Dynamic Competition with Network Externalities: Why History Matters*

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Abstract

This paper considers dynamic platform competition in a market with network externalities. A platform that dominated the market in the previous period becomes "focal" in the current period, in that agents play the equilibrium in which they join the focal platform whenever such equilibrium exists. We ask whether a low-quality but focal platform can maintain its focal position along time, when it faces a higher quality competitor. Under finite horizon, we find that when platforms are patient enough, the unique equilibrium is efficient. With infinite horizon, however, there are multiple equilibria in which either the low or the high quality platform dominates. If qualities are stochastic, the more platforms care about the future, the platform with the better average quality wins more often than the other. As a result, social welfare can decrease when platforms become more forward-looking.

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1 Introduction

Platform competition typically involves not only network effects, but also repeated interaction. We often observe that the platform that was dominant in the recent past has the advantage of customers favorable expectations in that customers expect that today the platform will be successful in also attracting other customers. We call such a platform a *focal* platform. For example, following its previous success with the iPhone 4, Apple's pre-orders of its iPhone 5 pre-orders topped 2 millions, only one day after its launch in September 2012, even though at that time there were no applications that could take the advantage of the new features. Moreover, analysts predicted that 50 millions users would buy the new smartphone within 3 months of its launch.[1] Similar dynamics was present during the release of iPhone 6, where sales topped 4 million in the first 24 hours.[2] In contrast, Blackberry and Windows phones did not enjoy such advantage in the same period. Despite the new Blackberry phones – Q10 and Z10 – receiving very good reviews, the absence of the positive expectation made it difficult for Blackberry to gain substantial market share. Application developers were skeptical about its ability to attract users. The sales were indeed sluggish, due to small number of apps available.[3] We can ascribe the skepticism and resulting lack of apps to the recent history.

At the same time, we do observe that platforms may lose the market even if they were winning in the past, if facing higher quality competitor. In the market for smartphones, for example, Nokia dominated the early stage, along with RIM, with smartphones based on physical keyboard. Apple then revolutionized the industry by betting on the new touch screen technology and its new operating system. Few years later, Samsung, managed to gain a substantial market share (though not strict dominance) by betting on smartphones with large screens. Industry leader changes were also common in the history of the video-game consoles market. Nintendo, Sony and Microsoft alternated in being the market leader.[4] That means that platforms may sometimes overcome the unfavorable expectations of the market. We are interested in when is it profitable for platforms facing unfavorable position to invest

[1] Faughnder and Satariano (2012) Faughnder, Ryan and Adam Satariano, Apple iPhone 5 Pre-Orders Top 2 Million, Doubling Record, Bloomberg (2012). Available at: http://www.bloomberg.com/news/2012-09-17/apple-iphone-5-pre-orders-top-2-million-double-prior-record-1-.html

[2] See: http://www.cnet.com/news/apple-iphone-6-iphone-6-plus-preorders-top-4m-in-first-24-hours

[3] On the quality and launch of the Blackberry phones see Bunton (2013) and Austin (2012).

[4] Hagiu and Halaburda (2009)

in winning the market, and when it is profitable for a platform facing a favorable position to invest in keeping the market, when both platforms know that it may impact their future position.

We know that without taking into account the impact on the future, it may not be profitable for the non-focal platform to overcome the expectations disadvantage even if it were to offer higher quality than the focal platform. This is because the presence of network effects provides the focal platform with a short-term competitive advantage — in a one-time interaction a focal platform can use its focal position for dominating the market even when competing against higher quality platform. In the long run, however, one may expect that when platforms care about the future, a high-quality but non-focal platform can overcome its beliefs disadvantage because it can afford to incur losses in the short run to become focal in the future. At the same time, the low-quality focal platform who cares about the future also has incentive to invest in keeping the focal position. While one of the platforms has the quality advantage, the other has expectations advantage of the market.

In this paper, we ask whether a low-quality platform that benefits from the focal position can keep dominating the market in a dynamic setting when platforms are forward-looking and take into account the future benefits of winning the market today. For example, is it possible for Apple to keep dominating the market for tablets even if competitors (Samsungs Tab, Microsofts Surface, etc.) were to offer tablets of higher base quality? Is it possible that a videogame console would maintain market leadership for several generations, even if facing more advanced competitors? Specifically, we ask whether it becomes more likely that the best platform wins as future matters more for the platforms. The identity of the winning platform also affects social welfare, which increases when it is more likely that the better platform wins.

To investigate our research question, we analyze a model of dynamic competition between two platforms. In each period, one of the platforms wins the market. In order to focus on the dynamic aspects of the model, we assume homogeneous consumers.[5] Hence the winning platform captures the whole market. The consumers base their behavior in the current period on the observation of the past outcomes. Specifically, the platform that won the market in the previous period becomes focal in the current period. In such a case, winning the market in one period gives the platform an advantage in the future periods. Hence, a non-focal platform may be willing to sacrifice current profit to gain better future market position.

[5] We discuss the consequences of this assumption in the conclusion.

We start with the case in which each the platforms stand-alone qualities are constant for all periods and the time horizon is finite. We show that when platforms do not care about future profits, the focal platform maintains its focal position even though it offers a lower quality than the non-focal platform, as long as the quality gap is sufficiently small. But when the future is valuable for the two platforms and the quality gap is sufficiently high, the superior quality platform wins the market at the start of the game and maintains its leadership. The reason is that a high quality platform can earn higher profits as focal in the last period of the game than the low-quality one. Taking it into account, the higher quality platform has a larger incentive to fight for focality in the early stages of the game.

We then consider infinite horizon. We find that when platforms moderately care about the future, there is a unique equilibrium in which a high-quality platform wins the market. But when the platforms put a lot of value in the future, then both the incentive of the focal platform to keep the focal position and the incentive of the non-focal platform to win the focal position increase, which give rise to multiple Markov equilibria: in some of these equilibria the focal can maintain its leadership infinitely even if it is of low-quality, but in one Markov equilibrium the high-quality platform wins the focal position in the first period and then maintains it infinitely even if it starts as a non-focal platform.

For social welfare, these results indicate that starting from a static case, the more platforms care about the future, the higher is social welfare because the market moves from the equilibrium in which the low-quality platform wins to the one in which the high-quality platform overcomes its non-focal position and then maintain the focal position infinitely. However, the effect of a further increase in the platforms awareness of future profits on welfare is ambiguous because then there are multiple equilibria.

When qualities are constant, our model finds that the same platform dominates the market in all periods. However, we observe that in some cases, platforms "take turns" in being the dominant platform, as in the case of Sonys, Nintendos and Microsofts videogame consoles mentioned earlier. We therefore study how focality and the importance of the future affect changes in market leadership and efficiency.

To study this question, we then consider the case where the platforms qualities change stochastically each period, which is consistent with continuous technology improvements in the markets like videogame consoles and smartphones. In this setting it is possible for a low-quality platform today to be of higher quality tomorrow. However, we assume that in expectations, one of the platforms has a higher quality.

Unlike the fixed quality case, we find a unique Markov equilibrium. It is possible for each platform to win the market in each period, when its quality realization is sufficiently high. However, we find that the more platforms care about the future, it may become more likely that a focal platform can win the market even when its quality realization is lower then that of the non-focal one. In some cases, a focal platform can lose the market even when its quality realization is higher than that of the non-focal one. That means that inefficiency may increase the more platforms care about the future.

The intuition for this result is that the platform with higher quality in expectations has a high probability of being able to defend its focal position in the future, because of its superior expected quality. Therefore, the more platforms care about the future, this platform has a larger incentive to compete aggressively to win focality in the current period, even if the quality realization happens to be low. At the same time, the competing platform has smaller incentive to win the market.

This result indicates that the changes in market leaderships, following technological improvements, which we observe in several markets for platforms (i.e., videogame consoles, smartphones), may not necessarily result in outcome in which the platform with superior quality wins.

The main conclusion of our paper is that long-term consideration may not always lead to an outcome in which the best platform wins. For example, there has been a disagreement in economic literature whether the presence of network effects allows for a long-term market inefficiency. In 1985 article Paul David used an example of QWERTY keyboard as an example of such long-term inefficiency. He showed evidence that Dvorak keyboard is better, in that it allows for faster typing after shorter training. But QWERTY prevails because of network effects. Since virtually all keyboards produced have QWERTY layout, people learn to touch-type in this layout. And therefore it makes little sense for the manufacturers to produce other layouts than QWERTY.

Leibovitz and Margolis (1990) criticize David's argument claiming that in a long term competitive will result in the efficient outcome, also in environments with network effects. Therefore, a long-term success of QWERTY is due to its quality superiority over Dvorak and not network effects. They base their arguments on a case study, and do not offer a theoretical argument for this result. Hossain and Morgan (2009) report that in an experiment the efficient platform always wins over time, supporting Leibovitz and Margolis's argument.

Our paper adds to the debate whether a market with network effects can achieve efficiency over a long term. We show that both efficient and inefficient equilibria are possible in the long-term, depending on the conditions. Unlike in the papers on QWERTY, in our model platforms are strategic. They actively set prices to compete for the users. This does not happen in the case of competing keyboard layouts. In both cases, the impact of network effects on the long-term efficiency remains in the center of the issue.

As another example that involves strategic platforms, Blackberry's inability (to date) to gain a substantial foothold in the market for smartphones, in spite of receiving good reviews, can be explained as an inefficient equilibrium that according to our model may prevail even when platforms cares about the future. Likewise, in the market for tablets, our model suggests that Apple can potentially defend it's market dominancy of the iPad even when facing potential higher quality competitors and even when platforms are forward looking.

Most theoretical analyses of platform competition focus on static games. Caillaud and Jullien (2001, 2003) consider competition between undifferentiated platforms, where one of them benefits from favorable beliefs. Hagiu (2006) considers undifferentiated platform competition in a setting where sellers join the platform first, and only then buyers. Lopez and Rey (2009) consider competition between two telecommunication networks when one of them benefits from customers' "inertia," such that in the case of multiple responses to the networks' prices, consumers choose a response which favors one of the networks. Halaburda and Yehezkel (forthcoming) consider competition between platforms when one of them has only partial beliefs advantage. While all those papers acknowledge the dynamic nature of the platform competition, they aim at approximating the characteristics of the market in static models. Halaburda and Yehezkel (forthcoming) explore how platform's strategies affect their future profits in a simple multi-period setup where the beliefs advantage depends on the history of the market. Markovich (2008) analyzes hardware standardization in a dynamic market where software firms invest in new product innovation. But the dynamics of platform competition is still underexplored. Cabral (2012) develops a dynamic model of competition with forward looking consumers but where only one consumer chooses at a time avoiding the coordination issue we focus on. Networks in Cabral (2012) have a long-term incentive to build an installed base because attracting a consumer in a current period increases the size of the network in future periods which in turn increases the network's attractiveness to future consumers. Our paper contributes to Cabral's work by exploring platforms' long-term incentive to affect beliefs and win a future focal position.

Bligaiser, Crémer and Dobos (2013) consider dynamic competition in the presence of switching costs. Our model share with theirs the feature that success provides an incumbency advantage. But the intertemporal linkage and the demand dynamics differ between the two models. While it is possible to think of network externalities as a type of switching costs, there is a qualitative difference between the two. With network externalities, consumers pay switching costs only of they join the "wrong" platform. If all consumers join the same platform, they do not pay switching costs even if they move from one platform to another. Consequently, consumers in our model do not need to form beliefs about the identity of the focal platform in future periods. In particular, this distinction enables consumers to switch from one platform to another when qualities are stochastic regardless of probability that a platform will remain focal in the future. Our real-life examples (i.e., the markets for smartphones and videogames) may include both the traditional switching costs (coming from the need to adjust to a new operating system, for example) and network externalities. While Bligaiser, Crémer and Dobos (2013) focus on the former, our paper complements their paper by focusing on the latter type of costs. A second main difference between the two papers is that they consider switching cost heterogeneity, while we focus on quality differential.

Argenziano and Gilboa (2012) consider a repeated coordination game where players use history to form beliefs regarding the behavior of other players. Our paper adopts the same approach in the context of platform competition and study how platforms should compete given such belief formation by consumers. In our paper, platforms can alter beliefs by wining and shifting consumers' coordination in their favor.

Our paper is related to ongoing work by Biglaiser and Crémer (2012) trying to define a notion of consumer inertia creating an history dependency. We do not try to model how history dependency emerges but its implications for competition.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we consider the benchmark of a dynamic game with finite horizon. Section 4 characterizes Markov equilibria under infinite time horizon. Section 5 considers the case where platforms qualities change stochastically over time.

2 The Model

Consider a homogeneous[6] population of size 1 and two competing platforms, $i = A, B$, with the same cost normalized to 0. There are T periods, $t = 1, 2, ..., T$, where T may be finite or infinite. Each platform i offers a stand-alone value, $q_i > 0$, which we call quality.[7] Additionally, consumers benefit from network effects. Consumers' utility from joining platform i is $q_i + \beta n_i - p_i$, where n_i is a measure of other consumers that joined platform i, β is strength of network effects, and p_i is the price of platform i.[8]

Every period each platform i sets a price $p_i(t)$, and then consumers decide which platform to join for the current period. In what follows, prices can be negative, interpreted as price below cost.[9] The two platforms operate for the T periods and discount future profits by δ, where $0 \leq \delta \leq 1$. Since there are no switching costs, consumers decide which platform to join each period independently of other periods and consequently independently of their discount factor.

The issue with competition in an environment with network effects is that there is a multiplicity of equilibria. Indeed consider the allocation of consumers that emerges for given prices. If $q_i - p_i(t) > q_j - p_j(t) + \beta$, then all consumers would join platform i. But if

$$|q_A - q_B + p_B(t) - p_A(t)| < \beta, \tag{1}$$

there are two possible allocations, all consumers join A or all join B. This multiplicity creates a major difficulty to discussing dynamic competition in environments with network effects, and several solutions have been proposed to address this issue. In this paper we rely on the idea of pessimistic beliefs and focal platform as developed in Caillaud-Jullien (2003), Hagiu (2006) and Jullien (2000). We say that platform i is *focal* if under condition (1), the consumers join platform i. We assume that at any date there is a focal platform.

In a dynamic model with $t = 1, \ldots, T$, the identity of the focal platform in $t > 1$ may be related to the history of the market. In this paper we explore the issue how allowing for such historical dependency affects the future outcomes of the market. To simplify the matters, we focus on one period dynamics.

[6]We discuss robustness of our results to the presence of hetergeneous agents in the conclusions (Section 6).

[7]We consider both cases of q_i's fixed over time (Sections 3 and 4), and of qualities that change between periods (Section 5).

[8]Since the consumers are homogeneous they all join the same platform.

[9]To allow for truly negative prices, we need to assume that agents who collect the subsidy indeed join the platform and provide the benefit to other users.

At every period t, let us summarize the market outcome by a pair (w_t, f_t), where $w_t \in \{A, B\}$ is the identity of the active platform, i.e., the platform who wins the market in t,[10] and $f_t \in \{A, B\}$ is the identity of the focal platform in t. It is possible for the non-focal platform to win the market, therefore these two do not need to be the same. Based on the observation of past outcome, consumers form conjectures about the platform most likely to win in the current period. These conjectures are assumed to converge to a single focal platform. In $t = 0$, one of the platforms is arbitrarily set as the focal platform. Call this platform A. At any date, the focal platform f_t is common knowledge and it is the only payoff relevant variable. The dynamics of the platform focality is then given by transition probabilities, $\Pr(f_t = i \mid w_{t-1}, f_{t-1})$. We consider a deterministic rule where the last winner of the market becomes focal, i.e., $\Pr(f_t = w_{t-1} \mid w_{t-1}, f_{t-1}) = 1$.

As a benchmark case for our analysis, consider a static, one-period game in this environment. Network externalities may give rise to inefficiency in equilibirum in a static game. Platform A is the focal platform, but it may be of higher or lower quality than platform B. When $q_A - q_B + \beta > 0$, in the equilibirium the platforms set $p_A = q_A - q_B + \beta, \quad p_B = 0$, and all customers join A. When $q_A - q_B + \beta < 0$, such strategy would yield platform A negative profits, so it is not an equilibrium. In such a case, in the equilibrium the platforms set $p_A = 0, \quad p_B = q_B - q_A - \beta > 0$, and all customers join platform B.

Thus, for q_A such that $q_A < q_B$ but $q_A > q_B + \beta$ platform A wins despite offering lower quality. It wins because it happens to be focal. This effect is called *excess inertia* and it creates inefficient outcome in equilibrium.

When there are multiple periods, a non-focal platform may find it worthwhile to win the market by setting negative price in an earlier period. While it would yield negative profit one period, the focal position could allow to recover those loses in future periods. In the static market no platform finds it optimal to win the market at negative prices, as there is no way to recover the loses. Thus, the focal platform has the upper hand even when it offers lower quality.

In the dynamic market, we could expect the higher quality non-focal platform to have an advantage, because with the higher stand-alone quality it can earn higher profits as a focal platform than the lower-quality one. Thus, it should be more worth the investment for the higher-quality platform to win the market than for the lower-quality platform to defend it.

[10] In this model there cannot be market sharing in equilibrium: at each date, a single platform attracts the whole population.

Of course, the focal platform anticipates this and strives to prevent the non-focal platform from taking the market.

3 Dynamic game with finite horizon

In this section we consider the case where the time horizon is finite. We show that with a finite horizon, there is a unique subgame perfect Nash equilibrium. In this equilibrium a high-quality but non-focal platform takes over the market in the first period and then maintains the focal position in all periods, when platforms sufficiently care about the future (i.e., δ is high or time horizon long) or the quality gap is sufficiently high. Otherwise, when a low-quality platform is focal in the first period, this platform can maintain the focal position in all periods.

Suppose that the time horizon is $T \geq 2$. We define the price $p_i^f(t)$ and the value function $V_i^f(t)$ as the equilibrium price and the expected future discounted profit of platform i at time t when platform f is focal at time t. We focus our analysis below on the interesting case where network effects are sufficiently high in comparison with the quality gap: $\beta > |q_i - q_j|$, such that in a one-period game the focal platform wins even with lower quality than the non-focal one. Lemma 1 and Proposition 1 below provide a general solution for all values of β, q_A and q_B. Looking for subgame perfect equilibria, we solve the game backwards.

Consider the last period, $t = T$. Since there is no future, the subgame equilibrium is identical to the one-period benchmark of Section 2. When platform i is focal, it wins the market regardless of the quality gap and earns $V_i^i(T) = q_i - q_j + \beta$ while platform j earns $V_j^i(T) = 0$. Notice that the last-period profits of a focal high-quality platform are higher than the last-period profits of a focal low-quality platform, i.e., $V_i^i(T) > V_j^j(T)$ when $q_i > q_j$.

Next, we turn to the period before the last one, $t = T - 1$. To facilitate notations, suppose that in this period platform A is focal. Both platforms take into account that by wining the market in this period, they will become focal in the next period and earn an additional profit of $\delta V_i^i(T)$. Consider first a subgame equilibrium where platform A wins the market. The lowest price that the losing platform B is willing to charge at time $T - 1$ is $p_B^A(T - 1) = -\delta V_B^B(T)$. To win, platform A needs to charge a price such that $q_A + \beta - p_A^A(T-1) > q_B - p_B^A(T-1)$ and earn the value function $V_A^A(T-1) = p_A^A(T-1) + \delta V_A^A(T)$

while platform B earns $V_B^A(T-1)=0$. Solving for $V_A^A(T-1)$ yields

$$V_A^A(T-1)=\beta-(q_B-q_A)(1+2\delta), \tag{2}$$

and this equilibrium exists if (2) is positive.

In general, the losing platform $-w$ sets the lowest price it can afford, $p_{-w}^f(t)=-\delta V_{-w}^{-w}(t+1)$. The winning platform w sets the maximal price that allows it to win the market, which is $p_w^f(t)=q_w-q_{-w}+p_{-w}^f(t)+\beta$ if $w=f$, and $p_w^f(t)=q_w-q_{-w}+p_{-w}^f(t)-\beta$ if $w\neq f$. We apply the same analysis for a subgame equilibrium in which the non-focal platform B wins at time $T-1$. In this equilibrium the losing platform A earns $V_A^A(T-1)=0$, while the winning platform B earns

$$V_B^A(T-1)=-\beta+(q_B-q_A)(1+2\delta), \tag{3}$$

This subgame equilibrium exists if (3) is positive. Comparing (2) and (3) shows that in $T-1$ exactly one of those subgame equilibria exists for any set of parameters. In this unique equilibrium platform A wins at $T-1$ if it is focal when

$$q_A>q_B-\frac{\beta}{1+2\delta}. \tag{4}$$

Otherwise, platform A loses the market in $T-1$, even when it is focal. Condition (4) shows that in contrast with the one-period case (and in contrast with the last period of the dynamic game), a non-focal platform B can win the market in $T-1$ even when $q_B-q_A<\beta$. The intuition for this result is that since platforms care about the future and since platform B's profit from being focal in the last period is higher than that of platform A, platform B's incentive to win the focal position at $T-1$ might be stronger than the incentive of platform A to maintain it's focal position. In particular, if the quality gap, q_B-q_A, and δ are sufficiently high, then a high-quality but non-focal platform can win the market at $T-1$, and then maintain the focal position at time T. In such a case, forward-looking platforms solve the inefficiency that may emerge because of coordination problem.

If $T>2$, there are periods before $T-1$, $t=1,...,T-2$. Suppose again that platform A is focal in $T-2$. In a subgame equilibrium where it loses the market in $T-2$, it sets $p_A^A(T-2)=-\delta V_A^A(T-1)$. Notice that if condition (4) does not hold, $V_A^A(T-1)=0$ and the price platform A sets is 0. This is because when condition (4) does not hold, platform A would lose the market in $T-1$ even if it won in $T-2$ and still be focal in $T-1$. By similar logic, platform A also loses the market and sets price 0 for all periods before

$T-2$. Nonetheless, it would still set a strictly negative price at T, as derived above. When condition (4) does not hold, there is no subgame equilibrium in $T-2$ where platform A wins the market profitably.

When condition (4) holds, platform A may either win or lose the market in $T-2$. In a subgame equilibrium where it loses the market, it sets $p_A^A(T-2) = -\delta V_A^A(T-1)$ which is strictly negative since (4) holds. But if A loses the market, its discounted future profit is $V_A^A(T-2) = 0$. Thus, for all periods before $T-2$, it sets price 0 and lose the market.

In a subgame where platform A wins in $T-2$, it sets the highest price with which it can win, $p_A^A(T-2) = q_A - q_B + p_B^A(T-2) + \beta$, where $p_B^A(T-2) = -\delta V_B^B(T-1)$, which in turn may be negative or 0. Again, exactly one subgame is possible: A wins in $T-2$ when

$$q_A > q_B - \frac{\beta}{1+2\delta+(2\delta)^2}\,,$$

and otherwise A loses. Notice that in the latter case, platform A wins each period when the time horizon is $T \leq 2$, but loses the market forever in $t=1$ when $T \geq 3$. We call T_A the shortest time horizon in which it is not worthwhile for platform A to win the market, even if it starts focal. In this case, $T_A = 3$. In the case when (4) does not hold, but $q_A - q_B < \beta$, $T_A = 2$. When $q_A - q_B > \beta$, $T_A = 1$. In general, T_A depends on the parameters of the model and may be an arbitrary number. For some parameter values there does not exist a finite T_A, i.e., for any T it is worthwhile for platform A to win the market if it is focal. We can similarly find T_B for platform B.

The following lemma characterizes how the equilibrium outcome for an arbitrary finite horizon depends on the parameters. All proofs are in the Appendix.

Lemma 1 (Subgame perfect equilibrium for arbitrary finite T) *For any set of parameters q_A, q_B, β, δ, there exists a unique subgame perfect equilibrium for arbitrary finite T. In the equilibrium the same platform wins the market in all periods. The identity of the winning platform, and its future discounted profit depend on the parameters:*

1. $|q_A - q_B| < \beta\frac{1-2\delta}{1-(2\delta)^T}$
   Then A wins every period because it is initially focal, and earns the total profit of
   $$(q_A - q_B)\frac{1-(2\delta)^T}{1-2\delta} + \beta\,.$$
2. $q_A - q_B > \beta\frac{1-2\delta}{1-(2\delta)^T}$
   Then $T_B < T$. Platform A wins every period, because it has sufficient quality advantage

and earns

$$(q_A - q_B + \beta)\frac{1-\delta^{T-T_B}}{1-\delta} + \delta^{T-T_B}\left((q_A - q_B)\frac{1-(2\delta)^{T_B}}{1-2\delta} + \beta\right).$$

3. $q_B - q_A > \beta\frac{1-2\delta}{1-(2\delta)^T}$
   Then $T_A < T$. Platform B wins every period because it has sufficient quality advantage, and earns

$$(q_B - q_A + \beta)\frac{1-\delta^{T-T_A}}{1-\delta} + \delta^{T-T_A}\left((q_B - q_A)\frac{1-(2\delta)^{T_A}}{1-2\delta} + \beta\right) - 2\beta.$$

The losing platform's profits are 0 in all cases.

The main qualitative results of Lemma 1 are the following. First, in all periods the same platform wins the market so an outcome in which the non-focal platform wins the market can only occur at the first period. Second, platform B wins the market only if it has quality advantage. But platform A may win either because it has a quality advantage, or it can win despite offering lower quality, because it started with a focal position. The latter happens when $0 < q_B - q_A < \beta\frac{1-2\delta}{1-(2\delta)^T}$, and it is an inefficient outcome. In all other cases, the higher quality platform wins, so the equilibrium outcome is efficient. Notice that the set of parameters for which the equilibrium outcome is inefficient is decreasing as T and δ increase, i.e., when future is more important for the platforms.

Thus, competition over multiple periods yields efficient equilibrium outcome for parameters that in a static model resulted in lower-quality platform winning. In this sense, there is less inefficiency when the time horizon increases. We could expects then that the inefficiency would disappear altogether if the time horizon was extended to infinity. However, this is not always the case.

To show that, we directly extrapolate the equilibrium outcome in Lemma 1 for $T \to \infty$. We need to recognize, however, that the ratio $\frac{1-(2\delta)^T}{1-2\delta}$ converges to $\frac{1}{1-2\delta}$ for $\delta < \frac{1}{2}$, and converges to ∞ for $\delta > \frac{1}{2}$.

Proposition 1 (Subgame perfect equilibrium extrapolated for $T \to \infty$) *For $T \to \infty$:*

1. $|q_A - q_B| < \beta(1-2\delta)$ *or* $q_A = q_B$
   Then platform A wins every period because it is initially focal, and earns the total profit of

$$V_A^A(1) = \frac{q_A - q_B}{1-2\delta} + \beta.$$

2. $q_A - q_B > \max\{\beta(1-2\delta), 0\}$

Then platform A wins every period, because it has quality advantage. And the platform earns

$$V_A^A(1) = \frac{q_A - q_B + \beta}{1-\delta}\,.$$

3. $q_B - q_A > \max\{\beta(1-2\delta), 0\}$

Then platform B wins every period because it has sufficient quality advantage. And the platform earns

$$V_A^B(1) = \frac{q_B - q_A + \beta}{1-\delta} - 2\beta\,.$$

The losing platform's profits are 0 in all cases.

No matter how long the time horizon is, the outcome of a subgame perfect equilibrium may be inefficient. When $0 < q_B - q_A < \beta(1-2\delta)$, platform A wins despite lower quality. But the problem of inefficiency due to excessive inertia occurs less often in with longer time horizons. And the inefficiency disappears altogether when platforms care about future more than about the present, i.e., $\delta > \frac{1}{2}$.

In following sections, we explore other ways why inefficient outcome may occur in equilibrium.

4 Markov perfect equilibria under infinite time horizon

In Proposition 1, we characterized an equilibrium in the infinite game by extrapolating the subgame perfect equilibrium of an arbitrary finite game. Infinite time horizon, however, may give rise to other equilibria as well. In this section we identify Markov perfect equilibria in the infinite game. The subgame perfect equilibrium identified in Proposition 1 is a Markov perfect equilibrium. But we find that there are also other Markov perfect equilibria that cannot arise from extrapolating any finite-game solution. Those new equilibria often result in inefficient outcomes for parameters where Proposition 1 equilibrium is efficient.

Every period t of the infinite game is characterized by the state variable at time t, f_t. A Markov perfect equilibrium is characterized by the strategies of both platforms in all possible states, and the outcome in each state. We consider three pure strategy equilibria outcomes:

(i) platform A wins in both states, (ii) platform B wins in both states, and (iii) the focal platform wins.[11]

In what follows we characterize the strategies supporting those equilibria outcomes, and parameter conditions under which each equilibrium exists. We define the value function V_i^f as the equilibrium expected discounted profit of platform i when platform f is focal.

Consider first the equilibrium outcome where platform A wins in both states. In this equilibrium, the value functions for platform B are $V_B^B = V_B^A = 0$, because platform B never sells. Platform B sets price $p_B^f = 0$, because in no situation platform B would like to win with price $p_B < 0$, given that it cannot count on future profits to justify the "investment" in taking over the market. When A is focal, it optimally sets $p_A^A = q_A - q_B + \beta$. Similarly, were B focal, platform A sets $p_A^B = q_A - q_B - \beta$, and platform B sets $p_B^B = 0$. Were A to set a higher price, platform B would keep the market and make non-negative profits. In such a case

$$V_A^A = q_A - q_B + \beta + \delta V_A^A \qquad \text{and} \qquad V_A^B = q_A - q_B - \beta + \delta V_A^A\,.$$

Moreover, incentive compatibility for platform A requires that

$$V_A^A \geq \delta V_A^B \qquad \text{and} \qquad V_A^B \geq 0\,.$$

Therefore, this equilibrium exists when $q_A - q_B \geq \beta(1 - 2\delta)$. With a similar analysis for platform B, we arrive at the following result.

Lemma 2 *There is an equilibrium where platform i wins in both states if $q_i - q_j \geq \beta(1 - 2\delta)$.*

Lemma 2 shows that a non-focal platform B can win a focal position and maintain it in all future periods when $q_B - q_A \geq \beta(1 - 2\delta)$. This inequality holds when quality of B is substantially superior than the quality of A, when platforms are very forward looking, such that δ is high, or when the network effects, β, are weak. Notice, however, that for $\delta > 1/2$, the left-hand-side of the inequality becomes negative, and that means that initially non-focal platform B can win the market every period even when $q_B < q_A$. Such equilibrium constitutes excess momentum and is inefficient. This equilibrium does not arise in the subgame perfect equilibrium even for $T \to \infty$.

In a finite case with $\delta > 1/2$, the platform with lower quality could never profitably set negative prices, as it would not regain this investment later. Therefore, the aggressive

[11] The fourth possibility of a pure strategy outcome, that non-focal platform wins, cannot be supported by any strategy.

strategy of setting $p_B^A = q_B - q_A - \beta$ when non-focal would not be credible, even if the last period was deferred to infinity. However, when there is no last period, a low quality platform can profitably use the aggressive strategy. Given the credibility of the strategy, the best response to the aggressive strategy is accommodation. This allows for the inefficient equilibria to exist.

Similarly, platform A can keep dominating the market forever under $\delta > 1/2$ even if $q_A < q_B$, as long as $q_A - q_B \geq \beta(1-2\delta)$. This equilibrium, characterized by excess inertia, is also inefficient, and also occurs only in truly infinite game. It is based on the same credible aggressive strategy of platform A and accommodating response of platform B.

The remaining equilibrium to consider is one where the focal platform wins. Recall that p_i^f denotes the price of platform i when f is focal in such an equilibrium. Since the wining platform anticipates it will stay active and focal from the new period on, we have value functions

$$V_i^i = \frac{p_i^f}{1-\delta}, \quad V_i^j = 0$$

The benefits of selling at a given date is $p_{it} + \delta V_i^i$. It follows that the minimal profit that platform i is willing to sacrifice today to gain the market is $-\delta V_i^i$. In such an equilibrium the focal platform sets a price $p_i^i \leq q_i - q_j + \beta - \delta V_j^j$, otherwise the competing platform would set a price above $-\delta V_i^i$ and win the market. Ruling out cases where $p_j < -\delta V_j^j$ because wining at this price would not be profitable for firm j, we obtain equilibrium prices[12]

$$p_i^i = q_i - q_j + \beta - \delta V_j^j,\ p_j^i = -\delta V_j^j.$$

This leads to values function in such an equilibrium solutions of

$$\begin{aligned}(1-\delta)\, V_A^A + \delta V_B^B &= q_A - q_B + \beta \\ (1-\delta)\, V_B^B + \delta V_A^A &= q_B - q_A + \beta\end{aligned}$$

yielding

$$V_A^A = \frac{q_A - q_B}{1-2\delta} + \beta; \quad V_B^B = \frac{q_B - q_A}{1-2\delta} + \beta.$$

We then conclude that:

Lemma 3 *There is an equilibrium where the focal platform wins in every state if* $\beta\,|1-2\delta| > |q_B - q_A|$.

[12]This is innocuous for existence argument

Proof. For this to be an equilibrium it is necessary and sufficient that $V_A^A > 0$ and $V_B^B > 0$. ■

Since platform A is arbirtarly focal in the first period, in equilibrium were the focal platform wins in every state platform A wins every period. By the condition in Lemma 3, platform A can maintain its focal position in all future periods even when it offers a lower quality than platform B. This is another instance of ineffecient, "excess intertia" equilibrium. To see the intuition for this result, consider first the case of $\delta \leq 1/2$. As Lemma 3 shows, the equilibrium holds in this case if δ and the quality gap, $q_B - q_A$, are sufficiently low. Intuitively, suppose that q_B increases. This has two effects on V_B^B. First, a *direct* effect – since $p_B^B = q_B - q_A + \beta - \delta p_A^B$, taking V_A^A as given, platform B can now attract agents with a higher p_B^B, implying that V_B^B will increase. Second, a *strategic* effect – since $p_B^A = -\delta V_B^B$, platform A knows that even if it is focal, it will compete against a more aggressive platform B, because platform B has more to gain by becoming focal. This reduces V_A^A, which in turn increases V_B^B because platform A will not compete aggressively to gain a focal position when it is not focal. Both the direct and the strategic effects work in the same direction of increasing V_B^B and decreasing V_A^A, when $\delta < 1/2$. If the gap $q_B - q_A$ is sufficiently wide, V_A^A becomes negative, implying that platform A cannot maintain its focal position when competing against a superior quality platform. As δ increases, platform B cares more about future profit so it will have a stronger incentive to win the market when it is not focal, and maintain its focal position when it if focal.

Now suppose that $\delta > 1/2$. As Lemma 3 reveals, in this case the equilibrium is completely reversed. Now, if $q_B > q_A$, then $V_A^A > V_B^B$, and as q_B increases, V_B^B decreases while V_A^A increases. However, the equilibrium in the case of $\delta > 1/2$ relies on the somewhat unusual feature where platforms "overreact", such that as q_B increases, while the direct effect increases V_B^B (as in the case of $\delta < 1/2$), the strategic effect works in the opposite direction and is stronger than the direct effect. To see how, suppose that platform B is focal and q_B increases. The equilibrium holds when platform B expects that as a response to the increase in q_B, platform A will over-react in the opposite direction than in the case of $\delta < 1/2$, by becoming very aggressive and decreasing its price when it is not focal, p_A^B. In this case, V_B^B increases since q_B increases (direct effect), but decreases since p_A^B decreases (strategic effect). The strategic effect outweighs the direct effect, and the overall effect is to decrease V_B^B and to increase V_A^A. In this equilibrium however, platform A reduces its price p_A^B because it anticipates that it will benefit once focal from competing with a more efficient rival, a rather peculiar feature.

Notice that if we rule out the possibility of overreaction, then the equilibrium does not adjust to small changes in quality. Without overreaction, e.g., with iterated best response dynamics, a small change to q_B in this equilibrium under $\delta > 1/2$ results in convergence to an equilibrium described in Lemma 2. In this sense, we can say that equilibrium where the focal platform wins in every state is unstable for $\delta > 1/2$. This is not the case for this equilibrium under $\delta \leq 1/2$, nor for equilibria described in Lemma 2 for any δ.

Proposition 2 below summarizes the results of Lemmas 2 and 3.

Proposition 2 (Markov perfect equilibria) *Suppose that platform A is focal at period t=1. Then,*

(i) for $q_B - q_A > \beta|1 - 2\delta|$ there exists a unique equilibrium, and in that equilibrium platform B wins;

(ii) for $\beta(1-2\delta) < q_B - q_A < \beta|1-2\delta|$, which occurs only for $\delta > 1/2$, there exist multiple equilibria, and in one of those equilibria platform B wins;

(iii) for $q_B - q_A < \beta(1 - 2\delta)$, platform A wins in all equilibria.

Proof.

This follows from the assumption that A is initially focal and from Lemma 2 and Lemma 3. ■

Equilibrium active platform is depicted in Figure 1. The figure shows that for low discount factor and low quality differential, there is a unique equilibrium in which focal platform A wins. Intuitively, in this case the same qualitative results of a static game follows to the dynamic game. For positive quality differential $q_B - q_A$ and intermediate values of δ, there is a unique equilibrium in which the most efficient platform B takes over the market and maintain its position infinitely. But for high discount factors and low quality differential, there are multiple equilibria in which either platform A or B win. Notice that disregarding the equilibria of Lemma 3 (for being unlikely to emerge) would not restore efficiency of the equilibrium in this parameter region as there are also two equilibria — including one where the low-quality platform wins — arising from Lemma 2. In both of these equilibria,

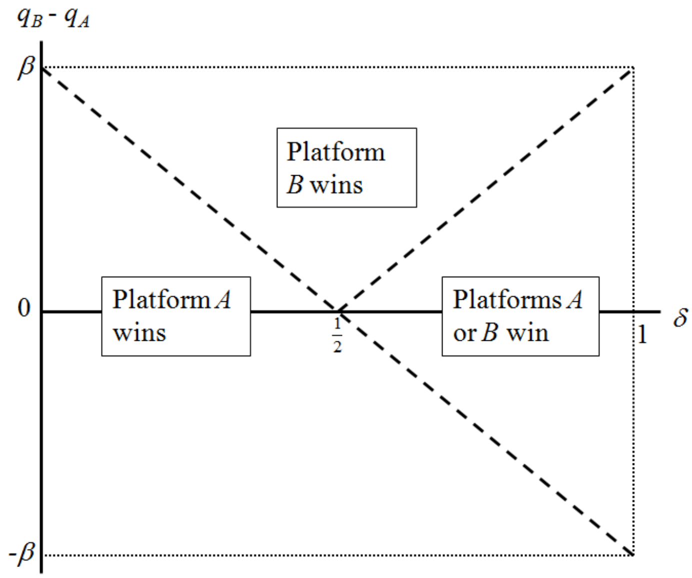

Figure 1: Equilibrium configuration

one platform expects low competitive pressure while the other renounces wining the market because it expects high competitive pressure, and these expectations are self-fulfilling. Thus at high discount factors, the prospect of gaining a focal position is not sufficient to outweigh firms' self-fulfilling expectations about the competitive pressure they will face.

5 Stochastic qualities

The previous section focused on the case where the qualities of the two platforms are constant for infinity. Consequently, in any equilibrium the same platform wins the market in all periods. In many markets for platforms there is a shift in leadership every few years, parallel to technology improvements. In this section we consider the more realistic case in which qualities are stochastic. We show that there is an equilibrium in which each platform can win in each period with some probability. The main conclusion of this section is that unlike the constant-qualities case, social welfare under stochastic qualities may decrease with δ.

Suppose that qualities change randomly in each period. At the beginning of each period, both platforms observe the realization of their qualities for this particular period. Then, the two platforms compete by setting prices.

The results of the previous sections showed that the equilibrium depends on the difference between the qualities of the two platforms, and not their absolute values. Suppose then, without loss of generality, that $q \equiv q_B - q_A$ change randomly in each period with full support on the real line according to a probability function $f(q)$, with a cumulative distribution function $F(q)$. Our assumption that the support is infinite ensures that there is an equilibrium in which each platform can win the market with a positive probability.[13] Suppose that q has a mean $\mu > 0$ such that on average, platform B is of superior quality than platform A. The case of $\mu < 0$ is symmetric.

Let $\bar{q}^A$ and $\bar{q}^B$ denote equilibrium cutoffs such that if platform A is focal in time t, it wins if $q \leq \bar{q}^A$ and platform B wins otherwise. Likewise, if platform B is focal in time t, it wins if $q \geq \bar{q}^B$ and platform A wins otherwise.[14] This equilibrium has the feature that when platform A is the focal platform, it will win in every period as long as $q < \bar{q}^A$. Then, once there is a realization with $q > \bar{q}^A$, platform B takes over the market and becomes focal. Platform B will maintain its focal position in future periods as along as $q \geq \bar{q}^B$, until eventually in a certain period there is a realization of q with $q < \bar{q}^B$, and platform A wins back its focal position. The game then repeats itself infinitely, with platforms "taking turns" in winning depending on the realization of q.

Let V_i^f denote the expected value function of platform i when platform f is focal. To solve for the equilibrium, suppose that platform A is focal in time t and the quality difference has some realization, q. The lowest price platform B is willing to charge in order to win the market is $-\delta V_B^B + \delta V_B^A$. This is because platform B will earn the expected value of V_B^B from becoming focal in the next period, and earn the expected value of V_B^A from remaining non-focal. Given the price of platform B, the highest price that enables platform A to win the market is $p_A = \beta - q - \delta V_B^B + \delta V_B^A$. Platform A earns $p_A + \delta V_A^A$ if indeed it wins (when $q \leq \bar{q}^A$) and earns $0 + \delta V_A^B$ if it loses (when $q > \bar{q}^A$). Therefore:

$$V_A^A = \int_{-\infty}^{\bar{q}^A} (\beta - q - \delta V_B^B + \delta V_B^A + \delta V_A^A) f(q) dq + \int_{\bar{q}^A}^{\infty} \delta V_A^B f(q) dq.$$

Suppose now that platform A is non-focal. The lowest price platform B is willing to charge to maintain its focal position is $p_B^B = -\delta V_B^B + \delta V_B^A$. Again, if platform A wins, it sets p_A^B that ensures that $-p_A^B \geq \beta - p_B^B + q$, or $p_A^B = -\beta - q - \delta V_B^B + \delta V_B^A$. Platform A

[13] We should note that this is a stronger assumption than what we need, as our results hold even with a finite support, as long as it is wide enough. Our assumption of infinite support facilitates the analysis and enables us to avoid corner solutions.

[14] It is straigtforward to see that any Markov equilibrium must have this form.

earns $p_A^B + \delta V_A^A$ if indeed it wins the market (when $q \leq \bar{q}^B$), and earns $0 + \delta V_A^B$ if it loses the market (when $q > \bar{q}^B$). Therefore:

$$V_A^B = \int_{-\infty}^{\bar{q}^B} (-\beta - q - \delta V_B^B + \delta V_B^A + \delta V_A^A) f(q) dq + \int_{\bar{q}^B}^{\infty} \delta V_A^B f(q) dq.$$

The cases of V_B^B and V_B^A are symmetric, as platform B wins the market if $q \geq \bar{q}^B$ when it is focal, and if $q > \bar{q}^A$ when it is not. Moreover, q positively affects the profit of platform B. Therefore:

$$V_B^B = \int_{\bar{q}^B}^{\infty} (\beta + q - \delta V_A^A + \delta V_A^B + \delta V_B^B) f(q) dq + \int_{-\infty}^{\bar{q}^B} \delta V_B^A f(q) dq,$$

$$V_B^A = \int_{\bar{q}^A}^{\infty} (-\beta + q - \delta V_A^A + \delta V_A^B + \delta V_B^B) f(q) dq + \int_{-\infty}^{\bar{q}^A} \delta V_B^A f(q) dq.$$

Next consider the equilibrium $\bar{q}^A$ and $\bar{q}^B$. The equilibrium $\bar{q}^A$ is such that for $q = \bar{q}^A$, a focal platform A is exactly indifferent between wining the market or not, taking the equilibrium future value functions and the price of platform B as given. That is:

$$\beta - \bar{q}^A - \delta V_B^B + \delta V_B^A + \delta V_A^A = \delta V_A^B.$$

Notice that the condition for making the non-focal platform B indifferent between winning and not is equivalent to the condition above. Turning to $\bar{q}^B$, the equilibrium $\bar{q}^B$ should be such that for $q = \bar{q}^B$, a non-focal platform A is exactly indifferent between wining the market or not, taking the equilibrium future value functions and the price of platform B as given. That is:

$$-\beta - \bar{q}^B - \delta V_B^B + \delta V_B^A + \delta V_A^A = \delta V_A^B.$$

Again notice that the condition for making the focal platform B indifferent between winning and not is equivalent to the condition above.

The set of the six equations above define the equilibrium V_A^A, V_A^B, V_B^B, V_B^A, $\bar{q}^A$ and $\bar{q}^B$. Using the above equations, the following proposition provides a sufficient condition for unique equilibrium values of $\bar{q}^A$ and $\bar{q}^B$:

Proposition 3 (Unique solutions to $\bar{q}^A$ and $\bar{q}^B$) *Suppose that $4\beta \max f(q) < 1$. There are unique equilibrium values of $\bar{q}^A$ and $\bar{q}^B$, with the following features:*

(i) for $\delta = 0$, $\bar{q}^A = \beta$ and $\bar{q}^B = -\beta$;

(ii) $\bar{q}^A - \bar{q}^B = 2\beta$ for all δ;

Proof. See Appendix.

The condition $4\beta \max f(q) < 1$ requires that the quality gap is sufficiently disperse and network effects are not too high. The intuition for these conditions is that they ensure that a non-focal platform can always overcome its competitive disadvantage if its quality is sufficiently high and therefore there are unique equilibrium values of $\bar{q}^A$ and $\bar{q}^B$. Proposition 3 also shows that evaluated at $\delta = 0$, $\bar{q}^A = \beta$ and $\bar{q}^B = -\beta$. Intuitively, at $\delta = 0$, the equilibrium is identical to the one-period benchmark in which a focal platform wins as long as its quality gap is higher than the network effects.

Next, we turn to study the effect of δ, β and μ on the equilibrium values of $\bar{q}^A$ and $\bar{q}^B$. To this end, we make the simplifying assumption that $f(q)$ is symmetric and unimodal around μ. That is, $f(\mu+x) = f(\mu-x)$ and $f(q)$ is weakly increasing with q for $q < \mu$, and decreasing with q for $q > \mu$. This is a sufficient but not necessary condition for the results below. The results may hold even when $f(q)$ is not strictly symmetric and unimodal as long as $f(q)$ places higher weights on positive values of q then negative values, such that platform B has higher probability to be focal in future periods. We also assume the uniqueness condition of Proposition 3 that $4\beta f(\mu) < 1$. With these assumptions, we have:

Proposition 4 (The effect of δ, β and μ on $\bar{q}^A$ and $\bar{q}^B$) *Suppose that $f(q)$ is symmetric and unimodal around μ and $4\beta f(\mu) < 1$. Then:*

(i) $\bar{q}^A$ and $\bar{q}^B$ are decreasing with δ. If $F(0) < 1/4$ then $\bar{q}^A < 0$ when δ is sufficiently high;

(ii) $\bar{q}^A$ and $\bar{q}^B$ are decreasing with μ (holding constant the distribution of $q - \mu$);

(iii) $\bar{q}^A$ is increasing with β and $\bar{q}^B$ is decreasing with β if $\delta < 1/2$. If $F(0) < 1/4$ and δ is close to 1, then $\bar{q}^A$ is decreasing with β.

Proof. See Appendix.

Figure 2 illustrates part (i) of Proposition 4. The figure reveals that an increase in δ does not necessarily increase the probability that the current-period higher quality platform wins. To see why, consider first the case where platform B is focal. Then, if $\delta = 0$, $\bar{q}^B = -\beta$ and $\bar{q}^B$ decreases with δ. Therefore, as δ increases, a focal platform B is more likely to win

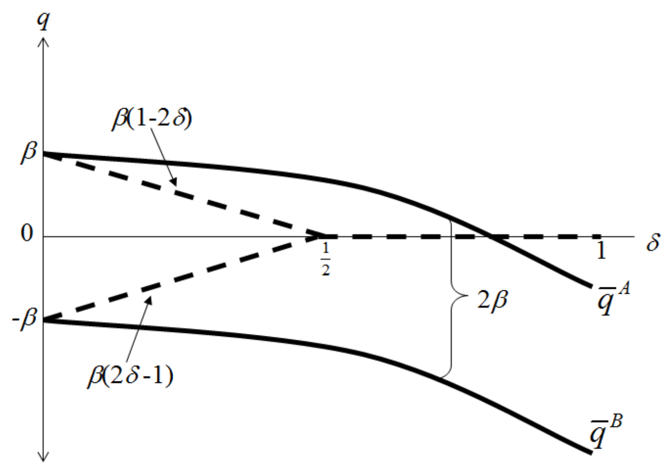

Figure 2: The effect of δ on $\bar{q}^A$ and $\bar{q}^B$ (when $F(0) < 1/4$)

the market with a lower quality than platform A, implying that the probability that the "wrong" platform wins increases with δ. Next, consider the case where platform A is focal. Then, when δ is low, an increase in δ makes it less likely that a focal platform A will be able to maintain it's focal position with a lower quality than platform B, in that $\bar{q}^A$ decreases with δ. However, when δ is sufficiently high and $F(0) < 1/4$, $\bar{q}^A$ crosses the 0 line, becomes negative and decreases further below 0 as δ increases. Now, platform A can lose the market even if it is focal and of superior quality than platform B (for realizations $\bar{q}^A < q < 0$). Therefore, the probability that the lower-quality platform wins increases with δ when either platform A or B are focal.

The intuition for these results is the following. Recall that a platform's expected profit includes current profit and the probability of maintaining its focal position in future periods. Since $\mu > 0$, platform B is more likely to have in future periods a higher quality than platform A. As δ increases, platform A internalizes that it is less likely to win in future periods and will therefore have less of an incentive to compete aggressively in the current period. Platform B internalizes that it is more likely to win the market in future periods and will therefore have more of an incentive to compete aggressively to win the market in the current period. This in turn provides platform B with a stronger competitive advantage over platform A, even when the current quality of platform B is inferior than the quality of platform A. If $F(0) < 1/4$, then μ is sufficiently high and platform B's competitive advantage is strong enough to deter platform A from winning the market even when it is focal and offer a higher quality than platform B.

The intuition above also explains the intuition behind part (ii) of Proposition 4. As μ increases, platform B is more likely to have higher quality in future periods. This provides platform B with a higher incentive to win in the current period, and as a result $\bar{q}^A$ and $\bar{q}^B$ decrease.

Part (iii) of Proposition 4 shows that if δ is not too high, then an increase in the degree of the network effect makes it more likely that the focal platform wins. This result is similar to the myopic case. An increase in the network effects increases the strategic advantage of being focal, as it becomes easier for the focal platform to attract consumers. However, when δ is sufficiently high and $F(0) < 1/4$, then an increase in network effects decreases the ability of a focal platform A to win the market. Intuitively, in such a case an increase in the network effects increases the incentives of the non-focal platform B to take over the market because of its superior expected quality which implies that platform B has a higher probability of maintaining its focal position.

Next we turn to social welfare. We first ask whether social welfare is higher when platform B is focal than when platform A if focal. Since platform B has a higher expected quality than platform A, it is intuitive to expect that social welfare is higher when platform B is focal. However, Proposition 4 showed that when δ increases, the probability that platform B wins when platform A has a superior quality increases, which may offset the first effect.

To this end, we normalize $q_A = 0$ and therefore $q_B = q$. Let $\bar{W}^i$, $i = A, B$, denote the recursive expected social welfare when platform i is focal in period t, where:

$$\bar{W}^A = \int_{-\infty}^{\bar{q}^A} (\beta + \delta \bar{W}^A) f(q) dq + \int_{\bar{q}^A}^{\infty} (\beta + q + \delta \bar{W}^B) f(q) dq,$$

$$\bar{W}^B = \int_{\bar{q}^B}^{\infty} (\beta + q + \delta \bar{W}^B) f(q) dq + \int_{-\infty}^{\bar{q}^B} (\beta + \delta \bar{W}^A) f(q) dq,$$

and let $W^i = (1 - \delta)\bar{W}^i$ denote the one-period expected welfare. Comparing W^A with W^B, we obtain the following:

Proposition 5 (The effect of δ on social welfare) *Suppose that $f(q)$ is symmetric and unimodal around μ and $4\beta f(\mu) < 1$. Then,*

(i) evaluated at $\delta = 0$, $W^B \geq W^A$ and W^A is increasing with δ while W^B is decreasing with δ ;

(ii) *There is a cutoff of* δ'', $0 \leq \delta'' \leq 1$, *such that* $W^B > W^A$ *for* $\delta \in (0, \delta'')$ *and* $W^A > W^B$ *for* $\delta \in (\delta'', 1)$. *A sufficient condition for* $\delta'' < 1$ *is* $F(0) < \frac{1}{4}$;

(iii) *evaluated at* $\delta = 1$, $W^A = W^B$.

Proof. See Appendix.

Notice that the case where q is distributed uniformly along a finite interval is a special case of symmetric and unimodal distribution in which $f(q)$ is constant. In this case $\delta'' = 0$ such that $W_A > W_B$ for $\delta \in (0, 1)$ and $W_A = W_B$ for $\delta = 0, 1$.

Part (i) of Proposition 5 shows that at low values of δ, W_B is larger than W_A. Intuitively, in this case the two cutoffs, $\bar{q}^A$ and $\bar{q}^B$, are close to their myopic levels: a focal platform A wins if $q < \beta$ and a focal platform B wins if $q > -\beta$. Since it is more likely that q will be positive, it is welfare-maximizing when platform B starts as a focal. However, part (i) of Proposition 5 also shows that for low values of δ, W_B is decreasing with δ while W_A is increasing with δ. This is because when platforms become more patient it becomes more likely that a focal platform B will win when its quality is inferior to platform A, which reduces welfare. Part (ii) finds that when platforms are sufficiently patient (δ is sufficiently high) and it is most likely that platform B has a superior quality ($F(0) < 1/4$), social welfare is higher when platform A starts as focal because the focal position provides platform B with a too strong competitive advantage such that platform B wins more than it should. Notice that the results above do not argue that social welfare is maximized when platform A is focal in all periods. They imply that in the first stage only, it is welfare maximizing to start the dynamic game with platform A as focal, even though platform B has on average higher quality.

The results above suggest that social welfare in the myopic case might be higher than when platforms are very patient. The comparison between social welfare evaluated at $\delta = 0$ and $\delta = 1$ for the general distribution function is inconclusive. We therefore make the simplifying assumption that q is uniformly distributed.

Corollary 1 (Welfare under uniform distribution) *Suppose that* q *is uniformly distributed along the interval* $[\mu - \sigma, \mu + \sigma]$ *and* $\sigma > \frac{1}{2}(\mu + 3\beta) + \frac{1}{2}\sqrt{(\mu^2 + 6\mu\beta + \beta^2}$. *Then,*

$$\bar{q}^A = \beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}, \quad \bar{q}^B = -\beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}.. \tag{5}$$

Moreover, $W^A|_{\delta=0} = W^B|_{\delta=0} > W^A|_{\delta=1} = W^B|_{\delta=1}$.

Proof. See Appendix.

The result is illustrated in Figure 3.

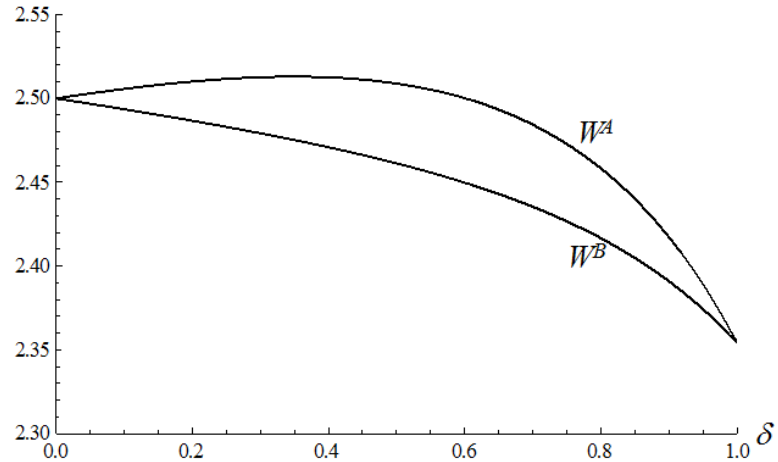

Figure 3: The effect of δ on welfare for a uniform distribution

6 Conclusions

In platform competition, having a superior quality over a competing platform may not be enough to dominate the market. In the presence of network externalities, a platform's success depends on the consumers' beliefs that other consumers will join it. In a static model a focal platform that benefits from such beliefs advantage may dominate the market even when it offers lower quality, resulting in an inefficient equilibrium. We study whether this inefficiency is eliminated in a long-horizon dynamic game.

In a model with very long but finite time horizon, we find that indeed when future matters the better platform wins and the efficient outcome is achieved. In particular, a higher-quality entrant can overcome the network effect advantage of the incumbent. Future matters when the time horizon is long enough and when the discount factor is high. For low discount factors, the platforms do not care sufficiently about the future, the game is closer to the static game and the inefficiency may remain even for time horizon extended to infinity. As a result, social welfare (weakly) increases the more platforms are forward-looking because then it is more likely that the better quality platform serves all consumers. Intuitively, a high-quality platform has more to gain by being focal in the final period and will therefore have more of an incentive to compete aggressively to gain or maintain the focal position in early periods.

Once we modify the model to capture more realistic features, however, we find new sources of inefficiency even for high discount factor. The finite horizon requires the platforms to known when the last period occurs. When this is not known, it is better to model it as an infinite horizon. Markov equilibria in the infinite game replicate the finite horizon game extended to infinity. But additional, inefficient Markov equilibria arise for high discount factors. In those new equilibria lower quality platform dominates the market in all periods.

We also consider a scenario where the platforms' qualities change stochastically from period to period. This allows each platform to win every period with some probability. In this setting, the more platforms are forward looking, it is less likely that a high-quality platform will be able to overcome a non-focal position. Intuitively, if one of the platforms has a higher average quality than the other platform, dynamic consideration provide it with a stronger incentive to win the market and maintain a focal position in the current period, even with a current-period inferior quality. At an extreme case, a focal and current-period high quality platform can still lose the market, if platforms are sufficiently forward looking.

This result indicates that with stochastic qualities, social welfare can decrease the more platforms are forward looking.

Our paper considers homogeneous consumers which raises the question of how the presence of heterogeneous consumers would affect our results. Previous literature suggests that when consumers are heterogeneous, focal position becomes less important for consumers. Armstrong (2006), for example, considers a continuum of consumers that differ in their preferences for two competing platforms. He shows that when the two platforms are sufficiently horizontally differentiated, then given the platforms' access prices there is a unique allocation of consumers between platforms evolving a positive market share to the two competing platforms.[15] Ambrus and Argenziano (2009) consider platform competition when consumers differ in their utility from network externality. They consider a different equilibrium concept than ours, but their main conclusion is that heterogeneous preferences give raise to equilibria with two active platforms such that one platform is cheaper and larger on one side, while the other platform is cheaper and larger on the other side. Halaburda and Yehezkel (forthcoming) show that focal position becomes less important for two competing platform the more consumers are loyal to a specific platform. Applying the intuition behind the above mentioned papers, it is reasonable to expect that the more consumers in our model become heterogeneous, the less focal position affects the platforms' profits, and thus platforms will have less of an incentive to compete in a current period for future focal position.

In real life, consumers are obviously heterogeneous. Nevertheless, our motivating examples show that in many markets for platforms, coordination problems between consumers and the platform's focal position plays an important role. Since the focus of this paper is on the effect of dynamic considerations on the platforms' focal position, our assumption of homogeneous consumers enables us to highlight the net effect of focal position in a tractable model.

Our model also abstracts from the presence of installed base and switching costs. Many markets with network effects experience the effects of installed base and switching costs, which are additional forces that can drive excess inertia and result in an equilibrium in which the lower quality platform wins for a long time. We abstract from them in order to highlight the role that coordination problem can also be a meaningful force that can drive

[15] Armstrong assumes that consumers preferences are distributed along the Hotelling line and the two platforms are located at the two extremes of the line. He finds that there is a unique allocation of consumers between the two platforms when consumers transportation costs are sufficiently high.

excess inertia. While all markets with network effects are affected by all these factors — expectations, installed base and switching costs — they are affected to a different degree. In the market for video game consoles, for example, we expect that the main driving force for excess inertia is indeed expectations. New generations are clearly distinguished from the old ones by technological jumps, with limited appeal of backward compatibility. In other markets, like smartphones or computer operating systems installed base and switching costs play a more important role. And indeed we observe more market leadership changes in the video game consoles market than in smartphones and computer operating systems. Nonetheless, expectations and thus focality are also present in those environments as a force affecting dynamics. And we show that it may lead to excess inertia under some conditions independently of installed base and switching costs.

A Proofs

Proof of Lemma 1

Proof. Let $\Pi_i^f(T)$ be the total discounted profit of platform i when platform f is focal at date $t = 1$ and there are T periods.

To win in $t = 1$, the focal plaform A needs to set $p_{A1} \leq p_{B1} + q_A - q_B + \beta$, and set such p_{A1} that would force $p_{B1} \leq -\delta\Pi_B^B(T-1)$. That is, platform A wins when it sets

$$p_{A1} \leq p_{B1} + q_A - q_B + \beta = -\delta\Pi_B^B(T-1) + q_A - q_B + \beta\,.$$

And then it earns

$$\Pi_A^A(T|\, A \text{ wins in } t = 1) = q_A - q_B - \delta\Pi_B^B(T-1) + \delta\Pi_A^A(T-1) \tag{6}$$

Notice that calculated in such a way the profit under the condition of winning may be negative. Then, the optimal action is to cede the market and earn 0 profit. Therefore, the profit from unconditionally optimal actions is $\Pi_i^f(T) = \max\{\Pi_i^f(T|\, i \text{ wins in } t = 1), 0\}$.
Using similar logic,

$$\Pi_B^A(T|\, B \text{ B wins in } t = 1) = q_B - q_A - \beta - \delta\Pi_A^A(T-1) + \delta\Pi_B^B(T-1) \;\equiv -\Pi_A^A(T|\, A \text{ wins in } t = 1)\,. \tag{7}$$

Let $\hat{\Pi}_i^f(T) \equiv \Pi_i^f(T|\, i \text{ wins in } t=1)$. Then

Suppose that $\hat{\Pi}_i^i(k) > 0$ for both $i = A$, B and $k = 1, \ldots, T-1$. Then from (6) we get[16]

$$\hat{\Pi}_i^i(T) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T-1) + \delta\hat{\Pi}_i^i(T-1) = (q_i - q_j)\sum_{k=1}^{T}(2\delta)^{k-1} + \beta = (q_i - q_j)\frac{1-(2\delta)^T}{1-2\delta} + \beta\,. \tag{8}$$

The fraction $\frac{1-(2\delta)^T}{1-2\delta}$ is positive and increasing with T. Therefore, $\hat{\Pi}_i^i(T)$ is also monotonic. When $q_i - q_j > 0$, then $\hat{\Pi}_i^i(T)$ is positive and increasing. Conversely, when $q_i - q_j < 0$, then $\hat{\Pi}_i^i(T)$ is decreasing, and when $q_i - q_j < -\beta\frac{1-2\delta}{1-(2\delta)^T}$, it may even be negative.[17] And once it is negative, it stays negative for all larger T's.

Now, suppose $(q_i - q_j)\frac{1-(2\delta)^T}{1-2\delta} + \beta < 0$. By the monotonic properties of $\frac{1-(2\delta)^T}{1-2\delta}$, it may only happen for $q_i < q_j$, and there exists $T_i \leq T$ such that $(q_i - q_j)\frac{1-(2\delta)^{T_i}}{1-2\delta} + \beta < 0$ and either $(q_i - q_j)\frac{1-(2\delta)^{T_i-1}}{1-2\delta} + \beta > 0$ or $q_i - q_j + \beta < 0$. In the latter case, $T_i = 1$. That is, T_i is the shortest time horizon for which it is not worth winning the market. For time horizon T_i and shorter, $\hat{\Pi}_i^i(T)$ for $i = A, B$ can be calculated using (8). But not for longer horizons.

Lemma 4 *If $\hat{\Pi}_i^i(T) < 0$, then for all $T' > T$, $\hat{\Pi}_i^i(T') < 0$.*

Proof. Suppose $T_i > 1$. By definition of T_i, $\hat{\Pi}_i^i(T_i - 1) > 0$ (and given by (8)), and

$$\hat{\Pi}_i^i(T_i) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T_i - 1) + \delta\hat{\Pi}_i^i(T_i - 1) < 0\,. \tag{9}$$

$\hat{\Pi}_i^i(T)$ for $T > T_i$ can no longer be calculated using (8). We need to apply (7) directly:

$$\hat{\Pi}_i^i(T_i + 1) = q_i - q_j + \beta - \delta\Pi_j^j(T_i) + \delta\Pi_i^i(T_i) = q_i - q_j + \beta - \delta\hat{\Pi}_j^j(T_i)$$

since $\Pi_j^j(T_i) = \hat{\Pi}_j^j(T_i)$ and $\Pi_i^i(T_i) = 0$.

By properties of (8), $\hat{\Pi}_j^j(T_i) > \hat{\Pi}_j^j(T_i - 1)$. By $\hat{\Pi}_i^i(T_i) < 0$, $\delta\Pi_j^j(T_i - 1) > q_i - q_j + \beta + \delta\hat{\Pi}_i^i(T_i - 1) > q_i - q_j + \beta$. Thus $\delta\hat{\Pi}_j^j(T_i) > q_i - q_j + \beta$ and $\hat{\Pi}_i^i(T_i + 1) < 0$. And so on for each $T > T_i$. ■

[16] Follows from applying the same formulas recursively in

$$\hat{\Pi}_i^i(T-1) - \hat{\Pi}_j^j(T-1) = 2(q_i - q_j) + 2\delta[\hat{\Pi}_i^i(T-2) - \hat{\Pi}_j^j(T-2)] = 2(q_i - q_j)\sum_{k=1}^{T-1}(2\delta)^{k-1}\,.$$

[17] This also implies that one of the $\Pi_i^i(T)$ must be positive. A negative $\hat{\Pi}_i^i(T)$ for some T implies $q_i - q_j < 0$, and $q_j - q_i > 0$ implies $\hat{\Pi}_j^j(T) > 0$ for all T.

Thus, for $T > T_i$, $\Pi_i^i(T) = 0$. Moreover, $\Pi_j^j(T) = \hat{\Pi}_j^j(T)$ also can no longer be calculated using (8). Applying (7) directly:

$$\Pi_j^j(T_i+1) = q_j - q_i + \beta + \delta \Pi_j^j(T_i)$$
$$\Pi_j^j(T_i+2) = q_j - q_i + \beta + \delta(q_j - q_i + \beta) + \delta^2 \Pi_j^j(T_i)\,.$$

And more generally, for any $T > T_i$

$$\Pi_j^j(T) = (q_j - q_i + \beta)\sum_{t=1}^{T-T_i} \delta^{t-1} + \delta^{T-T_i}\Pi_j^j(T_i) = (q_j - q_i + \beta)\frac{1-\delta^{T-T_i}}{1-\delta} + \delta^{T-T_i}\left((q_j - q_i)\frac{1-(2\delta)^{T_i}}{1-2\delta} + \beta\right)$$

Notice that for the case when $T_i = 1$, $\Pi_j^j(T)$ reduces to $(q_j - q_i + \beta)\frac{1-\delta^T}{1-d}$.

Now, using those properties of $\Pi_i^i(T)$, for $i = A, B$, we can consider following cases:

(1) $|q_A - q_B| < \beta\frac{1-2\delta}{1-(2\delta)^T}$
Then both $\hat{\Pi}_A^A(k)$ and $\hat{\Pi}_B^B(k)$ are positive for all $k = 1, \ldots, T$. Since platform A is focal in $t = 0$ and $\hat{\Pi}_A^A(T)$ is positive, the platform never cedes the market. And its profit is $\Pi_A^A(T) = \hat{\Pi}_A^A(T) = (q_A - q_B)\frac{1-(2\delta)^T}{1-2\delta} + \beta$, by (8).

(2) $q_A - q_B > \beta\frac{1-2\delta}{1-(2\delta)^T}$
That is, $(q_B - q_A)\frac{1-(2\delta)^T}{1-2\delta} + \beta < 0$, and thus, by the arguments above, there exists $T_B < T$. That means that $\hat{\Pi}_B^B(T) < 0$, i.e., platform B would not find it worthwhile to win the market even if it was focal, given A's quality advantage. Platform A wins the market, but the prices it charges and profit depend on T_B, as derived earlier:

$$\Pi_A^A(T) = (q_A - q_B + \beta)\frac{1-\delta^{T-T_B}}{1-\delta} + \delta^{T-T_B}\left((q_A - q_B)\frac{1-(2\delta)^{T_B}}{1-2\delta} + \beta\right)\,.$$

(3) $q_B - q_A > \beta\frac{1-2\delta}{1-(2\delta)^T}$
Now, there exists $T_A < T$. That is $\hat{\Pi}_A^A(T) < 0$, i.e., it is not worthwhile for platform A to defend the market in $t = 1$, given the quality advantage of platform B. Then platform B wins the market in $t = 1$, becomes the focal platform and keeps the market for the rest of the time horizon. To win the market, in $t = 1$, platform B sets $p_{B1}^A = q_B - q - A - \beta$, while platform A sets $p_{A1}^A = 0$. In the next period, platform B is the focal platform with quality advantage and with $T - 1$ period time horizon. Thus, the discounted total profit is as that of platform A in case (2), with relabeling the

platforms and length of the time horizon. That is:

$$\Pi_B^A(T) = q_B - q_A - \beta + \delta\left[(q_B - q_A + \beta)\frac{1-\delta^{T-1-T_A}}{1-\delta} + \delta^{T-1-T_A}\left((q_A - q_B)\frac{1-(2\delta)^{T_A}}{1-2\delta} + \beta\right)\right]$$
$$= (q_B - q_A + \beta)\frac{1-\delta^{T-T_A}}{1-\delta} + \delta^{T-T_A}\left((q_B - q_A)\frac{1-(2\delta)^{T_A}}{1-2\delta} + \beta\right) - 2\beta\,.$$

This completes the proof of Lemma 1. ■

Proof of Proposition 3

Proof. Directly from the formulas for V_A^A, V_A^B, V_B^B, V_B^A, and conditions for $\bar{q}^A$ and $\bar{q}^B$, we obtain

$$\bar{q}^A - \bar{q}^B = 2\beta\,.$$

Moreover,

$$V_A^A = \int_{-\infty}^{\bar{q}^A} \left(\bar{q}^A - q\right) f(q)\,dq + \delta V_A^B,$$
$$V_A^B = \int_{-\infty}^{\bar{q}^B} \left(\bar{q}^B - q\right) f(q)\,dq + \delta V_A^B = \frac{1}{1-\delta}\int_{-\infty}^{\bar{q}^B} \left(\bar{q}^B - q\right) f(q)\,dq,$$

and

$$V_B^B = \int_{\bar{q}^B}^{+\infty} \left(q - \bar{q}^B\right) f(q)\,dq + \delta V_B^A,$$
$$V_B^A = \frac{1}{1-\delta}\int_{\bar{q}^A}^{+\infty} \left(q - \bar{q}^A\right) f(q)\,dq,$$

The optimality condition is then

$$\bar{q}^A = \beta - \delta V_B^B + \delta V_B^A + \delta V_A^A - \delta V_A^B$$

which can be written

$$\bar{q}^A = \beta + \delta\phi\left(\bar{q}^A\right) \tag{10}$$

where

$$\phi\left(\bar{q}^A\right) = \int_{\bar{q}^A}^{+\infty} \left(q-\bar{q}^A\right) f\left(q\right) dq + \int_{-\infty}^{\bar{q}^A} \left(\bar{q}^A - q\right) f\left(q\right) dq - \int_{-\infty}^{\bar{q}^B} \left(\bar{q}^B - q\right) f\left(q\right) dq - \int_{\bar{q}^B}^{+\infty} \left(q-\bar{q}^B\right) f\left(q\right) dq.$$

Integrating by parts:

$$\phi\left(\bar{q}^A\right) = -2\beta + 2\int_{\bar{q}^A-2\beta}^{\bar{q}^A} F(q)dq. \tag{11}$$

We have

$$\begin{aligned} \phi'\left(\bar{q}^A\right) &= 2\left(F\left(\bar{q}^A\right) - F\left(\bar{q}^A-2\beta\right)\right) \\ \phi\left(-\infty\right) &= -2\beta \\ \phi\left(+\infty\right) &= 2\beta \end{aligned}$$

This implies that for $\bar{q}^A = \infty$, $\bar{q}^A > \beta + \delta\phi(\bar{q}^A)$ and for $\bar{q}^A = -\infty$, $\bar{q}^A < \beta + \delta\phi(\bar{q}^A)$. Therefore, there is a unique solution to $\bar{q}^A$ if $\bar{q}^A - \beta - \delta\phi(\bar{q}^A)$ is increasing with $\bar{q}^A$, or $\delta\phi'\left(\bar{q}^A\right) < 1$. We notice that $\delta\phi'\left(\bar{q}^A\right) < 1$ when

$$2\delta \max\left(F\left(q\right) - F\left(q-2\beta\right)\right) < 1.$$

In this case the equilibrium is unique. This is the case for all δ and if $4\beta \max f\left(q\right) < 1$.

Finally, notice that evaluated at $\delta = 0$, the solution to $\bar{q}^A = \beta + \delta\phi(\bar{q}^A)$ is $\bar{q}^A = \beta$. ■

Proof of Proposition 4

Proof.

Proof of part (i): Since $\bar{q}^A = \beta + \delta\phi(\bar{q}^A)$,

$$\frac{\partial \bar{q}^A}{\partial \delta} = \frac{\phi(\bar{q}^A)}{1-\delta\phi'(\bar{q}^A)}.$$

From the proof of Proposition 3, if $4\beta f(\mu) < 1$ then $1 - \delta\phi'(\bar{q}^A) > 0$. To see that $\phi(\bar{q}^A) < 0$ for all $\bar{q}^A \leq \beta$, suppose first that $\bar{q}^A < \mu$. Then:

$$\phi(\bar{q}^A) = -2\int_{\bar{q}^A-2\beta}^{\bar{q}^A} \left(\frac{1}{2} - F(q)\right) dq < 0,$$

where the inequality follows because symmetric and unimodal distribution (S-U thereafter) implies that for all $q < \mu$, $F(q) < 1/2$. Next, consider $\mu < \bar{q}^A \leq \beta$. Then:

$$\phi(\bar{q}^A) = -2\int_{\bar{q}^A-2\beta}^{\mu-(\bar{q}^A-\mu)} \left(\frac{1}{2} - F(q)\right) dq - 2\int_{\mu-(\bar{q}^A-\mu)}^{\mu+(\bar{q}^A+\mu)} \left(\frac{1}{2} - F(q)\right) dq < 0,$$

where the first term is negative because $\bar{q}^A > \mu > 0$ and S-U implies that $F(\mu-(\bar{q}^A-\mu)) < F(\mu) = \frac{1}{2}$ and the second term equals to 0 because S-U implies that $F(\mu+q) - \frac{1}{2} = \frac{1}{2} - F(\mu+q)$. Since $\phi(\bar{q}^A) < 0$, $\frac{\partial \bar{q}^A}{\partial \delta} < 0$ and since $\bar{q}^B = \bar{q}^A - 2\beta$, $\frac{\partial \bar{q}^B}{\partial \delta} < 0$.

Next, $\bar{q}^A$ is less than 0 if

$$0 > \beta + \delta\phi\left(0\right),$$

which holds for δ large if

$$-\beta > \phi\left(0\right) = -2\beta\left(1-2F\left(-2\beta\right)\right) + \int_{-2\beta}^{0} \left(-2q\right) f\left(q\right) dq = -2\beta + 2\int_{-2\beta}^{0} F\left(q\right) dq,$$

or if

$$\beta > 2\int_{-2\beta}^{0} F\left(q\right) dq.$$

This is true for all β if $F\left(0\right) < 1/4$.

Proof of part (ii): Let $F(q;\mu)$ denote the $F(q)$ given μ. We have:

$$\frac{\partial \bar{q}^A}{\partial \mu} = \frac{2\int_{\bar{q}^A-2\beta}^{\bar{q}^A} \left(\frac{\partial F(q;\mu)}{\partial \mu}\right) dq}{1-\delta\phi'(\bar{q}^A)} < 0,$$

where the inequality follows because S-U implies that $F(q;\mu)$ is decreasing with μ.

Proof of part (iii): We have:

$$\frac{\partial \bar{q}^A}{\partial \beta} = \frac{1-2\delta+4\delta F(\bar{q}^A-2\beta)}{1-\delta\phi'(\bar{q}^A)} > 0,$$

where the inequity follows because $1-2\delta+4\delta F(\bar{q}^A-2\beta) > 0$ if $\delta < \frac{1}{2}$. Since $\bar{q}^B = \bar{q}^A - 2\beta$, we have:

$$\frac{\partial \bar{q}^B}{\partial \beta} = -\left[\frac{1+\delta(2-4F(\bar{q}^A))}{1-2\delta(F(\bar{q}^A)-F(\bar{q}^A-2\beta))}\right] < 0,$$

where the inequality follows because the numerator in the squared brackets is positive when $\delta < \frac{1}{2}$ because $F(\bar{q}^A) < 1$ and the denominator is positive when $\delta < \frac{1}{2}$ because $F(\bar{q}^A) -$

$F(\bar{q}^A - 2\beta) < 1$. When $F(0) < 1/4$ and $\delta = 1$, we have:

$$\frac{\partial \bar{q}^A}{\partial \beta}|_{\delta=1} = \frac{-1 + 4F(\bar{q}^B)}{1 - \phi'(\bar{q}^A)} < \frac{-1 + 4\frac{1}{4}}{1 - \phi'(\bar{q}^A)} = 0,$$

where the inequality follows because $F(\bar{q}^B) < F(0) < 1/4$. ■

Proof of Proposition 5

Proof.

Solving for W^A and W^B:

$$W^A = \beta + \frac{(1 - \delta + \delta F(\bar{q}^B) \int_{\bar{q}^A}^{\infty} qf(q)dq + \delta(1 - F(\bar{q}^A)) \int_{\bar{q}^B}^{\infty} qf(q)dq}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)},$$

$$W^B = \beta + \frac{\delta F(\bar{q}^B) \int_{\bar{q}^A}^{\infty} qf(q)dq + (1 - \delta F(\bar{q}^A)) \int_{\bar{q}^B}^{\infty} qf(q)dq}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)}.$$

Consider first W^A. Solving the derivative of W^A with respect to δ and then evaluating at $\delta = 0$ yields:

$$\frac{\partial W^A}{\partial \delta}|_{\delta=0} = (1 - F(\bar{q}^A)) \left(\int_{\bar{q}^B}^{\infty} qf(q)dq - \int_{\bar{q}^A}^{\infty} qf(q)dq \right) - f(\bar{q}^A)\bar{q}^A \frac{\partial \bar{q}^A}{\partial \delta}$$
$$= (1 - F(\beta)) \int_{-\beta}^{\beta} qf(q)dq - f(\beta)\beta \frac{\partial \bar{q}^A}{\partial \delta},$$

where the equality follows from substituting $\bar{q}^A = \beta$ and $\bar{q}^B = -\beta$. By our assumption of *S-U*, $\int_{-\beta}^{\beta} qf(q)dq \geq 0$ (proof available upon request, implying the the first term is non-negative. Since Proposition 4 shows that $\bar{q}^A$ is decreasing in δ, the second term is positive implying that $\frac{\partial W^A}{\partial \delta}|_{\delta=0} > 0$.

Next, consider W^B. Solving the derivative of W^B with respect to δ and then evaluating at $\delta = 0$ yields:

$$\frac{\partial W^B}{\partial \delta}|_{\delta=0} = -F(\bar{q}^B)) \left(\int_{\bar{q}^B}^{\infty} qf(q)dq - \int_{\bar{q}^A}^{\infty} qf(q)dq \right) - f(\bar{q}^B)\bar{q}^B \frac{\partial \bar{q}^B}{\partial \delta}$$
$$= -F(-\beta) \int_{-\beta}^{\beta} qf(q)dq + f(-\beta)\beta \frac{\partial \bar{q}^B}{\partial \delta},$$

where the equality follows from substituting $\bar{q}^B = -\beta$ and $\bar{q}^A = \beta$. By our assumption of *S-U*, $\int_{-\beta}^{\beta} qf(q)dq \geq 0$, implying the the first term is non-positive. Since Proposition 2 shows that $\bar{q}^B$ is decreasing in δ, the second term is also negative implying that $\frac{\partial W^B}{\partial \delta}|_{\delta=0} < 0$.

Next, consider the gap $W^B - W^A$:

$$W^B - W^A = \frac{(1-\delta)(\int_{\bar{q}^B}^{\infty} qf(q)dq - \int_{\bar{q}^A}^{\infty} qf(q)dq)}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)} = \frac{(1-\delta)}{1 - \delta F(\bar{q}^A) + \delta F(\bar{q}^B)} M(\bar{q}^A),$$

where

$$M(\bar{q}^A) = \int_{\bar{q}^A - 2\beta}^{\bar{q}^A} qf(q)dq.$$

Since $1 \geq F(q) \geq 0$ and $0 \leq \delta \leq 1$, $\text{sgn}\big(W^B - W^A\big) = \text{sgn}\big(M(\bar{q}^A)\big)$.

Consider first $\delta = 0$ such that $\bar{q}^A = \beta$. Then, *S-U* implies $M(\beta) = \int_{-\beta}^{\beta} qf(q)dq \geq 0$ and $W^B - W^A \geq 0$. Second, consider $\delta = 1$. Then, $W^B - W^A = \frac{0}{1} M(\bar{q}^A)$, where $M(\bar{q}^A)$ is finite thus $W^B - W^A = 0$.

Next, we turn to $1 > \delta''$. We distinguish between two case, $F(0) < 1/4$ and $F(0) > 1/4$ that we analyze in turn.

Case 1: $F(0) < 1/4$. In this case, Proposition 2 implies that there is a cutoff, δ' where δ' is the solution to $\bar{q}^A = 0$, such that $\bar{q}^A > 0$ for $\delta \in [0, \delta']$ and $\bar{q}^A < 0$ for $\delta \in [\delta', 1]$. For all $\delta \in [\delta', 1]$, $M(\bar{q}^A) < 0$ because $q < 0$ for all $q \in [\bar{q}^A - 2\beta, \bar{q}^A]$. For $\delta \in [0, \delta']$, $M(\bar{q}^A)$ is decreasing with δ. To see why:

$$\frac{\partial M(\bar{q}^A)}{\partial \delta} = \left[\bar{q}^A f(\bar{q}^A) - (\bar{q}^A - 2\beta) f(\bar{q}^A - 2\beta)\right] \frac{\partial \bar{q}^A}{\partial \delta}.$$

The term inside the squared brackets is positive for all $\delta \in [0, \delta']$ because $\bar{q}^A \geq 0$, $f(q) > 0$ and because $\bar{q}^A \leq \beta$ implies that $\bar{q}^A - 2\beta \leq \beta - 2\beta = -\beta < 0$. Since $\bar{q}^A$ is decreasing with δ, $\frac{\partial M(\bar{q}^A)}{\partial \delta} < 0$

To summarize, $M(\bar{q}^A) \geq 0$ for $\delta = 0$, $M(\bar{q}^A)$ is decreasing with δ for $\delta \in [0, \delta']$ and $M(\bar{q}^A) < 0$ for $\delta \in [\delta', 1]$. Therefore, there is a unique cutoff $\delta'' < \delta'$ such that $M(\bar{q}^A) > 0$ for $\delta \in [0, \delta'']$ and $M(\bar{q}^A) < 0$ for $\delta \in [\delta'', 1]$. Since $\text{sgn}(W^B - W^A) = \text{sgn}\ M(\bar{q}^A)$, this implies that $W_B > W_A$ for $\delta \in [0, \delta'']$ and $W_B < W_A$ for $\delta \in [\delta'', 1)$.

Case 2: $F(0) > 1/4$. In this case, $\bar{q}^A > 0$ at $\delta = 1$. Notice that $M(\bar{q}^A)$ is decreasing with δ for all $\delta \in [0, 1]$ (the proof that $\frac{\partial M(\bar{q}^A)}{\partial \delta} < 0$ requires only that $\bar{q}^A > 0$ which holds in case 2 for all $\delta \in [0, 1]$). However, unlike case 1, now $M(\bar{q}^A)$ at $\delta = 1$ can be either positive or negative. It will be positive if $\bar{q}^A$ at $\delta = 1$ is sufficiently higher than 0, in which case for all $\delta \in [0,1]$, $M(\bar{q}^A) > 0$ and consequently $W_B > W_A$ for all $\delta \in [0,1)$. In this case $\delta'' = 1$. $M(\bar{q}^A)$ can be negative at $\delta = 1$ if $\bar{q}^A$ at $\delta = 1$ is sufficiently close to 0, in which case at $\delta = 1$, $M(\bar{q}^A) < 0$ and consequently $W_B > W_A$ for $\delta \in [0, \delta'']$ and $W_B < W_A$ for $\delta \in [\delta'', 1)$, as in case 1.

Remark on uniform distribution: with uniform distribution, $M(\bar{q}^A) = 0$ at $\delta = 0$ and $M(\bar{q}^A) < 0$ otherwise. This implies that $W_A > W_B$ for all $\delta \in (0,1)$ and $W_A = W_B$ otherwise. ■

Proof of Corollary 1

Proof.

Substituting $F(q) = \frac{q+\sigma}{2\sigma}$ into (11), yields (5). To ensure that $\bar{q}^B > \mu - \sigma$, we need that σ is high enough such that $\sigma > \frac{1}{2}(\mu + 3\beta) + \frac{1}{2}\sqrt{(\mu^2 + 6\mu\beta + \beta^2}$. Notice that this assumption implies that $\sigma > 2\beta$. The recursive expected social welfare functions are:

$$\bar{W}^A = \int_{\mu-\sigma}^{\beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}} (\beta + \delta\bar{W}^A)\frac{1}{2\sigma}dq + \int_{\beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}}^{\mu+\sigma} (\beta + q + \delta\bar{W}^B)\frac{1}{2\sigma}dq,$$

$$\bar{W}^B = \int_{-\beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}}^{\mu+\sigma} (\beta + q + \delta\bar{W}^B)\frac{1}{2\sigma}dq + \int_{\mu-\sigma}^{-\beta - \frac{2\delta\mu\beta}{\sigma - 2\delta\beta}} (\beta + \delta\bar{W}^A)\frac{1}{2\sigma}dq.$$

Hence,

$$W^A = (1-\delta)\bar{W}^A = \frac{1}{4}\left(4\beta - \frac{\beta^2}{\sigma} + \sigma + \frac{\mu(4\delta^2\beta^2(2\beta - 3\sigma) - \sigma^2(\mu + 2\sigma) + \delta\beta\sigma(5\mu - 4\beta + 10\sigma)))}{(\delta\beta - \sigma)(\sigma - 2\delta\beta)^2}\right)$$

$$W^B = (1-\delta)\bar{W}^B = \frac{1}{4}\left(4\beta - \frac{\beta^2}{\sigma} + \sigma + 2\mu + \frac{(\mu(8(-1+\delta)\delta^2\beta^3 + \delta\beta(5\mu - 4(-1+\delta)\beta)\sigma - \mu\sigma^2))}{(\delta\beta - \sigma)(\sigma - 2\delta\beta)^2}\right)$$

The gap $W^A - W^B$ is:

$$W^A - W^B = \frac{2(1-\delta)\delta\mu\beta^2}{(\sigma - \delta\beta)(\sigma - 2\delta\beta)}.$$

Since by assumption $\sigma > 2\beta$, $W^A - W^B > 0$ for all $0 < \delta < 1$ and $W^A - W^B = 0$ for $\delta = 0$ and $\delta = 1$. Moreover:

$$W^A|_{\delta=0} - W^A|_{\delta=1} = \frac{\mu^2\beta^2(2\sigma-\beta)}{\sigma(\sigma-\beta)(\sigma-2\beta)^2} > 0.$$

where the inequality follows because by assumption $\sigma > 2\beta$ and $\mu > 0$. ■

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