Private Money Creation and Equilibrium Liquidity*

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Abstract

We study the joint supply of public and private liquidity using a simple macroeconomic model. In a frictionless, competitive financial market, private intermediation creates riskless securities (safe assets) and satiates the liquidity needs of the economy without any regulation, and the economy achieves the first best. However, if issuing equity is more costly than debt, financial intermediaries increase leverage and instead supply risky securities which make the economy vulnerable to liquidity crunches. We use our framework to revisit, in the context of the modern financial system, some classic proposals on liquidity supply: real-bills doctrine, free banking, and narrow banking theories. We also compare these proposals to more standard interventions implemented in recent times, such as capital requirements and bailouts. Some government programs increase welfare but might entail subtle trade-offs.

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1 Introduction

The recent financial crisis has unveiled the importance of a shadow-banking sector that for years has been able to provide some form of money-like assets (i.e., private money). Suddenly, transacting parties realized that several types of these money-like assets were not completely safe because of a lack of appropriate backing in intermediaries' balance sheets. Thus, what had been acceptable to satisfy liquidity needs became inadequate. The subsequent shortage of liquid assets produced a disruption in the real economy and a deep recession.

Swings in the creation and destruction of private money are not just a recent phenomenon. In addition to the 2008 meltdown, they have characterized almost every deep financial crisis throughout much of monetary history, with different names given to the intermediaries and their assets and liabilities.[1]

Economists have long debated two questions related to the supply of liquid assets. First, should the supply of liquidity be left to a monopolist acting under government responsibility or run privately under competition with no regulation? And second, which kinds of backing should the suppliers of liquidity hold?

Old and prominent theories, such as real-bills doctrine, free banking theory, and narrow banking theory, have tried to answer to the above questions. This paper revisits this debate and enriches it with insights coming from recent macroeconomic theory. It also compares these classical proposals to some interventions implemented in recent times to support liquidity provision, such as bailouts and capital requirements.

We propose a simple macroeconomic model to study equilibrium liquidity. The model features a financial friction which limits the securities that are liquid and thus can be exchanged for goods. In line with the historical evidence discussed by Gorton (2016), different types of debt serve now as liquid assets.[2] The key distinction in our framework is that riskless debt (what we call safe assets) always provides liquidity whereas risky debt (pseudo-safe assets) only does so when not in default. Government and private financial intermediaries can both manufacture debt securities. Government debt is always safe and backed either by taxes or the earnings on central bank's portfolio. Financial intermediaries can instead back their debt by investing in risky projects or raising equity, and are subject to a limited liability constraint. The key feature of our framework is that the type of debt security that financial intermediaries create – safe or pseudo safe – is endogenous and depends on market competition and government policy.

The first result of our model is that frictionless intermediation under unregulated competition enables private financial intermediaries to issue risk-free debt – safe assets – and reach the efficient supply of liquidity. This finding is perfectly in line with

[1]For a comprehensive perspective on the debate, see Aguirre (1985), Aguirre and Infantino (2013), Sargent (2011).

[2]This is also in line with the view of Hayek (1976) that there is not 'one clearly defined thing called "money"...different kinds of money can differ from one another in two distinct although not wholly unrelated dimensions: acceptability (or liquidity) and the expected behaviour (stability or variability) of its value' (Hayek, 1976, p. 57).

the view of Hayek (1976) or other extreme theories of free banking, emphasizing the social benefits of deregulation, and it has two important implications. First, in contrast with the old real-bills doctrine, there is no need to restrict the assets that intermediaries should hold.[3] In our model, intermediaries invest in risky real securities and optimally choose to supply safe debt by raising enough equity to absorb any loss on their assets. Second, private incentives for intermediaries' borrowing are perfectly aligned with social objectives. But why is issuing risk-free debt in intermediaries' self interest? If there is a shortage of risk-free debt, which provides liquidity even during financial crises, consumers are willing to pay a premium to hold such assets. For intermediaries, the premium reduces borrowing costs and boosts rents, and thus it creates incentives to issue risk-free debt. Since competition eliminates all rents in equilibrium, issuance of risk-free debt continues up to the point in which the shortage of safe assets is eliminated and the liquidity premium is driven to zero, achieving efficiency.

With respect to *laissez faire,* a government monopoly over the supply of liquidity has no benefits, and possible shortcomings.[4] One way to implement a government monopoly is Friedman's (1960, ch. 4) proposal of narrow banking. This system prescribes that intermediaries have to satisfy a 100% reserve requirement and thus liquidity is *de facto* determined by the supply of central bank's reserves. At most, if the government is benevolent, the same welfare as *laissez faire* can be achieved. If instead the government restricts the supply of liquidity to drive up the liquidity premium and extract rents, welfare is lower. Indeed, in the words of Hayek, monopoly 'prevents the discovery of better methods of satisfying a need for which a monopolist has not incentive' (Hayek, 1976, p. 28). In our context the 'need' is the efficient supply of liquidity, the 'incentive' of a monopolist is to reduce the provision of liquidity to earn rents, and the 'discovery of better methods' is private money creation by financial intermediaries.

Our previous results change significantly if there are frictions in the financial sector. We introduce a simple amendment to the above framework by assuming that it is relatively more costly to raise equity than debt. This is motivated by a long strand of literature that has pointed out that financial intermediaries face higher financing costs through equity rather than debt financing. The important implication of adding this friction in our model is that, under unregulated competition, intermediaries have incentives to produce pseudo-safe debt (i.e., securities that are defaulted on in bad states) rather than safe, riskless securities. This result is a direct consequence of the higher cost of issuing equity. Since equity is more costly than debt, intermediaries

[3] Hundreds of years ago John Law, a Scottish financier, was one of the first proponents of the so called real-bills doctrine. Under this view, liabilities of intermediaries acting without barriers to competition can satisfy the liquidity needs of the economy, as long as the assets of these intermediaries are free of risk ("real bills"), guaranteeing the safety of liabilities themselves. Adam Smith (1976, p. 323) presents also a similar theory with a bit more emphasis on the ultimate convertibility of real bills into gold to avoid excessive money creation.

[4] In our model, some positive but small issuance of public debt is necessary to determine the price level as in the fiscal theory of the price level overcoming the difficulties that Friedman (1960) had with the intedeterminacy of the price level when money and credit markets are not separated. See also Sargent and Wallace (1982) and Sargent (2011).

issue more debt and less equity, thus increasing their leverage. The higher leverage has no effect in good states in which the return on intermediaries' assets is high, and thus pseudo-safe securities do not default and so provide liquidity services. However, the higher leverage forces intermediaries to default in bad states. In this case, the securities they supply lose liquidity value, causing an overall shortage of liquidity.

The results of the *laissez-faire* equilibrium are related to an inefficiency driven by the cost of equity. This inefficiency implies that private incentives are no longer aligned with social objectives. That is, there is a pecuniary externality similar to Stein (2012), but with a crucial difference.[5] In Stein (2012), intermediaries issue only safe assets and a fire-sale externality implies an overissuance of this type of debt. In contrast, in our model, intermediaries issue not only safe assets but also pseudo-safe assets. As a result, the cost of issuing equity gives rise to an overissuance of pseudo-safe securities (i.e., risky securities) and underissuance of safe, risk-free securities. In this sense, our model is consistent with the narrative of the run-up to the 2008 financial crisis. At that time, the supply of AAA asset-backed securities (i.e., pseudo-safe securities) was very high and such securities were widely used as collateral for repo transactions, representing a source of liquidity and financing for many players in the economy. These securities, though, were not as safe as Tresuries or other government-guaranteed assets (i.e., truly safe assets). When the financial crisis erupted, AAA asset-backed securities lost their liquidity value, causing substantial distress.

In contrast to the baseline model with frictionless intermediation, the cost of issuing equity opens up a role for the government to improve upon *laissez faire*. We consider three policies: government provision of liquidity, government bailouts, and capital requirements. We first present the results of the policy analysis in the context of the model and then provide some discussion related to their practical implementation.

The first policy we consider is government provision of liquidity. In the spirit of Friedman's proposal, a benevolent government can improve upon *laissez faire* and achieve the first best, as long as it appropriately backs its supply of debt. If fiscal capacity is large, the government can simply use taxes to back debt. If instead fiscal capacity is limited, the government can purchase pseudo-safe securities created by private intermediaries (either directly or through the central bank) and use them to back its debt. That is, the return received on private pseudo-safe securities provides a stream of revenues which in turn is used to pay interest on public debt.[6] In this case, a sufficiently large fiscal capacity is only needed in the unfavorable states in which private debt is defaulted on. Although richer models may impose some limit on the government purchases of risky debt (say, by emphasizing costs of default or moral-hazard arguments), our analysis suggests that some increase in central bank's balance sheet through the purchases of private debt might still be optimal even in normal times.

[5] The result of Stein (2012) is in turn related to Lorenzoni (2008).

[6] If the central bank buys only Treasury bills and issues interest-bearing reserves, the overall government supply of liquidity remains unchanged and its backing is only provided by taxes.

The second policy, government bailouts during financial crises, allows the economy to achieve the first best as well. By implicitly or explicitly promising to bail out financial intermediaries, the government makes intermediaries' debt riskless. As a result, private debt can provide the efficient supply of liquidity as in the baseline model with frictionless intermediation. Similar to the previous policy, the government achieves the first best by providing more public backing in the contingencies in which private backing is insufficient. However, more general models can point out the bailout costs (such as moral hazard) that are not captured by our analysis. Nevertheless, we argue that some form of bailouts might still be optimal even in broader frameworks, so that the spirit of our result would be unchanged.

The third proposal that we consider is capital requirements. This policy is a natural candidate in models in which intermediaries issue excessive, risky debt and their default reduces the availability of liquid assets. However, since capital requirements force intermediaries to reduce leverage and thus debt, welfare might decrease due to a lower supply of liquidity in good states. As a result, capital requirements are neither necessary nor sufficient to achieve the first best. Nevertheless, if the probability of a crisis is sufficiently large and the supply of public liquidity is sufficiently low, capital requirements increase welfare with respect to *laissez faire*.

Our framework provides a unifying rationale for three key policies that have been implemented in response to the 2008 financial crises: central bank's asset purchases and expansion of public liquidity provision, bailouts, and capital requirements. In addition to implementing these policies during crises, our results suggest that a somewhat larger central bank's balance sheet is useful even during normal times if private intermediation alone is unable to fulfill the liquidity needs of the economy.

1.1 Related literature

Our analysis complements a recent literature that has studied the role of liquidity in macro models by providing detailed specifications of financial intermediaries. Examples include Bianchi and Bigio (2016), Bigio (2015), Gertler and Kiyotaki (2010), Moreira and Savov (2016), and Quadrini (2014). In particular, Quadrini (2014) underlines the non-pecuniary benefits that the liabilities of financial intermediaries can provide to the economy and the implications of their shortages for poor macroeconomic outcomes. In his model, the value of liquidity depends on the self-fulfilling expectations of the private sector on whether banks' liabilities are or are not liquid. Bigio (2015) emphasizes the role of liquidity to relax limited-enforcement constraints and the importance of fluctuations of liquidity for macroeconomic outcomes. The creation of liquidity in his model is endogenous and is affected by the cost of liquidating assets under asymmetric information. Liquidity is also endogenous in our model but depends on the costs of equity financing and the overall supply of public debt, rather than asymmetric information. Moreira and Savov (2016) provide a model in which the liquidity transformation of the banking sector can produce both safe and pseudo-safe securities due to adverse selection problems. However, the main objective of their analysis is to study the macroeconomic consequences of shortages of liquidity due to shifts in the probability of default.

With respect to the above literature the novelty of our paper is to analyze the coexistence between private and public liquidity and efficiency of one form of liquidity over the other. A related paper by Sargent and Wallace (1982) compares the real-bills doctrine with the quantitative theory of money in an overlapping generation model.[7] However, the tension they emphasize is between achieving efficiency in the supply of inside money versus stabilizing the price level. Our focus is instead on the creation of private securities and on their contribution at achieving efficiency, as a function of financial market frictions and of the policy environment. In particular, one of the main contributions of our model is to endogenize the default risk embedded in private securities, which in turn affects welfare.

The banking literature is rich in models that analyze liquidity creation in the spirit of the seminal contribution of Gorton and Pennacchi (1990). The closest papers are Greenwood, Hanson, and Stein (2015) and Magill, Quinzii, and Rochet (2016). These works assume that liquidity services are provided only by risk-free securities, whereas in our framework risky securities can also be liquid. As a result, our model can study the determination of the liquidity and risk properties of private debt jointly as a function of the characteristics of financial intermediaries and the policy environment.[8] In addition, there are some other important differences with respect to the above two papers. In Magill, Quinzii, and Rochet (2016), only private debt can provide liquidity services and, therefore, the focus of their analysis is to study how government policies can enhance the supply of private liquidity. In our model, instead, government debt has also liquidity value. As a result, we focus on the complementarity and trade offs between private and public provision of liquidity. Despite these differences, both models predict that the central bank can achieve the first best by issuing safe securities and backing them by purchasing risky assets. In our model, this is a consequence of the direct liquidity role of public debt, whereas in their context it is a way to increase the funds channeled to investments. In Greenwood, Hanson, and Stein (2015), government short-term debt has liquidity value whereas long-term debt does not; however, short-term debt entails refinancing risk. Nevertheless, tilting the maturity structure by overissuing short-term government debt is optimal. More short-term government debt lowers the liquidity premium on liquid assets, which in turn reduces a pecuniary externality related to private money creation. This externality leads to overissuance of safe assets by private intermediaries and fire sale costs, as in Stein (2012).[9] In our context, public liquidity can also overcome the negative externality of private money creation, but with a key difference. The externality in our model leads to underissuance of safe assets and overissuance of risky pseudo-safe assets, as explained in the introduction. Moreover, we consider a broader set

[7] Bullard and Smith (2003) also use an overlapping generations model to study the role of outside and inside money in achieving efficiency.

[8] In Gennaioli et al. (2012), debt can also be risky; however, due to a behavioral assumption, investors perceive private debt to be risk-free. In their model, a crisis happens when investors become aware of the tail risk that they had neglected.

[9] Woodford (2016) also argues that quantitative-easing policies provide a channel that can mitigate incentives for risk taking of private intermediaries by reducing the liquidity premia in the economy.

of government policies in comparison to those studied by Greenwood, Hanson, and Stein (2015): provision of public liquidity backed by central bank's earnings on its portfolio of assets, bailouts, and capital requirements. These policies can lessen the tax burden required to back public money creation without impacting the ability of the government to improve welfare.

Finally, our work is also motivated by the recent literature spurred by the work of Caballero (2006) that has emphasized the shortage of safe assets as a key determinant of the imbalances of the global economy. Examples include Caballero and Farhi (2016), Caballero and Simsek (2017), and Farhi and Maggiori (2016). As in Caballero and Farhi (2016), we stress the importance of fiscal capacity for the supply of safe government securities and in general the role of other forms of backing (assets, equity) as the primary source of liquidity creation. Farhi and Maggiori (2016) study the supply of safe assets from a global perspective emphasizing the strategic devaluation decisions of a monopolist in an international context. They also consider multiple issuers and the limiting case of perfect competition in which all issuers can provide safe assets. Our model has instead underlined the endogeneity of the process of creation of safe assets depending on the frictions in financial intermediation and the policy environment.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 discusses the equilibrium under frictionless intermediation and unregulated competition. Section 4 studies the implications of the model with frictional intermediation adding an additional cost of issuing equity versus debt. Section 5 discusses the role of government intervention in improving upon the *laissez-faire* equilibrium of Section 4. Section 6 concludes.

2 Model

We present a simple infinite-horizon, stochastic, general-equilibrium model in which we show all our results analytically. The economy features three sets of actors: households, financial intermediaries and a government.

The model combines a fixed supply of capital that produces a stochastic output (Lucas tree) with a liquidity constraint that restricts the type of assets that can be used to finance some consumption expenditure. Households and financial intermediaries have the same ability to invest in the productive asset. Liquidity services are provided by riskless debt securities (safe assets) and, to a lesser extent, by risky debt securities (pseudo-safe assets). Debt securities can be issued by the government and by financial intermediaries. We explain in more detail the liquidity properties of pseudo-safe assets in Section 2.2.

2.1 Production

In a generic period t, output Y_t is produced by a fixed amount of capital, $\overline{K}$, through the production function $Y_t = A_t\overline{K}$ where A_t is aggregate productivity which is the only exogenous disturbance in the model. For simplicity, there are two states of nature, h and l, that are related to the realization of aggregate technology A_t according to

$$A_t = \begin{cases} A_h & with\, probability\ 1-\pi \\ A_l & with\, probability\ \pi \end{cases} \tag{1}$$

so that A_t is i.i.d. over time. Capital $\overline{K}$ can be held by both households and financial intermediaries. We denote K_t^H and K_t^I to be the stock of capital held by households and financial intermediaries, respectively, so that $K_t^H + K_t^I = \overline{K}$.

Output is purchased and consumed by households in two distinct markets that open sequentially during period t. In the first subperiod, households purchase C_t subject to a liquidity constraint (see next section); in the second subperiod, households purchase X_t. Output Y_t can be sold in both subperiods, so that $Y_t = C_t + X_t$. Alternatively, this setting can be described as a cash-credit model à la Lucas and Stokey (1987), where C_t is the cash good and X_t is the credit good. Since output Y_t can be sold in both submarkets, the price of C_t and X_t is the same and denoted by P_t.[10]

The nominal return on capital i_t^K is defined by

$$1 + i_t^K \equiv \frac{Q_t^K + P_t A_t}{Q_{t-1}^K}, \tag{2}$$

where the payoff is given by the price of capital, Q_t^K, and the nominal proceeds from selling goods, $P_t A_t$. Accordingly the nominal return on capital can be high or low depending on the respective state of nature: $1 + i_h^K$ if $A_t = A_h$, and $1 + i_l^K$ if $A_t = A_l$.

2.2 Households

Households are infinitely lived and have the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + X_t\right], \tag{3}$$

where E_0 is the expectation operator at time 0 and β is the intertemporal discount factor with $0 < \beta < 1$. C_t and X_t denote consumption of the same good but during different subperiods within period t: C_t is consumed in the first subperiod, X_t in the second.

A financial friction puts barriers on which securities can be used to purchase consumption goods C_t in the first subperiod. First, we assume that only debt can provide liquidity services, whereas other securities such as equity cannot. Second, in each state of nature, a debt security is liquid only if it is not defaulted on in that state. This second assumption can be justified by the existence of some time requirement to complete the default procedure; such delays prevent the use of the securities in trading goods in the first subperiod. Both our assumptions are in line with the discussion of Gorton (2016) that starting from the eighteenth century certain

[10] This result is the same as in Lucas and Stokey (1987).

types of debt have come to serve as liquid assets, including privately produced debt with some risk of default.[11]

As a result, in the first subperiod, households are subject to the following liquidity constraint

$$P_t C_t \leq B_{t-1} + \int_{j \in \mathcal{J}} (1 - I_t(j)) D_{t-1}(j) dj. \tag{4}$$

The liquidity constraint in (4) is based on the assumption that two classes of securities can potentially provide liquidity services: a publicly issued security B_{t-1}, which has the interpretation of Treasury debt or interest-bearing central bank reserves;[12] and financial intermediaries' debt $D_{t-1}(j)$, with $j \in \mathcal{J}$.[13] Each type of private debt j is identified by the state-contingent default rate $\chi_t(j) \in [0, 1]$ so that the payoff of the security j at time t is $1 - \chi_h(j)$ and $1 - \chi_l(j)$ in the high and low state, respectively. The indicator function $I_t(j)$ is related to the assumption, discussed above, that liquidity services can be provided only if the security is not defaulted on when used for transactions. That is, $I_t(j)$ takes the value of one if the security j is defaulted on at time t, and zero otherwise. Thus, security j can be used to purchase C_t only if $I_t(j) = 0$.

The set $\mathcal{J}$ includes different types of private money, that can be grouped into three broad categories: safe securities, which are never defaulted; pseudo-safe securities, which are defaulted on only in the low state; and unsafe securities, which are defaulted on both in the high and low state. While the set $\mathcal{J}$ includes all these securities, market interactions between households and intermediaries determine the ones that are going to be supplied. Indeed, a key aspect of our model is that financial intermediary can choose which type of security to provide.[14]

In principle, government debt B_{t-1} can also lose liquidity value if the government defaults. However, the government is always solvent in equilibrium, as we explain in Section 2.4.

In the second subperiod, households choose consumption goods, X_t, and make portfolio decisions regarding intermediaries' debt, $D_t(j)$ for each j, government bonds,

[11] Gorton (2016) argues that privately produced information-insensitive debt, what he defines as safe assets, can carry credit risk and that agencies' ratings can indicate the distance to information sensitivity. To capture this idea, we could clearly put a threshold on the level of riskiness above which debt securities will never be accepted for liquidity purposes. This approach will not have consequences for the generality of our result. Our assumptions are also related to Gorton and Pennacchi (1990), Stein (2012), Farhi and Maggiori (2016), and Woodford (2016), although they only restrict liquidity services to be provided by completely riskless securities.

[12] Our assumption that B_t provides liquidity services is based on the observation that government debt is widely used as a collateral in repo transactions and is the main asset held by a large class of money market mutual funds. These observations are line with the empirical evidence of Krishnamurthy and Vissing-Jorgensen (2012).

[13] Our assumption of perfect substitution between the liquidity provided by B_{t-1} and private intermediaries' debt $D_{t-1}(j)$ is motivated by the results of Nagel (2014), who estimates a high elasticity of substitution between public and private liquidity.

[14] To clarify further the notation, note that the index j identifies the default rate of a security, rather than the intermediary issuing it. As a result, a security of type j can be supplied by more than one intermediaries and potentially by infinitely many.

B_t, capital, K_t^H, and net worth of financial intermediaries, $N_t(j)$ for each j ($N_t(j)$ denotes the amount of net worth issued by an intermediary that supplies security j). Their budget constraint is

$$P_t X_t + Q_t^B B_t + \int_{j\in\mathcal{J}} Q_t^D(j) D_t(j) dj + Q_t^K K_t^H + \int_{j\in\mathcal{J}} N_t(j) dj \leq W_t - P_t T_t, \tag{5}$$

where Q_t^B, $Q_t^D(j)$, and Q_t^K are the nominal prices of government bonds, private debt of type j, and capital, respectively; W_t is the nominal wealth of households; and T_t are real lump-sum taxes. Wealth W_t is defined by:

$$\begin{aligned} W_t &= Q_{t-1}^K K_{t-1}^H \left(1 + i_t^K\right) + \int_{j\in\mathcal{J}} N_{t-1}(j)(1 + i_t^N(j)) dj \\ &\quad + \left[B_{t-1} + \int_{j\in\mathcal{J}} (1 - I_t(j)) D_{t-1}(j) dj - P_t C_t\right] + \int_{j\in\mathcal{J}} I_t(j)(1 - \chi_t(j)) D_{t-1}(j) dj. \end{aligned}$$

Households' wealth W_t depends on four components: (i) capital bought in $t-1$, K_{t-1}^H, plus a nominal return, i_t^K; (ii) dividends from holding equity in financial intermediaries, $N_{t-1}(j)\left(1 + i_t^N(j)\right)$, for each $j \in \mathcal{J}$; (iii) any liquidity not used to buy consumption goods C_t in the first subperiod, $B_{t-1} + \int (1 - I_t(j)) D_{t-1}(j) dj - P_t C_t$, which is zero if the liquidity constraint (4) holds with equality; and (iv) intermediaries' debt in default, $I_t(j)(1-\chi_t(j))D_{t-1}(j)$ for each $j \in \mathcal{J}$, and thus cannot be used for transactions in the first subperiod, but becomes available in the second subperiod.

To define the return on net worth, note that by investing $N_{t-1}(j)$ into financial intermediaries supplying the security of type j, households are entitled to receive a share of dividends $\Pi_t^D(j)$ from the intermediaries.[15] Accordingly, the return on net worth is defined by

$$1 + i_t^N(j) \equiv \frac{\Pi_t^D(j)}{N_{t-1}(j)}. \tag{6}$$

Consumption and portfolio choices are implied by the maximization of (3) under the constraints (4) and (5) and an appropriate borrowing-limit condition. Households are risk-averse in the consumption of C_t but risk-neutral in the consumption of X_t. This quasi-linear utility simplifies the problem of households, because the marginal utility of wealth is just given by $\lambda_t = 1/P_t$, where λ_t is the Lagrange multiplier of the budget constraint (5). Thus, the optimality conditions for the demand of capital and the supply of net worth are

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(1 + i_{t+1}^K\right) \right\}, \tag{7}$$

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(1 + i_{t+1}^N(j)\right) \right\} \; for\ each\ j \in \mathcal{J}. \tag{8}$$

[15] The return on net worth invested in intermediaries is given only by dividends and does not include any capital gains since, as will be detailed in the next section, intermediaries live for only two periods.

A further implication of the utility function is that the demand for goods in the first subperiod is

$$C_t = \frac{1}{1+\mu_t} \tag{9}$$

where μ_t/P_t is the Lagrange multiplier associated with the constraint (4). Since $\mu_t \geq 0$, thus $C_t \leq 1$ and at the first best $C_t = 1$. The first-best allocation follows from the fact that the marginal utility of consumption of X_t in the second subperiod is one, whereas the marginal utility of consumption in the first subperiod is $1/C_t$. Therefore, since the price of C_t and X_t is the same, the first best is achieved by $C_t = 1$.

To conclude the characterization of the household's problem, we derive the demand for government debt and intermediaries' debt. This demand is affected by the liquidity value provided by these assets, captured by the Lagrange multiplier μ_{t+1} on the constraint (4):

$$Q_t^B = \beta E_t \left\{ \frac{P_t}{P_{t+1}}(1+\mu_{t+1}) \right\}, \tag{10}$$

$$Q_t^D(j) = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[I_{t+1}(j)(1-\chi_{t+1}(j)) + (1-I_{t+1}(j))(1+\mu_{t+1}) \right] \right\} \tag{11}$$

for each $j \in \mathcal{J}$.[16] Private debt $D_t(j)$ provides liquidity services, captured by the variable μ_{t+1} if positive, only when they are not defaulted on, $I_{t+1}(j) = 0$. An implication of (10) and (11) is that $Q_t^B \geq Q_t^D(j)$, with strict inequality when intermediaries' debt defaults in some contingency. Crucially, liquidity services provide benefits not only to households but also to the issuer of the debt security because they lower borrowing costs. We return to this point later in the analysis.

Finally, a transversality condition applies imposing an appropriate limit on the rate of growth of assets held by households:

$$\lim_{\tau\to\infty} \frac{\beta^\tau}{P_{t+\tau}} \left(Q_{t+\tau}^B B_{t+\tau} + \int_{j\in\mathcal{J}} Q_{t+\tau}^D(j) D_{t+\tau}(j) dj + Q_{t+\tau}^K K_{t+\tau}^H + \int_{j\in\mathcal{J}} N_{t+\tau}(j) dj \right) = 0. \tag{12}$$

Equation (12) holds almost surely, looking forward from each time t and in each contingency at time t.

2.3 Financial Intermediaries

We make the simplifying assumption that financial intermediaries live for only two periods in an overlapping way. There is an infinite number of small financial intermediaries that can choose the type of debt security $j \in \mathcal{J}$ that they want to issue. Since intermediaries are small and thus marginal with respect to the supply of each $j \in \mathcal{J}$, they take prices $Q_t^D(j)$ as given. Without loss of generality we assume that

[16] See in particular Lagos (2010) on how the liquidity value of securities affects standard asset-pricing conditions.

each intermediary can supply only one type of security j, although a given security can be supplied by infinitely many intermediaries.

The price $Q_t^D(j)$ reflects the default characteristics of security j, captured by the state-contingent default rate $\chi_{t+1}(j)$. Default on debt can arise in our model because intermediaries are subject to a limited liability constraint which is modelled by a non-negativity constraint on their profits in the second period of their life. This constraint captures the limited backing that typically characterizes the supply of private money. Intermediaries' shareholders do not accept negative dividends or, equivalently, are not willing to infuse additional equity if the return on intermediaries' assets is too low. Therefore, default depends on the relative size of equity and debt initially issued by the intermediary (that is, leverage). Next, we show that intermediaries' choice of which security j to issue, and thus of $\chi_{t+1}(j)$, is equivalent to choosing the level of leverage at which they start their activity. To this end, we first define the budget constraint of intermediaries in period t and their profits in $t+1$.[17]

The intermediary collects funds by issuing debt $D_t(j)$ and raising net worth $N_t(j)$. Debt is issued in the form of one-period zero-coupon bonds with price $Q_t^D(j)$. The intermediary invests these resources into capital $K_t^I(j)$ at price Q_t^K given the budget constraint:

$$Q_t^K K_t^I(j) = Q_t^D(j) D_t(j) + N_t(j). \tag{13}$$

In the following period $t+1$, gross profits $\Pi_{t+1}(j)$ are given by

$$\Pi_{t+1}(j) = \left(1 + i_{t+1}^K\right) Q_t^K K_t^I(j) - (1 - I_{t+1}(j)) D_t(j) - I_{t+1}(j) \left(1 - \chi_{t+1}(j)\right) D_t(j)\,, \tag{14}$$

reflecting the return on capital and the cost of repaying debt.

Limited liability of intermediaries is modelled as a non-negativity constraint on profits in period $t+1$, $\Pi_{t+1}(j) \geq 0$. We next show that this constraint is the relevant condition that determines the initial leverage ratio of intermediaries. Using the definition of profits, (14), and the budget constraint of intermediaries, (13), $\Pi_{t+1}(j) \geq 0$ implies the following inequality for leverage:

$$\frac{N_t(j)}{D_t(j)} \geq \frac{(1 - I_{t+1}(j)) + I_{t+1}(j)\left(1 - \chi_{t+1}(j)\right)}{\left(1 + i_{t+1}^K\right)} - Q_t^D(j), \tag{15}$$

to be satisfied at each contingency at time $t+1$ and with equality when $\chi_{t+1}(j) > 0$. To clarify the role of (15), consider for example a security that is never defaulted on, $\chi_l(j) = \chi_h(j) = 0$. In this case, (15) implies:

$$\begin{aligned} \frac{N_t(j)}{D_t(j)} &\geq \max\left\{\frac{1}{(1+i_l^K)}, \frac{1}{(1+i_k^K)}\right\} - Q_t^D(j) \\ &= \frac{1}{(1+i_l^K)} - Q_t^D(j) \end{aligned}$$

[17] We assume that net worth and debt of each intermediary are observable and that intermediaries cannot abscond with net worth. These assumptions enure that intermediaries issuing security j have indeed a default rate given by $\chi_{t+1}(j)$. Alternatively, we could assume that intermediaries issuing security j have the ability to commit to the default rate $\chi_{t+1}(j)$.

If the measure of leverage $N_t(j)/D_t(j)$ satisfies the above condition, intermediaries are indeed solvent in all states in $t+1$.

Consider, instead, a generic pseudo-safe security j – that is, a security with default rates $\chi_h(j) = 0$ and $\chi_l(j) > 0$. In this case, $I_h(j) = 0$ and $I_l(j) = 1$ and thus (15) evaluated with equality implies the initial leverage ratio:

$$\frac{N_t(j)}{D_t(j)} = \frac{(1-\chi_l(j))}{(1+i_l^K)} - Q_t^D(j)$$

Indeed, if the intermediary chooses the above leverage ratio, it defaults at a rate $\chi_l(j)$ in state l whereas it is solvent in state h.

We can now characterize the decision problem of intermediaries using a two-step procedure. In the first stage, intermediaries choose the type of security $j \in \mathcal{J}$ to issue with default characteristic $\chi_{t+1}(j)$ and the appropriate leverage ratio. In the second stage, they choose how much physical capital to hold and how much debt and net worth to issue, given the leverage ratio. We now analyze this problem by preceding backward.

Consider an intermediary that has decided to issue security j. Its objective is to maximize expected discounted rents, $\mathcal{R}_t(j)$, defined as

$$\mathcal{R}_t(j) \equiv E_t\left\{\beta\frac{P_t}{P_{t+1}}(\Pi_{t+1}(j) - \Pi_{t+1}^D(j))\right\},$$

where the difference between profits $\Pi_{t+1}(j)$ and dividends $\Pi_{t+1}^D(j)$ captures intermediaries' rents. Using equations (6)-(8), (13) and (14), we can rewrite $\mathcal{R}_t(j)$ as:

$$\mathcal{R}_t(j) = Q_t^D(j)D_t(j) - \beta E_t\left\{\frac{P_t}{P_{t+1}}\left[(1-I_{t+1}(j)) + I_{t+1}(j)\left(1-\chi_{t+1}(j)\right)\right]\right\}D_t(j). \quad (16)$$

Expected discounted rents are the difference between the resources that the intermediary can collect by issuing debt, $Q_t^D(j)D_t(j)$, and the present-discounted value of the expected repayments to debt holders. The intermediary chooses $D_t(j)$ to maximize (16) taking as given the leverage ratio $N_t(j)/D_t(j)$ defined by (15) and therefore the default rate $\chi_{t+1}(j)$. The intermediary is willing to supply $D_t(j) > 0$ provided that the price of security j exceeds the expected discounted repayment:

$$Q_t^D(j) \geq \beta E_t\left\{\frac{P_t}{P_{t+1}}\left[(1-I_{t+1}(j)) + I_{t+1}(j)\left(1-\chi_{t+1}(j)\right)\right]\right\}. \quad (17)$$

Otherwise, if (17) does not hold, the intermediary chooses $D_t(j) = 0$. As a result, intermediaries' expected rents are nonnegative at the optimum, $\mathcal{R}_t(j) \geq 0$. To complete the intermediary's problem, we go back to the first stage where the type of security to supply is decided. Intermediaries choose security j if and only if

$$\mathcal{R}_t(j) = \max_{j' \in \mathcal{J}} \mathcal{R}_t(j') \quad (18)$$

taking into account prices $Q_t^D(j')$, the default characteristics $\chi_{t+1}(j')$ and the optimal choices $D_t(j')$, $N_t(j')$, $K_t(j')$ for each other security $j' \in \mathcal{J}$.

2.4 Government

The government includes both the treasury and the central bank. For expositional simplicity, here we consider the simple case in which the balance sheet of the government is composed by only liabilities, short-term zero-coupon bonds B_t, which can be interpreted as Treasury debt or central bank's reserves. Later, we discuss the case in which the government invests in privately-issued securities, possibly through the central bank.

The liabilities of the government, B_t, are free of risk because they are different from the liabilities of other agents in the economy. If B_t is interpreted as central bank's reserves, then B_t defines the unit of account of the monetary system and is thus free of risk by definition; that is, the central bank can repay its liabilities by "printing" new reserves.[18] If B_t is instead interpreted as Treasury debt, there could be in principle a risk of default. However, we assume that the Treasury is implicitly backed by the central bank so that Treasury debt is riskless as well.[19]

At time $t-1$, the government has to pay back B_{t-1} using newly issued securities B_t at the price Q_t^B and collecting real lump-sum taxes T_t at the price P_t. Therefore, its flow budget constraint is

$$B_{t-1} = Q_t^B B_t + P_t T_t$$

Iterating forward the last expression and combining it with (10), we get

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left(T_{t+\tau} + \beta \frac{\mu_{t+1+\tau}}{P_{t+1+\tau}} B_{t+\tau} \right) \right\} + \lim_{\tau \to \infty} \beta^\tau E_t \left\{ \frac{Q_{t+\tau}^B B_{t+\tau}}{P_{t+\tau}} \right\}. \tag{19}$$

Let us first focus on the second term on the right-hand side. Households' transversality condition (12), together with the balance sheet of intermediaries (13) and the market clearing condition for capital, implies that

$$\lim_{\tau \to \infty} \beta^\tau E_t \left\{ \frac{Q_{t+\tau}^B B_{t+\tau}}{P_{t+\tau}} \right\} = - \lim_{\tau \to \infty} \beta^\tau E_t \left\{ \frac{Q_{t+\tau}^K}{P_{t+\tau}} \overline{K} \right\}.$$

If we focus only on equilibria in which the real price of capital is stationary, the second term on the right-hand side of (19) is zero and the intertemporal budget constraint of the government simplifies to

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left(T_{t+\tau} + \beta \frac{\mu_{t+1+\tau}}{P_{t+1+\tau}} B_{t+\tau} \right) \right\}.$$

Another way to write the above intertemporal budget constraint is to use (10) and to define $Q_t^f \equiv \beta E_t \{P_t/P_{t+1}\}$ to be the price of a fictitious risk-free bond that does not provide liquidity services, so that:

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left(T_{t+\tau} + (Q_{t+\tau}^B - Q_{t+\tau}^f) \frac{B_{t+\tau}}{P_{t+\tau}} \right) \right\}. \tag{20}$$

[18] See Woodford (2000, 2001).

[19] Our analysis rely on the fact that B_t is not defaulted on in equilibrium; that is, our results are unchanged under different monetary-fiscal regime, as long as B_t is free of risk in equilibrium.

In equilibrium, the real value of outstanding government debt B_{t-1}/P_t has to be equal to the sum of the present-discounted value of real taxes, the first term on the right-hand side, and the liquidity premia on outstanding debt, as reflected in the second term on the right-hand side of (20). Liquidity premia lower the cost of borrowing and enhance the ability to repay debt; this effect is captured by a positive difference between the price of bonds and that of similar risk-free but illiquid securities.

The government chooses the path of two policy instruments, nominal debt and real taxes $\{B_t, T_t\}_{t=0}^{+\infty}$, given an initial condition on B_{-1}. To simplify our analysis, we assume that the tax rule is of the form

$$T_t = \begin{cases} (1-\beta)\,\overline{T}_h - \left(Q_t^B - Q_t^f\right)\frac{B_t}{P_t} & if\ A_t = A_h \\ (1-\beta)\,\overline{T}_l - \left(Q_t^B - Q_t^f\right)\frac{B_t}{P_t} & if\ A_t = A_l \end{cases} \tag{21}$$

where $\overline{T}_h$ and $\overline{T}_l$ are two constants, not necessarily equal. The tax rule (21) implies that, in each period and contingency, real taxes depend on a constant (i.e., $\overline{T}_h$ and $\overline{T}_l$ in the high and low state, respectively) and fall proportionally to the real value of public debt (i.e., B_t/P_t). The proportionality factor is captured by the liquidity premium $Q_t^B - Q_t^f$. The tax rule greatly simplifies our analysis; once it is substituted into (20), it yields

$$\frac{B_{t-1}}{P_t} = (1-\beta)\,E_t\left\{\sum_{\tau=0}^{\infty}\beta^{\tau}\overline{T}_{t+\tau}\right\}. \tag{22}$$

A further simplification is to assume that public debt is in constant supply, $B_t = B$, and that $\overline{T}_t$ is also constant and equal to $\overline{T}$ for all t (i.e., $\overline{T}_h = \overline{T}_l = \overline{T}$). It then follows that the specification of the monetary-fiscal policy determines a unique constant price level, $P = B/\overline{T}$.[20] Later, in Section 5, we relax the assumption that $\overline{T}_t$ is constant and we also introduce some limit to the amount of taxes that the government can collect.

Note that the above analysis shows the key role of $\overline{T}$ in the ability of the government to supply liquidity in real term. That is, the real supply of government liquidity, B/P, is a direct function of $\overline{T}$. This parameter is going to be crucial in the analysis.

3 Equilibrium

We have already characterized some equilibrium results, namely that the price level is constant given the monetary-fiscal policy regime. Using the latter result, the demand of capital (7), together with (2), allows us to solve for the real price of capital:

$$\frac{Q^K}{P} = \frac{\beta}{1-\beta}A,$$

[20] One of the main concerns of Friedman (1960) against private-money creation is the possible instability of the price level. The approach we use, based on the the fiscal theory of the price level and the risk-free property of central bank's reserves, overcomes these difficulties because it uniquely determines the price level.

which is also constant, where $A \equiv (1-\pi)A_h + \pi A_l$ is the unconditional expectation of A_t. The nominal return on capital (2) simplifies to

$$1 + i_t^K = \frac{\beta A + (1-\beta)A_t}{\beta A}. \tag{23}$$

Note that real and nominal returns on capital are equal since prices are constant. Denoting r_h^K and r_l^K to be the real returns on capital, respectively, in the high and low state, then

$$1 + r_h^K \equiv \frac{\beta A + (1-\beta)A_h}{\beta A}, \qquad 1 + r_l^K \equiv \frac{\beta A + (1-\beta)A_l}{\beta A}. \tag{24}$$

The following set of equations determines the remaining variables. The liquidity constraint (4) simplifies to

$$B + \int_{j \in \mathcal{J}} (1 - I_t(j))D_{t-1}(j)dj \geq PC_t, \tag{25}$$

while first-subperiod consumption and the Lagrange multiplier μ_t are related through (9). In particular (25) holds with equality whenever $\mu_t > 0$. With constant prices, the demand for government bonds (10) implies the following relationship between their price Q_t^B and the Lagrange multiplier μ_t:

$$Q_t^B = \beta E_t \{1 + \mu_{t+1}\}. \tag{26}$$

The demand for private debt (11) simplifies to

$$Q_t^D(j) = \beta E_t \{I_{t+1}(j)(1 - \chi_{t+1}(j)) + (1 - I_{t+1}(j))(1 + \mu_{t+1})\}, \tag{27}$$

for each security $j \in \mathcal{J}$.

We denote $\mathcal{E}_t \subseteq \mathcal{J}$ to be the subset of the securities that are supplied in equilibrium at time t. As shown in the previous section, intermediaries' optimality conditions imply that rents are nonnegative for all these securities, i.e. $\mathcal{R}_t(j) \geq 0$ for each $j \in \mathcal{E}_t$. Moreover, free entry eliminates all rents and therefore $\mathcal{R}_t(j) = 0$ for each $j \in \mathcal{E}_t$. As a result, $\mathcal{R}_t(j) = 0$ and (16) imply:

$$Q_t^D(j) = \beta E_t \{I_{t+1}(j)(1 - \chi_{t+1}(j)) + (1 - I_{t+1}(j))\}, \tag{28}$$

for each $j \in \mathcal{E}_t$. That is, supply is perfectly elastic at the above price.

Combining the demand for and the supply of private debt, (27) and (28), we obtain

$$E_t \{(1 - I_{t+1}(j))\mu_{t+1}\} = 0, \tag{29}$$

for $j \in \mathcal{E}_t$. Equation (29) states that if a security of type j is supplied, there must be complete satiation of liquidity $\mu_{t+1} = 0$ in the contingencies in which it is not in default (i.e. when $I_{t+1}(j) = 0$). Moreover, equation (15) simplifies under constant prices and implies the level of net worth of each security $j \in \mathcal{E}_t$

$$\frac{N_t(j)}{D_t(j)} \geq \frac{(1 - I_{t+1}(j)) + I_{t+1}(j)\,(1 - \chi_{t+1}(j))}{(1 + r_{t+1}^K)} - Q_t^D(j) \tag{30}$$

with strict equality whenever $\chi_{t+1}(j) > 0$.

Finally the set of securities $\mathcal{E}_t \subseteq \mathcal{J}$, which are in positive supply in equilibrium (i.e., with $D_t(j) > 0$) is such that $j \in \mathcal{E}_t$ if and only if (18) holds.

The next definition summarizes the concept of equilibrium.

Definition 1 *Give a state-contingent rate of default $\chi_t(j) \in [0,1]$ for each security $j \in \mathcal{J}$ and each time t, an equilibrium is a set of stochastic processes $\{C_t,\ \mu_t,\ Q_t^B,\ Q_t^D(j),\ D_t(j),\ N_t(j)\}$ such that:*

- *the set $\mathcal{E}_t \subseteq \mathcal{J}$, at each t, is such that $j \in \mathcal{E}_t$ if and only if (18) holds,*
- *$D_t(j) = N_t(j) = 0$ for $j \in \mathcal{J} \backslash \mathcal{E}_t$ and each t,*
- *conditions (9), (25), (26), hold at each t,*
- *conditions (27) hold for each $j \in \mathcal{J}$ and at each time t,*
- *conditions (29), (30) hold for each $j \in \mathcal{E}_t$ and at each time t. In particular, (30) holds with equality if $\chi_{t+1}(j) > 0$.*

We now solve for the equilibrium. To this end, we use the index $s \in \mathcal{J}$ to denote a privately-issued safe security (i.e. the one with zero default in all states $\chi_h(s) = \chi_l(s) = 0$). We show that an unregulated competitive market can provide a sufficient amount of these privately-issued safe assets and reach the efficient supply of liquidity in all states of nature.

Proposition 2 *In the frictionless intermediation model, there is complete satiation of liquidity, $\mu_h = \mu_l = 0$, and first-best consumption is delivered, $C_h = C_l = 1$. The quantity of financial intermediaries' safe debt is given by*

$$\frac{D(s)}{P} \geq \max\left(1 - \frac{B}{P}, 0\right) = \max\left(1 - \overline{T}, 0\right) \tag{31}$$

which is issued at the price $Q_t^D(s) = \beta$; intermediaries' net worth is $N_t(s) \geq \overline{N} > 0$, where

$$\overline{N} = D(s)\left[\left(1 + r_l^K\right)^{-1} - \beta\right]. \tag{32}$$

The economy achieves the first-best because the supply of safe assets is sufficiently large. There are two ways to achieve the efficient supply of safe assets. If $\overline{T} \geq 1$, the fiscal capacity of the government is high enough and thus the government can back a large supply of public money (more precisely, a large supply of public money in real terms, that is, a large B/P). In this case, whether or not financial intermediaries issue private money is irrelevant for welfare. If instead $\overline{T} < 1$, the government does not have enough fiscal capacity to back a large supply of public money. Thus, private money issued by intermediaries is crucial to complement the supply of public liquidity and achieve the first-best.

When $\overline{T} < 1$, the efficiency result of Proposition 2 is a direct implication of the competition mechanism of the model, which allows financial intermediaries to decide the type of money to supply. To understand this point and prove the Proposition, suppose by contradiction that there is no supply of privately-issued safe securities. Instead, assume that intermediaries only provide pseudo-safe assets j (that is, debt with default rate $\chi_l(j) > 0$). As a result, pseudo-safe securities default in the low state and thus consumption can be financed with public liquidity B_t only. Using (4) and (9):

$$\mu_l = \frac{1}{B/P} - 1 = \frac{1}{\overline{\overline{T}}} - 1 > 0$$

and thus there is a shortage of liquidity in the low state. In contrast, the market-equilibrium condition in (29) implies that the supply of j is large enough to satiate liquidity needs in the high state. Thus, there is no shortage of liquidity in that state, $\mu_h = 0$. Consider now a generic intermediary deciding which security $j \in \mathcal{J}$ to issue. Suppose that the intermediary chooses to issue safe debt s, which never defaults. Consumers attach a high value to safe securities, because the liquidity premium in the low state is positive; this high value is reflected in the price $Q_t^D(s) = \beta(1 + \pi\mu_l)$. The high $Q_t^D(s)$ implies that the intermediary can borrow at a lower cost. As a result, rents (16) from supplying the security s are positive and given by

$$\mathcal{R}_t(s) = \beta\pi\mu_l > 0.$$

Thus, issuing safe securities s is profitable. This result contradicts the initial conjecture that there exist an equilibrium in which safe debt is not supplied by any intermediary.

Thus, intermediaries supply safe private securities up to the point in which the liquidity premium is driven to zero in all states, $\mu_h = \mu_l = 0$. That is, free entry in the market ensures that all rents are eliminated. As a result, the price of a privately-issued safe asset is equal to that of government bonds, $Q_t^D(s) = Q_t^B = \beta$. Moreover, the supply of safe securities is enough to complement the amount of public liquidity (as described by (31)) and reach efficiency, $C_h = C_l = 1$. To supply safe assets, intermediaries choose to enter the market with a level of net worth that makes them solvent in all states of nature, i.e. with $N_t(s) \geq \overline{N}$, where $\overline{N}$ is given by (32).[21]

Finally, note that the supply of pseudo-safe securities (i.e., securities with default $\chi_h = 0$ and $\chi_l > 0$) and of completely unsafe securities (i.e., securities with default $\chi_h > 0$ and $\chi_l > 0$) can be positive. These securities are priced so that their expected return equals the inverse of the discount factor, $1/\beta$; that is, their price is just given by the present-discounted value of their payoffs. However, the supply of these assets is irrelevant for welfare.

3.1 Discussion

The results of the benchmark model with completely frictionless financial intermediation confirms the view of Hayek (1974). That is, the process of competition can lead

[21] More precisely, the lower bound on net worth in (32) is derived by imposing that intermediaries are solvent in the low state. Moreover, if intermediaries are solvent in the low state, they must be solvent in the high state as well.

the private sector to supply a sufficiently large quantity of the best available type of liquid assets, namely, safe assets. The competitive market structure in our model is indeed in the spirit of Hayek's (1974, p.43).[22] If safe securities were not provided, households would attach a premium to them because such securities relax the liquidity constraint during crises (i.e., when the low state in the model realizes). Therefore, intermediaries would find it convenient to supply safe debt because the premium paid by households reduces intermediaries' financing costs. Free entry then ensures that there are enough safe securities so that the households' liquidity constraint is never binding. As a result, the interest of households is perfectly aligned with that of financial intermediaries. Indeed the premium on safe assets, which reflects a lack of liquidity from a society point of view, creates incentives for profit-maximizing intermediaries to supply safe securities. To this end, intermediaries will raise enough equity to absorb any loss they can incur on their risky assets.

Unfettered competition achieves efficiency without the need of any type of regulation. We compare our result with the real-bills doctrine and with the view of Friedman (1960) about the separation between money and credit markets.

According to the real-bills doctrine, intermediaries should hold safe (and possibly illiquid) assets to back the supply of private money. This is not necessary in our framework. Even if intermediaries hold risky assets, competition forces them to raise enough equity to absorb any possible loss. As a result, the supply of private money is safe.

Our analysis can also be used to study the separation between money and credit markets advocated by Friedman (1960). According to this view, the government should have the monopoly power in the supply of liquidity. This objective can be reached if the government passes regulation to achieve a narrow banking system; that is, intermediaries are forced to satisfy a 100% reserve requirement. In the context of our model, intermediaries would buy government safe debt B_t instead of capital, so the budget constraint (13) would be replaced by $Q_t^B B_t = Q^D(s)D_t(s)$. If this were the case, private intermediaries would not perform any liquidity creation, because their debt would be backed by liquid government reserves, instead of illiquid, risky investments. As a result, the overall supply of liquid assets in the economy would be determined solely by the amount of government debt, B_t. Note that in turn the government has to back its debt and interest payments, which is achieved by collecting taxes (in Section 5, we explore an additional approach to provide backing, based on the active management of the central bank's balance sheet). A benevolent government that implements a narrow banking system can nevertheless achieve the first-best, by setting taxes $\overline{T} \geq 1$, so that government debt is in the amount $B/P \geq 1$. However, if the government is not benevolent, it would have a self-interest in reducing liquidity in order to drive up the liquidity premium, μ_t, and obtain rents.

To sum up, the views of Hayek and Friedman differ between achieving efficiency through forces of private competition or through the benevolence of a government monopoly. But if the monopolist makes its decision based on its self interest, it will not achieve the first best. Thus, the baseline model supports Hayek's proposal. It

[22]See also Hayek (1948, ch. V) for a critical analysis of the assumption of perfect competition.

is better to free up the forces of private competition that also meet society needs. However, this conclusion changes significantly when we make a simple amendment to the above framework by adding a friction in the financial market. We turn to this analysis in the next section.

4 Costs of issuing equity

This section considers a small departure from the benchmark model of Sections 2 and 3 by assuming that intermediaries face an additional cost of issuing equity. A cost of issuing equity or a constraint on the amount of outside equity is a standard feature in the literature on financial intermediation (see e.g. Bernanke and Gertler; 1989, Brunnermeier and Sannikov, 2014; Holmstrom and Tirole, 1997; Jermann and Quadrini; 2012). Diamond (2016) provides a microfundation for a cost of issuing equity similar to the one that we use, based on an agency friction between financial intermediaries and outside investors. The cost of issuing equity critically affects the results derived in the frictionless model.

The main result of this section is that intermediaries do not issue enough safe assets, unlike the frictionless intermediation model. Equity financing is now more expensive than debt financing, which in turn raises the cost of issuing safe assets.[23] As a result, the amount of privately-issued liquidity in this equilibrium is lower than in the frictionless-intermediation economy.

To keep the model tractable, we assume that for each dollar of net worth that is issued by an intermediary, only a fraction $1-\tau$ can be used to buy capital.[24] Thus, the flow budget constraint of a generic intermediary in market j changes to:

$$Q_t^K K_t^I(j) = Q_t^D(j) D_t(j) + (1-\tau) N_t(j).$$

Repeating the previous steps, we obtain the following expression for rents:

$$\begin{aligned}\mathcal{R}_t(j) = &-\tau N_t(j) + Q_t^D(j) D_t(j) \\ &- \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[(1 - I_{t+1}(j)) + I_{t+1}(j)\,(1-\chi_{t+1}(j))\right] \right\} D_t(j),\end{aligned}$$

which, using (27), can be written as

$$\mathcal{R}_t(j) = -\tau N_t(j) + \beta E_t \left\{ (1 - I_{t+1}(j)) \mu_{t+1} \right\} D_t(j). \tag{33}$$

[23] Note that not all frictions in financial intermediation reduce the incentives to supply safe assets. For instance, we show in Appendix A.6 that a model with a fixed cost to operate and monopolistic competition does not alter the results of our frictionless intermediation model.

[24] We assume that the cost $\tau N_t(j)$ is transferred lump-sum to households. For instance, this cost captures the idea that managers need to earn a rent in order to exert effort in running the firm. With this formulation, the cost of equity does not affect the amount of resources available for consumption and thus the constrained first best is equivalent to the first best; see Appendix A.5. Most of the results are unchanged if $\tau N_t(j)$ is wasted from the point of view of the society and thus reduces the total amount of resources available for consumption. However, in this case, the constrained first best differs from the first best and thus the analysis is more complicated.

As in the benchmark model, once intermediaries decide to supply security j, they choose the amount of debt and net worth to issue in order to maximize expected rents subject to the limited liability constraint. Under constant prices, the cost of issuing equity implies that equation (15) becomes

$$\frac{N_t(j)}{D_t(j)} \geq \frac{(1 - I_{t+1}(j)) + I_{t+1}(j)\,(1 - \chi_{t+1}(j))}{(1-\tau)\left(1 + r_{t+1}^K\right)} - \frac{Q_t^D(j)}{(1-\tau)}, \tag{34}$$

with equality if $\chi_{t+1}(j) > 0$.

The main implication of this framework is that the private sector now has incentive to supply pseudo-safe debt (i.e., debt that is not defaulted on in the high state but is partially defaulted on in the low state). Depending on the supply of public liquidity, it might be convenient for intermediaries to supply some safe debt as well. In what follows, we use $p \in \mathcal{J}$ to denote a pseudo-safe security with default rate $\chi_h(p) = 0$ and $\chi_l(p) > 0$. In equilibrium, intermediaries choose to hold no equity at all, $N_t(p) = 0$, and thus $\chi_l(p)$ is determined by (34) evaluated with equality at $N_t(p) = 0$.

We now present the equilibrium, focusing on the case in which the government raises a limited amount of taxes (i.e., $\overline{T} < 1$). We return to the analysis of government policy in Section 5.

Proposition 3 *If financial intermediaries face a cost $\tau > 0$ per unit of net worth raised and the government sets taxes $\overline{T} < 1$, then:*

1. *In the high state, a large amount of liquidity is available and thus consumption is efficient, $\mu_h = 0$ and $C_h = 1$; in the low state, a low amount of liquidity is available and thus consumption is not efficient*

$$C_l = \max\left(\frac{\pi}{\pi + \tau\gamma_l}, \overline{T}\right) < 1 \quad \textit{and} \quad \mu_l = \frac{1}{C_l} - 1 > 0,$$

   where:

$$\gamma_l \equiv \frac{1}{\beta(1 + r_l^K)} - 1;$$

2. *The price of safe securities s and pseudo-safe securities p is*

$$Q_t^D(s) = \beta\left(1 + \pi\mu_l\right) > \beta, \qquad Q_t^D(p) = \beta\left[(1-\pi) + \pi\frac{1 + r_l^K}{1 + r_h^K}\right] < \beta.$$

   Their supply is

$$\begin{aligned} \frac{D(s)}{P} &= \max\left(\frac{\pi}{\pi + \tau\gamma_l} - \overline{T}, 0\right), \\ \frac{D(p)}{P} &\geq 1 - \overline{T} - \frac{D(s)}{P}. \end{aligned}$$

   The net worth of intermediaries is

$$\begin{aligned} N(s) &= \frac{D(s)}{(1 + r_l^K)(1-\tau)}\left[1 - \beta\left(1 + \pi\mu_l\right)(1 + r_l^K)\right], \\ N(p) &= 0, \end{aligned}$$

and the default rate on pseudo-safe securities is:

$$\chi_h(p) = 0, \qquad \chi_l(p) = 1 - \frac{1 + r_l^K}{1 + r_h^K};$$

3. *The supply of public liquidity is* $B/P = \overline{T}$.

Proof. See Appendix. ■

The cost τ of raising equity breaks the efficiency of the competitive process that was at work in the benchmark model of Sections 2 and 3. In the frictionless intermediation model, intermediaries have the proper, social incentives to supply the best and safest type of debt. In contrast, as shown by Proposition 3, intermediaries now have the incentive to enter the market with the lowest possible level of net worth, $N_t(p) = 0$, and to supply pseudo-safe debt.

When the cost of equity τ is positive, an equilibrium with only safe securities s does not exist. We explain this result by contradiction. If all intermediaries issue only safe securities, they must raise equity and pay the cost τ. To offset this cost, the liquidity premium on their debt must be positive, but this positive premium gives rise to a profitable deviation. An intermediary can decide to offer pseudo-safe securities and thus earn some of the liquidity premium, because pseudo-safe securities relax the liquidity constraint (4) in the high state. As a result, the intermediary can earn positive rents because pseudo-safe securities are supplied with zero net worth, avoiding the cost τ. Thus, the only possibility is to have an equilibrium with a positive supply of pseudo-safe securities p.

Note, however, that there is room for private safe debt to be supplied in equilibrium, in addition to pseudo-safe securities. Indeed, if pseudo-safe debt is supplied, there is a lack of liquidity in the low state. As a result, securities that provide liquidity in the low state will trade at a premium. If this premium is large enough to cover the cost τ, intermediaries issue safe securities; otherwise, they will not issue them. Whether the premium on safe intermediaries' debt is large or not depends in turn on the amount of public liquidity. A large supply of public liquidity implies a low liquidity premium on safe debt (recall that public liquidity is risk-free); thus, issuing safe debt is not profitable for intermediaries. That is, a sufficiently high level of public debt crowds out the production of privately-issued safe money by influencing the liquidity premium on default-free obligations. Put differently, a low supply of public liquidity creates a profitable opportunity for intermediaries to issue some safe debt.

The fact that pseudo-safe assets are supplied in equilibrium can generate a crisis scenario characterized by a liquidity shortage when the realized return on capital is low. In good times, the economy runs at the efficient level with ample supply of liquidity; in bad times few securities provide liquidity service and then consumption C_t falls. As we will see in the next sections, this inefficiency points out to a role for government intervention.

The *laissez-faire* equilibrium in Proposition 3 is not efficient due to a pecuniary externality. This externality arises because the private incentives of financial intermediaries are not aligned with the social objectives. From the perspective of a private intermediary, there is an incentive to use the liquidity premium to lower the borrowing costs (which is in line with social objectives of creating liquidity) and save on the cost of issuing equity (which is instead needed to make debt safe). This trade-off is optimally exploited by issuing pseudo-safe securities and reducing equity to zero, i.e. $N_t(p) = 0$.[25]

The externality in our model is similar to Lorenzoni (2008) and Stein (2012). There are, however, some important differences. We focus here on the comparison between our model and Stein (2012), which shares a focus on the role of intermediaries in issuing assets that have liquidity value. In Stein (2012), intermediaries issue only riskless securities; as a result, the externality in his model gives rise to an overissuance of riskless securities. In contrast, in our model, two types of debt can arise in equilibrium: safe debt s and pseudo-safe debt p; crucially, the externality in our model implies an overissuance of the pseudo-safe (risky) debt and an underissuance of the safe (riskless) debt.

5 Government intervention

The model with costly equity is characterized by an inefficiency that opens up a role for government intervention. In the *laissez faire* equilibrium, the private sector has an incentive to supply pseudo-safe assets rather than safe assets. As a consequence, the amount of liquidity is large enough only in the high state, while the economy experiences a liquidity crunch in the low state of nature.

The first intervention that we consider is a large supply of public liquidity. This intervention crowds out entirely the production of safe private debt but achieves efficiency. This policy requires an adequate backing that can be obtained in two ways. One option is simply to rely on taxes. If instead the government does not want to or cannot rely too much on taxes, it can purchase pseudo-safe securities issued by intermediaries, perhaps through the central bank. This portfolio produces a return in the high state, allowing the government to reduce taxes in that contingency. Instead, in the low state, private pseudo-safe securities held by the central bank default, and thus backing through taxes is still required in that event.

A second policy option is to leave the supply of liquidity to intermediaries and bail them out in the event of a crisis. This option requires backing through taxes in bad times as well, similar to the case in which private liquidity is backed by a portfolio of private pseudo-safe securities held by the central bank.

The last policy we discuss is capital requirements. This intervention might seem natural since intermediaries issue excessive risky debt and their default reduces the amount of liquidity. We show that capital requirements improve upon *laissez faire* only if the probability of a crisis is not small. However, capital requirements are neither necessary nor sufficient to achieve the first best.

We now analyze these policies in more detail. We also briefly discuss the possible shortcomings that might arise in richer versions of our model.

[25]We provide more details on the analysis of the externality in the Appendix.

5.1 Optimal government policy with no limit on taxes

We first characterize the optimal government policy when there is no limit on the ability to raise lump-sum non-distortionary taxes. In this case, the optimal policy imposes large taxes $\overline{T}_t$ in order to back a sufficiently large supply of real public money, B/P. As a result, households can attain the first-best level of consumption using public liquidity only. The next Proposition formalizes this result.

Proposition 4 *If financial intermediaries face a cost $\tau > 0$ per unit of net worth raised and if the government follows a tax rule (21), issues nominal debt $B_t = B > 0$ and has the objective to achieve price stability, $P_t = P$ for all t, then the optimal government policy is to set $\overline{T}_h = \overline{T}_l \geq 1$, achieving the first best.*

With no limit on lump-sum taxes, the government has an advantage in supplying liquidity. That is, the government can reach the efficient allocation, whereas *laissez faire* leads only to a second-best solution. Moreover, optimal issuance of public liquidity crowds out entirely the supply of privately-issued safe assets. The reason is that intermediaries' safe debt, $D(s)$, is costly since it requires a backing through expensive equity, whereas government's safe money has no costs associated to backing with taxes. This solution is in the spirit of Friedman's proposal (Friedman, 1960). The government can provide interest-bearing liquidity and pay it through taxes. Moreover abundant public liquidity eradicates any return wedge among securities with the same risky characteristics. This is indeed the Friedman rule which can be achieved at a constant price level because the securities issued by the government pay an interest rate.

It is worth emphasizing that the solution of this subsection relies on two critical assumptions: first, that the government is benevolent; second, that it does not face any limit on raising taxes. The next subsections propose solutions which can overcome possible constraints on raising taxes.

5.2 Optimal government policy with limit on taxes: central bank's balance sheet

We now turn to the analysis of the optimal government supply of liquidity when raising taxes is costly which is modelled by an upper bound on the average taxes that can be collected:

$$(1-\pi)\,\overline{T}_h + \pi\,\overline{T}_l < 1 \tag{35}$$

Notwithstanding this limit, we show that an appropriate policy of asset purchases allows the economy to achieve the first best, $C_h = C_l = 1$. In particular, consider a policy in which the government, through the central bank, purchases private intermediaries' pseudo-safe debt. This policy is related to the second proposal of Friedman (1960), who suggested to back the supply of interest-bearing reserves (in our model, B_t) through the portfolio of assets held by the central bank (in our model, private intermediaries' pseudo-safe debt).[26] This policy reduces the reliance on taxes to back

[26]Even though our policy proposal is related to that of Friedman, it is slightly different because the original proposal considered only the possibility of investing in safe securities.

the overall amount of real public money B_t/P_t. As a result, the government can increase the supply of real public money, allowing the economy to achieve the first best.

Let the central bank purchase the quantity $D_t^c(p)$ of pseudo-safe securities of type p (i.e., intermediaries' debt that is defaulted on state l, providing a repayment $1-\chi_l(p)<1$ in such a state). The flow budget constraint of the government is:

$$B_{t-1}-(1-\chi_t(p))D_{t-1}^c(p)=Q_t^B B_t - Q_t^D(p)D_t^c(p)+P_tT_t.$$

As a result, the intertemporal government budget constraint in (20) is replaced by:

$$\begin{aligned}\frac{B_{t-1}}{P_t}-(1-\chi_t(p))\frac{D_{t-1}^c(p)}{P_t} &= \\ E_t\left\{\sum_{\tau=0}^{\infty}\beta^\tau\left[T_{t+\tau}+(Q_{t+\tau}^B-Q_{t+\tau}^f)\left(\frac{B_{t+\tau}}{P_{t+\tau}}-(1-I_{t+\tau+1}(p))\frac{D_{t+\tau}^c(p)}{P_{t+\tau}}\right)\right]\right\}&. \quad (36)\end{aligned}$$

By investing in private securities, the government can supply a higher level of real public liquidity, B_{t-1}/P_t, with the same level of taxes and thus satiate the liquidity needs of the economy using only public money, even under the bound on taxes (35). The next proposition formalizes this result.

Proposition 5 *If financial intermediaries face a cost $\tau>0$ per unit of net worth raised and if the government follows the tax rule (21) subject to the limit in (35), issues nominal debt $B_t=B>0$, and has the objective to achieve price stability, $P_t=P$ for all t, the government can supply the efficient level of liquidity $B/P=1$ with an appropriate choice of $\overline{T}_h$, $\overline{T}_l$, and $D_t^c(p)$, achieving the first best $C_h=C_l=1$. In particular, $\overline{T}_h$ can be chosen arbitrarily as long as it satisfies the restriction $\overline{T}_h<1$ whereas*

$$\overline{T_l} = \overline{T}_h+\chi_l(p)\frac{1-\overline{T}_h}{1-\beta+\pi\beta\chi_l(p)}>\overline{T}_h, \quad (37)$$

$$\frac{D^c(p)}{P} = \frac{(1-\beta)\left(1-\overline{T}_h\right)}{1-\beta+\pi\beta\chi_l(p)}>0, \quad (38)$$

in which $\chi_l(p)$ is the same as in Proposition 3, $\chi_l(p)=1-\left(1+r_h^K\right)/\left(1+r_l^K\right)$.

Proof. See Appendix. ■

The Proposition shows that the government can achieve efficiency even if there is a limit on average taxes. Allowing for a state-contingent level of taxation, as specified by (21), is crucial to obtain the result. In the high state, pseudo-safe assets are fully repaid and thus they provide a backing for public liquidity B/P. In the low state, instead, pseudo-safe securities are defaulted on and thus provide an insufficient backing for public liquidity. In this case, the way to achieve the first best is to increase taxes to back public liquidity. To see this result, we rewrite the intertemporal government budget constraint, (36), using $B_t=B$, $P_t=P$, and (21) evaluated at $Q_t^B = Q_t^f$ (because the liquidity premium is zero under the efficient supply of liquidity):

$$\frac{B}{P} = \begin{cases} \frac{D_{t-1}^c(p)}{P} + (1-\beta)\,\overline{T}_h + E_t\left\{\sum_{j=1}^{\infty} \beta^j T_{t+j}\right\} & if\ A_t = A_h \\ (1-\chi_l(p))\frac{D_{t-1}^c(p)}{P} + (1-\beta)\,\overline{T}_l + E_t\left\{\sum_{j=1}^{\infty} \beta^j T_{t+j}\right\} & if\ A_t = A_l. \end{cases}$$

As a result, $\overline{T}_l$ must necessarily be higher than $\overline{T}_h$ because $0 < \chi_l(p) < 1$.

Note further that the average level of taxation can be made arbitrarily small. With an appropriate choice of $T_h < 0$ (i.e., transfers to households in the high state), the government can achieve efficiency even if the average level of taxes is zero or negative.

Corollary 6 *If*

$$\overline{T}_h \leq -\frac{\pi\chi_l(p)}{(1-\beta)\,(1-\pi\chi_l(p))}$$

and $\overline{T}_l$, $D^c(p)/P$ are given by (37) and (38), respectively, the economy achieves the first best, $C_h = C_l = 1$, and average taxes are zero or negative, $(1-\pi)\overline{T}_h + \pi\overline{T}_l \leq 0$.

The result of the previous corollary is based on two features. With low and possibly negative taxes, the backing of B/P in good times is provided exclusively by the return paid on the pseudo-safe assets $D^c(p)$ held by the government. In low states of nature, it is crucial that the payoff of pseudo-safe securities (that is $1 - \chi_l(p)$) is not zero. As a result, a sufficiently large stock of $D^c(p)$ provides enough backing, even in bad times.

Next, we discuss the robustness of Proposition 5 to the assumptions of the model. Even if the result of Proposition 5 might not be identical in some extensions of our model, we argue that the spirit of the exercise is preserved.

To clarify our point, we sketch an extension showing that it might not be desirable under optimal policy to reduce to zero the liquidity premium, $\mu_h = \mu_l = 0$, and achieve the allocation $C_h = C_l = 1$. Consider an economy with the cost of issuing equity, $\tau > 0$, in which we further assume that intermediaries' default is costly (i.e., there are deadweight losses associated with bankruptcy processes). In this case, it might not be optimal for the government to have a lower demand for pseudo-safe assets $D_t^c(p)$, in order to lower the resources that are lost in the event of default. As a result, under a constraint on taxes, the optimal supply of public liquidity might be smaller so that reaching the allocation $C_h = C_l = 1$ would not be optimal. Nonetheless, the spirit of Proposition 5 is unchanged. The main implication of Proposition 5 is that the government should actively engage in the supply of public money using privately-issued intermediaries' debt D_t^c as partial backing. The optimal holding of D_t^c is most likely not zero for reasonable extensions of our model.

Another possible constraint that can limit the purchases of private risky debt arises if we separate the central bank from the treasury. By purchasing risky securities, the central bank faces income losses in the low state where the risky assets default, while still paying interest on reserves. Therefore, it needs to be recapitalized by the treasury.

If treasury's support is not automatic, an additional trade-off between maintaining price stability and achieving the efficient supply of liquidity could emerge.[27]

5.3 Optimal government policy with limit on taxes: bailouts

In this section, we propose an alternative government policy that allows the economy to achieve the first best by satisfying the limit on taxes (35): bailouts. We first study the policy in the context of the model and then elaborate on some limitations and extensions.

Consider a government that supplies a constant, low level of public debt which is not sufficient to satiate the demand for liquidity of the economy. In addition, the government commits to bail out financial intermediaries in the low state. To raise resources for bailouts, the government needs to increase taxes in the low state, $\overline{T}_l$, to guarantee intermediaries' debt. As a result, intermediaries' debt is safe and therefore always provides liquidity services.

Proposition 7 *If financial intermediaries face a cost $\tau > 0$ per unit of net worth raised and if the government follows the tax rule (21) subject to the limit in (35), issues nominal debt $B_t = B > 0$, and has the objective to achieve price stability, $P_t = P$ for all t, the government can achieve the first best, $C_h = C_l = 1$, with an appropriate choice of $\overline{T}_h$ and $\overline{T}_l$ coupled with bailouts of financial intermediaries in the low state, and with $B/P < 1$. In particular, $\overline{T}_h$ can be chosen arbitrarily as long as it satisfies the restriction $\overline{T}_h < 1$ whereas*

$$\overline{T}_l = \overline{T}_h + \frac{\chi_l(p)\left(1 - \overline{T}_h\right)}{1 - \beta + \pi\beta\chi_l(p)}$$

in which $\chi_l(p)$ is the same as in Proposition 3, $\chi_l(p) = 1 - \left(1 + r_h^K\right) / \left(1 + r_l^K\right)$.

Proof. See Appendix. ■

A shortcoming of government bailouts is related to the moral hazard they might generate. We discuss this issue in the context of the model, and then consider possible extensions.

In the model, bailouts do not create moral hazard for two reasons. First, intermediaries that issue pseudo-safe assets decide to set their net worth to zero; thus, leverage is already at its maximum (i.e., infinity) and therefore the expectation of a bailout has no effect. Second, intermediaries invest in capital, whose riskiness is exogenously given, and thereby it is not possible for intermediaries to direct their lending to more risky projects.

In a more general model, though, the expectation of a bailout may create an incentive for intermediaries to both increase leverage and seek more risky projects.[28] As a result, bailouts might not achieve the first-best in such models.

[27] See among others Sims (2000).

[28] For a model in which government guarantees affect the riskiness of intermediaries' investments, see, for instance, Dempsey (2017).

Nonetheless, the spirit of Proposition 7 might survive even in some more general models, similar to what we discussed in the context of Proposition 5. For instance, if there is a cost of intermediaries' default, a bailout policy will trade off the moral hazard costs created by the bailout with two benefits: saving on the default cost and achieving the efficient supply of liquidity through privately-issued safe debt, by backing intermediaries' debt in the low state.[29]

5.4 Capital requirements

We now turn our attention to capital requirements. This policy intervention is fundamentally different from those of Sections 5.1-5.3. Government provision of liquidity and bailouts require an adequate fiscal backing in the low state, even if the government buys assets through the balance sheet of the central bank. These policies primarily work by complementing the insufficient private backing of liquidity (provided by equity) with more public backing (provided by taxes). They also have an influence on financial markets because they affect asset prices. In this way, they have an indirect effect on intermediaries. Differently, capital requirements directly alter the functioning of the market in which financial intermediaries operate, but do not require any fiscal capacity.

The main shortcoming of capital requirements, though, is that they can only achieve a second-best outcome, whereas the policies of Sections 5.1-5.3 achieve the first best in the context of our model. Capital requirements force intermediaries to hold more equity or reduce debt, but both options have negative impact on welfare. In particular, issuing equity may reduce welfare because banks must earn rents to pay for the cost of equity, and rents can be earned only if the demand for liquidity is not fully satiated.

We show that capital requirements increase welfare with respect to *laissez faire* under two conditions: the supply of public liquidity must be sufficiently low and the probability of a crisis, π, must be sufficiently large. This result is the consequence of a trade off. On one hand, capital requirements avoid the liquidity crunch in the low state. On the other hand, capital requirements are costly because they require intermediaries to raise enough equity, which is more expensive than debt due to the cost τ. Therefore, intermediaries reduce the overall supply of liquidity. If public liquidity is very high, the economy is either already at the first best (as in Sections 5.1 and 5.2) or close to it. Therefore, the gains from avoiding a liquidity crunch in the low state are small enough to be offset by the cost of issuing equity. If the probability of the low state is small, crises are very infrequent and thus the ex-ante benefits of avoiding them is small.

To model capital requirements, consider a regulatory restriction that imposes a lower bound on the equity-to-debt ratio of the intermediaries of the form

$$\frac{N_t(j)}{D_t(j)} \geq \frac{1-\beta\,(1+\mu)\,(1+r_l^K)}{(1+r_l^K)\,(1-\tau)}. \tag{39}$$

[29] See among others Bianchi (2016).

This requirement is sufficient to insure that intermediaries remain solvent in all states. Thus, the only type of debt that intermediaries can issue is safe debt.[30] However, in order to participate to the market, rents must be sufficiently large to cover the equity costs imposed by τ. As a result, it could be possible that intermediaries do not operate in equilibrium. Indeed, this is the case for a sufficiently high level of public liquidity.

Proposition 8 *Assume that financial intermediaries face a cost τ per unit of equity raised and $\tau\gamma_l$ is sufficiently small. If*

$$\frac{B}{P} \geq \frac{1}{1+\tau\gamma_l} \tag{40}$$

the capital requirement in (39) reduces welfare. If instead

$$\frac{B}{P} < \frac{1}{1+\tau\gamma_l} \tag{41}$$

there exists a $\pi^* \in (0,1)$ such that the capital requirement in (39) improves welfare if and only if $\pi > \pi^*$.

Proof. See Appendix. ■

The results of the above Proposition are derived under the assumption that the costs τN paid by financial intermediaries are a transfer from the point of view of the society as a whole. [31] Therefore, these costs are not taken into account when evaluating welfare. Details of the proof are left to the Appendix.

When public debt is sufficiently high (more precisely, above the threshold (40)), welfare under *laissez faire* is higher than in the regulated economy. Under *laissez faire*, in state l, a high level of public debt dampens the adverse effect due to the shortage of private liquidity, ensuring a sufficiently high consumption; in state h, pseudo-safe securities of type p provide liquidity and thus consumption is at the efficient level. In contrast, regulation prohibits intermediaries from issuing pseudo-safe securities p and thus the efficient level of consumption is not achieved in the high state. Moreover, the high supply of public debt implies that intermediaries' rents earned by issuing safe debt are not large enough to offset the cost of equity, and thus intermediary do not issue any debt. That is, with large public debt, capital requirements reduce liquidity in the high state and do not affect consumption in the low state.

As the supply of public debt diminishes, capital requirements improve ex-ante welfare only if the probability π of realization of the low state is relatively high. If instead π is small, capital requirements reduce welfare. This is because, as before, capital requirements prohibits intermediaries from issuing pseudo-safe securities p and thus reduce liquidity in the high state. Since the probability of the high state is $1-\pi$, this negative effect of capital requirements dominates when π is small.

[30] Formally, the set of private debt securities that are traded in equilibrium is restricted to be $\mathcal{E} = \{s\}$.

[31] The results can be extended to the case in which the cost τN is wasted.

6 Conclusion

We have presented a framework for studying equilibria with private money creation in a model in which both public and private liquidity play a role for transactions. If the financial market is frictionless, private incentives and public objectives are fully aligned and thus the efficient level of liquidity is supplied without any need of government regulation. If instead the financial sector is characterized by a friction that makes equity financing more costly than debt financing, the demand for liquidity is satiated only in good times, whereas the economy is subject to crises and to liquidity shortages in times of economic distress.

Within this framework, we have explored several policies to improve welfare. The government can supply a large amount of liquidity backed by central bank's holding of private securities or by taxes. Alternatively, the government can bail out financial intermediaries during crises. In the context of the model, these policies achieve the first best, but we argue that an active role of the government in providing and supporting liquidity (either directly or through bailouts) is likely to be optimal in richer models. Finally, we show that capital requirements increase welfare if public liquidity is low and the probability of crisis is relatively high.

We are aware that we have omitted some important real-world features, but we consider our model as a first step in addressing the important topic of private and public liquidity determination. This debate has been at the center of economists' thoughts for hundreds of years but has received little attention in modern economic analysis. The trade offs that we have highlighted in the policy analysis of Section 5 deserve further investigation in a richer quantitative model.

We see at least three possible extensions of our framework. First, we have limited the focus of our analysis only on the consequences that financial disruption has on the liquidity market. There can be, however, important effects on the supply of credit with interesting spillovers between credit and money markets that could be explored in more complicated frameworks. Second, our analysis could be extended to markets with informational asymmetries between depositors and intermediaries or to other market structures in which intermediaries have some market power, like in the monopolistic-competition model detailed in the Appendix. Third, our framework could also be applied to understand equilibria with parallel currencies and the supply of liquidity in open economies. We leave these extensions for future work.

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A Appendix

In this appendix, we collect the proofs of Propositions 3, 5, 7 and 8. We also add the discussion of the constrained first best in relationship with Section 5.4. Finally, we discuss the extension of the framework of Section 2 to a market of monopolistic competition.

A.1 Proof of Proposition 3

First we show that it is not possible to have an equilibrium where only safe securities are supplied. Suppose by contradiction that this equilibrium exists. Given free entry in the market, rents (33) in supplying the safe security should be zero, i.e.

$$\mathcal{R}_t(s) = -\tau N_t(s) + \beta\mu D_t(s) = 0 \tag{A.1}$$

where $\mu_h = \mu_l = \mu$ is the Lagrange multiplier of the liquidity constraint, (4). Moreover, to supply safe assets, net worth should be high enough to avoid insolvency in the low state,

$$N_t(s) = \frac{D_t(s)}{(1+r_l^K)(1-\tau)}\left[1-\beta(1+\mu)(1+r_l^K)\right], \tag{A.2}$$

which follows from (34) where $Q_t^D(s) = \beta(1+\mu)$.

Combining (A.1) and (A.2) reveals that the marginal value of liquidity is positive, $\mu = \tau\gamma_l > 0$ where γ_l is defined in Proposition 3. In this candidate equilibrium consumption is equalized across the two states of nature but is lower than in the first best, because of the cost of raising equity:

$$C_l = C_h = \frac{1}{1+\tau\gamma_l} < 1.$$

To be an equilibrium, intermediaries should not find it profitable to issue any other type of securities in $\mathcal{J}$. Suppose to the contrary that a generic intermediary enters the market of the pseudo-safe security of type p. Since rents (A.1) are decreasing in net worth, securities of type p are the most profitable pseudo-safe securities (i.e., the most profitable securities among those that default in the low state) because they require zero net worth. If an intermediary supplies security p, its rents (33) are positive since the security carries a liquidity premium in the high state

$$\mathcal{R}_t(p) = \beta(1-\pi)\mu D_t(p) > 0.$$

It is therefore optimal from the point of view of the single intermediary to enter in the market of security p. This contradicts the assumption that there exists an equilibrium in which only safe securities are supplied.

However, it is possible that securities of type s and p coexist in equilibrium. For this to be the case, both intermediaries issuing s and p should not find any incentive to deviate. This turns out to be possible if liquidity is completely satiated in the high state, $\mu_h = 0$, which requires the overall supply of private liquidity to complement that of public liquidity

$$\frac{D(s)}{P} + \frac{D(p)}{P} \geq \max\left(1 - \frac{B}{P}, 0\right) = \max\left(1 - \bar{T}, 0\right).$$

Free entry implies that the rents of the intermediary supplying the security p are zero

$$\mathcal{R}_t(p) = -\tau N_t(p) + \beta(1-\pi)\mu_h D_t(p) = 0$$

which is the case since the optimal choice of equity is zero and $\mu_h = 0$. Free entry also implies that rents earned by issuing security s are zero

$$\mathcal{R}_t(s) = -\tau N_t(s) + \beta\pi\mu_l D_t(s) = 0, \tag{A.3}$$

where net worth is chosen at the minimum level necessary to avoid insolvency in the low state

$$N_t(s) = \frac{D(s)}{\left(1 + r_l^K\right)(1-\tau)}\left[1 - \beta\left(1 + \pi\mu_l\right)\left(1 + r_l^K\right)\right], \tag{A.4}$$

which follows from (34) with equality in state l, having used $Q_t^D(s) = \beta\left(1 + \pi\mu_l\right)$. Equations (A.3) and (A.4) can be solved to obtain the liquidity premium in the low state, $\mu_l = \tau\gamma_l/\pi$. Therefore consumption is at the first best in the high state, $C_h = 1$, but lower in the low state, $C_l = 1/(1+\mu_l) = \pi/(\pi + \tau\gamma_l)$. Using the latter expression and the binding liquidity constraint in the low state $D(s) + B = PC_l$, we can then derive the supply of the safe security s

$$\frac{D(s)}{P} = \frac{\pi}{\pi + \tau\gamma_l} - \frac{B}{P}.$$

This is positive when public debt is relatively low. For high levels of debt, the liquidity premium on safe securities is low and thus rents obtained by issuing safe securities are negative. As a result, only pseudo-safe securities of type p are supplied and consumption in the low state is solely determined by public liquidity $C_l = B/P$.

Finally, the default rate on pseudo-safe securities is determined by (34) holding with equality, and using $N_t(p) = 0$ (because intermediaries that issue pseudo-safe securities choose zero net worth) and $Q_t^D(p) = \beta\left[(1-\pi) + \pi\left(1 - \chi_l(p)\right)\right]$ (which follows from equation (28)).

A.2 Proof of Proposition 5

We need to prove that it is possible to implement the efficient allocation $C_h = C_l = 1$ with $P_t = P$ while satisfying the constraint (35) by appropriately choosing $D_t^c(p)$ and B_t. First, rewrite the intertemporal government budget constraint, (36), using $D_t^c(p) = D^c(p)$, $B_t = B$, $P_t = P$, and (21) evaluated at $Q_t^B = Q_t^f$ (because the liquidity premium is zero under the efficient supply of liquidity):

$$\frac{B}{P} = \begin{cases} \frac{D^c(p)}{P} + (1-\beta)\,\overline{T}_h + E_t\left\{\sum_{j=1}^{\infty}\beta^j T_{t+j}\right\} & if\ A_t = A_h \\ \left(1 - \chi_l(p)\right)\frac{D^c(p)}{P} + (1-\beta)\,\overline{T}_l + E_t\left\{\sum_{j=1}^{\infty}\beta^j T_{t+j}\right\} & if\ A_t = A_l. \end{cases} \tag{A.5}$$

We can rewrite the two equations in (A.5) – that is, one equation for the high state and one equation for the low state – evaluating them at $B/P = 1$ because $B/P = 1$ is the real supply of public liquidity that allows the economy to achieve efficiency:

$$
\begin{aligned}
1 &= \frac{D^c(p)}{P} + (1-\beta)\,\overline{T}_h + \beta\left[(1-\pi)\,\overline{T}_h + \pi\overline{T}_l\right] \\
1 &= (1-\chi_l(p))\,\frac{D^c(p)}{P} + (1-\beta)\,\overline{T}_l + \beta\left[(1-\pi)\,\overline{T}_h + \pi\overline{T}_l\right]
\end{aligned}
$$

where we have used the fact that (21) implies $E_t(T_{t+j}) = (1-\beta)\left[(1-\pi)\,\overline{T}_h + \pi\overline{T}_l\right]$. This is a system of two equations in two unknowns, where the unknowns are $D^c(p)/P$ and $\overline{T}_l$, as a function of $\overline{T}_h$ and the parameters of the model. The solution is given by $D^c(p)/P$ and $\overline{T}_l$ stated by the Proposition.

The restriction $\overline{T}_h < 1$ is required to satisfy (35). The inequalities $D^c(p)/P > 0$ and $\overline{T}_l > \overline{T}_h$ follow from $\overline{T}_h < 1$, $\chi_l(p) \in (0,1)$, and $\beta < 1$. The default rate on pseudo-safe debt, $\chi_l(p)$, can be computed following the same steps as in Proposition 3.

A.3 Proof of Proposition 7

We first note that if the government bail outs intermediaries in the low state, then intermediaries are able to issue safe debt s even with zero net worth, $N_t(s) = 0$. Thus, $N_t(s) = 0$ is optimal for all $\tau > 0$.

Given a level of public debt B/P, we guess (and later verify) that the amount of safe debt issued by intermediaries is:

$$
\frac{D_t(s)}{P} = 1 - \frac{B}{P} > 0. \tag{A.6}
$$

Similar to the proof of Proposition 5, we can rewrite (36), using $B_t = B$, $P_t = P$, and (21) evaluated at $Q_t^B = Q_t^f$ (because the liquidity premium is zero under the efficient supply of liquidity):

$$
\frac{B}{P} = \begin{cases} (1-\beta)\,\overline{T}_h + E_t\left\{\sum_{j=1}^{\infty}\beta^j T_{t+j}\right\} & if\ A_t = A_h \\ -\chi_l(p)\frac{D_t(s)}{P} + (1-\beta)\,\overline{T}_l + E_t\left\{\sum_{j=1}^{\infty}\beta^j T_{t+j}\right\} & if\ A_t = A_l \end{cases} \tag{A.7}
$$

where

$$
\chi_l(p) = 1 - \frac{1+r_h^K}{1+r_l^K}. \tag{A.8}
$$

That is, in the low state, the securities $D_t(s)$ would default without a government bailout, and their rate of default would be the same as pseudo-safe securities p (i.e., securities that are issued by intermediaries that have zero net worth as well, but no government bailout). As a result, for each dollar of deposits, the government transfers $\chi_l(p)$ dollars to the intermediary in the event of default, for a total of $\chi_l(p)\frac{D_t(s)}{P}$ real dollars of equity infusion.

Using (A.6) and (A.8), and proceeding as in the proof of Proposition 5, we can rewrite the two equations in (A.7) as:

$$\begin{aligned}
\frac{B}{P} &= (1-\beta)\,\overline{T}_h + \beta\left[(1-\pi)\,\overline{T}_h + \pi\overline{T}_l\right] \\
\frac{B}{P} &= -\chi_l(p)\left(1-\frac{B}{P}\right) + (1-\beta)\,\overline{T}_l + \beta\left[(1-\pi)\,\overline{T}_h + \pi\overline{T}_l\right]
\end{aligned}$$

This is a system of two equations in two unknowns where the two unknowns are $\overline{T}_l$ and B/P, as a function of $\overline{T}_h$ and the other parameters of the model. The solution for $\overline{T}_l$ is the one in the statement of the Proposition. The inequalities $\overline{T}_h < 1$ and $\overline{T}_l > \overline{T}_h$ can be proved as in Proposition 5.

As a last step, we verify the guess in (A.6). From the previous system of equations, the solution for B/P is

$$\frac{B}{P} = \frac{\overline{T}_h(1-\beta) + \pi\beta\chi_l(p)}{1-\beta+\pi\beta\chi_l(p)} < 1$$

where the inequality follows from $\overline{T}_h < 1$. Since $C_h = C_l = 1$, the liquidity constraint, (4), implies (A.6), and the inequality $D(s)/P > 0$ follows from $B/P < 1$.

A.4 Proof of Proposition 8

We start by solving for the equilibrium under capital requirements. In this case, if intermediaries operate, they issue debt of type s. We next show that intermediaries' debt is:

$$\frac{D(s)}{P} = \max\left\{0, \frac{1}{1+\tau\gamma_l} - \frac{B}{P}\right\}.$$

Consider first the case $B/P < 1/\left(1+\tau\gamma_l\right)$. Intermediaries' rents are:

$$\mathcal{R}_t = -\tau N_t(s) + \beta\mu D_t(s) \tag{A.9}$$

and net worth is chosen at the minimum level that guarantees the supply of a safe security, i.e., $\chi_l = 0$. Using the fact that the price of a safe security is $Q^D(s) = \beta\left(1+\mu\right)$, we have:

$$N_t(s) = \frac{D_t(s)}{(1+r_l^K)(1-\tau)}\left[1-\beta\left(1+\mu\right)\left(1+r_l^K\right)\right].$$

Therefore, plugging $N_t(s)$ into the expression for rents, (A.9), we can solve for the equilibrium value of μ that implies zero rents. This value is given by $\mu = \tau\gamma_l$.

As a result, consumption $C_h = C_l = C$ is given by

$$C = \frac{1}{1+\tau\gamma_l}$$

and, using $C = B/P + D(s)/P$, we have

$$\frac{D(s)}{P} = \frac{1}{1+\tau\gamma_l} - B/P.$$

If instead $B/P > 1/(1+\tau\gamma_l)$, one can follow similar steps and conclude that the non-negativity constraint on consumption implies $D(s)/P = 0$.

We now compare welfare with and without capital requirements, starting with the case

$$\frac{1}{(1+\tau\gamma_l)} < \frac{B}{P} < 1.$$

In this case, the equilibrium without regulation is characterized by $C_h = 1$ and $C_l = B/P$ (see Proposition 3). As a result, welfare is:

$$\begin{aligned}\mathcal{W}^{NR} &= (1-\pi)\left[\log C_h + X_h\right] + \pi\left[\log C_l + X_l\right] \\ &= A\overline{K} - 1 + \pi\left[\log\left(\frac{B}{P}\right) - \frac{B}{P} - (\log(1) - 1)\right].\end{aligned}$$

For future reference, note that $\mathcal{W}^{NR}$ is strictly decreasing in π because the term in square brackets is negative, using the fact that $\log x - x$ is increasing in x for $x \leq 1$ and $B/P < 1$.

With capital requirements, welfare is instead given by:

$$\mathcal{W}^{R} = \log\frac{B}{P} + A\overline{K} - \frac{B}{P}.$$

Thus, it follows that $\mathcal{W}^{NR} > \mathcal{W}^{R}$ because

$$\log\frac{B}{P} - \frac{B}{P} < \log(1) - 1$$

and $B/P < 1$.

If instead public debt is:

$$\frac{\pi}{(\pi+\tau\gamma_l)} < \frac{B}{P} < \frac{1}{(1+\tau\gamma_l)}$$

then welfare in the unregulated economy is unchanged whereas welfare with regulation is

$$\mathcal{W}^{R} = \log\left(\frac{1}{1+\tau\gamma_l}\right) + A\overline{K} - \frac{1}{1+\tau\gamma_l}.$$

To compare welfare with and without regulation, we use the fact that $\mathcal{W}^{NR}$ is strictly decreasing in π, as discussed before, and thus we can just look at the extreme case $\pi \to 1$ and $\pi \to 0$. Using once more the property of $\log x - x$, we have

$$\begin{aligned}\mathcal{W}^{NR} > \mathcal{W}^{R} \qquad as \quad \pi \to 0 \\ \mathcal{W}^{NR} < \mathcal{W}^{R} \qquad as \quad \pi \to 1\end{aligned}$$

As a result, there exists a unique $\tilde{\pi} \in (0,1)$ such that capital requirements are welfare improving if and only if $\pi > \tilde{\pi}$.

If instead public debt is:

$$\frac{B}{P} < \frac{\pi}{(\pi+\tau\gamma_l)}$$

and thus, using the results of Proposition 3 about consumption C_h and C_l, welfare without regulation is:

$$\begin{aligned}\mathcal{W}^{NR} &= (1-\pi)\left[\log C_h + X_h\right] + \pi\left[\log C_l + X_l\right] \\ &= A\overline{K} - 1 + \pi\left[\log\left(\frac{\pi}{\pi+\tau\gamma_l}\right) - \frac{\pi}{\pi+\tau\gamma_l} + 1\right].\end{aligned}$$

Note first that $W^{NR}(\pi = 0) > \mathcal{W}^R$ while $W^{NR}(\pi = 1) = \mathcal{W}^R$. We need to show that there exists a unique value of $\pi \in (0, 1)$, denoted by π^*, such that $\mathcal{W}^{NR}(\pi^*) = \mathcal{W}^R$. This boils down to showing that there is a unique $\pi^* \in (0, 1)$ that solves:

$$\pi^* \log\left(\frac{\pi^*}{\pi^*+\tau\gamma_l}\right) - \log\left(\frac{1}{1+\tau\gamma_l}\right) - \frac{\tau^2\gamma_l^2}{(1+\tau\gamma_l)}\frac{(1-\pi^*)}{(\pi^*+\tau\gamma_l)} = 0$$

Let $L(\pi)$ denote the left-hand side of the above equation. We have:

$$L'(\pi) = \log\left(\frac{\pi}{\pi+\tau\gamma_l}\right) + \left(\frac{\tau\gamma_l}{\pi+\tau\gamma_l}\right) + \frac{\tau^2\gamma_l^2}{(\pi+\tau\gamma_l)^2}$$

$$L''(\pi) = \left(\frac{\tau\gamma_l}{\pi+\tau\gamma_l}\right)^2 \frac{1}{\pi} - \frac{2\tau^2\gamma_l^2}{(\pi+\tau\gamma_l)^3}$$

To show the result, we show that $L(\pi)$ has the shape represented in Figure 1. First of all, note that $L(\pi = 0) > 0$ because $W^{NR}(\pi = 0) > \mathcal{W}^R$, whereas $L(\pi = 1) = 0$ because $W^{NR}(\pi = 1) = \mathcal{W}^R$.

Next, we show that $L'(\pi = 1) > 0$ so that there must be at least one $\pi^* \in (0, 1)$ such that $L(\pi^*) = 0$. We have:

$$L'(\pi = 1) = \log\left(\frac{1}{1+\tau\gamma_l}\right) + \left(\frac{\tau\gamma_l}{1+\tau\gamma_l}\right) + \frac{\tau^2\gamma_l^2}{(1+\tau\gamma_l)^2}$$

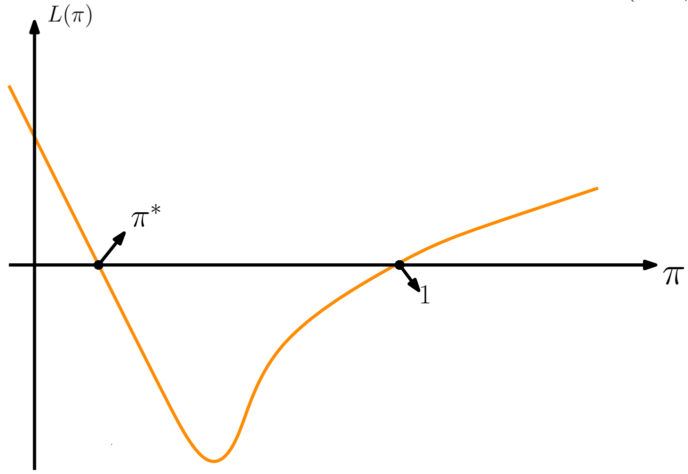
which is positive because $\tau\gamma_l$ is small by assumption; that is, $L'(\pi = 1) \to 0$ as $\tau\gamma_l \to 0$ and:

$$\frac{\partial}{\partial(\tau\gamma_l)}\left[L'(\pi = 1)\right] = 0, \quad \frac{\partial^2}{\partial(\tau\gamma_l)^2}\left[L'(\pi = 1)\right] = 1.$$

In other words, $L'(\pi = 1)$ (i) has a value of zero evaluated at $\tau\gamma_l = 0$, (ii) is flat, and (iii) is convex; thus, it must be positive for $\tau\gamma_l > 0$ and small.

Finally, we prove that π^* is unique, by showing that $L(\pi)$ is convex for $\pi \to 0$ and concave for $\pi \to 1$; that is, the uniqueness follows from the fact that the second derivative shifts sign only once, and from the fact that the slope of $L(\pi)$ is positive at $\pi \to 1$, as shown before. The sign of $L''(\pi^*)$ is equal to the sign of $\tau\gamma_l - \pi$. For $\pi \to 0$, $L''(\pi^*) > 0$, whereas $L''(\pi^*) < 0$ for $\pi \to 1$ because $\tau\gamma_l$ is small by assumption; that is, $L(\pi)$ is convex for $\pi < \tau\gamma_l$ and concave for $\pi > \tau\gamma_l$.

Figure 1: Proof capital requirements, case $\frac{B}{P} < \frac{\pi}{(\pi+\tau\gamma_l)}$



A.5 Constrained first-best and pecuniary externality

We now study welfare in the *laissez faire* equilibrium in the economy with a cost of issuing equity. We show that there is a pecuniary externality that precludes the economy from achieving the first best. That is, this externality implies an overissuance of pseudo-safe (risky) debt by financial intermediaries and an underissuance of safe (riskless) debt. To do so, we characterize the constrained first-best allocation of this economy and we show that is different from the allocation that arises in the market equilibrium.

Consider a social planner that can dictate choices to intermediaries and households but takes as given the monetary-fiscal policy stance (and thus takes as given the price level $P_t = P$). The social planner is subject to the same frictions imposed in the market economy, namely, the liquidity constraint, (4), and the cost of issuing equity $\tau > 0$. However, the social planner internalizes that $\tau N_t(j)$ is redistributed back to households and thus is not lost from the point of view of the society.[32]

Thus, the social planner problem is given by:

$$\max_{C_h, C_l} \left\{ \pi \left[\log C_h + A_h \overline{K} - C_h \right] + (1-\pi) \left[\log C_l + A_l \overline{K} - C_l \right] \right\},$$

considering that output in state h, $A_h \overline{K}$, can be used for the first subperiod consumption, C_h, and the second subperiod consumption $X_h = A_h \overline{K} - C_h$; similarly in state l. The first-order conditions of this problem imply:

$$C_h = 1, \qquad C_l = 1.$$

Therefore, the constrained first best differs from the allocation that arises in the market equilibrium; see Proposition 3. Moreover, the constrained first best is the same as first best in the model with frictionless intermediation (Sections 2 and 3).

[32]If $\tau N_t(j)$ is a physical cost that reduces the amount of resources that are available to be consumed, the analysis of the constrained first best is similar, but the algebra is more complicated. An externality still arises, provided that the amount of liquidity supplied by the government is not too large.

The literature has highlighted some approaches to correct for the externality. Lorenzoni (2008) and Stein (2012) suggest two approaches. One possibility is to use a system of taxes and subsidies on intermediaries' debt; the objective is to align the private incentives to the social ones. The second approach is a cap-and-trade system in which banks are granted a permission to create a given amount of money; each bank can either use such a permission, or sell it to the other intermediaries. The main shortcomings of these approaches is related to their practical implementation. Whereas taxes or cap-and-trade system can have a simple representation in theoretical models, they must also be robust to real-world features that are not included in the model. Differently, our proposed intervention to achieve the first best is based on the government provision of liquidity, as discussed in Sections 5.1 and 5.2.

There is an additional motivation for looking at the analysis of government policies. In our model, welfare in the constrained first best is the same as in the first best without the cost of equity. However, in richer models, welfare in the constrained first best can be lower than in the first best.[33] The policies of taxes or cap-and-trade achieve only the constrained first best, whereas the government interventions that we analyze in Sections 5.1 and 5.2 achieve the first best.

A.6 Monopolistic competition

This appendix presents the results of a model with monopolistic competition in the financial sector and a real fixed cost $\Phi > 0$ to enter that market. The cost $\Phi > 0$ adds to the budget constraint in the following way

$$Q_t^K K_t(j) + P\Phi = Q_t^D(j) D_t(j) + N_t(j).$$

Unlike perfect competition, financial intermediaries internalize the effects of the quantity supplied on the marginal value of liquidity and therefore on the price of the security. Indeed, the fixed real cost Φ ensures that they are not small and that in equilibrium there is a finite number of financial intermediaries operating in the market.

Proposition 9 *When financial intermediaries face a real fixed cost to enter the market Φ (with $0 < \Phi < \beta$) and act under monopolistic competition, the only private securities supplied in equilibrium are safe assets. Consumption is equal to*

$$C_h = C_l = \max\left(1 - \sqrt{\frac{\Phi}{\beta}}, \overline{T}\right),$$

assuming $\overline{T} < 1$, whereas the overall supply of safe securities is

$$\frac{D(s)}{P} = \max\left(1 - \overline{T} - \sqrt{\frac{\Phi}{\beta}}, 0\right);$$

[33] For instance, this is the case in Lorenzoni (2008) and Stein (2012). A similar results arise in our model if the cost $\tau N_t(j)$ is a physical cost that is lost, instead of being redistributed to the society.

the number Z of intermediaries in the market is

$$Z = \sqrt{\frac{\beta}{\Phi}} \frac{\max\left(1 - \overline{T} - \sqrt{\frac{\Phi}{\beta}}, 0\right)}{1 - \sqrt{\frac{\Phi}{\beta}}}$$

and each intermediary enters with a level of net worth given by

$$N(s) \geq \frac{D(s)}{Z} \left[\left(1 + r_l^K\right)^{-1} - \beta \right].$$

As the barrier to entry Φ becomes arbitrarily small, the *laissez-faire* equilibrium approaches the first best. To understand why only safe assets are privately supplied in equilibrium, note that the marginal value of liquidity is positive in equilibrium, that is, $\mu_h = \mu_l = \mu > 0$, because consumption is not at the first best, $C_h = C_l < 1$. Entry in market s implies zero rents taking into account the entry cost, $\mathcal{R}_t(s) = \beta\mu D(s) - P\Phi = 0$. Instead, supplying pseudo-safe securities of type j, rents would be negative because the intermediary would lose liquidity premium associated to the low state without any benefit. Similarly, if only a market of pseudo-safe securities exists, intermediaries have incentives to supply safe securities s in order to exploit the liquidity premium in the low state. Therefore, only safe securities are supplied in equilibrium.

More formally, the budget constraint of a generic intermediary i issuing debt of type j is now

$$Q_t^K K_t^i(j) + P\Phi = Q_t^D(j) D_t^i(j) + N_t^i(j),$$

taking into account the fixed cost of entry into the market. Consumer's demand of a generic security of type j is

$$Q_t^D(j) = \beta E_t \left\{ \left[I_{t+1}(j)(1 - \chi_{t+1}(j)) + (1 - I_{t+1}(j))(1 + \mu_{t+1}) \right] \right\}, \tag{A.10}$$

where we preserve the assumption of a constant price level P.

Equation (14) that describes intermediary i's gross profits issuing securities of type j is given by

$$\begin{aligned} \Pi_{t+1}^i(j) &= \left(1 + r_{t+1}^K\right) Q_t^K K_t^i(j) \\ &\quad -(1 - I_{t+1}(j)) D_t^i(j) - I_{t+1}(j) (1 - \chi_{t+1}(j)) D_t^i(j), \end{aligned} \tag{A.11}$$

where we have also used the fact that the nominal return on capital is equal to the real return, $i_{t+1}^K = r_{t+1}^K$, because prices are constant.

The discounted value of profits is

$$\begin{aligned} E_t \left\{ \beta \Pi_{t+1}^i(j) \right\} &= E_t \left\{ \beta \Pi_{t+1}^{D,i}(j) \right\} - P\Phi + D_t^i(j) Q_t^D(j) + \\ &\quad -\beta E_t \left\{ 1 - I_{t+1}(j) + I_{t+1}(j) (1 - \chi_{t+1}(j)) \right\} D_t^i(j). \end{aligned} \tag{A.12}$$

We turn now to characterize the optimal choice for a generic intermediary i. The objective is to maximize expected rents $\mathcal{R}_t^i(j)$, defined as the difference between expected profits and expected dividends:

$$\mathcal{R}_t^i(j) \equiv \beta E_t \left\{ \Pi_{t+1}^i(j) - \Pi_{t+1}^{D,i}(j) \right\}, \tag{A.13}$$

taking into account the demand schedule (A.10). It follows that

$$\mathcal{R}_t^i(j) = \beta E_t \left\{ (1 - I_{t+1}(j))\mu(B, (1 - I_{t+1}(j))D_t^i(j), D_{t+1}(-i)) \right\} D_t^i(j) - P\Phi. \tag{A.14}$$

The key feature of our monopolistically competitive market is that intermediary i is no longer small and therefore internalizes the effects of its choices on the liquidity premium $\mu(B, (1 - I_{t+1}(j))D_t^i(j), D_{t+1}(-i))$ and on the market price of securities. In particular,

$$\mu(B, (1 - I_{t+1}(j))D_t^i(j), D_{t+1}(-i)) = \frac{1}{\frac{B}{P} + \frac{D_{t+1}(-i) + (1 - I_{t+1}(j))D_t^i(j)}{P}} - 1 \tag{A.15}$$

where

$$D_{t+1}(-i) \equiv \sum_{z \neq i}^{Z} \int_{j \in \mathcal{J}} (1 - I_{t+1}(j))D_t^z(j)dj.$$

capturing the supply of other intermediaries, under the assumption that each intermediary can issue only one type of security; that is, $D_t^z(k) = 0$ for each $k \neq j$, with $k \in \mathcal{J}$, if $D_t^z(j) > 0$. The term Z is the total number of intermediaries supplying positive debt in the various markets.

The optimization problem of a generic intermediary i can be decomposed into two stages. In the first stage, the intermediary chooses the type of security j to issue. In the second stage, the intermediary chooses $D_t^i(j)$ and the level of net worth $N_t^i(j)$ to maximize $\mathcal{R}_t^i(j)$ considering (A.15) and taking $D_{t+1}(-i)$ as given. It is also subject to the limited liability constraint which can be written as a lower bound on the level of net worth

$$N_t^i(j) \geq \frac{[(1 - I_{t+1}(j)) + I_{t+1}(j)\,(1 - \chi_{t+1}(j))]}{\left(1 + r_{t+1}^K\right)} D_t^i(j) - Q_t^D(j)D_t^i(j) + P\Phi. \tag{A.16}$$

We first characterize the equilibrium in which all operating intermediaries supply safe securities and then show that indeed only safe securities are supplied in equilibrium. If all intermediaries supply security of type s, then

$$D_{t+1}(-i) \equiv \sum_{z \neq i}^{Z} D_t^z(s).$$

The optimal choice $D_t^i(s)$ of intermediary i that maximizes rents (A.14) given (A.15) implies the following first-order condition

$$\frac{D_t^i(s)}{P} = \left(\frac{B + D_{t+1}(-i)}{P} \right)^{\frac{1}{2}} - \left(\frac{B + D_{t+1}(-i)}{P} \right).$$

In a symmetric equilibrium, $D_t^i(s) = D(s)/Z$ and $D_{t+1}(-i) = (Z-1)D(s)/Z$ where $D(s)$ is the aggregate level of intermediaries' debt. We can then write the above condition as

$$\frac{D(s)}{P} + \frac{B}{P} = \left(\frac{B + \frac{Z-1}{Z}D(s)}{P}\right)^{\frac{1}{2}}. \tag{A.17}$$

Intermediaries enter the market until all rents are eliminated, $\mathcal{R}_t(s) = 0$, implying from (A.14) that

$$\beta\left(\frac{1}{\frac{B}{P} + \frac{D(s)}{P}} - 1\right)\frac{D(s)}{Z} = P\Phi. \tag{A.18}$$

Equations (A.17) and (A.18) can be solved for the equilibrium level of $D(s)$ and Z:

$$\frac{D(s)}{P} = \max\left(1 - \frac{B}{P} - \sqrt{\frac{\Phi}{\beta}}, 0\right),$$

$$Z = \sqrt{\frac{\beta}{\Phi}}\frac{\max\left(1 - \frac{B}{P} - \sqrt{\frac{\Phi}{\beta}}, 0\right)}{1 - \sqrt{\frac{\Phi}{\beta}}}.$$

At this point, it is important to use the restriction $\Phi < \beta$. Combining the above two results, we get that the amount issued by each intermediary, when positive, is:

$$\frac{D^i(s)}{P} = \sqrt{\frac{\Phi}{\beta}} - \frac{\Phi}{\beta},$$

which becomes very small as the fixed cost goes to zero. The level of net worth of intermediaries that issue security s is

$$N^i(s) \geq D^i(s)\left[\left(1 + r_l^K\right)^{-1} - \beta\right].$$

The key question is why it is optimal to just enter the market of security s and not to supply other securities. Suppose that intermediary i instead enters a generic market of a pseudo-safe security j while all other intermediaries are supplying security s, so that $\hat{D}_{t+1}(-i) \equiv (Z-1)\hat{D}_t(s)$. In market s, rents will be zero if intermediary i enters the market supplying the optimal quantity $\hat{D}_t(s)$. Instead, by supplying a generic pseudo-safe security j, rents are

$$\mathcal{R}(B, D_t^i(j), \hat{D}_{t+1}(-i)) = \beta\left\{(1-\pi)\,\mu(B, D_t^i(j), \hat{D}_{t+1}(-i))\right\}D_t^i(j) - P\Phi.$$

It follows then that the optimal quantity of security j to supply is exactly the same as the one that the intermediary would supply if it had chosen to issue security s; that is, $\hat{D}_t^i(j) = \hat{D}_t(s)$. Therefore, the maximum rent that the intermediary can get by issuing security j is negative and thus less than the rent that the intermediary would earn by issuing securities of type s:

$$\begin{aligned}
\mathcal{R}(B, \hat{D}_t^i(j), \hat{D}_{t+1}(-i)) &< \mathcal{R}(B, \hat{D}_t^i(s), \hat{D}_{t+1}(-i)) = \\
&= \beta\left\{\mu(B, \hat{D}_t^i(s), \hat{D}_{t+1}(-i))\right\}\hat{D}_t^i(s) - P\Phi \\
&= 0.
\end{aligned}$$

Since this is true for any generic pseudo-safe security j, it is optimal to issue seurities of type s.

Consider now the case in which all other intermediares in the market are supplying pseudo-safe securities. Without loss of generality, assume that they are all supplying a security of type j and therefore $\hat{D}_{t+1}(-i) \equiv (Z-1)(1-I_{t+1}(j))\hat{D}_t(j)$. If intermediary i issue securities of type j, its optimal choice is to supply $\hat{D}_t^i(j)$ and its rent will be zero. If instead the intermediary chooses securities of type s, its rents are

$$\begin{aligned}\mathcal{R}(B, D_t^i(s), \hat{D}_{t+1}(-i)) \;\; = \;\; & \beta\left\{(1-\pi)\,\mu_h(B, D_t^i(s), \hat{D}_{t+1}(-i)) + \pi\mu_l(B, D_t^i(s))\right\} D_t^i(s) \\ & -P\Phi,\end{aligned}$$

because only securities of type s are liquid in the low state. Define $\hat{D}_t^i(s)$ to be the optimal quantity that maximizes the above rents and $\tilde{D}_t^i(s)$ the quantity of security s that is equal to the quantity of security j, that is, $\tilde{D}_t^i(s) = \hat{D}_t^i(j)$. The rents obtained by issuing s are always positive because

$$\begin{aligned} & \mathcal{R}(B, \hat{D}_t^i(s), \hat{D}_{t+1}(-i)) \\ \geq \; & \mathcal{R}(B, \tilde{D}_t^i(s), \hat{D}_{t+1}(-i)) \\ = \; & \beta\left\{(1-\pi)\,\mu_h(B, \tilde{D}_t^i(s), \hat{D}_{t+1}(-i)) + \pi\mu_l(B, \tilde{D}_t^i(s))\right\} \tilde{D}_t^i(s) - P\Phi \\ = \; & \beta\left\{(1-\pi)\,\mu_h(B, \hat{D}_t^i(j), \hat{D}_{t+1}(-i)) + \pi\mu_l(B, \hat{D}_t^i(j))\right\} \hat{D}_t^i(j) - P\Phi \\ = \; & \beta\left\{\pi\mu_l(B, \hat{D}_t^i(j))\right\} \hat{D}_t^i(j) > 0. \end{aligned}$$

The second line follows from the fact that $\hat{D}_t^i(s)$ is optimally chosen. The fourth line follows by using the assumption that $\tilde{D}_t^i(s) = \hat{D}_t^i(j)$. The last line follows by noting that the zero-rent condition for securities of type j implies $\mu_h(B, \hat{D}_t^i(j), \hat{D}_{t+1}(-i)) = P\Phi$. Therefore, it is optimal to issue securities of type s because rents are positive.