RECENT DEVELOPMENTS IN DYNAMIC CAPITAL STRUCTURE

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(Joint work with Dirk Hackbarth)*

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Early “barrier” structural models of debt and capital structure

1. Debt maturity is exogenously given (or infinite):
   ➢ But choice of optimal maturity is a key part of financial choice:
     affects default, optimal leverage, and the extent of agency costs

2. Debt has perfect liquidity
   ➢ But Huang & Huang (2003, 2012) noted that liquidity costs needed to explain bond spreads (but didn’t explicitly introduce; a residual)
   ➢ Agency costs may also explain spreads; costs increase with maturity

3. Dynamics: assumed both
   ➢ Full roll-over of short term debt with constant maturity; and prior debt must be fully retired if additional debt issued. Relaxed by
     - Admati et al on “The Leverage Ratchet Effect” (ADHP, 2013-16),
     - Dangl & Zechner “Debt Maturity and Dynamics of Leverage (DZ, 2016),
     - DeMarzo & He “Leverage Dynamics without Commitment” (DH, 2016)
OUR GOALS:

- Develop a model with (almost) closed form solutions that includes
  - A simple jump-diffusion process for firm value
  - Endogenous maturity choice as well as leverage
  - An illiquidity (or agency cost) premium for corporate debt
  - Extension of static model to dynamic (preliminary)

Earlier models have introduced subsets of these aspects but not all, particularly illiquidity which affects optimal maturity


- Use this model to consider
  - Optimal leverage and debt maturity choice, and their joint sensitivity to exogenous parameters
  - Debt dynamics (particularly “Ratcheting” of Admati et al.) and interaction of original maturity choice and restructuring levels
1. A simple risk-neutral jump-diffusion process for firm value

Value of after-tax unlevered cash flows follows a simple Jump-Diffusion process with "rare" disaster (e.g. Barro (2006)):

\[
dV_t = \mu V_t dt + \sigma V_t dZ_t \quad \text{if no jump at or prior to } t
\]
\[
= -kV_{t-} \quad \text{if jump at } t_- \text{ and firm defaults}
\]

- Diffusion is standard log diffusion process with constant mean, risk
- Jump occurs with constant intensity \( \lambda \) (\( \approx 0.60\% \); physical \( \approx 0.24\% \))
- Later assume \( k = 1 \) ("total disaster") to simplify math
- Implies \( \mu = (r - \delta + \lambda k) \), where payouts (interest + dividends) = \( \delta V_t \)
- Same as simplest case in Merton (1976)

We calibrate \( \lambda \) to match Moody’s default statistics 1970-2010 and recovery rates on bonds; also calibrate to 1920-2011 default rates

➢ The jump process to default is similar to a “reduced form” approach
  --but there’s also diffusion to an endogenous default barrier

*Main role of jump here:* short term rates, defaults don’t go to zero
2. Finite average maturity debt

We use the standard “exponentially declining” debt model (Leland (1994))

- Firm issues debt with principal $P$ and a coupon rate $C$, infinite life
  - At initiation ($t = 0$), $C$ set so debt sells at par: $D(V(0)) = P$

- A constant fraction $m$ of outstanding debt principal is retired through time (e.g. through a sinking fund)
  - Debt extant at $t = 0$ receives cash flow $e^{-mt}(C + mP)dt$ at $t$
  - Average life of debt $T = 1/m$ (half-life = $0.69/m$)
  - Retired debt is fully replaced continuously, but new debt sold at then-current market value $D(V(t))$ -- Dangl-Zechner modifies

- Default occurs when $V(t)$ falls to $V_B$, set by $\frac{dE(V)}{dV} |_{V=V_B} = 0$
  - In default, debt receives $(1 - \alpha)V_B$
  - $V_B$ depends both on leverage AND maturity

- A Problem: In virtually all studies, $m = 0$ optimizes firm value
  - Infinite life debt minimizes debt service, default
3. (II)liquidity of debt: The “Credit Spread Puzzle”

- Huang & Huang (2003; 2012): Historical default rates and risk premia cannot account for the size of spreads
  - Confirmed by Longstaff, Mithal, & Neis (2005) who estimate a “Non-Default Component of spread” across ratings of 50-72 bps
  - Strongly related to illiquidity measures.
  - Strong maturity effect: increases 3 bps/yr of maturity
  - Spread could also reflect agency costs: increase with debt maturity (e.g. Myers (1977), Leland & Toft (1996), Hackbarth & Mauer (2012))

- We introduce a simple additional “liquidity discount” to corporate bonds: their payouts are discounted at $r_f + h$
  - $h = h_0 + h_1 \times T$ ($T$ = average debt maturity)
  - Higher discount rate disadvantages longer-term debt
  - Optimal maturity balances this disadvantage with lower debt service advantage (lower endogenous default barrier)
Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riskfree Rate</td>
<td>5.00%</td>
<td>(HH)</td>
</tr>
<tr>
<td>Payout Rate</td>
<td>4.00%</td>
<td>(HH)</td>
</tr>
<tr>
<td>Asset Std Dev</td>
<td>24.00%</td>
<td>(FS)</td>
</tr>
<tr>
<td>Jump Risk</td>
<td>0.60% *</td>
<td></td>
</tr>
<tr>
<td>Q-Diffusion Return</td>
<td>1.60%</td>
<td></td>
</tr>
<tr>
<td>Risk Premium on Assets</td>
<td>4.00%</td>
<td>(based on 6% equity risk premium, 33% leverage)</td>
</tr>
</tbody>
</table>

*RN rate, calibrated to default. Physical = RN/2.5 (Driessen 2004) = 0.24%

Model Outputs

(Closed form solutions for debt and equity values, simple search for optimal leverage and maturity—easily implemented in Excel! Model on Conference site)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Leverage</td>
<td>34%</td>
<td>(vs. median SF 36%; HH = 43%)</td>
</tr>
<tr>
<td>Average Maturity (T = 1/m)</td>
<td>8.3 yrs</td>
<td>Half life = 5.74 yrs ≈ 11.5 yr regular debt maturity (Choi et al. 2016)</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>159 bps</td>
<td>(vs. median BBB spread SF= 148bps; HH = 158-194)</td>
</tr>
<tr>
<td>Recovery Rate</td>
<td>51%</td>
<td>(vs. HH 51.3%; SF 37%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Years after Issuance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical Default Probs</td>
<td>0.18%</td>
<td>0.51%</td>
<td>0.93%</td>
<td>1.95%</td>
<td>3.03%</td>
<td>4.90%</td>
<td>6.37%</td>
<td>8.85%</td>
<td>12.41%</td>
</tr>
<tr>
<td>Model Default Probs</td>
<td>0.24%</td>
<td>0.48%</td>
<td>0.76%</td>
<td>1.65%</td>
<td>2.97%</td>
<td>5.33%</td>
<td>6.93%</td>
<td>9.23%</td>
<td>12.63%</td>
</tr>
</tbody>
</table>

(See Appendix for calibration to Moody’s 1920-2011 Default rates)
COMPARATIVE STATICS: Leverage $L$ % in blue; Maturity $T$ yrs. in red

**Growth Rate $\mu$**

**Default Costs $\alpha$**

**Asset Riskiness $\sigma$**

**Tax Rate $\tau$**

**Liquidity/Agency Cost $h_0$**

**Jump Intensity $\lambda$**
Application 1: The Capital Structure of Growth Firms

**Contention:** *Growth firms (high MB ratio) have less leverage, shorter maturity*

- Smith and Watts (SW, 1992): regress leverage on MB, negative relation
- Barclay & Smith (BS, 1995): regress maturity on MB, negative relation
  - But both studies consider M/B, which reflects endogenous leverage
  - Their independent variables don’t include risk $\sigma$ or default costs $\alpha$

...MB is not a direct input in our model--it reflects endogenous firm value $M$

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How do we describe a “growth firm”? Any or all of the below:

- **High drift $\mu$:** more investment opportunities, less payouts
  - Shorter maturity but higher leverage

- **High default costs (“alpha”):** lost growth opportunities (high MB)
  - Longer maturity but lower leverage
  - Could also be related to (low) asset tangibility (Rampini & Viswanawathan (2013)):
    --Leverage decreases with fewer tangible assets
- **High risk ("sigma")**
  - Longer maturity but lower leverage

- **Low effective tax rates ("tau")**
  - Shorter maturity and lower leverage

- **Large Illiquidity or agency costs ("h")**
  - Shorter maturity and lower leverage

<table>
<thead>
<tr>
<th></th>
<th>Optimal Leverage (%)</th>
<th>Average (1/m) Maturity (yrs)</th>
<th>Yield Spread</th>
<th>5-yr Default Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BASE CASE:</strong></td>
<td>33.9%</td>
<td>8.3</td>
<td>159</td>
<td>1.65%</td>
</tr>
<tr>
<td><strong>Base</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth Rate (Mu)</td>
<td>1.00%</td>
<td>45.6%</td>
<td>7.5</td>
<td>165</td>
</tr>
<tr>
<td>Default Costs (Alpha)</td>
<td>30.00%</td>
<td>80.00%</td>
<td>16.6%</td>
<td>11.5</td>
</tr>
<tr>
<td>Asset Risk (Sigma)</td>
<td>24.00%</td>
<td>50.00%</td>
<td>15.3%</td>
<td>10.9</td>
</tr>
<tr>
<td>Effective Tax Rate (tau)</td>
<td>20.00%</td>
<td>5.00%</td>
<td>6.3%</td>
<td>3.5</td>
</tr>
<tr>
<td>Total liquidity discount (h), bps</td>
<td>65.00 =&gt; 100.00</td>
<td>29.5%</td>
<td>5.3</td>
<td>180</td>
</tr>
<tr>
<td><strong>ALL OF THE ABOVE</strong></td>
<td>0.00%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Application 2: The “Ratchet Effect” Admati et al. (2016)

CLAIM: After initial debt is issued to maximize total firm value,

- Equity holders will never find it advantageous to reduce leverage
  - Debt reduction decreases equity value—even when it increases firm value
  - Debt increases help equity by devaluing older debt—even with absolute priority
  - Agency problem similar to “asset substitution”, where greater volatility helps equity at expense of debtholders and firm value if no precommitment

- Leverage will therefore increase monotonically--eventually to point where tax benefits are entirely exhausted
  - Only taxes determine ultimate leverage, no default cost tradeoff
  - Generalizes a local result in Leland (1994), who also suggests that debt can be reduced only by a re-negotiation of initial-debt terms
  - Unlike Fischer, Heinkel, Zechner (1989) and Goldstein Ju Leland (2001), they do not require current debt must be fully retired before new issue
ADHP model quite simplistic:

- Jump-only process, no endogenous default, zero recovery in bankruptcy
- Nonetheless, we can examine their conclusions by using our model (v. 1) (assuming absolute priority of prior debt; no foresight)

LOOKS CONCERNING! BUT NOTE AFTER INITIAL DEBT ISSUANCE:

- New rounds create tiny increases in equity value (5.5 down to 1.5 bps); and have increasing spreads (likely to be rated lower than original debt)
- Very long maturities: calibrated to half-life, bond maturities ≈ 30-40 yrs
More realistically, limit $T_{max}$ to 15 (regular bond maturity $\approx 20$ yrs)

<table>
<thead>
<tr>
<th>ROUNDS</th>
<th>Optimal Leverage</th>
<th>% Equity Increase</th>
<th>Total Firm Value</th>
<th>Coupon</th>
<th>Principal</th>
<th>Average Maturity $T$</th>
<th>Total Debt Principal</th>
<th>Credit Spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33.85</td>
<td>3.356</td>
<td>103.36</td>
<td>2.34</td>
<td>34.98</td>
<td>8.64</td>
<td>34.98</td>
<td>169</td>
</tr>
<tr>
<td>2</td>
<td>36.80</td>
<td>0.038</td>
<td>103.32</td>
<td>0.23</td>
<td>3.07</td>
<td>15.00</td>
<td>38.05</td>
<td>245</td>
</tr>
<tr>
<td>3</td>
<td>38.56</td>
<td>0.014</td>
<td>103.27</td>
<td>0.14</td>
<td>1.82</td>
<td>15.00</td>
<td>39.87</td>
<td>256</td>
</tr>
</tbody>
</table>

Perhaps more realistically, limit $T$ to original debt $T$ (maturity $\approx 12$ yrs)

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<td>169</td>
</tr>
<tr>
<td>2</td>
<td>34.94</td>
<td>0.006</td>
<td>103.33</td>
<td>0.08</td>
<td>1.13</td>
<td>8.86</td>
<td>36.11</td>
<td>217</td>
</tr>
</tbody>
</table>

Even one subsequent issue yields less than 1 bp increase in equity value!

(Even less new debt if foresight of subsequent rounds)
CONCLUSIONS

Conclusions re Ratcheting: In the context of our model

- ADHP correct that firm will never want to reduce debt
  - But see Dangl-Zechner (2016), who argue short term debt precommits to debt reduction through repayment of principal
  - It may not be optimal for firm to fully roll over expiring debt

- Given miniscule equity gains (3.8 bps or less) and likely lower bond ratings, the ADHP concern of additional debt issuance seems very unlikely

More Generally

- We have developed a model that allows the joint determination of leverage and maturity
  - Leverage affects maturity, and vice-versa
  - Considering either in isolation can create misleading results
Appendix: Fitting the Model to 1920-2011 Default Data

Feldhutter & Schaefer (2016): When fitted to long-term default data, structural models don’t need jumps, liquidity discounts to match spreads. No “Credit Puzzle”!

- But calibration with Black-Cox model → \( T = \infty \). No optimization
- Need jumps to explain short term default, spreads

Our model can match 1920-2011 data with minor Input Changes

(i) Increase jump frequency: \( \lambda = 0.006 \rightarrow 0.011 \) (still “rare”)
(ii) Increase liquidity discount: \( h = 40+3*T \rightarrow h = 70+3*T \) bps
(iii) All other inputs remain same \((\sigma, \delta, \alpha)\)

- Optimal Leverage ≈ constant: 34% (Graham et al. 2012 w/preferred stock)
- Optimal Maturity falls from 8.3 years to 5.9 years (Choi et al. 2016)

Default Frequencies now match 1920-2011 data quite well

<table>
<thead>
<tr>
<th>Years after Issuance</th>
<th>Historical vs. Predicted Default Rates - Moody's 1920-2011</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Historical Default Probs</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Historical Default Probs</td>
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</tr>
<tr>
<td>Model Default Probs</td>
<td>0.44%</td>
</tr>
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