Phasing out the GSEs *

Vadim Elenev — NYU Stern
Tim Landvoigt — UT Austin
Stijn Van Nieuwerburgh — NYU Stern, NBER, and CEPR

February 25, 2016

Abstract

We develop a new model of the mortgage market that emphasizes the role of the financial sector and the government. Risk tolerant savers act as intermediaries between risk averse depositors and impatient borrowers. Both borrowers and intermediaries can default. The government provides both mortgage guarantees and deposit insurance. Underpriced government mortgage guarantees lead to more and riskier mortgage originations and higher financial sector leverage. Mortgage crises occasionally turn into financial crises and government bailouts due to the fragility of the intermediaries' balance sheets. Foreclosure crises beget fiscal uncertainty, further disrupting the optimal allocation of risk in the economy. Increasing the price of the mortgage guarantee "crowds in" the private sector, reduces financial fragility, leads to fewer but safer mortgages, lowers house prices, and raises mortgage and risk-free interest rates. Due to a more robust financial sector and less fiscal uncertainty, consumption smoothing improves and foreclosure rates fall. While borrowers are nearly indifferent to a world with or without mortgage guarantees, savers are substantially better off. While aggregate welfare increases, so does wealth inequality.

JEL: G12, G15, F31.
Keywords: financial intermediation, housing policy, government bailouts

*First draft: February 2015. We thank seminar participants at UT Austin, NYU Stern, George Washington-Federal Reserve Board joint seminar, U.C. Berkeley Haas, the Einaudi Institute in Rome, the ECB-Bundesbank-Goethe Joint Lunch seminar, the Frankfurt School of Management and Finance, the University of Michigan, the University of Missouri, Harvard Business School, Arizona State University, Johns Hopkins University, and the Philadelphia Federal Reserve Board, and conference participants at the Yale Cowles Foundation conference, the UBC ULE Summer Symposium, the Society for Economic Dynamics in Warsaw, the National Bureau of Economic Research Summer Institute Public Economics and Real Estate Group, the World Congress of the Econometric Society in Montreal, the Liquidity conference at Wharton School, the Carnegie-Rochester conference in Pittsburgh, the Federal Reserve Bank of Atlanta Real Estate Finance Conference and the American Economic Association conference in San Francisco. We also thank Ralph Koijen, Edward Kung, Dirk Krueger, Kurt Mitman, Jack Favilukis, David Scharfstein, and Paul Willen for detailed comments.

1 Introduction

Government and quasi-government entities dominate mortgage finance in the U.S. Over the past five years, the government-sponsored enterprises, Fannie Mae and Freddie Mac, and the Federal Housing Administration have stood behind 80% of the newly originated mortgages.[1] Ever since the collapse of the GSEs in September of 2008 and the conservatorship which socialized housing finance, there have been many proposals to bring back private capital.[2] The main idea of these policy proposals is to dramatically reduce the size and scope of the government guarantee on standard (conforming) mortgages. Because the proposed reform would turn a largely public into a largely private housing finance market, there is both uncertainty and concern about its impact on house prices, the stable provision of mortgage credit, financial sector stability, and ultimately welfare.

Understanding the economic impact of mortgage finance reform of the kind currently debated requires a general equilibrium model. Such a model must recognize the important role that residential real estate and mortgage markets have come to play in the financial system and the macro-economy of rich countries (Jorda, Schularick, and Taylor (2014)). It must also recognize the large footprint of the government. This paper proposes a new general equilibrium model of the housing and mortgage markets where the interaction of the financial sector and the government plays a central role. As in the real world, mortgages in the model are long-term, prepayable, and defaultable. The government provides not only guarantees to banks' mortgage assets but also to its liabilities.[3]

In our model, the government provides mortgage default insurance at a fixed cost. The financial sector issues mortgages to borrowers and decides for how many of those mortgages to

[1]Currently, of the $9.85 trillion stock of residential mortgages, 57% are Agency Mortgage-backed Securities guaranteed by Fannie Mae, Freddie Mac, and Ginnie Mae. Private-label mortgage backed securities make up less than 8% of the stock. The rest is unsecuritized first liens held by the GSEs and the banking sector (28%) and second liens (7%). Acharya, Richardson, Van Nieuwerburgh, and White (2011) provide an in-depth discussion of the history of the GSEs, their growth, and collapse.

[2]The Obama Administration released a first report along these lines in February 2011. The bills proposed -but not passed- by Corker-Warner in 2013 and Johnson-Crapo in 2014 provide the most recent attempts at legislative reform.

[3]In our model, the bailout guarantee to banks' liabilities is equivalent to deposit insurance. Deposit insurance stands in for any explicit and implicit government guarantees to all short-term liabilities of the levered financial sector. Indeed, the government stepped in to bail out asset-backed commercial paper, repo, and money market mutual fund markets in 2008-09. Deposit insurance is an important feature of any financial system that the literature on mortgage finance has not considered hitherto.

buy the government insurance. When the cost (guarantee fee or g-fee) is set to the value observed in the pre-2013 data, guaranteed mortgages dominate banks' portfolios. Underpriced mortgage guarantees induce financial sector risk taking. The guarantee lowers the risk of banks' assets and prompts banks to dial up leverage in pursuit of a higher return portfolio. The favorable regulatory capital treatment of guaranteed mortgages enables high leverage. And deposit insurance makes banks' depositors insensitive to the risk of a collapse. The model generates the observed leverage ratio of the financial sector alongside the average loan-to-value ratio of mortgage borrowers.

Underpriced guarantees also induce banks to grow the size of their mortgage portfolio and increase its riskiness: they originate more and higher debt-to-income and loan-to-value ratio mortgages. A larger and riskier mortgage portfolio produces higher mortgage foreclosure rates and larger deadweight costs of foreclosures, a first source of welfare losses for the economy. Because of the increase in mortgage credit risk they induce, underpriced guarantees increase the mortgage spread between the mortgage rate and the risk-free interest rate. In sum, the government's underpriced mortgage guarantee distorts financial sector leverage and leads to a larger financial sector and lower underwriting standards. Given banks' low net worth and high leverage, housing crises occasionally turn into financial crises, defined as bank insolvencies. Thus, the economy with underpriced mortgage guarantees generates financial sector fragility. It also displays high house price volatility.

There is an important general equilibrium effect on the risk-free interest rate. In mortgage crises, the government must make good on the mortgage insurance it sold to the financial sector. It increases government debt to pay for this outlay. Risk averse depositors absorb the additional government debt in equilibrium, forcing them to save in bad states of nature. The mortgage guarantee program shifts the risk of mortgage losses from the intermediaries to the depositors whose preferences make them less suitable to bear this risk. More generally, fluctuations in the government's financing needs induce fluctuations in the depositors' consumption. This generates a strong precautionary saving's motive and a low equilibrium interest rate in the model with underpriced guarantees. The low risk-free interest rate in the low g-fee economy outweighs the high mortgage spread, so that the mortgage rate is low. Low equilibrium mortgage rates make borrowers willing to take on higher mortgage debt. House prices are inflated.

Our main policy experiment is to increase the g-fee from the low level observed until recently.

Naturally, higher guarantee fees "crowd in" the private sector: They induce a shift in the composition of banks' assets from guaranteed to private mortgage bonds. With banks now bearing more of the mortgage credit risk, their portfolio risk increases. They reduce leverage (by 7.3 percentage points at a g-fee high enough to crowd out mortgage guarantees completely), increase net worth, shrink the size of their mortgage book (by 8.7%), and make mortgages that are less risky. With sufficient capital, banks are able to lend during mortgage crises. The provision of mortgage credit is as stable as in the economy with underpriced guarantees. No GSEs are needed to provide stable access to mortgage finance. Mortgage crises are less likely to turn into financial crises because of the sturdier bank balance sheets. In sum, banks are better able to perform their function of intermediating between borrowers and depositors. The lower mortgage foreclosure rates (-40%) bring more stability to borrowers' consumption. The lower quantity of mortgage insurance bought and the lower loss rates on mortgages combine to dramatically reduce fiscal uncertainty. Depositors who absorb these changes in government debt issuance achieve a less volatile consumption stream and reduce precautionary savings. The private sector equilibrium features higher risk-free interest rates (+ 70 basis points). The higher risk-free interest rate more than offsets the decline in the mortgage spread, due to lower mortgage defaults and a lower mortgage risk premium, so that mortgage interest rates are higher (+ 23 basis points) in the private sector economy. Higher mortgage rates result in lower house prices (-6.2%).

The overall effect of phasing out the GSEs is an increase in social welfare of 0.63% in consumption equivalence units. Borrower welfare increases only marginally (+0.04%) while depositors (+1.30%) and intermediaries (+1.69%) gain substantially. Borrowers benefit from the improved risk sharing, but they lose their mortgage subsidy, face higher mortgage rates, tighter lending standards, and lower house prices. Depositors benefit from higher interest rates and less fiscal uncertainty. Thus, while abolishing the GSEs is a Pareto improvement, it increases wealth inequality.

These results are steady-state comparisons. We also compute a transition from the low to the high g-fee economy and find that borrowers suffer in the short-run from the reform. Prices adjust quickly but stocks of debt and wealth take longer to adjust so that the losses from higher mortgage rates and lower house prices hit immediately while the benefits from improved risk sharing only accrue gradually.

We study an intermediate economy where the guarantee fee is similar to the value charged by the GSEs in 2015. At even higher levels for the g-fee, we find that the government guarantee is only taken up in bad times. This dovetails with the "mortgage insurer of last resort" option in the 2010 Obama Administration proposal which envisions activating the government only in crises (Scharfstein and Sunderam (2011)). These intermediate g-fee economies, including ones where the g-fee is higher in good times than in bad, improve on the benchmark low g-fee economy. But they still have lower welfare than the fully private sector economy.

Finally, we use our model to quantitatively evaluate 2014 the Johnson-Crapo bill which proposes to change the nature of the government guarantee to one that only kicks in when losses are catastrophic. We find that the proposal would generate a substantial welfare gain of 0.66%. A much larger private sector loss absorption eliminates most moral hazard while intermediaries enjoy the benefit of the government insurance during very severe mortgage crisis. This (slightly) improves risk sharing compared to the laissez-faire equilibrium.

In sum, mortgage guarantees were arguably introduced in order to enhance the stable provision of mortgage credit especially in crises. We find that they perform this role in the model, but they induce moral hazard by the financial sector. For standard guarantees, we find that the cost of moral hazard outweighs the benefit of stability. The Johnson-Crapo experiment demonstrates that a well-designed government mortgage insurance can improve welfare at least as much as the private sector solution.[4]

2 Related Literature

Our paper contributes to several strands of the literature on housing, finance, and macroeconomics. Unlike recent quantitative work that explores the causes and consequences of the housing boom,[5] this paper focuses on the current and future state of the housing finance system and the role the government plays in this system. It shares a focus on quantitative considerations with this work. House prices and interest rates are determined in equilibrium rather than ex-

[4]As in any incomplete market model, adding a non-redundant security like the mortgage guarantee may or may not improve welfare (e.g., Hart (1975)). We identify a reason why it may not.

[5]Examples include Favilukis, Ludvigson, and Van Nieuwerburgh (2015), Landvoigt, Piazzesi, and Schneider (2015), Landvoigt (2012), Chu (2014). See Davis and Van Nieuwerburgh (2015) for a recent review of the literature.

ogenously specified. We simplify by working in an endowment economy with a constant housing stock.[6]

Like another strand of the literature, our model features borrowers defaulting optimally on their mortgages.[7] Unlike most of that literature, our lenders are not risk-neutral but risk averse. Unlike most of the literature, our mortgage contract is a long-term contract, allowing us to capture crucial features of the U.S. 30-year fixed-rate mortgage. We calibrate our mortgage contract to exhibit the same amount of interest rate risk as the outstanding pool of agency mortgage-backed securities. Our setting is ideally suited to study the interaction of default and prepayment risk. Government policy affects the quantity and price of both of these risks.

We contribute to a recent literature in asset pricing that has emphasized the central role of financial intermediaries in the crisis.[8] In this class of models, the net worth of the financial sector is the key state variable which governs risk sharing and asset prices. In our model, intermediary wealth is also an important state variable, but it is not the only one. The wealth of the depositors, the wealth of the borrowers, and the outstanding amount of government debt all have important effects on equilibrium allocations and prices. We explicitly model the intermediary's decision to default and transfer its assets and liabilities to the government. We ask how government intervention in the form of asset (mortgage) and liability (deposit) guarantees affect financial fragility and welfare.

Finally, our paper contributes to the literature that quantifies the effect of government policies in the housing market.[9] When house and rental prices are determined endogenously, such policy changes tend to lower house prices and price-rent ratios and cause an increase in home ownership rates. Our model has no renters and misses the home ownership channel. However, the results from this literature suggest that the welfare gains from a GSE phaseout would be further amplified in a model like ours but with renters. The lower house price-rent ratios following a GSE phase-out

[6] The role of housing supply and construction are studied in Favilukis, Ludvigson, and Van Nieuwerburgh (2015), Chatterjee and Eyigungor (2009), Hedlund (2014), among others.

[7] Recent examples of equilibrium models with default are Corbae and Quintin (2014), Garriga and Schlagenhauf (2009), Chatterjee and Eyigungor (2009), Jeske, Krueger, and Mitman (2013), Landvoigt (2012), Arslan, Guler, and Taskin (2013), and Hedlund (2014).

[8] Recent examples include Brunnermeier and Sannikov (2014), He and Krishnamurty (2013), Adrian and Boyarchenko (2012), and Drechsler, Savov, and Schnabl (2014).

[9] Most work focuses on studying the effect of abolishing the mortgage interest rate tax deductibility and the tax exemption of imputed rental income of owner-occupied housing. For example, Gervais (2002), Chambers, Garriga, and Schlagenhauf (2009), and Sommer and Sullivan (2013).

would benefit renters wanting to become home owners.

Like abolishing the fiscal benefits from home ownership, Jeske, Krueger, and Mitman (2013) find that eliminating GSE subsidies would not only increase overall welfare but also reduce inequality.[10] Our work predicts that GSE reform would raise inequality between borrowers, banks, and savers. Jeske et al. (2013) emphasize heterogeneity among borrowers who face (partially) uninsurable idiosyncratic income risk, but no aggregate risk. Our borrowers are able to perfectly insure idiosyncratic risk with one another, and we focus instead on how aggregate risk is spread between borrowers, banks, and depositors. While we do not investigate whether GSEs provide insurance against idiosyncratic risk, we find that underpriced guarantees result in a higher foreclosure rate. Whatever missing insurance role the GSEs play would have to be strong enough to offset this effect.

3 The Model

3.1 Endowments, Preferences, Technology, Timing

Endowments The model is a two-good endowment economy with a non-housing and a housing Lucas tree. The fruit of the non-housing tree, *output* Y_t, grows and its growth rate is subject to aggregate shocks. The different households are endowed with a fixed and non-tradeable share of this tree. This endowment can be interpreted as labor income. The size of the housing tree (housing stock) grows at the same stochastic trend as output. The total quantity of housing shares is fixed and normalized to $\bar{K}$. The housing stock yields housing services proportional to the stock.

Preferences The model features a government and three groups of households. Impatient households will play the role of borrowers in equilibrium (denoted by superscript B), while patient households will turn out to be savers. There are two type of savers, differentiated by their risk aversion coefficient; we refer to the less risk averse savers as "risk takers" (denoted by superscript R) and the more risk averse as "depositors" (denoted by D). Thus, for the rate of

[10] But, in a model similar to Jeske, Krueger, and Mitman (2013), Gete and Zecchetto (2015) argue that the poor, high-credit risk households suffer a disproportionate increase in the cost of mortgage credit from an abolition of GSEs, offsetting the reduction in inequality emphasized by Jeske et al.

impatience $\beta_R = \beta_D > \beta_B$, and for the coefficient of relative risk aversion $\sigma_R < \sigma_B \leq \sigma_D$. All agents have Epstein-Zin preferences with the same elasticity of intertemporal substitution ν over a Cobb-Douglas aggregate of housing and non-housing consumption with aggregation parameter θ.

$$U_t^j = \left\{(1-\beta_j)\left(u_t^j\right)^{1-1/\nu} + \beta_j\left(\mathrm{E}_t\left[(U_{t+1}^j)^{1-\sigma_j}\right]\right)^{\frac{1-1/\nu}{1-\sigma_j}}\right\}^{\frac{1}{1-1/\nu}} \tag{1}$$

$$u_t^j = \left(C_t^j\right)^{1-\theta}\left(A_K K_{t-1}^j\right)^{\theta} \tag{2}$$

C_j^t is numeraire non-housing consumption and the constant A_K specifies the housing services from owning the housing stock, expressed in units of the numeraire.

Figure 1 depicts the balance sheets of the different agents in the economy and the flows of funds between them.

Technology In addition to housing, there are three assets in the economy. The first is a one-period short-term bond. The second is a mortgage bond, which aggregates the mortgage loans made to all borrower households. The third is mortgage insurance which the government sells to the private market. The guarantee turns a defaultable long-term mortgage bond into a default-free government-guaranteed mortgage bond. The government exogenously sets the price of the guarantee.

Borrowers experience housing depreciation shocks and may choose to default on their mortgage. There is no recourse; savers and possibly the government (ultimately the tax payers) bear the loss depending on whether mortgage loans are held in the form of private or government-guaranteed mortgage bonds, respectively. A novel model ingredient is that risk takers may also choose to default and declare bankruptcy. Default wipes clean their negative wealth position with no further consequences; the losses are absorbed by the government in a "financial sector bailout."

Timing The timing of agents' decisions at the beginning of period t is as follows:

1. Income shocks for all types of agents and housing depreciation shocks for borrower households are realized.

2. Risk takers (financial intermediaries) decide on a bankruptcy policy. In case of a bankruptcy, their financial wealth is set to zero and they incur a utility penalty. At the time of the decision, the magnitude of the penalty is unknown.[11] Risk takers know its probability distribution and maximize expected utility by specifying a binding decision rule for each possible realization of the penalty.[12]

3. Borrowers decide on mortgage default.

4. Risk takers' utility penalty shock is realized and they follow their bankruptcy decision rule from step 2. In case of bankruptcy, the government picks up the shortfall in repayments to debt holders (depositors).

5. Borrowers choose how much of the remaining mortgage balance to prepay (refinance). All agents solve their consumption and portfolio choice problems. Markets clear. All agents consume.

Each agent's problem depends on the wealth of others; the entire wealth distribution is a state variable. Each agent must forecast how that state variable evolves and predict the bankruptcy decisions of borrowers and risk takers. We now describe each of the three types of household problems and the government problem in detail.

3.2 Borrower's Problem

Mortgages As in reality, mortgage contracts are long-term, defaultable, and prepayable. The mortgage is a long-term contract, modeled as a perpetuity. Bond coupon (mortgage) payments decline geometrically, $\{1, \delta, \delta^2, \ldots\}$, where δ captures the duration of the mortgage. Because the payment per unit of mortgage bond is 1 in the current period, A_t^B is both the mortgage payment and the number of outstanding units of the mortgage bond. A mortgage can default, in which case the lenders have recourse to the housing collateral. We introduce a "face value" $F = \frac{\alpha}{1-\delta}$, a fixed fraction α of the mortgage payments (per unit of mortgage bond), at which the mortgage

[11] Introducing a random utility penalty is a technical assumption we make for tractability. It makes the value function differentiable and allows us to use our numerical methods which rely on this differentiability. Without it, there would be a discontinuity in the value function. This randomization assumption is common in labor market models (Hansen (1985)). Additionally, uncertainty about the consequences of a systematic banking crisis and insolvency may seem quite reasonable.

[12] The assumption of making a binding default decision is necessitated in the presence of Epstein-Zin preferences.

can be prepaid. Prepayment incurs a cost detailed below. Mortgage payments can be deducted from income for tax purposes at a rate $\tau_t^m = (1-\alpha)\tau_t$, where τ_t is the income tax rate and the fraction $(1-\alpha)$ reflects the interest component of mortgage payments.

Borrower Default There is a representative family of borrowers, consisting of a measure one of members. Each member receives the same stochastic labor income $Y_t^B \propto Y_t$, chooses the same quantity of housing k_t^B s.t. $\int_0^1 k_t^B di = K_t^B$, and the same quantity of outstanding mortgage bonds a_t^B s.t. $\int_0^1 a_t^B di = A_t^B$.

After having received income and having chosen house and mortgage size, each family member draws an idiosyncratic housing depreciation shock $\omega_{i,t} \sim F_\omega(\cdot)$ which proportionally lowers the value of the house by $(1-\omega_{i,t})p_t k_{t-1}^B$. The value of the house after stochastic depreciation is $\omega_{i,t} p_t k_{t-1}^B$. We denote the cross-sectional mean and standard deviation by $\mu_\omega = \mathrm{E}_i[\omega_{i,t}]$ and $\sigma_{t,\omega} = (\mathrm{Var}_i[\omega_{i,t}])^{0.5}$, where the latter varies over time. The variable $\sigma_{t,\omega}$ governs the mortgage credit risk in the economy; it is the second exogenous aggregate state variable.

Each borrower household member then optimally decides whether or not to default on the mortgages. The houses that the borrower family defaults on are turned over to (foreclosed by) the lender. Let the function $\iota(\omega) : [0,\infty) \to \{0,1\}$ indicate the borrower's decision to default on a house of quality ω. We conjecture and later verify that the optimal default decision is characterized by a threshold level ω_t^*, such that borrowers default on all houses with $\omega_{i,t} \leq \omega_t^*$ and repay the debt for all other houses. Using the threshold level ω_t^*, we define $Z_A(\omega_t^*)$ to be the fraction of debt *repaid* to lenders and $Z_K(\omega_t^*)p_t K_{t-1}^B$ to be the value to the borrowers of the residual (non-defaulted) housing stock after default decisions have been made. We have:

$$Z_A(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \geq \omega_t^*], \tag{3}$$

$$Z_K(\omega_t^*) = \int_0^\infty (1-\iota(\omega))\omega\, f_\omega(\omega) d\omega = \Pr[\omega_{i,t} \geq \omega_t^*]\ \mathrm{E}[\omega_{i,t} \,|\, \omega_{i,t} \geq \omega_t^*] \tag{4}$$

After making a coupon payment of 1 per unit of remaining outstanding mortgage, the amount of outstanding mortgages declines to $\delta Z_A(\omega_t^*) A_t^B$.

Prepayment Next, the households can choose to prepay a quantity of the outstanding mortgages R_t^B by paying the face value F per unit to the lender. We denote by $Z_t^R \equiv R_t^B / A_t^B$ the ratio of prepaid mortgages to beginning-of-period mortgages. Prepayment incurs a monetary cost Ψ. We use an adjustment cost function $\Psi(R_t^B, A_t^B)$ that is convex in the fraction prepaid Z_t^R, capturing bottlenecks in the mortgage refinance infrastructure when too large a share of mortgages are prepaid at once.

Borrower Problem Statement The borrower family's problem is to choose consumption C_t^B, housing K_t^B, default threshold ω_t^*, prepayment quantity R_t^B, and new mortgage debt B_t^B to maximize life-time utility U_t^B in (1), subject to the budget constraint:

$$C_t^B + (1-\tau_t^m) Z_A(\omega_t^*) A_t^B + p_t K_t^B + F R_t^B + \Psi(R_t^B, A_t^B) \leq (1-\tau_t) Y_t^B + G_t^{T,B} + Z_K(\omega_t^*) p_t K_{t-1}^B + q_t^m B_t^B, \tag{5}$$

an evolution equation for outstanding mortgage debt:

$$A_{t+1}^B = \delta Z_A(\omega_t^*) A_t^B + B_t^B - R_t^B, \tag{6}$$

a maximum loan-to-value constraint:

$$F A_{t+1}^B \leq \phi p_t K_t^B. \tag{7}$$

and a double constraint on the amount of mortgages that can be refinanced:

$$0 \leq R_t^B \leq \delta Z_A(\omega_t^*) A_t^B. \tag{8}$$

Outstanding mortgage debt at the end of the period (equation 6) is the sum of the remaining mortgage debt after default and new borrowing B_t^B minus prepayments. The borrower household uses after-tax labor income, net transfer income from the government ($G_t^{T,B}$), residual housing wealth, and new mortgage debt raised to pay for consumption, mortgage debt service net of mortgage interest deductibility, new home purchases, prepayments $F R_t^B$ and associated prepayment costs $\Psi(R_t^B, A_t^B)$. New mortgage debt raised is $q_t^m B_t^B$, where q_t^m is the price of one unit of mortgage bond in terms of the numeraire good.

The borrowing constraint in (7) caps the face value of mortgage debt at the end of the period, $F A_{t+1}^B$, to a fraction of the market value of the underlying housing, $p_t K_t^B$, where ϕ is the maximum loan-to-value ratio. Declines in house prices (in bad times) tighten borrowing constraints. It is the first of two occasionally binding borrowing constraints .

The refinancing constraints in equation (8) ensure that the amount prepaid is between 0 and the outstanding balance after the default decision was made. Equivalently, the share prepaid, Z_t^R, must be between 0 and $\delta Z_A(\omega_t^*)$.

3.3 Risk Takers

Next we study the problem of the risk taker households, who lend to borrower households and borrow from depositor households. Hence, we refer to this household type as intermediaries.[13]

Risk-taker Default After shocks to income and housing depreciation have been realized, the risk taker (financial intermediary) chooses whether or not to declare bankruptcy. Risk takers who declare bankruptcy have all their assets and liabilities liquidated.[14] They also incur a stochastic utility penalty ρ_t, with $\rho_t \sim F_\rho$, i.i.d. over time and independent of all other shocks. At the time of the bankruptcy decision, risk takers do not yet know the realization of the bankruptcy penalty. Rather, they have to commit to a bankruptcy decision rule $D(\rho) : \mathbb{R} \to \{0, 1\}$, that specifies the optimal decision for every possible realization of ρ_t. Risk takers choose $D(\rho)$ to maximize expected utility at the beginning of the period. We conjecture and later verify that the optimal default decision is characterized by a threshold level ρ_t^*, such that risk takers default for all realizations for which the utility cost exceeds the threshold. As we explain below, risk taker default leads to a government bailout. After the realization of the penalty, risk takers execute their bankruptcy choice according to the decision rule. They then face a consumption and portfolio choice problem, where they allocate their wealth between a short-term risk-free bond, a private mortgage bond, and a government-guaranteed mortgage bond.

Private Mortgage Bond A private mortgage bond is a simple pass-through vehicle, aggregating the mortgages of the borrowers. The coupon payment on performing mortgages in the current period is $A_t^B Z_A(\omega_t^*)$, which is the number of mortgage bonds times the fraction that is performing

[13]Note that we could separately model risk taker households as the shareholders of the banks and the banks they own. For simplicity we combine the two balance sheets.

[14]The mortgages are bonds that trade in a competitive market. They are sold during the liquidation and bought by the banks that start off the following period with zero financial wealth and the exogenous income stream.

times the coupon payment of 1 per unit of performing bond. For mortgages that go in foreclosure, the risk taker repossesses the homes. These homes are worth $(1-\zeta)\left(\mu_\omega - Z_K(\omega_t^*)\right) p_t K_{t-1}^B$, where ζ is the fraction of home value destroyed in a foreclosure. It represents a deadweight loss to the economy. Thus, the total payoff per unit of private mortgage bond is:

$$M_{t,P} = Z_A(\omega_t^*) + \frac{(1-\zeta)(\mu_\omega - Z_K(\omega_t^*))p_t K_{t-1}^B}{A_t^B},$$

while the price is q_t^m per unit.

Government-guaranteed Mortgage Bond A government-guaranteed bond is a security with the same duration (maturity and cash-flow structure) as a private mortgage bond. The only difference is that it carries no mortgage default risk because of the government guarantee. To prevent having to keep track of an additional state variable, we model guarantees as one-period default insurance.[15] Combining one unit of a private mortgage bond with one unit of default insurance creates a mortgage bond that is government-guaranteed for one period. One unit of a government-guaranteed mortgage bond has the following payoff:

$$M_{t,G} = 1 + (1 - Z_A(\omega_t^*))\delta F$$

The first term is the coupon of 1 on all loans in the pool, performing and non-performing. The second term is compensation for the loss in principal of defaulted loans. Owners of guaranteed loans receive a principal repayment F. The price of the bond is $q_t^m + \gamma_t$ per unit. The government sets the price of insurance γ_t per unit of bond to the government. The choice between guaranteed and private mortgage bonds is the main choice of interest in the paper.

Intermediary wealth Denote risk-taker financial wealth at the start of period t by W_t^R. This financial intermediary net worth is a key state variable.

$$W_t^R = (M_{t,P} + \delta Z_A(\omega_t^*)q_t^m - Z_t^R[q_t^m - F])A_{t,P}^R + (M_{t,G} + \delta Z_A(\omega_t^*)q_t^m - Z_t^R[q_t^m - F])A_{t,G}^R + B_{t-1}^R.$$

[15]Rolling over default insurance every period for the life of the loan is the equivalent to the real-world long-term guarantees provided by Fannie Mae and Freddie Mac. Having the choice of renewal each period makes our guarantees more flexible, and hence more valuable, than those in the real world.

It consists of the market value of the portfolio of private and guaranteed bonds bought last period, as well as the short-term bonds from last period which mature this period. When the holdings of short-term bonds are negative, the last term is short-term debt which must be repaid this period. Since the mortgage guarantee is valid for only one period, both private and government-guaranteed bonds bought last period trade for the same price q_t^m. Mortgage prepayments "come in at par" F. Since such prepayments only happen when $q_t^m > F$, they represent a loss to the intermediary. If the portfolio consists entirely of guaranteed bonds, prepayments are an important driver of risk taker net worth.

Consumption-Portfolio Choice Problem While intertemporal preferences are still specified by equation (1), intraperiod utility u_t^j depends on the bankruptcy decision and penalty:

$$u_t^R = \frac{\left(C_t^R\right)^{1-\theta}\left(A_K K_{t-1}^R\right)^\theta}{\exp\left(D(\rho_t)\rho_t\right)}.$$

Entering with wealth W_t^R, the risk taker's problem is to choose consumption C_t^R, holdings of private mortgage bonds $A_{t+1,P}^R$, holdings of government-guaranteed mortgage bonds $A_{t+1,G}^R$, and short-term bonds B_t^R to maximize life-time utility U_t^R in (1), subject to the budget constraint:

$$C_t^R + q_t^m A_{t+1,P}^R + (q_t^m + \gamma_t) A_{t+1,G}^R + q_t^f B_t^R + (1-\mu_\omega) p_t K^R \leq (1-\tau_t) Y_t^R + G_t^{T,R} + W_t^R, \quad (9)$$

and the following constraints:

$$\begin{aligned} A_{t+1,P}^R &\geq 0, && (10)\\ A_{t+1,G}^R &\geq 0, && (11)\\ -q_t^f B_t^R &\leq q_t^m \varkappa(\xi_P A_{t+1,P}^R + \xi_G A_{t+1,G}^R). && (12) \end{aligned}$$

The budget constraint (9) shows that the risk-taker uses after-tax labor income, net transfer income, and beginning-of-period wealth to pay for consumption, purchases of private and government-guaranteed mortgage bonds and short-term bonds, and for housing repairs which undo the effects of depreciation. Risk takers own a fixed share K^R of the housing stock. We do not allow for negative positions in either long-term mortgage bond (equations 10 and 11).

A key constraint in the model is (12). A negative position in the short-term bond is akin to the risk taker issuing short-term bonds, or equivalently deposits. The negative position in the short-term bond must be collateralized by the market value of the risk taker's holdings of long-term mortgage bonds. The parameters ξ_P and ξ_G together with $\varkappa$ determine how useful private and government-guaranteed mortgage bonds are as collateral. In the calibration, we will assume that guaranteed mortgages are better collateral: $\xi_G > \xi_P$. The constraint captures the reality of Basel II/III-type risk weights that restrict intermediary leverage.[16]

3.4 Depositors

The second type of savers, depositors, receive labor income, $Y_t^D \propto Y_t$, own a fixed share of the housing stock K^D, and solve a standard consumption-savings problem. Entering with wealth $W_t^D = B_{t-1}^D$, the depositor's problem is to choose consumption C_t^D and holdings of short-term bonds B_t^D to maximize life-time utility U_t^D in (1), subject to the budget constraint:

$$C_t^D + q_t^f B_t^D + (1-\mu_\omega)p_t K^D \leq (1-\tau_t)Y_t^D + G_t^{T,D} + W_t^D, \tag{13}$$

and short-sales constraints on bond holdings:

$$B_t^D \geq 0. \tag{14}$$

The budget constraint (13) is similar to that of the risk taker. We also do not allow depositor's to take a negative position in the short-term bond (14), consistent with our assumption that the depositor must not declare bankruptcy.

[16] The short-term borrowing is akin to a repo contract. It allows the intermediary to buy a mortgage bond by borrowing a fraction ξ of the purchase price while only using a fraction $1-\xi$ of the purchase price, the margin requirement, of her own capital. One can think of the guaranteed bond as a private mortgage bond plus a government guarantee (a credit default swap or mortgage insurance). Implicit in constraint (12) is the assumption that the government guarantee itself is an off-balance sheet item that cannot be collateralized.

3.5 Government

We model the government as set of exogenously specified tax, spending, bailout, and debt issuance policies.[17] Government tax revenues, T_t, are labor income tax receipts minus mortgage interest deduction tax expenditures plus mortgage guarantee fee income:

$$T_t = \tau_t Y_t - \tau_t^m Z_A(\omega_t^*) A_t^B + \gamma_t (A_{t,G}^R + A_{t,G}^D)$$

Government expenditures, G_t are the sum of payoffs on mortgage guarantees, financial sector bailouts, other exogenous government spending, G_t^o, and government transfer spending G_t^T:

$$G_t = (M_{t,G} - M_{t,P}) A_{t,G}^R - D(\rho_t) W_t^R + G_t^o + G_t^T$$

The mortgage guarantee pays to the risk takers the difference in cash-flow between a guaranteed and a private mortgage bond, for each unit of guaranteed bond they purchase. The bailout to the financial sector equals the negative of the financial wealth of the risk taker, W_t^R, in the event of a bankruptcy ($D(\rho_t) = 1$). By bailing out the intermediaries, the government renders intermediaries' liabilities, deposits, risk-free. In the model, risk taker bankruptcies, limited liability for risk takers, and deposit insurance are equivalent.

The government issues one-period risk-free debt. Debt repayments and government expenditures are financed by new debt issuance and tax revenues, resulting in the budget constraint:

$$B_{t-1}^G + G_t \leq q_t^f B_t^G + T_t \tag{15}$$

We impose a transversality condition on government debt:

$$\lim_{u \to \infty} \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+u}^D B_{t+u}^G \right] = 0$$

where $\tilde{\mathcal{M}}^D$ is the SDF of the depositor.[18] Because of its unique ability to tax and repay its debt, the government can spread out the cost of mortgage default waves and financial sector rescue operations over time. We are interested in understanding whether the government's ability to

[17] We consolidate the role of the GSEs and that of the Treasury department into one government, reflecting the reality as of September 2008.

[18] We show below that the risk averse saver is the marginal agent for short-term risk-free debt.

tax and issue debt leads to an increased stability of mortgage credit provision in the world with government guarantees.

Government policy parameters are $\Theta_t = (\tau_t, \gamma_t, G^o_t, \phi, \xi_G, \xi_P, \mu_\rho)$. The parameters ϕ in equation (7) and (ξ_G, ξ_P) in equation (12) can be thought of as macro-prudential policy tools which govern household and intermediary leverage. We added the parameter μ_ρ that governs the mean utility cost of bankruptcy to risk takers to the set of policy levers, since the government may have some ability to control the fortunes of the financial sector in the event of a bankruptcy. This cost directly affects the strength of deposit insurance but could also include reputational costs of bank defaults.

3.6 Equilibrium

Given a sequence of aggregate income and house valuation shocks $\{Y_t, \sigma_{\omega,t}\}$ and utility costs of default shocks $\{\rho_t\}$, and given a government policy $\{\Theta_t\}$, a competitive equilibrium is an allocation $\{C^B_t, K^B_t, B^B_t, R^B_t\}$ for borrowers, $\{C^R_t, A^R_{t,P}, A^R_{t,G}, B^R_t\}$ for risk takers, $\{C^D_t, B^D_t\}$ for depositors, default policies $\iota(\omega_{it})$ and $D(\rho_t)$, and a price vector $\{p_t, q^m_t, q^f_t\}$, such that given the prices, borrowers, depositors, and risk-takers maximize life-time utility subject to their constraints, the government satisfies its budget constraint, and markets clear.

The market clearing conditions are:

$$\text{Risk-free bonds: } B^G_t = B^D_t + B^R_t \tag{16}$$

$$\text{Mortgages: } A^B_t = A^R_{t,G} + A^R_{t,P} \tag{17}$$

$$\text{Housing: } K^B_t = \bar{K} - K^R - K^D \tag{18}$$

$$\text{Consumption: } Y_t = C^B_t + C^R_t + C^D_t + (1-\mu_{t,\omega})p_t\bar{K} + G^o_t + \zeta(\mu_{t,\omega} - Z_K(\omega^*_t))\frac{p_t K^B_{t-1}}{A^B_t} + \Psi(R^B_t, A^B_t)$$

The last equation states that total non-housing resources equal the sum of non-housing consumption expenditures and home renovations by the households, discretionary spending by the government, and lost resources due to the deadweight costs of foreclosure and mortgage refinancing.

3.7 Welfare

In order to compare economies that differ in the policy parameter vector Θ_t, we must take a stance on how to weigh the different agents. We propose a utilitarian social welfare function summing value functions of the agents according to their population weights ℓ: $\mathcal{W}_t(\cdot;\Theta_t) = \ell^B V_t^B + \ell^D V_t^D + \ell^R V_t^R$, where the $V^i(\cdot)$ functions are the value functions defined in the appendix. A nice feature of value functions under Epstein-Zin preferences is that they are homogeneous of degree one in consumption. Thus, a $\lambda\%$ increase in the value function from a policy change is also a $\lambda\%$ change in consumption units.

3.8 Computation

Appendix A presents the Bellman equations for each of the three household types and derives first-order conditions for optimality. Appendix A.5 contains an intuitive discussion of the main first-order conditions.

The presence of occasionally binding constraints and the default option for both borrowers and risk takers make this a challenging problem to solve. Appendix B discusses our non-linear global solution method.

4 Calibration

The parameters of the model and their targets are summarized in Table 1.

Aggregate Income The model is calibrated at annual frequency. Aggregate endowment or labor income Y_t follows:

$$Y_t = Y_{t-1}\exp(g_t), \qquad g_t = \rho_g g_{t-1} + (1-\rho_g)\bar{g} + \epsilon_t, \quad \epsilon_t \sim iid\,\mathcal{N}(0,\sigma_g)$$

We scale all variables by permanent income in order render the problem stationary. Given the persistence of income growth, g_t becomes a state variable. We discretize the g_t process into a 5-state Markov chain using the method of Rouwenhorst (1995). The procedure matches the mean, volatility, and persistence of GDP growth by choosing both the grid points and the transition probabilities between them. We use annual data on real per capita GDP growth from the BEA NIPA tables from 1929-2014 excluding the war years 1940-1945. The resulting mean is 1.9%, the standard deviation is 3.9%, and the persistence is 0.42. The states, the transition probability matrix, and the stationary distribution are listed in Appendix C.1.

Foreclosure crises The stochastic depreciation shocks or idiosyncratic house value shocks, $\omega_{i,t}$, are drawn from a Gamma distribution characterized by shape and a scale parameters $(\chi_{t,0}, \chi_{t,1})$. $F_\omega(\cdot; \chi_{t,0}, \chi_{t,1})$ is the corresponding CDF. We choose $\{\chi_{t,0}, \chi_{t,1}\}$ to keep the mean μ_ω constant at 0.975, implying annual depreciation of housing of 2.5%, a standard value (Tuzel (2009)), and to let the cross-sectional standard deviation $\sigma_{t,\omega}$ follow a 2-state Markov chain. Fluctuations in $\sigma_{t,\omega}$ govern the aggregate mortgage credit risk and represent the second source of exogenous aggregate risk. We refer to states with the high value for $\sigma_{t,\omega}$ as mortgage crises or foreclosure crises.

We set the two values $(\sigma_{H,\omega}, \sigma_{L,\omega}) = (0.10, 0.14)$ and the deadweight losses of foreclosure $(\zeta_H, \zeta_L) = (0.25, 0.425)$ in order to match the mortgage default rates and severities (losses given default) in normal times and in mortgage crises. In the benchmark model with low guarantee fees, and given all other parameter choices, these parameters imply equilibrium mortgage default rates of 1.6% in normal times and 12.7% in mortgage crises. The unconditional default rate is 2.7%. They imply equilibrium severities of 28.2% in normal times and 47.0% in crises. Mortgage default and severity rates combine to produce unconditional mortgage loss rates of 1.0% per year; 0.46% in normal times and 6.1% in crises. Appendix C.2 discusses the empirical evidence and argues that these numbers are a good match for the data. We note that the values for σ_ω are in line with standard values for individual house price shocks (Landvoigt, Piazzesi, and Schneider (2015)). Our unconditional severities are 30%, in line with typical values in the literature (Campbell, Giglio, and Pathak (2011)).

To pin down the transition probabilities of the 2-state Markov chain for $\sigma_{t,\omega}$, we assume that when the aggregate income growth rate in the current period is high (g is in one of the top three income states), there is a zero chance of transitioning from the $\sigma_{L,\omega}$ to the $\sigma_{H,\omega}$ state and a 100% chance of transitioning from the $\sigma_{H,\omega}$ to the $\sigma_{L,\omega}$ state. Conditional on low growth (g is in one of the bottom two income states) we calibrate the two transition probability parameters (rows have to sum to 1), p^ω_{LL} and p^ω_{HH}, to match the frequency and length of mortgage crises. Based on the argument by Jorda et al. (2014) that most financial crises after WW-II are related to the mortgage market and the historical frequency of financial crises in Reinhart and Rogoff (2009), we target a 10% probability of mortgage crises. Conditional on a crisis, we set the expected length to 2 years, based on evidence in Jorda et al. and Reinhart and Rogoff. Thus, the model implies that not all recessions are mortgage crises, but all mortgage crises are recessions.[19]

Population and wealth shares To pin down the labor income and housing shares for borrowers, depositors, and risk takers, we calculate a net fixed-income position for each household in the Survey of Consumer Finance (SCF).[20] Net fixed income equals total bond and bond-equivalent holdings minus total debt. If this position is positive, we consider a household to be a saver, otherwise it is a borrower. For savers, we calculate the amount of risky assets, defined as their holdings of stocks, business wealth, and real estate wealth, as well as the share of these risky assets in total wealth. We define risk takes as households that are within the top 5% of risky asset holdings and have a risky asset share of at least 75%. This delivers population shares of $\ell^B = 47\%$, $\ell^D = 51\%$, and $\ell^R = 2\%$. Based on this classification and the same SCF data, borrowers receive 38% of aggregate income and own 39% of residential real estate. Depositors receive 52% of income and 49% of housing wealth. Finally, risk takers receive 10% of income and 12% of housing wealth. By virtue of the calibration, the model thus matches basic aspects of the observed income and wealth inequality.

Mortgages In our model, a government-guaranteed MBS is a geometric bond. The issuer of one bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. The same is true if the borrower refinances the mortgage. We estimate values for δ and F such that the duration of the geometric mortgage in the model matches the duration of the portfolio of outstanding mortgage-backed securities, as measured by the Barclays MBS Index, across a range of historically observed

[19]In a long simulation, 33% of recessions are mortgage crises. This compares to a fraction of 6/22 (≈27%) in Jorda et al. (2014). The correlation between $\sigma_{t,\omega}$ and g_t is -0.42. The model generates persistence in the mortgage default rate of 0.02 in the low g-fee economy and 0.08 in the high g-fee economy. The persistence depends on, among other things, the persistence of $\sigma_{t,\omega}$ which is 0.48.

[20]We use all survey waves from 1995 until 2013 and average across them.

mortgage rates. This novel procedure, detailed in Appendix C.3, recognizes that the mortgage in the model represents the pool of all outstanding mortgages of all vintages. We find that values of $\delta = 0.95$ and $\alpha = 0.52$ imply a relationship between price and mortgage rate for the geometric mortgage that closely matches the price-rate relationship for a real-life MBS pool consisting of fixed-rate mortgages issues across a range of vintages. The average duration of the mortgage (pool) in model and data is about 4 years.

Borrowers can obtain a mortgage with face value up to a fraction ϕ of the market value of their house. We set the LTV ratio parameter $\phi = 0.65$ to match the average mortgage debt-to-income ratio for borrowers in the SCF of 130%. The calibration produces an unconditional mortgage debt-to-income ratio among borrowers of 148% in the benchmark model, somewhat overshooting the target. In the benchmark model, borrowers' mean loan-to-value ratios are 63.8% in book value and 76.2% in market value terms.

We set the marginal prepayment cost parameter ψ so that the benchmark model generates reasonable conditional prepayment speeds. We target an average rate of 15% annually, which is the historical average.

Government parameters Government fiscal policy consists of mortgage guarantee policy, a financial sector bailout policy, and general taxation and spending policies. To have a quantitatively meaningful model, we also match basic elements of non-housing related fiscal policy.[21]

Starting with the guarantee policy, our parameter γ specifies the cost of a guarantee, expressed in the same units as the price of the mortgage. Real-world guarantee fees are expressed as a surcharge to the interest rate. We consider several values for γ with implied g-fees ranging from 20 to 300 basis points. Freddie Mac's management and g-fee rate was stable at around 20bps from 2000 to 2012. Similarly, Fannie Mae's single-family effective g-fee was also right around 20bps between 2000 and 2009. Thus, our benchmark model is the 20 basis point g-fee economy. The main policy experiment in the paper is to raise γ and investigate the effects from higher guarantee fees. Interestingly, Freddie Mac has increased its g-fee gradually from 20bps at the start of 2012 to 32 bps at the end of 2014, while Fannie Mae has increased its g-fee from 20bps at the start of 2009 to 41 bps at the end of 2014 (Urban Institute Housing Finance Policy Center, December

[21] After the conservatorship of Fannie Mae and Freddie Mac in September 2008, the merger of the GSEs and the Treasury Department became a reality.

2014 update). Fannie's g-fees on new single-family originations currently average 63bps.

We set the proportional income tax rate equal to $\tau = 20.4\%$ in order to match average discretionary tax revenue to trend GDP in U.S. data. The discretionary tax revenue in the 1946-2013 data of 19.97% is after mortgage interest deductions, which is about 0.43% of trend GDP. Hence, we set a tax rate before MID of 20.4%.[22] As explained before, the model features mortgage interest rate deductibility at the income tax rate.[23] Tax revenues are pro-cyclical, as in the data. Every dollar of income is taxed at the same tax rate. Risk takers are only 2% of the population but pay 10% of the income taxes since they earn 10% of the income.

We set exogenous government spending equal to $G^o = 0.163$ (times trend GDP of 1) in order to match average exogenous government spending to trend GDP in the 1946-2013 U.S. data of 16.3%.[24] This exogenous spending is wasted. We also allow for transfer spending of 3.18% of GDP, which equals the net transfer spending in the 1946-2013 data. This spending is distributed lump-sum to the agents in proportion to their population share. As a fraction of *realized* GDP, expenditures fluctuate, mimicking their counter-cyclicality in the data.

We can interpret the risk-taker borrowing constraint parameters, $\varkappa$, ξ_G and ξ_P as regulatory capital constraints set by the government. Under Basel II and III, "first liens on a single-family home that are prudently underwritten and performing" enjoy a 50% risk weight and all others a 100% risk weight. Agency MBS receive a 20% risk weight. Given that we think of the non-guaranteed mortgage market as the subprime and Alt-A market, a capital charge of 8% (100% risk weight) seems most appropriate for ξ_P. Given that the government guaranteed mortgages are the counterpart to agency MBS, we set a capital charge of 1.6% (20% risk weight) for ξ_G. We set

[22] In our numerical work, we keep the ratio of government debt to GDP stationary by decreasing tax rates τ_t when debt-to-GDP threatens falls below $\underline{b^G} = 0$ and by increasing tax rates when debt-to-GDP exceedd $\overline{b^G} = 1.2$. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -30%. Tax rates are gradually and convexly increased until they hit 50% ay a debt-to-GDP ratio of 160%. Our simulations never reach the -30% and +160% debt/GDP states. These profligacy and austerity tax policies do not affect the amount of resources that are available for private consumption in the economy.

[23] Because our geometric mortgages do not distinguish between interest and principal payments, we assume that the entire mortgage payment is deductible but at a lower rate, $\tau^m = (1-\alpha) \times \tau$. As discussed in Appendix C.3, the sum of all mortgage payments is $1/(1-\delta)$ and $F = \alpha/(1-\delta)$ is the payment of "principal." Hence, the fraction of "interest payments" is the fraction $(1-\alpha)$. In the equilibrium with low g-fees, the mortgage interest deductibility expense is 0.46% of trend GDP, very close to the target.

[24] The data are from Table 3.1 from the BEA. Exogenous government spending is defined as consumption expenditures (line 18) plus subsidies (line 27) minus the surplus of government enterprises (line 16). It excludes interest service on the debt and net spending on social security and other entitlement programs. Government revenues are defined as current receipts (line 1), which excludes social security tax receipts. Trend GDP is calculated with the Hodrick Prescott Filter.

the additional margin $\varkappa$ to match average leverage ratios of the financial sector, given all other parameters. Since mortgage assets are predominantly held by leveraged financial institutions, we calculate leverage for those kinds of institutions. The average ratio of total debt to total assets for 1985-2014 is 95.6%.[25] Since the non-mortgage portfolio of these institutions have higher risk weights than their mortgage portfolio, we find that $\varkappa < 1$.

Utility cost of risk-taker bankruptcy The model features a random utility penalty that risk takers suffer when they default. Because random default is mostly a technical assumption, it is sufficient to have a small penalty. We assume ρ_t is normally distributed with a mean of μ_ρ =1, i.e., a zero utility penalty on average, and a small standard deviation of $\sigma_\rho = 0.05$. The mean size of the penalty affects the frequency of financial sector defaults (and government bailouts). The lower μ_ρ, the lower the resistance to declare bankruptcy, and the higher the frequency of bank defaults. The standard deviation affects the correlation between negative financial intermediary wealth and bank defaults. Given those parameters, the frequency of financial crises (government bailouts of the risk-taker) depends on the frequency of foreclosure crises, and the endogenous choices (asset composition and liability choice) of the risk taker.

Preference parameters Preference parameters are harder to pin down directly by data since they affect many equilibrium quantities and prices simultaneously. However, the discussion of the first-order conditions above helps us connect the various parameters to specific equilibrium objects they have a disproportionate effect on.

The coefficients of risk aversion are $\sigma_R = 1$, $\sigma_B = 8$, and $\sigma_D = 20$. The annual subjective time discount factors are $\beta_R = \beta_D = 0.98$ and $\beta_B = 0.88$. Risk aversion and the time discount factor of the depositor disproportionately affect the short-term interest rate and its volatility. The

[25] Specifically, we include U.S. Chartered Commercial Banks and Savings Institutions, Foreign Banking offices in U.S., Bank Holding Companies, Banks in U.S. Affiliated Areas, Credit Unions, Finance Companies, Security Brokers and Dealers, Funding Corporations (Fed Bailout entities e.g. Maiden Lanes), GSEs, Agency- and GSE-backed Mortgage pools (before consolidation), Issuers of ABS, REITs, and Life and Property-Casualty Insurance Companies. Krisnamurthy and Vissing-Jorgensen (2011) identify a group of financial institutions as net suppliers of safe, liquid assets. This group is the same as ours except that we add insurance companies and take out money market mutual funds, since we are interested in leveraged financial firms. For comparison, leverage for the Krisnamurthy and Vissing-Jorgensen institutions is 90.7% for the 1985-2014 sample. The group of excluded, non-levered financial institutions are Money Market Mutual Funds, other Mutual Funds, Closed-end funds and ETFs, and State, Local, Federal, and Private Pension Funds. Total financial sector leverage, including these non-levered institutions, is 60.6%.

benchmark model generates a mean one-year real risk-free interest rate of 1.1% with a standard deviation of 3.0%. In the data, the mean real interest rate is 1.2% with a volatility of 2.0% over the period 1985-2014.[26] The borrower's discount factor governs mortgage debt and ultimately house prices. In the model, housing wealth to trend GDP is 2.24, while in the Flow of Funds data (1985-2014) it is 2.41.[27] Borrower risk aversion is set to target the volatility of the annual change in household mortgage debt to GDP (Flow of Funds and NIPA), which is 4.2% in the 1985-2014 data. Our low g-fee economy produces a volatility of 2.8%. The risk takers have log period utility and their subjective discount factor is set equal to that of the depositors. We set the elasticity of inter-temporal substitution equal to 1 for all agents, a common value in the asset pricing literature.

5 Main Results: Phasing out the GSEs

The main experiment in the paper is to compare an economy with and without government-guaranteed mortgages. Specifically, we compute a sequence of economies that only differ by the mortgage guarantee fee γ_t set by the government. We compare equilibrium prices, quantities, and ultimately welfare across economies. All economies feature a government bailout guarantee to the financial sector, or equivalently, deposit insurance. We simulate each economy for 10,000 periods and report unconditional means and standard deviations across the simulations in Tables 2 and 3.

Our benchmark model is one where the government provides the mortgage guarantee relatively cheaply. We set γ_t to a value that implies an annual rate spread of 20bps.[28] The benchmark "low g-fee" case represents the period between the late 1990s and the late 2000s when g-fees were around 20bps. In the interest of space, we only report detailed results for two intermediate economies: the 55bps and 100bps g-fee cases. The former is of particular interest since it reflects

[26] To calculate the real rate, we take the nominal one year constant maturity Treasury yield (FRED) and subtract expected inflation over the next 12 months from the Survey of professional Forecasters. The mean interest rate is sensitive to the sample period. Over the period 1990-2014, the mean is 0.72% and over the period 1998-2014, it is only 0.21%.

[27] The number in the data includes the real estate owned by the corporate sector since our model is a model of the entire economy but does not include real estate-owning firms. Real estate owned by the household and non-corporate sector is 1.51 times GDP on average over this period.

[28] The interest rate on private bonds can be calculated as $r_{P,t} = \log\left(\frac{1}{q_t^m} + \delta\right)$, and the rate on guaranteed bonds is $r_{G,t} = \log\left(\frac{1}{q^m + \gamma_t} + \delta\right)$. The effective g-fee, quoted as a difference in rates, is therefore given by $r_{P,t} - r_{G,t}$.

the level of g-fees observed today. The latter is of interest because, at a 100bp g-fee, guaranteed mortgages turn out to dominate during mortgage crises while private bonds are dominant in normal times. This outcome is reminiscent of Option B in the Obama Administration's policy document of February 2010 which envisions setting the g-fee high enough so that it is only taken up in crises. Finally, we report on a "high g-fee" economy, the 275bps g-fee economy, where guarantees are expensive enough that they are never bought. In this last economy, the GSEs are "phased out" and the mortgage market functions as a private market without government intervention (except for deposit insurance).

5.1 Prices

The first panel of Table 2 shows that risk-free and mortgage interest rates are low and house prices are high in the benchmark low g-fee economy. As g-fees rise, risk-free and mortgage interest rates rise while house prices fall. The cheap mortgage guarantees lead to mortgage interest rates of 3.52%, 23 basis points lower than in the high g-fee economy. This magnitude of subsidy to mortgage rates is similar to what the empirical research has inferred form the spread between conforming and jumbo mortgage loans.

To show that the mortgage guarantee is indeed underpriced, we compute the actuarially fair guarantee fee. It is the fee that a hypothetical risk-neutral agent with the same degree of patience as the savers would charge for the mortgage guarantee payment upon a default ($M_G - M_P$). The actuarially fair g-fee depends on the model in which it is computed. In the 20bp g-fee economy, whose equilibrium displays financial fragility, the actuarially fair g-fee is 76bps.

Short-term interest rates vary more substantially across economies, with 1.2% annual real interest rates in the low g-fee economy, 70 bps lower than in the high g-fee economy. The key reason for low real interest rates is that the low g-fee economy is riskier, as explained further below. Depositors, who often are the marginal agents in the risk-free bond market and who are the most risk averse of all agents, have strong precautionary savings motives which push down interest rates.

The entire rise in mortgage rates is due to the rise in risk-free interest rates. Mortgage spreads, the difference between mortgage and risk-free interest rates, go down as the g-fee rises, a reflection of declining mortgage default rates.

Faced with low mortgage rates, borrowers who are the marginal agents in the housing sector demand more housing. Given a fixed housing supply (relative to trend growth), increased housing demand results in higher house prices. The low g-fee economy's house prices are 6.2% higher than in the high g-fee economy. Thus, phasing out the GSEs would lead to a non-trivial decline in house prices.

House prices are also more volatile in the low g-fee economy: 0.14 annual standard deviation compared to 0.12 in the high g-fee economy. This is a consequence of the higher volatility in the demand for mortgage debt in the low g-fee economy, as discussed further below.

5.2 Borrowers

Faced with high house prices and low mortgage rates, borrowers demand more mortgage debt. The steady state stock of mortgages outstanding is high in the low g-fee equilibrium (0.053 units A^B or 63.3% of GDP in market value terms). The average borrower LTV ratio is 63.7% and borrowers' mortgage debt-to-income is 1.49 on average, both are close to the averages for borrowers in SCF data. When g-fees and mortgage rates rise, the size of the mortgage market shrinks. The mortgage market also becomes safer: the mortgage debt-to-income ratio drops by 9% points.

We recall that the optimal mortgage default policy for the borrower family depends on the mark-to-market LTV ratio (equation 65). That ratio is the highest and thus mortgages are the riskiest in the low g-fee economy. The average mortgage default rate is 2.7% while the average severity rate (loss given default) is 30.2%. Both match the data. They deliver a mortgage loss rate which is 1.0% on average. Both default and loss rates fluctuate substantially across aggregate states of the world (output growth and mortgage crisis vs. normal times). Loss rates from mortgage defaults are 6.0% in housing crises but less than 0.5% in normal times.

In the high g-fee economy, the mortgage default rate is 1.7%, a reduction by almost 40% compared to the low g-fee economy. Since the severity rate is the same across economies, this translates in a loss rate of 0.7% unconditionally. The lower mark-to-market LTV ratio implies a lower mortgage default rate. It is itself the result of lower mortgage prices q^m a and smaller amount of mortgage debt and occurs despite lower house prices. The first sense in which the private sector economy is safer is that borrowers have more home equity and mortgages default less frequently. Fewer foreclosures lead to less deadweight losses from foreclose, a reduction in deadweight losses. The reduction in mortgage loss rates results in lower mortgage spreads in the high g-fee economy.

In the benchmark economy, mortgage debt is not only higher, it is also more volatile across aggregate states of the world. Specifically, there is a larger drop in mortgage credit during crises episodes (high σ_ω states). An oft-invoked rationale for government guarantees is to ensure the stable provision of mortgage credit at all times.[29] Our measure of the stability of the provision of mortgage credit is the standard deviation of mortgage debt to income growth. This volatility is 2.8% in the low g-fee economy. Surprisingly, this volatility initially *decreases* to 2.7% as the g-fee increases to 55bp. As we will see below, banks are better capitalized in the 55bp economy. The volatility inches back up as g-fees increase further and banks take on more credit risk. As we approach the private market economy, the volatility reaches a level equal to that in the low g-fee economy. Even in crisis periods, the decline in mortgage credit relative to income is smaller in absolute value in the high g-fee economy than in the low g-fee economy. The popular fear that a private mortgage system would lead to large swings in the availability of mortgage credit, especially in bad times, is unwarranted in our model.

In terms of their prepayment decisions, borrowers prepay 15.8% of non-defaulted mortgages on average, matching historical data. As the g-fee rises, prepayment rates go down slightly. Higher equilibrium mortgage rates reduce the benefit from refinancing. Financial intermediaries face more mortgage credit risk but less prepayment risk in the private sector economy.

5.3 Risk Takers

The third panel of Table 2 reports on the risk takers. As financial intermediaries, they make long-term mortgage loans to impatient borrower households and borrow short-term from patient depositor households. They play the traditional role of maturity transformation. Given their low risk aversion, they are the most willing to bear fluctuations in their net worth among all agents. Given sufficient intermediary capital, they can absorb a disproportionate amount of aggregate risk.

[29] Indeed, Fannie Mae was founded in the Great Depression when a massive default wave of banks threatened the supply of mortgage credit. By guaranteeing mortgages, it is widely believed to make mortgage markets more liquid thereby ensuring that banks are willing to lend even in bad times.

Low g-fees In the low g-fee economy, risk takers hold nearly all of their assets in the form of government-guaranteed bonds. They buy mortgage guarantees both in normal times and in mortgage crises (high σ_ω) states, taking advantage of the cheap mortgage guarantees provided by the government. As a result, the asset side of their balance sheet is largely shielded from mortgage default risk. Bearing little default risk on their assets and facing a low interest rate on safe deposits, banks use substantial leverage in order to achieve their desired risk-return combination. The intermediary leverage constraint allows banks who exclusively hold guaranteed mortgage bonds to have a maximum leverage ratio of 96.4%. Banks hit that constraint in 32.6% of the periods. The average bank leverage (market value of debt to market value of assets) ratio is 95.6%, matching the data. Average risk taker wealth is modest, at 2.9% of trend GDP. Banks have little "skin in the game." The constraint binds more frequently in normal times (34.7%) than in crises (14.5%) because of precautionary deleveraging in crises. Leverage is pro-cyclical in the low g-fee economy.

In addition to taking risk through leverage, banks in the low g-fee economy have a larger balance sheet of mortgages. Due to the combination of low risk taker wealth, high leverage, and a large and risky mortgage portfolio, the banking system is fragile. When adverse income or mortgage credit shocks hit, risk taker net worth falls. The reason this happens despite the prevalence of guaranteed mortgages on intermediaries' balance sheets is mark-to-market losses on guaranteed bonds. Mortgage defaults act as prepayments for holders of guaranteed bonds. Since agency bonds trade above par prior to prepayment, but prepayments come in at par, prepayments constitutes a loss for the holder or agency MBS.[30] The average mark-to-market loss rate given a prepayment is 10.7% (12.6% in crises). The overall loss rate on banks' mortgage portfolio is 0.37% (0.22% in normal times and 1.65% in crises). Losses are nearly entirely due to prepayment-related (both default- and rate-induced) losses rather than due to credit losses/mortgage arrears.

When intermediary net worth turns negative, which occurs in 0.27% of the simulation periods, the government steps in to bail out the financial sector. Thus mortgage crises can trigger financial crises, a relationship documented in the work of Jorda et al. (2014). The table reports the return on risk-taker wealth. It is 3.2% excluding the bankruptcy events, but 2.9% including such events (unreported). This difference illustrates the option value introduced by the possibility to go bankrupt. The return on risk-taker wealth is 13% in a crisis, consistent with the result in

[30] Put differently, prepayments happen when interest rates are low and reinvestment opportunities are poor.

intermediary-based asset pricing literature that risk premia increase when intermediary capital is scarce.

Higher g-fees As g-fees rise, the composition of the risk taker portfolio shifts towards private bonds. In the 55bp economy, guaranteed bonds still make up 98.4% of the portfolio. This is consistent with the situation today, where g-fees have risen to about 55bp and yet guaranteed bonds continue to dominate.

When g-fees go up to 100 bps, banks guarantee only 11% of their portfolio. This dramatic reversal occurs because risk takers buy the guarantee only in crises, when guaranteed bonds constitute 94% of their portfolio. In good times they prefer an all-private portfolio. The state uncontingent g-fee is too cheap to forego in bad times but too expensive in normal times. This 100 basis point g-fee economy is reminiscent of Option B of the Obama Administration housing reform plan, which envisions a g-fee level that is high enough so that it would only be attractive in bad times (Scharfstein and Sunderam, 2011).

In the high g-fee economy, risk takers shift exclusively towards holding private MBS. They do not buy default insurance from the government, neither in good nor in bad times. The 275 basis point g-fee is high enough to "crowd-in" the private sector at all times. The 275bp g-fee economy implements Option A in the Obama plan which envisions an entirely private mortgage market.

Our main result is that increasing the g-fee lowers the riskiness of the financial sector. Risk taker leverage falls from 95.6% in the 20bp and 95.2% in the 55bp economy to 89.0% in the 100bp economy, and 88.2% in the high g-fee economy. As the portfolio shifts towards private mortgages, the risk taker's collateral constraint becomes tighter because private mortgages carry higher regulatory capital requirements ($\xi_P > \xi_G$). But this is not the main driver of the lower leverage. Rather, banks choose to stay away from their leverage constraint in most periods; their leverage constraint binds in less than 20% of periods compared to 33% in the low g-fee economy. Having to bear mortgage credit risk, banks attain their desired high risk-high return portfolio without the need of having to lever up as much.

The financial sector is well-enough capitalized (7.0% equity) and has low enough leverage that it can guarantee the stable provision of mortgage credit in good and bad times, relative to a system with a government backstop where banks are poorly capitalized and prone to occasional collapses. Higher g-fees reduce the incidence of bank insolvencies and concomitant bank bailouts. Interestingly, risk taker leverage becomes counter-cyclical for higher g-fees as the intermediaries are sufficiently strong to desire an increase in lending in bad times.[31] In sum, low g-fees create moral hazard: faced with cheap mortgage guarantees which offload mortgage credit risk onto the taxpayer, banks endogenously increase leverage, make more and riskier mortgages. The incidence of foreclosure increases and ultimately increases the fragility of the financial system.

The model endogenously generates a negative relationship between the amount of default and prepayment risk. In the high g-fee economy, banks face less prepayment risk (CPRs are 4% points lower) but more of the credit risk. However, there is less credit risk to be born because the mortgage default rate is lower and the mortgage portfolio smaller. In the low g-fee economy there is endogenously more credit risk and more prepayment risk created, but banks only bear the latter.

5.4 Depositors and Government

Depositors have high risk aversion and thus loathe large fluctuations in consumption across states of the world. Their strong precautionary savings demand makes them willing to lend to intermediaries and the government at low interest rates. Deposit insurance is important because it makes depositors' claims on the banks risk-free, irrespective on the riskiness of the bank's balance sheet.

Low G-fees The high mortgage loss rates are mostly absorbed by the government, as are the occasional bank insolvencies. Both lead to a surge in government expenditures, financed with government debt. Government debt to trend GDP is 15.9% on average with a high standard deviation of 22.6%. Tax revenues only slightly exceed government discretionary and transfer spending, and in the aftermath of a severe mortgage crisis it takes many years of small surpluses as well as the absence of a new mortgage crisis to reduce government debt back to steady state levels.

[31] Our result is a cousin of the volatility paradox in Brunnermeier and Sannikov (2014) where an increase in the fundamental volatility of the asset endogenously reduces the risk appetite of the intermediaries.

Depositors must hold not only the debt issued by the banks (deposits) but also the debt issued by the government. High risk taker leverage and high government debt in the low g-fee economy lead to high equilibrium holdings of short-term debt by depositors. All else equal, the large supply should result in a low price of short-term debt, or equivalently, a high interest rate to induce the depositor to hold all that debt.[32] Yet on average the low g-fee economy exhibits a *low* average equilibrium interest rate. The reason is that the precautionary savings demand more than offsets the supply effect.

During mortgage crises government debt shoots up as the government pays out on mortgage guarantees and occasionally on bank bailouts. This increase in the supply of short-term debt by the government is only somewhat offset by lower risk taker leverage, so that on net the supply of bonds grows during crises. The ultra low real interest rates in crises make debt issuance attractive for the risk taker and government alike. The depositor absorbs this debt increase in equilibrium by increasing savings and reducing consumption.

In sum, by protecting the financial sector from mortgage defaults, the government shifts more of the consumption fluctuations across states of the world onto the depositor. By virtue of the depositor's high risk aversion, she is more unwilling to bear such consumption fluctuations than the risk taker. The depositor responds to the "fiscal uncertainty" by saving a lot more at all times to absorb at least some of the fluctuations in government debt with existing savings. The result is a low equilibrium interest rate and low average financial income for the depositor. The low interest rate is a signature of the large amount of risk in the low g-fee economy.

High G-fees The high g-fee economy witnesses the same fraction of mortgage crises. But these crises result in much lower mortgage loss rates. Furthermore, the mortgage losses are no longer borne by the government but rather absorbed by the intermediaries' balance sheet. Average government debt and its volatility fall precipitously. The lower equilibrium supply of both government debt and risk taker debt (deposits) would result in a lower interest rate if savers were risk neutral.[33] But overall interest rates are *higher* in the high g-fee economy because the precautionary savings effect again dominates given the high risk aversion of the depositors. A

[32]Indeed, the interest rate during income contractions that coincide with mortgage crises (-1.7%) is a percentage point higher than the interest rate during contractions that do not coincide with crises (-2.7%).

[33]Indeed, the large difference between crisis and non-crisis interest rates attributable to additional supply of risk-free debt, holding aggregate income constant, entirely disappears.

safe economy without financial fragility and therefore with low and predictable government debt induces depositors to scale back their precautionary saving demand. The fall in demand for safe assets exceeds the decline in the supply, explaining higher equilibrium real interest rates.

In summary, when the g-fee is high enough, risk takers are well enough capitalized and their intermediation capacity is rarely impaired. They bear and hence internalize all mortgage default risk. In contrast, in the low g-fee economy, mortgage crisis episodes frequently disrupt risk takers' intermediation function. During these crises, the risk free rate drops sharply and government debt increases sharply, effectively making depositors bear a greater part of the mortgage default risk.

5.5 Welfare

We measure aggregate welfare as the population-weighted average of the value functions of the three types of agents.[34] The first row of Table 3 shows that it is 0.63% higher in the high g-fee economy than in the benchmark low-g-fee economy. In unreported results for a series of intermediate g-fee economies, we find that aggregate welfare increases monotonically in the g-fee. We consider the 0.63 percent improvement in consumption equivalence terms from GSE reform to be a substantial effect.

There are two effects that help understand the aggregate welfare gain: an improvement in risk sharing and a reduction in deadweight losses. First, risk sharing between the different types of agents generally improves as g-fees increase. To measure the extent of the improvement, we compute the ratios of (log) marginal utilities between the different types. If markets were complete, agents would be able to achieve perfect risk sharing by forming portfolios that keep these ratios constant. Hence, larger volatilities of these marginal utility (MU) ratios indicate worse risk sharing between the different types of agents. Table 3 lists the average MU ratios and their volatilities for borrowers/risk takers, and risk takers/depositors, as these are the pairs of agents that directly trade with each other. The volatilities of both ratios is lower in the high g-fee economy than in the low g-fee economy. The MU ratio volatility between borrower and risk taker falls by 22.6%. The decline in the MU ratio volatility between risk taker and depositor is

[34] With unit EIS, the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore, increases in aggregate welfare can be directly interpreted as consumption-equivalence gains.

7.9%. We see similarly large improvements in risk sharing if we look at consumption volatility for the three types of agents: -9.2% for the borrower, -20.5% for the depositor, and -9.1% for the risk taker.[35] Intermediary wealth is a crucial driver of the overall degree of risk sharing between the agents in the economy. In the private economy, banks are better able to provide consumption smoothing services to both borrowers and depositors because they are better capitalized and less fragile. Improved risk sharing also increases the risk-free rate and therefore mean consumption for savers.

The second source of the welfare gain is a reduction in deadweight losses. The first deadweight loss is the one associated with mortgage foreclosures. Lower deadweight costs leave more resources for private consumption each period. This deadweight loss is 0.58% of GDP in the 20bps economy and 0.35% of GDP in the 275bp economy. While the deadweight loss falls by 39%, it remains modest. The reduction in DWL from foreclosure accounts for 53% of the overall welfare gain from a GSE phaseout.[36] The economy also benefits from lower deadweight losses from prepayment costs in high g-fee economies since prepayment rates are lower. These losses are of the same magnitude as those from foreclosures and fall by about the same percentage. Finally, there is a reduction in housing maintenance costs going from the low to the high g-fee economy because maintenance expenses are proportional to the value of the house. A reduction in housing consumption leaves more resources for non-housing consumption. Combined, the increases in resources benefits the depositor and risk taker, both of which increase mean consumption. The borrower's consumption declines because she faces higher mortgage interest rates in the high g-fee economy.

What are the distributional consequences of a mortgage market privatization? Close inspection of the value function of each of the three household types shows that the borrower's welfare stays almost constant between the low and the high g-fee economies (+0.04%), while depositor

[35] While the risk sharing between borrower and risk taker improves monotonically as g-fees rise, the risk sharing between the risk taker and the depositor is highest at an intermediate g-fee closest to the actuarially fair g-fee (around 65bp). Similarly, the volatility of consumption for the risk taker is the lowest at that g-fee. At low g-fee levels, the high leverage and risk-taking of intermediaries makes their consumption volatile. As the g-fee rises from 20bps to about 65bps, leverage falls sharply but the risk taker's portfolio is still largely protected by government guarantees. However, as g-fees rise further above 65bps, the risk taker portfolio tilts towards private bonds, and this makes consumption volatility rise again, despite further reductions in leverage.

[36] To obtain this number, we solve a model where the DWLs from foreclosure are redistributed to all agents in lump-sum fashion, thereby increasing their mean consumption and welfare proportionally. We find a welfare gain from phasing out the GSEs of 0.30% excluding the DWLs from foreclosure, compared to a baseline welfare gain of 0.63%. Thus, the reduction in DWLs from foreclosure that accompanies a GSE phase out accounts for 53% of the welfare gain.

welfare (+1.30%) and risk taker welfare (+1.69%) both increase substantially.[37] The near-absence of a welfare loss for borrowers is surprising since taking away underpriced mortgage guarantees increases mortgage rates and lowers property values. The important offset to a decline in her consumption comes from the improvement in risk sharing. In conclusion, while GSE reform is a Pareto improvement, it redistributes wealth from borrowers to savers thereby raising inequality.

In Figure 2, we study a transitional experiment rather than a steady-state comparison. We assume that the economy starts in the 20-bp g-fee equilibrium at typical values of the state variables. The economy then undergoes a once-and-for-all change to the 275bps g-fee economy. We find that prices adjust rapidly, while state variables such as the wealth distribution adjust gradually. As a result of the sudden rise in interest rates and decline in house prices, borrowers' value function falls upon impact. It only slowly recovers as the intermediary sector's wealth accumulation gradually facilitates better risk sharing. Depositors and risk takers gain upon impact as well as in the long-run. Aggregate welfare rapidly stabilizes.

6 Alternative Policy Experiments

6.1 State-contingent Guarantees

Figure 3 shows the actuarially fair g-fee for the low and high g-fee economies as well as several intermediate economies. The solid line shows the unconditional average, the dashed line the actuarially fair g-fee in crises, and the dotted line the fair g-fee in normal times. The figure also draws in the 45-degree line along which actual and actuarially fair g-fees are equal. We note that the actuarially fair g-fee is decreasing in the exogenous g-fee (solid line with circles). The higher the g-fee, the safer the mortgages are, and the more stable the financial sector. To break even, a risk neutral insurer could charge a lower average rate. The actuarially fair g-fee declines from 76 bps in the 20bp economy to 53 bps in the 275bp economy. The fixed point where the actual and fair g-fees equate is around 60bp. Relative to the risk-neutral benchmark, mortgage guarantees are overpriced *on average* in the economies with a g-fee above 60bp and

[37]Given the homogeneity properties of the value function, log changes in value functions are directly interpretable as consumption equivalence changes and hence directly comparable across agents.

unconditionally underpriced in economies with g-fees below 60bp.[38]

The figure also makes clear that the actuarially fair g-fee is state contingent. During mortgage crises (high σ_ω states), the mortgage loss rate is a lot higher and a much higher g-fee must be charged to break even (dashed line with squares). For g-fees below 150bp, risk takers would always want to buy guarantees in crisis times since risk takers are not risk neutral but risk averse. We must go above 275bp to make guarantees expensive enough so that they are (almost) never bought in any of the states of the world. The 100bp economy is an interesting case. Risk takers overwhelmingly buy the mortgage guarantee in crisis periods but overwhelmingly hold private mortgage bonds in normal times. The actuarially fair g-fee in that economy is 155bp in crisis times while it is 46bp in normal times. Thus, the 100bp non-state contingent guarantee fee is attractively priced only in crises.

We study a policy experiment where the government charges a high g-fee of 100bps in good times (expansions and normal mortgage credit risk states) and a low g-fee of 55bps in bad times (mortgage crises and recessions). This policy has been proposed by Scharfstein and Sunderam (2011) as well as in the Obama Administration's policy paper of 2010, known as long-run Option B. The results are reported in columns 5 and 6 of Table 4. There is not much to be gained by making g-fees time-varying. Welfare is higher than in the fixed 55bps economy, and lower to than in the fixed 100bps economy. Comparing the riskiness of the fixed 55bp and the counter-cyclical g-fee economy, there are two effects. On one hand, higher g-fees in booms reduce risk, consistent with our previous experiments. On the other hand, time variation in g-fees introduces a new source of risk, because next period's g-fee affects next period's prices. The offset from the latter reduces the appeal of a counter-cyclical g-fee.

6.2 Catastrophic Insurance

Our framework is well-suited to quantitatively evaluate a recent legislative proposal in the U.S. Senate Banking Committee, the Johnson-Crapo or JC proposal.[39] The proposal envisions chang-

[38] Appendix D.5 discusses whether there is there scope for welfare-enhancing private mortgage insurance in our model.

[39] The "Housing Finance Reform and Taxpayer Protection Act of 2014" introduced by senators Corker and Warner preceded the draft bill introduced by Senators Johnson and Crapo and voted in the Senate Committee on Banking, Housing, and Urban Affairs on May 15, 2014. The 13-9 vote was not strong enough to force a full Senate floor vote.

ing the nature of the government-provided mortgage guarantee by mandating mortgage lenders to hold a substantial buffer of private capital. Having more private capital at risk, mortgage underwriting would be more prudent and intermediary moral hazard would be diminished. Under the proposal, the private sector would shoulder a 10% mortgage loss rate. Only losses above 10% would be absorbed by the government. Our paper is the first to provide a detailed quantitative analysis of Johnson-Crapo, including all the general equilibrium effects on risk taking, interest rates, house prices, and the distributional effects for the various types of tax payers.

In the JC model, if the loss on a guaranteed mortgage bond is less than 10%, the guarantee is worthless and the guaranteed bond has the same payoff as a private mortgage bond. If the loss is higher than the threshold, the guaranteed bond pays out an amount equal to the losses above the threshold. For ease of comparison, we keep the regulatory capital advantages of guaranteed bonds from the benchmark economy. We assume that the government offers the catastrophic insurance at a g-fee of 20bp.[40] We compute the actuarially fair cost of the catastrophic guarantee, at the new equilibrium. The last two columns of Table 4 present the results.

The JC economy is similar to the high g-fee economy in several aspects. It has lower house prices, higher mortgage rates, a smaller mortgage market, and lower mortgage default rates. The JC guarantee's actuarially fair cost is 2 bps. Insurance is quite *over*priced at 20bp. As a result, risk takers hold fewer guaranteed bonds (28% of the portfolio). The risk takers' portfolio loss rate is 0.55%, which is 48% higher than in the benchmark economy, but 16% lower than in the high g-fee economy. The guaranteed bonds are still substantially safer than uninsured bonds. Absent the severely underpriced guarantee, borrower leverage is lower at 91.6%. The protection offered by the catastrophic guarantee increases banks' appetite to resume lending after a mortgage crisis. They run more often into binding bank capital constraints (76% on average, 87% in crises).

The welfare gain from transitioning from the low g-fee to the JC economy is 0.66%. This gain is similar to, and in fact slightly larger than that in our benchmark policy experiment where it was 0.63%. Borrowers' welfare changes in the almost identical way as from a phase-out of the guarantee (+0.06%). Depositors gain slightly more than in the main experiment (+1.36% vs. +1.30%), while risk takers gain less (+1.24% versus +1.69%). The biggest difference is for

[40] This assumption does not significantly affect results. Appendix D.2 reports results from two additional experiments. In the first one, the catastrophic insurance is priced at 5bp. The second one keeps the g-fee at 20 bp but provides insurance for losses in excess of 5%, rather than 10% percent. The real estate industry has retorted that a 10% private loss rate is too high and proposed 5% instead. Results are qualitatively similar.

risk takers and arises from the additional improvement in their consumption smoothing. The catastrophic guarantee protects the banks in very adverse states of the world. While in the high g-fee economy the 0.1-percentile of risk taker consumption is 0.049, the same percentile is 0.055 in the JC economy (mean risk taker consumption is approximately 0.075 in both economies). The better insurance banks enjoy makes the economy less risky and raise the equilibrium interest rate even (6bp) higher than in the high g-fee economy.

7 Great Recession Experiment

This section explores implications of the model for foreclosures and house prices in periods of low aggregate economic growth and high credit risk, lie the Great Recession in 2008-10 in the United States.

In the benchmark model, all borrowers with negative equity default right away (recall equation 65). To capture the foreclosure delays, which were rampant in judiciary states, we modify our model slightly.[41] In the extension, we still assume that borrowers with negative equity miss their mortgage payment. But if the economy is in a mortgage crisis, we assume only a fraction of those default right away. We set this fraction to half. If a borrower is still underwater the following period and the the economy is still in the high σ_ω state, there is again a 50% probability of foreclosure. This mechanism spreads out foreclosures over multiple years during a prolonged crisis. It also reduces the total foreclosure rate relative to the benchmark model.

Once the extended low g-fee model is solved, we feed in a particular sequence of aggregate shocks mimicking the economic conditions of the period 2001–2013.[42] The left panel of Figure 4 plots GDP (Y) resulting from the exogenous growth dynamics we fed into the model. GDP peaks in 2007 and bottoms out in 2010.

The middle panel shows the (endogenous) loss rate on mortgages. As in the data, there are

[41]The average length of a foreclosure processes increased dramatically during and after the financial crisis from about 200 days nationwide in 2007 to 630 days in 2015. Foreclosure processes lasting 2.5-3 years were not uncommon in judiciary foreclosure states such as New Jersey, Florida, and New York according to Realty Trac.

[42]Specifically, we assume the state variable σ_ω takes on its normal-times value in 2001–2007 and 2011–2013, but its higher crisis value in 2008-2010. For the second aggregate state variable, economic growth g, we assume the following sequence. In 2005–2007, the economy is in a strong expansion (highest of 5 grid points). In 2002–04 and 2011–13, the economy is in a mild expansion. In 2001 and 2010, the economy is in a mild recession, and in 2008 and 2009, it is in a severe recession.

almost no mortgage losses until 2007. In 2008, the foreclosure crisis starts and mortgage loss rates spike. Because of the foreclosure delay mechanism, foreclosures and mortgage losses remain elevated in 2009 and 2010. The right panel plots the house prices. It shows that the model is able to generate a 22% drop in house prices between 2007 and 2008. This accounts for about 2/3rds of the observed house price drop in the data and shows that the economy has enough risk to generate meaningful house price dynamics. As in the data, house prices are back to 2003 levels by 2013.

8 Conclusion

Underpriced, government-provided mortgage default insurance leads to moral hazard in the financial sector. Banks become too levered, make too many mortgages, and make riskier mortgages. House prices are too high, mortgage and risk-free interest rates are too low. Mortgage default and loss rates are too high and mortgage crises may lead to the insolvency of the financial sector. Even though the government can mitigate the fallout from such crises by spreading out the costs over time by issuing bonds, savers must buy these bonds at inopportune times. The resulting allocation of risk is suboptimal.

We document a substantial welfare gain from moving to a private mortgage system, a transition which can be effectuated by raising the cost of the government mortgage guarantees. The private market provides a safer financial sector with fewer mortgage foreclosures and better intermediation between borrowers and savers. It features lower fiscal volatility. While the policy change is a Pareto improvement, it benefits depositors and bankers more and raises wealth inequality. We find that recent policy proposals in which the government only provides catastrophic loss insurance behind private loss-bearing capacity strike a good balance between keeping banks' moral hazard at bay while providing some backstop for the financial system in very bad states of the world.

References

Acharya, V. V., M. Richardson, S. Van Nieuwerburgh, and L. J. White (2011): *Guaranteed To Fail: Freddie, Fannie, and the Debacle of U.S. Mortgage Finance*. Princeton University Press.

Adelino, M., A. Schoar, and F. Severino (2014): "Credit Supply and House Prices: Evidence from Mortgage Market Segmentation," working Paper MIT Sloan and Duke Fuqua.

Adrian, T., and N. Boyarchenko (2012): "Intermediary leverage cycles and financial stability," *Working paper*.

Arslan, Y., B. Guler, and T. Taskin (2013): "Joint Dynamics of House Prices and Foreclosures," Working Paper, Indiana University.

Brunnermeier, M. K., and Y. Sannikov (2014): "A macroeconomic model with a financial sector," *The American Economic Review*, 104(2), 379–421.

Campbell, J. Y., S. Giglio, and P. Pathak (2011): "Forced Sales and House Prices," *American Economic Review*, 101, 2108–31.

Chambers, M., C. Garriga, and D. E. Schlagenhauf (2009): "Housing Policy and the Progressivity of Income Taxation," *Journal of Monetary Economics*, 56(8), 1116–1134.

Chatterjee, S., and B. Eyigungor (2009): "Foreclosures and House Price Dynamics: A Quantitative Analysis of the Mortgage Crisis and the Foreclosure Prevention Policy," Federal Reserve Bank of Philadelphia Working Paper 09-22.

Chu, Y. (2014): "Credit Constraints, Inelastic Supply, and the Housing Boom," *Review of Economic Dynamics*, 17(1), 52–69.

Corbae, D., and E. Quintin (2014): "Leverage and the Foreclosure Crisis," *Journal of Political Economy*, forthcoming.

Davis, M. A., and S. Van Nieuwerburgh (2015): *Handbook of Regional and Urban Economics*chap. Housing, Finance, and the Macroeconomy, p. Chapter 12. North Holland.

Deng, Y., J. M. Quigley, and R. Van Order (2000): "Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options," *Econometrica*, 68(2), 275–307.

Drechsler, I., A. Savov, and P. Schnabl (2014): "A Model of Monetary Policy and Risk Premia," *Working paper*.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh (2015): "The Macroeconomic Effects of Housing Wealth, Housing Finance and Limited Risk Sharing in General Equilibrium," *Journal of Political Economy*, Unpublished paper, New York University.

Garriga, C., and D. E. Schlagenhauf (2009): "Home Equity, Foreclosures, and Bailouts," Working Paper, Federal Reserve Bank of St. Louis.

Gervais, M. (2002): "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, 49(7), 1461–1489.

Gete, P., and F. Zecchetto (2015): "Distributional Implications of Government Guarantees in Mortgage Markets," Working Paper Georgetown.

Hansen, G. D. (1985): "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309–327.

Hart, O. (1975): "On the Optimality of Equilibrium when the Market Structure is Incomplete," *Journal of Economic Theory*, 11, 418–443.

He, Z., and A. Krishnamurty (2013): "Intermediary asset pricing," *American Economic Review*, 103 (2), 732–770.

Hedlund, A. (2014): "The Cyclical Dynamics of Illiquid Housing, Debt, and Foreclosures," Working Paper, University of Missouri.

Jeske, K., D. Krueger, and K. Mitman (2013): "Housing, Mortgage Bailout Guarantees and the Macro Economy," *Journal of Monetary Economics*, 60(8).

Jorda, O., M. Schularick, and A. Taylor (2014): "Betting the House," NBER Working Paper No. 20771.

Judd, K. L. (1998): *Numerical Methods in Economics*, vol. 1 of *MIT Press Books*. The MIT Press.

Judd, K. L., F. Kubler, and K. Schmedders (2002): "A solution method for incomplete asset markets with heterogeneous agents," Working Paper, SSRN.

Kubler, F., and K. Schmedders (2003): "Stationary Equilibria in Asset-Pricing Models with Incomplete Markets and Collateral," *Econometrica*, 71(6), 1767–1793.

Landvoigt, T. (2012): "Aggregate Implications of the Increase in Securitized Mortgage Debt," Working Paper, Stanford University.

Landvoigt, T., M. Piazzesi, and M. Schneider (2015): "The Housing Market(s) of San Diego," *American Economic Review*, 105(4), 1371–1407.

Reinhart, C. M., and K. Rogoff (2009): *This Time is Different*. Princeton University press.

Rouwenhorst, K. G. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *rontiers of Business Cycle Research*, ed. by T. Cooley. Princeton University Press.

Scharfstein, D., and A. Sunderam (2011): *The Future of Housing Finance*chap. The Economics of Housing Finance Reform. Brookings Institution Press.

Sommer, K., and P. Sullivan (2013): "Implications of U.S. Tax Policy for House Prices, Rents and Homeownership," Working Paper, Federal Reserve Board of Governors.

Tuzel, S. (2009): "Corporate Real Estate Holdings and the Cross Section of Stock Returns," Unpublished paper, Marshall School, University of Southern California.

Table 1: Calibration

This table reports the parameter values of our model.

Parameter	Description	Value	Target
		Exogenous Shocks	
$\bar{g}$	mean income growth	1.9%	Mean rpc GDP gr 29-13
σ_g	vol. income growth	3.9%	Vol rpc GDP gr 29-13
ρ_g	persistence income growth	0.41	AC(1) rpc GDP gr 29-13
μ_ω	mean idio. depr. shock	2.5%	Housing depreciation Census
σ_ω	vol. idio. depr. shock	{0.10,0.14}	Mortgage default rates (Appendix B.2)
p^ω_{LL}, p^ω_{HH}	transition prob	0.2,0.99	Frequency and duration of mortgage crises
		Population, Income, and Housing Shares	
ℓ^i	pop. shares $i \in \{B, D, R\}$	{47,51,2}%	Population shares SCF 95-13
Y^i	inc. shares $i \in \{B, D, R\}$	{38,52,10}%	Income shares SCF 95-13
K^i	housing shares $i \in \{B, D, R\}$	{39,49,12}%	Housing wealth shares SCF 95-13
		Mortgages	
ζ	DWL of foreclosure	{0.25,0.425}	Mortgage severities (Appendix B.2)
δ	average life mortgage pool	0.95	Duration Fcn. (Appendix B.3)
α	guarantee payout fraction	0.52	Duration Fcn. (Appendix B.3)
ϕ	maximum LTV ratio	0.65	Borrowers' mortg. debt-to-inc. SCF 95-13
ψ	refinancing cost parameter	8	Mean Conditional Prepayment Rate
		Preferences	
σ^B	risk aversion B	8	Vol househ. mortgage debt to GDP 85-14
β^B	time discount factor B	0.88	Mean housing wealth to GDP 85-14
θ^B	housing expenditure share	0.20	Housing expend. share NIPA
σ^D	risk aversion D	20	Vol. risk-free rate 85-14
$\beta^D = \beta^R$	time discount factor D, R	0.98	Mean risk-free rate 85-14
σ^R	risk aversion R	1	Standard Value
ν	intertemp. elasticity of subst.	1	Standard Value
		Government Policy	
τ	income tax rate	19.83%	BEA govt. revenues to trend GDP 46-13
G^o	exogenous govmt spending	15.8%	BEA govt. spending to trend GDP 46-13
G^T	govmt transfers to agents	3.41%	BEA govt. net transfers to trend GDP 46-13
$\varkappa$	margin	98%	Fin. sector leverage Flow of Funds 85-14
ξ_G	margin guaranteed MBS	1.6%	Basel reg. capital charge agency MBS
ξ_P	margin private MBS	8%	Basel reg. capital charge non-agency mortg.

Table 2: Phasing Out the GSEs: Main Results

	20 bp g-fee		**55 bp g-fee**		**100 bp g-fee**		**275 bp g-fee**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
				Prices				
Risk free rate	1.17%	3.00%	1.25%	2.91%	1.55%	2.99%	1.88%	3.13%
Mortgage rate	3.52%	0.24%	3.60%	0.25%	3.68%	0.26%	3.75%	0.26%
House price	2.239	0.142	2.199	0.130	2.152	0.122	2.100	0.120
Actuarially Fair g-fee	0.76%	0.42%	0.66%	0.39%	0.57%	0.38%	0.53%	0.30%
				Borrower				
Mortgage debt	0.053	0.001	0.052	0.001	0.051	0.001	0.050	0.001
Borrower LTV	63.74%	3.90%	63.75%	3.65%	63.76%	3.45%	63.74%	3.49%
Market value of debt LTV	75.59%	6.44%	74.98%	6.19%	74.35%	6.02%	73.77%	6.00%
Borrower debt to income	1.487	0.040	1.461	0.029	1.430	0.024	1.395	0.024
Debt/income growth	0.04%	2.84%	0.04%	2.72%	0.04%	2.79%	0.04%	2.87%
Mortgage default rate	2.66%	6.03%	2.25%	5.00%	1.92%	4.10%	1.70%	3.81%
Severity rate	30.19%	5.75%	30.14%	5.73%	30.10%	5.72%	30.06%	5.71%
Mortgage loss rate	1.02%	2.68%	0.86%	2.19%	0.73%	1.78%	0.66%	1.68%
Rate-induced prepayment rate	15.77%	4.24%	14.43%	4.27%	13.05%	4.42%	11.86%	4.37%
				Risk-Taker				
Market value of bank assets	0.633	0.018	0.617	0.013	0.598	0.013	0.578	0.013
Fraction guaranteed	99.96%	0.53%	98.42%	5.44%	10.86%	28.90%	0.51%	5.14%
Risk taker leverage	95.58%	0.92%	95.21%	1.22%	88.95%	1.63%	88.24%	1.78%
Risk taker wealth	0.029	0.012	0.031	0.013	0.068	0.016	0.070	0.016
Fraction $\lambda^R > 0$	32.60%	46.88%	35.09%	47.73%	28.60%	45.19%	19.75%	39.81%
MTM Loss Given Prepayment	10.74%	2.59%	9.91%	2.65%	9.04%	2.80%	8.27%	2.81%
Loss rate guaranteed	0.37%	0.85%	0.30%	0.70%	0.24%	0.55%	0.20%	0.48%
Loss rate portfolio	0.37%	0.85%	0.30%	0.71%	0.59%	1.52%	0.65%	1.67%
Bankruptcy frequency	0.27%	5.19%	0.06%	2.45%	0.16%	4.00%	0.22%	4.69%
Return on RT wealth[a]	3.17%	35.79%	3.10%	36.13%	3.77%	17.62%	3.77%	17.93%
				Government				
Government debt / GDP	15.88%	22.63%	−0.47%	10.15%	−4.69%	4.90%	−6.15%	3.42%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The model in the first 2 columns has a mortgage guarantee fee of 20 bps (20 bp g-fee), the model in columns 3 and 4 has an average g-fee of 55 bps, the model in columns 5 and 6 has an average g-fee of 100 bps, and the model in the last two columns has an average g-fee of 275 bps.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

Table 3: Phasing Out the GSEs: Welfare and Risk Sharing

	50 bp g-fee		**100 bp g-fee**		**275 bp g-fee**	
	mean	stdev	mean	stdev	mean	stdev
Aggregate Welfare[a]	+0.13%	+0.30%	+0.19%	+0.15%	+0.63%	+0.22%
Value function borrower[a]	+0.06%	−0.26%	+0.05%	−0.96%	+0.04%	−1.39%
Value Function depositor[a]	+0.21%	−0.39%	+0.35%	0.00%	+1.30%	+0.83%
Value function risk taker[a]	+0.57%	+4.29%	+1.10%	+4.45%	+1.69%	+7.08%
Consumption borrower	−0.19%	−3.86%	−0.46%	−7.97%	−0.55%	−9.20%
Consumption depositor	+0.79%	−3.96%	+1.49%	−13.56%	+2.12%	−20.50%
Consumption risk taker	+0.43%	−11.51%	+1.49%	−16.23%	+1.82%	−9.07%
MU ratio borrower/risk taker[b]	−2.13%	−5.19%	−19.48%	−18.04%	−14.17%	−22.64%
MU ratio risk taker/depositor[b]	+1.16%	−11.29%	+1.18%	−20.90%	+1.15%	−7.85%
DWL from Foreclosure	−16.61%	−18.17%	−30.31%	−34.34%	−39.47%	−40.00%
DWL from Prepayment	−16.49%	−9.25%	−31.28%	−16.86%	−43.49%	−25.95%
Maintenance costs	−1.79%	−23.27%	−3.88%	−35.36%	−6.19%	−34.34%

The table reports percent changes in unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of three different models relative to the 20 bp g-fee benchmark. The model in the first 2 columns has a mortgage guarantee fee of 55 bps (50 bp g-fee). The model in columns 3 and 4 has an average g-fee of 100 bps, and the model in the last two columns has an average g-fee of 275 bps.

[a]: With unit EIS the value functions are in units of composite consumption $C^{1-\rho}K^{\rho}$. Therefore differences in values have a direct interpretation as consumption-equivalent welfare differences.

[b]: Marginal utility ratios are calculated as the difference of the logarithm of marginal utilities.

Table 4: The Role of Countercyclical Charges and Catastrophic Insurance

	20 bp g-fee		**275 bp g-fee**		**CC g-fees**		**JC 10%**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
Prices								
Risk free rate	1.17%	3.00%	1.88%	3.13%	1.38%	2.92%	1.94%	3.23%
Mortgage rate	3.52%	0.24%	3.75%	0.26%	3.64%	0.26%	3.75%	0.27%
House price	2.239	0.142	2.100	0.120	2.185	0.123	2.101	0.119
Actuarially Fair g-fee	0.76%	0.42%	0.53%	0.30%	0.59%	0.42%	0.02%	0.01%
Borrower								
Mortgage debt	0.053	0.001	0.050	0.001	0.052	0.001	0.050	0.001
Borrower LTV	63.74%	3.90%	63.74%	3.49%	63.75%	3.43%	63.75%	3.45%
Market value of debt LTV	75.59%	6.44%	73.77%	6.00%	74.66%	6.02%	73.77%	6.01%
Borrower debt to income	1.487	0.040	1.395	0.024	1.452	0.023	1.396	0.024
Debt/income growth	0.04%	2.84%	0.04%	2.87%	0.04%	2.79%	0.04%	2.96%
Mortgage default rate	2.66%	6.03%	1.70%	3.81%	2.03%	4.27%	1.70%	3.77%
Severity rate	30.19%	5.75%	30.06%	5.71%	30.12%	5.72%	30.06%	5.71%
Mortgage loss rate	1.02%	2.68%	0.66%	1.68%	0.78%	1.84%	0.66%	1.65%
Rate-induced prepayment rate	15.77%	4.24%	11.86%	4.37%	13.72%	4.44%	11.82%	4.47%
Risk-Taker								
Market value of bank assets	0.633	0.018	0.578	0.013	0.610	0.013	0.579	0.014
Fraction guaranteed	99.96%	0.53%	0.51%	5.14%	11.62%	30.33%	28.44%	29.91%
Risk taker leverage	95.58%	0.92%	88.24%	1.78%	89.12%	1.52%	91.57%	2.41%
Risk taker wealth	0.029	0.012	0.070	0.016	0.068	0.015	0.050	0.019
Fraction $\lambda^R > 0$	32.60%	46.88%	19.75%	39.81%	29.56%	45.63%	76.36%	42.49%
MTM Loss Given Prepayment	10.74%	2.59%	8.27%	2.81%	9.46%	2.79%	8.24%	2.88%
Loss rate guaranteed	0.37%	0.85%	0.20%	0.48%	0.27%	0.60%	0.20%	0.48%
Loss rate portfolio	0.37%	0.85%	0.65%	1.67%	0.61%	1.51%	0.55%	1.42%
Bankruptcy frequency	0.27%	5.19%	0.22%	4.69%	0.15%	3.87%	0.35%	5.91%
Return on RT wealth[a]	3.17%	35.79%	3.77%	17.93%	3.86%	17.46%	2.74%	26.24%
Government								
Government debt / GDP	15.88%	22.63%	−6.15%	3.42%	−3.23%	6.54%	−6.37%	3.43%
Welfare								
Aggregate Welfare	0.279	0.008	+0.63%	+0.22%	+0.15%	+0.20%	+0.66%	+0.47%
Value Function borrower	0.319	0.010	+0.04%	−1.39%	+0.05%	−0.77%	+0.06%	−1.19%
Value Function depositor	0.249	0.006	+1.30%	+0.83%	+0.25%	−0.13%	+1.36%	+1.15%
Value function risk taker	0.083	0.000	+1.69%	+7.08%	+1.23%	+2.15%	+1.24%	+42.72%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The first two models (first 4 columns) are the benchmark and high g-fee models from Table 2. The model in columns 6 and 7 two columns has a capital charge for guaranteed bonds set to 8% (same as for private bonds). The last 2 columns report results for an economy where the government guarantees only losses in excess of 10%. Like in the benchmark economy, guarantees in the last two models are both prices at 20 bp.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

Figure 1: Balance sheets of agents in model economy

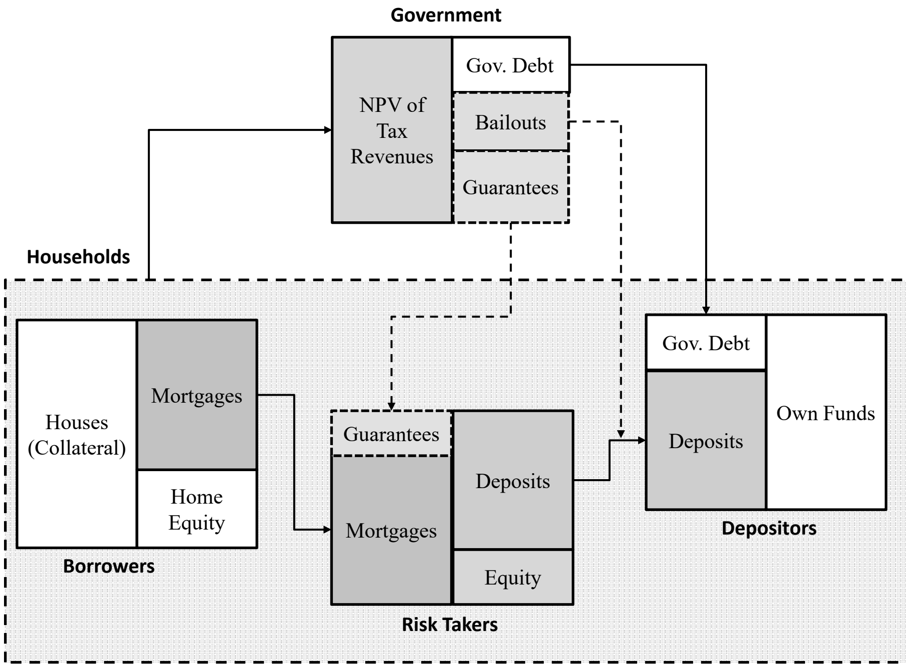
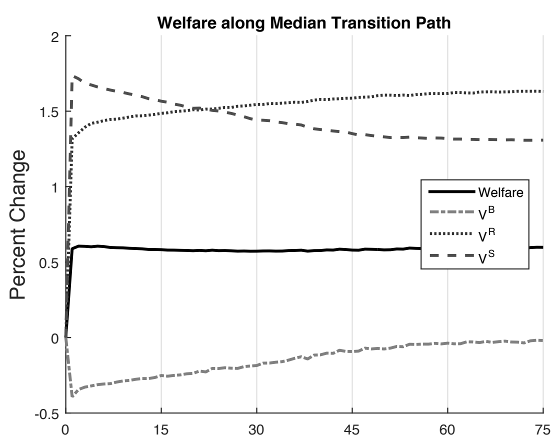
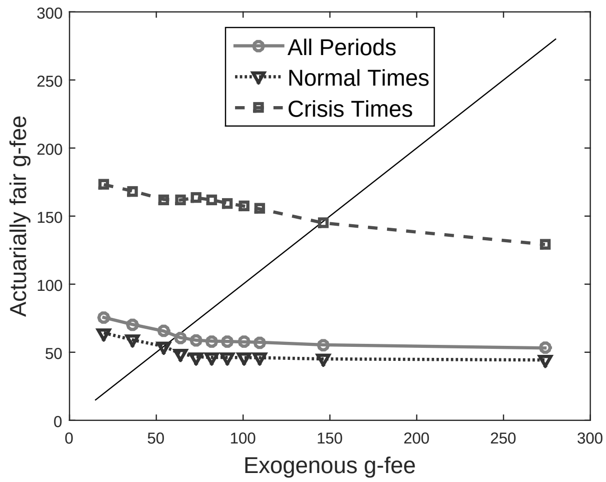


Figure 2: Transition



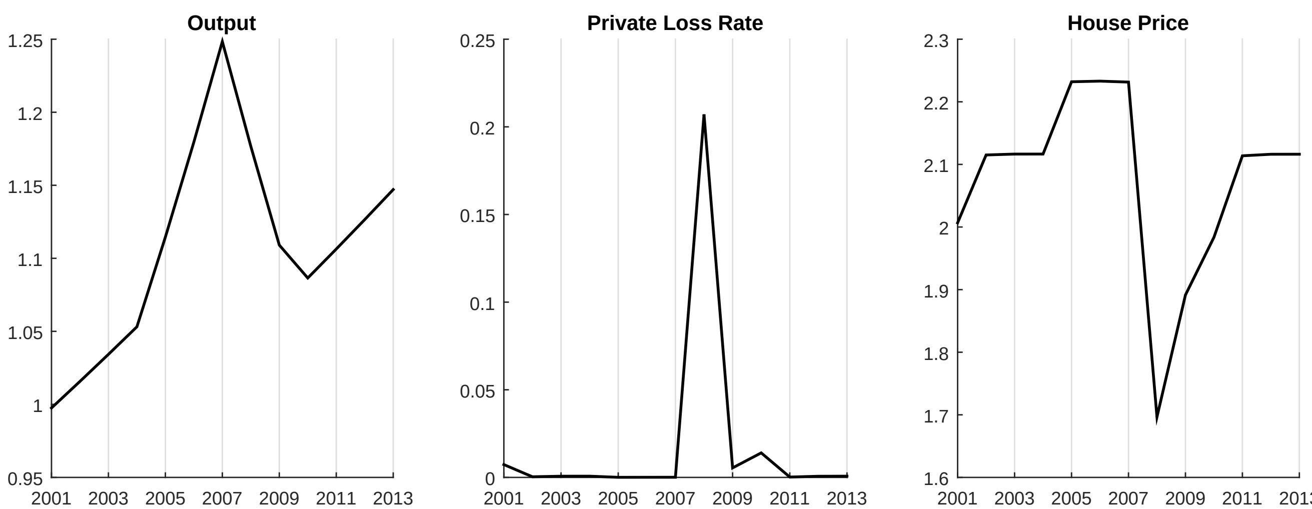
The graph plots aggregate welfare as well as the value functions of the three types of agents along the median transition path from the low g-fee economy to the high g-fee economy. The economy switches from the 20bps to the 275bps g-fee at time 0. The median transition path is computed based on 10,000 simulations.

Figure 3: Actuarially Fair G-Fees



The graphs show the actuarially fair g-fee (y-axis) for seven economies that differ by their exogenously given g-fee (x-axis). The solid line with circles denotes the average g-fee across all periods in a long simulation. The dotted line with triangles denotes the average g-fee during normal times whereas the dotted line with squares denotes the average g-fee during mortgage crises (high σ_ω) times.

Figure 4: Great Recession in the Model



The left panel plots the exogenously assumed path for GDP. We start the low g-fee economy in the year 2000 at typical values for the state variable and in a mild expansion. The exact shock sequence we feed in for the years from 2001 until 2013 is given in the main text. The middle plots the resulting loss rate on mortgages. The right panel shows the house prices.

Online Appendix "Phasing Out the GSEs"
Vadim Elenev, Tim Landvoigt, and Stijn Van Nieuwerburgh

A Model Appendix

We reformulate the problem of risk taker, depositor, and borrower to ensure stationarity of the problem. We do so by scaling all variables by permanent income.

A.1 Borrower problem

A.1.1 Preliminaries

We start by defining some preliminaries.

$$Z_A(\omega_t^*) = [1 - F_\omega(\omega_t^*; \chi)]$$
$$Z_K(\omega_t^*) = [1 - F_\omega(\omega_t^*; \chi)]\mathrm{E}\left[w_{i,t} \mid \omega_{i,t} \geq \omega_t^*; \chi\right]$$

and $F_\omega(\cdot; \chi)$ is the CDF of $\omega_{i,t}$ with parameters χ. Assume $\omega_{i,t}$ are drawn from a Gamma distribution with shape and scale parameters $\chi = (\chi_0, \chi_1)$ such that

$$\mu_\omega = \mathrm{E}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1$$
$$\sigma_{t,\omega}^2 = \mathrm{Var}_i[\omega_{i,t}; \chi_0, \chi_1] = \chi_0 \chi_1^2$$

From Landsman and Valdez (2004, equation 22), we know that

$$\mathrm{E}[\omega | \omega \geq \bar{\omega}] = \mu_\omega \frac{1 - F_\omega(\bar{\omega}; \chi_0 + 1, \chi_1)}{1 - F_\omega(\bar{\omega}; \chi_0, \chi_1)}$$

so the closed form expression for Z_K is

$$Z_K(\omega_t^*) = \mu_\omega \left[1 - F_\omega(\omega_t^*; \chi_0 + 1, \chi_1)\right]$$

It is useful to compute the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$:

$$\frac{\partial Z_K(\omega_t^*)}{\partial \omega_t^*} = \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_\omega(\omega) d\omega = -\omega_t^* f_\omega(\omega_t^*),$$
$$\frac{\partial Z_A(\omega_t^*)}{\partial \omega_t^*} = \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_\omega(\omega) d\omega = -f_\omega(\omega_t^*),$$

where $f_\omega(\cdot)$ is the p.d.f. of a Gamma distribution with parameters (χ_0, χ_1).

Prepayment Cost Let

$$\Psi(R_t^B, A_t^B) = \frac{\psi}{2} \left(\frac{R_t^B}{A_t^B}\right)^2 A_t^B$$

Then partial derivatives are

$$\Psi_R(R^B_t, A^B_t) = \psi \frac{R^B_t}{A^B_t} \tag{19}$$

$$\Psi_A(R^B_t, A^B_t) = -\frac{\psi}{2}\left(\frac{R^B_t}{A^B_t}\right)^2 \tag{20}$$

A.1.2 Statement of stationary problem

Let $\mathcal{S}^B_t = \left(g_t, \sigma_{\omega,t}, W^R_t, W^D_t, B^G_{t-1}\right)$ represent state variables exogenous to the borrower's decision. We consider the borrower's problem in the current period after income and house depreciation shocks have been realized, after the risk taker has chosen a default policy, and after the risk taker's random utility penalty is realized. Then the borrower's value function, transformed to ensure stationarity, is:

$$\begin{aligned} V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t) = \max_{\{C^B_t, K^B_t, \omega^*_t, R^B_t, B^B_t\}} & \Bigg\{(1-\beta_B)\left[\left(C^B_t\right)^{1-\theta}\left(A_K K^B_{t-1}\right)^{\theta}\right]^{1-1/\nu} + \\ & + \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\Bigg\}^{\frac{1}{1-1/\nu}} \end{aligned}$$

subject to

$$C^B_t = (1-\tau_t)Y^B_t + G^{T,B}_t + Z_K(\omega^*_t)p_t K^B_{t-1} + q^m_t B^B_t - (1-\tau^m_t)Z_A(\omega^*_t)A^B_t - p_t K^B_t - F R^B_t - \Psi(R^B_t, A^B_t) \tag{21}$$

$$A^B_{t+1} = e^{-g_{t+1}}\left[\delta Z_A\left(\omega^*_t\right)A^B_t - R^B_t + B^B_t\right] \tag{22}$$

$$\phi p_t K^B_t \geq F\left[\delta Z_A\left(\omega^*_t\right)A^B_t - R^B_t + B^B_t\right] \tag{23}$$

$$0 \leq R^B_t \leq \delta Z_A\left(\omega^*_t\right)A^B_t \tag{24}$$

$$\mathcal{S}^B_{t+1} = h(\mathcal{S}^B_t) \tag{25}$$

where the functions Z_K and Z_A are defined in the preliminaries above.

The continuation value $\tilde{V}^B(\cdot)$ must take into account the default decision of the risk taker at the beginning of next period. We anticipate here and show below that that default decision takes the form of a cutoff rule:

$$\begin{aligned} \tilde{V}^B\left(K^B_{t-1}, A^B_t, \mathcal{S}^B_t\right) &= F_\rho(\rho^*_t)\mathrm{E}_\rho\left[V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t)\,|\,\rho < \rho^*_t\right] + (1-F_\rho(\rho^*_t))\,\mathrm{E}_\rho\left[V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t)\,|\,\rho > \rho^*_t\right] \\ &= F_\rho(\rho^*_t)V^B(K^B_{t-1}, A^B_t, \mathcal{S}^S_t(\rho_t < \rho^*_t)) + (1-F_\rho(\rho^*_t))\,V^B(K^B_{t-1}, A^B_t, \mathcal{S}^S_t(\rho_t > \rho^*_t)), \end{aligned} \tag{26}$$

where (26) obtains because the expectation terms conditional on realizations of ρ_t and ρ^*_t only differ in the values of the aggregate state variables.

Denote the value function and the partial derivatives of the value function as:

$$\begin{aligned} V^B_t &\equiv V(K^B_{t-1}, A^B_t, \mathcal{S}^B_t), \\ V^B_{A,t} &\equiv \frac{\partial V(K^B_{t-1}, A^B_t, \mathcal{S}^B_t)}{\partial A^B_t}, \\ V^B_{K,t} &\equiv \frac{\partial V(K^B_{t-1}, A^B_t, \mathcal{S}^B_t)}{\partial K^B_{t-1}}. \end{aligned}$$

Therefore the marginal values of borrowing and of housing of $\tilde{V}^B(\cdot)$ are:

$$\tilde{V}^B_{A,t} = F_\rho(\rho^*_t)\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t < \rho^*_t))}{\partial A^B_t} + (1 - F_\rho(\rho^*_t))\,\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t > \rho^*_t)}{\partial A^B_t}$$
$$\tilde{V}^B_{K,t} = F_\rho(\rho^*_t)\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t < \rho^*_t))}{\partial K^B_{t-1}} + (1 - F_\rho(\rho^*_t))\,\frac{\partial V^B(K^B_{t-1}, A^B_t, \mathcal{S}^B_t(\rho_t > \rho^*_t)}{\partial K^B_{t-1}}$$

Denote the certainty equivalent of future utility as:

$$CE^B_t = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}$$

Recall that

$$u^B_t \,=\, \left(C^B_t\right)^{1-\theta}\left(A_K K^B_{t-1}\right)^{\theta}$$

A.1.3 First-order conditions

New mortgages The FOC for new mortgage loans B^B_t is:

$$\begin{aligned}
0 = \frac{1}{1-1/\nu}&\left\{(1-\beta_B)\left[\left(C^B_t\right)^{1-\theta}\left(A_K K^B_{t-1}\right)^{\theta}\right]^{1-1/\nu} + \right.\\
&\left.+\ \beta_B \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\right\}^{\frac{1}{1-1/\nu}-1} \times\\
\times&\left\{(1-1/\nu)(1-\beta_B)\left[\left(C^B_t\right)^{1-\theta}\left(A_K K^B_{t-1}\right)^{\theta}\right]^{-1/\nu}(1-\theta)(A_K K^B_{t-1})^{\theta}(C^B_t)^{-\theta}q^m_t + \right.\\
&+\ \beta_B\frac{1-1/\nu}{1-\sigma_B}\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}-1}\times\\
&\left.\times\ E_t\left[(1-\sigma_B)\left(e^{g_{t+1}}\tilde{V}^B(K^B_t, A^B_{t+1}, \mathcal{S}^B_{t+1})\right)^{-\sigma_B} e^{g_{t+1}}\tilde{V}^B_{A,t+1}e^{-g_{t+1}}\right]\right\} - \lambda^B_t F
\end{aligned}$$

where λ^B_t is the Lagrange multiplier on the borrowing constraint.

Simplifying, we get:

$$\begin{aligned}
&q^m_t\frac{1-\theta}{C^B_t}(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu} = \\
&\quad\lambda^B_t F - \beta_B\mathrm{E}_t[(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\tilde{V}^B_{A,t+1}](CE^B_t)^{\sigma_B-1/\nu}(V^B_t)^{1/\nu}
\end{aligned} \tag{27}$$

Observe that we can rewrite equation (27) as:

$$q^m_t = \frac{C^B_t}{(1-\theta)(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}\left\{\lambda^B_t F - \beta_B\mathrm{E}_t[(e^{g_{t+1}}\tilde{V}^B_{t+1})^{-\sigma_B}\tilde{V}^B_{A,t+1}](CE^B_t)^{\sigma_B-1/\nu}(V^B_t)^{1/\nu}\right\}.$$

We define the rescaled Lagrange multiplier of the borrower as the original multiplier divided by marginal utility of current consumption:

$$\tilde{\lambda}^B_t = \lambda^B_t\frac{C^B_t}{(1-\theta)(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}.$$

Then we can solve for the mortgage price as:

$$q_t^m = \tilde{\lambda}_t^B F - \beta_B \frac{C_t^B \left\{ \mathrm{E}_t[(e^{g_{t+1}} \tilde{V}_{t+1}^B)^{-\sigma_B} \tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu} \right\}}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}. \tag{28}$$

Houses The FOC for new purchases of houses K_t^B is:

$$\begin{aligned} 0 = \frac{1}{1-1/\nu}(V_t^B)^{1/\nu} \times \Big\{ &-(1-1/\nu)(1-\beta_B)(u_t^B)^{-1/\nu}(1-\theta)(A_K K_{t-1}^B)^\theta (C_t^B)^{-\theta} p_t + \\ &+ \frac{1-1/\nu}{1-\sigma_B}\beta_B (CE_t^B)^{\sigma_B - 1/\nu} \mathrm{E}_t[(1-\sigma_B)(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B} e^{g_{t+1}} \tilde{V}_{K,t+1}^B] \Big\} + \lambda_t^B \phi p_t. \end{aligned}$$

Simplifying, we get:

$$\begin{aligned} &p_t \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ &\qquad \lambda_t^B \phi p_t + \beta_B \mathrm{E}_t[e^{(1-\sigma_B)g_{t+1}}(\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu} \end{aligned} \tag{29}$$

Default Threshold Taking the first-order condition with respect to $\omega_t^\star$ and using the expressions for the derivatives of $Z_K(\cdot)$ and $Z_A(\cdot)$ in the preliminaries above yields:

$$\begin{aligned} &f_\omega(\omega_t^*) \left[\omega_t^* p_t K_{t-1}^B - (1-\tau_t^m) A_t^B\right] \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ &\qquad \delta A_t^B f_\omega(\omega_t^*) \left\{ \lambda_t^B F - \lambda_t^{RB} - \beta_B \mathrm{E}_t \left[\left(e^{g_{t+1}} \tilde{V}_{t+1}^B \right)^{-\sigma_B} \tilde{V}_{A,t+1}^B \right] \times (CE_t^B)^{\sigma_B - 1/\nu} \left(V_t^B\right)^{1/\nu} \right\}. \end{aligned}$$

This can be simplified by replacing the term in braces on the right-hand side using the FOC for new loans (28) and solving for ω_t^* to give:

$$\omega_t^* = \frac{A_t^B(1-\tau_t^m + \delta q_t^m - \delta \tilde{\lambda}_t^{RB})}{p_t K_{t-1}^B}, \tag{30}$$

where the rescaled Lagrange multiplier on the upper refinancing bound is:

$$\tilde{\lambda}_t^{RB} = \lambda_t^{RB} \frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Prepayment The FOC for repayments R_t^B is:

$$\begin{aligned} &[F + \Psi_R(R_t^B, A_t^B)] \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} = \\ &\qquad \mu_t^{RB} - \lambda_t^{RB} + \lambda_t^B F - \beta_B \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}, \end{aligned} \tag{31}$$

where λ_t^{RB} is the Lagrange multiplier on the upper bound on R_t^B and μ_t^{RB} is the Lagrange multiplier on the lower bound. Combining with (28), we obtain:

$$\Psi_R(R_t^B, A_t^B) = q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB},$$

where we defined the lower bound Lagrange multiplier on refinancing as the original multiplier divided by marginal utility of consumption:

$$\tilde{\mu}_t^{RB} = \mu_t^B \frac{C_t^B}{(1-\theta)(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}.$$

Recall the definition $Z_t^R = R_t^B / A_t^B$. Using the functional form of Ψ_R from (19), the optimal prepayment fraction is:

$$Z_t^R = \frac{1}{\psi}\left(q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB}\right) \tag{32}$$

A.1.4 Marginal Values of State Variables and SDF

Mortgages Taking the derivative of the value function with respect to A_t^B gives:

$$\begin{aligned} V_{A,t}^B = &-\left(1 - \tau_t^m + \frac{\Psi_A(R_t^B, A_t^B)}{Z_A(\omega_t^*)}\right) Z_A(\omega_t^*) \frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu} \\ &- \delta Z_A(\omega_t^*)\{\lambda_t^B F - \lambda_t^{RB} - \beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{A,t+1}^B] \times (CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu}\}. \end{aligned}$$

Note that we can substitute for the term in braces using equation (27) and for Ψ_A using (20):

$$V_{A,t}^B = -Z_A(\omega_t^*)\left(1 - \tau_t^m - \frac{\psi\left(Z_t^R\right)^2}{2Z_A(\omega_t^*)} + \delta q_t^m - \delta\tilde{\lambda}_t^{RB}\right)\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}. \tag{33}$$

Houses Taking the derivative of the value function with respect to K_{t-1}^B gives:

$$V_{K,t}^B = \left[p_t Z_K(\omega_t^*) + \frac{\theta C_t^B}{(1-\theta)K_{t-1}^B}\right]\frac{1-\theta}{C_t^B}(V_t^B)^{1/\nu}(1-\beta_B)(u_t^B)^{1-1/\nu}. \tag{34}$$

SDF Define the borrower's intertemporal marginal rate of substitution between t and $t+1$, conditional on a particular realization of ρ_{t+1} as:

$$\begin{aligned} \mathcal{M}_{t,t+1}^B(\rho_{t+1}) &= \frac{\partial V_t^B / \partial C_{t+1}^B}{\partial V_t^B / \partial C_t^B} = \frac{\partial V_t^B}{\partial V_{t+1}^B} e^{-g_{t+1}} \frac{\partial V_{t+1}^B / \partial C_{t+1}^B}{\partial V_t^B / \partial C_t^B} \\ &= (V_t^B)^{1/\nu}\beta_B(CE_t^B)^{\sigma_B - 1/\nu}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\frac{\frac{1-\theta}{C_{t+1}^B}(1-\beta_B)(V_{t+1}^B)^{1/\nu}(u_{t+1}^B)^{1-1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}} \\ &= \beta_B e^{-\sigma_B g_{t+1}}\left(\frac{C_{t+1}^B}{C_t^B}\right)^{-1}\left(\frac{u_{t+1}^B}{u_t^B}\right)^{1-1/\nu}\left(\frac{V_{t+1}^B}{CE_t^B}\right)^{-(\sigma_B - 1/\nu)} \end{aligned}$$

We can then define the stochastic discount factor (SDF) of borrowers as:

$$\tilde{\mathcal{M}}_{t,t+1}^B = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*),$$

where $\mathcal{M}_{t,t+1}^B(\rho_{t+1} < \rho_{t+1}^*)$ and $\mathcal{M}_{t,t+1}^B(\rho_{t+1} > \rho_{t+1}^*)$ are the IMRSs, conditional on the two possible realizations of state variables.

A.1.5 Euler Equations

Mortgages Recall that $\tilde{V}_{A,t+1}^B$ is a linear combination of $V_{A,t+1}^B$ conditional on ρ_t being below and above the threshold, and with each $V_{A,t+1}^B$ given by equation (33). Substituting in for $\tilde{V}_{A,t+1}^B$ in (28) and using the SDF expression, we get the recursion:

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*)\left(1 - \tau^m - \frac{\psi\left(Z_{t+1}^R\right)^2}{2Z_A(\omega_{t+1}^*)} - \delta\tilde{\lambda}_{t+1}^{RB} + \delta q_{t+1}^m\right)\right]. \tag{35}$$

Houses Likewise, observe that we can write (29) as:

$$p_t\left[1-\tilde{\lambda}_t^B\phi\right]=\frac{\beta_B \mathrm{E}_t[e^{g_{t+1}}(e^{g_{t+1}}\tilde{V}_{t+1}^B)^{-\sigma_B}\tilde{V}_{K,t+1}^B](CE_t^B)^{\sigma_B-1/\nu}(V_t^B)^{1/\nu}}{\frac{1-\theta}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu}(u_t^B)^{1-1/\nu}}$$

Recall that $\tilde{V}_{K,t+1}^B$ is a linear combination of $V_{K,t+1}^B$ conditional on ρ_t being below and above the threshold, and with each $V_{K,t+1}^B$ given by equation (34). Substituting in for $\tilde{V}_{K,t+1}^B$ and using the SDF expression, we get the recursion:

$$p_t\left[1-\tilde{\lambda}_t^B\phi\right]=\mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^B e^{g_{t+1}}\left\{p_{t+1}Z_K(\omega_{t+1}^*)+\frac{\theta C_{t+1}^B}{(1-\theta)K_t^B}\right\}\right] \tag{36}$$

A.2 Risk Takers

A.2.1 Statement of stationary problem

Denote by W_t^R risk taker wealth at the beginning of the period, before their bankruptcy decision. Then wealth after realization of the penalty ρ_t is:

$$\tilde{W}_t^R=(1-D(\rho_t))W_t^R,$$

and the effective utility penalty is:

$$\tilde{\rho}_t=D(\rho_t)\rho_t.$$

Let $\mathcal{S}_t^R=\left(g_t,\sigma_{\omega,t},W_t^D,A_t^B,B_{t-1}^G\right)$ denote all other aggregate state variables exogenous to risk takers.

After the default decision, risk takers face the following optimization problem over consumption and portfolio composition, formulated to ensure stationarity:

$$\begin{aligned}V^R(\tilde{W}_t^R,\tilde{\rho}_t,\mathcal{S}_t^R)=\max_{C_t^R,A_{t+1,P}^R,A_{t+1,P}^R,B_t^R}&\left\{(1-\beta_R)\left[\frac{(C_t^R)^{1-\theta}(K_{t-1}^R)^\theta}{e^{\tilde{\rho}_t}}\right]^{1-1/\nu}\right.\\&\left.+\beta_R\mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R,\mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1-1/\nu}{1-\sigma_R}}\right\}^{\frac{1}{1-1/\nu}}\end{aligned} \tag{37}$$

subject to:

$$(1-\tau^S)Y_t^R+\tilde{W}_t^R+G_t^{T,R}=C_t^R+(1-\mu_\omega)p_tK_{t-1}^R+q_t^mA_{t+1,P}^R+(q_t^m+\gamma_t)A_{t+1,G}^R+q_t^fB_t^R, \tag{38}$$

$$\begin{aligned}W_{t+1}^R=e^{-g_{t+1}}&\left[(M_{t+1,P}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F])A_{t+1,P}^R\right.\\&\left.+(M_{t+1,G}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F])A_{t+1,G}^R+B_t^R\right],\end{aligned} \tag{39}$$

$$q_t^fB_t^R\geq -q_t^m(\xi_PA_{t+1,P}^R+\xi_GA_{t+1,G}^R), \tag{40}$$

$$A_{t+1,G}^R\geq 0, \tag{41}$$

$$A_{t+1,P}^R\geq 0, \tag{42}$$

$$\mathcal{S}_{t+1}^R=h(\mathcal{S}_t^R). \tag{43}$$

The continuation value $\tilde{V}^R\left(W_{t+1}^R,\mathcal{S}_{t+1}^R\right)$ is the outcome of the optimization problem risk takers face at the beginning of the following period, i.e., before the decision over the optimal bankruptcy rule. This continuation value function is given by:

$$\tilde{V}^R(W_t^R,\mathcal{S}_t^R)=\max_{D(\rho)}\mathrm{E}_\rho\left[D(\rho)V^R(0,\rho,\mathcal{S}_t^R)+(1-D(\rho))V^R(W_t^R,0,\mathcal{S}_t^R)\right] \tag{44}$$

Define the certainty equivalent of future utility as:

$$CE_t^R = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^R\left(W_{t+1}^R, \mathcal{S}_{t+1}^R\right)\right)^{1-\sigma_R}\right]^{\frac{1}{1-\sigma_R}}. \tag{45}$$

and the composite within-period utility (evaluated at $\rho = 0$) as:

$$u_t^R = (C_t^R)^{1-\theta}(A_K K_{t-1}^R)^{\theta}.$$

A.2.2 First-order conditions

Optimal Default Decision The optimization consists of choosing a function $D(\rho) : \mathbb{R} \to \{0, 1\}$ that specifies for each possible realization of the penalty ρ whether or not to default.

Since the value function $V^R(W, \rho, \mathcal{S}_t^R)$ defined in (37) is increasing in wealth W and decreasing in the penalty ρ, there will generally exist an optimal threshold penalty ρ^* such that for a given W_t^R, risk-takers optimally default for all realizations $\rho < \rho^*$. Hence we can equivalently write the optimization problem in (44) as

$$\begin{aligned}\tilde{V}^R(W_t^R, \mathcal{S}_t^R) =& \max_{\rho^*} \mathrm{E}_\rho\left[\mathbb{1}[\rho < \rho^*]\ V^R(0, \rho, \mathcal{S}_t^R) + (1 - \mathbb{1}[\rho < \rho^*])\, V^R(W_t^R, 0, \mathcal{S}_t^R)\right] \\ =& \max_{\rho^*} F_\rho(\rho^*)\ \mathrm{E}_\rho\left[V^R(0, \rho, \mathcal{S}_t^R)\,|\,\rho < \rho^*\right] + (1 - F_\rho(\rho^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R).\end{aligned}$$

The solution ρ_t^* is characterized by the first-order condition:

$$V^R(0, \rho_t^*, \mathcal{S}_t^R) = V^R(W_t^S, 0, \mathcal{S}_t^R).$$

By defining the partial inverse $\mathcal{F} : (0, \infty) \to (-\infty, \infty)$ of $V^S(\cdot)$ in its second argument as

$$\left\{(x, y) : y = \mathcal{F}(x) \Leftrightarrow x = V^R(0, y)\right\},$$

we get that

$$\rho_t^* = \mathcal{F}(V^R(W_t^R, 0, \mathcal{S}_t^R)), \tag{46}$$

and by substituting the solution into (44), we obtain

$$\tilde{V}^R(W_t^R, \mathcal{S}_t^R) = F_\rho(\rho_t^*)\mathrm{E}_\rho\left[V^R(0, \rho, \mathcal{S}_t^R)\,|\,\rho < \rho_t^*\right] + (1 - F_\rho(\rho_t^*))\, V^R(W_t^R, 0, \mathcal{S}_t^R). \tag{47}$$

Equations (37), (46), and (47) completely characterize the optimization problem of risk-takers.

To compute the optimal bankruptcy threshold ρ_t^*, note that the inverse value function defined in equation (46) is given by:

$$\mathcal{F}(x) = \begin{cases}\log((1 - \beta_R)u_t^R) - \frac{1}{1-1/\nu}\log\left(x^{1-1/\nu} - \beta_R(CE_t^R)^{1-1/\nu}\right) & \text{for } \nu > 1 \\ (1 - \beta_R)\log(u_t^R) + \beta_R\log(CE_t^R) - \log(x) - (1 - \beta_R) & \text{if } \nu = 1.\end{cases}$$

Optimal Portfolio Choice The first-order condition for the short-term bond position is:

$$\begin{aligned}& q_t^f \frac{1-\theta}{C_t^R}(1 - \beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu} = \\ & \lambda_t^R q_t^f + \beta_R \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R](CE_t^R)^{\sigma_R - 1/\nu}(V_t^R)^{1/\nu}\end{aligned} \tag{48}$$

where λ_t^R is the Lagrange multiplier on the borrowing constraint (40).

The first order condition for the government-guaranteed mortgage bond position is:

$$
\begin{aligned}
&(q_t^m+\gamma_t)\frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu}=\lambda_t^R\xi_G q_t^m+\mu_{G,t}^R\\
&+\beta_R\mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R\left(M_{G,t+1}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F]\right)](CE_t^R)^{\sigma_R-1/\nu}(V_t^R)^{1/\nu},
\end{aligned}
\tag{49}
$$

where $\mu_{t,G}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (41).

The first order condition for the private mortgage bond position is:

$$
\begin{aligned}
&q_t^m\frac{1-\theta}{C_t^R}(1-\beta_R)(V_t^R)^{1/\nu}(u_t^R)^{1-1/\nu}=\lambda_t^R\xi_P q_t^m+\mu_{P,t}^R\\
&+\beta_R\mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^R)^{-\sigma_R}\tilde{V}_{W,t+1}^R\left(M_{P,t+1}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F]\right)](CE_t^R)^{\sigma_R-1/\nu}(V_t^R)^{1/\nu},
\end{aligned}
\tag{50}
$$

where $\mu_{t,P}^R$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (42).

A.2.3 Marginal value of wealth and SDF

Differentiating (47) gives the marginal value of wealth

$$
\tilde{V}_{W,t}^R=(1-F_\rho(\rho_t^*))\,\frac{\partial V^R(W_t^R,0,\mathcal{S}_t^R)}{\partial W_t^R},
$$

where

$$
\frac{\partial V^R(W_t^R,0,\mathcal{S}_t^R)}{\partial W_t^R}=\frac{1-\theta}{C_t^R}(1-\beta_R)(V^R(W_t^R,0,\mathcal{S}_t^R))^{1/\nu}(u_t^R)^{1-1/\nu},
$$

The stochastic discount factor of risk-takers is therefore

$$
\mathcal{M}_{t,t+1}^R=\beta_R e^{-\sigma_R g_{t+1}}\left(\frac{V^R(W_{t+1}^R,0,\mathcal{S}_{t+1}^R)}{CE_t^R}\right)^{-(\sigma_R-1/\nu)}\left(\frac{C_{t+1}^R}{C_t^R}\right)^{-1}\left(\frac{u_{t+1}^R}{u_t^R}\right)^{1-1/\nu},
$$

and

$$
\tilde{\mathcal{M}}_{t,t+1}^R=(1-F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^R
$$

A.2.4 Euler Equations

It is then possible to show that the FOC with respect to B_t^R, $A_{t+1,G}^R$, and $A_{t+1,P}^R$ respectively, are:

$$
q_t^f=q_t^f\tilde{\lambda}_t^R+\mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\right],\tag{51}
$$

$$
q_t^m+\gamma_t=q_t^m\xi_G\tilde{\lambda}_t^R+\tilde{\mu}_{t,G}+\mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{G,t+1}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F]\right)\right],\tag{52}
$$

$$
q_t^m=q_t^m\xi_P\tilde{\lambda}_t^R+\tilde{\mu}_{t,P}+\mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{P,t+1}+\delta Z_A(\omega_{t+1}^*)q_{t+1}^m-Z_{t+1}^R[q_{t+1}^m-F]\right)\right].\tag{53}
$$

A.3 Depositor

We state here a slightly more general problem than in the main text whereby we allow the depositor to also invest in government-guaranteed mortgage bonds in addition to short-term government bonds. The problem in the main text then arises as a special case where we impose the additional constraint that the guaranteed mortgage bond holdings must be non-positive. The Lagrange multiplier on this constraint tells us whether the depositor in the restricted problem would want to hold guaranteed bonds, evaluated at the equilibrium allocation of the restricted model.

A.3.1 Statement of stationary problem

Let $\mathcal{S}_t^D = \left(g_t, \sigma_{\omega,t}, W_t^R, A_t^B, B_{t-1}^G\right)$ be the depositor's state vector capturing all exogenous state variables. Scaling by permanent income, the stationary problem of the depositor -after the risk taker has made default her decision and the utility cost of default is realized- is:

$$
\begin{aligned}
V^D(W_t^D, \mathcal{S}_t^D) = \max_{\{C_t^D, B_t^D, A_{t+1,G}^D\}} \Bigg\{ & (1-\beta_D)\left[\left(C_t^D\right)^{1-\theta}\left(A_K K_{t-1}^D\right)^{\theta}\right]^{1-1/\nu} + \\
& + \beta_D \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W_{t+1}^D, \mathcal{S}_{t+1}^D)\right)^{1-\sigma_D}\right]^{\frac{1-1/\nu}{1-\sigma_D}} \Bigg\}^{\frac{1}{1-1/\nu}}
\end{aligned}
$$

subject to

$$
\begin{aligned}
C_t^D &= (1-\tau_t^S)Y_t^D + G_t^{T,D} + W_t^D - (q_t^m + \gamma_t)A_{t+1,G}^D - q_t^f B_t^D - (1-\mu_{t,\omega})p_t K_{t-1}^D && (54)\\
W_{t+1}^D &= e^{-g_{t+1}}\left[(M_{t+1,G} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F])A_{t+1,G}^D + B_t^D\right] && (55)\\
B_t^D &\geq 0 && (56)\\
A_{t+1,G}^D &\geq 0 && (57)\\
\mathcal{S}_{t+1}^D &= h(\mathcal{S}_t^D) && (58)
\end{aligned}
$$

As before, we will drop the arguments of the value function and denote marginal values of wealth and mortgages as:

$$
\begin{aligned}
V_t^D &\equiv V_t^D(W_t^D, \mathcal{S}_t^D),\\
V_{W,t}^D &\equiv \frac{\partial V_t^D(W_t^D, \mathcal{S}_t^D)}{\partial W_t^D},
\end{aligned}
$$

Denote the certainty equivalent of future utility as:

$$
CE_t^D = \mathrm{E}_t\left[\left(e^{g_{t+1}}\tilde{V}^D(W_t^D, \mathcal{S}_t^D)\right)^{1-\sigma_D}\right],
$$

and the composite within-period utility as:

$$
u_t^D = (C_t^D)^{1-\theta}(A_K K_{t-1}^D)^{\theta}.
$$

Like the borrower, the depositor must take into account the risk-taker's default decisions and the realization of the utility penalty of default. Therefore the marginal value of wealth is:

$$
\tilde{V}_{W,t}^D = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.
$$

A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$
\begin{aligned}
& q_t^f \frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} = \\
& \lambda_t^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu} \qquad (59)
\end{aligned}
$$

where λ_t^D is the Lagrange multiplier on the no-borrowing constraint (56).

The first order condition for the government-guaranteed mortgage bond position is:

$$(q_t^m + \gamma_t)\frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu} =$$
$$\mu_{G,t}^D + \beta_D \mathrm{E}_t[(e^{g_{t+1}}\tilde{V}_{t+1}^D)^{-\sigma_D}\tilde{V}_{W,t+1}^D \left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)](CE_t^D)^{\sigma_D - 1/\nu}(V_t^D)^{1/\nu}, \quad (60)$$

where $\mu_{t,G}^D$ is the Lagrange multiplier on the no-shorting constraint for guaranteed loans (57).

A.3.3 Marginal Values of State Variables and SDF

Marginal value of wealth is:

$$V_{W,t}^D = \frac{1-\theta}{C_t^D}(1-\beta_D)(V_t^D)^{1/\nu}(u_t^D)^{1-1/\nu}, \quad (61)$$

and for the continuation value function:

$$\tilde{V}_{W,t}^D = F_\rho(\rho_t^*)\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t < \rho_t^*))}{\partial W_t^D} + (1 - F_\rho(\rho_t^*))\,\frac{\partial V^D(W_t^D, \mathcal{S}_t^D(\rho_t > \rho_t^*)}{\partial W_t^D}.$$

Defining the SDF in the same fashion as we did for the borrower, we get:

$$\mathcal{M}_{t,t+1}^D(\rho_t) = \beta_D e^{-\sigma_D g_{t+1}}\left(\frac{V_{t+1}^D}{CE_t^D}\right)^{-(\sigma_D - 1/\nu)}\left(\frac{C_{t+1}^D}{C_t^D}\right)^{-1}\left(\frac{u_{t+1}^D}{u_t^D}\right)^{1-1/\nu},$$

and

$$\tilde{\mathcal{M}}_{t,t+1}^D = F_\rho(\rho_{t+1}^*)\mathcal{M}_{t,t+1}^D(\rho_{t+1} < \rho_{t+1}^*) + (1 - F_\rho(\rho_{t+1}^*))\mathcal{M}_{t,t+1}^D(\rho_{t+1} > \rho_{t+1}^*).$$

A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (59) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\right] \quad (62)$$

where $\tilde{\lambda}_t^D$ is the original multiplier λ_t^D divided by the marginal value of wealth.

Similarly, from (60) we get the Euler Equation for guaranteed mortgages:

$$q_t^m + \gamma_t = \tilde{\mu}_{G,t}^D + \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^D\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right] \quad (63)$$

A.4 Equilibrium

The optimality conditions describing the problem are (21), (30), (32), (67) and (36) for borrowers, (38), (51), (52), and (53) for risk takers, and (54), (62), and (63) for depositors. We add complementary slackness conditions for the constraints (23) and (24) for borrowers, (40), (41), and (42) for risk-takers, and (56) and (57) for depositors. Together with the market clearing conditions (16), (17), and (18), these equations fully characterize the economy.

A.5 Discussion of First Order Conditions

We now discuss some of the key first-order conditions in some more detail.

Borrower FOCs First, since borrowers are the only households freely choosing their housing position, their choice pins down the price of housing in the economy. Recall that $\tilde{\mathcal{M}}_{t,t+1}^i$ is the intertemporal marginal rate of substitution (or stochastic discount factor) for agent $i \in \{B, D, R\}$. At the optimum, house prices satisfy the recursion:

$$p_t \left[1 - \tilde{\lambda}_t^B \phi\right] = \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^B e^{g_{t+1}} \left\{ p_{t+1} Z_K(\omega_{t+1}^*) + \frac{\theta C_{t+1}^B}{(1-\theta) K_t^B} \right\}\right]. \tag{64}$$

The marginal cost of housing on the left-hand side consists of the house price p_t minus a term which reflects the collateral benefit of housing; an extra unit of housing relaxes the maximum LTV constraint (7). The right hand side captures the expected discounted future marginal benefits which depends on the resale value of the non-defaulted housing stock as well as on the dividend from housing, which is the intratemporal marginal rate of substitution between housing and non-housing goods.

Second, the borrower's optimal default decision results in the following threshold:

$$\omega_t^* = \frac{(1 - \tau_t^m + \delta q_t^m - \delta \tilde{\lambda}_t^{RB}) A_t^B}{p_t K_{t-1}^B}. \tag{65}$$

At the depreciation threshold level ω_t^*, the cost from foreclosure (and mortgage debt relief), which is the loss of a house valued at $\omega_t^* p_t K_{t-1}^B$, exactly equals the expected cost from continuing the service the mortgage (including the option to default in the future which is encoded in q_t^m) and keeping the house. The cutoff has an intuitive interpretation. It is the aggregate loan-to-value ratio of the borrowers, with both mortgage debt and housing valued at market prices. When the *market* leverage of the borrower increases, the house value threshold ω_t^* rises and default becomes more likely. Note also that when the borrower exercises her prepayment option to its maximum extent, $\tilde{\lambda}_t^{RB} > 0$ and default becomes less likely. Hence the default option and the prepayment option interact. A valuable refinancing option gives the borrower incentives to postpone a default decision as in Deng, Quigley, and Van Order (2000).

Third, the optimal share of outstanding mortgages that the borrower chooses to prepay, $Z_t^R = R_t^B / A_t^B$ is given by:

$$\psi Z_t^R = q_t^m - F + \tilde{\mu}_t^{RB} - \tilde{\lambda}_t^{RB}. \tag{66}$$

This balances the marginal cost of refinancing on the left-hand side with the marginal benefit on the right-hand side. For an internal prepayment choice, the marginal benefit is to increase the value of mortgage debt raised by $q_t^m - F$. Intuitively, when current mortgage rates are lower than when the mortgage was originated, the mortgage is a premium bond and trades at a price q^m above par value F. By refinancing a marginal unit of debt, the borrower gains $q^m - F$. If $q^m - F$ is large enough, the borrower will want to refinance all outstanding debt ($Z_t^R = \delta Z_A(\omega^*)$). The multiplier on the refinancing upper bound activates ($\tilde{\lambda}_t^{RB} > 0$). Conversely, when $q_t^m < F$, refinancing is not useful and the multiplier on the lower refinancing bound, $\tilde{\mu}_t^{RB} > 0$, turns positive to keep $Z_t^R = 0$.

Fourth, from the borrower's first order condition for A^B, we can read off the demand for mortgage debt.

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^B Z_A(\omega_{t+1}^*) \left(1 - \tau^m - \frac{\psi \left(Z_{t+1}^R\right)^2}{2 Z_A(\omega_{t+1}^*)} - \delta \tilde{\lambda}_{t+1}^{RB} + \delta q_{t+1}^m \right)\right]. \tag{67}$$

A unit of mortgage debt obtained generates an amount q_t^m today but uses up some borrowing capacity, which is costly when the borrower's loan-to-value constraint binds ($\tilde{\lambda}_t^B > 0$). The non-defaulted part of the debt must be serviced in future periods, modulo a mortgage interest tax deduction, as long as it is not prepaid.

Depositor FOC The risk averse saver buys short-term debt issued by the risk taker. This debt is equivalent to government debt by virtue of the deposit insurance. The depositor's first-order condition for the short-term bond, assuming the short-sales constraint is not binding, is:

$$q_t^f \quad = \quad \mathrm{E}_t \left[\tilde{\mathcal{M}}_{t,t+1}^D\right].$$

The depositor's precautionary savings incentives are a crucial force determining equilibrium risk-free interest rates.

Risk Taker FOCs Next, we turn to the risk taker's default decision. The risk taker will optimally default whenever the utility costs of doing so is sufficiently small: $\rho_t < \rho_t^*$. The threshold depends on her wealth W_t^R and the state variables $\mathcal{S}_t^R$ that are exogenous to the risk taker, including the wealth of the borrower and of the depositor, and the outstanding amount of government debt. At the threshold, she is indifferent between defaulting and offloading her (negative) wealth onto the government or carrying on:

$$V^R(0, \rho_t^*, \mathcal{S}_t^R) = V^R(W_t^S, 0, \mathcal{S}_t^R),$$

where the value function is defined earlier in this appendix.

Second, the risk taker can invest in both government guaranteed and private MBS. The respective first-order conditions are:

$$\begin{aligned} q_t^m + \gamma_t &= \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{G,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right] + q_t^m \varkappa \xi_G \tilde{\lambda}_t^R \\ q_t^m &= \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{P,t+1} + \delta Z_A(\omega_{t+1}^*)q_{t+1}^m - Z_{t+1}^R[q_{t+1}^m - F]\right)\right] + q_t^m \varkappa \xi_P \tilde{\lambda}_t^R. \end{aligned}$$

Absent binding risk taker borrowing constraints ($\tilde{\lambda}_t^R = 0$), the marginal cost of a guaranteed mortgage bond is the price q_t^m plus the guarantee fee γ_t (expressed as a price) while the benefit is the expected discounted value of the bond tomorrow, which consists of the coupon payment and the repayment of principal in case of default (both are in M_G) plus the resale value of the non-defaulted portion of the mortgage bond. When there are prepayments, the market value of the bond is adjusted for the difference between the market value and the face value, on the share of mortgages that gets prepaid. If the collateral constraint is binding, the benefit is increased by the relaxation of the borrowing constraint, and depends on the haircut ξ_G for guaranteed mortgages. The first-order condition for private mortgages is similar, without the guarantee fee term, with a different collateral requirement term (ξ_P), and a different mortgage payoff M_P.

An equivalent way of restating the risk taker's choice is in terms of how many units of mortgages to originate to borrowers, and for how much of these holdings to buy default insurance from the government. The optimal amount of default insurance to buy solves:

$$\gamma_t = \mathrm{E}_t\left[\tilde{\mathcal{M}}_{t,t+1}^R\left(M_{G,t+1} - M_{P,t+1}\right)\right] + \tilde{\lambda}_t^R \varkappa q_t^m \left(\xi_G - \xi_P\right).$$

Risk takers will buy insurance until the marginal cost of insurance on the left equals the marginal benefit. An extra unit of default insurance increases the payoff of the mortgage and it increases the collateralizability of a mortgage, a benefit which only matters when the borrowing constraint binds. A binding risk taker leverage constraint increases demand for mortgage bonds, and especially for guaranteed bonds given their low risk weight (high ξ_G).

B Computational Solution

The computational solution of the model is implemented using what Judd (1998) calls "time iteration" on the system of equations that characterizes the equilibrium of the economy defined in appendix section A.4. Policy functions, prices, and Lagrange multipliers are approximated as piecewise linear functions of the exogenous and endogenous state variables. The algorithm solves for a set of non-linear equations including the Euler equations and the Kuhn-Tucker conditions expressed as equalities. The general solution approach for heterogeneous agent models with incomplete markets and portfolio constraints that we employ in this paper is well described by Kubler and Schmedders (2003). They show that there exist stationary equilibria in this class of models when all exogenous state variables follow Markov chains, as is the case in our model as well.

The procedure consists of the following steps

1. **Define approximating basis for the unknown functions.** The unknown functions of the state variables that need to be computed are the set of endogenous objects specified in the equilibrium definition. These are the prices, agents' choice variables, and the Lagrange multipliers on the portfolio constraints. There is an equal number of unknown functions and nonlinear functional equations. To approximate the unknown functions in the space of the two exogenous state variables $[Y_t, \sigma_{\omega_t}]$ and four endogenous state variables $[A^B_t, W^R_t, W^S_t, G_t]$, we discretize the state space and use multivariate linear interpolation (splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation). One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents' budget constraints, conditional on any three other state variables. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding, and we test the accuracy of the approximation by computing relative Euler equation errors.

2. **Iteratively solve for the unknown functions.** Given an initial guess $\mathcal{C}^0(\mathcal{S})$ to compute tomorrow's optimal policies as functions of tomorrow's states, solve the system of nonlinear equations for the current optimal policies at each point in the discretized state space. Expectations are computed using quadrature methods. Using the solution vector for current policies, compute the next iterate of the approximation $\mathcal{C}^1(\mathcal{S})$ and repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Judd, Kubler, and Schmedders (2002) show how Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose.

3. **Simulate the model for many periods using approximated policy functions.** To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 500 periods. We verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed.

In a long simulation, errors in the nonlinear equations are low. Table 1 reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 99.5^{th} percentiles.

C Calibration Appendix

C.1 States and Transition Probabilities

After discretizing the aggregate real per capita income growth process as a Markov chain using the Rouwenhorst method, we obtain the following five states for g:

$$[0.943, 0.980, 1.018, 1.058, 1.101]$$

with 5×5 transition probability matrix:

$$\begin{bmatrix} 0.254 & 0.415 & 0.254 & 0.069 & 0.007 \\ 0.103 & 0.381 & 0.363 & 0.134 & 0.017 \\ 0.042 & 0.242 & 0.430 & 0.242 & 0.042 \\ 0.017 & 0.134 & 0.363 & 0.381 & 0.103 \\ 0.007 & 0.069 & 0.254 & 0.415 & 0.254. \end{bmatrix}$$

We discretize the process for σ_ω^2 into a two-state Markov chain that is correlated with income growth g. The two states are:

$$[.078, .203]$$

The transition probability matrix, conditional on being in one of the bottom two g states is:

$$\begin{bmatrix} 0.80 & 0.20 \\ 0.01 & 0.99 \end{bmatrix}$$

The transition probability matrix, conditional on being in one of the top three g states is:

$$\begin{bmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \end{bmatrix}$$

The stationary distribution for the joint Markov chain of g and σ_ω^2 is

State	1	2	3	4	5	6	7
g	0.943	0.943	0.980	0.980	1.018	1.058	1.101
σ_ω^2	0.078	0.203	0.078	0.203	0.078	0.078	0.078
Prob.	0.039	0.023	0.167	0.081	0.372	0.255	0.063

From a long simulation, we obtain the following mean, standard deviation, and persistence for g: 1.019, .039, and .42, respectively. We obtain the following mean, standard deviation, and persistence for σ_ω^2: .092, .039, and .46, respectively. We obtain a correlation between g and σ_ω of -0.42.

C.2 Evidence on default rates and mortgage severities

Since not all mortgage delinquencies result in foreclosures (loans can cure or get modified), we use the fraction of loans that 90-day or more delinquent or in foreclosure as the real world counterpart to our model's default rate. Some loans that were 90-day delinquent or more received a loan modification, but many of these modifications resulted in a re-default 12 to 24 months later. Given that our model abstracts from modifications, using a somewhat broader criterion of delinquency than foreclosures-only seems warranted.

The observed 90-day plus (including foreclosures) default rate rose from 2% at the start of 2007 to just under 10% in 2010.Q1. Since then, the default rate has been gradually falling back, to 4.7% by 2014.Q3 (Mortgage Bankers Association and Urban Institute). The slow decline in foreclosure rates in the data is partly due to legal delays in the foreclosure process, especially in judicial states like New York and Florida where the average foreclosure process takes up to 1000 days. In other part it is due to re-defaults on modified loans. Since, neither is a feature of the model, it seems reasonable to interpret the abnormally high default rates of the post-2013 period as due to such delays, and to reassign them to the 2010-2012 period. If we assume that the foreclosure rate will return to its normal 2% level by the end of 2016, then such reassignment delivers an average foreclosure rate of 8.5% during the 2007-2012 foreclosure crisis. Absent reassignment, the average default rate would be 5.9% over the 2007-2016 period. Jeske et al. (2014) target only a 0.5% foreclosure rate, but their calibration is to the pre-2006 sample. The evidence from the post-2006 period dramatically raises the long-term mean default rate. Our benchmark model generates a default rate of 1.5% in normal times and 12.5% in mortgage crises, for a combined unconditional default rate of 2.65%. These numbers are reasonable in light of the experience over the 2000-2015 period.

Fannie Mae's 10K filings for 2007 to 2013 show that severities, or losses-given-default, on conventional single-family loans were 4% in 2006, 11% in 2007, 26% in 2008, 37% in 2009, 34% in 2010, 35% in 2011, 31% in 2012, and 24% in 2013. Severities on Fannie's non-conforming (mostly Alt-A and subprime) portfolio holdings exceed 60% in all these years. If anything, the severity rate on Fannie's *non-conforming* holdings is lower than that of the overall non-conforming market due to advantageous selection (Adelino, Schoar, and Severino (2014)). Given that the non-conforming market accounted for half of all mortgage originations in 2004-2007, the severities on conventional loans are too low to accurately reflect the market-wide severities. To take account of this composition effect, we target a market-wide severity rate of 45-50% in the crisis (2007-2012). We target a severity rate of 25-30% in non-crisis years (pre-2007 and post-2012), based on Fannie's experience in that period and the much smaller size of the non-conforming mortgage market in those years.

Combining a default rate of 1.5% in normal times with a severity of 30%, we obtain a loss rate of 0.45% in normal times. Combining the default rate of 12.5% during a foreclosure crisis with the severity of 50% in crises, we obtain a 6% loss rate. The unconditional average loss rate in the benchmark model is 1.0%.

C.3 Long-term mortgages

Our model's mortgages are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. If the borrower defaults on the mortgage, the government guarantee entitles the holder to receive a "principal repayment" $F = \frac{\alpha}{1-\delta}$, a constant parameter that does not depend on the value of the collateral or any state variable of the economy. Real life mortgages have a finite maturity (usually 30 years) and a principal payment. They also have a vintage (year of origination), whereas our mortgages combine all vintages in one variable. This appendix explains how to map the geometric mortgages in our model into real-world mortgages.

Our model's mortgage refers to the entire pool of all outstanding mortgages. In reality, this pool not only consists of newly issued 30-year fixed-rate mortgages (FRMs), but also of newly issued 15-year mortgages, other mortgage types such as hybrid adjustable-rate mortgages (ARMs), as well as all prior vintages of all mortgage types. This includes, for example, 30 year FRMs issued 29 years ago. The Barclays U.S. Mortgage Backed Securities (MBS) Index is the best available measure of the overall pool of outstanding government-guaranteed mortgages. It tracks agency mortgage backed pass-through securities (both fixed-rate and hybrid ARM) guaranteed by Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC). The index is constructed by grouping individual TBA-deliverable MBS pools into aggregates or generics based on program, coupon and vintage. For this MBS index we obtain a time series of monthly price, duration (the sensitivity of prices to interest rates), weighted-average life (WAL), and weighted-average coupon (WAC) for January 1989 until December 2014.

Our calibration strategy is to choose values for δ and F so that the relationship between price and interest rate (duration) is the same for the observed Barclays MBS Index and for the model's geometric bond. We proceed in two steps. In the first step, we construct a simple model to price a pool of MBS bonds and calibrate it to match the observed time series of MBS durations. With this auxiliary model in hand, we then choose the two parameters to match the price-rate curve in the auxiliary model and the geometric mortgage model.

C.3.1 Step 1: A simple MBS pricing model

Changes in duration of the Barclays MBS index are often driven by changes in the index composition. As mortgages are prepaid and new ones are issued with different coupons, both the weighted-average-life and weighted-average-coupon of the Index change significantly. Any model that wants to have a chance at matching the observed durations must account of these compositional changes.

For simplicity, we assume that all mortgages are 30-year fixed-rate mortgages. We construct a portfolio of MBS with *remaining* maturities ranging from 1 to 360 months. Each month, a fraction of each MBS prepays. We assume that the prepayment rate is given by a function $CPR(c-r)$ which depends on the "prepayment incentive" of that particular MBS, defined as the difference between the original coupon rate of that mortgage and the current mortgage rate. We assume that every prepayment is a refinancing: a dollar of mortgage balance prepaid result in a dollar of new mortgage balance originated at the new mortgage rate. In addition, each period an exogenously given amount of new mortgages are originated with a coupon equal to that month's mortgage rate to reflect purchase originations (as opposed to refinancing originations).

In a given month t, each mortgage i has starting balance bal_t^i, pays a monthly mortgage pmt_t^i of which int_t^i is interest and $prin_t^i$ is scheduled principal, where i is the remaining maturity of the mortgage, i.e., the mortgage was originated at time $t-(360-i)-1$. Denote the unscheduled principal payments, or prepayments, by prp_t^i. Let SMM_t^i be the prepayment rate in month t on that mortgage. The evolution equations for actual mortgage cash flows are:

$$\begin{aligned}
int_t^i &= \frac{c_{t-(360-i)-1}}{12} \times bal_t^i \\
prin_t^i &= pmt_t^i - int_t^i \\
prp_t^i &= SMM_t^i(bal_t^i - prin_t^i) \\
bal_{t+1}^{i-1} &= (1-SMM_t^i)(bal_t^i - prin_t^i) \\
pmt_{t+1}^{i-1} &= (1-SMM_t^i)pmt_t^i
\end{aligned}$$

The initial payment is given by the standard annuity formula, normalizing the amount borrowed to 1.

$$\begin{aligned}
pmt_t^{360} &= \frac{\frac{c_{t-1}}{12}}{1-(1+c_{t-1}/12)^{-360}} \\
bal_t^{360} &= 1 + \sum_{i=1}^{360} prp_{t-1}^i
\end{aligned}$$

The last equation says that the initial balance of new 30-year FRMs is comprised on 1 unit of purchase originations, an exogenously given flow of originations each period, plus refinancing originations which equal all prepayments from the previous period.

Furthermore, at every month t we compute *projected* cash flows on each mortgage assuming mortgage rates stay constant from t until maturity i. These projected cash flows follow the same evolution equations as presented above. Denote these projected cash flows with a tilde over the variable.

We can then compute the price P_t, (modified) duration Dur_t, and weighted-average-life WAL_t of the MBS *portfolio* comprised of all vintages:

$$\begin{aligned}
P_t &= \sum_{i=1}^{360} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1+r_t/12)^s} \\
Dur_t &= \frac{1}{1+\frac{r_t}{12}} \sum_{i=1}^{360} \frac{1}{P_t^i} \sum_{s=0}^{i} \frac{\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}}{(1+r_t/12)^s} s \\
WAL_t^i &= \sum_{i=1}^{360} \frac{\sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s}) s}{\sum_{i=1}^{360} \sum_{s=0}^{i} (\tilde{pmt}_{t+s}^{i-s} + \tilde{prp}_{t+s}^{i-s})}
\end{aligned}$$

What remains to be specified is our prepayment model delivering the single-month mortality SMM_t^i used above. Following practice, we assume an annual constant prepayment rate (CPR) which is a S-shaped function of the rate incentive: $CPR_t^i = CPR(r_t - c_{t-(360-i)-1})$:

$$CPR(x) = \underline{CPR} + (\overline{CPR} - \underline{CPR})\left(1 - \frac{\exp(\psi(x-\bar{x})}{1+\exp(\psi(x-\bar{x}))}\right)$$

The annual CPR implies a monthly SMM $SMM_t^i = factor_i \times (1-(1-(CPR_t^i)^{1/12})$. The multiplicative $factor_i$ allows us to deal with slow prepayments early in the life of the mortgage (the "ramp-up" phase) and late in the life of the mortgage (the "burn-out" phase). For simplicity, we make $factor_i$ linearly increasing from 0 in month 1 (when $i = 360$) to 1 in month 30, flat at 1 between month 30 and month 180 and linearly decreasing back to 0 between months 180 and month 360. We choose the CPR curve parameters $\{\underline{CPR}, \overline{CPR}, \psi, \bar{x}\}$ to minimize the sum of squared errors between the time series of model-implied duration $\{Dur_t\}$ and observed duration on the Barclays index.

To produce the time-series of model-implied duration $\{Dur_t\}$, we feed in the observed 30-year conventional fixed rate mortgage rate (MORTGAGE30US in FRED), $\{r_t\}$. We initialize the portfolio many years before the start of our time series data to ensure that the model is in steady state by the time our time series for the Barclays index starts. Specifically, we start the computation in April 1903 by issuing 1 MBS. By March 1933, we have a complete portfolio of 360 fixed-rate amortizing mortgages, maturing any month from April 1933 to March 1963.

The left panel of Figure 1 shows the observed time series of duration on the Barclays MBS index plotted against the model-implied duration on the MBS pool. The two time series track each other quite closely despite several strong modeling assumptions. The resulting CPR curve looks close to historical average prepayment behavior on agency MBS, as prepayment data from SIFMA indicate. CPR is slightly above 40% when the rate incentive is 200 bps or more, about 15% when the rate incentive is zero, and slightly above 5% when the rate incentive is below -200 bps.

C.3.2 Step 2: Matching MBS pool to perpetual mortgage in our model

With a well-calibrated auxiliary model for a MBS pool, we now proceed to match key features of that auxiliary model's MBS pool to the mortgage in our model, which is a geometrically declining perpetuity.

We start by computing the price $P(r)$ of a fixed-rate MBS with maturity T and coupon c as a function of the current real MBS rate r, using the constant prepayment rate function $C\hat{P}R(r) = CPR(r-c)$ obtained from step 1. For T and c we use the time-series average of the weighted-average maturity and weighted-average real coupon, respectively, from the model-implied MBS pool obtained in step 1.[43]

We can write the steady-state price of a guaranteed geometric mortgage with parameters (δ, F) and a per-period fee γ paid for the life of the loan recursively as:

$$Q(r,\gamma) + \gamma = \frac{1}{1+r}\left(1 + C\hat{P}R(r)\delta F + (1 - C\hat{P}R(r))\delta(Q(r,\gamma)+\gamma)\right)$$

Solving for $Q(r,\gamma)$, we get

$$Q(r,\gamma) = \frac{1 + C\hat{P}R(r)\delta F}{1 + r - \delta(1 - C\hat{P}R(r))} - \gamma. \tag{68}$$

Note that the fee γ in equation 68 is quoted in units of the guaranteed bond's price. However, in the data MBS pool we observe a guarantee and servicing fee of approximately 50 bp on average that is charged as a spread on top of a bond's yield. During the calibration, we thus need to use the net-of-fees rate for the MBS pool and the gross-of-fees rate for the geometric bond.

The stage 2 calibration determines how many units X of the geometric mortgage with parameters (δ, F) one needs to sell to hedge one unit of the MBS against parallel shifts in interest rates, across the range of historical mortgage rates:

$$\min_{\delta,F,X,\gamma} \int [P(r) - XQ(r+0.005,\gamma)]^2 dr,$$

[43] To get real mortgage rates from nominal mortgage rates, we subtract realized inflation over the following year. To get real coupons and MBS rates from real mortgage rates, we subtract 50 bps to account for servicing and guarantee fees.

subject to

$$\log\left(\frac{1}{Q}+\delta\right)=\log\left(\frac{1}{Q+\gamma}+\delta\right)+0.005. \tag{69}$$

The equality constraint 69 determines the price-fee γ that corresponds to the 50 bps rate-fee. The LHS is the gross-of-fees mortgage rate and the RHS is the equivalent net-of-fees mortgage rate plus the 50 bps fee[44]. Generally the equivalent price-fee will depend on the level of the price, which is endogenous to the minimization problem. Thus the constraint determines γ as the equivalent price-fee when the MBS trades at par (with price 1) so that $Q = 1/X$.

We estimate values of $\delta = 0.948$, $F = 9.910$, which implies $\alpha = 0.520$, and $X = 0.1080$. For the model calibration, we only need δ and α. The right panel of Figure 1 shows that the fit is excellent. The average error is only 0.34% of the MBS pool price.

In conclusion, despite its simplicity, the perpetual mortgage in the model captures all important features of real life mortgages (or MBS pools). The relationship between price and interest rate is convex when rates are high and concave ("negative convexity") when rates are low, which is when the prepayment option is in the money. It matches the interest rate risk (duration) of real-life mortgages, for different interest rate scenarios.

D Additional Experiments

Our main policy experiment consisted of raising the g-fees. In the main body of the paper, we also reviewed two alternative experiments. The first one introduced a state-contingent g-fee. The second considered a legislative proposal to to "put private capital in front of a government guarantee" by limiting guarantees to losses in excess of 10%. We now study three more policy experiments and compare their welfare consequences to those in the main policy experiment. The first experiment explores the effect of limited liability. The next two experiments study alternate ways to make guarantees operative only when losses are catastrophic. These exercises help to further illuminate the interaction of government guarantees, deposit insurance, and risk taker leverage.

D.1 Limited Liability

We consider a policy that weakens deposit insurance. The knowledge that they (and their depositors) will be bailed out by the government if their net worth turns negative leads banks to take on more risk. We weaken deposit insurance, or equivalently weaken limited liability for banks, by increasing the mean (μ_ρ) of the utility penalty that banks incur for insolvency. The third and fourth columns of Table 2 labeled "high μ_ρ" report the results.

Guarantees remain very valuable and dominate the portfolio. Portfolio delinquency and loss rates are close to the benchmark low g-fee economy. One big difference to the main experiment is that weaker limited liability does not lead to a meaningful reduction in intermediary leverage. Risk taker net worth only increases marginally. Still, this small increase in net worth, combined with the higher utility cost of bankruptcy is enough to eliminate all bankruptcies. As a result of the high leverage and government debt, depositors must hold substantial amounts of safe assets and interest rates are a bit higher than the benchmark model. The real short rate increases by 2bps to 1.15%.

We find no significant effect on aggregate welfare from this policy. It has an aggregate welfare loss of 0.02%. Borrowers' welfare is unaffected and both risk takers and depositors lose slightly. In sum, while increasing the costs of bank bankruptcy is successful at eliminating bank bankruptcies, it has a small negative aggregate welfare effect. This demonstrates that intermediary bankruptcies are not the driving force behind our welfare results. The key issue rather is the underpricing of the guarantee, which is as paramount in this economy as in the low g-fee benchmark.

[44]The yield of a geometric bond with price Q and duration parameter δ is $r = \log\left(\frac{1}{Q}+\delta\right)$.

D.2 Catastrophic Insurance

Columns 6 and 7 of Table 2 report results for a catastrophic insurance policy which "kicks in" at losses of 5%. Welfare increases are smaller than when the private sector loss is capped at 10% (+0.56% vs +0.67%). This increase in welfare is smaller than that from a complete phase-out. Thus the largest welfare gains are obtained when the private sector bears enough losses to reduce its risk taking, but not so much as to debilitate its intermediation function which is important to achieve the best distribution of aggregate risk in the economy.

The last two columns of Table 2 report results for a catastrophic insurance experiment in which losses are capped at 10% but where the insurance is offered at a much lower price of 5 bp instead of the 20bp discussed in the main text. This policy has higher welfare gains of +0.69%, compared to +0.67% for the 20bp catastrophic guarantee and +0.63% for the full phase-out. The main difference with the more expensive catastrophic guarantee is that because the guarantee is cheaper (and closer to the actuarially fair cost of 2bp), risk takers are much more likely to purchase it. This protects them better to unexpected catastrophic shocks than in the 20bp JC economy. It further improves risk sharing and raises interest rates.

D.3 Lowering Depositor Risk Aversion

In the baseline calibration we chose depositor risk aversion and patience to match the risk-free rate level and volatility. How sensitive are our results to the high coefficient of relative risk aversion for the depositor? Will the risk-taker's role as intermediary between borrower and depositor still arise even when the spread in risk aversion between borrower and depositor and between depositor and risk taker is much lower? Will the high intermediary leverage result survive in the low g-fee economy? Ultimately, this robustness check gets at the question of how sensitive our preference-based modeling of intermediation is. We find qualitatively similar results when we lower depositor risk aversion, σ_D, from 20 to 10.

As expected, the depositors precautionary savings incentives are much reduced. This results in higher equilibrium depositor consumption and higher risk-free interest rates (3.06% versus 1.13%). Depositor welfare in consumption equivalent units nearly doubles. Higher risk-free interest rates are passed through in the form of higher mortgage rates. Faced with a higher cost of borrowing, borrowers take out fewer mortgages, have lower LTVs, and default rates fall considerably (from 2.74% to 1.44%). Given higher mortgage rates and a smaller mortgage market, house prices are naturally lower. Borrower consumption and welfare are lower. Faced with lower mortgage default and prepayment risk, the risk taker continues to choose high leverage (94.7% versus 95.6% in the benchmark). In other words, our key result of high intermediary leverage is surprisingly insensitive to lower σ_D. Our intuition is that the risk taker faces a low mortgage spread and chooses to keep leverage high given her desired risk-return profile. Finally, because the risk-free rate goes up, the governments cost of borrowing goes up considerably. It takes the government longer to pay off debt accumulated in crises, and the steady-state level of government debt is higher (128% of trend GDP on average). Taxes are higher on average as well since the model spends more time in the austerity region where tax rates are increased. Higher average taxes lower every agents consumption.

D.4 Progressive Taxation

In the models considered in the main text, all agents' labor income is taxed at the same rate τ. We now investigate how the model changes with progressive taxation, and in particular whether progressive taxation affects our main conclusion regarding the change in welfare from phasing out the GSEs. We recall that per capita income is highest for risk takers, followed by savers, and lowest for borrowers. To implement progressive taxation, we solve our economy for a different tax rate for each agent. Correspondingly, we set the tax rate at 30% for risk takers, keep the tax rate at 20.3% for depositors (the tax rate that applies to all agent in the baseline calibration), and lower the tax rate to 18% for the borrowers. This choice of tax rates keeps overall tax revenue unchanged compared to our benchmark economy. We find that the effects of increasing the price of the government guarantee in the economy with progressive taxation are quite similar to that same transition in the flat tax economy. Specifically, the welfare gain from privatization (increasing the g-fee from 20bps to 275bps) is +0.57% compared to +0.63% in the benchmark exercise. All intuition carries over.

As an aside, introducing progressive taxation by itself, holding fixed the g-fee at 20bps, increases welfare by +1.19%. There is less consumption inequality, with borrowers having higher and risk-takers lower average consumption. Higher taxes lead risk takers to accumulate more wealth, but intermediary leverage is very similar to the benchmark economy with flat taxes. Government debt level and volatility as well as the risk-free interest rates are similar as well.

D.5 Private Mortgage Insurance

An interesting question is whether a private market for mortgage insurance may achieve the same outcome as an economy with government-provided guarantees. Our take on this interesting, but complicated question is as follows. Private mortgage insurance companies (like AIG) would be part of the (levered) financial sector. Like banks deposit insurance, they would enjoy (implicit) bailout guarantees. This would introduce the same kind of moral hazard considerations that we have explored for the banks. Like banks, they would be subject to regulatory capital regulations, in this case Solvency II rather than Basel II. A complete treatment of private mortgage insurers would require adding one more balance sheet, going from 4 to 5 balance sheets, and add one more state variable, namely the net worth of the insurance sector. The added numerical complexity means that we do not pursue this route. In terms of the economics, we speculate that the too-big-to-fail guarantees that the PMI sector would inevitably enjoy would lead to underpriced mortgage insurance. A bad aggregate state of the world with severe mortgage losses might wipe out the PMI sectors net worth and offload its excess liabilities onto the government/tax payers. This situation is not unlike the catastrophic insurance model we already study where the government directly bears the default risk in bad states of the world.

A much simpler answer is to read the current model as one that has private mortgage insurance already incorporated, but assets and liabilities of banks and private mortgage insurers are consolidated as part of the levered financial sector. Because PMI is a liability of the insurers and an asset of the banks; it cancels out on the aggregate financial sector balance sheet, no matter what the quantity of insurance demanded and supplied. All other equilibrium quantities and prices would be the same as in the economy we currently compute. Even though this model does not pin down the quantity of private mortgage insurance, it can speak to the price of PMI. Specifically, we can calculate in the private sector equilibrium where no government guarantees are purchased (high g-fee equilibrium), what price the financial sector would charge for mortgage insurance, using the SDF of the financial sector. We find that the unconditional value of mortgage insurance as an asset (to a bank buying it from an insurer) is 58 bps. It is much higher in crises (132bps) than in normal times (49bps). Interestingly, the price is a bit lower as a liability (to a private mortgage insurer): 52bps versus 58bps. This happens because because with probability F(rho) the insurer defaults and the payout is made by the government instead. Even though F(rho) is very small in almost all states, the states where it is not are precisely the states where mortgage payouts are high. So the product of the probability of a bailout and the cost of the PMI bailout to the taxpayer is not negligible.

Table 1: Computational Errors

	Percentile				
	50th	75th	95th	99th	Max
(67)	0.0013	0.0024	0.0099	0.0137	0.0254
(36)	0.0005	0.0013	0.0063	0.0089	0.0162
(53)	0.0008	0.0050	0.0158	0.0266	0.0702
(51)	0.0007	0.0055	0.0166	0.0293	0.0758
(62)	0.0002	0.0004	0.0008	0.0017	0.0081
(23)	0.0006	0.0007	0.0042	0.0062	0.0309
(40)	0.0003	0.0006	0.0025	0.0035	0.0086
(56)	0.0031	0.0042	0.0085	0.0094	0.0242
(16)	0.0002	0.0004	0.0013	0.0028	0.0073
(42)	0.0000	0.0000	0.0000	0.0007	0.0114
(24)	0.0000	0.0001	0.0003	0.0015	0.0105
(52)	0.0007	0.0053	0.0160	0.0283	0.0749
(41)	0.0001	0.0001	0.0001	0.0002	0.0028
(39)	0.0001	0.0002	0.0005	0.0015	0.0269
(15)	0.0004	0.0006	0.0017	0.0030	0.0507

The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the 20 bps g-fee model. The first 13 equations define policy functions. They are a subset of the 22 equations that define the equilibrium. The last two equations define evolutions of risk-taker wealth and government debt, respectively.

Table 2: The Role of Limited Liability and Catastrophic Insurance

	20 bp g-fee		**High μ_ρ**		**JC 5%**		**JC 10%, 5 bp**	
	mean	stdev	mean	stdev	mean	stdev	mean	stdev
				Prices				
Risk free rate	1.17%	3.00%	1.20%	3.07%	1.86%	3.21%	1.96%	3.20%
Mortgage rate	3.52%	0.24%	3.52%	0.24%	3.73%	0.26%	3.74%	0.25%
House price	2.239	0.142	2.239	0.142	2.114	0.121	2.106	0.122
Actuarially Fair g-fee	0.76%	0.42%	0.76%	0.42%	0.10%	0.08%	0.02%	0.02%
				Borrower				
Mortgage debt	0.053	0.001	0.053	0.001	0.050	0.001	0.050	0.001
Borrower LTV	63.74%	3.90%	63.74%	3.88%	63.76%	3.47%	63.72%	3.49%
Market value of debt LTV	75.59%	6.44%	75.59%	6.44%	73.93%	6.02%	73.82%	5.98%
Borrower debt to income	1.487	0.040	1.487	0.040	1.405	0.024	1.399	0.024
Debt/income growth	0.04%	2.84%	0.04%	2.88%	0.04%	2.88%	0.04%	2.75%
Mortgage default rate	2.66%	6.03%	2.67%	6.01%	1.77%	3.90%	1.70%	3.69%
Severity rate	30.19%	5.75%	30.19%	5.75%	30.07%	5.72%	30.06%	5.71%
Mortgage loss rate	1.02%	2.68%	1.02%	2.66%	0.68%	1.71%	0.65%	1.60%
Rate-induced prepayment rate	15.77%	4.24%	15.75%	4.30%	12.16%	4.42%	12.01%	4.28%
				Risk-Taker				
Market value of bank assets	0.633	0.018	0.633	0.018	0.583	0.013	0.580	0.012
Fraction guaranteed	99.96%	0.53%	99.92%	0.88%	54.18%	34.20%	75.18%	24.32%
Risk taker leverage	95.58%	0.92%	95.27%	1.06%	93.28%	2.34%	94.89%	1.57%
Risk taker wealth	0.029	0.012	0.031	0.013	0.040	0.018	0.031	0.014
Fraction $\lambda^R > 0$	32.60%	46.88%	27.06%	44.43%	84.93%	35.78%	98.81%	10.84%
MTM Loss Given Prepayment	10.74%	2.59%	10.73%	2.62%	8.46%	2.83%	8.37%	2.74%
Loss rate guaranteed	0.37%	0.85%	0.37%	0.86%	0.21%	0.51%	0.20%	0.48%
Loss rate portfolio	0.37%	0.85%	0.37%	0.86%	0.41%	1.10%	0.33%	0.81%
Bankruptcy frequency	0.27%	5.19%	0.00%	0.00%	0.02%	1.41%	0.44%	6.62%
Return on RT wealth[a]	3.17%	35.79%	2.47%	34.73%	2.47%	30.80%	3.22%	36.84%
				Government				
Government debt / GDP	15.88%	22.63%	16.39%	22.98%	−6.03%	3.89%	−6.05%	4.17%
				Welfare				
Aggregate Welfare	0.279	0.008	−0.02%	+0.06%	+0.56%	+0.48%	+0.69%	+0.41%
Value Function borrower	0.319	0.010	0.00%	+0.02%	+0.06%	−1.03%	+0.04%	−1.16%
Value Function depositor	0.249	0.006	−0.04%	+0.14%	+1.14%	+0.95%	+1.44%	+0.95%
Value function risk taker	0.083	0.000	−0.24%	+6.48%	+0.96%	+40.22%	+1.02%	+9.03%

The table reports unconditional means and standard deviations of the main outcome variables from a 10,000 period simulation of four different models. The first two columns are the benchmark model from Table 2. The next two columns have a higher mean utility cost of default μ_ρ. The model in columns 6 and 7 report results for an economy where the government offers catastrophic insurance i.e. guarantees only losses in excess of 5%. The last 2 columns report results for a catastrophic insurance economy where the attachment point is 10%, like in Table 4, but at a lower price of 5 bp.

[a]: Return on wealth is the return on the risk takers total portfolio i.e. their positive position in mortgages and negative position in deposits. Return on wealth is computed by excluding simulation periods when risk takers declare bankruptcy.

Figure 1: Matching Mortgages in Model to Data

The left panel plots the observed time series of duration on the Barclays MBS index (solid line) plotted against the duration on the model-implied MBS pool (dashed-line). The right panel plots the mortgage price-interest rate relationship for the model-implied MBS pool (solid line) and the model-implied geometrically declining perpetual mortgage (dashed line). Prices on a $100 face value mortgage are on the vertical axis, while interest rates are on the horizontal axis. The Barclays MBS index data are from Bloomberg for the period 1989 until 2014 (daily frequency). The calculations also use the 30-year fixed-rate mortgage rate from FRED.