An American Call IS Worth More than a European Call

Stephen Figlewski*
A Familiar Classroom Demonstration

An American Call on a Non-Dividend Paying Stock

- Should never be exercised early
- Is therefore worth the same as a European call

To liquidate an American call position early, don't exercise

- sell it in the market for its fair value (the European call price)
- if there is no options market, sell short the stock and delta-hedge until expiration
But in the Real World...

...the market's bid price for an in-the-money option is very rarely at fair value, and is usually lower than the option's intrinsic value.

Example: Exxon-Mobil, Monday, April 24, 2017, 1:57 P.M.

<table>
<thead>
<tr>
<th></th>
<th>BID</th>
<th>ASK</th>
<th>S-X</th>
<th>EBV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exxon stock</td>
<td>81.16</td>
<td>81.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XOM MAY 75-strike Call (25 days)</td>
<td>5.85</td>
<td>6.40</td>
<td>6.16</td>
<td>0.31</td>
</tr>
<tr>
<td>XOM JUN 75-strike Call (53 days)</td>
<td>5.90</td>
<td>6.45</td>
<td>6.16</td>
<td>0.26</td>
</tr>
</tbody>
</table>

These bid prices are exercise boundary violations (EBVs). Exercise is the only way to liquidate an American call position and recover even intrinsic value. (You still lose the option's time value.)

Unfortunately, EBVs in equity options are the norm, not the exception.
What Can a Trader Do When the Best Bid is Too Low?

1. Sell in the market at $C_{Bid}$, below the intrinsic value $(S - X)$

2. Exercise (receive $S - X$)

3. Hold the option but exit the position synthetically by delta hedging to expiration.

In theory, the third strategy returns the full value $C_{EUR}$.

But...
But...in practice, delta hedging to expiration requires

- borrowing and selling short the stock,
- paying transactions costs for hedge rebalancing
- earning interest on the short sale proceeds
- extra costs and risk if the stock is "on special"

Jensen and Pedersen (JFE 2016) show that this alternative is expensive and is easily dominated by early exercise. Using closing prices, they show that replication costs are frequently high enough that early exercise is rational, but they can't look at intraday bid quotes directly.
Transactions Costs

Transactions costs differ widely between market makers and retail investors, and across retail brokerage firms.

But trading costs for "discount" retail brokers are typically well below levels that would make exercise unprofitable, or subject to significant timing risk.

Example: Interactive Brokers (a large, low-cost online brokerage)
- Option trades: $0.70 per contract
- Option exercise: No charge; stock is available in account immediately
- Stock trades: $0.005 per share plus $2.18 per $100,000 of trade value

IB's commission to exercise and liquidate an option contract on a $100 stock is

\[ = 100 \times 0.005 + 10,000 \times \frac{2.18}{100,000} \]

\[ = 0.72, \text{ less than 1 cent per share} \]
Results from Battalio, Figlewski and Neal (2017)


Examined intraday data for all single stock call and put option bid and ask quotes during March 2010.

- all in the money contracts
- ex-dividend days excluded – no "rational" call exercises in the sample
- intraday at one-minute intervals
- ~125,000 option contracts; ~670 million observations
Battalio et al. find that

Early exercise boundary violations (EBVs) are

- common
- large
- persistent
- some investors trade options at EBV prices (~$39 million left on the table in a single month)
- some rationally exercise American options early, both calls and puts
Deep ITM (30 – 50%) calls: >95% violation, half by > $0.20
Mid ITM (10 – 30%) calls: >85% violation, half by > $0.17
Bids at least $0.20 below intrinsic value:
Deep ITM: >50%; Mid ITM: ~25%
Consider an American call that must be liquidated before expiration, on a specific future date $t < T$.

Assume the price in the options market is the Black-Scholes value minus half of the bid-ask spread of $2B$.

- Selling at date $t$ yields: $C_{BS}(S_t, X, T-t) - B$.

- An American call can be exercised for: $S_t - X$

It is possible to replicate the payoff of the optimal sell or exercise decision with a portfolio of optional contracts that can all be priced at date 0.
American Call Early Exercise Premium

\( C_{BS}(S,X,T) \) is the date 0 Black-Scholes price for a European call maturing at date \( T \).

There is a level of the date \( t \) stock price \( S_t^* \) such that the proceeds from selling at the market's bid price or exercising are the same.

\[
C_{BS}(S_t^*,X,T-t) - B = S_t^* - X
\]

- Above \( S_t^* \) it is better to exercise (the option's remaining time value is less than \( B \))

- Below \( S_t^* \) it is better to sell in the market
Finding the Early Exercise Boundary $S^*$ with $X = 90$

![Graph showing the relationship between stock price and option value with the early exercise boundary](image-url)
Finding the Early Exercise Boundary $S^*$ with $X = 90$

$C(S^*,X,T-t) - B = S^* - X$

- **Call Value**
- **Best Bid** = $C - B$
- **Intrinsic Value**
- **$S^* = 95.86$**
Replicating Portfolio for "American" Call with Single Early Exercise Date $t$

1. Buy a European call with strike $X$ and maturity $T$: $C_{BS}(S,X,T)$

2. Buy a European call with strike $S_t^*$ and maturity $t$: $C_{BS}(S,S_t^*,t)$

3. Write a compound call on the call in step 1, with strike $S_t^* - X + B$:
   
4. Borrow present value of $B$ with repayment at date $t$:
   
   $- C_{call}(C_{BS}(S,X,T),S_t^* - X + B, t)$
   
   $- B e^{-rt}$
American Call Early Exercise Premium

These are the payoffs of this portfolio on date $t$ for the two cases.

<table>
<thead>
<tr>
<th>Position</th>
<th>Liquidation value if $S_t \leq S_t^*$&lt;br&gt; Sell the American call</th>
<th>Liquidation value if $S_t &gt; S_t^*$&lt;br&gt; Exercise the American call</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$C_{BS}(S, X, T)$</td>
<td>$C_{BS}(S_t, X, T - t)$</td>
</tr>
<tr>
<td>2</td>
<td>$C_{BS}(S, S_t^*, t)$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$-C_{Call}(C_{BS}(X, T), S_t^* - X + B, t)$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Borrow $B e^{-rt}$</td>
<td>$-B$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$C_{BS}(S_t, X, T - t) - B$</td>
<td>$S_t - X$</td>
</tr>
</tbody>
</table>

$C_{BS}$ denotes the Black-Scholes call option price, $C_{Call}$ denotes the call option price of another call option, and $B$ denotes borrowed funds.
Closed form valuation

A similar replicating portfolio can be constructed for every possible early exercise date $t$.

Given a schedule of risk neutral probabilities $\{ p_1, p_2, \ldots, p_t, \ldots, p_{T-1} \}$ for liquidation on each future date, the American option can be valued in closed form.
American Call Early Exercise Premium

Closed form valuation

Example: Assume the probability of liquidating an existing option position early is given by a Poisson process with intensity $\lambda$. The probability that there is no exercise prior to date $t$ is given by a survival function $G(t)$:

$$G(t) = e^{-\lambda(t-1)}$$

The unconditional probability of a liquidation on date $t$ is therefore

$$L(t) = G(t) - G(t+1) = e^{-\lambda(t-1)} - e^{-\lambda t}$$

To allow for options that are held to maturity, I assume $\lambda$ is such that $G(T) = 0.25$, i.e., for a call purchased at time 0, there is 25% probability it will not be liquidated early.

$$L(T) = G(T) = e^{-\lambda T} = 0.25$$
American Call Early Exercise Premium

The European option value at time 0 for an investor subject to early liquidation at date \( t \), where \( D(0, \tau) \) is the price of a date \( t \) zero coupon bond:

\[
C_0^E = E_0 \left[ \sum_{\tau=1}^{T-1} D(0, \tau) L(\tau) (C_{Eur}(S_{\tau}, X, T - \tau) - B) \right] \\
+ D(0, T) L(T) Max(S_T - X, 0)
\]

The American option value at time 0 for an investor subject to early liquidation:

\[
C_0^A = E_0 \left[ \sum_{\tau=1}^{T-1} D(0, \tau) L(\tau) Max(C_{Eur}(S_{\tau}, X, T - \tau) - B, S_{\tau} - X) \right] \\
+ D(0, T) L(T) Max(S_T - X, 0)
\]
American Call Early Exercise Premium

**Additional Assumptions**

The continuously compounded riskless rate is \( r \):

\[
D(s,t) = e^{-r(t-s)}
\]

The stock price dynamics under risk neutrality are given by:

\[
\frac{dS}{S} = \mu dt + \sigma dz
\]

with volatility \( \sigma \) a known constant.

The American call valuation equation becomes

\[
C_0^A = \sum_{\tau=1}^{T-1} e^{-r\tau} \left( e^{-\lambda(t-1)} - e^{-\lambda t} \right) E_0 \left[ \text{Max} \left( C_{BS}(S_{\tau}, X, T - \tau) - B, S_{\tau} - X \right) \right] + e^{-\lambda(T-1)} C_{BS}(S_0, X, T)
\]
In the real world, the bid-ask spread on an option is not a fixed constant. It varies with liquidity and moneyness.

To get a more accurate view of option market liquidity, I looked at 1-month call options on 10 stocks and fitted the following model to the quoted bid-ask spreads (adjusted to $S_\tau = 100$):

$$B \equiv (B_0 + B_1 \max(S_\tau - X, 0))/2$$

$B_0$ is the minimum spread and the slope $B_1$ determines how fast the spread widens as the option goes deeper in the money. $B$ is the half-spread, as before.
Replicating Portfolio for "American" Call with Single Early Exercise Date $t$
with Bid-Ask Spreads that Depend on Moneyness

Let $B \equiv (B_0 + B_1 \max(S_t - X, 0))/2$

1. Buy a European call with strike $X$ and maturity $T$ $C_{BS}(S, X, T)$
2. Buy $(1 + B_1/2)$ European calls with strike $S_t^*$ and maturity $t$ $C_{BS}(S, S_t^*, t)$
3. Write a compound call on the call in step 1, with strike $S_t^* - X + B$ $- C_{call}(C_{BS}(S, X, T), S_t^* - X + B, t)$
4. Borrow present value of $B_0/2$ with repayment at date $t$ $- B_0 e^{-rt/2}$
5. Write $B_1/2$ European calls with maturity $t$ and strike $X$. $-B_1 C_{BS}(S, X, t)/2$
American Call Early Exercise Premium

These are the payoffs of this portfolio on date $t$ for the two cases.

<table>
<thead>
<tr>
<th>Position</th>
<th>Liquidation value if $S_t \leq S_t^*$</th>
<th>Liquidation value if $S_t &gt; S_t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sell the American call</td>
<td>Exercise the American call</td>
</tr>
<tr>
<td>1</td>
<td>$C_{BS}(S, X, T)$</td>
<td>$C_{BS}(S_t, X, T - t)$</td>
</tr>
<tr>
<td>2</td>
<td>$(1 + \frac{B_1}{2})C_{BS}(S, S_t^*, t)$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$-C_{Call}(C_{BS}(S, X, T - t), S_t^* - X + B(S_t^*), t)$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Borrow $B_0 e^{-rt}/2$</td>
<td>$-B_0/2$</td>
</tr>
<tr>
<td>5</td>
<td>$-B_1C_{BS}(S, X, t)/2$</td>
<td>$-B_1Max(S_t - X, 0)/2$</td>
</tr>
<tr>
<td>Total</td>
<td>$C_{BS}(S_t, X, T - t) - B(S_t)$</td>
<td>$S_t - X$</td>
</tr>
</tbody>
</table>
American Call Early Exercise Premium

I used a ranking of option stocks by trading volume from the OCC and selected:

- Four with high option trading volume (in top 10): AAPL, FB, BAC, XOM
- Three with medium option trading volume (ranked 200-300): AIG, COST, GT
- Three with low option trading volume (ranked 500-600): TRIP, TMUS, CAKE

I picked two weeks with low and high market volatility:
  - June 22-26 (VIX = 13.23 on average)
  - Aug. 24-28, 2015 (VIX = 31.85 on average)
American Exercise Premium in High Volatility Regime ($\sigma=0.75$)

- **High Volume Options**: $B_0 = 0.135$, $B_1 = 0.036$
- **Medium Volume**: $B_0 = 0.391$, $B_1 = 0.092$
- **Low Volume**: $B_0 = 0.651$, $B_1 = 0.195$
### American Call Early Exercise Premium
#### Low Volatility ($\sigma = 0.25$) Regime

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>1 week</th>
<th>2 week</th>
<th>1 month</th>
<th>2 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.183</td>
<td>0.133</td>
<td>0.051</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>0.633</td>
<td>0.583</td>
<td>0.465</td>
<td>0.265</td>
</tr>
<tr>
<td></td>
<td>1.166</td>
<td>1.117</td>
<td>1.000</td>
<td>0.750</td>
</tr>
<tr>
<td>90</td>
<td>0.100</td>
<td>0.044</td>
<td>0.020</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>0.302</td>
<td>0.222</td>
<td>0.128</td>
<td>0.090</td>
</tr>
<tr>
<td></td>
<td>0.640</td>
<td>0.550</td>
<td>0.357</td>
<td>0.243</td>
</tr>
<tr>
<td>100</td>
<td>0.037</td>
<td>0.020</td>
<td>0.011</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>0.103</td>
<td>0.072</td>
<td>0.057</td>
<td>0.051</td>
</tr>
<tr>
<td></td>
<td>0.297</td>
<td>0.203</td>
<td>0.151</td>
<td>0.132</td>
</tr>
<tr>
<td>105</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>0.001</td>
<td>0.004</td>
<td>0.008</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>0.010</td>
<td>0.021</td>
<td>0.034</td>
</tr>
</tbody>
</table>
### American Call Early Exercise Premium
#### High Volatility ($\sigma = 0.75$) Regime

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>1 week</th>
<th>2 week</th>
<th>1 month</th>
<th>2 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.220</td>
<td>0.144</td>
<td>0.110</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>0.711</td>
<td>0.518</td>
<td>0.377</td>
<td>0.335</td>
</tr>
<tr>
<td></td>
<td>1.570</td>
<td>1.305</td>
<td>0.984</td>
<td>0.861</td>
</tr>
<tr>
<td>90</td>
<td>0.073</td>
<td>0.063</td>
<td>0.061</td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td>0.242</td>
<td>0.203</td>
<td>0.197</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>0.602</td>
<td>0.504</td>
<td>0.487</td>
<td>0.525</td>
</tr>
<tr>
<td>100</td>
<td>0.039</td>
<td>0.040</td>
<td>0.045</td>
<td>0.051</td>
</tr>
<tr>
<td></td>
<td>0.124</td>
<td>0.126</td>
<td>0.142</td>
<td>0.168</td>
</tr>
<tr>
<td></td>
<td>0.300</td>
<td>0.303</td>
<td>0.345</td>
<td>0.413</td>
</tr>
<tr>
<td>105</td>
<td>0.009</td>
<td>0.015</td>
<td>0.024</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>0.028</td>
<td>0.046</td>
<td>0.073</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td>0.064</td>
<td>0.106</td>
<td>0.172</td>
<td>0.258</td>
</tr>
</tbody>
</table>
American Exercise Premium: $S=100$, low $\sigma = .25$, hi $\sigma = .75$

- low vol, $X=80$, hi $\sigma$
- low vol, $X=85$, hi $\sigma$
- low vol, $X=90$, hi $\sigma$
- low vol, $X=95$, hi $\sigma$
- low vol, $X=100$, hi $\sigma$
- low vol, $X=105$, hi $\sigma$
American Exercise Premium: $S=100$, low $\sigma = .25$, hi $\sigma = .75$

- hi vol, $X=80$, low $\sigma$
- mid vol, $X=80$, low $\sigma$
- low vol, $X=80$, low $\sigma$
- hi vol, $X=90$, low $\sigma$
- mid vol, $X=90$, low $\sigma$
- low vol, $X=90$, low $\sigma$
Comparison with Standard Early Exercise Values

The American Put:
In theory, an American put should be optimally exercised when the extra interest gained by investing the strike price offsets the optionality component of time-value that is lost.

The maximum value of early put exercise is the interest that could be earned on the strike over the remaining life of the option: $X(e^{r(T-t)} - 1)$, treating the lost optionality as having zero value.

At the 5% interest rate we have assumed, these amounts for 1 week or one month are, respectively, $0.077$ and $0.330$ for an 80-strike put. For a 100-strike contract the comparable figures are $0.096$ and $0.413$.

The liquidity-based early exercise premia are the same order of magnitude and in many cases larger than these maximum exercise premia for American puts in the standard framework.
An American Call on a Dividend-Paying Stock:

In theory, an American call should be exercised early, just before the stock goes ex-dividend.

The dollar amount of the dividend is the maximum value of early exercise. This can only occur if the call that would be left if the option were not exercised would have zero remaining time value.

A 30-day call on a $100 stock that pays a quarterly dividend of $0.50. Ex-dividend is in 15 days. 40% volatility

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>American exercise for dividend</th>
<th>American exercise for liquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Hi volume</td>
</tr>
<tr>
<td>80</td>
<td>0.228</td>
<td>0.053</td>
</tr>
<tr>
<td>90</td>
<td>0.192</td>
<td>0.024</td>
</tr>
<tr>
<td>95</td>
<td>0.044</td>
<td>0.015</td>
</tr>
</tbody>
</table>
Conclusion

Real world option markets are not perfectly liquid, unlike the "perfect" markets of option theory.

The best bid in the market for an in the money option with less than a few months to expiration is below intrinsic value much of the time.

In that case, the holder of an American call who liquidates prior to expiration will typically do better to exercise than to sell in the market.

It is possible to value the right to exercise the American call and recover intrinsic value under standard Black-Scholes assumptions.

An American call IS worth more than a European call, by an amount that can easily exceed the value of theoretically correct early exercise of an American put or an American call on a dividend-paying stock.
Clearly, More Research Is Needed

Several major questions remain:

1. Why are the bids so low in the options market?

2. Why do option holders sell American options at prices below intrinsic value instead of exercising?

3. Why do we finance professors persist in telling our students that an American call on a non-dividend paying stock should never be exercised early, and is worth the same as a European call?
THANKS!