Financial Technology, Unpredictability and Illiquidity in the Long Run

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Abstract

In most sectors, technological progress boosts efficiency. But financial technology and the associated data-intensive trading strategies have been blamed for market volatility, illiquidity and inefficiency. We adopt the lens of growth theory and point it at the financial economy to understand how technological progress in data processing might shape financial activity. When the financial sector becomes more efficient at processing data, it alters the value of information about future dividends (fundamentals) relative to the value of information about order flow (non fundamental trading). Thus unbiased technological change can explain why financial analysis has shifted from primarily investigating the fundamental profitability of firms to primarily acquiring and processing client order flow data. Growth in financial technology can also reconcile two seemingly contradictory trends in asset markets: the increase in price informativeness and the stagnation of market liquidity.

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> "[I]ts all about getting as much customer order flow as possible ... The more trades these sophisticated machines get to see, the better they become [at] making money for their creators." (Reuters, August 14, 2009)

In most sectors, technological progress boosts efficiency. But in finance, information technology and the new data-intensive trading strategies it has spawned have been blamed for market volatility, illiquidity and inefficiency.At the same time, the nature of financial analysis and trading has shifted from "kicking the tires" of a firm and investigating its business model and profitability, to recognizing patterns in how others are trading and developing algorithms to profit from the information revealed by others' trades. We develop a new set of tools to explore the effects of growth in data processing efficiency on the financial sector. Our model explains why a shift in the type of analysis took place and clarifies what changes in the asset market may have resulted from this shift.

What effect more information has depends crucially on what the content of the information is. Broadly speaking, there are two types of information that technology can help investors access. The first type is fundamental information, such as earnings reports, business model simulations, macro announcements etc., that help to predict the future value of a firm. The second type of information is information extracted from the trades (order-flow) of others. It is the second type of information that is often blamed for market malfunction.

We develop a two-sector growth model of the financial economy with two key features. First, we allow investors to choose between styles of financial analysis and then observe the information produced from that analysis, before they invest. This features enables us to explore sectoral shifts between fundamental information acquisition and order-flow information extraction. Second, we incorporate long-lived assets. This feature is essential to understand why technological progress can also undermine market liquidity.

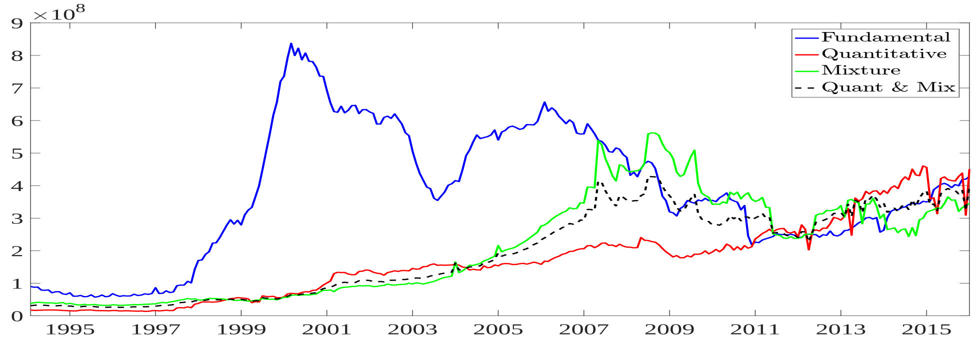
The driving force behind the model is technological change in the total flow of information the sector can process. Of course, other trends, such as a decline in fees, entry of new investors or assets, digitization, changes in covariance or improvements in order flow execution are operating during this period as well. We want to take one simple trend, unbiased technological progress, which has been studied in many other contexts, and see how much that alone can explain. We see this simple driving force as a solid foundation for a new benchmark model of financial technology growth, to which many other ingredients and trends might eventually be added.

We find that an increase in total information creates an endogenous change in the relative value of fundamental versus order-flow information. When information is scarce, it is very valuable to know what the fundamental value of an asset is. But when most investors are well-informed, it becomes more valuable to identify and trade against the remaining non-informational trades. Order flow analysis allows investors to target these more profitable trades.

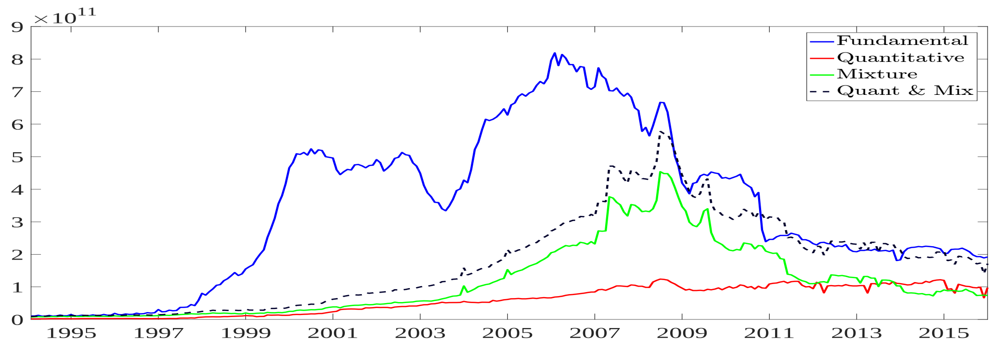
At the same time, market liquidity may deteriorate. Contrary to popular wisdom, it is not order flow trading that makes markets less liquid. Rather, it is the expectations that future traders will be well-informed that makes future asset prices – and today's returns – more uncertain. Thus, the rise in order-flow trading and a rise in return uncertainty, which in turn reduces liquidity, emerge as two concurrent trends with financial technology improvements as their common cause.

Suggestive evidence of the trend we are exploring comes from the change in the mix of hedge funds. Many hedge funds report that their fund is "fundamental", "mixture," or "quantitative." Figure 1 illus-

Figure 1: Hedge Funds are Shifting Away from Fundamental Analysis
Sources: Lipper TASS. Data is monthly from 1994-2015. Database reports on 17,534 live and defunct funds.



(a) Assets Under Management per Fund



(b) Total Assets Under Management

trates the evolutions of assets under management, by fund, and in total, for these different styles of funds. While the overall trend of growth and then shrinkage in hedge funds following the financial crisis is most dominant, the other clear trend is that fundamental analysis is waning, in favor of strategies based on market data. This shift in reported style suggests a transformation in the way information technologies are used in finance.

Another quite different indicator that points to the growing importance of order flow data comes from the frequency of web searches. Google trends reports the frequency of searches that involve specific search terms. Figure 2 shows that from 2004 to 2016, the frequency of searches for information about "order flow" has risen roughly 3-fold. This is not an overall increase in attention to asset market information. In contrast, the frequency of searches for information about "fundamental analysis" fell by about one-half over the same time period.

Much of the trade against order flow takes the form of algorithmic trading. This happens for a couple of reasons. First, while firm fundamentals are slow-moving, order flow can reverse rapidly. Therefore, mechanisms that allow traders to trade quickly are more valuable for fast-moving order flow based strategies. Second, while fundamental information is more likely to be textual, partly qualitative, and varied in nature, order flow is more consistently data-oriented and therefore more amenable to algorithmic analysis. Hendershott, Jones, and Menkveld (2011) measure algorithmic trading and find that it has increased, but

Figure 2: Google trends: Fraction of Google searches involving "order flow" or "fundamental analysis." Source: Google trends. Data is the weekly fraction of searches involving these search terms. Series is normalized to make the highest data point equal to 100.

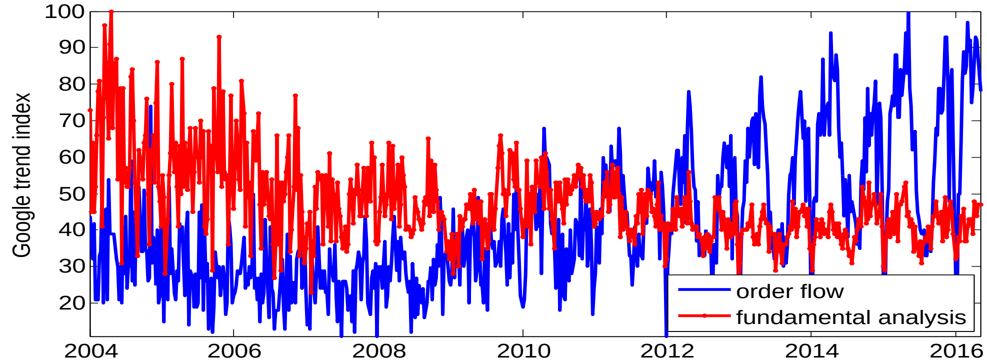
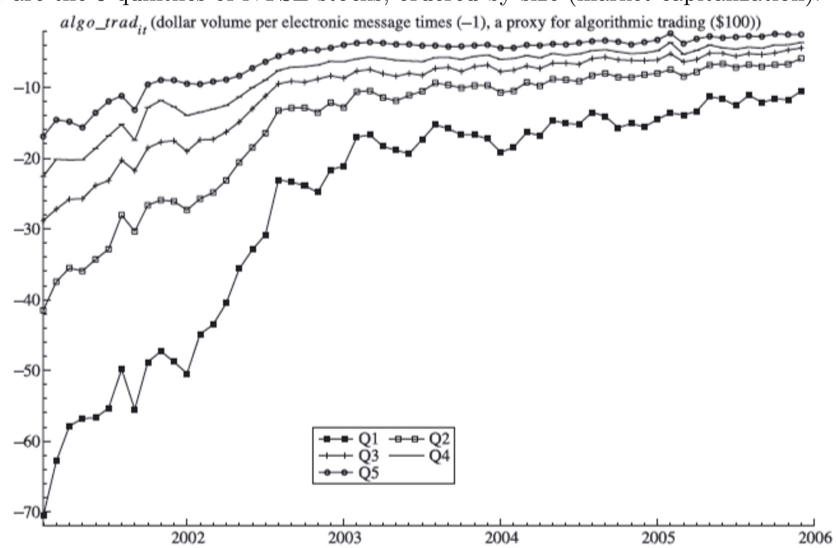


Figure 3: Algorithmic Trading Growth 2001-2006. Source: Hendershott, Jones, and Menkveld (2011). Their proxy for algorithmic trading is the dollar volume of trade per electronic message. The rise is more pronounced for largest market cap (Q1) stocks. Q1-Q5 are the 5 quintiles of NYSE stocks, ordered by size (market capitalization).



it increased most rapidly during the period between the start of 2001 and the end of 2005. During this six-year window, average trade size fell and algorithmic trading increased, about seven-fold (Figure 3). This rapid transition is another feature of the data we'd like our model to explain.

Our goal is to explore how simple technological progress in information production, and the resulting shifts in analysis styles affects market liquidity, volatility and informational efficiency. Section 1 describes our model and solution method. Section 2 describes our main results. We show analytically how and why the financial sector shifts from doing mostly fundamental analysis to doing mostly order flow analysis as its overall information processing productivity improves. Section 3 describes our choice of parameters for our numerical results. Section 4 uses the numerical example to illustrate the main results of Section 2. Section 5 extends the model to explore two possible spillovers this financial sector transformation has for real macroeconomic outcomes.

Contribution to the existing literature Our model combines features from a few disparate literatures. Long run trends in finance are featured in Asriyan and Vanasco (2014), Biais, Foucault, and Moinas (2015), and Glode, Green, and Lowery (2012), who model growth in fundamental analysis or an increase in its speed. Davila and Parlatore (2016) explore a decline in trading costs. Philippon (2015) argues that increased issuance can explain the growth of the financial sector. The idea that there is long-run growth in information processing is supported by the rise in price informativeness documented by Bai, Philippon, and Savov (2013).

A small, growing literature examines order-flow information in equilibrium models. In Yang and Ganguli (2009), agents can choose whether or not to purchase a fixed bundle of fundamental and order-flow information. In Yang and Zhu (2016) and Manzano and Vives (2010), the precision of fundamental and order-flow information is exogenous. Babus and Parlatore (2015) examine intermediaries who observe the order flow of their customers. Our order flow signals also resemble Angeletos and La'O (2014)'s sentiment signals about other firms' production, **?**'s signals about motives for trade, or the signalling by Zhiguo (2009)'s intermediaries. But none of these papers examines the choice that is central to this paper: The choices of whether to process more about asset payoffs or to analyze more order flow. Without that trade-off, these papers cannot explore how the incentives to process each type of information change as productivity improves. Also this paper adds a long-lived asset in a style of model that has traditionally been static. The long-lived asset causes growth in future information processing to have effects on uncertainty and information choices today.

In the microstructure literature, our model contributes a new perspective on what high-frequency traders do, which complements work by Du and Zhu (2017), Crouzet, Dew-Becker, and Nathanson (2016) and others. Empirically, Hendershott and Menkveld (2014) and Hendershott, Jones, and Menkveld (2011) use natural experiments to measure how fundamental and algorithmic trading affects liquidity. By contributing theory to this discussion, we can understand why the shift is taking place and run counter-factual experiments that show what would have happened if the nature of the information shock was different.

1 Model

To explore the dynamic evolution of financial analysis style and its consequences, we start with a dynamic model with long-lived assets and asymmetric information, as in Wang (1993). The long-lived asset assumption is more realistic than the standard static framework. But more importantly, it teaches us why static models may deliver misleading predictions about the role of information in market liquidity. On top of this foundation, we add information choice. The choice of fundamental information precision resembles that in static or repeated static models such as Kacperczyk, Nosal, and Stevens (2015). But acquiring fundamental information trades off with extracting of information from order flow. That information trade-off is a new piece of the model. Of course, it would be simpler to assume that the mix of information changes exogenously. But that would not inform us about why the nature of financial markets are changing. If we took that approach, we might wrongly attribute the decline in market liquidity to an increase in order flow information extraction, instead of understanding both as outcomes of a common cause.

One of the challenges is how to model information extraction from order flow, which, in practice, can take many forms. Extraction often takes the form of high-frequency trading, where the information of an imminent trade is used to predict a future price and trade before the new price is realized. But it also take the form of "partnering," a practice where brokers sell their order flow to hedge funds, who systematically trade against, what are presumed to be uninformed traders. Finally, it may mean looking at price trends, often referred to as technical analysis, in order to discern what information others may be trading on. All of these practices have in common that they are not uncovering orignal information about the future payoff of an asset. Instead, they are using information to profit from what others already know (or don't know). We capture this general strategy, while abstracting from many of its details, by allowing investors to observe a signal about the non-informational trades of other traders. This allows our traders to profit by trading against uninformed order flow (as Citadel seems to do with E-trade customers). This signal also allows investors to remove noise from the equilibrium price. This clearer price signal reveals more of what others know. In that way, it resembles strategies that try to infer the information of others. Finally, just like high-frequency traders, our investors who extract information from order flow will be able to better predict future prices, buy before price rises and sell before it falls.

1.1 Setup

Investor preferences and endowments At the start of each date t, a measure-one continuum of overlapping generations investors is born. Investors born at time t have constant absolute risk aversion utility over total, end of period t consumption $\tilde{c}_t$:

$$U(\tilde{c}_t) = -e^{-\rho \tilde{c}_t} \tag{1}$$

where ρ is absolute risk aversion.We adopt the convention of using tildes to indicate t-subscripted variables that are not in the agents' information set when they make time-t investment decisions.

Each investor i born at date t is endowed with an exogenous income that is $\tilde{e}_t$ units of consumption goods at the end of period t. Investors can pledge their labor income to buy risky assets. But they cannot trade shares of or any assets contingent on their income.[1]

There is a single tradeable asset.[2] It's supply is one unit per capita. It is a claim to an infinite stream of dividend payments d_t:

$$\tilde{d}_t = \mu + G\tilde{d}_{t-1} + \tilde{y}_t. \tag{2}$$

where μ and $G < 1$ are known parameters, d_t is paid out and revealed at the end of each period t and $\tilde{y}_t \sim N(0, \tau_0^{-1})$ is likewise revealed at the end of period t.

An investor born at date t, sells his assets at price p_{t+1} to the $t+1$ generation of investors, collects dividends $\tilde{d}_t$ per share, combines that with the endowment that is left $(e_{it} - q_{it}p_t)$, times the rate of time preference $r > 1$, and consumes all those resources.[3] Thus the cohort-t investor's budget constraint is

[1]We'll interpret this income as labor income, which is in practice difficult to ensure. However, we don't call it labor income because this is an endowment economy, with no labor or production.

[2]We describe a market with a single risky asset because our main effects do not require multiple assets. However, we have some results for the generalized, multi-asset setting.

[3]The value of c_{t+1} can be thought of as an aggregate of $(e_{it} - p_t q_{it})$ consumed at the start of time t, weighted by $r > 1$ and $(q_{it}(p_{t+1} + \tilde{d}_t))$ consumed at the end of time t, with a weight of 1. Note that negative consumption is allowed, both at the start and the end of the period. That allows young agents to buy assets from old agents at the start of t at any possible price p_t by consuming a negative amount. This almost never happens in the simulations. One can think of this negative consumption as if it were borrowing from future consumption. However, no such debt markets are explicitly in the model.

$$\tilde{c}_t = r(e_{it} - q_{it}p_t) + q_{it}(p_{t+1} + \tilde{d}_t) \tag{3}$$

where q_{it} is the shares of the risky asset that investor i purchases at time t and $\tilde{d}_t$ are the dividends paid out at the start of period $t+1$. Since we do not prohibit $c_t < 0$, all pledges to pay income for risky assets are riskless.

The value of endowments is correlated with the firm's dividend: $e_{it} = \bar{e} + h_{it}\tilde{y}_t + \tilde{\epsilon}_{eit}$, where $\bar{e}$ is known and $\tilde{e}_{it} \sim N(0, \tau_e^{-1})$ is independent across agents and independent of all the other shocks in the economy. The variable h_{it} governs the correlation of agent i's endowment with output. That variable has a common component and an investor-specific component: $h_{it} = \tilde{x}_t + \tilde{\epsilon}_{hit}$ where $\tilde{x}_t \sim N(0, \tau_x^{-1})$ and $\tilde{\epsilon}_{hit} \sim N(0, \tau_h^{-1})$.[4]

The reason for this rich, correlated endowment process is that it preserves a motive to acquire information. For information to have value, prices must not perfectly aggregate asset payoff information. We inject noise in prices by giving investors both informational and non-information reasons for trade. They have non-financial income that they want to hedge with financial assets. Shocks to this non-financial income will create shocks to their hedging demand, which is our source of noise in prices.

Information Choice If we want to examine how the nature of financial analysis has changed over time, we need to have at least two types of analysis to choose between. Financial analysis in this model means signal acquisition. This acquisition could represent the cost of researching and uncovering new information. But it could also represent the cost of interpreting and computing optimal trades based on information that is readily available from public sources.

Investors choose how much information to acquire or process about each of two random variables: They can choose how much to learn about the next-period dividend innovation $\tilde{y}_t$, and also choose how much to learn about $\tilde{x}_t$, the hedgers' demand shocks, which is the source of non-fundamental fluctuations in prices. We call $\eta_{fit} = \tilde{y}_t + \tilde{\epsilon}_{fit}$ a fundamental signal and $\eta_{xit} = \tilde{x}_t + \tilde{\epsilon}_{xit}$ an order-flow signal. What investors are choosing is the precision of these signals. In other words, if the signal errors are distributed $\tilde{\epsilon}_{fit} \sim N(0, \Omega_{fit})$ and $\tilde{\epsilon}_{xit} \sim N(0, \hat{\Omega}_{xit})$, then the precisions Ω_{fit} and Ω_{xit} are choice variables for investor i. For notational convenience, we define $\Omega_{xit} = \tau_h + \hat{\Omega}_{xit}$. Instead of choosing $\Omega_{xit} \geq 0$, we then allow the investor choose $\Omega_{xit} \geq \tau_h$. Then Ω_x represents the joint signal precision that the investor has both from order-flow analysis and from observing his own endowment exposure to systemic financial risk.

The constraint that investors face when choosing information is

$$\Omega_{fit}^2 + \chi_x \Omega_{xit}^2 \leq K_t. \tag{4}$$

This represents the idea that getting more and more precise information about a given risk is tougher and tougher. But acquiring information about multiple risks is just linear. An investor could hire another equal size staff to perform the other kind of analysis with the same precision at the same cost.

The main force in the model is technological progress in information analysis. Specifically, we assume that K_t is a deterministic, increasing process.

[4] The fact that the mean of h_{it} is zero is just for simplification. Assuming a non-zero mean affects the average asset price. But we have done robustness checks to ensure it does not affect our main results.

Information sets and equilibrium First, we recursively define two information sets. The first is all the variables that are known at the end of period $t-1$. This information is $\{\mathcal{I}_{t-1}, y_{t-1}, d_{t-1}, x_{t-1}\} \equiv \mathcal{I}_{t-1}^{+}$. This is what investors know when they choose what signals to acquire. The second information set is $\{\mathcal{I}_{t-1}, y_{t-1}, d_{t-1}, x_{t-1}, \eta_{fit}, \eta_{xit}, h_{it}, p_t\} \equiv \mathcal{I}_{t-1}^{+}$. This includes the two signals the investor chooses to see, information contained in equilibrium prices and the information conveyed by one's endowed income. This is the information set the investor has when they make investment decisions. The time 0 information set includes the entire sequence of information capacity: $\mathcal{I}_0 \supset \{K_t\}_{t=0}^{\infty}$.

An equilibrium is a sequence of symmetric information choices $\{\Omega_{fit}\}$, $\{\Omega_{xit}\}$ and potentially heterogeneous portfolio choices $\{q_{it}\}$ by investors such that

1. Investors choose signal precisions Ω_{fi} and Ω_{xi} to maximize $E[ln(E[U(c_{i,t+1})|\mathcal{I}_t])|\mathcal{I}_{t-1}^{+}]$, where U is defined in (1), taking the choices of other agents as given. This choice is subject to (4), $\Omega_{fi} \geq 0$ and $\Omega_{xi} \geq 0$. We focus on symmetric information choice equilibria, where the precisions of signals are equal across agents.

2. Investors choose their risky asset investment q_{it} to maximize $E[U(c_{it})|\eta_{fit}, \eta_{xit}, h_{it}, p_t]$, taking the asset price and the actions of other agents as given, subject to the budget constraint (3).

3. At each date t, the risky asset price clears the market:

$$\int q_{it} di = 1 \qquad \forall t. \tag{5}$$

Interpreting Order Flow Trading Of course, real order flow traders are not taking their orders, solving some equilibrium pricing model, and inverting the whole price system of all risky assets to try and infer future dividends. But another way to interpret the order flow trading strategy is that investors with order flow information use it to identify non-information trades to trade against. Now, in this model, we made all investors symmetric and they all have both informational and hedging motives for trade. We did that to simplify the solution. But the same forces emerge if the hedging trades are done by a different class of agents, which we might call uninformed retail investors. These types of trades might alternatively represent liquidations by pension funds. In that world, the order flow trading strategy amounts to finding the uninformed investors and systematically taking the opposite side of their trades. If we interpret order flow trading in this way, we again see why it becomes more valuable over time. If there is very little information capacity at the start, then informed and uninformed trades are not very different. But when informed traders become very informed, distinguishing dumb from smart money before taking the other side of a trade becomes essential. That is the evolution this setup captures.

1.2 Solving the Model

There are four main steps to solve the model. Since we have just assumed that information choices are symmetric, we now use Ω_{ft} and Ω_{xt} to represent the symmetric fundamental and order flow precision choices of all investors.

Step 1: Solve for the optimal portfolios, given information sets. Each investor i at date t chooses a number of shares q_{it} of the risky asset to maximize expected utility (1), subject to the budget constraint

(3). The first-order condition of that problem is

$$q_{it} = \frac{E[p_{t+1} + \tilde{d}_t | \mathcal{I}_{it}] - rp_t}{\rho_{it} Var[p_{t+1} + \tilde{d}_t | \mathcal{I}_{it}]} - h_{it} \tag{6}$$

Step 2: Clear the asset market. Given this optimal investment choice, we can impose market clearing (5) and obtain a price function that is linear in past dividends d_{t-1}, the t-period dividend innovation $\tilde{y}_t$, and the aggregate component of the hedging shocks $\tilde{x}_t$:

$$p_t = A_t + Bd_{t-1} + C_t\tilde{y}_t + D_t\tilde{x}_t \tag{7}$$

where A_t is in the appendix and the coefficients B, C_t and D_t are the solution to the following set of equations:

$$B = \frac{G}{r - G} \tag{8}$$

$$C_t = \frac{1}{r - G}\left(1 - \tau_0 Var[\tilde{y}_t | \mathcal{I}_{it}]\right) \tag{9}$$

$$rD_t = -\rho Var[p_{t+1} + \tilde{d}_t | \mathcal{I}_{it}] + \frac{r}{r - G} Var[\tilde{y}_t | \mathcal{I}_{it}] \frac{C_t}{D_t} \tau_x \tag{10}$$

where Ω_{pt} is the precision of the information extracted from prices about t_{t+1}, $Var[\tilde{y}_t|\mathcal{I}_{it}] = (\tau_0 + \Omega_{ft} + \Omega_{pt})^{-1}$ is the posterior uncertainty about next-period dividend innovations and the resulting uncertainty about asset returns is proportional to $Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}] = C_{t+1}^2 \tau_0^{-1} + D_{t+1}^2 \tau_x^{-1} + (1 + B)^2 Var[\tilde{y}_t|\mathcal{I}_{it}]$.

Step 3: Compute ex-ante expected utility. When choosing information to observe, investors do not know what signal realizations will be, nor do they know what the equilibrium price will be. The relevant information set for this information choice is $\mathcal{I}_{t-1}^+$.

After we substitute the optimal portfolio choice (6) and the equilibrium price rule (7) into utility (1) and take the beginning of time-t expectation (before observing signals or prices), we get:

$$\begin{aligned} &- E[ln(E[\exp(\rho c_{it})|\eta_{fit}, \eta_{xit}, h_{it}, p_t])|\mathcal{I}_{t-1}^+] = \\ &\rho\, r\, e_{it} + \rho E[q_{it}(E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - p_t r)|\mathcal{I}_{t-1}^+] - \frac{\rho^2}{2} E[q_{it}^2 Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]^{-1}|\mathcal{I}_{t-1}^+]. \end{aligned} \tag{11}$$

Appendix A shows that, since asset demand $q_{it} = 1/\rho Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]^{-1}(E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - pr)$ and the $E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t]$ and p terms depend on information choices, but their expected values do not depend on the precision of any given investor's information choices, the agent's choice variables Ω_{ft} and Ω_{xt} show up only through the conditional precision of payoffs, $Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]^{-1}$. So, information choices amount to maximizing this precision, or minimizing the variance, subject to the constraint. The payoff variance, in turn, has a bunch of terms the investor takes as given, plus a term that depends on $Var[\tilde{y}_t|\mathcal{I}_{it}] = (\tau_0 + \Omega_{ft} + \Omega_{pt})^{-1}$. Price information precision is $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt} + \tau_h)$, which is linear in Ω_{xt}. Thus expected utility is a function of the sum of Ω_{ft} and $(C_t/D_t)^2\Omega_{xt}$.

Therefore, for each generation t, optimal information choices maximize the weighted sum of fundamental and order-flow precisions:

$$\max_{\Omega_{ft},\Omega_{xt}} \ \Omega_{ft} + \left(\frac{C_t}{D_t}\right)^2 \Omega_{xt} \tag{12}$$
$$\text{s.t. (4)}, \quad \Omega_{ft} \geq 0, \ \text{and } \Omega_{xt} \geq \tau_h.$$

Step 4: Solve for information choices. The first order conditions are

$$\Omega_{ft} = \frac{1}{2\lambda_t} \tag{13}$$

$$\Omega_{xt} = \left(\frac{C_t}{D_t}\right)^2 \frac{1}{2\lambda_t \chi_x} \tag{14}$$

where λ_t is the lagrange multiplier on the information processing constraint (4). Note that because of the quadratic form of the information processing constraint, the marginal cost of processing the first unit of either type of information is zero. So the non-negativity conditions never bind.

While we can characterize the solutions analytically, the information choices are a function of pricing coefficients, like C and D, which are in turn functions of information choices. To determine the evolution of analysis and its effect on asset markets, we need to compute a fixed point to a highly non-linear set of equations. After substituting in the first order conditions for Ω_{ft} and Ω_{xt}, we can write the problem as two non-linear equations and two unknowns and prove that when capacity K_t is near zero, the marginal value of order flow information is zero as well.

2 Analytical Results: A Secular Shift in Financial Analysis

Now that we have a solution to the model, we can use that solution to understand what happens when there is technological progress in information processing. We begin by exploring what happens in the neighborhood near no information processing limit $K \approx 0$. We show that all investors prefer to acquire only fundamental information in this region. This helps explain why most capacity is devoted to fundamental information processing at the start of the growth trajectory. Next, we prove that, away from $K = 0$, an increase in aggregate information processing increases the value of order flow information, relative to fundamental information. In other words, fundamental information has diminishing relative returns. However, we show conditions under which order flow information does not have diminishing returns. What does this mean for the evolution of analysis over time, as total information processing grows? Since the value of order flow information starts at zero and rises, and the relative value of fundamental information falls, the economy starts out doing fundamental analysis and then gradually shifts to order flow analysis. We explore this mechanism in more detail in the following propositions.

2.1 Analysis Choices when Information Is Scarce

In order to understand why investors with little information capacity use it all on fundamental information, we start by thinking about what makes each type of information valuable. Fundamental information is valuable because it informs an investor about whether the asset is likely to have a high dividend payoff tomorrow. Since prices are linked to current dividends, this also predicts a high asset price tomorrow and thus a high return. Knowing this allows the investor to buy more of the asset in times when its return will be high and less when return is likely to be low.

In contrast, order flow information is not directly relevant to future payoff or future price. But one can still profit from trading on order flow. An investor who knows that hedging demands are high will systematically profit by selling the asset because high hedging demands will make the price higher than the fundamental value, on average. In colloquial terms, order flow signals allow one to trade against "dumb money." The next result proves that if the price has very little information embedded in it, because information is scarce (K_t is low), then getting order flow data to extract price information is not very valuable. In other words, if the market is all "dumb," then identifying the uninformed trades has no value.

Result 1 ***When information is scarce, order flow analysis has zero marginal value:***
As $K_t \to 0$, for any future path of prices (A_{t+j}, B_{t+j}, C_{t+j} and D_{t+1}, $\forall j > 0$), $dU_1/d\Omega_{xt} \to 0$.

Mathematically, this order-flow trading strategy is represented as an investor who uses order flow signals to extract more information from today's equilibrium asset price. Order flow shocks are the noise in the asset price. Knowing something about order flow allows the investor to remove some of that noise from prices and obtain a clearer signal about the future dividend innovation, $\tilde{y}_t$. Order flow information is only valuable in conjunction with the current price p_t because it allows one to extract more information from price. The proof (in Appendix B) establishes two key claims: 1) that when $K \approx 0$, there is no information in the price: $C_t = 0$ and 2) that the marginal value of order flow information is proportional to $(C_t/D_t)^2$. Thus, when the price contains no information about future dividends ($C_t = 0$), then analyzing order flow is has no marginal value $(C_t/D_t)^2 = 0$. Order flow trading is like removing noise from a signal that has no information content.

This results explains why order flow analysis starts very low when financial analysis productivity is low. In contrast, when prices are highly informative, order flow information is like gold because it allows one to identify exactly the price fluctuations that are not informative and are therefore profitable to trade on. The next results explain why order flow analysis increases with productivity growth and why it may eventually start to crowd out fundamental analysis.

2.2 How Analysis Shifts Affect Price Information and Liquidity

Next, we explore how the value of information changes as information processing capacity grows. Do to this, we examine the effect of marginal changes in the amount of fundamental and order flow analysis. We begin by exploring how each type of analysis changes each price coefficient (C_t, D_t) separately. Then, we turn to the question of how it affects the ratio $(C/D)^2$, which governs the marginal value of order flow versus fundamental analysis. Taken together, these results paint a picture of order flow analysis that starts low and then takes off as fundamental analysis improves and as more order flow analysis feeds on itself. The proofs are in Appendix B.

Result 2 ***Both fundamental and order flow analysis increase the information content of prices.*** *If* $r - g > 0$ *and* $(\tau_x + \Omega_{xt})$ *is sufficiently small, then* $\partial C_t / \partial \Omega_{ft} > 0$ *and* $\partial C_t / \partial \Omega_{xt} > 0$.

The more information investors have, the more information is reflected in the risky asset price. While the idea that dividend (fundamental) information improves price informativeness is unsurprising, the question of whether order-flow speculation improves or reduces price informativeness is not obvious. It turns out that they increase the information content because by selling the asset when the price is high for non-fundamental reasons and buying when the price is erroneously low, they make it easier to extract information from prices. Better informed traders who learn both from independent signals and from prices, therefore have better information, take more agressive positions which in turn, cause the price so reveal even more information.

Liquidity here is the impact a non-informational trade has on price. A liquid market is one where one can buy or sell large quantities, in a way that is not correlated with dividends, without moving price by much. The next two results together show that information today and information tomorrow have opposite effects on today's liquidity. These opposite results are why it was important to use a dynamic model to think about the long run effects on increasing information technology.

Result 3 ***If order flow is not too volatile, then both fundamental and order flow analysis at date*** t ***improve date-***t ***liquidity.*** *If* $\tau_x > \rho r/(r - g)$*, then* $\partial D_t / \partial \Omega_{ft} > 0$ *and* $\partial D_t / \partial \Omega_{xt} > 0$.

The contemporaneous effect is that both types of analysis can increase liquidity. This is the effect that static models identify and explains why the typically predict that price informativeness and liquidity move in lock step. The rationale is that order flow traders identify trades that are not likely to be informational and take the opposite position. Investors are eager to trade against non-informational trades. Doing so yields profits, on average. So, the more identifiable non-informational traders are, the better the price they'll get. Fundamental traders buy when the price is low, relative to their fundamental information. This is exactly the same states where hedgers are selling. Conversely, they sell when hedgers are buying because the price is too high, relative to their signal. By taking the other side of the hedging trade, they mitigate its price impact.

Why would this result be reversed if τ_x was too low? A low τ_x means that prices are very noisy. In such an economy, when information increases, noise trades might be mis-attributed to agents having fundamental information, and prices might move. In other words, the presence of informed fundamental or order flow traders makes others more hesitant to trade against hedging trades and thus causes the hedging trades to have a larger price impact.

Another way of understanding the same phenomenon is to think about it all as risk. More information of either type makes the asset less risky – lower conditional variance. It one share of the asset involves bearing less risk, then market investors don't need much price concession to induce them to hold a little extra risk. When one share is riskier, then inducing the market to buy one more share requires them to take on lots of risk, which requires a large price concession. This effect shows up in (10), the formula for D_t, which depends negatively on $Var[p_{t+1} + \tilde{d}_t | \mathcal{I}_{it}]^{-1}$, the variance of the asset payoff. Assets with more uncertain payoffs have more negative D_t, which means selling or buying a share has more price impact. This risk-based interpretation helps explain the next result about how future information affects today's liquidity.

Result 4 ***More information at date*** $t+1$ ***reduces date-***t ***liquidity.*** *If* $|C_{t+1}/D_{t+1}|$ *is sufficiently large, then* $\partial D_t/\partial Var[\tilde{y}_{t+1}|\mathcal{I}_{i(t+1)}] < 0$*.*

The reason that future information can reduce liquidity is because it makes future price p_{t+1} more sensitive to future information and thus harder to forecast today. The future price is an important component of the return to a date t asset. If the asset's return is harder to forecast, then the asset is effectively riskier. Invoking the logic above, a riskier asset has a less liquid market. Thus, future information reduces today's liquidity.

We can see this logic in the formula for the variance of the asset payoff:

$$Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]^{-1} = C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1} + (1+B)^2 Var[\tilde{y}_t|\mathcal{I}_{it}] \tag{15}$$

We know that time-t information increases information content of prices at t. Similarly, time $t+1$ information increases C_{t+1}. Future information may increase or decrease D_{t+1}. But as long as C_{t+1}/D_{t+1} is large enough, the net effect of $t+1$ information is to increase $C_{t+1}^2\tau_0^{-1} + D_{t+1}^2$. Since future information cannot affect today's divident uncertainty $Var[\tilde{y}_t|\mathcal{I}_{it}]$, the net effect of future information is to late today's payoff variance. What this means economically is that tomorrow's prices will be more responsive to tomorrow's fundamental and order flow shocks. That makes the price more uncertain today.

In our dynamic model, information improves today and improves again tomorrow. That means the static effect and dynamic effect are competing.[5] The net effect of the two is sometimes positive, sometimes negative. But it is never as clear-cut as what a static information model would suggest. What we learn is that information technology efficiency and liquidity are not synonymous. If fact, IT can make markets function in a less liquid way.

Changes in the marginal value of information We know that both types of information can increase price information and increase liquidity. But if both C_t and D_t rise, it is not clear what happens to their ratio C_t/D_t. This object is important because it is the relative marginal utility of order flow, relative to fundamental information. The next result shows that, as long as price information is low or order flow analysis is not too large, both types of analysis increase price informativeness (the ratio of the information content C to the noise D), which increases the marginal value of order flow information, relative to fundamental information. Thus, fundamental analysis complements order flow information and order flow information complements itself.

Result 5 ***Complementarity in order flow analysis:***
If $\Omega_{xt} < \tau_0 + \Omega_{ft}$ *and either*

1. C_t/D_t *is smaller in absolute value than* $(2\,Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}])^{-1}$*, or*

2. $Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}] < \sqrt{3}$

then $\frac{\partial (C_t/D_t)^2}{\partial \Omega_{ft}} > 0$ *and* $\frac{\partial (C_t/D_t)^2}{\partial \Omega_{xt}} \geq 0$*.*

Unlike fundamental analysis, the rise in order-flow analysis can increase the value of further order-flow analysis. For fundamental information, the increase in $|C/D|$ makes additional fundamental information

[5] This part of our effect is similar to the effect that arises in Cai (2016a), with only fundamental information.

less valuable. This result resembles the strategic substitutability in information identified by Grossman and Stiglitz (1980), in a model with a different information structure. But for order flow information, the effect is the opposite. More precise average order flow information (higher Ox) can increase $(C_t/D_t)^2$, which is the marginal value of order flow information. That's complementarity.

Complementarity comes from a rise in price informativeness. Recall that the value of order flow information, relative to fundamental information comes from the ratio of price coefficients $(C_t/D_t)^2$. This is like a signal-to-noise ratio from prices. C_t is the coefficient on dividend innovations $\tilde{y}_t$. When C_t is high, price contains lots of information about future dividends. From (9), we know that C_t is proportional to $1 - \tau_0 Var[\tilde{y}_t|\mathcal{I}_{it}]$. As either type of information precision (Ω_{ft} or Ω_{xt}) improves, the uncertainty about next period's dividend innovation $Var[\tilde{y}_t|\mathcal{I}_{it}]$ declines, and C_t increases. In other words, if investors know more, the information content (C_t) of the price increases.

D_t is the coefficient on noise $\tilde{x}_t$. The price impact of uninformative trades D_t may also increase with information. But conditions (1) and (2) guarantee that D_t does not rise at a rate faster than C_t so that the ratio C_t/D_t, which is the signal-to-noise ratio of prices, increases with more information.

Higher signal-to-noise (more informative) prices encourage order flow trading because the value of order flow analysis comes from the ability to better extract the signal from prices. In this model (as in most information processing problems), it is easier to clear up relatively clear signals than very noisy ones. So the aggregate level of order-flow analysis improves the signal clarity of prices, which makes order-flow analysis more valuable.

This set of results above are comparative statics with respect to current information choices. They do not describe what happens if future information choices differ as well. Because these results are simpler, they give us clearer insight about exactly what the competing forces are when one time-t and one future $t+1$ information choice changes. When we consider a marginal change in analysis choice in the infinite future (a change in the steady state), the results are similar, but with more complex necessary conditions.

2.3 Analysis and Price in the Long-Run

The result that order flow analysis feeds on itself suggests that in the long run, order flow analysis will crowd out fundamental analysis. But that does not happen. When order flow precision (Ω_x) is high, the necessary conditions for Proposition 5 break down. The next result tells us that, in the long run as information becomes abundant, growth in fundamental and order-flow analysis becomes balanced.

Result 6 (High-Information Limit) *As $K_t \to \infty$, both analysis choices Ω_f and Ω_x tend to ∞ such that*

(a) Ω_f/Ω_x does not converge to 0;

(b) Ω_f/Ω_x does not converge to ∞; and

(c) if τ_0 is sufficiently positive, there exists an equilibrium where Ω_f/Ω_x converges to finite, positive constant.

See Appendix B for the proof and an expression (94) for the lower bound on τ_0.

It is not surprising that fundamental analysis will not out-strip order flow analysis (part (a)). We know that more fundamental analysis lowers the value of additional fundamental analysis and raises the value of order flow analysis. This is the force that prompts order flow analysis to explode at lower levels of information K.

What is puzzling is: What force restrains the growth of order flow analysis when information technology is more advanced? The reason that fundamental analysis cannot become a negligible fraction of order flow analysis (part (b)) is that, if it did, the price signal-to-noise ratio $(C_t/D_t)^2$ would fall; this would reduce the incentive to acquire order flow information. Specifically, when information precision is growing, future prices become harder to predict. Higher uncertainty about the future asset payoff becomes large; this makes price impact D_t high; so the magnitude of D_t rises faster than C_t (eq.s (9) and (10)). High D_t/C_t means that current prices have a lower signal-to-noise ratio. So using order flow information to mine the signal from prices becomes less valuable. In sum, if fundamental analysis is too scarce, the value of mining order flow falls, and brings the two types of analysis back to some fixed proportion.

The relatively faster growth in D_t is a dynamic effect. It arises from the expectations of high levels of future analysis. If tomorrow, many investors will trade on precise $(t+1)$ information, then tomorrow's price will be very sensitive to tomorrow's dividend innovation y_{t+1} and tomorrow's order flow shock x_{t+1}. In other words, both C_{t+1} and D_{t+1} will be high. But investors today learn only about today's shocks, y_t and x_t. They know nothing about the $t+1$ shocks. Therefore, tomorrow's analysis makes tomorrow's price (p_{t+1}) extremely sensitive to shocks that today's investors are uninformed about. Because tomorrow's price is a component of the payoff to the asset purchased at date t, today's investors face high asset payoff risk (high Ω^{-1}). Assets with more risk have more price impact. This effect can be seen in equation (10) for D_t, where the second term is increasing in payoff uncertainty Ω^{-1}. That same uncertainty term does not show up in the equation (9) for C_t. Thus, high information makes both pricing coefficients grow. But the additional effect of high payoff uncertainty, which becomes particularly large at high levels of analysis, causes D_t to grow faster than C_t.

Lemma 4 in the appendix makes this link between D_t/C_t and future payoff risk more formal. It bounds the magnitude of D_t from below. The result says that $|D_t|/C_t > \rho\left((r-G)/r\right)(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1})$. The first term is just fixed parameters. The second term, $(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1})$ is the part of tomorrow's price variance that is unknowable today. This component of the payoff is unknowable today because it is the part of tomorrow's price that depends on future shocks, x_{t+1} and y_{t+1}, for which no signals are available today. This unknowable future price risk, or put differently, the risk created by future information flow is what creates the wedge between D_t and C_t. What makes this unknowable risk high is when price sensitivity to future shocks C_{t+1} and D_{t+1} are large in magnitude. These price sensitivity coefficients are high when agents have precise information. So, as information technology improves, the risk of future price changes grows, and this makes the sensitivity of price to today's non-fundamental shocks D_t high as well. This noise in prices is the force pushing the value of order flow analysis down in the long run. It works against the rise in C_t that comes from more fundamental information in prices, which pushes the value of order flow information up.

In sum, if order flow analysis grows faster than fundamental analysis (Ω_f/Ω_x falls), then at high levels of information (high K), price impact $|D_t|$ rises faster than price information C_t, and the ratio $(C_t/D_t)^2$ falls. According to the first order conditions (13) and (14), if $(C_t/D_t)^2$ falls, agents would choose to acquire less order flow information over time. But choosing less order flow analysis over time contradicts the initial supposition that, as the total amount of analysis K grows, order flow analysis grows faster than fundamental analysis.

The only solution that reconciles the first order condition, with the equilibrium price coefficients, is one where (Ω_f/Ω_x) stabilizes and converges to a constant (result (c)). If fundamental analysis grows proportionately with order flow analysis, the rise in the amount of fundamental analysis makes prices more informative about dividends: C_t increases. Proportional growth in fundamental and order flow analysis allows C_t to keep up with the rise in D_t, described above. By the first order condition, $(C_t/D_t)^2$ determines the ratio of fundamental analysis Ω_f to order flow analysis Ω_x that agents choose to do. Therefore, as information technology grows ($K \to \infty$), a stable C_t/D_t rationalizes information choices (Ω_x, Ω_f) that grow proportionately, so that Ω_x/Ω_f converges to a constant.

Economically, what is happening is that if lots of data is being processed, today's payoffs (future prices) start becoming very sensitive to future data. Because of this sensitivity to shocks that are unknowable today, it is harder to use today's data to reliably predict returns. In other words, future price risk grows.

This raises the question: What if order flow shocks were not independent, or some information about future dividend shocks was available to participants today? Would this overturn the effect that future information processing increases risk? This is an extension we are currently exploring. Such assumptions would indeed allow today's information to resolve some of the risk that is unknowable in the current setup. But at the same time, it would introduce new risks. Tomorrow's price would depend on the new information, learned tomorrow about shocks that will materialize in $t + 2$ or $t + 3$. That new information observed in $t + 1$ will affect $t + 1$ prices. That new future information, only released in $t + 1$ cannot be known at time t. So, new sources of unlearnable risk would arise. The general point is that as long as new information is constantly arriving, whether it pertains to current or future events, it creates risk. The risk is that before the information arrives, one does not know it and can not know it, no matter how much analysis is done. And yet, this information yet to arrive will affect future prices in a uncertain way. When information processing technology is poor, market participants will not process it well and cannot trade on it correctly anyway. Such poorly-processed information has little price effect. Thus with low information technology, future information poses little risk. When more information is being processed efficiently that risk of unknown future information grows.

3 Parameter Choice

The results so far, sketch out important features of the dynamic path of analysis. We know that when information was hard to process, only fundamental analysis was done. We know that as information processing became more abundant, order flow analysis took off and fed on itself. Finally, we know that eventually both types of analysis grow proportionately. But this does not trace out the entire dynamic path. It also doesn't tell us how much any of this matters quantitatively for price informativeness or liquidity. To explore these issues, we need to solve a calibrated model numerically.

First, we describe the data used for model calibration. Next, we describe moments of the data and model that we match to identify model parameters. In the next section, we report the results from our numerical simulations.

Data We use two datasets that both come from CRSP. The first is the standard S&P 500 market capitalization index based on the US stock market's 500 largest companies.[6] The dataset consists of three variables: the value-weighted price level of the index p_t, the value-weighted return total return, and the value-weighted return $(p_t + d_t)/p_{t-1}$, where d_t is dividends. All three are reported at a monthly frequency for the period 1999.12-2015.

Given returns and prices, we impute dividends per share as

$$d_t = \left(\frac{p_t + d_t}{p_{t-1}} - \frac{p_t}{p_{t-1}}\right) p_{t-1}.$$

Both the price series and the dividend series are seasonally adjusted and exponentially detrended. As prices are given in index form, they must be scaled to dividends in a meaningful way. The annualized dividend per share is computed for each series by summing dividends in 12 month windows. Then, in the same 12-month window, prices are adjusted to match this yearly dividend-price ratio.

Moments Using the price data and implied dividend series, we estimate the following two equations, both implied by our model:

$$d_t = \mu + G d_{t-1} + \tilde{y}_t$$

$$p_t = A + B d_{t-1} + C y_t + D x_t,$$

where Dx_t and $\tilde{y}_t$ are regression residuals. We can then map these estimates into the underlying model parameters G, τ_x^{-1}, τ_0^{-1}, μ and χ_x, using the model solutions:

$$A = \frac{1}{(r-1)} \frac{r\mu}{(r-G)} - \Omega \bar{x}$$

$$B = \frac{G}{(r-G)}$$

$$C = \frac{1}{(r-G)} (1 - \tau_0 \hat{V})$$

$$D = \frac{1}{(r-G)} \hat{V} \frac{C}{D} - \frac{\rho}{r} \Omega^{-1}$$

$$Var[p_t] = (C^2 + \frac{B^2}{1-G^2}) \tau_0^{-1} + D^2 \tau_x^{-1}$$

An important issue is that the price variable in the regressions above is really an index. Because this index is an average of prices, the volatility of the average will likely underestimate the true volatility of representative stock prices. In order to find an estimate for price volatility at the asset level, we construct a quarterly time series of the average S&P constituent stock price for the period 2000-2015. Compustat gives us the S&P constituent tickets for each quarter. From CRSP, we extract each company's stock price for that quarter.

[6] As a robustness check, we redo the calibration using a broader index: a composite of the NYSE, AMEX and Nasdaq. This is a market capitalization index based on a larger cross-section of the market - consisting of over 8000 companies (as of 2015). The results are similar. Moment estimates are within about 20% of each other. This is close enough that the simulations differ imperceptibly. Results are available upon request.

Table 1: Parameters

	low risk av	high risk av
G	0.9365	0.9365
μ	0.235	0.4153
τ_0^{-1}	0.2575	0.2445
τ_x^{-1}	1.9850	0.5514
χ_x	10.6625	0.6863
r	1.03	1.03
ρ	0.05	0.1

The first five parameters in Table 1 (those above the line) are calibrated to match the model and data values of the five equations above. This is an exactly identified system. The riskless rate is set to match a 3% net return. The last parameter is risk aversion. Risk aversion clearly matters for the level of the risky asset price. But it turns out if we change risk aversion, and then re-calibrate the mean, persistence and variance parameters to match price coefficients and variance at the new risk aversion level, the predictions of the model are remarkably stable.

We use the risk aversion $\rho = 0.10$ in what follows. But we show an example of an alternative parameterization in column 2 of Table 1 with even lower risk aversion that yields similar results. (See Appendix for plots.)

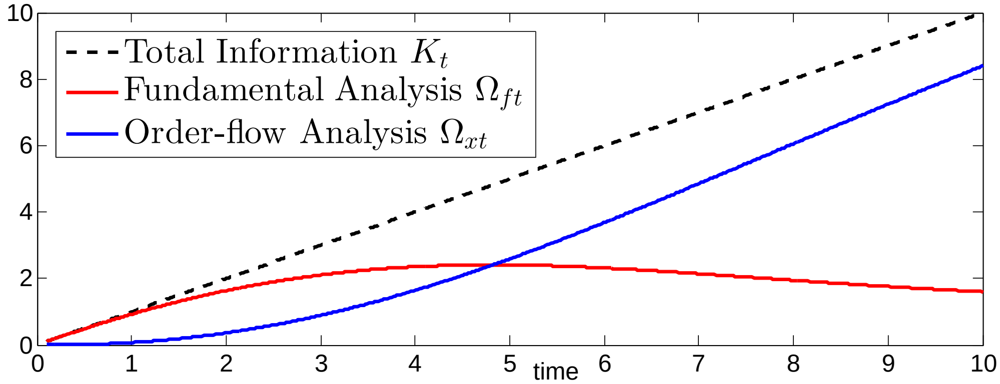
In addition, we need to choose ending values for the price coefficients A_T, C_T and D_T. The reason we need this is that our numerical solution method relies on backwards induction. Given $t+1$ parameters, we can solve the model and find t parameters. But we need the future parameters to initialize the algorithm. We obtain these ending parameters by solving for steady state. If we believed that forever after that information would remain constant $K_{t+1} = K_t$ and price would have stable coefficients, $A_{t+1} = A_t$, $C_{t+1} = C_t$ and $D_{t+1} = D_t$, what would these stable coefficients be? We find $A_T = 16.03$, $C_T = 7.865$ and $D_T = -5.7$ ($= -3.0$, for low risk aversion). Note that B_t is always stable because it is a simple function of fixed parameters.

We use these terminal values both for the calibration and to solve backwards. At the same time, we reduce information processing capacity, as we move back in time, to understand what effect information growth has had on financial markets. The exercise starts K_T at 10.[7]

Simulation The model is an infinite-horizon model where the solution to the pricing coefficients at each date t depends on the $t+1$ coefficients. Therefore, we solve by choosing a final date T and using our estimated price function parameters to initialize the backwards induction algorithm. In other words, we use the A_T, B_T, C_T and D_T from the data and our calibrated parameters to solve backwards for A_t, B_t, C_t and D_t, $t = T-1, \cdots, 1$. In a typical round, taking $t+1$ pricing coefficients as given we solve for time t price coefficients. These in turn, give us solutions for optimal information choices Ω_{ft} and Ω_{xt}. Then, we use the time-t solutions and our model solution to get $t-1$ information choices and price coefficients, and so forth. At each date, we are using a function minimization routine that finds the zeros of a non-linear

[7] We checked to the robustness of alternative K_T values and found that it makes no difference to our conclusions. For example, when we used $K_T = 5$, we found that the results look as if we'd simulated the results with $K_T = 10$ and truncated the time series plot where K_t reaches 5. The other calibrated parameters are identical when we vary K, except for χ_x, which falls by about one-half: 4.8 and 0.31 for low and high risk aversion parameterizations.

Figure 4: Evolution of fundamental analysis and order flow analysis.
What is driving the change over time is an increase in total information processing K. Fundamental information is the choice variable Ω_{ft}, scaled by fundamental variance τ_0^{-1}. Order flow information is the part of Ω_{xt} that the investor can choose, $\Omega_{xt} - \tau_x$, scaled by non-fundamental order flow variance τ_x^{-1}.



equation in $\frac{C_t}{D_t}$.

The one thing that changes at each date is the total information capacity K_t. We start the routine with $K_T = 5$. In each period prior to that, we reduce K_t by 0.01. So if the last period is denoted T, then $K_{T-1} = 4.99$ and $K_{T-2} = 4.98$. We simulate the model in this fashion for 500 periods.

Multiple Equilibria The non-linear equation in $\frac{C_t}{D_t}$ that we solve to get our solution is a third-order polynomial. There can be three solutions. It turns out, that for the parameter values we explore, this cubic equation has only one real root.

4 Numerical Results

Our main finding from the model is that the growth path is not balanced: Unbiased technological change in information processing, modeled as an increase in the information budget K_t, causes investors to allocate that total information capacity differently over time. Although the relative shadow cost of fundamental and technical information is not affected when K_t changes, their relative benefits change. When information is scarce, fundamental information is more valuable. As the ability to process information grows and fundamental information becomes more abundant, its value declines.

We begin by exploring the forces that make order flow information more valuable over time. This mechanism is not an inherently dynamic one. It comes from comparative statics in a two-period model as well. Then, we explore why the transition increases and then decreases the price impact of trades, i.e., reduces liquidity. For this question, the dynamic nature of the model with its long-lived assets is crucial.

4.1 Transition from Fundamental to Technical Analysis

Figure 4 shows that order flow analysis is scarce initially. Consistent with Result 1, we see that when information processing ability is limited, almost all of that ability is allocated to processing fundamental information. Only once fundamental information is sufficiently abundant, does order-flow analysis take off. After that inflection point, not only does order flow processing increase, it increases by so much that, despite the greater ability to acquire more total information, the amount of fundamental information actually declines. Once it takes off, order flow trading quickly comes to dominate fundamentals-based trading. This pattern suggests that order flow analysis might rise in an unbounded way. But recall that the nature of information choices is that there is a limit K_t on overall information processing. Thus there is a natural upper bound: $\Omega_{xt} \leq K_t$. Furthermore, we know that if we drive Ω_{xt} all the way to K_t, then Ω_{ft} goes to zero. When fundamental analysis Ω_{ft} goes to zero, the pricing equilibrium has prices become uninformative (C_t goes to zero). When C_t goes to zero, the marginal value of order flow information $(C_t/D_t)^2$ goes to zero as well. This is the opposing force that limits the amount of order flow information processing after the complementarity kicks in.

Exploring alternative parameter values reveals that this result is quite robust. Ω_{xt} consistently surpasses Ω_{ft} once C_t/D_t crosses $\sqrt{\chi_x}$. There are parameters for which C_t/D_t never exceeds $\sqrt{\chi_x}$, but even in those cases, Ω_{xt} increases faster, while Ω_{ft} is concave. Thus, over time, the growth of fundamental analysis is slowing down.

4.2 Price Informativeness

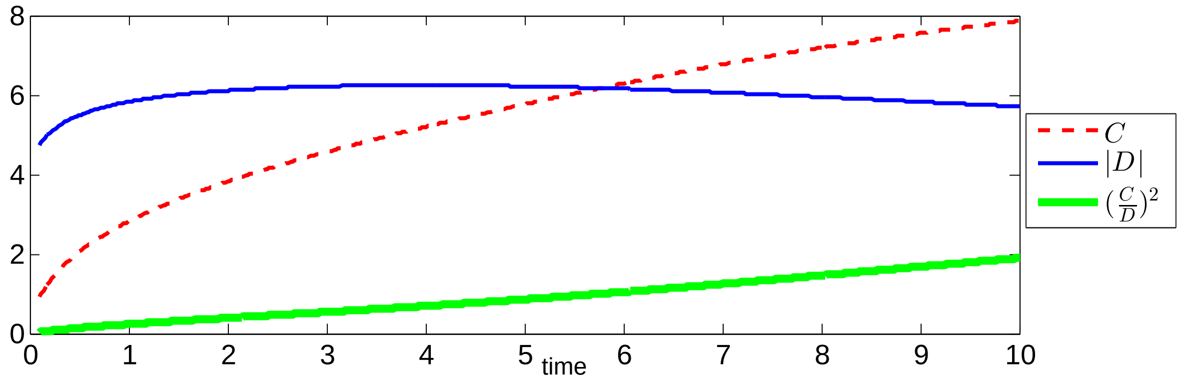
So far, we have explored why an increase in the productivity of information processing causes the value of additional fundamental information to fall and the value of additional order flow information to rise. But why does this transformation matter? We focus on two ways in which the nature of the information investors are analyzing matters for asset market participants. First, it alters the price impact of trades, often referred to as "market liquidity." Second, it changes what real investors can learn from prices. In the section that follows, we show how both of these asset market changes can affect the efficiency of the real economy as well.

The concept of financial market efficiency is based on the idea that prices of asset aggregate all the information known to market participants. In particular, prices are considered to be informative if they contain information about future firm fundamentals. Informative prices are important because they can inform firm managers and allow them to make profitable investment decisions. Informativeness also makes equity or equity price based compensation a useful incentive tool because it incentivizes managers to take actions that raise future firm value and thereby the asset price. Finally, informative prices direct new capital to the right firms, those who will use the capital most productively.

In our model, prices are informative if a change in future dividends is reflected in the price: $dp_t/d\tilde{y}_t$. Our equilibrium price solution (7) reveals that informativeness is C_t, which is plotted in Figure 5. Both fundamental analysis and order flow analysis have the same objective, to help investors better discern the true value of the asset. Thus as the productivity of financial analysis rises, and more information is acquired and processed, the informativeness of the price (C_t) rises. This is consistent with empirical work that documents such a rise over many decades (Bai, Philippon, and Savov, 2013).

The solid line in Figure 5 confirms that as financial analysis becomes more productive, the loading of the price on dividend innovations, C_t, rises. We know this because when fundamental analysis Ω_f is zero, C_t is zero (result 1). As order-flow is observed with greater precision, investors can use the price to infer what the true dividend innovation $\tilde{y}_t$ is more precisely. But then, higher dividend innovations increase the price; an increase in p_t reveals that the innovation $\tilde{y}_t$ is high, which further pushes up the price. So, by making the price more revealing to investors, order-flow information makes the price more responsive

Figure 5: Price Informativeness (C_t) and Price Impact of Trades ($-D_t$).
C_t is the impact of future dividend innovations on price. $(-D_t)$ is the price impact of a one-unit uninformed trade. $(C_t/D_t)^2$ tells us the marginal value of order-flow information, relative to fundamental information. The x-axis is time.



to fundamentals as well. This rise in price informativeness, (C_t), increases the relative marginal value of order-flow information $(C_t/D_t)^2$ and causes the secular shift in analysis.

4.3 Price Impact of Trades

Market liquidity is an important object of study in finance. It is a risk factor that helps to explain the cross-section of asset prices (Brunnermeier and Pedersen, 2008). The breakdown of liquidity played a central role in the 2007-08 financial crisis (Brunnermeier, 2009). Long before these ideas took hold, it was a central object of study in the market microstructure literature (Hasbrouck, 2007). A common metric of market liquidity is the sensitivity of an asset's price to a buy or sell order. If a buy order causes a large increase in the asset price and conversely a sell order causes a large fall, then buying and selling this asset is costly. In such a market, trading strategies that require frequent or large trades would have a harder time generating a profit.

In the context of our model, it makes sense to think of price impact as being the impact of a one-unit hedging trade $(dp_t/d(-\tilde{x}_t))$. The alternative means that we would consider, at least in part, the impact of an information-based trade. But if a measure of investors all bought or sold for informational reasons, then the fundamental (future dividend) much have actually changed to rationalized all these people seeing a higher or lower signal. That question of how much a change in the fundamental changes price is interesting and one we explored, but it is distinct from price impact. The linear price solution (7) reveals that price impact is $dp_t/d(-\tilde{x}_t) = -D_t$.

Looking at the dashed line in Figure 5, we see that the price impact of hedging trades, $-D_t$, rises in the early periods when only Ω_{ft} is increasing and then declines as information becomes more abundant. Price impact is the sum of two competing forces, the static force $(r/(r-G))Var[\tilde{y}_t|\mathcal{I}_{it}](C_t/D_t)$ and a dynamic force $-\rho Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]$. Both terms measure how uncertain the future value of the asset is to the average investor. When the future asset value is known almost for sure, the asset is nearly riskless. In these cases, demand elasticity is very high: When the asset's price exceeds that expected value, demand will be extremely low, and when price falls short of the expected value (by more than a tiny risk premium), demand will surge, pushing the price back up. This implies that price elasticity is low: changes in demand

$-\tilde{x}_t$ that are not related to future value, result in very small price changes. The price sensitivity $dp/d(-x)$ in our model is D_t. In contrast, when uncertainty about the future asset value is high, a change in hedging demand can have a large change in price because to offset that change in demand, another investor needs to bear lots of extra risk. The need to be compensated for bearing that risk with a higher expected return, which implies a lower price. This logic is at work in even the most basic static model with only fundamental information. In these simpler static models, the payoff is exogenous, just $\tilde{y}_t$ is in this model. More information reduces uncertainty about exogenous payoffs ($Var[\tilde{y}_t|\mathcal{I}_{it}]$), which in turn reduces the price impact of non-fundamental trades.

In our model, this static effect is reversed. More information increases price impact. The reason is that the payoff of the asset is endogenous. When information choices change, future price p_{t+1} becomes more volatile. Even if signals are more informative, those signals make the price more responsive to shocks and thus more volatile and more uncertain. In our numerical example, as information analysis rises, investors expect C_t and D_t to be higher tomorrow. Higher C_t and D_t means that tomorrow's price is more sensitive to $\tilde{y}_{t+1}$ and $\tilde{x}_{t+1}$ shocks. More sensitivity means more a volatile and therefore a more uncertain future price. High payoff uncertainty makes today's price noise have more impact (high D_t). For our parameters, the static force of lower $Var[\tilde{y}_t|\mathcal{I}_{it}]$ is overwhelmed by the dynamic price volatility effect of higher $Var[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]$. A similar effect can arise in a dynamic model with only fundamental analysis (see Cai (2016b)).

Intuitively, the reason that price impact is greater is that price volatility rises when information processing improves (if we learn nothing, there is very little price fluctuation). A rise in price volatility makes holding the asset risker. So a sale of one unit of the asset requires more and more risk to be absorbed by the rest of the market over time It will require a larger price adjustment to induce those investors to bear the additional risk. While our setting is quite different, the logic of the result is reminiscent of the Hirshleifer (1971) effect, where revelation of information inhibits investor risk sharing in a static model. In this setting, the mechanism is inherently dynamic. But the increase in information does work to make trading more costly and reduces the extent to which investors can trade risky assets to diversify their idiosyncratic risks.

Early on, this effect works against the rise in C_t to mitigate the increase in $(C_t/D_t)^2$ and reduce complementarity. But the effect on C_t is stronger so that $(C_t/D_t)^2$ consistently rises (dotted line, Figure 5). Later on, the fall in D_t works to increase the signal-to-noise ratio in prices. This rise in $(C_t/D_t)^2$ is what makes technical information relatively more valuable and causes it to crowd out fundamental analysis. Exploring different parameters or solutions, we see that the dynamics of market liquidity can very, being concave like here or convex. But what is consistent is that the changes are small compared to the change in price informativeness.

5 Real Economic Effects

If the growth in financial analysis has caused a transformation of the financial sector, it is natural to ask what the consequences are for real economic activity. In this section, we provide a sketch of two channels through which changes in informativeness and price impact can alter the efficiency of real business investment.

5.1 Manager incentive effects

Time is discrete and infinite. There is a single firm whose profits $\tilde{d}_t$ depend on a firm manager's labor choice l_t. Specifically, $\tilde{d}_t = g(l_t) + \tilde{y}_t$, where g is increasing and concave and $\tilde{y}_t \sim N(0, \tau_0^{-1})$ is unknown at t. Because effort is unobserved, the manager's pay w_t is tied to the equity price p_t of the firm: $w_t = \bar{w} + p_t$. However, effort is costly. We normalize the units of effort so that a unit of effort corresponds to a unit of utility cost. Insider trading laws prevent the manager from participating in the equity market. Thus the manager's objective is

$$U_m(l_t) = \bar{w} + p_t - l_t \tag{16}$$

The firm pays out all its profits as dividends each period to its shareholders. Firm equity purchased at time t is a claim to the present discounted stream of future profits $\{\tilde{d}_t, \tilde{d}_{t+1} \dots\}$.

The preferences, endowments, budget constraint and information choice sets of investors are the same as before. Order flow signals are defined as before. Fundamental analysis now generates signals of the form $\eta_{fit} = g(l_t) + \tilde{y}_t + \tilde{\epsilon}_{fit}$, where the signal noise is $\tilde{\epsilon}_{fit} \sim N(0, \Omega_f)$. Investors choose the precision Ω_f of this signal, as well as their order flow signal Ω_x. Equilibrium is defined as before, with the additional condition that the manager effort decision maximizes (16).

The key friction here is that the entrepreneur's investment choice is unobserved by equity investors. Because of this, real investment efficiency depends on asset price informativeness. The entrepreneur only has an incentive to invest to the extent that price reflects and responds to the true investment. Of course, this friction reflects the fact that w_t is not the optimal contract. The optimal compensation for the manager is to make him hold all equity in the firm. This sort of contract is not feasible and microfounding the nature of the constraints would distract us from our main point about the evolution of financial analysis. Regardless of its optimality properties, compensation contracts that tie wages to firm equity prices (e.g., options packages) are common in practice.

Solution The asset market equilibrium has a similar equilibrium price. Notice that since dividends are not persistent, d_t is not relevant for the t price, which is a claim to $\tilde{d}_t$. Thus, the terms that was Bd_t in the previous model becomes zero here:

$$p_t = A_t + C_t(g(l_t) + \tilde{y}_t) + D_t\tilde{x}_t \tag{17}$$

The firm manager chooses his effort to maximize (16). The first order condition is $C_t g'(l_t) = 1$, which yields an equilibrium effort level $l_t = (g')^{-1}(1/C_t)$. Notice that the socially optimal level would set the marginal utility cost of effort (1) equal to the marginal product $f'(l)$. Instead the manager sets this marginal product equal to $1/C_t$. When C_t is below one, managers under-provide effort, relative to the social optimum because their stock compensation moves less than one-to-one with the true value of their firm.

Similar to before, the equilibrium level of price informativeness C is

$$C_t = \frac{1}{r}\left(1 - \tau_0 Var[g(l_t) + \tilde{y}_t|\mathcal{I}_{it}]\right). \tag{18}$$

Thus, as more information is analyzed, $Var[g(l_t)+\tilde{y}_t|\mathcal{I}_{it}]$ falls, C_t rises and managers are better incentivized to exert optimal effort. While the model is stylized and the solution presented here is only a sketch, it is designed to clarify why trends in financial analysis matter for the real economy.

5.2 Equity Issuance Cost

The previous model suggested that trends in the financial sector are all positive for real economic efficiency because more analysis of either type makes price more informative and thereby improves incentives. In contrast, the second model highlights a possible downside of the growth in financial analysis. More information rises the risk of assets, which makes raising capital more expensive.

Suppose that a firm has a profitable investment opportunity and wants to issue new equity to raise capital for that investment. The firm chooses k to maximize the total revenue from the sale of $\bar{s}$ shares each at price p, minus a fixed investment cost:

$$E[\bar{s}p - c(\bar{s})|\mathcal{I}_f]$$

The firm makes its choice conditional on the same prior information that all the investors have. But does not condition on p. It does not take price as given. Rather, the firm chooses $\bar{s}$, taking into account its impact on the equilibrium price. The change in issuance is permanent and unanticipated. The rest of the model is the same as the dynamic model in section 1.

Solution Given the new asset supply $\bar{s}$, the asset market solution and information choice solution to the problem are the same as before. The only question is how the firm choose $\bar{s}$. This depends on how new issuance affects the asset price.

When the firm issues new equity, all asset market participants are aware that new shares are coming online. It is not like the unobserved hedging trades. Instead, equity issuance changes the known supply of the asset $\bar{s}$. Supply $\bar{s}$ enters the asset price in only one place in the equilibrium pricing formula, through A_t (see Appendix A for derivation):

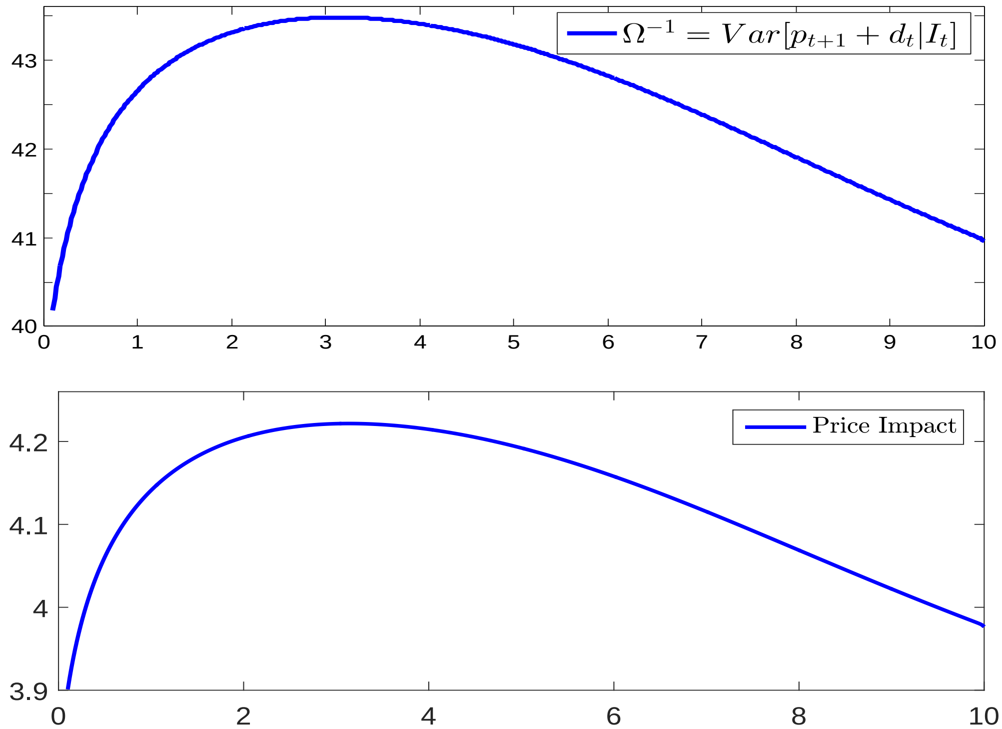
$$A_t = \frac{1}{r}\left[A_{t+1} + (1 + B_{t+1})\mu - \rho\Omega_t^{-1}\bar{s}\right] \tag{19}$$

Taking A_{t+1} as given for the moment, $dA_t/d\bar{s} = -\rho\Omega_t^{-1}/r$. The impact of a one-period change in asset supply depends on Ω_t^{-1}, which is the conditional variance (the uncertainty about) the future asset payoff, $p_{t+1} + \tilde{d}_t$. Recall from the discussion of price impact of trades in Section 4.3 that in a dynamic model, more information analysis can result in more uncertainty about future payoffs. As information analysis rises, investors expect C_{t+1} and D_{t+1} to be higher tomorrow, which means that tomorrow's price is more sensitive to $\tilde{y}_{t+1}$ and $\tilde{x}_{t+1}$ shocks. More sensitivity means more a volatile and therefore a more uncertain future price.

In this context, technological progress in information analysis – of either type – initially makes asset payoffs more uncertain, which makes it more costly to issue new equity. When we now take into account that the increase in asset supply is permanent, the effect of issuance is amplified, relative to the one-period (fixed A_{t+1}) case. But when analysis becomes sufficiently productive, issuance costs decrease again, as the risk-reducing power of more precise information becomes the dominant force.

Figure 6 plots the increase and decrease in payoff risk from this dynamic asset price effect and the

Figure 6: Payoff Risk and The Cost of Raising Capital.
The top panel shows payoff risk, which is $\Omega^{-1} \equiv Var[p_{t+1} + \tilde{d}_t | \mathcal{I}_{it}]$. The bottom panel shows the price impact of a one-unit change in issuance, normalized by the average level of dividends. This impact represents a change in the price-dividend ratio of between xx and xx, from a one-unit change in supply, where the baseline supply of the asset is $\bar{s} = 1$.



concurrent rise and fall in the price impact of issuing new equity. To give the units of the price impact some meaning, he issuance cost is scaled by the average dividend payment so that it can be interpreted as the change in the price-dividend ratio from a one-unit change in equity supply.

6 Conclusion

Technological progress is the driving force behind most if not all models of long-run economic growth. Yet it is surprisingly absent in models of the financial economy. We explore the consequences of a simple deterministic increase in productivity in the information processing of the financial sector. While studies have documented an increase in price informativeness (Bai, Philippon, and Savov, 2013), we know of no theories that explore the consequences of such changes on the equilibrium structure of the market.

We find that when the financial sector becomes more efficient at processing information, it changes the nature of the equilibrium asset prices. This, in turn, changes the incentives to acquire information about future dividends (fundamentals) versus order flow (non fundamental shocks to price). Thus a simple rise in information processing productivity can explain a transformation of financial analysis from a sector that primarily investigates the fundamental profitability of firms to a sector that does a little fundamental analysis but mostly concentrates on acquiring and processing client order flow. This is consistent with suggestive evidence that the nature of financial analysis has changed.

Of course, there are many other features one might want to add to this model to speak to other related trends in financial markets. One might make fundamental changes more persistent than order flow innovations so that different styles of trade were associated with different trading volumes. Another possibility is to explore regions in this model where the equilibrium does not exist and use the non-existence and the basis for a theory of market breakdowns or freezes. Another interesting extension would be to ask where order flow signals come from. In practice, people observe order flow because they intermediate trades. Thus, the value of the order flow information might form the basis for a new theory of intermediation. In such a world, more trading might well generate more information for intermediaries and faster or stronger responses of markets to changes in market conditions. Finally, one might regard this theory is a prescriptive theory of optimal investment, compare it to investment practice, and compute expected losses from sub-optimal information and portfolio choices. For example, a common practice now is to blend fundamental and order flow trading by first selecting good fundamental investment opportunities and then using order flow information to time the trade. One could construct such a strategy in this model, compare it to the optimal blend of trading strategies, see if the optimal strategy performs better inside the model, and then test it out-of-sample with market data.

While this project with its one simple driving force leaves many question unanswered, it also provides a tractable foundation on which to build, to continue exploring how and why asset markets are evolving, as financial technology improves.

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A Model Solution Details

A.1 Bayesian Updating

To form the conditional expectation, $E[f_{it}|\mathcal{I}_{it}]$, we need to use Bayes' law. But first, we need to know what signal investors extract from price, given their observed endowment exposure h_t and their order-flow signal η_x. We can rearrange the the linear price equation (7) to write a function of the price is the dividend innovation plus mean zero noise: $\eta_{pi} = \tilde{y}_t + (D_t/C_t)(\tilde{x}_t - E[\tilde{x}_t|\eta_x])$, where the price signal and the signal precision are

$$\eta_{pi} \equiv (p - A - Bd_t - D\,E[x|\eta_x])/C \tag{20}$$

$$\Omega_{pt} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xt}) \tag{21}$$

For the simple case of an investor who learned nothing about order flow ($E[x] = 0$) the information contained in prices is $(p - A - Bd_t)/C$, which is equal to $y + D/Cx$. Since x is a mean-zero random variable, this is an unbiased signal of the asset payoff f. The variance of the signal noise is $Var[D/Cx] = (D/C)^2\tau_x^{-1}$. The price signal precision Ω_{pt} is the inverse of this variance.

But conditional on h_t and η_x, x_t is typically not a mean-zero random variable. Instead, investors use Bayes' law to combine their prior that $x_t = 0$, with precision τ_x with their endowment and order flow signals: h_{it} with precision τ_h and η_{xit} with precision Ω_{xi}. The posterior mean and variance are

$$E[x|h_{it}, \eta_{xit}] = \frac{\tau_h h_{it} + (\Omega_{xi} - \tau_h)\eta_{xit}}{\tau_x + \Omega_{xi}} \tag{22}$$

$$V[x|h_{it}, \eta_{xit}] = \frac{1}{\tau_x + \Omega_{xi}} \tag{23}$$

Since that is equal to $f + D/C(x - E[x|\eta_x])$, the variance of price signal noise is $(D/C)^2Var[x|\eta_x]$. In other words, the precision of the price signal for agent i (and therefore for every agent since we are looking at symmetric information choice equilibria) is $\Omega_{pit} \equiv (C/D)^2(\tau_x + \Omega_{xi})$.

Now, we can use Bayes' law for normal variables again to form beliefs about the asset payoff. We combine the prior μ, the price/order-flow information η_{pi}, and the fundamental signal η_{fi} into a posterior mean and variance:

$$E[y_t|\mathcal{I}_{it}] = (\tau_0 + \Omega_{pit} + \Omega_{fi})^{-1}(\tau_0\mu + \Omega_{pit}\eta_{pi} + \Omega_{fi}\eta_{fi}) \tag{24}$$

$$V[y_t|\mathcal{I}_{it}] = (\tau_0 + \Omega_{pit} + \Omega_{fi}) \tag{25}$$

Average expectations and precisions: Next, we integrate over investors i to get the average conditional expectations. Begin by considering average price information. The price informativeness is $\Omega_{pit} \equiv (C/D)^2(\tau_x + \Omega_x)$. In principle, this can vary across investors. But since all are ex-ante identical, they make identical information decisions. Thus, $\Omega_{pi} = \Omega_p$ for all investors i. Since this precision is identical for all investors, we drop the i subscript in what follows. But the realized price signal still differs because signal realizations are heterogeneous. Since the signal precisions are the same for all agents, we can just integrate over signals to get the average signal: $\int \eta_{pi}di = (1/C)(p - A - Bd_t) - (D/C)Var(x|\mathcal{I})\Omega_x\tilde{x}_t$. Since $\Omega_p^{-1} = (D/C)^2Var(x|\mathcal{I})$, we can rewrite this as

$$\int \eta_{pi}di = \frac{1}{C}(p - A - Bd_t) - \frac{C}{D}\Omega_p^{-1}\Omega_x\tilde{x}_t \tag{26}$$

Next, let's define some conditional variance / precision terms that simplify notation. The first term, Ω, is the precision of future price plus dividend (the asset payoff). Is comes from taking the variance of the pricing equation (7). It turns out that the variance Ω^{-1} can be decomposed into a sum of two terms. The first, $\hat{V}$, is the variance of the dividend innovation. This variance depends on information choices Ω_f and Ω_x. The other term Z_t depends on future information choices through $t+1$ price coefficients.

$$\hat{V} \equiv Var(\tilde{y}_t|\mathcal{I}) = (\tau_0 + \Omega_f + \Omega_p)^{-1} = (\tau_0 + \Omega_f + (C/D)^2(\tau_x + \Omega_x))^{-1} \tag{27}$$

$$Z_t^{-1} \equiv Var[p_{t+1} + \tilde{d}_t|\mathcal{I}] = C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1} + (1+B)^2\hat{V} \tag{28}$$

$$Z_t = \frac{\rho}{r}(r-G)(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}) \tag{29}$$

$$\Omega_t^{-1} = \frac{r}{\rho(r-G)}Z_t + (\frac{r}{r-G})^2\hat{V} \tag{30}$$

The last equation (30) shows the relationship between Ω, $\hat{V}$ and Z_t. This decomposition is helpful because we will repeatedly take derivatives where we take future choices (Z_t) as given and vary current information choices ($\hat{V}$).

Next, we can compute the average expectations

$$\int E[\tilde{y}_t|\mathcal{I}_{it}]di = \hat{V}\left[\Omega_f\tilde{y}_t + \Omega_p\left(\frac{1}{C}(p-A-Bd_t) - \frac{C}{D}\Omega_p^{-1}\Omega_x\tilde{x}_t\right)\right] \tag{31}$$

$$= \hat{V}\left[\Omega_f\tilde{y}_t + \Omega_p\frac{1}{C}(p-A-Bd_t) - \frac{C}{D}\Omega_x\tilde{x}_t\right] \tag{32}$$

$$\int \frac{E[p_{t+1} + \tilde{d}_t|\mathcal{I}_{it}]}{d}i = A + (1+B)E[\tilde{d}_t|\mathcal{I}] \tag{33}$$

$$= A + (1+B)\left(\mu + Gd_t + E[\tilde{y}_t|\mathcal{I}_{it}]\right). \tag{34}$$

A.2 Solving for equilibrium prices

The new price conjecture is

$$p_t = A_t + B_td_t + C_t\tilde{y}_t + D_t\tilde{x}_t \tag{35}$$

where the sequence of pricing coefficients is known at every date. The signals η_{fit} and η_{xit} are the same as before, except that their precisions Ω_{ft} and Ω_{xt} may change over time if that is the solution to the information choice problem.

The conditional expectation and variance of $\tilde{y}_t$ (24) and (25) are the same, except that the Ω_p term gets a t subscript now because $\Omega_{pt} \equiv (C_t/D_t)^2(\tau_x + \Omega_{xt})$. Likewise the mean and variance of $\tilde{x}_t$ (22) and (23) are the same with a time-subscripted Ω_{xt}. Thus, the average signals are the same with t-subscripts:

$$\int \eta_{pi}di = \frac{1}{C_t}(p_t - A_t - B_td_t) - \frac{D_t}{C_t}Var(x|\mathcal{I})\Omega_{xt}\tilde{x}_t \tag{36}$$

Since $\Omega_{pt}^{-1} = (D_t/C_t)^2Var(x|\mathcal{I})$, we can rewrite this as

$$\int \eta_{pi}di = \frac{1}{C_t}(p_t - A_t - B_td_t) - \frac{C_t}{D_t}\Omega_{pt}^{-1}\Omega_{xt}\tilde{x}_t \tag{37}$$

Solving for non-stationary equilibrium prices To solve for equilibrium prices, start from the portfolio first-order condition for investors (6) and equate total demand with total supply. The total risky asset demand (excluding hedging shocks) is

$$\int q_{it}di = \frac{1}{\rho}\Omega_t\left[A_{t+1} + (1+B_{t+1})\left(\mu + Gd_t + \hat{V}_t\left[\Omega_{ft}\tilde{y}_t + \Omega_{pt}\frac{1}{C_t}(p_t - A_t - B_td_t) - \frac{C_t}{D_t}\Omega_{xt}\tilde{x}_t\right]\right) - p_tr\right]. \tag{38}$$

The market clearing condition equates the expression above to the residual asset supply $\bar{x} + \tilde{x}_t$. The model assumes the asset supply is 1. We use the notation $\bar{x}$ here for more generality because then we can apply the result to the model with issuance costs where asset supply is a choice variable. Rearranging the market clearing condition (just multiplying through by $\rho\Omega_t^{-1}$ and bringing p terms to the left) yields

$$[r - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]p_t = -\rho\Omega_t^{-1}(\bar{x} + \tilde{x}_t) + A_{t+1} \tag{39}$$

$$+(1+B_{t+1})(\mu + Gd_t) + (1+B_{t+1})\hat{V}_t\Omega_{ft}\tilde{y}_t - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}(A_t + B_td_t) - (1+B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}\tilde{x}_t$$

Solving for p and matching coefficients yields

$$A_t = [r - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}[A_{t+1} + (1+B_{t+1})\mu - \rho\Omega_t^{-1}\bar{x} - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t] \tag{40}$$

Multiplying both sides by the inverse term:

$$rA_t - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t = A_{t+1} + (1+B_{t+1})\mu - \rho\Omega_t^{-1}\bar{x} - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}A_t \tag{41}$$

and cancelling the $1+B$ term on both sides leaves

$$A_t = \frac{1}{r}\left[A_{t+1} + (1+B_{t+1})\mu - \rho\Omega_t^{-1}\bar{x}\right] \tag{42}$$

Matching coefficients on d_t yields:

$$B_t = [r - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}\left[(1+B_{t+1})G - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{B_t}{C_t}\right] \tag{43}$$

Multiplying on both sides by the inverse term

$$rB_t - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}B_t = (1+B_{t+1})G - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{B_t}{C_t} \tag{44}$$

and cancelling the last term on both sides yields

$$B_t = \frac{1}{r}(1+B_{t+1})G \tag{45}$$

As long as r and G don't vary over time, it seems that a stationary solution for B at least exists. That stationary solution would be (8).

Next, collecting all the terms in $\tilde{y}_t$

$$C_t = [r - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}(1+B_{t+1})\hat{V}_t\Omega_{ft} \tag{46}$$

multiplying both sides by the first term inverse:

$$rC_t - (1+B_{t+1})\hat{V}_t\Omega_{pt} = (1+B_{t+1})\hat{V}_t\Omega_{ft} \tag{47}$$

dividing through by r and collecting terms in $\hat{V}(1+B_{t+1})$

$$C_t = \frac{1}{r}(1+B_{t+1})\hat{V}_t(\Omega_{pt}+\Omega_{ft}) \tag{48}$$

using the fact that $\hat{V}^{-1} = \tau_0 + \Omega_p + \Omega_f$, we get

$$C_t = \frac{1}{r}(1+B_{t+1})(1-\tau_0\hat{V}_t) \tag{49}$$

Of course the $\hat{V}$ term has C_t and D_t in it. If we use the stationary solution for B (if r and G don't vary) then we can simplify

$$C_t = \frac{1}{r-G}(1-\tau_0\hat{V}_t). \tag{50}$$

Lemma 1 *If $\Omega_f > 0$, then $C_t > 0$.*

Proof: Using equation (50), it suffices to show that $1/(r-G) > 0$ and $(1-\tau_0\hat{V}_t) > 0$. From the setup, we assumed that $r > 1$ and $G < 1$. By transitivity, $r > G$ and $r - G > 0$. For the second term, we need to prove equivalently that $\tau_0\hat{V}_t < 1$ and thus that $\tau_0 < \hat{V}_t^{-1}$. Recall from (27) that $\hat{V}^{-1} = \tau_0 + \Omega_f + \Omega_p$. Since Ω_f and Ω_p are defined as precisions, they must be non-negative. Furthermore, we supposed that $\Omega_f > 0$. Thus, $\tau_0 < \hat{V}_t^{-1}$, which completes the proof. □

Finally, we collect terms in $\tilde{x}_t$.

$$D_t = [r - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{1}{C_t}]^{-1}[-\rho\Omega_t^{-1} - (1+B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt}] \tag{51}$$

multiply by the inverse term to get

$$rD_t - (1+B_{t+1})\hat{V}_t\Omega_{pt}\frac{D_t}{C_t} = -\rho\Omega_t^{-1} - (1+B_{t+1})\hat{V}_t\frac{C_t}{D_t}\Omega_{xt} \tag{52}$$

and the use $\Omega_{pt} = (C_t/D_t)^2(\tau_x + \Omega_{xt})$ to get

$$rD_t - (1+B_{t+1})\hat{V}_t\frac{C_t}{D_t}(\tau_x + \Omega_{xt}) = -\rho\Omega_t^{-1} - (1+B_{t+1})\frac{C_t}{D_t}\hat{V}_t\Omega_{xt} \tag{53}$$

Then, adding $(1+B)C/D\hat{V}\Omega_x$ to both sides, and substituting in B (stationary solution), we get

$$D_t - \frac{1}{r-G}\hat{V}_t\tau_x\frac{C_t}{D_t} = -\frac{\rho}{r}\Omega_t^{-1} \tag{54}$$

Of course, D_t still shows up quadratically, and also in $\hat{V}_t$. The future coefficient values C_{t+1} and D_{t+1} show up in Ω_t.

Lemma 2 $D_t < 0$

Proof: Start from equation (56) in the LongRunEvolution_Nov2016, substitute in (27) but does not set $\Omega_f = 0$. Since we will often treat the signal-to-noise ratio in prices as a single variable, we define

$$\xi \equiv \frac{C_t}{D_t} \tag{55}$$

Also let: $\alpha \equiv \frac{\rho r}{r-G}$. This gives the general version of (60):

$$\xi^3(Z_t\tau_x + Z_t\Omega_x) + \xi^2(\Omega_x) + \xi(\alpha + Z_t\tau_0 + Z_t\Omega_f) + \Omega_f = 0 \tag{56}$$

Then, use the budget constraint to express the first order conditions (13) and (14) as

$$\Omega_x = \frac{\xi^2\chi_f}{\chi_x}\Omega_f \tag{57}$$

which then one can solve for both Ω_x and Ω_f in terms of ξ:

$$\Omega_f = \Big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}} \tag{58}$$

$$\Omega_x = \Big(\frac{K}{\chi_x}\big(1 - \frac{1}{1+\frac{\chi_f}{\chi_x}\xi^4}\big)\Big)^{\frac{1}{2}} = \Big(\frac{K\frac{\chi_f}{\chi_x}}{\chi_x\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}}\xi^2 = \frac{\xi^2\chi_f}{\chi_x}\Big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}} \tag{59}$$

Now I can substitute both of these into equation (56), which fully determines ξ, in terms of exogenous variables.

$$\xi\Big(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + \xi^2\Omega_x(1+\xi Z_t) + \Omega_f(1+\xi Z_t) = 0 \tag{60}$$

First note that

$$\Omega_f + \xi^2\Omega_x = -\frac{\xi(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0)}{(1+\xi Z_t)}$$

where the left hand side is the objective function. So we know the maximized value of objective function solely as a function of $\xi = \frac{C}{D}$. Note that in this derivation I have already imposed condition (57), which is an optimality condition. So this latter equation holds only at the optimum.

Substituting in for Ω_f and Ω_x from (58) and (59) yields an equation that implicitly defines ξ as a function of primitives, K and future equilibrium objects, embedded in Z_t.

$$\xi\Big(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0\Big) + (1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)\Big(\frac{K}{\chi_f\big(1+\frac{\chi_f}{\chi_x}\xi^4\big)}\Big)^{\frac{1}{2}} = 0$$

$$\xi^3 Z_t\tau_x + \xi(\alpha + Z_t\tau_0) + (1+\xi Z_t)(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = 0 \tag{61}$$

The left hand side must equal zero for the economy to be in equilibrium. However, all the coefficients $K, \chi_f, \chi_x, \tau_0, \tau_x$ are assumed to be positive. Furthermore, Z_t is a variance. Inspection of (29) reveals that it must be strictly positive. Thus, the only way that the equilibrium condition can possibly be equal to zero is if $\xi < 0$. Recall that $\xi = C_t/D_t$. The previous lemma proved that $C_t > 0$. Therefore, it must be that $D_t < 0$.

A.3 Solving Information Choices

Details of Step 3: Compute ex-ante expected utility. Note that the expected excess return ($E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - p_t r$) depends on fundamental and supply signals, and prices, all of which are unknown at time $t = 0$. Because asset prices are linear functions of normally distributed shocks, $E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - p_t r$, is normally distributed as well. Thus, $(E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - p_t r)\Omega(E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - p_t r)$ is a non-central χ^2-distributed variable. Computing its mean yields the expression in the text.

Details of Step 4:

Solve for fundamental information choices. Note that in expected utility (12), the choice variables Ω_{ft} and Ω_{xt} enter only through the posterior variance Ω^{-1} and through $V[E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - p_t r|\mathcal{I}^+_{t-1}] = V[p_{t+1} + \tilde{d}_t - p_t r|\mathcal{I}^+_{t-1}] - \Omega_t^{-1}$. Since there is a continuum of investors, and since $V[p_{t+1} + \tilde{d}_t - p_t r|\mathcal{I}^+_{t-1}]$ and $E[E[p_{t+1} + \tilde{d}_t|\mathcal{I}_t] - p_t r|\mathcal{I}^+_{t-1}]$ depend only on $t-1$ variables, parameters and on aggregate information choices, each investor takes them as given. If the objective is to maximize an increasing function of Ω, then information choices must maximize Ω as well.

B Proofs

The next lemma proves the following: If no one has information about future dividends, then no one's trade is based on information about future dividends, thus the price cannot contain information about future dividends. Since C_t is the price coefficient on future dividend information, $C_t = 0$ means that the price is uninformative. In short, price cannot reflect information that no one knows.

Lemma 3 ***When information is scarce, price is uninformative****: As $K_t \to 0$, for any future path of prices (A_{t+j}, B_{t+j}, C_{t+j} and D_{t+1}, $\forall j > 0$), the unique solution for the price coefficient C_t is $C_t = 0$.*

Proof: Step 1: As $\Omega_f \to 0$, prove C_t is always a solution.

Start with the equation for D_t (10). Substitute in for Ω using (30) and $1 + B = r/(r-G)$ and rewrite it as

$$D_t = \frac{1}{r-G}\hat{V}_t\left[\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t\hat{V}_t^{-1}\right] \tag{62}$$

Then, express C_t from (50) as $C_t = 1/(r-G)\hat{V}_t(\hat{V}_t^{-1} - \tau_0)$ and divide C_t by D_t, cancelling the $\hat{V}_t/(r-G)$ term in each to get

$$\frac{C_t}{D_t} = \frac{\hat{V}_t^{-1} - \tau_0}{\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t\hat{V}_t^{-1}} \tag{63}$$

If we substitute in $\hat{V}_t^{-1} = \tau_0 + \Omega_p + \Omega_f$ from (27) and then set $\Omega_f = 0$, we get

$$\frac{C_t}{D_t} = \frac{\Omega_p}{\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \Omega_p)} \tag{64}$$

Then, we use the solution for price information precision $\Omega_p = (C/D)^2(\tau_x + \Omega_x)$ and multiply both sides by the denominator of the fraction to get

$$\frac{C_t}{D_t}\left[\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x))\right] = \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x) \tag{65}$$

We can see right away that since both sides are multiplied by C/D, as $\Omega_f \to 0$, for any given future price coefficients C_{t+1} and D_{t+1}, $C = 0$ is always a solution.

Step 2: prove uniqueness.

Next, we investigate what other solutions are possible by dividing both sides by C/D:

$$\tau_x\frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t(\tau_0 + \left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x)) - \left(\frac{C_t}{D_t}\right)(\tau_x + \Omega_x) = 0 \tag{66}$$

This is a quadratic equation in C/D. Using the quadratic formula, we find

$$\frac{C_t}{D_t} = \frac{\Omega_x \pm \sqrt{\Omega_x^2 - 4Z_t(\tau_x+\Omega_x)(\rho r/(r-G)+\tau_0 Z_t)}}{-2Z_t(\tau_x+\Omega_x)} \tag{67}$$

If we now take the limit as $\Omega_x \to 0$, the term inside the square root becomes negative, as long as $r - G > 0$. Thus, there are no additional real roots when $\Omega_x = 0$.

Similarly, if Ω_x is not sufficiently large, there are no real roots of (67), which proves that: As $\Omega_f \to 0$, if we take C_{t+1} and D_{t+1} as given, and Ω_x is sufficiently small, then the unique solution for the price coefficient C is $C = 0$. □

Proof of Result 1 From lemma 3, we know that as $C_t = 0$. From the first order condition for information (14), we see that the marginal utility of order flow information is a positive constant times $(C_t/D_t)^2$. If $C_t = 0$, then $\partial U_{it}/\partial\Omega_{xit}$ is a positive constant time zero, which is zero.

Proof of Result 2

Claim: If $r - g > 0$ and $(\tau_x + \Omega_{xt})$ is sufficiently small, then $\partial C_t/\partial\Omega_{ft} > 0$ and $\partial C_t/\partial\Omega_{xt} > 0$.

From (50), $C_t = \frac{1}{r-G}(1 - \tau_0\hat{V}_t)$.

From (27), $\hat{V}_t$ is defined as

$$\hat{V} = [\tau_0 + \Omega_{ft} + \frac{C_t}{D_t}^2(\tau_x + \Omega_{xt})]^{-1} \tag{68}$$

Notice that C_t shows up twice, once on the left side and once in $\hat{V}$. Therefore, we use the implicit function theorem to differentiate. If we define $F \equiv C_t - \frac{1}{r-G}(1-\tau_0\hat{V})$, then $\partial F/\partial C_t = 1 + \frac{1}{r-G}\tau_0\partial\hat{V}/\partial C_t$. Since τ_x and Ω_{xt} are both precisions, both are positive. Therefore, $\partial\hat{V}^{-1}/\partial C_t = 2C_t/D_t^2(\tau_x + \Omega_{xt})$. This is positive, since we know that $C_t > 0$. That implies that the derivative of the inverse is $\partial\hat{V}/\partial C_t = -\hat{V}^2 2C_t/D_t^2(\tau_x+\Omega_{xt})$, which is negative. The $\partial F/\partial C_t$ term is therefore one plus a negative term. The result is positive, as long as the negative term is sufficiently small: $\frac{2}{r-G}\tau_0\hat{V}^2 C_t/D_t^2(\tau_x+\Omega_{xt}) < 1$. We can express this as an upper bound on $\tau_x + \Omega_{xt}$ by rearranging the inequality to read: $(\tau_x+\Omega_{xt}) < 1/2(r-G)\tau_0^{-2}\hat{V}^{-2}D_t^2/C_t$.

Next, we see that $\partial\hat{V}^{-1}/\partial\Omega_{ft} = 1$. Thus, $\partial\hat{V}/\partial\Omega_{ft} < 0$. Since $\partial F/\partial\hat{V} > 0$, this guarantees that $\partial F/\partial\Omega_{ft} < 0$.

Likewise, $\partial\hat{V}^{-1}/\partial\Omega_{xt} = (C_t/D_t)^2$. Since the square is always positive, $\partial\hat{V}/\partial\Omega_{xt} < 0$. Since $\partial F/\partial\hat{V} > 0$, this guarantees that $\partial F/\partial\Omega_{xt} < 0$.

Finally, the implicit function theorem states that $\partial C_t/\partial\Omega_{ft} = -(\partial F/\partial\Omega_{ft})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial\Omega_{ft} > 0$. Likewise, $\partial C_t/\partial\Omega_{xt} = -(\partial F/\partial\Omega_{xt})/(\partial F/\partial C_t)$. Since the numerator is positive, the denominator is negative and there is a minus sign in front, $\partial C_t/\partial\Omega_{xt} > 0$. □

Proof of Result 3, part 1

Claim: If $\tau_x > \rho r/(r-G)$ and $D_t < 0$, then $\partial D_t/\partial\Omega_{ft} > 0$.

Proof:

From market clearing:

$$D_t = [r - (1+B)\hat{V} + \Omega_p\frac{1}{C}]^{-1}[-\rho\Omega_t^{-1} - (1+B)\frac{C}{D}\hat{V}\Omega_x] \tag{69}$$

Use $\Omega_p = (\frac{C}{D})^2(\Omega_x+\tau_x)$ to get $D_t r - (1+B)\hat{V}_t\frac{C}{D}(\tau_x) = -\rho\Omega_t^{-1}$. Then, use the stationary solution for $B : 1 + B = \frac{r}{r-G}$:

$$D_t - \frac{1}{r-G}\hat{V}_t\frac{C}{D}\tau_x = -\frac{\rho}{r}\Omega_t^{-1} \tag{70}$$

Then use (30) to substitute in for Ω_t^{-1}:

$$D_t = -\frac{1}{r-G}Z_t - \frac{r\rho}{(r-G)^2}\hat{V} + \frac{1}{r-G}\hat{V}_t\frac{C_t}{D_t}\tau_x \tag{71}$$

In the above, the RHS, less the last term, is the loading on X_{t+1}, and the last term represents price feedback. We then define $F \equiv$ L.H.S. of (71) $-$ R.H.S. of (71). So that we can apply the implicit function theorem as $\partial D_t/\partial\Omega_f = -\frac{\partial F}{\partial \Omega_f}/\frac{\partial F}{\partial D_t}$. We begin by working out the denominator.

$$\frac{\partial F}{\partial D_t} = 1 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial D_t} - \frac{1}{r-G}\frac{\partial \hat{V} + \frac{C_t}{D_t}}{\partial D_t}\tau_x \tag{72}$$

$$\frac{\partial \hat{V}}{\partial D_t} = \frac{\partial \hat{V}}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial D_t} = -\hat{V}^2[-\frac{2C_t^2}{D_t^3}(\tau_x + \Omega_x)] = 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) \tag{73}$$

$$\frac{\partial \hat{V}\frac{C_t}{D_t}}{\partial D_t} = \frac{C_t}{D_t}\frac{\partial \hat{V}_t}{\partial D_t} + \hat{V}(-\frac{C}{D^2}) \tag{74}$$

$$= \frac{C}{D^2}\hat{V}[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{75}$$

$$\frac{\partial F}{\partial D_t} = 1 + \frac{r\rho}{(r-G)^2}\cdot 2\frac{C^2}{D^3}\hat{V}_t^3(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C_t}{D_t}(\tau_x + \Omega_x) - 1] \tag{76}$$

$$\frac{\partial F}{\partial \Omega_f} = 0 - 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial \Omega_t} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\frac{\partial \hat{V}}{\partial \Omega_t} \tag{77}$$

Recall the definition $\hat{V}_t \equiv [\tau_0 + \Omega_{ft} + \frac{C_t}{D_t}^2(\tau_x + \Omega_x)]^{-1}$. Differentiating $\hat{V}$, we get

$$\frac{\partial \hat{V}}{\partial \Omega_f} = \frac{\partial \hat{V}_t}{\partial \hat{V}_t^{-1}}\cdot\frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2\frac{\partial \hat{V}_t^{-1}}{\partial \Omega_f} = -\hat{V}_t^2 \tag{78}$$

substituting this in to (77) yields

$$\frac{\partial F}{\partial \Omega_f} = \frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}] \tag{79}$$

Substituting in the derivative of $\hat{V}$, we get

$$\frac{\partial D_t}{\partial \Omega_f} = -\frac{\frac{1}{r-G}\hat{V}_t^2[\frac{C_t}{D_t}\tau_x - \frac{r\rho}{r-G}]}{1\frac{2r\rho}{(r-G)^2}\frac{C^2}{D^3}\hat{V}_t^2(\tau_x + \Omega_x) - \frac{\tau_x}{r-G}\frac{C}{D^2}\hat{V}_t[2\frac{C}{\rho}(\tau_x + \Omega_x) - 1]} \tag{80}$$

Observe that if $\frac{C_t}{D_t} < 0$, and $r > G$, then the numerator is positive (including the leading negative sign). The denominator is positive if the following expression is positive:

$$\frac{r-G}{\frac{C}{D^2}\hat{V}} + 2\rho\frac{r}{r-G}\frac{C_t}{D_t}\hat{V}_t(\tau_x + \Omega_x) - \tau_x\hat{V}_t[\frac{2C}{D}(\tau_x + \Omega_x - 1)] > 0 \tag{81}$$

This is equivalent to

$$\frac{r-G}{\hat{V}_t}\frac{D^2}{C} + 2\hat{V}_t\frac{C_t}{D_t}(\tau_x + \Omega_x)[\frac{r\rho}{r-G} - \tau_x] + \tau_x\hat{V}_t > 0. \tag{82}$$

Lemma 2 proves that $D < 0$. That makes the middle term potentially negative. However, if $[\frac{r\rho}{r-G} - \tau_x] < 0$ as well, the product of this and D is positive. Thus the middle term is positive. That inequality can be rearranged as $\tau_x > \frac{r\rho}{r-G}$. Since the rest of the terms are squares and precisions, the rest of the expression is positive as well.

Thus if $\tau_x > \frac{r\rho}{r-G}$, then $\frac{\partial D_t}{\partial \Omega_t} > 0$. $\square$

Proof of Result 3, part 2

If $\tau_x > \rho r/(r-g)$ and $D_t < 0$, then $\partial D_t/\partial \Omega_{xt} > 0$.

Proof: Begin with the implicit function theorem: $\partial D_t/\partial \Omega_x = -\frac{\partial F}{\partial \Omega_x} / \frac{\partial F}{\partial D_t}$. The previous proof already proved that if $\tau_x > \frac{r\rho}{r-G}$, the denominator is positive. All that remains is to sign the numerator.

$$\frac{\partial F}{\partial \Omega_x} = 0 + 0 + \frac{r\rho}{(r-G)^2}\frac{\partial \hat{V}}{\partial \Omega_x} - \frac{1}{r-G}\frac{C_t}{D_t}\tau_x \frac{\partial \hat{V}}{\partial \Omega_x}$$

where $\partial \hat{V}/\partial \Omega_x = -\hat{V}^2(C^2)/(D^2)$. Substituting the partial of $\hat{V}$ into the partial of F yields

$$\frac{\partial F}{\partial \Omega_x} = \hat{V}^2 \frac{C^2}{D^2}\left(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\right).$$

Combining terms,

$$\frac{\partial D_t}{\partial \Omega_x} = -\frac{\hat{V}^2 \frac{C^2}{D^2}\left(-\frac{r\rho}{(r-G)^2} + \frac{1}{r-G}\frac{C_t}{D_t}\tau_x\right)}{\frac{\partial F}{\partial D_t}}$$

We know from lemmas 1 and 2 that $\frac{C_t}{D_t} < 0$. Since $r > G$, by assumption, $\partial F/\partial \Omega_x$ is negative (i.e., the $\frac{C^2}{D^2}$ factor does not change the sign). Applying the implicit function theorem tells us that $\partial D_t/\partial \Omega_{xt} > 0$. $\square$

Proof of Result 5

The strategy for proving this result is to apply the implicit function theorem to the price coefficients that come from coefficient matching in the market-clearing equation. After equating supply and demand and matching all the coefficients on $\tilde{x}_t$, we arrive at (10). Rearranging that equation gives us the expression for C_t/D_t in (63). If we subtract the right side of (63) from the left, we are left with an expression that is equal to zero in equilibrium, which we'll name F:

$$F = \frac{C_t}{D_t} - \frac{\hat{V}_t^{-1} - \tau_0}{\tau_x \frac{C_t}{D_t} - \frac{\rho r}{(r-G)} - Z_t \hat{V}_t^{-1}}$$

We compute $\frac{\partial C/D}{\partial \Omega_x} = -\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial \Omega_x}$ and $\frac{\partial C/D}{\partial \Omega_f} = -\left(\frac{\partial F}{\partial C/D}\right)^{-1}\frac{\partial F}{\partial \Omega_f}$. In particular, we have:

$$\begin{aligned}
\frac{\partial F}{\partial C/D} &= 1 - \left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-1} \\
&\quad + (\hat{V}^{-1} - \tau_0)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-2}\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right) \\
&= 1 - \left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-2} \\
&\quad \left[\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) - (\hat{V}^{-1} - \tau_0)\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right)\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial F}{\partial \Omega_f} &= -(1)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-1} + (\hat{V}^{-1} - \tau_0)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-2}(-Z_t) \\
&= -\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-2}\left[\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0)\right]
\end{aligned}$$

We notice that $\frac{\partial F}{\partial \Omega_x} = \left(\frac{C_t}{D_t}\right)^2 \frac{\partial F}{\partial \Omega_f}$ since

$$\frac{\partial F}{\partial \Omega_x} = \frac{\partial F}{\partial \hat{V}^{-1}}\frac{\partial \hat{V}^{-1}}{\partial \Omega_x} = \frac{\partial F}{\partial \hat{V}^{-1}}\left(\frac{C_t}{D_t}\right)^2 \frac{\partial \hat{V}^{-1}}{\partial \Omega_f} = \left(\frac{C_t}{D_t}\right)^2 \frac{\partial F}{\partial \Omega_f}$$

.

Then:

$$\frac{\partial C/D}{\partial \Omega_f} = \frac{\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0)}{\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^2 - \left[\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) - (\hat{V}^{-1} - \tau_0)\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right)\right]} \tag{83}$$

Result 5, part 1: If $C/D \leq 0$, $\Omega_x < \tau_0 + \Omega_f$ and $C/D > -Z_t/2$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$

The numerator of (83) is

$$\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0) = \tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \tau_0 < 0$$

The inequality holds since we've proven that $C_t/D_t < 0$ and $r > G$.

In the denominator, however, not all the terms are negative. The denominator of (83), divided by by $\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) + Z_t(\hat{V}^{-1} - \tau_0)$ is:

$$\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right) - \left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right) + (\hat{V}^{-1} - \tau_0)\left(\tau_x - Z_t\left(2\frac{C_t}{D_t}(\tau_x + \Omega_x)\right)\right)\left(\tau_x \frac{C_t}{D_t} - \frac{\rho r}{r-G} - Z_t \hat{V}^{-1}\right)^{-1} \tag{84}$$

The only positive term is $-2\frac{C_t}{D_t}\Omega_x$. Then, is it easy to see that if C/D is sufficiently close to zero, then $-2\frac{C_t}{D_t}\Omega_x < \frac{\rho r}{r-G} + Z_t(\tau_0 + \Omega_f)$, so (84) is negative.

Thus, the numerator is negative and if C/D is sufficiently close to zero the denominator is positive, so $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} = \left(\frac{C_t}{D_t}\right)^2 \frac{\partial C/D}{\partial \Omega_f} < 0$ if $C/D < 0$ and $\frac{\partial C/D}{\partial \Omega_x} = 0$ if $C/D = 0$. □

Proof of Result 5, part 2 *Claim: If $C/D \leq 0$, and $C/D < -\frac{2Z_t^{-1}}{3}$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$*

To see this, we analyze if under these new condition inequality (84) holds. We have:

$$\begin{aligned} & -\frac{\rho r}{r-G} - Z_t(\tau_0 + \Omega_f) - 2\frac{C_t}{D_t}\Omega_x - 3Z_t\left(\frac{C_t}{D_t}\right)^2(\tau_x + \Omega_x) \\ = & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2 - 3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \end{aligned}$$

So if $C/D < -\frac{2Z_t^{-1}}{3}$, we can prove the above claim:

$$\begin{aligned} = & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - \frac{C_t}{D_t}\Omega_x\left(2 - 3Z_t\frac{C_t}{D_t}\right) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\ < & -\frac{\rho r}{r-G} - Z_t(\Omega_x) - 3Z_t\left(\frac{C_t}{D_t}\right)^2\tau_x \\ < & \ 0 \end{aligned}$$

Now, combining the two previous claims, we have that if $\Omega_x < \tau_0 + \Omega_f$ and $Z_t > \frac{1}{\sqrt{3}}$, then $\frac{\partial C/D}{\partial \Omega_f} < 0$ and $\frac{\partial C/D}{\partial \Omega_x} \leq 0$. The condition $Z_t > \frac{1}{\sqrt{3}}$ implies that $\frac{-Z_t}{2} < -\frac{2Z_t^{-1}}{3}$ so with claims 3, 4 and 5 we have guaranteed the result for the entire support of C/D and thus proved result 5.

Proof of Result 6a: Ω_f/Ω_x does not converge to 0

If Ω_f/Ω_x converges to ∞, then by the first order condition, it must be that $\xi \to \infty$. It is sufficient to show that $\xi \to \infty$ violates equation (61). Rearrange (61) to get

$$\left[\xi Z_t\left(\xi^2\tau_x + (\frac{K}{\chi_f})^{\frac{1}{2}}(1 + \frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} + \tau_0\right) + \xi\alpha\right] + (\frac{K}{\chi_f})^{\frac{1}{2}}(1 + \frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = 0 \tag{85}$$

The term in square brackets is negative and the one outside is positive. Assume $\xi \to \infty$. If Z_t does not go to zero, then the negative term grows faster and the equality cannot hold. So it must be that $Z_t \to 0$. Using equation (29) of the draft, that requires that both $C_{t+1} \to 0$ and $D_{t+1} \to 0$. In order for C_{t+1} to go to zero, $\hat{V} \to \tau_0^{-1}$. But since $\xi \to \infty$, from equation (27) in the main draft, $\hat{V} \to 0$, which is a contradiction.

Proof of Result 6b: As $K \to \infty$, Ω_f/Ω_x does not converge to ∞

If Ω_f/Ω_x did converge to ∞ as $K \to \infty$, then by the first-order condition (57), it would have to be that $\xi \to 0$. So it suffices to show that $\Omega_f/\Omega_x = \infty$ is inconsistent with $\xi = 0$, in equilibrium.

Start from the equilibrium condition (60), which must be zero in equilibrium. If $\xi \to 0$, then the first term goes to zero. The proof of lemma 4 proves, along the way, that $(1 + \xi Z_t) > 0$. (Otherwise, (60) can never be zero because it is always negative.) Thus the second term $\Omega_x \xi^2(1 + \xi Z_t)$ must be non-negative.

The third term $\Omega_f(1 + \xi Z_t)$ also converges to ∞ because $\Omega_f \to \infty$ and $(1 + \xi Z_t) > 0$. How do we know that $\Omega_f \to \infty$? In principle, Ω_f/Ω_x could become infinite either because Ω_f became infinite or because Ω_x goes to zero. But if Ω_x goes to zero and Ω_f is finite, then the information processing constraint (3), which requires that the weighted sum of Ω_f and Ω_x be K cannot be satisfied as $K \to \infty$.

Since one term of (60) becomes large and positive and the other two are non-negative in the limit, the sum of these three terms cannot equal zero. Therefore, $\Omega_f/\Omega_x \to \infty$ cannot be an equilibrium.

Proof of Result 6c: there exists an equilibrium where Ω_f/Ω_x converges to a constant.

By the first order condition (57), we know that Ω_f/Ω_x converges to a constant, if and only if ξ converges to a constant. Thus, it suffices to show that there exists a constant ξ that is consistent with equilibrium, in the high-K limit.

Suppose ξ and Z_t are constant in the high-K limit. In equation (61) as $K \to \infty$, the last term goes to infinity, unless $\xi \to \frac{1}{Z_t}$. If the last term goes to infinity and the others remain finite, this cannot be an equilibrium because equilibrium requires that the left side of (61) is zero. Therefore, it must be that $\xi \to \frac{-1}{Z_t}$. The question that remains is whether ξ and Z_t are finite constants, or whether one explodes and the other converges to zero, in the high-K limit.

Suppose $\xi = -\frac{1}{Z_t}$, which is constant ($\xi = \bar{\xi}$). Then $Z_t = \bar{Z}$ is constant too. The rest of the proof checks to see if such a proposed constant- $\bar{\xi}$ solution is consistent with equilibrium. We do this by showing that ξ does not explode on contract as K increases. In other words, for $\xi = \frac{-1}{Z_t}$ to be stable and thus the ratio of fundamental to technical analysis to be stable, we need that $\partial\xi/\partial K \to 0$, in other words, ξ and therefore Ω_f/Ω_x converges to a constant as $K \to \infty$.

Step 1: Derive $d\xi/dK$: Start from the equilibrium condition for ξ (61) and apply the implicit function theorem:

$$\Big(3Z_t\tau_x\xi^2 + A + Z_t\tau_0\Big)d\xi + \frac{1}{2}(\frac{1}{K\chi_f})^{\frac{1}{2}}(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}dK$$
$$+\left[\frac{1}{2}(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{-\frac{1}{2}}(4\frac{\chi_f}{\chi_x}\xi^3) + Z_t(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}\right]d\xi = 0$$

So we have

$$\frac{d\xi}{dK} = \frac{1}{2}(\frac{1}{K\chi_f})^{\frac{1}{2}}\frac{-(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}}{3Z_t\tau_x\xi^2 + A + Z_t\tau_0 + 2\frac{\chi_f}{\chi_x}(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{-\frac{1}{2}}\xi^3 + Z_t(\frac{K}{\chi_f})^{\frac{1}{2}}(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}}$$

Use equation 61 to write the numerator as

$$(1+\xi Z_t)(1+\frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = -(\frac{\chi_f}{K})^{\frac{1}{2}}\xi(\xi^2 Z_t\tau_x + A + Z_t\tau_0) \tag{86}$$

Now use this to rewrite $\frac{d\xi}{dK}$ as

$$\frac{d\xi}{dK} = \frac{1}{2K}\frac{1}{\frac{3Z_t\tau_x\xi^2 + A + Z_t\tau_0}{\xi(\xi^2 Z_t\tau_x + A + Z_t\tau_0)} - 2\frac{\chi_f}{\chi_x}(1+\frac{\chi_f}{\chi_x}\xi^4)^{-1}\xi^3 - \frac{Z_t}{(1+\xi Z_t)}} \tag{87}$$

Step 2: Show that $d\xi/dK \to 0$ as $K \to \infty$, as long as $X(\cdot) \not\to 0$

As $K \to \infty$, it is clear that $1/2K \to 0$. As long as the term that multiplies $1/2K$ stays finite, the product will converge to zero. Since the numerator is just 1, the second term will be finite, as long as the denominator does not go to zero. Define

$$X(\xi, Z_t) = \frac{3Z_t\tau_x\xi^2 + A + Z_t\tau_0}{\xi(\xi^2 Z_t\tau_x + A + Z_t\tau_0)} - 2\frac{\chi_f}{\chi_x}(1 + \frac{\chi_f}{\chi_x}\xi^4)^{-1}\xi^3 - \frac{Z_t}{(1+\xi Z_t)} \tag{88}$$

which is the denominator of the second fraction on the rhs of equation (87). Then if $X \not\to 0$, $1/X$ is finite, then $1/2K * 1/X$ goes to zero as K gets large. Thus, we get that $\partial\xi/\partial K \to 0$ as $K \to \infty$.

Step 3: $X(\cdot) \not\to 0$.
To complete the proof, we need to show that $\bar{\xi} = -\frac{1}{Z}$ which satisfies the equilirium condition (93) as $K \to \infty$, does not cause $X(\cdot) = 0$. We can check this directly: in equation (88), if $\xi = -\frac{1}{Z_t}$, the denominator of the last term becomes zero; so last term becomes infinite. The only term in (88) with opposite sign is the middle term, which is finite if $\xi = \frac{C}{D}$ is finite (the running assumption). If the last term of X tends to infinity and the only term of opposite sign is finite, the sum cannot be 0. Thus, for $\bar{\xi} = -\frac{1}{Z}$, which is the limit attained in the limit as $K \to \infty$, we have that $X(\bar{\xi}) \neq 0$.

Step 4: As $K \to \infty$, if (94) holds, the real, finite-ξ solution exists.

From equation (27-30) in the main draft, as $K \to \infty$ at least one of the two information choices goes to ∞, so with finite, non-zero $\frac{C}{D}$:

$$\lim_{K\to\infty} \hat{V} = 0 \tag{89}$$

$$\lim_{K\to\infty} \Omega_t^{-1} = \frac{r}{\rho(r-G)} Z_t = D_{t+1}^2(\xi_{t+1}^2\tau_0^{-1} + \tau_x^{-1}) \tag{90}$$

$$\lim_{K\to\infty} D_t = -\frac{\rho}{r}\Omega_t^{-1} = -\frac{1}{(r-G)} Z_t \tag{91}$$

A word of interpretation here: Equation (30), which defines Ω^{-1} is the total future payoff risk. As $\hat{V} \to 0$, it means the predictable part of this variance goes away as information capacity gets large. Z_t, which is the unpredictable part, remains and governs liquidity, D_t.

Next, solve (90) for D_{t+1}, backdate the solution 1 period, to get an expression for D_t, and equate it to the expression for D_t in (91). This implies that $\lim_{K\to\infty} D = \bar{D}$ is constant and equal to both of the following expressions

$$\bar{D}^2 = \frac{-rZ_t}{\rho(r-G)\bar{\xi}(\bar{\xi}^2\tau_0^{-1} + \tau_x^{-1})} = \frac{Z_t}{(r-G)^2\bar{\xi}^2} \tag{92}$$

We can cancel Z_t on both sides, which delivers a quadratic equation in one unknown in $\bar{\xi}$:

$$\bar{\xi}^2\tau_0^{-1} + \frac{r(r-G)}{\rho}\bar{\xi} + \tau_x^{-1} = 0. \tag{93}$$

In order for $\bar{\xi}$ to exist equation (93) requires that the expression inside the square root term of the quadratic formula (often written as $(b^2 - 4ac)$) not be negative. This imposes the parametric restriction

$$\left(\frac{r(r-G)}{\rho}\right)^2 - 4\tau_0^{-1}\tau_x^{-1} \geq 0. \tag{94}$$

Rearranging this to put τ_0 on the left delivers $\tau_0 \geq \underline{\tau}$, where $\underline{\tau} = 4\tau_x^{-1}\rho^2(r(r-G))^{-2}$.

Lemma 4 *$|D_t| \geq \frac{\rho(r-G)}{r} C_t \left(C_{t+1}^2\tau_0^{-1} + D_{t+1}^2\tau_x^{-1}\right)$, with strict inequality if $K > 0$.*

Proof. Use equation (61) to write

$$(1+\xi Z_t)(1 + \frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}} = -(\frac{\chi_f}{K})^{\frac{1}{2}}\xi(\xi^2 Z_t\tau_x + \alpha + Z_t\tau_0) \tag{95}$$

Since we've proven that $\xi \leq 0$ (lemma 2). And we know from lemma 1 that if $K > 0$, then $C_t > 0$ so that $\xi < 0$ with strict inequality. The other terms on the right side are strictly positive squares or positive constants, with a negative sign in front. Thus, the right hand side of the equation (95) is positive. On the left, since $(1 + \frac{\chi_f}{\chi_x}\xi^4)^{\frac{1}{2}}$ is a square root, and therefore positive, this implies that $(1 + \xi Z_t)$ must be positive as well for the equality to hold. $(1 + \xi Z_t) > 0$ implies that $Z_t < -1/\xi$ Substitute for Z_t to get the result. This result puts a bound on how liquid the price can be. The liquidity is bounded by the product of price informativeness and un-learnable, future risk. ∎

C Numerical Results

Lower risk aversion The steady state coefficients with low risk aversion $\rho = 0.05$ are We find $A_T = 16.03$, $C_T = 7.865$ and $D_T = -3.0$. A_T and C_T are unchanged, while D_T changed from $= -5.7$, for high risk aversion to 3.0.

Similarly, after re-calibrating, risk aversion makes only a minor difference. With $\rho = 0.05$, order flow analysis still outstrips fundamental analysis between periods 4 and 5. But if falls slightly more slowly. The ending value of Ω_f is 1.8, instead of 1.6.

Figure 7: Similar Results with Lower Risk Aversion ($\rho = 0.05$)

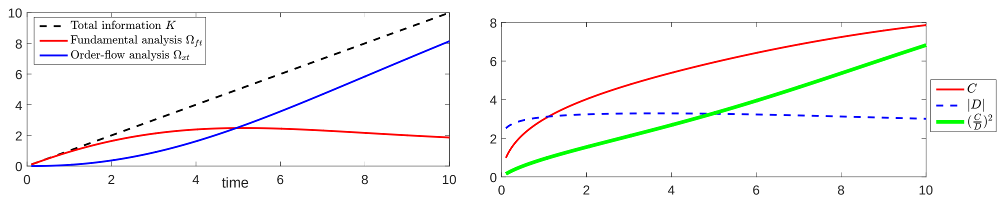
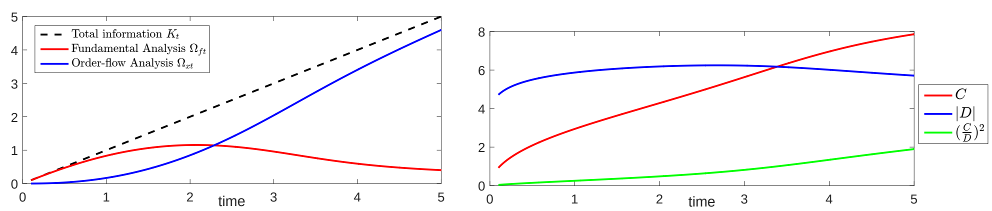


Figure 8: Similar Results with Lower Terminal Capacity ($K_t = 5$)



Lower terminal capacity

D Data Appendix

D.1 Hedge Fund Data

Lipper TASS Database **?** provides performance data on over 7,500 actively reporting hedge funds and funds of Hedge Funds and also provides historical performance data on over 11,000 graveyard funds that have liquidated or stopped reporting. In addition to performance data, data are also available on certain fund characteristics, such as investment approach, management fees, redemption periods, minimum investment amounts and geographical focus. This database is accessible from Wharton Research Data Services (WRDS).

Data Overview and Word of Caution Though the database provides a comprehensive window into the hedge fund industry, data reporting standards are low. There is a large portion of the industry (representing about 42% of assets) that simply do not report anything (Edelman, Fund, and Hsieh, 2013). Reporting funds regularly report only performing assets (Bali, Brown, and Caglayan, 2014). While any empirical analysis must be considered with caution, some interesting stylized facts about the current state and evolution of the hedge fund industry do exist in these data.

Data Description

All data is monthly and come from Lipper TASS. In total, the database reports on 17,534 live and defunct funds. Data are from 1994-2015, as no data was kept on defunct funds before 1994. A significant portion of this total consists of the same fund reported in different currency and thus are not representative of independent fund strategies (Bali, Brown, and Caglayan, 2014). Therefore, I limit the sample to only USD-based hedge funds. I also remove funds of funds. This limits the sample size to 10,305 funds. As the focus is to gain insight into the division between fundamental and quantitative strategy in the market, I further limit the sample to funds who explicitly possess these characteristics (which I explain below). This further limits the sample to 7093 funds. Firms are born and die with surprising regularity throughout the sample, highlighting the transient nature of the industry, and thus there are never more than 3000 existing, qualifying funds at any point in time. By the end of 2015, there were just over 1000 qualifying funds.

Lipper TASS records data on each fund's investment strategies. In total, there are 18 different classifications and most of these classifications have qualities of both fundamental and quantitative analysis.[8] However, 4 strategy classifications explicitly denote fund strategy as being fundamental or quantitative. They are:

- **Fundamental**: This denotes that the fund's strategy is explicitly based on fundamental analysis.
- **Discretionary**: This denotes that the fund's strategy is based upon the discretion of the fund's manager(s).
- **Technical**: This denotes that the fund deploys a technical strategy.
- **Systematic Quant**: This denotes that funds deploy technical/algorithmic strategy.

Using these classifications, it is possible to divide hedge fund strategy into three broad groups:

- **Fundamental**: Those funds whose strategy is classified as fundamental and/or discretionary, and *not* technical and/or sytematic quant.
- **Quantitative**: Those funds whose strategy is classified as technical and/or systematic quant, and *not* technical and/or sytematic quant.
- **Mixture**: Those funds whose strategy is classified as having at least one of fundamental or discretionary and at least one of technical or systematic quant.

From 2000-2015, the AUM has systematically shifted away from fundamental firms to firms that deploy some sort of quantitative analysis in their ivnestment approach. In mid-2000, the AUM per fundamental firm was roughly 8 times the size of that in a quantitative or mixture firm, but this had equalized by 2011, representing a true shift away from fundamental analysis and towards quantitative analysis in the hedge fund industry.

[8] An example of a strategy that could be considered both, "Macro: Active Trading strategies utilize active trading methods, typically with high frequency position turnover or leverage; these may employ components of both Discretionary and Systematic Macro strategies"