Local-Momentum Autoregression and the Modeling of the Interest Rate Term Structure

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This paper presents a new model for interest rates, to capture:

- Very long-term mean reversion
- Shorter-run autocorrelation
- Very short run momentum
US 3-month T-bill rate, Jan 1954 – Dec 2013
Clearly persistent, possibly mean-reverting, periods of momentum
The “LM-CTAR” model

- A standard AR process captures persistence:

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- The LM-CTAR model additionally allows for momentum:

\[ \Delta X_t = \kappa_x (\mu_t - X_{t-1}) + \omega \left( \bar{X}_{(t-1)|n} - X_{t-1} \right) + \sigma_x \epsilon_t \]
\[ \Delta \mu_t = \kappa_\mu (\bar{\mu} - \mu_{t-1}) + \sigma_\mu \epsilon_t \]
\[ \bar{X}_{(t-1)|n} \equiv \sum_{i=1}^{n} b_i X_{t-i} \]
Related work

- A time series “mean-reverting” to a time-varying central point:
  - Engle and Lee (1999): GARCH model with time-varying mean
  - Barndorff-Nielsen and Shephard (2002), and others: two factor models for stochastic volatility

- An autoregression with time-varying persistence:
  - Aït-Sahalia (1996) and Ang and Bekaert (2002): AR is close to random walk near “middle” of distribution, mean-reverting for extreme values
  - Diebold and Inoue (2001): regime switching AR processes can appear to have long memory
  - “Threshold AR” and “Smooth transition AR” models: see Granger and Teräsvirta (1993) for a survey
Some comments on the paper

- I like the paper, and it was interesting to think about how a time series model can try to capture the features of US interest rates.

- I have a few questions and comments for the author.
The “local momentum” component

The new term here is the LM term: $\bar{X}_{(t-1)|n} \equiv \sum_{i=1}^{n} b_i X_{t-i}$

A few questions about $\bar{X}_{(t-1)|n}$:

1. How do we optimally choose $n$? Author sets $n = 7$, which seems reasonable, but so would many other choices.

2. Intuition on how $n$ will change with the sampling frequency? Would optimal $n$ for daily data be 5 times larger than that for weekly data?

3. What is gained by the lag coefficients $\{b_i\}_{i=1}^{n}$? The author imposes these to be $1/n$ in estimation, which seems reasonable. Does he envisage a scenario when this would be relaxed?

4. If $\{b_i\}_{i=1}^{n}$ are not fixed, then they are unidentified when $\omega = 0$, making estimation and inference a bit trickier.
The models considered here may be stationary, but they are very close to being non-stationary.

- AR(1) coefficient in Table 2 is 0.9974
- LM-AR $\rho(B)$ coefficient is 0.9957
- Although LM-CTAR $\rho(B)$ coefficient for is only 0.8259 (why?)

These models may be inside the stationary region of the parameter space, but they are so close to the boundary that standard inference methods may be unreliable.

Perhaps run some simulations to see whether standard methods work, using the parameter estimates reported in the paper.
• This paper considers both a scalar time series process, and its generalization to the entire term structure (which is nice).

• What is the motivation for the additional AR(1) process that appears in this generalization (eq 14)?

\[ r_t = \underbrace{X_t}_{LM-CTAR} + \underbrace{\nu_t}_{AR(1)} + \underbrace{\epsilon_t}_{AR(0)} \]

• The LM-CTAR model already contains (i) an AR(1) for the central tendency factor, \( \mu_t \) (ii) an AR(1) for the interest rate (iii) an MA(7) for the local momentum effect

• The combined model for the term structure rates is thus quite complicated. Are all these AR/MA parameters are well identified?
Simple models w/ structural breaks vs. complicated models

One approach: keep generalizing a time series model until it fits the data over a (long) sample period

- Out-of-sample performance? Over-fitting?
- Stability of model parameters over a 50+ year period?
- Identification of parameters in the various components of the model?

Alternative: consider simpler time series models, estimated over shorter samples

- Formal tests for structural breaks (eg, Bai, 1995–now)
- Estimate using rolling window (eg, Fan, Farmen and Gijbels, 1998)
- Long-run predictions are harder, term structure extension may be hard

It would be interesting to see comparisons of the LM-CTAR model not only with models it nests (ie, with constant parameters) but some other alternatives as well.