

# RECENT DEVELOPMENTS IN DYNAMIC CAPITAL STRUCTURE

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## Early “barrier” structural models of debt and capital structure

### 1. *Debt maturity is exogenously given (or infinite):*

- But choice of *optimal maturity is a key part of financial choice*: affects default, optimal leverage, and the extent of agency costs

### 2. *Debt has perfect liquidity*

- But Huang & Huang (2003, 2012) noted that *liquidity costs* needed to explain bond spreads (but didn't explicitly introduce; a residual)
- Agency costs may also explain spreads; costs increase with maturity

### 3. *Dynamics: assumed both*

- Full roll-over of short term debt with constant maturity; *and* prior debt must be fully retired if additional debt issued. Relaxed by
  - Hackbarth & Mauer, “Optimal Priority, Capital Structure, and Investment (2012)
  - Admati et al on “The Leverage Ratchet Effect” (ADHP, 2013-16),
  - Dangl & Zechner “Debt Maturity and Dynamics of Leverage (DZ, 2016),
  - DeMarzo & He “Leverage Dynamics without Commitment” (DH, 2016)

## OUR GOALS:

- Develop a model with (almost) closed form solutions that includes
  - A simple jump-diffusion process for firm value
  - Endogenous maturity choice as well as leverage
  - An illiquidity (or agency cost) premium for corporate debt
  - Extension of static model to dynamic (preliminary)

*Earlier models have introduced subsets of these aspects but not all, particularly illiquidity which affects optimal maturity*

- *Extension of model I presented in Princeton Lectures in Finance (2006)*  
<https://www.princeton.edu/bcf/newsevents/events/lectures-in-finance/>
- Use this model to consider
  - **Optimal leverage and debt maturity choice**,  
and their joint sensitivity to exogenous parameters
  - **Debt dynamics** (particularly “Ratcheting” of Admati et al. ) and  
interaction of original maturity choice and restructuring levels

## 1. A simple risk-neutral jump-diffusion process for firm value

Value of after-tax unlevered cash flows follows a simple Jump-Diffusion process with “rare” disaster (e.g. Barro (2006)):

$$\begin{aligned}dV_t &= \mu V_t dt + \sigma V_t dZ_t && \text{if no jump at or prior to } t \\ &= -kV_{t-} && \text{if jump at } t_- \text{ and firm defaults}\end{aligned}$$

- Diffusion is standard log diffusion process with constant mean, risk
- Jump occurs with constant intensity  $\lambda$  ( $\approx 0.60\%$ ; physical  $\approx 0.24\%$ )
- Later assume  $k = 1$  (“total disaster”) to simplify math
- Implies  $\mu = (r - \delta + \lambda k)$ , where payouts (interest + dividends) =  $\delta V_t$
- Same as simplest case in Merton (1976)

We calibrate  $\lambda$  to match Moody’s default statistics 1970-2010 and recovery rates on bonds; also calibrate to 1920-2011 default rates

- The jump process to default is similar to a “reduced form” approach --but there’s also diffusion to an endogenous default barrier

*Main role of jump here:* short term rates, defaults don’t go to zero

## 2. Finite average maturity debt

We use the standard “exponentially declining” debt model (Leland (1994))

- Firm issues debt with principal  $P$  and a coupon rate  $C$ , infinite life
  - At initiation ( $t = 0$ ),  $C$  set so debt sells at par:  $D(V(0)) = P$
- A constant fraction  $m$  of outstanding debt principal is retired through time (e.g. through a sinking fund)
  - Debt extant at  $t = 0$  receives cash flow  $e^{-mt}(C + mP)dt$  at  $t$
  - Average life of debt  $T = 1/m$  (half-life =  $0.69/m$ )
  - Retired debt is fully replaced continuously, but new debt sold at then-current market value  $D(V(t))$  -- *Dangl-Zechner* modifies
- Default occurs when  $V(t)$  falls to  $V_B$ , set by  $\frac{dE(V)}{dV} \Big|_{V=V_B} = 0$ 
  - In default, debt receives  $(1 - \alpha)V_B$
  - $V_B$  depends both on leverage AND maturity
- **A Problem:** In virtually all studies,  $m = 0$  optimizes firm value
  - Infinite life debt minimizes debt service, default

### 3. (II)liquidity of debt: The “Credit Spread Puzzle”

- Huang & Huang (2003; 2012): Historical default rates and risk premia cannot account for the size of spreads
  - Confirmed by Longstaff, Mithal, & Neis (2005) who estimate a “Non-Default Component of spread” across ratings of 50-72 bps
  - Strongly related to illiquidity measures.
  - Strong maturity effect: increases 3 bps/yr of maturity
  - Spread could also reflect agency costs: increase with debt maturity (e.g. Myers (1977), Leland & Toft (1996), Hackbarth & Mauer (2012))
  
- We introduce a simple additional “*liquidity discount*” to corporate bonds: their payouts are discounted at  $r_f + h$ 
  - $h = h_0 + h_1 * T$  ( $T$  = average debt maturity)
  - Higher discount rate disadvantages longer-term debt
  - Optimal maturity balances this disadvantage with lower debt service advantage (lower endogenous default barrier)

## Calibration

$r$	Riskfree Rate	5.00%	(HH)		$\alpha$	Default Costs	30.0%	(AK; Glover)
$\delta$	Payout Rate	4.00%	(HH)		$\tau$	(Net) Tax Rate	20.0%	
$\sigma$	Asset Std Dev	24.00%	(FS)			Debt Issuance Costs	1.00%	
$\lambda$	Jump Risk	0.60%	*			Debt Rollover Costs	0.25%	
	Q-Diffusion Return	1.60%				Liquidity Discount (bps)	40 + 3*T	(LMN)
	Risk Premium on Assets	4.00%	(based on 6% equity risk premium, 33% leverage)					
		*RN rate, calibrated to default. Physical = RN/2.5 (Driessen 2004) = 0.24%						
HH:Huang/Huang (2012); FS:Feldhutter/Schaefer (2016); AK:Andrade/Kaplan (1998);Glover (2016); LMN: Longstaff/Mithal/Neis (2005)								

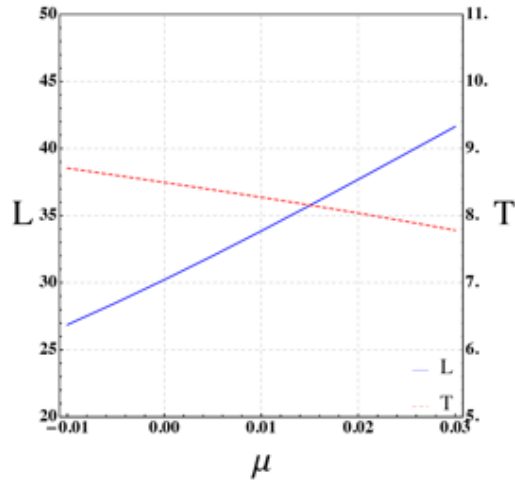
**Model Outputs** (Closed form solutions for debt and equity values, simple search for optimal leverage and maturity—easily implemented in Excel! Model on Conference site)

<b>Optimal Leverage</b>	<b>34%</b>	(vs. median SF 36%; HH = 43%)							
<b>Average Maturity (T = 1/m)</b>	<b>8.3 yrs</b>	Half life = 5.74 yrs $\approx$ 11.5 yr regular debt maturity (Choi et al. 2016)							
<b>Yield Spread</b>	159 bps	(vs. median BBB spread SF= 148bps; HH = 158-194)							
<b>Recovery Rate</b>	51%	(vs. HH 51.3%; SF 37%)							
		<b>Historical vs. Predicted Default Rates - Moody's 1970-2010</b>							
<b>Years after Issuance</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>5</b>	<b>7</b>	<b>10</b>	<b>12</b>	<b>15</b>	<b>20</b>
<b>Historical Default Probs</b>	0.18%	0.51%	0.93%	1.95%	3.03%	4.90%	6.37%	8.85%	12.41%
<b>Model Default Probs</b>	0.24%	0.48%	0.76%	1.65%	2.97%	5.33%	6.93%	9.23%	12.63%

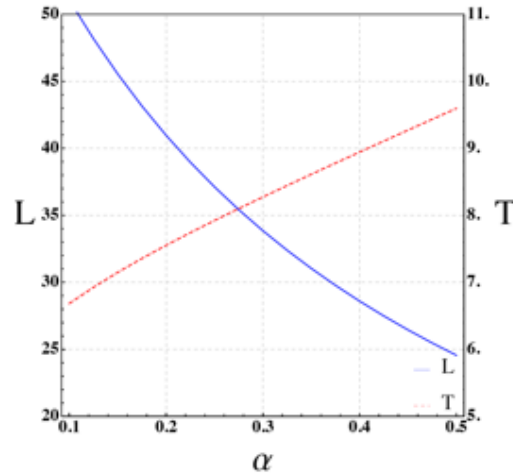
(See Appendix for calibration to Moody's 1920-2011 Default rates)

**COMPARATIVE STATICS: Leverage L % in blue; Maturity T yrs. in red)**

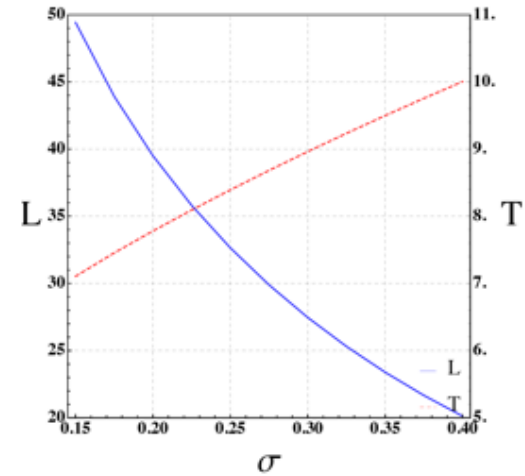
**Growth Rate  $\mu$**



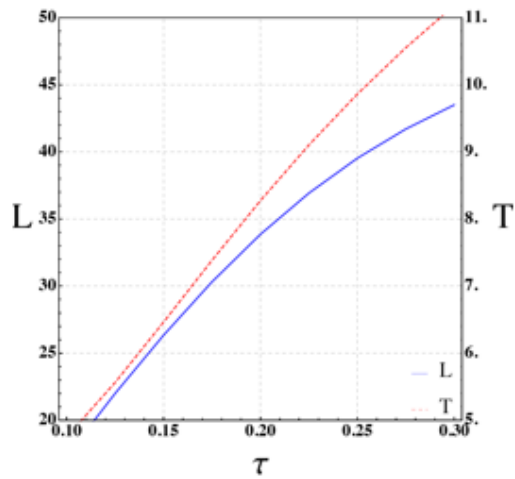
**Default Costs  $\alpha$**



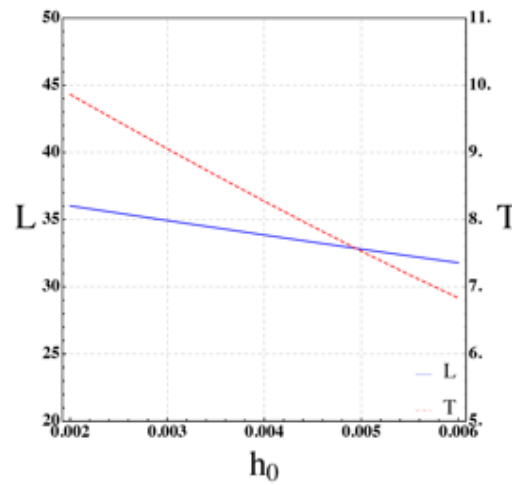
**Asset Riskiness  $\sigma$**



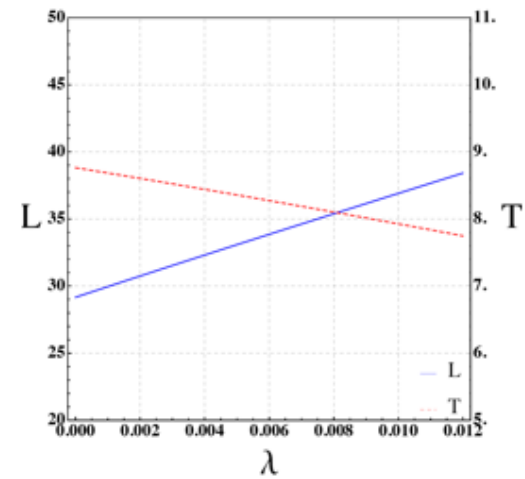
**Tax Rate  $\tau$**



**Liquidity/Agency Cost  $h_0$**



**Jump Intensity  $\lambda$**





## Application 1: The Capital Structure of Growth Firms

**Contention:** *Growth firms (high MB ratio) have less leverage, shorter maturity*

- Smith and Watts (SW, 1992): regress leverage on MB, negative relation
- Barclay & Smith (BS, 1995): regress maturity on MB, negative relation
  - But both studies consider M/B, which reflects endogenous leverage
  - Their independent variables don't include risk  $\sigma$  or default costs  $\alpha$

**...MB is not a direct input in our model--it reflects endogenous firm value M**

How do we describe a “growth firm”? Any or all of the below:

- **High drift  $\mu$ :** more investment opportunities, less payouts
  - Shorter maturity but higher leverage
- **High default costs (“alpha”):** lost growth opportunities (high MB)
  - Longer maturity but lower leverage
  - Could also be related to **(low) asset tangibility** (Rampini & Viswanawathan (2013)):  
--Leverage decreases with fewer tangible assets

➤ **High risk (“sigma”)**

- **Longer maturity** but **lower leverage**

➤ **Low effective tax rates (“tau”)**

- **Shorter maturity and lower leverage**

➤ **Large Illiquidity or agency costs (“h”)**

- **Shorter maturity and lower leverage**

					Optimal Leverage (%)	Average (1/m) Maturity (yrs)	Yield Spread	5-yr Default Probability
				<b>BASE CASE:</b>	<b>33.9%</b>	<b>8.3</b>	<b>159</b>	<b>1.65%</b>
		<b>Base</b>		<b>New</b>				
	<b>Growth Rate (Mu)</b>	1.00%	==>	4.00%	45.6%	7.5	165	2.80%
	<b>Default Costs (Alpha)</b>	30.00%	==>	80.00%	16.6%	11.5	152	1.20%
	<b>Asset Risk (Sigma)</b>	24.00%	==>	50.00%	15.3%	10.9	295	5.23%
	<b>Effective Tax Rate (tau)</b>	20.00%	==>	5.00%	6.3%	3.5	111	1.19%
	<b>Total liquidity discount (h), bps</b>	65.00	==>	100.00	29.5%	5.3	180	1.50%
		<b>ALL OF THE ABOVE</b>			0.00%	-	-	-

## Application 2: The “Ratchet Effect” Admati et al. (2016)

**CLAIM: After initial debt is issued to maximize total firm value,**

- Equity holders will *never* find it advantageous to reduce leverage
  - Debt reduction decreases equity value—even when it increases firm value
  - Debt increases help equity by devaluing older debt—even with absolute priority
  - Agency problem similar to “asset substitution”, where greater volatility helps equity at expense of debtholders and firm value *if no precommitment*
  
- Leverage will therefore increase monotonically--eventually to point where *tax benefits are entirely exhausted*
  - **Only taxes determine ultimate leverage, no default cost tradeoff**
  - Generalizes a local result in Leland (1994), who also suggests that ***debt can be reduced only by a re-negotiation of initial-debt terms***
  - Unlike Fischer, Heinkel, Zechner (1989) and Goldstein Ju Leland (2001), they do not require current debt must be fully retired before new issue

## ADHP model quite simplistic:

- Jump-only process, no endogenous default, zero recovery in bankruptcy
- Nonetheless, we can examine their conclusions by using our model (v. 1) (assuming absolute priority of prior debt; no foresight)

ROUNDS	Optimal Leverage	% Equity Increase	Total Firm Value	Coupon	Principal	Average Maturity T	Total Debt Principal	Credit Spread (bps)
1	33.85	3.356	103.36	2.34	34.98	8.64	34.98	169
2	37.74	0.055	103.32	0.31	4.03	27.81	39.02	262
3	41.13	0.048	103.22	0.28	3.53	26.97	42.54	282
4	44.13	0.043	103.09	0.25	3.11	26.02	45.66	301
5	46.78	0.038	102.93	0.23	2.75	25.03	48.41	320
6	49.11	0.033	102.75	0.20	2.42	24.05	50.82	338
7	51.14	0.028	102.56	0.18	2.11	23.12	52.93	356
8	52.89	0.023	102.37	0.16	1.82	22.26	54.75	372
9	54.40	0.019	102.19	0.14	1.56	21.49	56.31	387
10	55.67	0.015	102.02	0.12	1.33	20.80	57.64	400

## LOOKS CONCERNING! BUT NOTE AFTER INITIAL DEBT ISSUANCE:

- New rounds create tiny increases in equity value (5.5 down to 1.5 bps); and have increasing spreads (likely to be rated lower than original debt)
- Very long maturities: calibrated to half-life, bond maturities  $\approx$  30-40 yrs

- **More realistically, limit  $T$  max to 15** (regular bond maturity  $\approx$  20 yrs)

ROUNDS	Optimal Leverage	% Equity Increase	Total Firm Value	Coupon	Principal	Average Maturity T	Total Debt Principal	Credit Spread (bps)
1	33.85	3.356	103.36	2.34	34.98	8.64	34.98	169
2	36.80	0.038	103.32	0.23	3.07	15.00	38.05	245
3	38.56	0.014	103.27	0.14	1.82	15.00	39.87	256

- **Perhaps more realistically, limit  $T$  to original debt  $T$**  (maturity  $\approx$  12 yrs)

ROUNDS	Optimal Leverage	% Equity Increase	Total Firm Value	Coupon	Principal	Average Maturity T	Total Debt Principal	Credit Spread (bps)
1	33.85	3.356	103.36	2.34	34.98	8.64	34.98	169
2	34.94	0.006	103.33	0.08	1.13	8.86	36.11	217

- ***Even one subsequent issue yields less than 1 bp increase in equity value!***  
(Even less new debt if foresight of subsequent rounds)

## CONCLUSIONS

**Conclusions re Ratcheting:** In the context of our model

- ADHP correct that firm will never want to reduce debt
  - But see Dangl-Zechner (2016), who argue **short term debt precommits to debt reduction** through repayment of principal
  - It may not be optimal for firm to fully roll over expiring debt
- Given miniscule equity gains (*3.8 bps or less*) and likely lower bond ratings, *the ADHP concern of **additional debt issuance seems very unlikely***

### More Generally

- We have developed a model that allows the ***joint determination of leverage and maturity***
  - Leverage affects maturity, and vice-versa
  - Considering either in isolation can create misleading results

## Appendix: Fitting the Model to 1920-2011 Default Data

**Feldhutter & Schaefer (2016):** When fitted to *long-term* default data, structural models don't need jumps, liquidity discounts to match spreads. No "Credit Puzzle"!

- But calibration with Black-Cox model  $\rightarrow T = \infty$ . No optimization
- Need jumps to explain short term default, spreads

### Our model can match 1920-2011 data with minor Input Changes

- (i) Increase jump frequency:  $\lambda = 0.006 \rightarrow 0.011$  (still "rare")
  - (ii) Increase liquidity discount:  $h = 40+3*T \rightarrow h = 70+3*T$  bps
  - (iii) All other inputs remain same ( $\sigma, \delta, \alpha$ )
- **Optimal Leverage  $\approx$  constant: 34%** (Graham et al. 2012 w/preferred stock)
  - **Optimal Maturity falls from 8.3 years to 5.9 years** (Choi et al. 2016)

### *Default Frequencies now match 1920-2011 data quite well*

		Historical vs. Predicted Default Rates - Moody's 1920-2011									
Years after Issuance		1	2	3	5	7	10	12	15	20	
Historical Default Probs		0.29%	0.86%	1.55%	3.09%	4.63%	7.03%	8.62%	10.81%	13.63%	
Model Default Probs		0.44%	0.88%	1.36%	2.63%	4.24%	6.88%	8.60%	11.03%	14.63%	