency back to 1994. The indices are constructed using prices of liquid cash MBS that are deliverable in the to-be-announced (TBA) forward market. The TBA market constitutes the vast majority of MBS trading volume.⁸ We use hedged returns of coupon-level aggregates of Fannie Mae 30-year fixed-rate MBS pools. Hedged returns are computed by Barclays using a term structure-matched position in Treasuries based on a key-rate duration approach. In the Appendix, we report results for prepayment risk loadings using Barclays raw index returns hedged with a simple empirical hedging model.

A given coupon may trade at a premium or discount depending on current mortgage rates. Thus, we define securities by their coupon relative to the current par coupon in order to obtain securities with more stable exposures to prepayment risk. Specifically, we compute the difference between the coupon of each liquid MBS at each date, and the par coupon on that date. We compute the par coupon using the TBA prices of securities trading near par. We then use data from eMBS to compute the remaining principal balance (RPB) for each MBS relative coupon.

⁸See Vickery and Wright (2013) for a detailed description of the TBA market. Gao et al. (forthcoming) study the relation between the TBA and cash MBS market. Finally, Song and Zhu (2016) studies MBS financing rates implied by TBA market prices.

We utilize two sources for prepayment data. The first is Bloomberg’s monthly report of the median dealer prepayment forecast by coupon. Bloomberg collects these data via survey. We use forecasts for Fannie Mae 30-year fixed securities for the base rate scenario, since current forward rates should approximately reflect rate expectations over the month. Using realized rates requires conditioning on future rate realizations. However, because rates rarely move over 50bps within the month, results using the forecast for the realized rate scenario, available upon request, are very similar to those using the base rate scenario. We collect realized prepayment data for Fannie Mae 30-year fixed securities by coupon monthly from eMBS. To compute prepayment shocks, we also measure the moneyness of borrowers’ prepayment options for each MBS coupon. To do this, we collect data on weighted average coupons (WAC) for each MBS coupon. These WAC’s measure the underlying borrower loan rates. Then, we compare these rates to the current mortgage rate as reported weekly by Freddie Mac in their Primary Mortgage Market Survey (PMMS). We use a monthly average of the weekly primary mortgage rates as the current mortgage rate.

5 Empirical Analysis

Our model is

$$E_M[R^{ei}] = \lambda_{x,M}\beta_x^i + \lambda_{y,M}\beta_y^i.$$

We estimate our linear factor model using standard Fama and MacBeth (1973) techniques, while providing additional pooled time series, cross section OLS results for robustness. Our analysis proceeds in three steps, which we label zero, one, and two. Steps one and two consist of standard Fama McBeth regressions. The first step is a time series regression, run for each asset in the cross section, of asset returns on factor innovations. This first step yields estimated factor loadings for each asset. The second step is a series of cross section regressions, one for each date, of returns on estimated factor loadings. This second step generates prices of risk by averaging the estimated cross section coefficients of returns on on factor loadings over time. We use the terminology “Step 0” to describe the step in which we estimate the prepayment risk factors. The following sections describe the method and results for each step.

Additional details appear in the Appendix.

5.1 Step 0: Prepayment Risk Factors

In order to measure β_x^i and β_y^i using the time series regression for the first stage Fama McBeth regression, we need time series for shocks to x_t and y_t . We use shocks to the level and rate-sensitivity factors, since expected prepayments should not affect returns. To extract the prepayment shocks, we use the difference between forecasted and realized prepayments. Each month, dealers provide Bloomberg with their forecast for prepayments for each MBS coupon, and for several possible future interest rate scenarios. For our estimate of forecasted prepayments, we use the Bloomberg median forecast for the base interest rate scenario for each coupon.⁹ We obtain realized prepayments for each MBS coupon from eMBS. Realized prepayments are reported on the eMBS website on the 4th business day of the month for the prior month. The Appendix contains further details on the data and our methodology. We estimate innovations to the level and turnover prepayment risk factors as follows. First, we estimate the following cross section regression across available underlying borrower loan rates using the forecast data in each month:

$$ppmt_t^{i, \text{forecast}} = x_t^{\text{forecast}} + y_t^{\text{forecast}} \max(0, m^i - m_t^{\text{PMMS}}) + \epsilon_t^i. \quad (16)$$

We use the Weighted Average Coupon (WAC) of the loans underlying MBS with a particular coupon i to measure borrower loan rates m^i . The prevailing mortgage rate m_t^{PMMS} is obtained from the Freddie Mac Primary Mortgage Market Survey (PMMS). The second term is positive for MBS with underlying borrower loan rates which are above prevailing rates, and zero otherwise. In this regression, the estimated intercept, $\hat{x}_t^{\text{forecast}}$ measures the forecasted level of prepayments, while the forecasted slope on the rate incentive for borrowers' with in-the-money prepayment options is estimated by

⁹Results using the ex-post rate realization forecast are similar, since few rate realizations are more than 50bps different from the base interest rate. See also Carlin, Longstaff, and Matoba (2014), who forecast TBA returns using a measure of disagreement regarding prepayment rates across dealers.

$\hat{y}_t^{\text{forecast}}$. Next, we run the same regression in realized prepayment data for each month:

$$ppmt_t^{i,\text{realized}} = x_t^{\text{realized}} + y_t^{\text{realized}} \max(0, m^i - m_t^{\text{PMMS}}) + \epsilon_t^i. \quad (17)$$

For parsimony, we use the notation x_t and y_t to denote these innovations. Innovations in the realized relative to forecasted level of prepayments x_t are measured as

$$x_t = \hat{x}_t^{\text{realized}} - \hat{x}_t^{\text{forecast}}. \quad (18)$$

Similarly, innovations in the realized relative to forecasted rate-sensitivity of prepayments y_t are measured as:

$$y_t = \hat{y}_t^{\text{realized}} - \hat{y}_t^{\text{forecast}}. \quad (19)$$

Figure 3 presents a graphical representation of the estimation of x_t and y_t . Figure 4 presents four sample months of the forecast and realized prepayment curves that are used for estimation.

Figure 5 plots the time series for the two prepayment risk factors. We note that our time series estimates of the turnover and rate-sensitivity prepayment risk factors are an additional contribution of our work, since prior studies use model implied estimates, or principal components from pricing data to measure prepayment risk, whereas our series are estimated from differences between actual forecasted and realized prepayment data. The correlation between the innovations in x and y is low, at 0.13. The series are, however, autocorrelated (0.78 for x and 0.66 for y). We argue that despite this measured autocorrelation, these innovations should be considered “surprises” in the context of MBS price setting behavior. It is standard for dealers and investors to use statistical models to forecast prepayment. When data which is inconsistent with the model arrives, they face a tradeoff for updating their model. If they update the model too often, then it is not a model, but instead just a statistical description of current data. On the other hand, if the data consistently contradicts the model over a longer time period, parameters are updated. This behavior leads to slowly changing prepayment models, and persistent prepayment model errors. For example, the average autocorrelation of Barclays’ prepayment model implied Option Adjusted Spreads across coupons is 0.83. This high autocorrelation is consistent

with persistent prepayment model errors. Elevated OAS may reflect persistent poor performance of prepayment models which only feature stochastic interest rates when prepayment is being driven by other state variables. Despite being persistent, then, prepayment errors are correlated with returns because investors' prepayment model output feeds directly into MBS pricing on both the buy and sell side. Indeed, the fact that first stage Fama MacBeth regressions of returns on the estimated factors yield significant loadings supports the interpretation of the x_t and y_t series as shocks. The largest innovations also confirm this interpretation. The largest x_t innovation occurs in January of 2009, when prepayments declined in association with the financial crisis. The largest y_t innovation occurs in March of 2010, when prepayments increased due to Fannie Mae's buyouts of delinquent loans with higher coupons.

5.2 Step 1: Factor Loadings

With the level and rate-sensitivity factors in hand, we can estimate prepayment risk factor loadings using the following time series regression for each relative coupon i :

$$R_t^{ei} = a^i + \beta_x^i x_t + \beta_y^i y_t + \epsilon_t^i. \quad (20)$$

We use the Barclays MBS Index Excess Returns, available at the coupon level. Barclays uses a proprietary prepayment model to compute key-rate durations, and constructs hedged MBS returns using these key-rate durations and US treasury returns. Details regarding the index returns construction can be found in Phelps (2015). We also provide further detail in the Appendix, including the precise timing of measurement for each variable, and results using alternative data series.¹⁰ We define securities by their coupon relative to the par coupon, rather than by their absolute coupon. This is because the sensitivities of securities' values with respect to prepayment (the risk factor loadings) vary less over time for securities defined by their relative coupon than by their absolute coupon, as can be seen in Lemma 2. For example, an MBS with a 5% coupon has varied from being discount to par to being premium relative to par over our sample. When the 5% coupon was discount, its value increased with prepay-

¹⁰We provide results using short term prepayment forecasts from a single dealer, results for empirically rate-hedged returns, and results for Barclays excess returns hedged to rate volatility returns.

ment speeds, and vice versa when it became premium. In fact, we will show that the characteristic we use to define securities, relative moneyness, will have a monotonic relationship with factor loadings. This supports our model as well as using relative moneyness to define a “security”.

Table 3 displays summary statistics for each coupon relative to par, from -2% to 3.5%. Due to data limitations, we use full sample estimates for the factor loadings, however we provide evidence below for stable loadings for securities defined by their relative coupon. Table 4 presents our estimated loadings when we restrict $\beta_y^{\text{disc}} = 0$, as in a strict interpretation of our model.¹¹ The restricted estimates are exactly consistent with the results of Lemma 1, which predicts positive loadings for discount securities, and negative loadings for premium securities. Turning to the predictions of Lemma 2, which uses the prepayment model in Equation (10), we see that the results also closely match each of the more detailed predictions of the model stated in Lemma 2. Not only do the signs match the model’s predictions, but also the loadings for both x and y are monotonically decreasing in the absolute value of the relative coupon. Finally, we note that the loadings tend to be more significant in the tails of the relative coupon space, i.e. the pattern of significance follows the pattern of the absolute magnitude of the coefficients. Figure 6 plots the coefficients for a visual description of the fit between the model’s predictions and our empirical findings.

We present unrestricted results in Table 5. As can be seen, the results are very similar, and the R^2 do not change much between the unconstrained and constrained specifications. The signs for the loadings in Table 5 also match the predictions of Lemma 1. Lemma 1 uses only the pricing model, without a specific model for how x and y affect prepayments across the coupon stack.

5.3 Step 2: Prices of Risk

With our estimated loadings in hand, we turn to estimating the four prices of risk, $\lambda_{x,M}$ and $\lambda_{y,M}$, $M \in \{DM, PM\}$ using the following cross section regressions each month:

$$R_{t,M}^{ei} = a_{t,M} + \lambda_{t,x,M} \hat{\beta}_x^i + \lambda_{t,y,M} \hat{\beta}_y^i + \epsilon_t^i. \quad (21)$$

¹¹The intercepts in all regressions used to estimate factor loadings are less than 0.1%, and insignificant, for all securities, and so we do not report them.

Following Fama and MacBeth (1973), we then use average risk prices over time to estimate the risk prices, $\lambda_{x,M}$ and $\lambda_{y,M}$, using only data from either discount markets (DM), or premium markets (PM), to respectively measure each conditional risk price. We measure market type using beginning of month data. As described in Hypothesis 1, we expect that the signs of the prices of risk depend on the market composition. We measure market composition using the percent of remaining principal balance (RPB) that is discount at the beginning of the month. We discuss alternative measures in the Appendix. Figure 7 plots the market composition over time. We classify a month as discount if greater than 50% of the outstanding MBS balance trades at a discount, and premium otherwise. Table 6 presents summary statistics by relative coupons and for the subsamples defined by whether the market type is premium or discount. Figure 8 plots the average returns by relative coupon for all months, and then by averaging within discount, and within premium months only. We note the similarity between Figure 8 from the data, and Figure 2 from the model.

Note also that ignoring market type biases conditional return estimates towards zero for months in which conditional average returns are positive. This can be seen by the fact that, conditional on the market type leading to positive average returns for a particular security, the green solid line plotting unconditional returns is closer to zero than the line plotting returns conditional on market type. Thus, in discount markets, when discount securities earn higher average returns, the unconditional average return estimate is lower than the estimate conditional on months in which 50% or more of total remaining principal balance trades at a discount. Similarly, in premium markets, when premium securities earn higher average returns, the unconditional average return estimate is lower than the estimate conditional on months in which more than 50% of total remaining principal balance trades at a premium. This can also be seen by comparing the unconditional summary statistics in Table 3 to the summary statistics conditional on market type in Table 6. Using unconditional average returns biases discount security average returns downward in discount months, and biases premium security average returns downward in premium months.

One challenge with estimating $\lambda_{x,M}$ and $\lambda_{y,M}$ is that the loadings across factors are highly correlated for each security, leading to a multicollinearity problem. This can be seen in Table 4 and in Figure 6. To alleviate the multicollinearity somewhat,

when running the second stage regressions, we drop months in which the cross section correlation amongst factor loadings is greater than 0.90. The results from the second stage regression appear in Table 7. The signs are as predicted by Hypothesis 1. In terms of statistical significance, two of the risk prices are significant at the 85% significance level, and one is significant at the 90% level. This may seem relatively low in the context of cross section tests in equity markets, but it is important to note that we are restricted to a much smaller cross section. On average, we have seven securities per month. We have at least five coupons 97% of all months. Figure 9 plots the predicted returns from the model using Fama MacBeth estimates for the risk prices in the top panel, and realized average returns in the bottom panel. That is, we plot:

$$\widehat{E}_M[R^{ei}] = \hat{a}_M + \hat{\lambda}_{x,M} \hat{\beta}_x^i + \hat{\lambda}_{y,M} \hat{\beta}_y^i. \quad (22)$$

All intercepts are very close to zero, and are not statistically significant from zero. We use $M \in \{\text{DM,PM}\}$ to emphasize that we use risk prices which are estimated conditional on market type, as defined by the composition of total remaining principal balance between discount and premium securities. To compute unconditional averages, we weight by the empirical distribution over market types, i.e. we use the actual relative frequency of discount and premium market months that is observed in our sample, and used in Figure 8. Comparing Figure 9 to Figure 8 shows the relatively good fit of the model.

To confront the challenge presented by the multicollinearity of the factor loadings, we perform two additional tests. First, the fact that the loadings on both turnover risk and rate-sensitivity risk are monotonic in relative coupon suggests using the characteristic, relative moneyness, as a single “factor”. Note that this monotonicity is a prediction of Lemma 2, and thus this test also supports our model of priced risk factor loadings, despite using a characteristic as a factor (or, more precisely, a factor loading). In Table 8 we present the results from a second stage Fama MacBeth regression in which we use relative moneyness as the single risk factor. That is, we estimate the conditional risk prices, $\lambda_{c,M}$ using the following cross section regression at each date, and estimate risk prices using the conditional time series average by market type:

$$R_{t,M}^{ei} = a_{t,M} + \lambda_{t,c,M} (c^i - r) + \epsilon_t^i, \quad (23)$$

where, consistent with the notation in Section 3, r denotes the par coupon rate. Consistent with our theory, the price of risk for discount securities is positive in discount months and negative in premium months, and vice versa for premium securities. The bottom panel of Figure 9 plots the predicted returns from the model using relative moneyiness as a single characteristic/factor. That is, we plot

$$\widehat{E_M[R^{ei}]} = \hat{a}_M + \hat{\lambda}_{c,M} (c^i - r). \quad (24)$$

All intercepts are again very close to zero and statistically insignificantly different from zero. We use the empirical distribution over market types to compute unconditional average returns. As can be seen, the predictions of our model are very robust across the two specifications for prepayment risk exposure. Moreover, these results indicate that factor loadings are stable over time, which supports our estimates of β_x^i and β_y^i using the full sample of data.

As a second alternative estimation strategy for prepayment risk prices, we run a pooled time series cross section regression, with interaction terms to capture the effect of market type on the risk prices. Specifically, we run the following regression over all coupons and across all months:

$$R_t^{ei} = a + \kappa_x \beta_x^i + \kappa_y \beta_y^i + \delta_x \beta_x^i (\%RPB_{t,BoM}^{\text{disc}} - 50\%) + \delta_y \beta_y^i (\%RPB_{t,BoM}^{\text{disc}} - 50\%) + \epsilon_t^i. \quad (25)$$

We use BoM to denote observation at the beginning of the month, emphasizing that this is a predictive regression. When the market is perfectly balanced, $\%RPB^{\text{disc}} - 50\% = 0$, and κ_x and κ_y should thus be zero. Investors are naturally prepayment-risk-hedged from their balanced portfolio of discount and premium securities, and thus should not require prepayment risk premium. On the other hand, we expect that δ_x and δ_y should both be positive. In discount heavy months, $\%RPB^{\text{disc}} - 50\% > 0$, and since discount securities have positive loadings β_x^i , and zero β_y^i , a positive δ_x leads to the model-implied higher expected returns for discount securities in discount months. Similarly, in premium heavy months, $\%RPB^{\text{disc}} - 50\% < 0$, and since premium securities have negative loadings β_x^i and β_y^i , positive δ_x and δ_y lead to the model-implied higher expected returns for premium securities in premium months. Table 9 presents the

results. As predicted, the κ 's are zero, and the δ 's are positive. The δ 's are jointly significant at the 87% level. Thus, the pooled time series cross section results provide additional support for the model's implications. We present standard errors clustered by time.¹² Note that we predict monthly returns with an R^2 of 1% without time fixed effects. However, we acknowledge the fact that, in addition to the risk factors which change the shape of expected returns in the cross section, there are likely to be shocks or risk factors that move the entire coupon stack of returns. Thus, we also report results including a time fixed effect. Table 9 shows that the results with time fixed effects are qualitatively similar. The fixed level loadings remain very small and statistically insignificantly different from zero. Both of the interaction coefficients increase in magnitude, and the interaction between β_x^i and relative coupon is statistically significant at the 10% level. Including time fixed effects, the δ 's are jointly significant at the 97% level. Overall, these results are very consistent with our pricing model, and with the Fama MacBeth results.

Another way of assessing our pricing model is to compare it to MBS market models using constant risk prices. We consider two benchmark models. The first uses the return on the RPB weighted MBS market return as the single factor. That is, we estimate factor loadings using the following time series regression by coupon:

$$R_t^{ei} = a^i + \beta^{i, \text{VW}_{all}} R_t^{\text{VW}_{all}} + \epsilon_t^i, \quad (26)$$

where VW_{all} uses the hedged coupon return series, along with RPB by coupon, to construct a value weighted index. We then estimate risk prices $\lambda^{\text{VW}_{all}}$ using the average risk prices from the following cross section regressions each month:

$$R_t^{ei} = a_t + \hat{\beta}^{i, \text{VW}_{all}} \lambda_t^{\text{VW}_{all}} + \epsilon_t^i, \quad (27)$$

Predicted returns from this model, using the time series averages of the cross section

¹²In an asset pricing context, we expect it to be most important to cluster errors in the time dimension, see Petersen (2011). Standard errors are smaller using coupon and time clusters, however the size of the clusters becomes small.

intercepts and slopes, are:

$$\widehat{E[R^{ei}]} = \hat{a} + \hat{\beta}^{i, \text{vW}_{all}} \hat{\lambda}^{\text{vW}_{all}}. \quad (28)$$

The second uses the return on a spread asset constructed by going long the maximum coupon in each month, and short the minimum coupon in each month. We scale this spread asset so that its return has equal leg volatility and constant volatility over time. The intuition for this benchmark model is that it makes use of the monotonicity of the factor loadings, but not the time varying risk prices. Thus, by comparing our model to these two benchmark models we see that, (1) it is important to construct loadings on prepayment risk factors, rather than on MBS market returns, and (2) it is important to condition risk price estimation on market type. The second benchmark model is estimated using the following time series regression by coupon:

$$R_t^{ei} = a^i + \beta^{i, \text{Max-Min}} R_t^{\text{Max-Min}} + \epsilon_t^i, \quad (29)$$

where $R_t^{\text{Max-Min}}$ is the return from going long the maximum premium coupon and short the minimum discount coupon. We then estimate risk prices $\lambda^{\text{Max-Min}}$ using the average risk prices from the following cross section regressions each month:

$$R_t^{ei} = a_t + \hat{\beta}^{i, \text{Max-Min}} \lambda_t^{\text{Max-Min}} + \epsilon_t^i, \quad (30)$$

Predicted returns from this model, using the time series average of the cross section intercepts and slopes, are:

$$\widehat{E[R^{ei}]} = \hat{a} + \hat{\beta}^{i, \text{Max-Min}} \hat{\lambda}^{\text{Max-Min}}. \quad (31)$$

Figure 10 presents scatter plots of the results for the models in Equations (28) and (31) conditional on market type, and over the full sample. Each column is one model, and rows plot different market types. We compare these results to the results for the models implied by our theory. Figure 11 plots the results for the models described in Equations (22) and (24) conditional on market type, and over the full sample. The superior performance of the models implied by our theory is clear in the

figures. The left column of Figure 10, plots the benchmark model using the return on the RPB weighted market-level return to MBS as the single factor. The estimated β 's from this model are approximately equal to one for all relative coupons, thus, the predicted returns are nearly equal whereas the actual realized returns display substantial variation. The right column of Figure 10 plots the benchmark model using the return on a spread asset constructed by going long the maximum coupon in each month, and short the minimum coupon in each month. This factor creates more spread in β 's, and it performs slightly better. The improvement in performance is primarily in premium market months. This is because the loadings (not reported) are monotonically increasing in relative coupon, negative for discount securities and positive for premium securities. The estimated price of risk is positive. Then, in premium markets this model correctly predicts that premium securities should have higher expected returns. In discount markets, predicted returns are the same, however realized returns have the opposite pattern and this model gets the wrong sign for the slope of returns across relative coupons. As a result, the overall performance is poor, as can be seen in the plot for the full sample, in the bottom row of the figure. The left column of Figure 11 plots the results for the model described in Equation (22), with level and rate-sensitivity risk factors. Two things improve the fit of this model. First, this model produces a larger spread in β 's than either benchmark model. Second, allowing the price of risk to vary by market type allows the model to match the slope of average returns in the cross section of relative coupons in both market types, and hence in the full sample. The right column of Figure 11 plots the results for the model described in Equation (24), with relative coupon as the single factor/characteristic. This model is also implied by our theory, and has a good fit. Thus, the two models which are consistent with our theory appear to offer a substantial improvement over the benchmark models using passive indices. The better performance of the two models we propose can also be measured by the root mean squared errors for each model for the full sample, corresponding to the bottom row of Figures 10 and 11. These are 0.68% for the value weighted market model, 0.67% for the Max-Min model, 0.46% for the two factor model, and 0.21% for the relative moneyiness model.

5.4 Time Series Results: Timed Spread Asset

The results of our estimated model

$$E_M[R^{ei}] = \lambda_{x,M}\beta_x^i + \lambda_{y,M}\beta_y^i$$

suggest implementing an active strategy consisting of a long-short spread asset which changes direction with market type. Since loadings are monotonic in coupon, and given our estimated time varying risk prices, the results suggest going long the deepest discount security and short the most premium security in discount heavy markets, and vice versa in premium markets. Intuitively, this spread asset is designed to harvest the risk premium earned for bearing prepayment risk that is hard to hedge with US treasuries. To construct our active spread asset strategy, we restrict the asset to have a constant volatility over time, and to have equal volatility in the long and short legs, which is standard. The Sharpe ratio¹³ of this optimal spread asset is 0.76. This is 2.62 times the Sharpe ratio of a passive value weighted MBS index. Table 10 presents results for the Sharpe ratios of passive spread assets, the optimally timed spread asset, and passive indices over the full sample, and within discount and premium months. The final row, using the full sample, shows the superior performance of the optimally timed spread asset over all other strategies. The conditional Sharpe ratios are also informative, since the Sharpe ratio for any strategy that is always long discount securities has a Sharpe ratio that is positive in discount months, and negative in discount months. The converse is true for any strategy that is always long premium securities.

We also present Information ratios, a version of the active Sharpe ratio which controls for the correlation between the actively managed portfolio and the passive benchmark since it is the excess return relative to the standard deviation of the active return less the benchmark return:

$$\frac{E[R^{\text{Active}} - R^{\text{Benchmark}}]}{\sigma(R^{\text{Active}} - R^{\text{Benchmark}})}$$

where $R^{\text{Benchmark}}$ is the benchmark return. Table 12 displays the excess return, tracking

¹³Sharpe (1966).

error, and information ratio for our model-implied optimally timed portfolio relative to three passive benchmarks, namely, a passive long maximum premium coupon short minimum discount coupon portfolio with constant volatility and equal-leg volatility, a passive long maximum premium coupon short par portfolio with constant volatility and equal-leg volatility, and the remaining principal balance weighted MBS index. In all cases, the information ratio is about 0.3, which seems high for our simple strategy.

To study the magnitude of risk loadings and α 's with respect to passive benchmarks, we regress the optimally timed spread asset returns on four passive benchmarks. That is, we estimate:

$$R_t^{\text{model implied spread asset}} = \alpha + \beta^{\text{Benchmark}} R_t^{\text{Benchmark}} + \epsilon_t \quad (32)$$

where $R_t^{\text{Benchmark}}$ is one of four benchmark returns, namely, the remaining principal balance weighted MBS index, VW_{all} , the remaining principal balance weighted MBS index amongst premium securities only, VW_{prem} , an untimed long maximum premium coupon short minimum discount coupon portfolio with constant volatility and equal-leg volatility, Max - Min, and an untimed long maximum premium coupon short par coupon portfolio with constant volatility and equal leg volatility, Max - Par. The monthly α 's are all highly statistically significant. We note that, importantly, the returns to the active strategy are largely independent of the passive benchmark returns. In particular, the loading on the remaining principal balance weighted MBS market portfolio is -0.08 and the R^2 of this regression is only 1%. The highest loading of the optimally timed strategy, 0.45, is on the Max-Par benchmark, and this regression has an R^2 of 22%. The fact that the R^2 is higher for this benchmark relative to the Max - Min portfolio (9%) is reflective of the fact that despite the fact that it is always optimal to hold some form of the Max - Min portfolio, the optimal long and short legs switch position over time. All of these results are consistent with our finding that neglecting to control for the time varying prices of prepayment risks biases estimates of positive average returns towards zero.

Finally, we compute the cumulative returns from investing in the model-implied, optimally timed spread asset vs. the alternative cumulative returns from the three passive benchmark strategies with the next highest Sharpe ratios, namely, a passive

long maximum premium coupon short minimum discount coupon portfolio with constant volatility and equal-leg volatility, a passive long maximum premium coupon short par portfolio with constant volatility and equal-leg volatility, and the remaining principal balance weighted MBS index. Figure 12 plots the results, and shows that cumulative returns over the last twenty years have been almost double that of the next best strategy. Note that the difference in cumulative returns between the Max-Min strategy (blue line), and the optimal strategy (black line) is entirely driven by optimally switching the long and short legs, conditional on market type. The market has been dominated by premium securities since 2009, so the difference in cumulative returns over this time between these two strategies is constant. The market type will change to discount if rates increase in the future, and at that point the cumulative returns will again diverge. Recall also that these cumulative returns are net of treasury returns, and so are compensation for prepayment risk only.

6 Conclusion

We present a simple, linear asset pricing model for the cross section of MBS returns. We show that loadings on a turnover and rate-sensitivity risk factor are priced in the time series and cross section. We measure the turnover and rate-sensitivity factors using surprises in prepayment realizations relative to prepayment forecasts. Discount securities load positively on turnover prepayment risk, while premium securities load negatively on turnover and rate-sensitivity risk. The measured loadings are monotonic in securities' coupons relative to the par coupon. These predictions for risk loadings are precisely as predicted by the simple pricing model. Using the relative fraction of discount vs. premium securities in the overall MBS market, we show that the price of prepayment risk is positive in discount markets, and negative in premium markets. This leads to a downward sloping pattern of expected returns in the cross section in discount markets, and an upward sloping pattern in premium markets. Overall, in the pooled time series cross section, the resulting pattern for the cross section of returns is U-shaped in relative moneyness. As a result, failing to account for the market composition, and the associated prices of prepayment risk, leads to estimates of average returns, and risk premia, which are biased. In particular, estimates are

biased downwards when they are positive conditional on market type; discount securities' average returns are underestimated in discount markets and premium securities' average returns are underestimated in premium markets.

Our study provides new evidence of segmented markets for mortgage-backed securities, populated by specialized investors who price market-specific risks. In particular, we show that the price of prepayment risk is determined by whether prepayment is wealth increasing or wealth decreasing for a representative MBS investor who holds the MBS market.

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A. Appendix

Constructing Prepayment Risk Factors

This section provides a detailed description of the construction of the turnover and rate-sensitivity prepayment risk factors, as well as results using an alternative source of prepayment forecasts.

Prepayment Forecasts For our study, we use historical prepayment forecasts obtained from Bloomberg. Specifically, we use a Bloomberg-computed median of prepayment projections submitted by contributing dealers. Projections are available for generic TBA securities defined by agency/program/coupon. In this paper, we focus on prepayment projections for Fannie Mae 30-year TBA securities.

Dealers have the option of updating their prepayment projections on Bloomberg on a daily basis and do so at their own discretion. Bloomberg computes a daily median prepayment forecast based on whatever dealer projections are available at the time. On average, there are about 8-10 contributing dealers. Bloomberg median prepayment forecasts can be downloaded historically with a monthly frequency (i.e. a monthly snapshot on the 15th).

Dealer prepayment forecasts are available for a range of interest rate scenarios. In addition to the base case that assumes rates remain unchanged from current levels, forecasts are also made assuming parallel shifts in the yield curve of $\pm 50, 100, 200, 300$ basis points. We utilize the base case projection for our main analysis. Using realized rates requires conditioning on future rate realizations. However, because rates rarely move over 50bps within the month, results using the forecast for the realized rate scenario, available upon request, are very similar.

The dealer prepayment forecasts on Bloomberg are quoted according to the PSA convention. We convert that to an annualized constant prepayment rate (CPR) using the standard conversion formula:

$$CPR = PSA * \min(6\%, 0.2\% * \text{weighted-average loan age}).$$

For reference, we provide a more detailed description of PSA and CPR:¹⁴

- Constant Prepayment Rate (CPR) and the Securities Industry and Financial Markets Association's Standard Prepayment Model (PSA curve) are the most popular models used to measure prepayments.
- CPR represents the annualized constant rate of principal repayment in excess of scheduled principal amortization.

¹⁴See <http://www.fanniemae.com/resources/file/mbs/pdf/basics-sf-mbs.pdf>.

- The PSA curve is a schedule of prepayments that assumes that prepayments will occur at a rate of 0.2 percent CPR in the first month and will increase an additional 0.2 percent CPR each month until the 30th month and will prepay at a rate of 6 percent CPR thereafter (“100 percent PSA”).
- PSA prepayment speeds are expressed as a multiple of this base scenario. For example, 200 percent PSA assumes annual prepayment rates will be twice as fast in each of these periods; 0.4 percent in the first month, 0.8 percent in the second month, reaching 12 percent in month 30 and remaining at 12 percent after that.

Realized Prepayments Historical realized prepayment rates are obtained via eMBS. The realized prepayment rate is computed based on the pool factors that are reported by the agencies on the fourth business day of each month. The pool factor is the ratio of the amount of remaining principal balance relative to the original principal balance of the pool. Using the pool factors and the scheduled balance of principal for a pool, one can calculate the fraction of the pool balance that was prepaid, that is the unscheduled fraction of the balance that was paid off by borrowers. The prepayment rates reported on eMBS are a 1-month CPR measure. In other words, prepayments are measured as the fraction of the pool at the beginning of the month that was prepaid during that month, yielding a single monthly mortality (SMM) rate. The SMM is then annualized to get the constant prepayment rate (CPR).

Borrower Moneyness We define borrower moneyness or incentive to be the rolling 3-month average of the difference between the weighted-average coupon (WAC) of a Fannie Mae 30-year coupon aggregate and the Freddie Mac Primary Mortgage Market Survey (PMMS) rate for 30-year fixed-rate mortgages.

The Fannie Mae 30-year coupon aggregate is formed by grouping Fannie Mae 30-year MBS pools that have the same specified coupon. The WAC of a MBS pool is defined to be the weighted-average of the gross interest rates of the underlying mortgages in the pool, weighted by the remaining principal balance of each mortgage. Similarly, the WAC of the coupon aggregate is defined to be the weighted-average of the WAC of the underlying MBS pools, weighted by the remaining principal balance of each MBS pool. We obtain historical WAC data for Fannie Mae 30-year coupon aggregates from eMBS. The data is available with monthly frequency and represents an end-of-month snapshot.

The Freddie Mac Primary Mortgage Market Survey (PMMS) is used as an indicator of current mortgage rates. Since April 1971, Freddie Mac has surveyed lenders across the nation weekly to determine the average rates for conventional mortgage products. The survey obtains indicative lender quotes on first-lien prime conventional

conforming home purchase mortgages with a loan-to-value of 80 percent. The survey is collected from Monday through Wednesday and the national average rates for each product are published on Thursday morning. Currently, about 125 lenders are surveyed each week; lender types consist of thrifts, credit unions, commercial banks and mortgage lending companies. The mix of lender types surveyed is approximately proportional to the volume of mortgage loans that each lender type originates nationwide. In our study, we use the historical monthly average PMMS rate for 30-year fixed-rate mortgages, available from Freddie Mac’s website.¹⁵

We use a 3-month average to measure the borrower incentive because we recognize that there is a lag between a refinance application and the resulting closing and actual mortgage prepayment. Refinancing a mortgage can take a considerable amount of time due to the various steps involved, such as credit checks, income verification, and title search.¹⁶ Borrowers can choose to lock in their rate during this time by requesting a rate lock from their lender. The rate locks usually range from 30 to 90 days. In our regression in Equations (16) and (17), the borrower moneyness of a security is determined at the beginning of the month and we only include securities with at least USD 1bn outstanding in RPB as a liquidity filter.

Results using Single Dealer Forecasts Projections reported to Bloomberg, while heavily weighting the first month forward and with approximately exponential decay thereafter, cover the life of the security. We use these forecasts for the main analysis because we can remove dealer-specific noise by using the median forecast, and, even more importantly, because these forecasts are available for a broad cross section and for the entire time period covered by the Barclays MBS Index Returns. Correctly estimating prepayment shocks requires that we include data for a wide sample of coupons in both premium and discount markets. Short term forecasts can sometimes be obtained at the dealer level, but the quality, sample length, and coupon coverage, varies widely. The longest real-time time series of short term forecasts we are able to obtain come from a major dealer and cover the period from January 2001 to June 2016.¹⁷ For that time period, these data cover almost the same broad cross section as the Bloomberg forecast data. This dealer provided us daily data containing the short term forecasts for their models in real time under the assumption that interest rates follow the forward rate at the time of the forecast. Table A.1 presents the prepayment risk factor loadings using the single-dealer one month forward forecast from the 15th

¹⁵See <http://www.freddie.mac.com/pmms/pmms30.htm>.

¹⁶See Hayre and Young (2004).

¹⁷We also explored historical forecast data from other peer dealers. Electronically available data from one peer dealer’s API uses their current prepayment model rather than the model which was used on the historical date. A shorter sample of real-time forecasts from this dealer can be obtained from pdf files back to December 2008, however the cross section coverage is very limited.

of each month January 2001 to June 2016, and shows very similar results to our main analysis. Some significance is lost for discount securities due to the shorter sample which excludes the earlier years in which discount securities were more prevalent.

Table A.1: Factor loadings by relative coupon using single-dealer prepayment forecasts. β_y^{disc} is restricted to equal zero. Standard errors are reported using adjusted degrees of freedom to account for the estimated regressors. The following time series regression is estimated for each security, i :

$$R_t^{ei} = a^i + \beta_x^i x_t + \beta_y^i y_t + \epsilon_t^i \text{ with } \beta_y^{\text{disc}} \equiv 0.$$

Relative Coupon	β_x	t-stat $_x$	β_y	t-stat $_y$	n	r^2
-1.5%	0.1%	0.03	0	0	32	0.0%
-1.0%	0.6%	0.63	0	0	81	0.5%
-0.5%	1.2%	2.09	0	0	136	3.2%
0.0%	1.8%	2.15	1.5%	1.28	171	2.7%
0.5%	0.9%	1.21	1.2%	1.11	183	0.9%
1.0%	-0.1%	-0.10	0.5%	0.47	186	0.3%
1.5%	-1.0%	-1.40	-0.5%	-0.46	182	1.4%
2.0%	-2.1%	-2.39	-1.4%	-0.98	171	3.4%
2.5%	-2.2%	-1.84	-1.7%	-0.76	139	2.4%
3.0%	-2.6%	-2.03	-5.1%	-1.89	112	4.2%
3.5%	-3.2%	-1.93	-3.7%	-1.16	92	4.1%

Estimating Factor Loadings

We obtain monthly MBS returns from indices created by Bloomberg Barclays. The indices are constructed by grouping individual TBA-deliverable MBS pools into aggregates or generics based on their characteristics. As a liquidity filter, we also exclude monthly returns from coupons that have less than USD 1bn outstanding in RPB at the beginning of the month. The following is a brief description of the restriction that securities in the index are TBA-deliverable. More than 90 percent of agency MBS trading occurs in the to-be-announced (TBA) forward market. In a TBA trade, the buyer and seller agree upon a price for delivering a given volume of agency MBS at a specified future date. The characteristic feature of a TBA trade is that the actual identity of the securities to be delivered at settlement is not specified on the trade date. Instead, participants agree upon only six general parameters of the securities to be delivered: issuer, maturity, coupon, price, par amount, and settlement date. The exact pools to be delivered are “announced” to the buyer two days before settlement.

The pools delivered are at the discretion of the seller, but must satisfy SIFMA good delivery guidelines, which specify the allowable variance in the current face amount of the pools from the nominal agreed-upon amount, the maximum number of pools per \$1 million of face value, and so on. Because of these eligibility requirements, “TBA-deliverable” pools can be considered fungible because a significant degree of actual homogeneity is enforced among the securities deliverable into any particular TBA contract.¹⁸

Absolute coupon return series are converted into a relative coupon return series based on investor moneyness. We define investor moneyness to be the difference between the TBA coupon and the par coupon at the beginning of the month. The implied par coupon is determined from TBA prices by finding the TBA coupon that corresponds to a price of 100, linearly interpolating when needed. For example, if the 4.0 coupon has a price of 95 and the 4.5 coupon has a price of 105, the implied par coupon would be equal to 4.25. After computing the investor moneyness (x) for each absolute coupon, we map it to a relative coupon in increments of 0.5 centered around zero. For example:

- $-0.75 \leq x < -0.25$ maps to relative coupon -0.5%
- $-0.25 \leq x < 0.25$ maps to relative coupon 0.0% (par is centered around zero)
- $0.25 \leq x < 0.75$ maps to relative coupon 0.5%

For our study, we use Treasury-hedged returns of coupon-level aggregates of Fannie Mae 30-year fixed-rate MBS pools. Hedged returns are computed by Barclays using a term structure-matched position in Treasuries based on a key-rate duration approach. Results are similar using returns hedged using empirical durations. We construct the empirical-duration hedged series by starting with the Barclays MBS total index returns by absolute coupon. We compute empirical hedge ratios by estimating three year rolling betas for these index returns on 2 and 10 year US Treasury Futures returns. To extend the sample back to the start of the Barclays index return sample, we use 2 year Treasury Index returns from CRSP prior to 5/1996. Table A.2 displays the results for security loadings on empirically hedged returns, and shows that these results are very similar to those using the hedged series provided by Barclays. We note that the negative loadings for the -1.5% coupon are due to the shortened sample induced by the rolling window.

Our results are also robust to including an empirical hedge for interest rate volatility. Table A.3 reports factor loadings for Barclays excess returns hedged with respect to short volatility returns constructed using the returns from shorting three month

¹⁸See Vickery and Wright (2013), Hayre et al. (2010), or http://www.sifma.org/uploadedfiles/services/standard_forms_and_documentation/ch08.pdf?n=42389.

Table A.2: Factor loadings by relative coupon for empirically hedged returns. β_y^{disc} is restricted to equal zero. Standard errors are reported using adjusted degrees of freedom to account for the estimated regressors. The following time series regression is estimated for each security, i :

$$R_t^{ei} = a^i + \beta_x^i x_t + \beta_y^i y_t + \epsilon_t^i \text{ with } \beta_y^{\text{disc}} \equiv 0.$$

Relative Coupon	β_x	t-stat _x	β_y	t-stat _y	n	r^2
-2.0%	1.98%	0.35	0	0	32	0.40%
-1.5%	-4.03%	-1.49	0	0	71	3.13%
-1.0%	2.11%	1.42	0	0	110	1.84%
-0.5%	2.91%	2.56	0	0	130	4.88%
0.0%	0.67%	1.15	0.24%	0.22	181	0.82%
0.5%	-1.03%	-1.97	-0.86%	-0.77	209	2.31%
1.0%	-1.14%	-2.22	-1.85%	-1.60	207	3.94%
1.5%	-1.49%	-3.21	-1.14%	-1.02	197	5.76%
2.0%	-1.39%	-2.70	-4.67%	-3.47	172	10.57%
2.5%	-1.35%	-2.08	-7.44%	-4.44	139	15.03%
3.0%	-2.21%	-4.39	-7.20%	-6.23	112	32.92%
3.5%	-3.14%	-6.49	-4.11%	-4.03	92	36.47%

maturity straddles constructed using ten year maturity US swaptions. As with the results for the empirical rate hedge, the pattern of risk factor loadings is very similar to our baseline results. Again, we note that the negative loadings for the -1.5% coupon are due to the shortened sample induced by the rolling window. Note that, in unreported results, we also find that security return loadings on short volatility returns are highest around par, and decrease in the tail coupons. This is intuitive since vega is likely to be highest near par. We think that exposure to volatility risk is unlikely to be driving our results, since, in addition to par securities having the lowest average returns in both market types, the correlation between short volatility returns and our x and y factors is approximately zero (0.02 and 0.09 respectively over the period January 1994 to June 2016). Likewise, if we add short volatility returns as an additional factor in our step one regressions, the loadings on x and y display the same robust declining pattern as in our baseline specification. Again, the loadings on the short volatility returns are largest and most significant around par.

Finally, it is important to note that in Step 1 of our Fama-MacBeth regression, we regress returns against 1-month lagged prepayment risk factors. For example, if the LHS is the 1-month return for the month of January, we regress that against the prepayment shocks measured for the month of December. The reason for the lag is to

Table A.3: Factor loadings by relative coupon for Barclays excess returns, hedged to short volatility returns. β_y^{disc} is restricted to equal zero. Standard errors are reported using adjusted degrees of freedom to account for the estimated regressors. The following time series regression is estimated for each security, i :

$$R_t^{ei} = a^i + \beta_x^i x_t + \beta_y^i y_t + \epsilon_t^i \text{ with } \beta_y^{\text{disc}} \equiv 0.$$

Relative Coupon	β_x	t-stat _x	β_y	t-stat _y	n	r^2
-2.0%	8.23%	1.03	0	0	24	4.57%
-1.5%	-5.01%	-1.66	0	0	56	4.85%
-1.0%	2.36%	1.52	0	0	91	2.54%
-0.5%	2.37%	2.29	0	0	113	4.51%
0.0%	0.70%	1.49	-0.63%	-0.64	166	1.48%
0.5%	-0.57%	-1.62	-0.15%	-0.18	198	1.39%
1.0%	-0.78%	-1.81	-0.90%	-0.90	203	2.17%
1.5%	-1.13%	-2.64	-0.21%	-0.20	197	3.53%
2.0%	-1.06%	-1.83	-3.95%	-2.61	172	5.84%
2.5%	-0.98%	-1.29	-7.05%	-3.60	139	9.71%
3.0%	-1.89%	-2.41	-6.91%	-3.83	112	14.78%
3.5%	-3.33%	-3.46	-4.66%	-2.30	92	14.46%

account for the fact that the Bloomberg Barclays MBS Index convention uses same day settlement prices with paydowns estimated throughout the month, as opposed to the market's convention of PSA settlement. Because prepayment data for a given month is reported after index results have been calculated, paydown returns in the MBS Index are reported with a one-month delay. As an example, the paydown return for January will reflect December prepayment data (which were made available by the agencies during January) since complete factor (or prepayment) data for January will be not available until the middle of February (due to PSA settlement). The MBS Index reflects an estimate of paydowns in the universe on the first business day of the month and the actual paydowns after the 16th business day of a month. See Phelps (2015) for a detailed discussion of the index construction and timing conventions.

Determining Market Type

We define market type based on the market composition between discount and premium Fannie Mae 30-year MBS securities. At the beginning of each month, we measure the remaining principal balance (RPB) for each these two types of securities. If the total RPB for discount securities is greater than the total RPB for premium se-

curities, we classify that month as a discount market; otherwise the month is deemed to be a premium market. By this measure of market type, the market has been in a premium market state about 70% of the time during our sample period (Jan 1994 to June 2016).

We note that, although there are several ways one could classify market type, they are all highly correlated. We analyzed the following alternative measures of market type: (1) RPB weighted WAC relative to current mortgage rates, or “borrower moneyness”, (2) RPB weighted relative coupon, or “investor moneyness”, (3) RPB weighted relative coupon minus the ten year US treasury yield, as in Gabaix et al. (2007), and (4) innovations to the percentage of RPB that trades at a discount, measured by errors in an AR(1) regression. The correlation of these measures with the percentage of RPB that trades at a discount are 0.84, 0.89, 0.77 and 0.49, respectively. Thus, the correlation of measure of market type defined by percentage of RPB that trades at a discount with all other measures is very high.

Spread Assets

We scale all long short portfolios to have, in expectation, constant volatility and equal leg volatility. We predict monthly volatility for each leg, for each month using a six month equally weighted moving average of past realized monthly volatility. We predict correlations using a twelve month equally weighted moving average of past realized correlations. Correlations tend to be more stable than volatilities, hence we use the longer window. If any volatility or correlation is missing for a leg/month observation, we use the estimate of the closest coupon or coupon pair in that month to replace the missing value. Each leg in the spread assets are scaled to target 1% volatility, and each spread asset is scaled to target 1% volatility in each month.

Figures

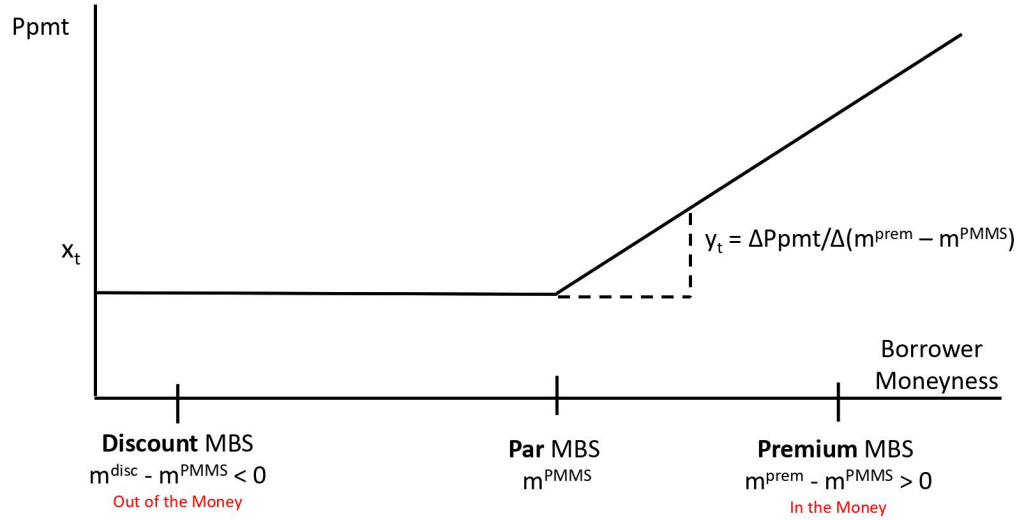


Figure 1: This figure plots prepayment as a function of borrower moneyiness and a realization of the turnover (x), and rate-sensitivity (y) prepayment factors.

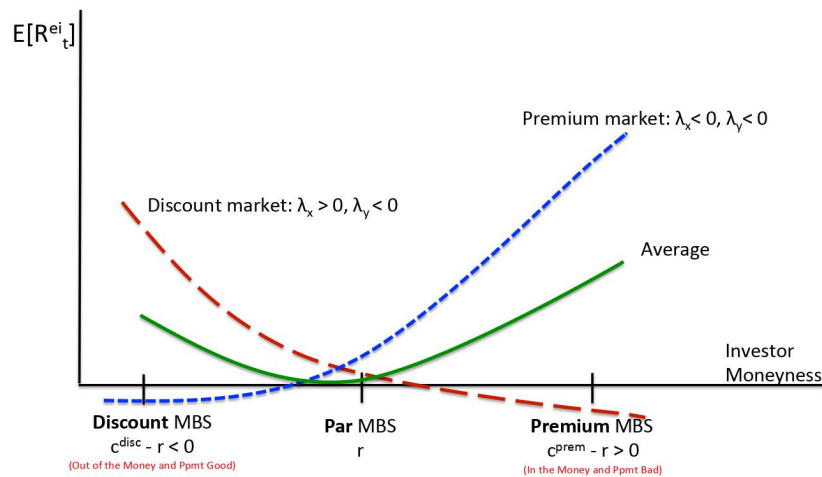


Figure 2: This figure summarizes the implications of Hypothesis 1 regarding the signs of λ_x and λ_y in the two market types, discount and premium. Expected returns are increasing in relative moneyiness in premium markets. In discount markets, expected returns may be decreasing in relative moneyiness, or U-shaped, depending on the magnitudes of the x and y loadings and risk prices.

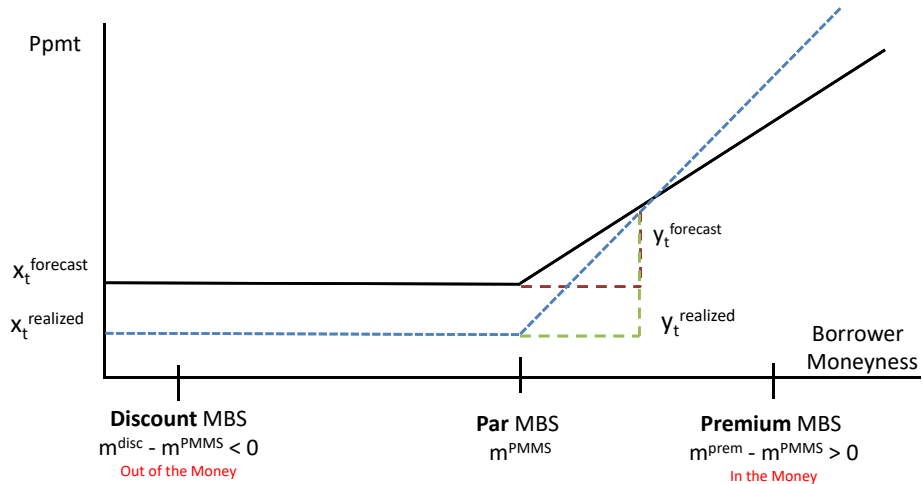


Figure 3: This figure plots forecast and realized prepayment as a function of borrower moneyness and a realization of the turnover (x), and rate-sensitivity (y) prepayment factors. Prepayment shocks are measured as the difference between realized and forecasted factors, $x_t = \hat{x}_t^{\text{realized}} - \hat{x}_t^{\text{forecast}}$, and $y_t = \hat{y}_t^{\text{realized}} - \hat{y}_t^{\text{forecast}}$.

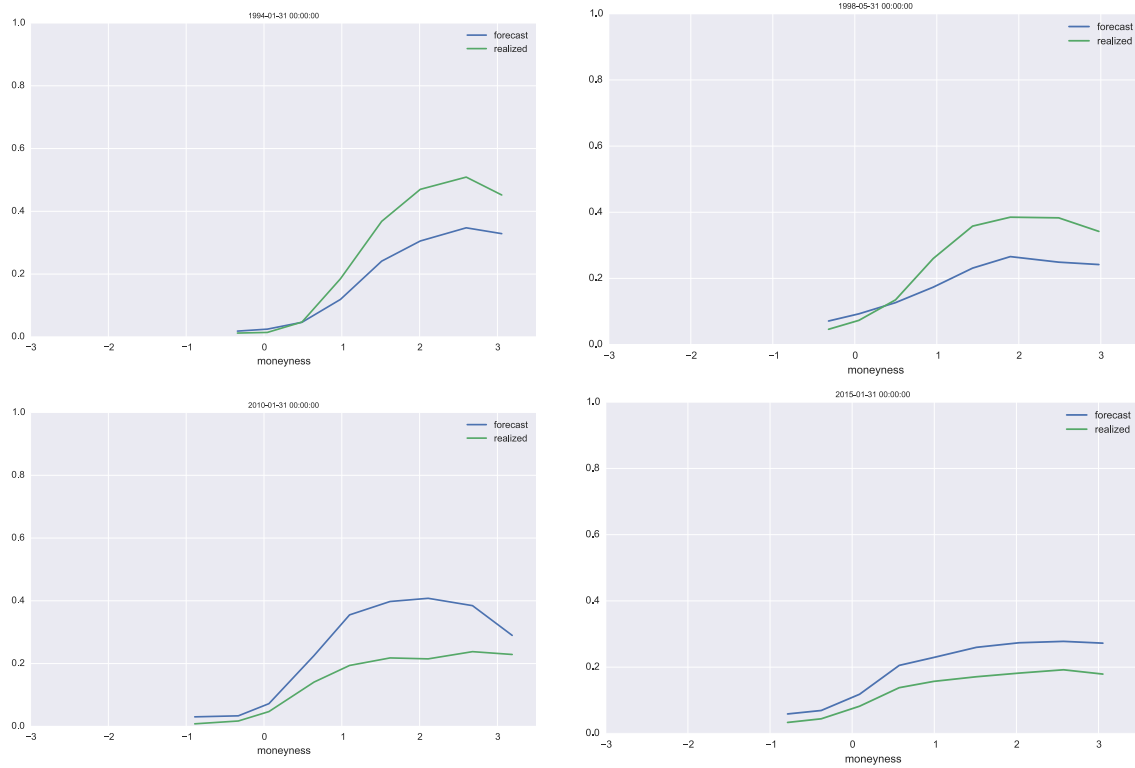


Figure 4: This figure plots four examples of the forecast and realized prepayment data used to estimate the innovations to the level and rate-sensitivity prepayment risk. The y-axis is prepayment rates in percent, and the x-axis is $m^i - m_t^{\text{PMMS}}$, or borrower moneyness. Note that borrower moneyness is typically 50-100bps above investor moneyness defined as $c^i - r$.

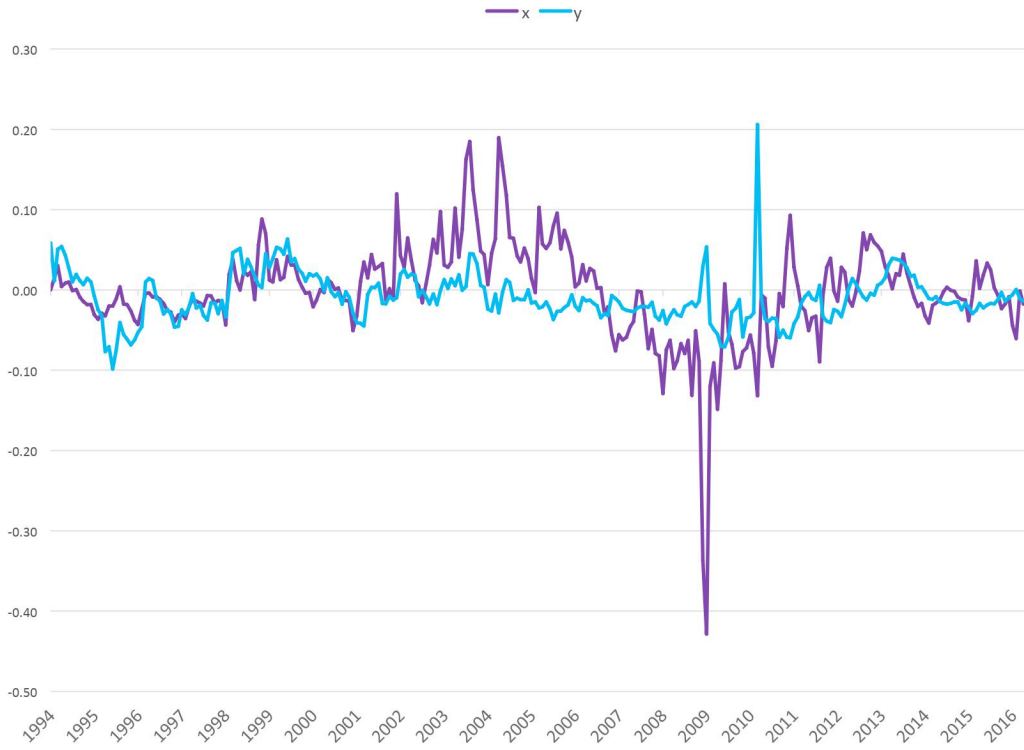


Figure 5: This figure plots the estimated time series' for the two prepayment risk factors, turnover (x), and rate-sensitivity (y).

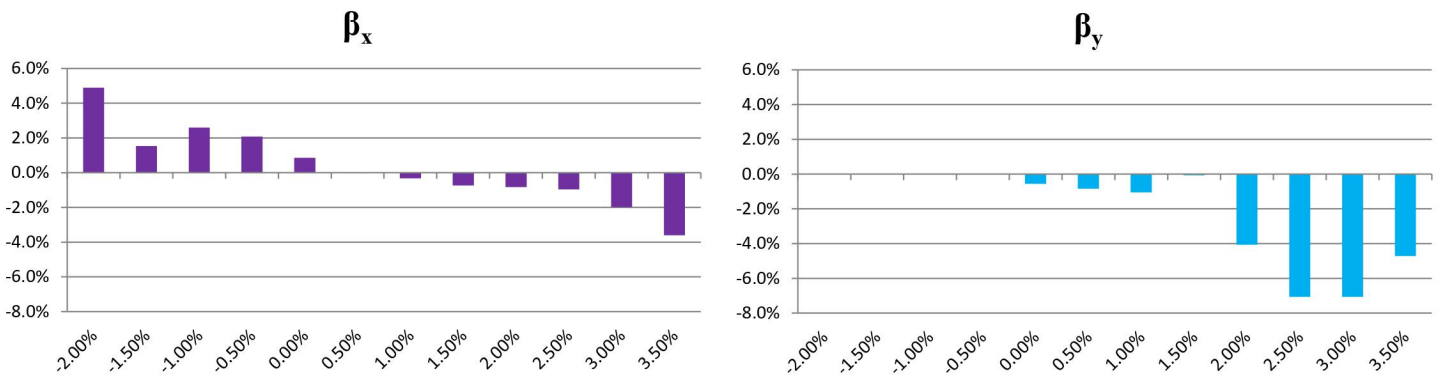


Figure 6: This figure plots the results for the loadings on the two prepayment risk factors, turnover (x), and rate-sensitivity (y), by relative coupon.

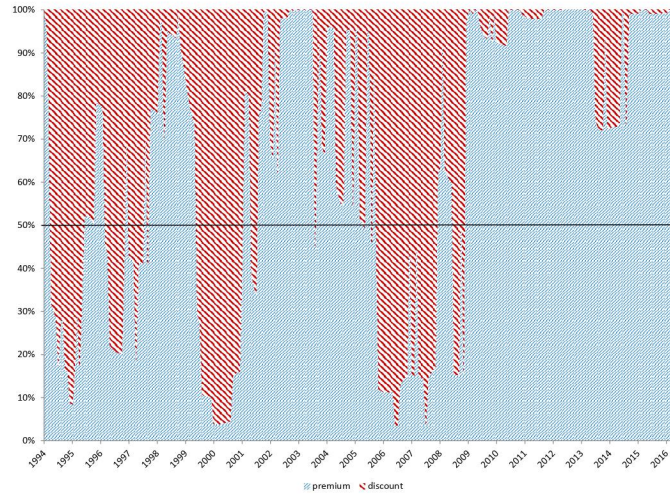


Figure 7: This figure plots the Fannie Mae 30 year MBS market composition between discount and premium securities. We define market type by classifying any month in which more than 50% of total remaining principal balance is discount as a discount market (DM). The remaining months are classified as premium markets (PM).



Figure 8: This figure plots annualized average monthly returns for the full sample, and within discount months and premium months only. The pattern of average returns is U-shaped overall, declining in discount markets, and increasing in premium markets. We exclude coupons which would require averaging over less than five observations in a particular market type.

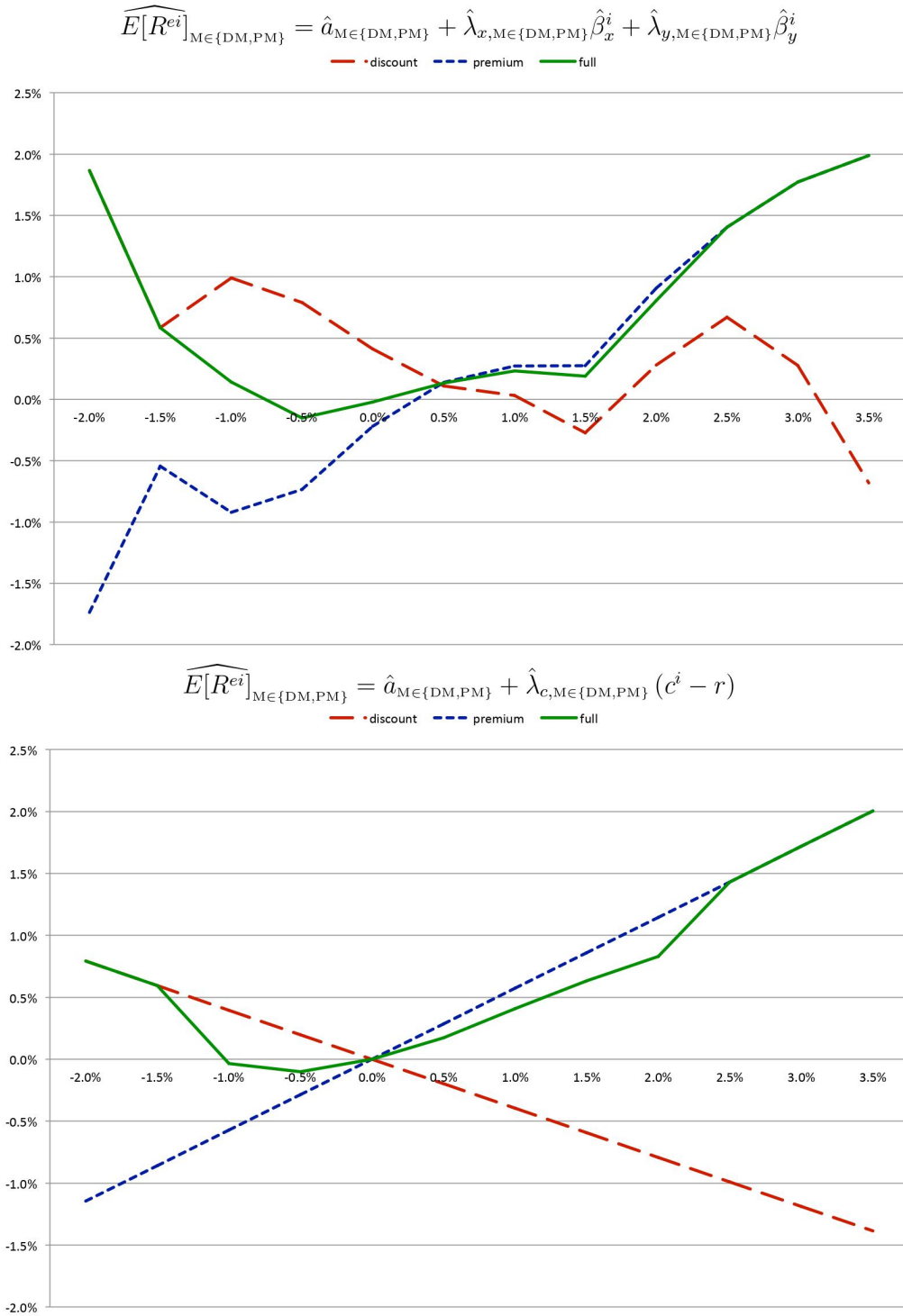


Figure 9: This figure plots predicted monthly returns for our model using the Fama MacBeth estimates for λ 's (top), and the estimates for λ 's from a single relative money-ness characteristic/factor model (bottom). Refer to Figure 8 for the empirical average monthly returns. All plots include unconditional average returns, and averages within discount months and premium months only.

$$\widehat{E[Re^i]} = \hat{a} + \hat{\beta}^{i, VW_{all}} \hat{\lambda}^{VW_{all}}$$

$$\widehat{E[Re^i]} = \hat{a} + \hat{\beta}^{i, \text{Max-Min}} \hat{\lambda}^{\text{Max-Min}}$$

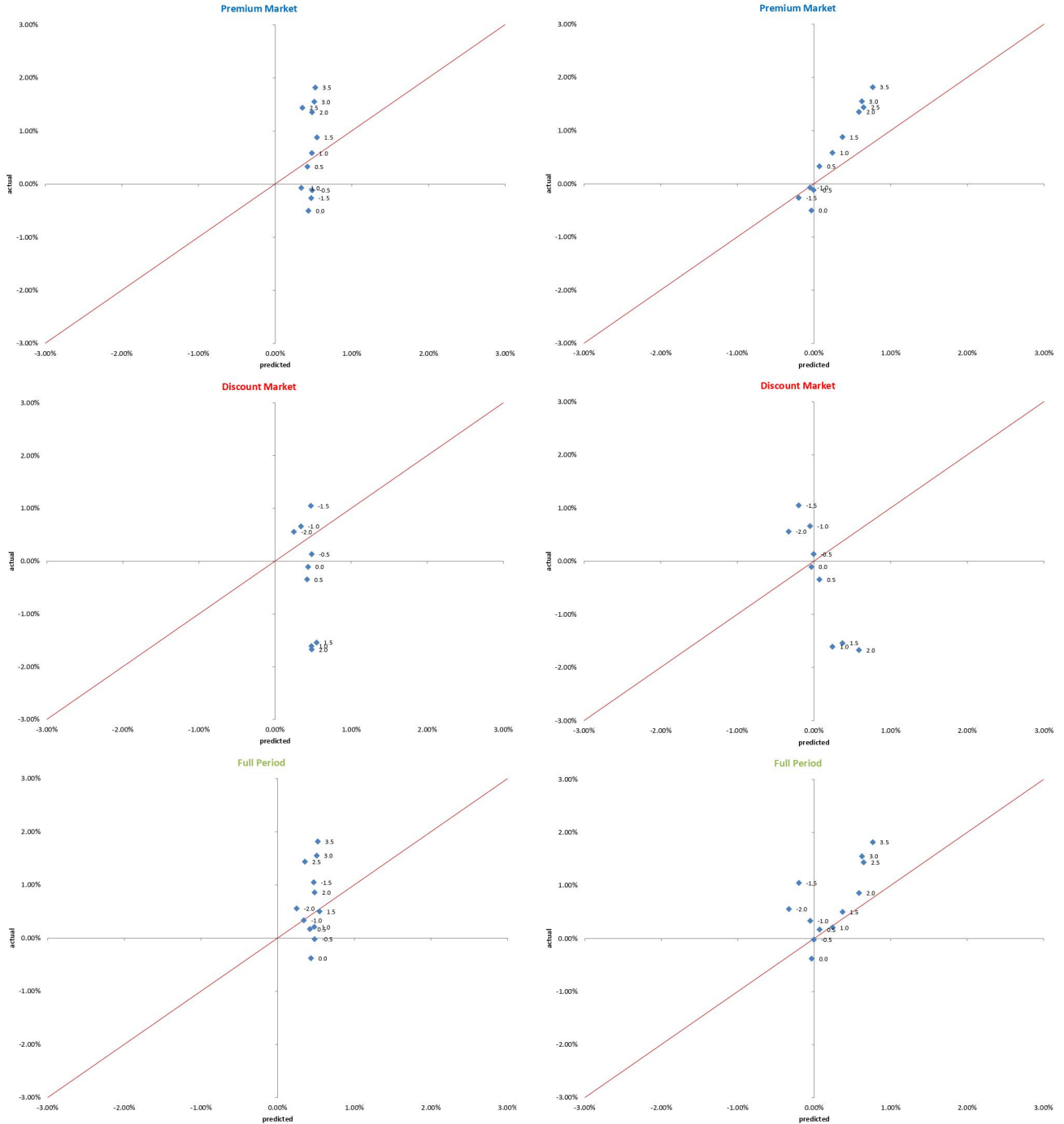


Figure 10: This figure plots annualized realized returns vs. predicted returns for two passive benchmark models, by market type, and for the full sample.

$$\widehat{E}[R^{ei}]_{M \in \{DM, PM\}} = \hat{a}_{M \in \{DM, PM\}} + \hat{\lambda}_{x, M \in \{DM, PM\}} \hat{\beta}_x^i + \hat{\lambda}_{y, M \in \{DM, PM\}} \hat{\beta}_y^i \quad \widehat{E}[R^{ei}]_{M \in \{DM, PM\}} = \hat{a}_{M \in \{DM, PM\}} + \hat{\lambda}_{c, M \in \{DM, PM\}} (c^i - c^{part})$$

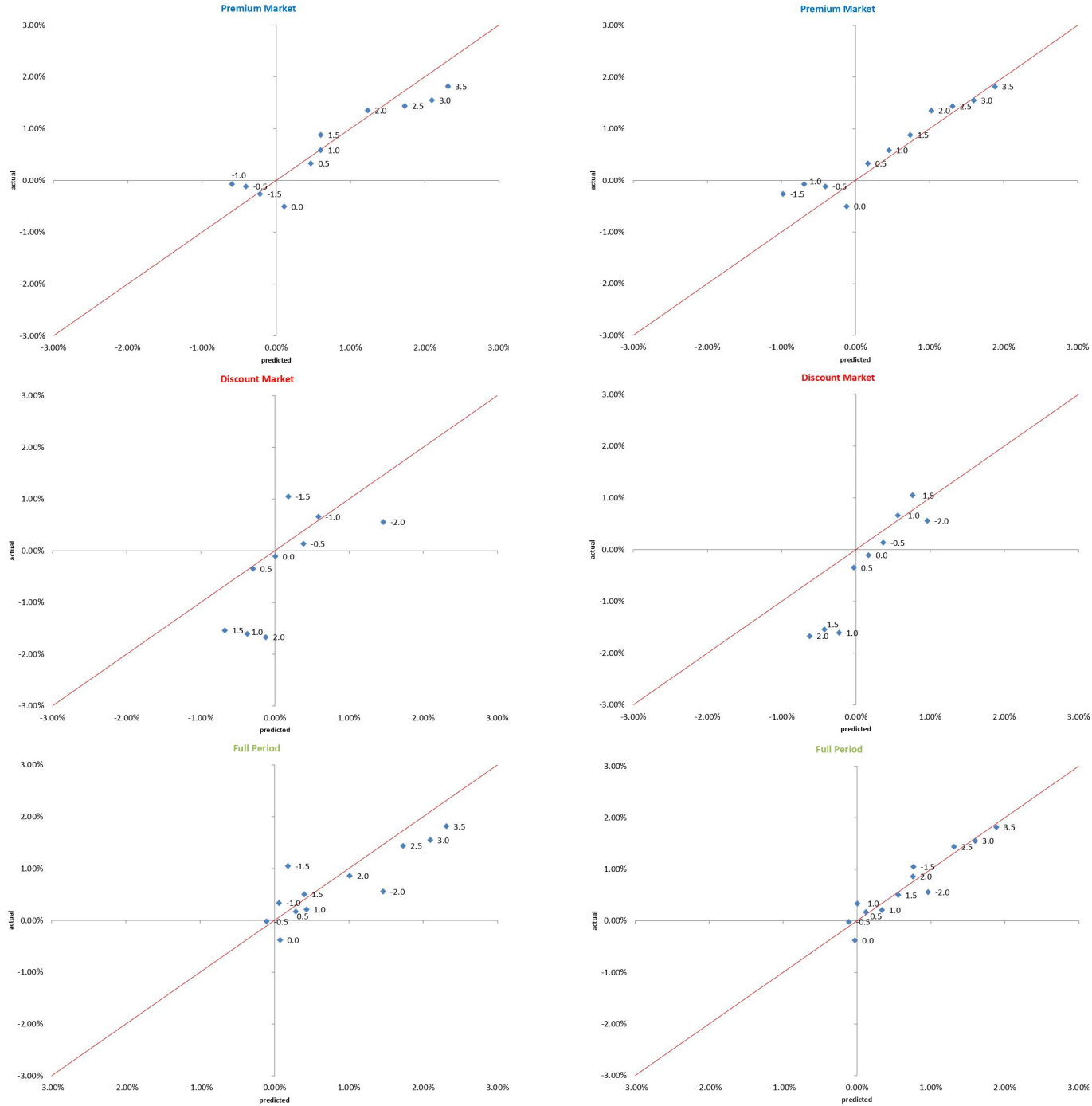


Figure 11: This figure plots annualized realized returns vs. predicted returns for the two and one factor models implied by our theory, by market type, and for the full sample.

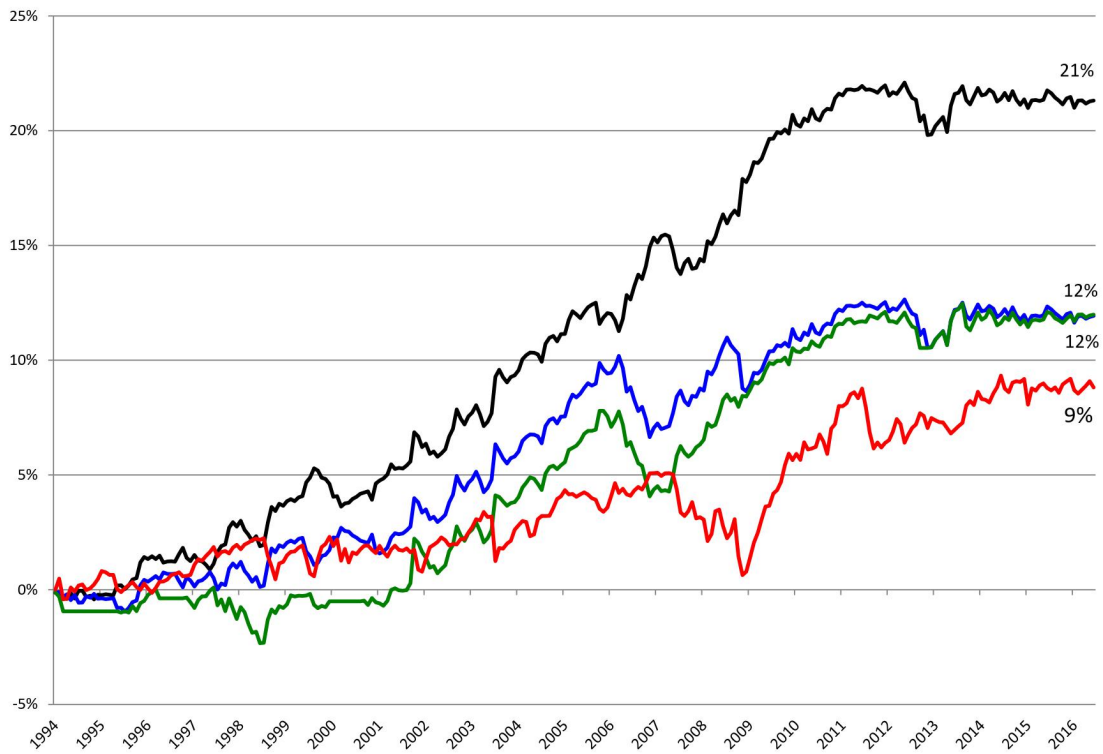


Figure 12: This figure plots cumulative returns for our model-implied optimally timed portfolio (black) relative to three passive benchmarks. Max - Min (blue) is a passive long maximum premium coupon short minimum discount coupon portfolio, Max - Par (green) is a passive long maximum premium coupon short par portfolio, VW_{all} (red) is the RPB weighted MBS index.

Tables

Table 1: This table summarizes the results in Lemmas 1 and 2 regarding the signs and magnitudes of the prepayment risk factor loadings.

	x_t	y_t
Discount Securities	$\beta_x^{i,\text{disc}} > 0$	$\beta_y^{i,\text{disc}} = 0$
Premium Securities	$\beta_x^{i,\text{prem}} < 0$	$\beta_y^{i,\text{prem}} < 0$

Table 2: This table summarizes the predictions in Hypothesis 1 regarding the signs of the prices of turnover and rate-sensitivity risk across market types.

	$\lambda_{x,M}$	$\lambda_{y,M}$
Discount Market (M=DM)	+	-
Premium Market (M=PM)	-	-

Table 3: Annualized returns, volatility, and Sharpe ratios, as well as number of observations for MBS by Relative Moneyiness, defined as own coupon relative to par coupon.

	-2.0%	-1.5%	-1.0%	-0.5%	0.0%	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%
ann. ret	0.56%	0.97%	0.34%	-0.02%	-0.38%	0.17%	0.21%	0.50%	0.86%	1.43%	1.55%	1.82%
ann. vol	1.70%	1.82%	1.87%	1.67%	1.78%	1.71%	1.63%	1.59%	1.97%	2.45%	2.10%	2.21%
SR	0.33	0.53	0.18	-0.01	-0.21	0.10	0.13	0.32	0.44	0.58	0.74	0.82
n	41	87	153	217	248	238	217	199	172	139	112	92

Table 4: Factor loadings by relative coupon. β_y^{disc} is restricted to equal zero. Standard errors are reported using adjusted degrees of freedom to account for the estimated regressors. The following time series regression is estimated for each security, i :

$$R_t^{ei} = a^i + \beta_x^i x_t + \beta_y^i y_t + \epsilon_t^i \text{ with } \beta_y^{\text{disc}} \equiv 0.$$

Relative Coupon	β_x	t-stat _x	β_y	t-stat _y	n	r^2
-2.0%	4.90%	1.10	0	0	41	3.00%
-1.5%	1.54%	0.94	0	0	87	1.02%
-1.0%	2.60%	3.20	0	0	153	6.34%
-0.5%	2.07%	3.83	0	0	216	6.42%
0.0%	0.86%	1.52	-0.57%	-0.54	247	1.00%
0.5%	-0.04%	-0.07	-0.84%	-0.8	237	0.30%
1.0%	-0.32%	-0.67	-1.05%	-0.98	216	0.70%
1.5%	-0.74%	-1.57	-0.07%	-0.07	198	1.30%
2.0%	-0.83%	-1.41	-4.07%	-2.65	172	5.20%
2.5%	-0.96%	-1.29	-7.07%	-3.69	139	10.10%
3.0%	-1.99%	-2.76	-7.07%	-4.27	112	18.00%
3.5%	-3.60%	-4.23	-4.72%	-2.63	92	19.60%

Table 5: Factor loadings by relative coupon. β_y^{disc} is unrestricted. Standard errors are reported using adjusted degrees of freedom to account for the estimated regressors. The following time series regression is estimated for each security, i :

$$R_t^{ei} = a^i + \beta_x^i x_t + \beta_y^i y_t + \epsilon_t^i.$$

Relative Coupon	β_x	t-stat _x	β_y	t-stat _y	n	r^2
-2.0%	2.73%	0.49	2.52%	0.66	41	4.10%
-1.5%	1.53%	0.86	0.04%	0.02	87	1.00%
-1.0%	2.42%	2.89	1.22%	0.95	153	6.90%
-0.5%	2.08%	3.79	-0.01%	-0.01	216	6.40%
0.0%	0.86%	1.52	-0.57%	-0.54	247	1.00%
0.5%	-0.04%	-0.07	-0.84%	-0.8	237	0.30%
1.0%	-0.32%	-0.67	-1.05%	-0.98	216	0.70%
1.5%	-0.74%	-1.57	-0.07%	-0.07	198	1.30%
2.0%	-0.83%	-1.41	-4.07%	-2.65	172	5.20%
2.5%	-0.96%	-1.29	-7.07%	-3.69	139	10.10%
3.0%	-1.99%	-2.76	-7.07%	-4.27	112	18.00%
3.5%	-3.60%	-4.23	-4.72%	-2.63	92	19.60%

Table 6: Annualized returns, volatility, and Sharpe ratios, as well as number of observations for MBS by Relative Moneyiness, defined as own coupon relative to par coupon, conditional on the market type. The market is defined as Premium if $> 50\%$ of RPB is premium, and discount otherwise.

premium (M=PM)	-2.0%	-1.5%	-1.0%	-0.5%	0.0%	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%
ann. ret		-0.26%	-0.07%	-0.12%	-0.50%	0.33%	0.58%	0.88%	1.35%	1.44%	1.55%	1.82%
ann. vol		0.84%	2.20%	1.87%	1.98%	1.80%	1.63%	1.52%	1.94%	2.48%	2.10%	2.21%
SR		-0.31	-0.03	-0.06	-0.25	0.18	0.36	0.58	0.70	0.58	0.74	0.82
n		5	68	134	170	182	180	168	144	136	112	92
discount (M=DM)	-2.0%	-1.5%	-1.0%	-0.5%	0.0%	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%
ann. ret	0.56%	1.05%	0.66%	0.13%	-0.11%	-0.35%	-1.61%	-1.54%	-1.67%	1.14%		
ann. vol	1.70%	1.87%	1.56%	1.29%	1.25%	1.36%	1.57%	1.85%	1.99%	0.47%		
SR	0.33	0.56	0.42	0.10	-0.09	-0.25	-1.03	-0.83	-0.84	0.42		
n	41	82	85	83	78	56	37	31	28	3		

Table 7: Prices of Risk, Fama MacBeth Estimation. Risk prices are time series averages of cross section regression coefficients conditional on market type $M \in (DM, PM)$. The following regression is estimated at each date t :

$$R_{t,M}^{ei} = a_{t,M} + \lambda_{t,x,M} \hat{\beta}_x^i + \lambda_{t,y,M} \hat{\beta}_y^i + \epsilon_t^i.$$

Market Type	λ_x	t-stat _x	λ_y	t-stat _y	n
Discount (M=DM)	3.17%	1.41	-1.22%	-0.44	62
Premium (M=PM)	-2.95%	-2.19	-1.26%	-1.57	168

Table 8: Prices of Risk, Relative Moneyiness Characteristic. Risk prices are time series averages of cross section regression coefficients conditional on market type $M \in (DM, PM)$. The following regression is estimated at each date t :

$$R_{t,M}^{ei} = a_{t,M} + \lambda_{t,c,M} (c^i - r) + \epsilon_t^i.$$

Market Type	λ_c	t-stat
Discount	3.30%	1.38
Premium	-4.77%	-2.66

Table 9: Prices of Risk, Pooled Time Series Cross Section Regression. F-statistics for joint statistical significance of δ_x and δ_y are computed using the Wald statistic.

$$R_t^{ei} = a + \kappa_x \beta_x^i + \kappa_y \beta_y^i + \delta_x \beta_x^i (\%RPB_{BoM}^{disc} - 50\%) + \delta_y \beta_y^i (\%RPB_{BoM}^{disc} - 50\%) + \epsilon_t^i.$$

	Coefficient	t-statistic clustering		Coefficient	t-statistic clustering		
		none	time		none	time	
κ_x	-0.3%	-0.31	-0.27	-0.15%	-0.18	-0.14	
κ_y	0.3%	0.27	0.20	0.73%	0.79	0.65	
δ_x	4.9%	2.11	1.49	6.88%	3.18	2.40	
δ_y	3.3%	1.45	0.92	3.81%	1.79	1.45	
a	0.00%	0.68	0.45				
time f.e.	no					yes	
n	1915					1915	
R^2	1%					60%	
F-stat δ_x and δ_y	87%					97%	

Table 10: Sharpe Ratios for Spread Assets and Indices. Max - Min is a passive long maximum premium coupon short minimum discount coupon portfolio, Max - Par is a passive long maximum premium coupon short par portfolio, Min - Par is a passive long minimum premium coupon short par portfolio, Optimally Timed is an active portfolio which is long maximum premium coupon short minimum discount coupon when $> 50\%$ of outstanding RPB is premium and long minimum discount coupon short maximum premium coupon otherwise. VW_{all} is the RPB weighted MBS index, VW_{ex-par} is the RPB weighted MBS index excluding par coupon, VW_{disc} is the RPB weighted MBS index of discount securities only, VW_{prem} is the RPB weighted MBS index of premium securities only. All long short portfolios are scaled to have constant volatility and equal leg volatility.

	Max - Min	Max - Par	Min - Par	Optimally Timed	VW_{all}	VW_{ex-par}	VW_{disc}	VW_{prem}
Discount (M=DM)	-0.47	0.28	0.49	0.47	0.12	0.18	0.27	-0.50
Premium (M=PM)	0.91	0.73	-0.42	0.91	0.36	0.41	-0.08	0.47
Full Sample	0.44	0.48	-0.02	0.76	0.29	0.35	0.03	0.26

Table 11: Excess returns, tracking errors, and information ratios for our model-implied optimally timed portfolio relative to three passive benchmarks. Max - Min is a passive long maximum premium coupon short minimum discount coupon portfolio, Max - Par is a passive long maximum premium coupon short par portfolio, VW_{all} is the RPB weighted MBS index.

Benchmark	Max - Min	Max - Par	VW_{all}
Active excess return: Optimally Timed	0.36%	0.41%	0.48%
Tracking Error	1.35%	1.22%	1.82%
Information Ratio	0.27	0.33	0.26

Table 12: Loadings of the model implied, optimally timed portfolio returns on, and α 's with respect to, four passive benchmarks, namely, the remaining principal balance weighted MBS index, VW_{all} , the remaining principal balance weighted MBS index amongst premium securities only, VW_{prem} , an untimed long maximum premium coupon short minimum discount premium portfolio with constant volatility and equal-leg volatility, Max - Min, and an untimed long maximum premium coupon short par portfolio with constant volatility and equal leg volatility, Max - Par.

Benchmark	α	t-stat $_{\alpha}$	$\beta^{\text{Benchmark}}$	t-stat $_{\beta}$	n	R ²
Max - Min	0.06%	3.11	0.30	5.16	270	9%
Max - Par	0.06%	3.12	0.45	8.14	238	22%
VW_{all}	0.07%	3.75	-0.08	-1.48	270	1%
VW_{prem}	0.08%	3.90	-0.15	-2.59	241	3%