What's in Your Wallet? Psychophysical Biases in the Estimation of Money

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November 1, 2016

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Abstract

The denomination effect (Raghubir and Srivastava 2009) suggests that individuals are less likely to spend a specific amount of money when it is represented by a single large denomination (e.g., a $10 bill) relative to many smaller denominations (e.g., ten $1 bills). This research explores the idea that consumers are reluctant to break large bills because smaller denominations are less easy to monitor and keep track of relative to larger denominations. This increases the likelihood of spending with smaller denominations. A series of four studies suggest that in estimating the contents of one's wallet, money is more difficult to recall, and recalled less accurately the smaller the denomination, and more the number of units of any denomination. Study 1 demonstrates the biases in recall in a simple memory-based task, study 2 adds a stimulus-based task, study 3 explores the effect of motivation to be accurate and task difficulty, and study 4 links the biases in recall to actual spending. We develop a descriptive psychophysical model of how individuals estimate the money they are carrying and fit the model using the data from the four studies. Together, the findings provide evidence for a *numerosity bias*, where a larger number of units of a denomination are less accurately recalled compared to a fewer number of units; and a *denomination bias*, where smaller denominations are recalled less accurately as compared to larger denominations.

Key words: Denomination effect, Economic psychology, Subjective value of money, Memory biases, Psychophysical functions.

Money is commonly used as a proxy for utility or a measure of value for comparing different goods and services. While the importance of money in standard economic theories cannot be overemphasized, and despite money being fungible, there is increasing evidence that different representations of an equivalent amount of money affect individuals' spending (and saving) decisions. Abundant research demonstrates that the principle of descriptive invariance (Kahneman and Tversky 1979; Tversky, Sattath, and Slovic 1988), or that preferences ought to be invariant across different representations of the same objective stimuli, is commonly violated in the realm of money (e.g., Kahneman and Tversky 1979; Shafir, Diamond, and Tversky 1997; Raghubir and Srivastava 2002, 2008, 2009). Descriptive theories, mostly rooted in prospect theory (Kahneman and Tversky 1979) and related mental accounting ideas (Thaler 1985), suggest that the value of money is subjective (Raghubir 2006) and is contingent on factors such as its face value (Raghubir and Srivastava 2002; Raghubir, Morwitz, and Santana 2012), form (Raghubir and Srivastava 2008), cleanliness (Di Muro and Noseworthy 2013), and denomination (Raghubir and Srivastava 2009).

Of particular relevance to the current investigation is the denomination effect (Raghubir and Srivastava 2009) wherein individuals are less likely to spend a specific amount of money when it is represented by a single large denomination (e.g., a $10 bill) relative to multiple smaller denominations (e.g., ten $1 bills). Raghubir and Srivastava (2009) suggest that the denomination effect occurs because large denominations are psychologically less fungible and individuals use large denominations as a way to control and regulate spending. Their explanation rests on the assumption that a large denomination is easier to manage since it facilitates better monitoring and tracking of behavior (spending) than an equivalent amount in many smaller denominations.

The current research explicitly examines how individuals' monitoring and tracking capabilities are influenced by the denomination of money. Since failures in self-control are more likely to occur when individuals' monitoring and tracking capabilities are attenuated (Baumeister, Heatherton, and Tice 1994; Ragubir and Srivastava 2009), to the extent that smaller denominations are harder to monitor and track, individuals will be more susceptible to spending when money is in the form of many small denominations relative to a large denomination. The current research thus suggests a complementary, memory-based route that affects individuals' higher inclination to spend money when it is in many small denominations than a single large denomination (see Srivastava and Raghubir (2002) for a memory-based account for differences in spending using cash versus credit cards). Specifically, we examine the idea that as the number of units of a denomination increases (e.g., 2 to 12), and the denomination becomes smaller (e.g., $1 vs. $20), the more difficult it is to accurately recall how much money one has. Said differently, money in the form of many small denominations is less likely to be accurately recalled. The inability to accurately monitor and track how much money one has is an antecedent of individuals' reluctance to spend and break a large denomination.

As such, the current research not only expands our insights into the underpinnings of the denomination effect but also adds to the overall literature on the subjective value of money. We report the results of four studies that examine a memory-based account for the denomination effect. We then propose a descriptive psychophysical model for the memory for money in one's wallet and estimate model parameters that best fit the experimental data. We conclude by discussing the implications of our findings and the psychophysical model.

CONCEPTUAL BACKGROUND

The denomination effect

Mishra, Mishra, and Nayakankuppam (2006) demonstrated a "bias for the whole" wherein a large denomination (e.g., one $100 bill) led to lower spending intentions than an equivalent amount of money in smaller denominations (five $20 bills). They argue that a single large denomination allows greater fluency in processing leading to more positive affect towards the money relative to many small denominations. The positive affect leads individuals to overvalue the whole, thus making them less likely to spend it compared to an equivalent amount in smaller parts. Another perspective argues that individuals prefer clean bills to unclean bills (Di Muro and Noseworthy 2013). Since smaller denominations tend to be used more often, and are more soiled than large denominations, individuals prefer to save large denominations and spend small denominations. Both these perspectives imply that different denominations are differentially valued, such that individuals intrinsically prefer a $5 bill to five $1 bills.

Raghubir and Srivastava (2009) offered an alternative explanation which does not invoke differential intrinsic valuation as a function of denomination. Instead, based on the idea that individuals often use various tactics to impose constraints on the self (e.g., Baumeister et al. 1994), a large denomination is less likely to be spent than many small denominations as a way to control and regulate one's spending. Money in the form of a large denomination is easier to monitor and track than an equivalent amount in smaller denominations. Since self-control failures in spending are more likely to occur when money is more difficult to monitor and track, individuals are more likely to spend money when it is represented by many smaller denominations than a large denomination. Building on this idea, the current research assesses the role of differential memory for relatively small (vs. large) denominations in affecting spending decisions. To the extent that small denominations are more difficult to monitor and track, the fear of losing monitoring and tracking capabilities if one breaks a large denomination may underlie the denomination effect. As such, in exploring a memory-based route, we measure the ease and the accuracy in recalling the overall amount as well as specific denominations in one's wallet.

Memory-based estimates as an antecedent to purchase likelihood

Prior research suggests that credit card expenses are less accurately recalled than cash expenses (e.g., Soman 2001). The differential accuracy in the recall of the expenses can be attributed to the finding that the memory trace associated with credit card expenses is weaker than that associated with cash, leading to underestimation of credit card expenses (Srivastava and Raghubir 2002). Since credit card purchases are decoupled from the actual payment, there is a lower "pain of paying" associated with credit cards (Prelec and Loewenstein 1998) and, hence, a weaker memory trace of the actual expense. Given that the weaker memory trace associated with credit card versus cash expenses has been shown to increase the likelihood of spending as well as the amount spent (Srivastava and Raghubir 2002), this research examines if there are systematic differences in memory traces across different denominations, and whether these are related to differences in likelihood of spending as a function of errors in the recall of the contents of one's wallet.

Specifically, this research examines whether differences in the number of units of any denomination carried as well as the denomination of the units themselves is related to the accuracy with which an individual recalls the amount of money one is carrying. To the extent that there is a difference in memory trace for the number of units of any denomination and the denomination itself, and this influences purchase behavior, the proposed memory-based account in this research lends credence and complements Raghubir and Srivastava's (2009) explanation for the denomination effect based on self-control and regulation. Importantly, it provides an additional explanation for why individuals are more likely to spend with smaller denominations (vs. a large denomination) beyond the suggested explanations based on perceptual fluency (Mishra et al. 2006) and cleanliness (Di Muro and Noseworthy 2013).

Recall of denomination and number of units of each denomination

In estimating (or recalling) the amount of money one is carrying, an individual may either make a holistic assessment of the money one is carrying, or attempt to recall and count the number of units of each denomination one has. Errors in the estimation of the total value one is carrying can be due to the over- or underestimation of the actual number of units of each denomination (e.g., underestimation of six quarters) and/or the subjective value associated with each denomination (e.g., devaluing a quarter as less than 25¢).

Focusing on memory effects, we propose that larger denominations are easier to recall. This is because larger denominations are likely to be more salient as errors in the recall of these have larger consequences. Said differently, it is more important for an individual to keep track of the number of $20 bills than the number of 25¢ coins, if they are trying to accurately estimate the overall amount of money they are carrying. The greater motivation to be accurate in recalling larger denominations should lead to greater ease of recall of these denominations.

We further propose that as the number of units of a denomination increases, it will be more difficult to recall the actual number of units and these will be recalled less accurately. For example, it is easier to recall that one has two $1 bills relative to eight $1 bills. The level of effort involved in recalling the number of units of a denomination increases with the actual number of units. As such, the greater the effort associated with recalling the number of units of any denomination, the lower the ease. The lower ease of recall leads to less accurate recall of the actual number of units. Therefore, as the number of units increase, estimation errors are likely to increase.

Note that although conceptually distinct, there is likely to be a relationship between denomination and the number of units (or numerosity) of the denomination such that individuals may have larger number of units of smaller denominations.[1] In our analyses, we attempt to control for this potential relationship by holding one constant while examining the other. In sum, while errors in the recall of small versus large denominations are primarily due to the motivation to be accurate, errors in the recall of the number of units of a denomination are primarily due to task difficulty. In a series of four studies, we explore the existence and robustness of the systematic biases in the recall of money one carries as a function of the denomination and the number of units of a denomination.

Study 1 examines how individuals recall money in a memory-based estimation task. Study 2 examines the effect of a stimulus- versus memory-based task on estimates. Study 3 examines the robustness of the systematic biases as a function of task difficulty and motivation. Study 4 examines whether the systematic errors carry over to spending behavior. We then propose and calibrate (using data from the four studies) a descriptive psychophysical model that incorporates the biases in the recall of money as a function of denomination and the number of units of a denomination.

EMPIRICAL APPROACH

Method

Study participants. In all four studies, undergraduate business students participated for partial course credit. Study 1 had 71 participants (65 completed all measures), study 2 had 100 participants (96 completed all measures), study 3 had 132 participants (127 completed all measures), and study 4 had 67 respondents. There was no overlap across the four studies.

Measures. Participants in all studies were asked to estimate the overall value of their wallet without looking at their wallets (V^e, where V denotes overall value and e denotes

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[1]The correlation between n and D is -.24, -.26, -.32, and -.38 for studies 1-4 respectively, indicating that there are fewer numbers (n) of larger denominations (D).

estimate). They were then asked to estimate the overall content of their wallet decomposed by denomination (N^e_i, where N denotes number of units, e denotes estimate, and i denotes ten different denominations = \$100, \$50, \$20, \$10, \$5, \$1, quarter, dime, nickel, and penny). Subsequently, participants rated how easy it was to recall each denomination (1 = easy and 3 = hard). Finally, participants were asked to count the actual contents of their wallet and record the number of units by denomination ($N^a_{i,}$, where a denotes actual). Thus, the actual overall value of the wallet (V^a) was easily computed.

The dependent measures included errors in the estimation of the total overall value ($V^a - V^e$), disaggregated wallet value ($V^a - \Sigma N^e_i D_i$), number of units of each denomination ($N^a_i - N^e_i$ for each denomination, i; also computed as a function of how many units, N^a, of that denomination there actually were), and ease of recall. Additional manipulations, measures, and tasks introduced in studies 2, 3, and 4 are described along with those studies.

Data analysis

The data was analyzed by ignoring the \$100 and \$50 bills as very few participants had these denominations (study 1 = 0 and 1, study 2 = 1 and 0, study 3 = 4 and 2, study 4 = 3 and 0 for \$100 and \$50 respectively). The results for ease and accuracy are stronger with the inclusion of these data as majority of the individuals accurately reported that they had no \$100 and \$50 bills and this was easy to recall.

Ease of recall. To test whether larger denominations are easier to recall, we first report the repeated-measures ANOVA on the ease of recall of the eight denominations, with denomination as the within-subjects factor. We report linear and quadratic contrasts to test whether ease increases with denomination, and then tapers off. Treating each participant's ease of recall and the number of units they actually had as a data point for each denomination, we then report a regression on the ease of recall as a function of the number of units carried and denomination to test whether difficulty increases as the number of units carried increase, controlling for denomination.

Overall estimation accuracy. We report paired t-tests for errors in the estimation of the total aggregate value ($V^a - V^e$) and the disaggregated wallet value ($V^a - \Sigma N^e_i D_i$).

Estimation inaccuracy as a function of denomination. A 2 (actual/estimate) x 8 (denominations) repeated measures ANOVA is reported on the number of units of each denomination. Descriptive statistics are reported in the first table of each study.

Additionally, the last two columns of the table with the descriptive statistics, report the results of two regression models for each estimate as a function of the actual number of units, separately for each denomination: a linear model ($N^e = \beta N^a$) and a power formulation ($N^e = N^{a\beta}$). $\beta < 1$ implies underestimation in both models. As $\beta \rightarrow 1$ in either model, estimates are accurate. As the explained variance (R^2_a) between the two models diverges, either one or the other model represents a better representation of how people recall their money. We expect $\beta \rightarrow 1$ for larger denominations in both models and the differences in the R^2_a between the two models to reduce as denominations increase.

Estimation inaccuracy as a function of numerosity. To test the idea that as the number of units of any given denomination increases, the greater the error in estimation, models 1-4 report regression analyses where the dependent variable is the error in estimation (N^a - N^e). Thus, positive (negative) numbers reflected underestimation (overestimation). The independent variable is the number of units (N^a) for any denomination including an intercept (model 1) as well as excluding the intercept (model 2). Model 3 includes a quadratic term (N^{a2}) to account for the possibility that as the number of units for any denomination increase, the errors become larger. Model 4 incorporates denomination to test for the effect of number of units after controlling for denomination. Models 5-8 use the same set of independent variables, but instead use the absolute value of the errors as the dependent variable to test for inaccuracy (rather than under- or over-estimation) as a function of the number of units and denomination. The eight regression models are presented in a second table in each study.

Psychophysical model fitting. A psychophysical model is then developed and the parameters are estimated using the data from the studies to capture the systematic biases in the recall of different denominations and the overall estimate of value. Three models are reported in the paper, and three additional formulations are reported in the Appendix. The model is intended to be descriptive rather than predictive, aiming to empirically capture the subjectivity in value assessment.

STUDY 1: WHAT'S IN YOUR WALLET?

Ease of recall

A repeated-measures ANOVA on ease of recall of the eight denominations revealed a main effect of denomination ($F(7, 420) = 18.73$, $p < .001$, $\eta^2 = .23$). The linear ($F(1, 60) = 32.58$, $p < .001$, $\eta^2 = .35$) and quadratic ($F(1, 60) = 21.40$, $p < .001$, $\eta^2 = .26$) coefficients were both significant, suggesting that difficulty increases with smaller denominations and then tapers off.

A regression on the ease of recalling the number of units of each denomination on the number of units the person had and the denomination revealed a significant effect of the number of units ($\beta = .055$), indicating that recall difficulty increased as number of units increased, and denomination ($\beta = -.041$), indicating that smaller denominations were more difficult to recall (t's = 3.62 and -8.26, p's < .001 for both, $R^2_a = .144$, $F(2, 512) = 44.07$, $p < .001$).

Table 1 displays the summary statistics of the initial estimates, actual amount, difference (estimate less actual), and ease of recall for the eight denominations.

Table 1 - Summary Statistics by Denomination (Study 1)

Denomination	N^e Estimate		N^a Actual		Ease of recall		Model β and (R^2_a) (separately for each denomination)	
(USD)	Mean	SD	Mean	SD	Mean	SD	$N^e = N^a$	Power Function[2]: Ln (N^e) = Ln(N^a)
20	0.97	2.82	1.04	2.89	1.08	0.27	.96* (.97)	.98* (.95)
10	0.34	0.61	0.25	0.58	1.26	0.48	.96* (.79)	.89* (.87)
5	0.43	0.65	0.49	0.75	1.43	0.67	.60* (.47)	.90* (.77)
1	2.03	2.22	2.04	2.70	1.66	0.73	.46* (.25)	.44* (.19)
0.25	1.25	1.62	1.70	2.72	1.79	0.84	.52* (.69)	.85* (.78)
0.10	0.82	1.65	1.10	1.76	1.90	0.91	.72* (.66)	.88* (.64)

[2]To fit a power function separately for each denomination, for each study, we added .00001 to all estimates and actual counts to allow us to take logs on both sides of the equation. The equation is Ln(estimate) = β Ln (Actual), where β is the exponent associated with the function Est = Act^{β} or $N^e = N^{a\beta}$. The dependent variable is Estimate for the first model and Ln(Est) for the power function. Cells provide beta-coefficients of the model and the R^2_a in parentheses. * = *p < .05*.

0.05	0.37	0.83	0.63	1.09	1.90	0.93	.42* (.35)	.92* (.69)
0.01	1.25	2.89	1.59	2.64	1.85	0.95	.11 (.00)	.54* (.23)

Overall estimation accuracy

The mean initial estimate of the value of the wallet (*M = $28.28, SD* = 60.26) was significantly lower than the decomposed estimate of value (*M* = $31.89, *SD* = 60.74) as well as the actual overall value of the wallet (*M* = $32.21, *SD* = 61.86), with the latter two not significantly different from each other. Decomposition is a common method to improve recall accuracy (Srivastava and Raghubir 2002).

Estimation inaccuracy as a function of denomination

A 2x8 repeated-measures ANOVA on the measure (estimate vs. actual counts) of the eight denominations revealed a main effect of denomination ($F(7, 462) = 12.28$, $p < .001$, η^2 = .16) as well as a main effect of measure ($F(1, 66) = 4.72$, $p < .001$, $\eta^2 = .06$), but there was no significant interaction ($F < 1$).[3] The non-significant interaction may suggest that the smaller denominations are more numerous (ignoring $100 and $50, $r = -.10$, $p < .05$). The next analyses, thus, report the errors as a function of the number of units.

The last two columns of table 1 show the β coefficients and adjusted R^2s associated with the regression models for the estimate as a function of the actual number of units separately for each denomination. The linear model ($N^e = \beta N^a$) can be compared with the power formulation ($N^e = N^{a\beta}$) to see whether the difference in the adjusted R^2s is greater for smaller (e.g., 1¢) as compared to larger ($20) denominations. This implies that the power formulation that captures underestimation for coefficients < 1, is particularly useful for smaller denominations, whereas for larger denominations, the coefficient → 1.

Estimation inaccuracy as a function of number of units

As shown in table 2, for all eight regression model specifications, the coefficient for the

[3] A 10 (denominations: $0.01.. $100) x 2 (estimate/actual) ANOVA reveals a significant interaction ($F(9, 576) = 2.09$, $p < 0.05$), while there is also a main effect of estimation ($F(1, 67)=10.08$, $p < 0.01$). The interaction is likely driven by the greater accuracy is responding about the estimated numbers of $100 and $50 bills.

actual number of units (N^a) was positive and significant, confirming greater underestimation (models 1-4) and greater inaccuracy (models 5-8) as the number of units increases. The negative coefficients on the quadratic terms (models 3 and 7) suggest that the inaccuracy in underestimation tapers off as the number of units increase. Although the numerosity bias is supported at the aggregate level, models 4 and 8 shows that the numerosity effects are robust even after controlling for denomination. As expected, model 8 revealed that the estimation errors are greater for smaller denominations.

Table 2: Different Regression Models Test for the
Numerosity Effect on Estimation Accuracy (Study 1)

Dep. Var	**(ACT-EST)**				**|ACT-EST|**			
Model	1	2	3	4	5	6	7	8
Constant	-0.44*	-	-0.56*	-0.61*	0.38*	-	0.28*	.55*
N^a	0.55*	0.47*	0.77*	.78*	0.49*	0.56*	0.68*	.63*
N^{a2}			-0.02*	-.02*			-0.02*	-.01*
D				.01				-.05*
F	F(1,554) 283.26*	F(1,555) 248.70*	F(2,553) 154.02*	F(3,552) 102.98*	F(1,554) 273.48*	F(1,555) 435.08*	F(2,553) 148.64*	F(3,552) 114.13*
R^2_a	0.34	0.31	0.35	0.35	0.33	0.44	0.35	.38

Observations = 71 participants x 8 denominations = 568, * = $p < 0.001$

Discussion

In a memory-based wallet estimation task, study 1 provides preliminary evidence for the idea that individuals subjectively estimate the overall value of their wallet and that these estimates are more accurate with fewer number of units and higher denominations. The data also indicate that smaller denominations are underestimated ($N^a < N^e$) and that a power formulation ($N^a = N^{e\beta}$) can better explain people's estimates of their wallet than a linear formulation ($N^a = \beta N^e$). Given that a memory-based task involves recalling each denomination and the number of units of each denomination, the likelihood of errors such as underestimating small denominations as well as number of units is likely to be higher than a stimulus-based task (Menon 1993). Thus, study 2 not only attempts to replicate the effects of study 1 but also examines whether the biases are stronger for a memory-based task than a stimulus-based task.

STUDY 2: HOW MANY COINS ARE IN THE BAG?

Procedure

The experiment consisted of eight sessions where participants conducted both a memory-based task (same steps as in study 1) and a stimulus-based task, with the order of the two tasks counterbalanced. In the stimulus-based task, participants estimated the contents of four bags of coins (each containing quarters, dimes, nickels, and pennies). To examine whether the estimates varied with the number of coins, participants were assigned randomly to one of eight sessions where the number of coins per bag was varied across sessions (ranging from 110 quarters/94 dimes/67nickels/326 pennies to 96 quarters/80 dimes/53 nickels/312 pennies). Note the content of the bags was thus reduced by two coins per denomination or 82¢ from session to session.

Memory-based ease of recall

A repeated-measures ANOVA on ease of recall of the eight denominations revealed a main effect of denomination ($F(7, 665) = 26.65$, $p < .001$, $\eta^2 = .22$). The linear ($F(1, 95) = 48.02$, $p < .001$, $\eta^2 = .34$), quadratic ($F(1, 95) = 14.88$, $p < .001$, $\eta^2 = .14$), and cubic ($F(1, 95) = 5.38$, $p < .05$, $\eta^2 = .05$) contrasts were significant, suggesting that difficulty increases with smaller denominations and then tapers off.

A regression on the ease of recalling the number of units of each denomination on the actual number of units and the denomination revealed a significant effect of number of units ($\beta = .09$), indicating that recall difficulty increased as number of units increased, as well as denomination ($\beta = -.032$), indicating that smaller denominations were more difficult to recall (t's = 10.22 and -7.66, p's < .001 for both, $R^2_a = .213$, $F(2, 765) = 104.58$, $p < .001$).

Table 3 displays the summary statistics of the initial estimates, actual amount, and ease of recall for the eight denominations, and regression estimates for each denomination separately.

Table 3 - Summary Statistics by Denomination (Study 2)

Denomination	N^e Estimate		N^a Actual		Ease of recall		Model β and (R^2_a) (separately for each denomination)	
(USD)	Mean	SD	Mean	SD	Mean	SD	$N^e = N^a$	$Ln(N^e) = Ln(N^a)$
20	.67	1.05	.70	1.11	1.25	0.50	.92* (.91)	.95** (.91)
10	.34	.56	.39	.62	1.33	0.59	.61** (.45)	.87** (.75)
5	.65	.86	.65	.92	1.56	0.71	.71** (.54)	.83** (.74)
1	2.14	1.92	2.30	2.64	1.77	0.81	.66** (.65)	.75** (.62)
0.25	1.69	2.27	2.58**	3.60	1.88	0.83	.51** (.64)	.84** (.76)
0.10	1.46	2.20	1.76*	2.75	2.06	0.93	.68** (.70)	.88** (.75)
0.05	1.29	2.01	1.10	1.59	2.10	0.95	.87** (.50)	.81** (.63)
0.01	2.69	4.00	4.25**	5.99	2.06	0.97	.55** (.71)	.86** (.62)

* = Estimate is different from actual using a paired t-test, $p < .10$, ** = $p < .05$. Regression is significant.

Memory-based overall estimation accuracy

The mean actual overall value of the wallet was $24.29 ($SD$ = 25.17). This value was not significantly different from the estimated overall value or the disaggregated overall estimate (M's = $24.07 and $24.57, both p's > .50).

Memory-based estimation inaccuracy as a function of denomination

A 2x8 repeated-measures ANOVA on the measure (estimate vs. actual) of the eight denominations revealed main effects of denomination ($F(7, 658) = 22.13$, $p < .001$, $\eta^2 = .19$), measure ($F(1, 94) = 13.11$, $p < .001$, $\eta^2 = .12$), and their interaction ($F(7, 658) = 7.97$, $p < .001$, $\eta^2 = .08$), which is consistent with the fact that estimation accuracy is greater for larger denominations (see table 3). Paired t-tests of mean differences in the actual and estimated number of units separately for each denomination show that the four bills were not significantly different, but three of the four coins were significantly underestimated. The regression models show that the explained R^2 is highest for the $20 bill, and that this is no different from the linear formulation.

Memory-based estimation inaccuracy as a function of number of units

As shown in table 4, study 1 results replicate across all eight regression models. For all models, the coefficient for the number of units is positive (and significant) confirming greater underestimation (models 1-4) and greater inaccuracy (models 5-8) as the number of units increases. The small, but significant, negative coefficients on the quadratic terms (models 3 and 7) suggest that the inaccuracy in underestimation tapers off as the number of units increases. As in study 1, models 4 and 8 show that the numerosity effects are robust to denomination, with model 8 demonstrating that estimation errors are greater for smaller denominations. The numerosity bias is, thus, supported at the aggregate level.

Table 4: Different Regression Models Test for the
Numerosity Effect on Estimation Accuracy (Study 2)

Dep. Var	**(ACT-EST)**				**|ACT-EST|**			
Model	1	2	3	4	5	6	7	8
Constant	-0.45*	-	-0.36*	-0.45*	0.11*	-	0.17*	.32*
N^a	0.47*	0.41*	0.36*	.37*	0.47*	0.48*	0.40*	.38*
N^{a2}			-0.01*	-.01*			-0.005*	-.005*
D				.02				-.03*
F	F(1,766)	F(1,767)	F(2,765)	F(3,764)	F(1,766)	F(1,767)	F(2,765)	F(3,764)
	757.80*	697.25*	391.92*	263.47*	1142.19*	1567.80*	148.64*	399.58*
R^2_a	0.50	0.48	0.50	0.51	0.60	0.67	0.35	.61

N = 96 participants x 8 observations each = 768, * = $p < 0.05$)

Memory-based and stimulus-based estimates

A 2 (Task: Wallet Recall/Stimulus) x 2 (Actual/Estimate) x 4 (Denomination: Quarters, dimes, nickels and pennies) repeated measures ANOVA was conducted to examine whether the number of units are also underestimated in a stimulus-based task. The ANOVA revealed main effects of task ($F(1, 94) = 1704.44$, $p < .01$, $\eta^2 = .95$) and denomination ($F(3, 282) = 593.88$, $p < .001$, $\eta^2 = .86$), with the latter merely reflecting the different number of coins of different denominations. The two-way interaction between denomination and task as well as denomination and measure were significant ($F(3, 282) = 564.15$ and 6.11, p's $< .01$, η^2's $= .86$ and $.06$, respectively), which were qualified by a significant three-way interaction ($F(3, 282) = 5.49$, $p < .001$, $\eta^2 = .055$), suggesting that the contingent effect of denomination on errors in the estimates was different across the memory- and stimulus-based tasks.

Figure 1: Estimates of Coins in Wallet and Stimulus-Based Tasks (Study 2)

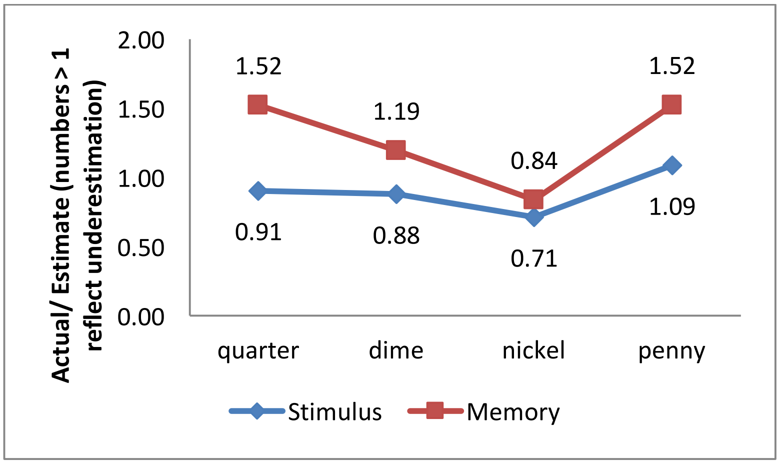


Figure 1 displays the interaction graphically using the measure of Actual/Estimate to show whether coins were over- (N < 1) or under-estimated (N > 1). The results suggest that there is underestimation for three of the four coins in the memory-based task but there is over-estimation in three of the four coins in the stimulus-based task.

Discussion

Study 2 largely replicated the results of study 1 in the memory-based wallet estimation task. It showed that as the number of units increase, and the denomination gets smaller, recall is more difficult and more inaccurate. Additionally, it examined estimation accuracy for four coins (quarters, dimes, nickels, and pennies) in a stimulus-based task and compared this to the estimation accuracy in the memory-based task. Although the two tasks are not comparable given that the number of coins differed across the two tasks, the findings are consistent with the idea that in a memory-based task, people underestimate the relatively smaller denominations (coins) they are carrying. As argued earlier, this could be due to the lower motivation and greater task difficulty associated with recalling more numerous units and smaller denominations. Whereas studies 1 and 2 use denomination and the number of units of every denomination to examine the effects of task difficulty and motivation, study 3 examines whether these variables can also be manipulated exogenously and moderate the effects of denomination and numerosity.

STUDY 3: ROLE OF MOTIVATION AND TASK DIFFICULTY

Procedure

Participants completed a memory-based task (as in study 1) among several other memory related tasks in four sessions. After completing several other non-memory related filler tasks, participants were asked to count the contents of their wallet. The study used a 2x2 design where motivation and task difficulty were manipulated. In the "*high motivation*" condition, participants were instructed that it was extremely important to be accurate in the recall of each item and that they would have to justify their responses at the end. They were also told that they would be entered in a sweepstake for a $100 cash reward based on the accuracy of their responses. In the "*low motivation*" condition, participants were told that "it is not at all important" to be accurate and that they would be entered in the sweepstakes based on study completion.

The time that participants had to complete the questionnaire in each session was varied to manipulate task difficulty. In the "*low task difficulty*" condition, there was no time limit for the task. In the "*high task difficulty*" condition, participants were given five minutes to complete the memory part of the questionnaire.

As a check for the manipulations, motivation and task difficulty were each measured by a single nine-point scale, where higher numbers reflect greater motivation and task difficulty. The task difficulty item also served as a measure of the difficulty of recalling different denominations and different numbers of units of each denomination.

Motivation

A 2 (motivation) x 2 (task difficulty) ANOVA on the motivation measure revealed no significant effects, suggesting that the manipulation was not successful. Nonetheless, we retain motivation as a factor in the analyses reported here.

Ease of recall

A 2 (motivation) x 2 (task difficulty) x 8 (denominations: \$20, \$10, \$5, \$1, 25¢, 10¢, 5¢, 1¢) repeated-measures ANOVA on ease of recall of the eight denominations revealed significant main effects of task difficulty ($F(1, 123) = 4.05, p < .05, \eta^2 = .032$) and denomination ($F(7, 861) = 22.31, p < .01, \eta^2 = .154$), which were qualified by a significant two-way interaction between task difficulty and motivation ($F(1, 123) = 4.99, p < .05, \eta^2 = .039$) and a marginal interaction between task difficulty and denomination (($F(7, 861) = 1.956, p = .058, \eta^2 = .016$). The main effect of denomination suggests that larger denominations were easier to recall (table 5). The linear ($F(1, 123) = 35.73, p < .001, \eta^2 = .22$) and quadratic ($F(1, 123) = 14.88, p < .001, \eta^2 = .08$) contrasts were significant, suggesting that difficulty increases as the denominations get smaller and then tapers off.

The motivation by task difficulty interaction revealed that when motivation was higher, the task was more difficult under limited time pressure ($M = 3.76$) as compared to unlimited time ($M = 2.51$), but task difficulty was not different in the low motivation condition (*M*'s = 2.68 vs. 2.75 for low vs. high task difficulty). Although task difficulty was higher for higher ($M = 3.22$) versus low task difficulty ($M = 2.63$), task difficulty was contingent on the level of motivation.

The denomination by task difficulty interaction revealed that limited time exacerbated the difficulty associated with recalling smaller denominations ($M_{\text{high task difficulty}}$ for \$20 = 1.64, \$10 = 2.20, \$5 = 2.62, \$1 = 3.65, 25¢ = 3.30, 10¢ = 3.95, 5¢ = 4.19, and 1¢ = 4.24) as compared to when participants had unlimited time ($M_{\text{low task difficulty}}$ for \$20 = 1.68, \$10 = 1.88, \$5 = 2.44, \$1 = 3.41, 25¢ = 2.35, 10¢ = 2.86, 5¢ = 3.03, 1¢ = 3.38).

A regression on the ease of recalling the number of units of each denomination on the actual number of units, denomination, task difficulty (1 = difficult), and motivation (1= high), revealed a significant effect of the actual number of units ($\beta = .23$), indicating that recall difficulty increased as the number of units increased, as well as the denomination ($\beta = -.08$), indicating that smaller denominations were more difficult to recall (*t*'s = 10.80 and -7.24, *p*'s < .001 for both, $R^2_a = .175$, $F(4, 1003) = 54.38, p < .001$). The effect of task difficulty was significant and in the expected direction while the effect of motivation was marginal (βs = .40

and .28, *t*'s = 2.68 and 1.90, $p < .05$ and $p = .058$, respectively). Incorporating the two- and three-way interaction terms with the between-subjects factors revealed that the results were robust (R^2_a = .227, $F(10, 997) = 30.50$, $p < .001$). The task difficulty factor interacted with both number of units ($\beta = -.48$, $t = -6.32$, $p < .01$) as well as denomination ($\beta = -.08$, $t = -2.56$, $p < .01$), suggesting that greater task difficulty exacerbated both effects.

Table 5 displays the summary statistics of the initial estimates, actual amount, and ease of recall for the eight denominations, as well as the estimates of the regressions.

Table 5 - Summary Statistics by Denomination (Study 3)

Denomination	N^e Estimate		N^a Actual		Ease of recall		Model β and (R^2_a) (separately for each denomination)	
(USD)	Mean	SD	Mean	SD	Mean	SD	$N^e = N^a$	Ln (N^e) = Ln(N^a)
20	.78	1.30	.81	1.29	1.66	1.27	.97** (.93)	.92** (.86)
10	.19	.43	.20	.44	2.03	1.72	.61** (.45)	.65** (.44)
5	.71	.94	.65	.91	2.52	1.99	.72** (.53)	.61** (.36)
1	2.13	2.23	2.00	2.12	3.53	2.58	.64** (.38)	.71** (.53)
0.25	1.09	1.94	1.28	2.52	2.83	2.51	.58** (.56	.78** (.62)
0.10	1.11	2.08	1.41*	2.80	3.37	2.84	.50** (.43)	.64** (.44)
0.05	1.21	2.10	0.88**	1.59	3.57	2.95	.71** (.44)	.73** (.47)
0.01	2.74	5.13	3.68*	5.99	3.80	3.15	.46** (.93)	.85** (.74)

* = Estimate is different from actual using a paired t-test, Regression is significant, $p < .10$, ** = $p < .05$.

Overall estimation accuracy

The mean actual value of the wallet was $28.74 (*SD* = 40.07). The actual value was not significantly different from the estimated overall value (*M* = $27.61, *SD* = 39.65) or the disaggregated overall estimate (M = $28.65, SD = 39.61; *p*'s > .10).

Estimation inaccuracy as a function of denomination

A 2 (task difficulty) x 2 (motivation) x 2 (measure: estimate/actual) x 8 (denominations) repeated-measures ANOVA revealed main effects of denomination ($F(7, 861) = 21.41, p < .001, \eta^2 = .15$) and task difficulty ($F(1, 123) = 6.51, p < .05, \eta^2 = .05$), as well as their two-way interaction ($F(7, 861) = 5.37, p < .001, \eta^2 = .04$). Effects including the key measure factor were the two-way interactions with task difficulty ($F(1, 123) = 5.92, p < .05, \eta^2 = .05$) and denomination ($F(7, 861) = 3.60, p < .01, \eta^2 = .03$), the three-way interactions with motivation and task difficulty ($F(1, 123) = 3.95, p < .05, \eta^2 = .03$), denomination and task difficulty ($F(7, 861) = 4.25, p < .01, \eta^2 = .03$), and the four-way interaction ($F(7, 861) = 5.02, p < .01, \eta^2 = .04$).

The measure by task difficulty interaction shows that when the task is not difficult, people overestimate (*M*'s = 1.06 vs. .97 for estimate and actual respectively, $p = .17$), whereas as the task becomes more difficult, recall inaccuracy increases and there is evidence of underestimation (*M*'s = 1.47 vs. 1.80 for estimate and actual respectively, $p < .05$). As in the earlier studies, the measure by denomination interaction shows that smaller denominations are more inaccurately recalled (*M*'s for estimate and actual for the $20, $10, $5 and $1= .77 vs. .80, .20 vs. .20, .73 vs. .65, 2.15 vs. 2.01; *M*'s for quarters, dimes, nickels, and pennies = 1.09 vs. 1.29, 1.13 vs. 1.44, 1.23 vs. .90, and 2.84 vs. 3.78).

The three-way interaction between measure, task difficulty and denomination shows that greater task difficulty exacerbates estimation inaccuracy for smaller denominations (see figure WA1 in the Web Appendix).

The three-way interaction depicted between estimate, motivation, and task difficulty shows that estimates are more inaccurate when the task is difficult, especially when there is higher motivation (see figure WA2 in the Web Appendix).

The four-way interaction suggests that the estimates are more inaccurate for coins versus bills (see figure WA3 in the Web Appendix). Given the significant two-way interaction between measure and denomination, table 5 reports individual pairwise differences between the estimate and actual for each of the eight denominations. Pennies and dimes are significantly underestimated, quarters are directionally underestimated (the nickel is overestimated), whereas estimates of the bills do not differ from the actual number of bills carried.

Estimation inaccuracy as a function of number of units

As shown in table 6, the eight regression models suggest that the results of studies 1 and 2 replicate. The coefficient for the number of units is positive and significant for all models, after controlling for denomination (models 4 and 8). There is also greater inaccuracy as denominations become smaller, controlling for number of units (model 8).

Table 6: Coefficients of Different Regression Models
Test for the Numerosity Effect on Estimation Accuracy (Study 3)

Dep. Var	**(ACT-EST)**				**|ACT-EST|**			
Model	1	2	3	4	5	6	7	8
Constant	-0.55*	-	-0.29*	-0.37*	0.06*	-	0.20*	.34*
N^a	0.48*	0.43*	0.18*	.18*	0.54*	0.54*	0.37*	.35*
N^{a2}			0.01*	.01*			0.006*	.006*
D				$.02^{\dagger}$				-.03*
F	F(1, 1014) 841.67*	F(1, 1015) 710.30*	F(2, 1013) 551.00*	F(3, 1012) 369.62*	F(1, 1014) 1691.42*	F(1, 1015) 1999.15*	F(2, 1013) 931.06*	F(3, 1012) 637.14*
R^2_a	0.45	0.41	0.52	0.52	0.625	0.66	0.65	.65

N = 127 participants x 8 observations each = 1016, * = $p < 0.05$, † = $p = .051$)

Discussion

Study 3 replicated the primary result that smaller denominations, and larger number of units, are more difficult to recall and recalled with greater inaccuracy, particularly as the task becomes more difficult. Thus, despite the task difficulty increasing endogenously with the number of units, exogenous contextual factors that affect task difficulty may have interactive (rather than mere additive) effects. Interestingly, the task difficulty manipulation also affected the denomination effect for both ease of recall as well as inaccuracy of estimates, with greater task difficulty making it increasingly difficult to recall smaller denominations, which were recalled with greater inaccuracy. This could be a reflection of the inverse relationship between denomination and the number of units carried, or be suggestive of the fact that larger denominations are less effortful to recall for reasons beyond motivation. Regardless, it is premature to draw a conclusion based solely on these results without replicating the effects of task difficulty on denomination, and controlling for the effects of number of units. The failure of the motivation manipulation check also makes it difficult to draw conclusions about whether the effects of motivation will also be interactive (or merely additive), further necessitating the need for replication.

In sum, study 3 showed that task difficulty makes it more difficult to recall smaller denominations, and larger numbers of these units, which leads to greater inaccuracy. Study 4 next examines the extent to which the inaccuracies in recall have a downstream consequence on purchase behavior, as per the denomination effect.

STUDY 4: BEHAVIORAL CONSEQUENCES OF UNDERESTIMATION

Procedure

Participants were given a $1 bill at the beginning of the study and were then asked to complete the wallet estimation task as in the earlier studies. Upon completion of the experimental session which included other unrelated experiments, participants were asked to come to the front of the room, one at a time, to sign out. At this stage they were given a surprise choice task where participants could exchange their $1 for a Starburst. The decision to retain their dollar or buy the Starburst was discreetly noted and the participant was signed out. The decision to buy or not to buy was the dependent variable, which was analyzed as a function of the difference in the estimate versus the actual number of units for each of the eight denominations.

Ease of recall

A repeated-measures ANOVA on ease of recall of the eight denominations revealed a significant effect of denomination ($F(7, 413) = 9.80, p < .05, \eta^2 = .14$). The linear ($F(1, 59) =$ 16,16, $p < .001, \eta^2 = .21$) and quadratic ($F(1, 59) = 13.85, p < .001, \eta^2 = .19$) contrasts were significant, suggesting that difficulty increases as denomination gets smaller and then tapers off.

A regression on the ease of recalling the number of units of each denomination on the actual number of units and the denomination of the instruments revealed a significant effect of number of units (β = .13) indicating recall difficulty increased as number of units increased, as well as denomination (β = -.006), indicating that smaller denominations were more difficult to recall (t's = 8.11, and -5.71, p's < .001 for both, R^2_a = .179, $F(2, 607)$ = 66.02, p < .001).

Overall estimation accuracy

Dropping one outlier for total estimated value ($9600), the mean total estimate is $31.56 ($SD$ = 35.28), with no means being significantly different from each other.

Table 7 shows summary statistics and regression results.

Table 7 - Summary Statistics by Denomination (Study 4)

Denomination	N^e Estimate		N^a Actual		Ease of recall		Model β and (R^2_a) (separately for each denomination)	
(USD)	Mean	Sd	Mean	sd	Mean	sd	$N^e = N^a$	Ln (N^e) = Ln(N^a)
20	.91	1.20	.89	1.14	1.22	.55	1.03* (.94)	.98** (.95)
10	.33	.50	.27	.54	1.39	.64	.61** (.45)	.74** (.45)
5	.81	1.05	.76	.99	1.54	.71	.59** (.82)	.77** (.58)
1	2.85	2.26	3.01	2.30	1.86	.85	.66** (.45)	.82** (.55)
0.25	1.40	2.10	1.85**	2.93	1.84	.94	.53** (.73)	.76** (.60)
0.10	.94	2.15	.78	1.35	1.92	.95	.68** (.51)	.54** (.29)
0.05	.97	2.22	.83	2.00	1.90	.94	.34** (.23)	.56** (.29)
0.01	1.93	4.06	2.11	3.41	1.80	.96	.74** (.53)	.71** (.54)

* = Estimate is different from actual using a paired t-test, p < .10, ** = p < .05. Regression is significant.

Estimation inaccuracy as a function of denomination

A 2 (measure: estimate/actual) x 8 (denomination) repeated-measures ANOVA on the measure (estimate vs. actual counts) of the eight denominations, revealed a main effect of denomination ($F(7, 448)$ = 17.37, p < .001, η^2 = .21), a marginal effect of measure ($F(1, 64)$ = 2.84, p = .097, η^2 = .04), and a marginal interaction (F(7, 448) = 1.78, p = .088, η^2 = .03) that captures the fact that only quarters were significantly underestimated. The linear and quadratic contrasts of the denomination factor were both significant reflecting that accuracy increases with denomination and tapers off as in earlier studies ($F(1, 64) = 4.16$ and 4.05, p's < .05).

Estimation inaccuracy as a function of number of units

Table 8 shows the regression results. Across models, the results show that the numerosity coefficient or its squared term is significant, suggesting that the greater the number of units, the greater the underestimation and overall estimation error. This is true controlling for denomination, which itself is negatively related to estimation accuracy: smaller denomination are more likely to be underestimated.

Table 8: Coefficients of Different Regression Models
Test for the Numerosity Effect on Estimation Accuracy (Study 4)

Dep. Var	**(ACT-EST)**				**|ACT-EST|**			
Model	1	2	3	4	5	6	7	8
Constant	-0.29*	-	-0.15*	-0.16ǂ	0.13*	-	0.18*	.30*
N^a	0.33*	0.27*	0.08	.08	0.38*	0.40*	0.29*	.28*
N^{a2}			0.03*	.03*			0.01*	.01*
D				.001				-.02*
F	F(1,530)	F(1,531)	F(2,529)	F(3,528)	F(1,530)	F(1,531)	F(2,529)	F(3,528)
	175.15*	154.10*	105.96*	70.51*	333.62*	507.09*	170.25*	120.36*
R^2_a	0.25	0.22	0.28	0.28	0.38	0.49	0.39	.40

N = 67 participants x 8 observations each = 536, * = $p < 0.05$, ǂ = p = .053)

Purchase decision

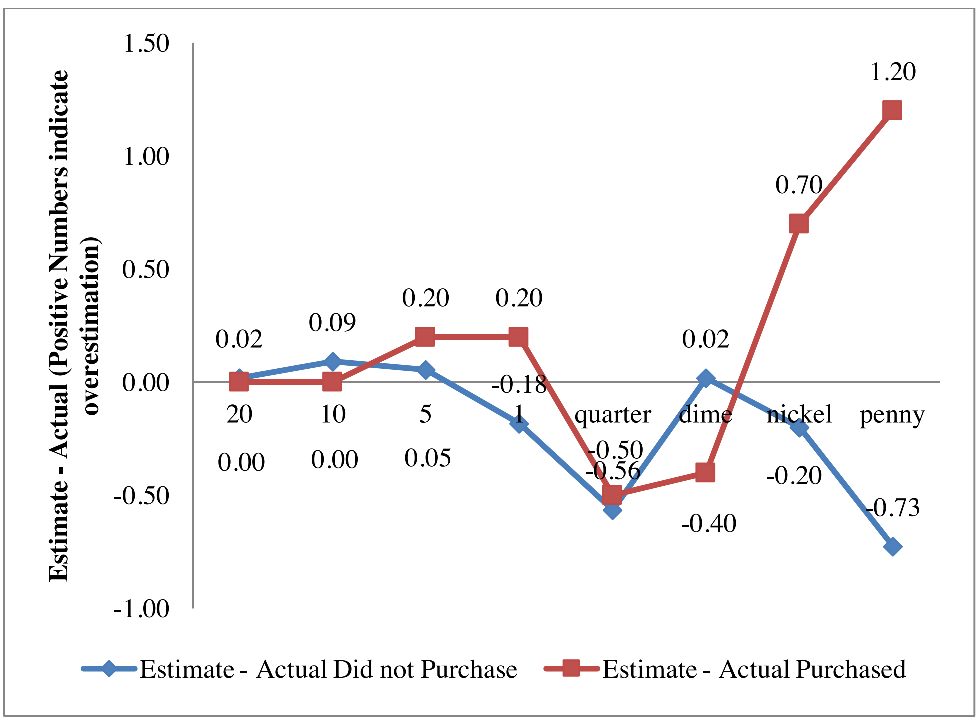
A logit analysis of the likelihood of purchase as a function of estimation errors revealed that participants were more likely to purchase as the difference between actual and estimated number of units decreased. Figure 2 shows that those who purchased the candy were more likely to have previously overestimated the contents of their wallet.

Table 9: Study 4 – Analysis of Likelihood of purchase

Dep. Var	Logit Analysis of the BUY (0/1) Decision		
	Model 1	Model 2	Model 3
Model	Actual under estimation	Model 1 with no constant	Model 1 only for coins
Constant	-1.55***	-	-1.59***
Actual – Estimate	β = -0.13*	β = -0.18***	β = -1.83*
LR-$\chi^2(1)$	3.02*	6.74***	3.84*
LL	-306.5	-457.7	-117.0
Num. Obs.	666	666	264
Pseudo R^2	0.005	0.009	0.016

* $p < .10$, *** $p < .001$

Figure 2: Overestimation is Associated with Greater Likelihood of Purchase (Study 4)



Discussion

Study 4 provides evidence consistent with the idea that the errors in the recall of the contents of one's wallet can have downstream consequences for purchase decisions, as people are more likely to spend when they have overestimated the contents of different denominations in their wallet. This evidence is the critical element that links the memory-based account proposed as an antecedent of the behavioral denomination effect. The argument is that people are less willing to spend using a larger denomination, because doing so would generate smaller denominations in change, and this change would be less easy to track. Using measures of actual estimation accuracy, study 4 results showed that those who overestimated the contents of their wallet in the earlier estimation task were more likely to spend their $1 on the Starburst. Note that this result should also be replicated to better assess what other exogenous factors contribute to recall inaccuracy (see General Discussion), and, ideally, by manipulating, rather than measuring recall inaccuracy.

Next, in tying the results of the four studies, we propose a descriptive psychophysical model that attempts to capture the errors related to the number of units and the denomination of the units using a parsimonious formulation.

A PSYCHOPHYSICAL MODEL OF WALLET ESTIMATION

The actual value of money available in a wallet (V^a) is an aggregation of the number of units of a specific denomination (i) multiplied by the denomination. A denomination (i) is a unit of currency, such as $5 and $1 bills, as well as 25¢ coins.

(1) $V^a = \sum_{i = 1, k} n_i D_i$

In equation 1,

n = number of units of a particular denomination,

D = denomination or face value, across the range of available denominations, i.

The subjective value, U(*V*), associated with this money is contingent on:

1. Whether the number of units, n, of each denomination are accurately recalled, and/or
2. Whether denominations, D, of different face value are equally valued.

The extent to which denominations are valued differentially as a function of its face value has been the focus of much attention. Prior research based on direct attitudinal measures (Mishra et al. 2006) and indirect measures of spending (Di Muro and Noseworthy 2013; Raghubir and Srivastava 2009) suggests that large denominations are preferred to smaller denominations. This research focuses on the first contingency wherein recall of denominations and the number of units of a denomination is contingent on the denomination itself.

A simple manner in which to capture the systematic biases in the estimates of the number of units of a denomination is to use a power function. The empirical law of sensation states that equal percentage changes in objective magnitude are perceived as equal percentage changes in subjective magnitudes (Stevens 1986). The analytical expression that captures this relationship is a power function of the form: $S = b\, I^c$, where S is the subjective magnitude (sensation), I is the objective magnitude (intensity), and b is a scaling parameter. The exponent, c, captures the concavity of the power function. With a few exceptions (such as the experience of pain), c is less than 1, which means that the power function is compressive. As stated by Kruger (1989) "*the true psychophysical function is approximately a power function whose exponents normally range between 0 and 1, and exceed 1 only in rather special cases*." For example, people perceive that brightness added by a second candle is less than the brightness added by a first candle (Chandon and Wansink 2007). The robustness of this empirical power law has been demonstrated in a variety of consumer settings, including judgments of area (Krider, Raghubir, and Krishna 2001), size (Teghtsoonian 1965), and calorie estimation (Chandon and Wansink 2007).

The extent to which wallet estimation or the process of recall-and-counting money follows the empirically robust law, the subjective value, U(*V*), may then be expressed as a variation of objective value, V, as below[4]:

(2) $U(V) = \sum_{i = 1, k} [n_i^{\alpha i}\, D_i^{\beta i}]$, such that $\alpha_{(i)}, \beta_{(i)} > 0$

The exponent, α, captures systematic biases in estimating the number of units carried of a specific denomination, n_i. If there are no biases in estimating the number of units, then $\alpha = 1$. The exponent, β, captures the systematic over or undervaluation of a specific denomination. For example, the β associated with a penny may be smaller than the β associated with a quarter or a dollar bill. Given that our focus is on investigating biases in the recall of the number of units of specific denomination (α), rather than deviations in the perceived value of different

[4] Note, variations in the subjective value of D, as suggested by Mishra et al. (2006), can be incorporated into this subjective value function by making β endogenously determined; that is, raising D to its own exponent β, such that $\beta = f(D)$. In other words, this formulation would capture that larger denominations may be valued more than smaller denominations, or U($5) > 5x U($1). See Appendix for Model 6.

denominations (β), it is assumed that $\beta = 1$ (i.e., the subjective and objective value of each denomination is identical across all denominations).

Exponents of the number of units that are lower than 1 ($0 < \alpha < 1$) imply underestimation whereas those that are greater than 1 ($\alpha > 1$) imply overestimation. For example, assume an individual has three quarters (n = 3, D = 25¢, nD = 75¢). If $\alpha > 1$, say 1.1, then the subjective value of the three quarters will be 84¢ (= 25¢*$3^{1.1}$) , an overestimate. If $\alpha < 1$, say .9, then the subjective value of the three quarters is 67¢ (=25¢*$3^{0.9}$), an underestimate. This is also true of non-fractional bills. In the case of three $1 bills (Actual = $3), an α of 1.1 gives a value of $3.35, an overestimate, and an α of .90 gives a value of $2.69, an underestimate. The size of the errors increases with the denomination (or face value). For three $20 bills, α's of 1.1 and 0.9 would lead to estimates of $66.97 and $53.76, respectively, representing deviations between $6 and $7 from the actual value of $60. Similarly, as the number of units increases, the cost of inaccuracy increases as well. For example, if an individual was carrying five $20 bills, then α's of 1.1 and 0.9 would lead to estimates of $117.46 and $85.14, respectively, representing deviations of over $10 from the actual value of $100. In sum, the higher the number of units carried in a specific denomination (n_iD_i), the greater the likelihood of an inaccurate estimate. This could be due to higher number of units of a denomination, n, or higher denominations, D. Thus, with large denominations, the cost of inaccuracy increases, and the likelihood of $\alpha \rightarrow 1$ increases. Said differently, it is more important to accurately recall the number of $20 bills one is carrying than the number of dimes, if one is trying to accurately estimate the value of one's wallet.[5]

The three common constructs that ought to affect the extent to which the estimates are accurate are an individual's ability, motivation, and opportunity to make an assessment (Petty and Cacioppo 1986). These are typically exogenously manipulated or measured. We propose that they may also be endogenous to the task. *Ability* is a function of the actual number of units (or

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[5]Note that making the values of α and β contingent on each denomination (that is, they both have subscript "I" relating to a denomination, i) makes the model descriptive rather than prescriptive in nature. A prescriptive model would require that we specify a functional form for α as a function of N and D. While external contextual factors (such as task difficulty and motivation) should affect the values of α ($\alpha \rightarrow 1$ as motivation increases, and task difficulty reduces), we propose that the intrinsic contents of the wallet itself affects the accuracy with which these contents will be recalled.

numerosity) of a denomination to be estimated, n. The greater the actual number of units, the more difficult the estimation task. Thus, as n increases, α should decrease, leading to an inverse relationship between n and α. The implication is a *numerosity bias* in estimates such that the greater the number of units, the higher the likelihood of underestimation. The numerosity bias was found across all four studies. Note that although this bias follows the basic prediction of the psychophysics of estimation, our contribution lies in *(a)* demonstrating the power law effect for monetary units and *(b)* demonstrating that the psychophysical law may operate in a recall-and-count (memory) task as well as in a stimulus-based estimation task.

The *motivation* to be accurate is likely to be function of the denomination of the number of units that needs to be estimated. The motivation to be accurate increases with the actual denomination such that D and α should be positively related. Model 8 supported this prediction in all studies. The *opportunity* to be accurate is also a function of task factors (whether the task is stimulus-based or memory-based, and time pressure). Thus, inaccuracies in estimates are likely to increase with time pressure. The results of study 3 provide support for this idea.

We use the excel solver function to estimate three different psychophysical models ranging from easier to harder to estimate.[6] An additional three models are described in the Appendix.

1. Model 1: $\sum_i n^{\alpha} D_i$, single α for all denominations, $\beta = 1$. This model implies that the number of units of all denominations are recalled with the same level of (in)accuracy. It serves as a base-line model.

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[6]We use the excel solver function to estimate the parameters. We first calculated the number of actual units per denomination, multiplied by the denomination, raised to an exponent, α, separately for each denomination. These were summed to form the total estimate as per the specified value function. This estimate was then subtracted from participants' *a priori* estimate of the amount they were carrying in their wallets. The difference (or estimation error) was then squared. After first initializing all alphas to 1, the sum of squares of (or the sum of the absolute value of) the error terms so generated were minimized subject to the constraint that all alpha coefficients were greater than .0000001. (Results are robust to constraints where the alpha value was lower or higher than .0000001). However, this solver has its limitations. It is not a completely stable solution. The more parameters that need to be estimated, the less stable it becomes (it crashes or it does not reach a point estimate). The same is true for studies that have fewer observations. For example, for model 2 it matters if the default number of α from which the estimation starts with solver is 1 or .9. Complete details are available from the authors. We present the results of the model parameters by minimizing both the sum of $(\text{error})^2$ and $|\text{error}|$. The latter method gives more stable solutions.

2. Model 2: $\sum_i n^{\alpha}_i D_i$, one α for each denomination, β = 1. This model, the primary focus of our research, implies that level of (in)accuracy in recall of the number of units is contingent on the denomination.
3. Model 3: $\sum_i n^{\alpha} D^{\beta}_i$, single α and single β for all denominations. Model 3 is a simplified improvement of the base model (model 1) as it can be fitted separately for bills and coins.

We present the results of models 1 and 3 (across denominations, ignoring \$100 and \$50, and separately just for coins) for studies 1-4 separately, and then for all studies in the aggregate (Table 10). We then present the results of model 2, the primary focus of this research (Table 11).

Table 10 – Model coefficients for Models 1 and 3 (aggregating across denominations)

Model	Denominations	Study 1	Study 2*	Study 3	Study 4	Studies 1-4 (pooled data)
Model 1: $\sum_i n^{\alpha} D_i$, single α for all denomination, β fixed=1						
Single α (β fixed=1)	w/o \$100 & \$50 (k=8)	α = 0.99	α = 0.62	α = 0.98	α = 0.43	α = 0.99
Single α (β fixed=1)	Only coins (k=4)	α = 0.69	α = 0.59	α = 0.85	α = 0.77	α = 0.76
Model 3: $\sum_i n^{\alpha} D_i^{\beta}$ single α and single β for all denominations						
Single α and single β	w/o \$100 & \$50 (k=8)	α = 1.10 β = 0.90	α = 0.61 β = 1.00	α = 1.05 β = 0.97	α = 1.52 β = 0.50	α = 1.03 β = 0.96
Single α and single β	Only coins (k=4)	α = 0.60 β = 0.91	α = 0.29 β = 0.72	α = 0.80 β = 0.75	α = 0.72 β = 0.94	α = 0.69 β = 0.92

*only when memory task was conducted first

Table 10 shows that the alpha coefficient for the set of four coins is reliably < 1 (Model 1: α = 0.76 across all studies, Model 3: α = 0.69 across all studies), indicating underestimation of coins. It also shows that the alpha associated with coins diverges from 1 to a greater extent than the alpha for the set of 8 denominations, reflecting greater inaccuracy in the estimates of coins. Model 3 results additionally show that coins are valued to a lower extent (β = 0.92 across all four studies) than are the entire set of denominations ($\beta = 0.96$). As such, Model 1 and 3 results support the analyses reported in the experiments that the greater the denomination, the more accurately it will be recalled.

Table 11 presents the results of model 2. Estimates based on the sum of the absolute value of the error are more stable than those based on the square of the sum of the error.

Table 11 – Model Fit by Denomination for Studies 1 – 4

Denomination	Estimate of α ($\alpha = 1$ is normative)				
	Study 1	Study 2	Study 3	Study 4	Studies 1-4 (pooled data)
Sum of the square of the error					
20	1.00	0.39	0.99	0.21	0.99
10	1.39	1.07	0.69	0.00	1.13
5	0.00	0.70	0.70	0.00	0.73
1	0.45	0.97	1.21	1.36	0.95
0.25	0.00	0.65	0.61	1.96	0.00
0.10	2.21	0.00	0.00	0.00	1.62
0.05	0.00	3.33	0.00	0.00	0.00
0.01	2.78	1.96	0.00	1.18	0.73
Sum of absolute value of the error					
20	1.00	0.57	1.00	0.00	1.00
10	1.10	1.05	0.79	0.00	0.96
5	0.90	0.92	0.99	0.69	0.94
1	0.56	1.00	1.05	1.01	0.99
0.25	0.94	0.92	0.76	2.01	0.73
0.10	1.02	0.98	0.89	0.86	1.00
0.05	0.99	1.00	1.01	0.64	1.08

0.01	1.00	1.00	1.00	0.97	1.10

α = 0.0 indicates solver did not reach point estimate or could not continue optimization

The parameters of the psychophysical model are more stable for the dependent variable sum of absolute value of the error (vs. sum of error squared). The parameters across all studies show that quarters are the most underestimated, whereas there is the greatest inaccuracy in the recall of nickels and pennies. The values of α are close to 1 for all four bills. The contribution of the psychophysical models is that they represent a conceptually appealing way of understanding how the contents of a wallet are estimated. Better estimation methods may lead to more stable solutions given this formulation.

GENERAL DISCUSSION

The current research contributes to the literature on the subjective value of money by examining the denomination effect (Raghubir and Srivastava 2009) and providing a memory-based explanation for the finding that individuals are less likely to spend money if it is in the form of a large denomination relative to an equivalent amount in multiple smaller denominations. In documenting the denomination effect, prior research suggests that individuals intrinsically value a larger denomination either due to the ease of processing and the resultant positive affect that generates (Mishra et al. 2006) or due to larger denominations less likely to be soiled as much as smaller denominations (Di Muro and Noseworthy 2013). Based on ideas of self-control and regulation, Raghubir and Srivastava (2009) suggested that individuals strategically choose to receive money in larger denominations under conditions in which they wish to save and exercise self-control in spending. A critical assumption underlying the pre-commitment explanation is that individuals are aware that monitoring and tracking capabilities may be attenuated when the money is represented by multiple smaller denominations than a large denomination. In fact, failure to monitor and keep track of one's behavior is an antecedent to failures in self-control (Baumeister et al. 1994).

This research, thus, explores a memory-based account of the denomination effect and examines the extent to which individuals' estimates of the money they carry is affected by the denomination and the number of units of a specific denomination. Although prior research suggests that individuals may be aware of their ability to monitor and track smaller denominations, this research explicitly examines the extent to which individuals are biased in their recall of money as a function of denomination and the number of units of any denomination. To the extent to which individuals are prone to errors in recalling money they are carrying, our results bolster lay beliefs that breaking a larger denomination into smaller denominations will hinder monitoring and tracking of the money. As such, individuals are reluctant to spend money when it is in the form of a large denomination than an equivalent amount in multiple small denominations. The memory-based explanation thus complements and supports Raghubir and Srivastava's (2009) explanation that the denomination effect occurs due to reasons of self-control and regulation.

Specifically, this research reports the results of four studies that examined the extent to which there were errors in individuals' estimates of the money they were carrying. Study 1 presented the results of a simple recall and count task of the contents of one's wallet. Study 2 extended the investigation to a stimulus-based task. Study 3 examined the extent to which the biases were robust under different motivation and task difficulty conditions. Study 4 examined the extent to which the bias in estimating the money one is carrying is linked to actual spending. Together, the results of four studies suggest that the more the number of units one is carrying and the smaller the denomination, the greater the difficulty in recall as well as recall errors.

Given the systematic empirical findings regarding the bias in the recall of money as a function of denomination, and that biases in recall of money have not been systematically examined, we develop and present a set of pyschophysical models that attempt to capture the subjectivity in the recall and valuation of the contents of one's wallet. Estimating the amount of money one is carrying involves recalling the different denominations one has and the number of units of each denomination. As such, errors in recall may creep into either the recall of the different denominations and/ or the number of units of each denomination. The results of the four studies provide evidence for a *numerosity bias*, where a larger number of units of a denomination is underestimated compared to a fewer number of units, as well as a *denomination bias*, where smaller denominations are recalled less accurately as compared to larger denominations. We use a simple power function, commonly used in capturing empirical regularities in perceptions, to capture the numerosity bias. Estimating the exponent, α, which captures the numerosity bias using the data from the four studies suggest that α deviates substantially from 1 for the recall-and-counting of smaller denominations (i.e., coins). Importantly, the robustness of the numerosity bias indicates that these effects extend to the memory-based model of recalling the money in one's wallet.

Despite the robustness of the findings, there are several factors that could cause α to deviate from 1. For example, the variance in the number of different denominations carried for a specific amount of money is likely to affect recall errors. A sum of money, say \$20, may be held in many different forms (e.g., a \$20 bill, four \$5 bills, two \$10 bills, one \$10 bills + two \$5 bills, one \$10 bill + one \$5 bill + five \$1 bills etc.). The larger the number of units and the higher the variance in the denominations carried, the more likely the errors in recall. Another factor causing money recall errors may be an overall wealth (V) effect - a natural hypothesis – given our findings would predict that as V increases, α diverges from 1.

Other possible moderators for the memory biases worth investigating include the storage salience of bills versus coins, individual differences in age, mathematical acuity, and psychographics. It would be interesting to examine whether predispositions to overestimate or underestimate the contents of one's wallet (which was shown to affect the likelihood of spending in study 4) are directly related to individual differences in the attitudes towards money and spending, such as the spendthrift-tightwad scale (Rick, Cryder, and Loewenstein 2008).

Another factor that may moderate the denomination effect is the extent to which the amount (or denomination) one has "matches" the amount one is considering spending. For example, the denomination effect is likely to be weaker when one is buying a \$5 cup of coffee with \$5, where there is no change being generated, relative to when one is buying a \$3.60 cup of coffee with \$5, generating a change consisting of a \$1 bill, a quarter, a dime, and a nickel. When there is a match, there is little fear of attenuating one's monitoring and tracking capabilities.

Further, mental accounting ideas suggest that people mentally track expenses against pre-assigned budgets for different expense categories. These ideas have been useful in uncovering and explaining the subjectivity in the valuation of money (Thaler 1985). The extent to which individuals assign a specific amount of money as "petty" cash, the rules governing the spending of such money may not be as stringent as those associated with other more serious labels. For example, an individual may withdraw $20 every week to spend on miscellaneous petty expenses. Such money may be less prone to the denomination effect. There are thus several avenues for future research.

Notwithstanding, this research provides a memory-based explanation for the denomination effect and complements the explanation that the denomination effect occurs due to reasons of self-control and regulation (Raghubir and Srivastava 2009). In exploring a memory-based explanation, it identifies a numerosity bias and a denomination bias. These biases occur presumably because of the difficulty to monitor and track many denominations, particularly when the denominations are small. Although we attempted to explore accuracy motivation and task difficulty, our manipulation of motivation was not as strong. Given the robust effects of factors relating to motivation, ability, and opportunity in many different domains, it is likely that these will play a role in the valuation of money, particularly when memory-based effects are considered. In general, this research identifies and tests different factors which affect how money is subjectively valued and thus has important implications for resource valuation and allocation decisions. Taking such different perspectives is likely to enhance our understanding of how individuals make these decisions.

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APPENDIX

There are three alternative formulations of the psychophysical model of the subjective value of money. These are:

4. Model 4: $\sum_i (n_i D_i)^{\alpha I}$, one α for the total amount held in each denomination. This model implies that people treat each set of denominations differently. For example, in a wallet consisting of four quarters and five $1 bills, people estimate their wallet separately in terms of the bills they have and the coins they are carrying. The model may have intriguing possibilities if there are systematic biases in the way in which people treat bills and coins as legal tender. It appears to have more direct implications for a wallet containing a range of currencies (e.g., 5 x $20, £10, and 10 x €10). In such a case, the value of the wallet could remain objective with the β coefficient playing the role of exchange rates. We do not fit this model as it is not pertinent to our context.

5. Model 5: $\sum_i n^{\alpha i} D_i^{\beta i}$, independent α and β for each denomination. This is the most complete and appropriate model capturing the fact that the accuracy of recall of the number of units and the value associated with any denomination are both contingent on the denomination itself. Table A1 presents preliminary evidence for this model, although its parameters are unstable.

Table A1: Model 5 Results: $\sum_i n^{\alpha i} D_i^{\beta I}$, independent α and β for each Denomination

Denom	Study 1		Study 2		Study 3		Study 4		Studies 1-4 (pooled data)	
Minimizing the sum of the squared errors										
	α	β	α	β	α	β	α	β	α	β
$20	1.11	0.88	0.44	1.00	1.03	0.98	2.93	0.00	1.03	0.96
$10	1.14	1.03	1.69	0.82	1.14	0.87	0.00	0.67	1.35	0.89

$5	0.00	1.04	0.61	0.89	0.90	0.89	0.00	1.55	0.52	1.19
$1	0.62	1.00	1.04	1.00	1.25	1.00	1.29	1.00	0.97	1.00
$0.25	0.00	4.97	0.24	0.00	0.43	0.98	1.32	0.00	0.00	4.07
$0.10	2.21	0.92	0.00	6.52	0.00	0.19	0.20	0.00	25.89	31.2
$0.05	0.00	3.40	1.31	0.00	0.00	0.17	0.00	1.44	0.00	0.20
$0.01	1.11	0.04	1.68	0.87	0.00	2.44	0.92	0.00	0.20	0.00
Minimizing the sum of the absolute value of the errors										
$20	1.02	0.97	0.57	1.00	1.00	1.00	0.00	0.58	1.00	1.00
$10	1.27	1.00	1.14	0.99	0.81	0.98	0.00	0.07	0.98	1.00
$5	0.78	1.13	0.92	1.00	0.96	1.00	0.00	1.45	0.93	1.01
$1	0.00	1.00	1.00	1.00	1.09	1.00	1.00	0.96	0.97	1.00
$0.25	0.78	1.05	0.71	0.68	0.58	0.63	1.52	0.14	0.43	0.42
$0.10	1.90	0.71	0.64	1.02	0.42	0.88	0.55	0.00	4.01	3.71
$0.05	0.96	1.08	1.06	0.96	1.10	1.59	0.00	0.39	5.64	5.15
$0.01	0.00	0.05	1.22	0.89	1.21	1.05	0.97	0.00	2.42	1.87

* For all numbers 0.0 estimation ends once the zero point is reached. Estimation is unstable (i.e. highly subjective to starting values for alpha and beta´s.)

6. Model 6: $\sum_i n^{\alpha_i} D_i^{\beta_I}$, where $\alpha_i = f(N_i, D_i)$ and $\beta_I = f(D_i)$. This model is an extension of model 4, where α and β are both endogenously determined as a function of the number of units and/or denomination. For example, one formulation could be $\alpha_i = \beta_i = \kappa_i D_i$, where κ's value is contingent on the denomination (e.g., $\kappa = 1/20$ for a $20 bill, allowing α and $\beta = 1$, but $\kappa = 3$ for a quarter, making α and $\beta = 0.75$, or undervalued and underestimated). Modeling α and β as a function of number of units and denomination is outside the scope of this paper.