

# Estimating and Understanding High-Dimensional Dynamic Stochastic Econometric Models

(For Volatility, Derivatives, and More...)

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# High Dimensionality

- ▶ Macro
- ▶ Finance
- ▶ Everywhere...

## Some Recent Work

Diebold, F.X. and Yilmaz, K. (2015), "Measuring the Dynamics of Global Business Cycle Connectedness," in S.J. Koopman and N. Shephard (eds.), *Unobserved Components and Time Series Econometrics: Essays in Honor of Andrew C. Harvey*, Oxford University Press, 45-89.

Demirer, M., Diebold, F.X., Liu, L. and Yilmaz, K. (2017), "Estimating Global Bank Network Connectedness," *Journal of Applied Econometrics*, in press.

Diebold, F.X., Liu, L. and Yilmaz, K. (2017), "Commodity Connectedness," Manuscript.

# A Very General Environment

$$x_t = B(L) \varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

Perhaps  $\dim(x) = 5$ , or 50, or 50000, or 5000000, or ...

# Many Interesting Issues / Choices

- ▶ x objects: Returns? **Return volatilities**? Return correlations?
- ▶ x universe: **How many and which ones**?
- ▶ x frequency: **Daily**? Monthly? Quarterly?
  
- ▶ Approximating model specification: **VAR**? Structural? DFM?
- ▶ Estimation: Classical? Bayesian? **Hybrid**?
  - ▶ Selection: Information criteria? Stepwise? **LASSO**?
  - ▶ Shrinkage: BVAR? Ridge? **LASSO**?
  - ▶ **Static** vs. **dynamic** (rolling, expanding, TVP modeling)?
- ▶ Identification: **Mechanical** (e.g. Cholesky)? SVAR? DSGE?
- ▶ Understanding I: Visualization via **network graphs**
- ▶ Understanding II: Summarization via **network degree distributions**

## Estimating the VAR: Regularization

Constrained estimation:

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \left( \sum_{t=1}^T \left( y_t - \sum_i \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^K |\beta_i|^q \right)$$

Concave penalty functions non-differentiable at the origin produce selection. Convex penalties produce shrinkage (e.g.,  $q = 2$  is ridge)

$q = 1$  is LASSO (concave and convex, selects and shrinks):

$$\hat{\beta}_{\text{LASSO}} = \underset{\beta}{\operatorname{argmin}} \left( \sum_{t=1}^T \left( y_t - \sum_i \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^K |\beta_i| \right)$$

– Immediately useful for forecasting

# Understanding the VAR: Variance Decomposition (Old Days, Circa 1980-2010)

- Parameters are not directly revealing
- So examine variance decomposition  $D$  - Saved by Sims!


$D$

	$x_1$	$x_2$	...	$x_5$
$x_1$	$d_{1,1}$	$d_{1,2}$	...	$d_{1,5}$
$x_2$	$d_{2,1}$	$d_{2,2}$	...	$d_{2,5}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$x_5$	$d_{5,1}$	$d_{5,2}$	...	$d_{5,5}$

But what if  $\dim(x) = 5000$ ?

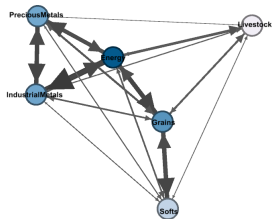
# A New Approach to Understanding the VAR, I: Variance Decomposition Summarization Via the Network Degree Distribution (Connectedness Perspective)

<i>D</i>					
	$x_1$	$x_2$	...	$x_N$	From Others
$x_1$	$d_{11}$	$d_{12}$	...	$d_{1N}$	$\sum_{j \neq 1} d_{1j}$
$x_2$	$d_{21}$	$d_{22}$	...	$d_{2N}$	$\sum_{j \neq 2} d_{2j}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$x_N$	$d_{N1}$	$d_{N2}$	...	$d_{NN}$	$\sum_{j \neq N} d_{Nj}$
To Others	$\sum_{i \neq 1} d_{i1}$	$\sum_{i \neq 2} d_{i2}$	...	$\sum_{i \neq N} d_{iN}$	$\sum_{i \neq j} d_{ij}$

Connectedness: Pairwise, total “from,” total “to,” system-wide 



# A New Approach to Understanding the VAR, II: Variance Decomposition Visualization Via the Network Graph



- ▶ Node shading/thickness: Total directional connectedness “to others”
- ▶ Node location: Average pairwise directional connectedness
- ▶ Link thickness: Average pairwise directional connectedness
- ▶ Link arrow sizes: Pairwise directional “to” and “from”

# Commodity Return Volatilities

19 sub-indices (based on futures contracts)

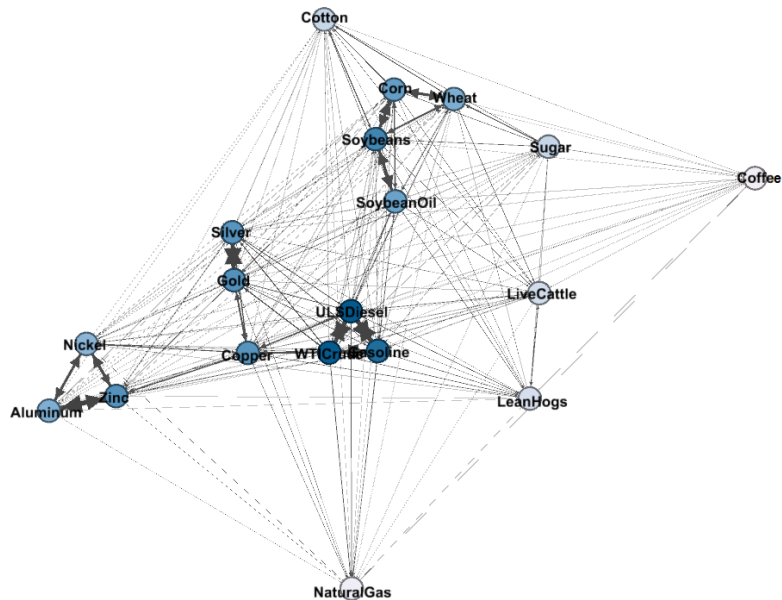
underlying the Bloomberg Commodity Price Index:

- 4 energies (crude oil, heating oil, natural gas, unleaded gasoline)
- 2 precious metals (gold, silver)
- 4 industrial metals (aluminum, copper, nickel, zinc)
- 2 livestock (live cattle, lean hogs)
- 4 grains (corn, soybeans, soybean oil, wheat)
- 3 "softs" (coffee, cotton, sugar)

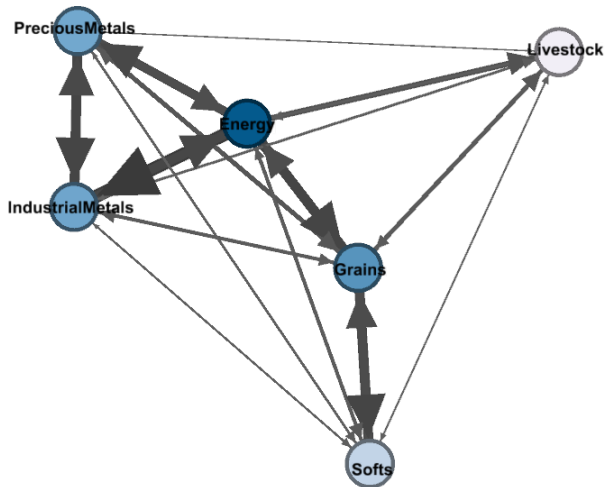
Garman-Parkinson-Klass range-based daily realized volatility

May 2006 - January 2016

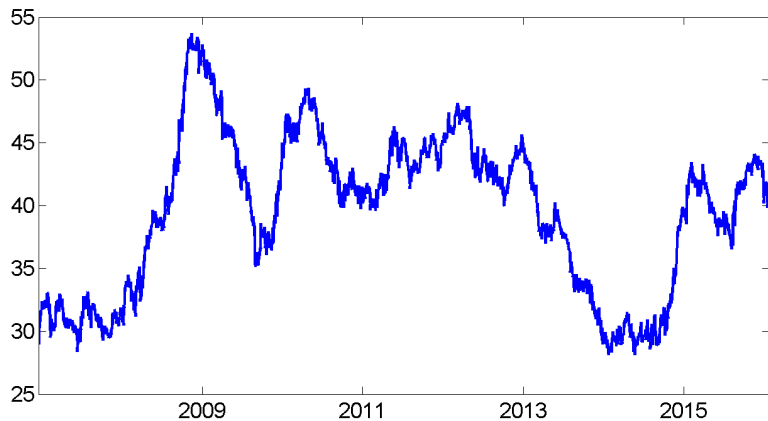
# Full-Sample Network Graph



# Full-Sample Network Graph, Six-Group Aggregation



# Rolling-Sample System-Wide Connectedness



# Conclusion

*THERE'S NOTHING NEW UNDER THE SUN...*

- Standard time-series dynamic econometric modeling
- VAR estimation, forecasting, understanding, ...

*...BUT NEW TOOLS ARE REQUIRED FOR  
BIG-DATA ENVIRONMENTS:*

- Regularization methods for estimation
- Network methods for understanding