Securitization Networks and Endogenous Financial Norms in U.S. Mortgage Markets

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PRELIMINARY DRAFT

Abstract

We develop a theoretical model of a network of intermediaries, which we apply to the U.S. mortgage supply chain. In our model, heterogeneous financial norms and systemic vulnerabilities arise endogenously. Intuitively, the optimal behavior of each intermediary, in terms of its attitude toward risk, the quality of the projects that it undertakes, and the intermediaries it chooses to interact with, is influenced by the behavior of its prospective counterparties. These network effects, together with intrinsic quality differences between intermediaries, jointly determine financial health and systemic vulnerability at the aggregate level as well as for individual intermediaries. We apply our model to the mortgage-origination and securitization network of financial intermediaries, using a large data set of more than twelve million mortgages originated and securitized through the private-label supply chain from 2002–2007. We then track the ex-post foreclosure performance of each loan in the network and compare the evolution of credit risk by vintage with the model’s predictions. We find that credit risk evolves in a concentrated manner among highly linked nodes, defined by the geography of the network and the interactions between originator and counterparty over time. This confirms that network effects are of vital importance for understanding the U.S. mortgage supply chain.

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1 Introduction

Several recent studies highlight the importance of network linkages between intermediaries and financial institutions in explaining systemic risk in the financial system (see, for example, Allen and Gale, 2000; Allen, Babus, and Carletti, 2012; Cabrales, Gottardi, and Vega-Redondo, 2014; Glasserman and Young, 2013; Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2013; Elliott, Golub, and Jackson, 2014). These studies show that financial networks may create resilience against shocks in a market via diversification and insurance, but also contagion and systemic vulnerabilities by allowing shocks to propagate and amplify. The network structure is thus a pivotal determinant of the riskiness of a financial market.

These theoretical studies of networks and risk in financial markets typically focus on how the network redistributes risk between participants, and on the consequences for the system’s solvency and liquidity after a shock. This is an ex post effect of the financial network. One may also expect ex ante effects to be important. Specifically, the presence and structure of a financial network should affect—and be affected by—the actions of individual intermediaries and financial institutions, even before shocks are realized. Understanding the equilibrium interaction between network structure, the actions taken by market participants, and the market’s riskiness is the focus of our study.

We build upon the approach in Stanton, Walden, and Wallace (2014), who study the mortgage market from a network perspective, showing that network linkages defined by contractual relationships are an important factor in the U.S. single-family residential-mortgage market and find that, despite the large total number of firms, the market is highly concentrated, with significant inter-firm linkages—even between seemingly independent institutions—once account is taken of the wholesale lending mechanisms used to fund mortgage origination. They represent the mortgage market as a network of mortgage firms, where links are represented by loan flows, and show that the performance of an individual node is closely related to the performance of the node’s neighbors in the network.

We introduce a model with multiple agents representing financial intermediaries, who are connected in a network. Network structure in our model, in addition to determining the ex post riskiness of the financial system, also affects—and is affected by—what we call the financial norms in the network, inspired by the literature on influence and endogenous evolution of opinions and social norms in networks (see, for example, Friedkin and Johnsen, 1999; Jackson and López-Pintado, 2013; López-Pintado, 2012). These financial norms represent the quality and riskiness of the actions agents take.

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1The most important of these funding mechanisms are master repurchase agreements, a form of repo, and extendable asset-backed commercial paper programs.
Our model is parsimonious, in that the strategic action space of agents as well as the contract space is quite limited. Links in the network represent risk-sharing agreements, as in Allen et al. (2012). Agents may add and sever links, in line with the concept of pairwise stability in games on networks (see Jackson and Wolinsky, 1996), and also have the binary decision of whether to invest in a costly screening technology that improves the quality of the projects they undertake.

The equilibrium concept used is subgame-perfect Nash. In an equilibrium network each agent optimally chooses to accept the network structure, as well as whether to invest in the screening technology, having correct beliefs about all other agents’ actions and risk. Shocks are then realized and distributed among market participants according to a clearing mechanism similar to that introduced in Eisenberg and Noe (2001).

As in Elliott et al. (2014), we assume that there are costs associated with the insolvency of an intermediary, potentially creating contagion and propagation of shocks through the clearing mechanism, and thereby making the market systemically vulnerable. The model is simple enough to allow us to computationally analyze the equilibrium properties of large-scale networks using approximation methods.

Our model has three general implications. First, network structure influences financial norms. Given that an agent’s actions influence and are influenced by the actions of those that the agent interacts with, this result is natural and intuitive. Importantly, an agent’s actions affect not only others to whom the agent is directly linked but also those who are indirectly connected through a sequence of links. As a consequence, there is a rich relationship between equilibrium financial norms and network structure, in turn suggesting a deeper relationship between the network and the financial health of the market beyond the mechanical relationship generated by shock propagation.

Second, heterogeneous financial norms may coexist in the network, in equilibrium. Thus, two intermediaries that are ex ante identical may be very different when their network position is taken into account, not just in how they are influenced by the rest of the network but also in their actions. Empirically, this suggests that network structure is an important determinant not only of the aggregate properties of the economy but also of the actions and performance of individual intermediaries.

Third, proximity in the network is related to financial norms: nodes that are close tend to develop similar norms, just like in the literature on social norms in networks. This result suggests the possibility of decomposing the market’s financial network into “good” and “bad” parts, and address vulnerabilities generated by the latter.

We analyze the mortgage-origination and securitization network of financial intermediaries, using a large data set of more than twelve million mortgages originated and securitized
through the private-label supply chain from 2002–2007. Our approach is to use loan flows to identify the network structure of this market. We then use ex-post foreclosure rates of these loans as a measure of performance, and use the model to estimate the evolution of credit risk by vintage. Our analysis suggests that credit risk evolves in a concentrated manner among highly linked nodes, defined by the geography of the network and the interactions between originator and counterparty over time, and more generally that both ex ante and ex post network effects are of vital importance for understanding the U.S. mortgage supply chain and its systemic vulnerabilities.

Our supply-chain representation of the private-label mortgage market allows us to highlight several key characteristics of the organizational structure of this market that importantly differentiate this paper from prior work. Rather than focusing on possible relationships between risk taking and aggregate measures of market concentrations (see Allen and Gale, 2004; Beck, Coyle, Seabright, and Freixas, 2010; Claessens, 2009; Scharfstein and Sunderam, 2013), we instead focus on the economic and organizational forces that underlie the system of unintegrated exchange that characterizes the supply chain of this industry (see Bresnahan and Levin, 2012; Jacobides, 2005). We follow Stanton et al. (2014) and represent this system of unintegrated exchange as a network in which the related activities of origination, funding for origination, aggregation for securitization, and the sale of mortgage-backed securities to investors are largely undertaken by independent firms, whose actions are coordinated explicitly through contracts (such as funding commitments) or implicitly through standardized institutional structures and norms (such as the underwriting quality choices of originators within the network) that are difficult to monitor contemporaneously.

There are two important differences between the unintegrated exchange systems of the private-label supply chains considered here and the GSE supply chains of the Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporation (FHLMC). First, the GSE’s apply ex ante standardized screening protocols, called Loan-Prospector (FHLMC) and Desktop Underwriter (FNMA), to determine which loans meet quality standards for securitization. Second, the GSEs apply long-term, ex post performance evaluations of originators’ loans to identify poor-quality originators who can be sanctioned, either by permanent expulsion from the insured securitized markets or by long-term putback risks. Ex post exercise of putback rights, allowing the trustees of the loan pools to require that the originator buy back at par any poorly performing loans within 60 days of origination, are, and were pre-crisis, enforceable through the pooling and servicing agreements of

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2We, however, also show in Stanton et al. (2014) that the private-label supply chain shares all but a few of the same characteristics with the government sponsored enterprise (GSE) supply chain for residential prime loans.
both GSE and private-label securitized pools. In most other respects the two unintegrated exchanges share similar data standards for performance reporting and standardized institutional infrastructure such as the treatment of foreclosures under state laws and the legal standing of loan sales along the supply chain under U.S. Universal Commercial Code, Article 9 (see Follain and Zorn, 1990; Jacobides, 2005; Hunt, Stanton, and Wallace, 2014; Stanton et al., 2014).

The rest of the paper is organized as follows. Section 2 describes the structure of the U.S. residential mortgage market and available data. Section 3 introduces the model. Section 4 analyzes the properties of equilibrium, and Section 5 discusses how to estimate equilibrium from observed data in a large-scale network. Section 6 lays out our approach to identifying the mortgage-securitization network from loan flows. Section 7 shows some preliminary results for the 2006 U.S. private-label mortgage market, and Section 8 concludes. The appendix contains a detailed description of the network game used in the model.

2 Structure of the U.S. residential mortgage market

There is a very small literature that considers the economic factors that lead to unintegrated exchange systems, either in theoretical treatments (see Jacobides, 2005; Chen, 2005; Bresnahan and Levin, 2012) or empirical studies (see Langlois and Robertson, 1992; Holmes, 1999; Chen, 2005) or even studies of the sociological and institutional norms or pre-conditions needed to engender unintegrated market structures (see Cooter, 2000; MacKenzie and Millo, 2003; Fligstein, 2001; Jacobides, 2005). In his famous essay on the nature of the firm, Coase (1937) describes why and how economic activity divides between firms and markets. He argues that firms exist to reduce the costs of transacting through markets. Building on Coase’s seminal ideas, Williamson won a Nobel prize for his development of the transaction cost theory of integration (see Williamson, 1971, 1975, 1979). A key element of this theory is that market contracts are inherently incomplete and this limitation of explicit contracts may be especially severe when complexity or uncertainty make it difficult to specify contractual safeguards, or when parties cannot walk away without incurring substantial costs. Transaction cost theory therefore argues that vertical integration can be an effective response when these features are present. A related rationale for integration is that it might mitigate potential holdups by suppliers (see Joskow, 2005; Williamson, 2010).

The property rights theories of vertical integration (see Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995) have focused on how integration changes the incentives to make specific investments and find that ownership strengthens a party’s bargaining position. Incentive theories (see Holmström and Milgrom, 1994; Holmström, 1999) have shown that
under certain conditions, asset ownership by the agent (e.g., non-integration) can be complementary to providing high-powered financial incentives. In contrast to the contracting and transaction cost literatures, theories in organizational economics have focused more directly on the determination of horizontal market structure, typically based in firm-level costs or in strategic interaction among firms. Stigler (1951) argued that in the early and innovative phases of an industry, firms have to be vertically integrated because there are no markets for the relevant inputs – the costs of organizing those markets, he assumes, are higher than the costs of coordinating the production of the inputs within the firm. As the industry becomes larger, what had formerly been internal inputs are supplied by new, vertically unintegrated industries. In addition to the trade-off between efficient horizontal scale and vertical market power, Stigler’s theory adds the additional idea that market institutions that are needed to support unintegrated trade are themselves endogenous and have to be developed over time.

The point made by Stigler (1951) that market institutions need to be invented before a wide variety of firms can participate in them remains only partially explored in organizational economics, however Jacobides (2005) builds on these ideas based on an extensive case study of vertical disintegration in the U.S. mortgage banking industry from 1970 through 1998. He argues that transaction costs theories, decision and incentive theories provide an inadequate explanation for how the mortgage banking industry and its value chain structures evolved. Both Jacobides (2005) and Bresnahan and Levin (2012) argue that transaction costs depend on existing market institutions — institutions that facilitate search and matching, and institutions that facilitate contractual and pricing arrangements. As a result, an unintegrated market structure, and particularly the creation of an unintegrated industry with frequent arms-length exchange, often requires the creation of market institutions: standards for products and contracts, and mechanisms for matching buyers and sellers, determining prices, and disseminating information (see Jacobides, 2005; Bresnahan and Levin, 2012). Jacobides (2005) also points out that the creation of market institutions is facilitated by having a degree of standardization in the underlying products.

According to Jacobides (2005), the disintegration of the highly vertically integrated mortgage origination and funding systems that existed prior to 1970 was caused by three institutional changes. First, the Federal government introduced standardized securitization systems through the GSEs and the Government National Mortgage Association (GNMA) and allowed non-depository mortgage banks to issue and service loans under GSE criteria. To fund loans temporarily before sales to the GSE securitizers, mortgage banks would use lines of credit obtained from commercial banks (see Fabozzi and Modigliani, 1992). The second major change occurred as the result of the recession of 1979-81, when banks and S&Ls laid off their loan origination staff and then re-established long-term relationships, often with the same
staff, as independent loan brokers. The loan brokerage vertical disintegration was funded with lines of credit from mortgage banks, commercial banks or S&Ls or by an alternative model whereby the brokers merely served as agents that matched borrowers with loan products without making the underwriting or funding decisions. The third major change was the vertical disintegration of loan servicing from loan origination through the creation of a market for mortgage servicing rights.

Each of these stages of vertical disintegration in the mortgage market led to the creation of highly specialized entities: mortgage brokers with highly specialized local market knowledge; mortgage bankers with specialized knowledge concerning capital market funding and managing pre-securitization pipeline risk; depository institutions who did some origination but increasing specialized in funding downstream originators through short-term lines of credit and repurchase agreements that require the management of liquidity and roll-over risks; mortgage servicers specialized in the management of interest rate and prepayment risk; and mortgage securitizers with specialized capital market knowledge associated with accessing the mortgage investors needed to purchase the mortgage backed securities. In addition, the mortgage market became characterized by potential gains from trade, resulting from the imbalances specialization along the value chain of existing participants or gains that could be obtained if new, vertically specialized entrants, such as specialized technology vendors or investment banks, were to enter the market.

As discussed above, the highly co-specialized and coordinated agents within the private-label mortgage supply chain appear to be well characterized as a network in which the actions of independent firms are coordinated explicitly through contracts, such as funding commitments, or implicitly through standardized institutional structures and norms. Existing theories in the economics of transactions costs, contracting, and industrial organization provide limited insight into competitive outcomes in vertically unintegrated markets in which agents can act strategically when entering into contractual agreements among themselves; are influenced by the actions of others to whom they are only indirectly connected potentially through quality norms with the network; and make unobservable quality choices that impact outcomes, locally as well as globally in the network. We first characterize the lender, securitization shelf, and holding company funders of this unintegrated supply chain in the following sections and then in Section 3 we develop a network model of the strategic interactions of the unintegrated private-label mortgage market.
2.1 The lenders

The residential-mortgage origination market comprises thousands of firms and subsidiaries, including commercial banks, savings banks, savings and loan institutions (S&Ls), mortgage companies, real estate investment trusts (REITs), mortgage brokers and credit unions. The industry operates within the dual (state and federal) supervisory system of the banking industry established by the National Bank Act (1863). Under this system, there is a federal system based on national bank charters and a state system based on state charters. There are three different types of bank charter, corresponding to the three different primary federal regulators: the Office of the Comptroller of the Currency (OCC), the Federal Deposit Insurance Corporation (FDIC), and the Federal Reserve System (FRS). Federally chartered banks and their branches are known as National Banks (N.A.), and are primarily chartered and supervised by the OCC (see Engel and McCoy, 2011). The FDIC regulates state-chartered banks that are not members of the Federal Reserve System. Prior to October 19, 2010, and after the passage of the Financial Institutions, Reform, Recovery and Enforcement Act (1989), S&Ls were regulated by the Office of Thrift Supervision (OTS). Mortgage companies are the most diverse group of mortgage originators. They include mortgage bankers, mortgage brokers that use their own money for origination, and Real Estate Investment Trusts. Since 2004, mortgage companies, even those affiliated with large regulated bank or thrift holding companies, tend to be regulated by the Department of Housing and Urban Development (HUD) (see Engel and McCoy, 2011). Finally, there are also mortgage brokers who are regulated by the states (see Pahl, 2007).

In the early 1990s, all banks and thrifts had to obey state mortgage and consumer protection laws, and non-bank mortgage companies had to comply with the same laws. In 1996 the OTS issued two preemption rules, under which federal thrifts and their subsidiaries were exempted from many state mortgage laws. In 2004, the OCC issued a preemption rule giving national banks the ability to exercise “incidental powers” for activities such as lending and deposit taking, thus preempting all state laws that “obstruct, impair or condition” the business of banking. Again, many of these laws involved consumer protection. The mixing of federal preemption and charter competition among the various regulatory agencies led to inconsistencies in the implementation of examination rules for mortgage lending (see Agarwal, Lucca, Seru, and Trebbi, 2012). It also allowed mortgage originators to actively shop for regulators (see Rosen, 2003, 2005) and to engage in a “race to the bottom” in an effort to seek out the least restrictive regulatory charter (see Kane, 1989; Calomiris, 2006).

A second distinction among the firms in the industry is between retail and wholesale

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originators. For retail originators, the underwriting and funding processes are carried out by the labor and capital of either a single originator or the consolidated subsidiary of a single originator. In contrast, the origination and underwriting processes of wholesale originators are handled in whole or in part by the labor and capital of another party. Wholesale originators are also distinguished by the degree of autonomy that the originating party exercises over the underwriting and funding processes. Wholesale broker lending usually involves a more limited level of autonomy, because brokers generally do not make the final credit decision and neither do they fund the loan. Correspondent wholesale originators — who can be subsidiaries of mortgage companies, REITs, or depositories — originate and deliver loans determined by defined underwriting standards (usually an advance commitment on the loan structure and price), and they exercise full control over the underwriting and funding processes of loan origination. They are also legally the creditor of record.

The result of the U.S. system of dueling regulatory charters for pre-crisis mortgage originators was that it was not uncommon for large bank holding companies, thrift holding companies, and large mortgage REITs to acquire, or internally develop, subsidiaries that had different functional roles, such as retail or wholesale origination, and operated under different regulatory charters. Thus, for example from 2002–2007, the Wells Fargo Bank holding company was composed of: depository banks (National Associations, or N.A.s) and their branches that were engaged in retail mortgage origination under OCC charters; Wells Fargo Home Loan, Inc. an affiliated mortgage company operating under a HUD charter; and Wells Fargo Funding Inc., a correspondent lender (a lender to originators) operating under an OCC charter.

Figure 1 shows the organizational structure for the residential mortgage origination market for loans securitized by entities other than Freddie Mac or Fannie Mae: the private-label securitized market. Mortgage origination flows are organized within five strata of influence: 1) the independents, either depositories or non-depository mortgage companies; 2) the depositories and subsidiaries; 3) the bank and thrift holding companies; 4) the regulators; and 5) the securitization channels that were owned by investment banks, banks and finance companies. Direct ownership (or partial ownership) channels between these strata are shown by red dotted lines. Black dotted lines connect the regulators to their respective regulated entities. Blue dotted lines are the primary securitization channels, and green dotted lines represent the contractual mortgage-origination funding channels (lines of credit structured as repo and/or ABCP) from the wholesale lenders, the warehouse lenders, and/or the bank holding companies to the independent mortgage companies and depositories, who originated mortgages with borrowed capital. These contractual funding channels introduced important elements of systemic risk exposures associated with the short-term liquidity risk of the loans.
and the counterparty exposures among the mortgage originators and their funders.\textsuperscript{4}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Structure of the U.S. private-label mortgage market}
\end{figure}

\section{2.2 Securitization}

As is well known, securitization was a key feature of the pre-crisis structure of the residential-mortgage origination system. Mortgage originators would sell their newly originated mortgages to sponsors, who would pool them into so-called special purpose entities (SPEs). The SPEs would in turn issue and sell bonds to fund the purchase of the mortgage assets held by the SPEs, the principal and interest payments of which were used to service the bond debt of the SPE. Each SPE would be an independent legal entity with its own capital structure. However, each SPE also belongs to what is called a shelf registration.\textsuperscript{5} The sponsor first files

\textsuperscript{4} The mortgage companies and independent depositories used revolving credit lines to fund the mortgages that they originated. Their warehouse lenders then owned an interest in the newly originated mortgages that were subject to a commitment by the originator to repurchase the loan within thirty days. Once the mortgage originator sold the loans into the securitized market, the sales proceeds were used to repay the warehouse lender, releasing the capacity of the facility for future lending.

\textsuperscript{5} The SPEs are organized as a form of business trust called Real Estate Mortgage Investment Conduits (REMIC). The REMIC securities of private-label MBS are subject to the registration requirements of the federal securities laws. To offer and sell these securities, the sponsor must file a registration statement with the SEC following the procedural requirements of the Securities Act. When private-label issuers file a registration statement to register an issuance of a REMIC security, they typically use a “shelf registration” (see Simplification of Registration Procedures for Primary Securities Offerings, Release No. 33-6964, Oct. 22, 1992, and SEC Staff Report: Enhancing Disclosure in the Mortgage-Backed Securities Markets, January, 2003, \url{http://www.sec.gov/news/studies/mortgagebacked.htm#secii}).
a disclosure document, known as the “core” or “base” prospectus, that outlines the parameters of the various types of securities offerings to be sold from the sponsor’s shelf registration. The registration statement will also contain the form of a prospectus supplement outlining the format of deal-specific information that will be disclosed when a shelf offering actually occurs.\(^6\) The shelf registration defines common contractual features among the SPEs in the shelf, such as common terms of the pooling and servicing agreements, as well as similarities in the deal structuring for the bonds to be issued by the SPEs. Hundreds of SPEs were issued annually under these unique shelf registration entities, which define a given securitization strategy. Each of the separate securitization shelves are issued and controlled by a third-party holding company.

As an example of this supply chain of agents and counterparties, a loan might be originated by a mortgage company in King County, Washington. It would then be purchased by an aggregator/sponsor, such as EMC Mortgage, a subsidiary of Bear Stearns Companies, Inc., who would then securitize the mortgage in a pool, or SPE, with the registration name of “Bear Stearns ALT-A Trust 2005-2”. This unique SPE, Bear Stearns ALT-A Trust, 2005-2, is a distinct and independent legal entity, a REMIC. Bear Stearns ALT-A Trust, 2005-2 is also a member of a unique shelf registration, called Bear Stearns ALT-A Trust, and as a member of this shelf it shares common characteristics with other SPEs within that shelf registration. The holding company that controls the shelf registration is Bear Stearns Companies, Inc. Over our analysis period, Bear Stearns controlled five different shelves, each with unique features. Each shelf was used to issue hundreds of SPEs with common structures, and these SPEs in turn contained thousands of mortgages.

### 3 Network Model

In this section we introduce the equilibrium network model with the fundamental properties that agents 1) act strategically when entering into contractual agreements among themselves, 2) are influenced by the actions of others to whom they are only indirectly connected, 3) make unobservable quality choices that impact outcomes, locally as well as globally in the network.

We introduce the model in several steps. We first describe the risk environment and possible actions of agents that affect project payoffs, and analyze the outcome when agents

act in isolation. We then study in detail the case when two agents interact, before analyzing equilibrium in the general $N$-agent model. A detailed description of the strategic game between agents is provided in the appendix.

3.1 Intermediaries and projects

There are $N$ intermediaries with limited liability, each owned by a different risk-neutral agent. Each intermediary invests in a project that generates risky cash flows at $t = 1$, $\tilde{CF}_{1}^{n}$, and may moreover incur some costs at $t = 0$. The one-period discount rate is normalized to 0. Agent $n$’s objective is to maximize the expectation at $t = 0$ of the value of the intermediary at $t = 1$, net any costs incurred at time 0.

$$V^{n} = E_{0}[\tilde{CF}_{1}^{n}] - C_{0}^{n}.$$  

The risky project has scale $s^{n} > 0$, and there are two possible returns, represented by the Bernoulli-distributed random variables $\xi^{n}$, so that $R^{n} = R_{H}$ if $\xi^{n} = 1$, and $R^{n} = R_{L}$ if $\xi^{n} = 0$. The probability, $p$, that $\xi^{n} = 0$ is exogenous, with $0 < p \ll 1$. We assume complete symmetry in that the probability is the same for each project of the $N$ projects.\footnote{7Each project may be viewed as a representative project for a portfolio of a large number of small projects with idiosyncratic risks that cancel out, and an aggregate risk component measured by $\xi^{n}$.}

Each agent has the option to invest a fixed amount, $C_{0}^{n} = cs^{n} > 0$ at $t = 0$, to increase the quality of the project. This cost is raised externally at $t = 0$. If the agent invests, then in case of the low realization, $\xi^{n} = 0$, the return on the project is increased by $\Delta R > c$, to $R_{L} + \Delta R$. This investment cost could, for example, represent an investment in a screening procedure that allows the agent to filter out the parts of the project that are most vulnerable to aggregate shocks. We represent this investment choice by the variable $q^{n} \in \{0, 1\}$, where $q^{n} = 1$ denotes that the intermediary invests in quality improvement. Intermediaries that choose $q = 1$ are said to be of high quality, whereas those that choose $q = 0$ are said to be of low quality. For the time being, we assume that $c$ is the same for all intermediaries. We will subsequently allow $c$ to vary across intermediaries, representing exogenous quality variation (as opposed to the quality differences that arise endogenously because of intermediaries’ investment decisions).\footnote{8Several variations of the model are possible, e.g., assuming that screening costs, $c$, are part of the $t = 1$ cash flows, and fixing the scale of all intermediaries to $s \equiv 1$, all leading to qualitatively similar results. The version presented here was chosen for its tractability in combination with empirical relevance.}

There is a threshold, $d > 0$, such that if the return on the investment for an intermediary goes below $d$, then additional costs are immediately imposed, and no cash flows can be recov-
For simplicity, we call these costs “insolvency costs,” in line with Nier, Yang, Yorulmazer, and Alentorn (2008), who note that systemic events typically originate from insolvency shocks, although they are often also associated with liquidity constraints. We view these additional costs of insolvency as wasteful, in that they impose a real cost on society rather than being a transfer between agents.

It will be convenient to define the functions

\[ X(z) = \begin{cases} 
1, & z > d, \\
0, & z \leq d,
\end{cases} \quad \text{and} \quad Y(z) = X(z)z. \]

We also make the following parameter restrictions:

\[ 0 \leq R_L < d < 1 < (1 - p)R_H, \quad \text{and} \quad R_L + \Delta R < R_H. \]

The first restriction, among other things, implies that there will be insolvency costs for a low-quality intermediary after a low realization. The second restriction states that the outcome in the high realization is always higher than in the low realization, even for a high-quality intermediary.

### 3.2 Isolated intermediaries

We first focus on the situation in which intermediaries do not interact, and study the choice of an intermediary whether to be of high or low quality in this setting. The cash flows generated by project \( n \in \{1, 2\} \) are then

\[ \tilde{CF}_P^n(\xi, q) \overset{\text{def}}{=} \begin{cases} 
S^nR_H, & \xi = 1, \\
S^n(R_L + q\Delta R), & \xi = 0,
\end{cases} \]

the total cash flows generated to the owner, after insolvency costs are accounted for, are

\[ \tilde{CF}_1^n(\xi, q) = S^nY \left( \frac{\tilde{CF}_P^n(\xi, q)}{S^n} \right) = \begin{cases} 
S^nY(R_H), & \xi = 1, \\
S^nY(R_L + q\Delta R), & \xi = 0,
\end{cases} \]

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9 This assumption, similar to assumptions made, for example, in Elliott et al. (2014) and Acemoglu et al. (2013), is a stylized way of modeling the additional costs related to risk for insolvency, e.g., direct costs of bankruptcy, costs of fire sales, loss of human capital, customer and supplier relationship capital, etc. It could also more generally represent other types of convex costs of capital faced by a firm with low capitalization, along the lines described in Froot, Scharfstein, and Stein (1993).
and the $t = 0$ value of the intermediary is

$$V^n = V^n(q) = E_0[\overrightarrow{CF}_1^n] - C_0^n = s^n((1 - p)Y(R_H) + pY(R_L + q\Delta R) - qc). \quad (5)$$

Given the parameter restrictions (1), we have $V^n(0) = s^n(1 - p)R_H$. We assume that if an intermediary is indifferent between being high and low quality, it chooses to become low quality. Therefore, $q = 1$, if and only if $V^n(1) > s^n(1 - p)R_H$, immediately leading to the following result:

**Proposition 1.** An intermediary chooses to be of high quality, $q = 1$, if and only if

$$R_L + \Delta R > \max \left( d, \frac{c}{p} \right). \quad (6)$$

Qualitatively, the result above is very intuitive, implying that increases in costs of information acquisition, the probability of a high outcome, and costs of being insolvent, make it less attractive for an intermediary to be of high quality. The first argument in the maximum function on the RHS ensures that a high-quality firm avoids insolvency in the low realization. If the condition is not satisfied, there is no benefit to being high quality even in the low realization. The second argument ensures that the expected increase of cash flows in the low state outweighs the cost of investing in quality. The value of the intermediary when acting in isolation and following the rule (6) is then $V^n_I = s^nV_I$, where

$$V_I = \begin{cases} pR_H, & q = 0, \\ pR_H + (1 - p)(R_L + \Delta R) - c, & q = 1. \end{cases}$$

Note that the objective functions of the agents coincide with that of society in this special case. Specifically, given that society has the social welfare function, $V = \sum_n V^n$, under the constraint that intermediaries act in isolation, the socially optimal outcome is realized by the intermediaries’ joint actions. We obviously do not expect this to be the case in general, when agents interact.

### 3.3 Two intermediaries

We now explore the case with two interacting intermediaries, allows us to gain intuition in a fairly simple setting, before introducing the general $N$-agent network model. Intermediaries may enter into contracts that transfer risk. These contracts are settled according to a market-clearing system along the lines of that in Eisenberg and Noe (2001). Because of the high
dimensionality of the problem when we allow agents to act strategically, we necessarily have to assume a very limited contract space between intermediaries. Specifically, we assume that the contracts available are such that intermediaries swap claims on the aggregate cash flows generated by their projects in a one-to-one fashion, similar to Allen et al. (2012).

We focus on the case when the two intermediaries have the same scale \( s^1 = s^2 = 1 \). The contract is then such that intermediary 1 agrees to deliver \( \pi \times \widehat{CF}_P^1 \) to intermediary 2 at \( t = 1 \), and in turn receive \( \pi \times \widehat{CF}_P^2 \) from intermediary 2, for some \( 0 \leq \pi \leq 1 \). Our focus is on the two cases when project risks are shared equally (\( \pi = 0.5 \)) and when intermediaries act in isolation (\( \pi = 0 \)). We use the general \( \pi \) notation, since in the general case with \( N \) agents that we will analyze subsequently, \( \pi \) will typically take on other values.

The probabilities for the possible realizations of \((\xi^1, \xi^2)\) are

\[
\begin{align*}
\mathbb{P}(\xi^1 = 0, \xi^2 = 0) &= p_2, & \mathbb{P}(\xi^1 = 1, \xi^2 = 0) &= p_1, \\
\mathbb{P}(\xi^1 = 0, \xi^2 = 1) &= p_1, & \mathbb{P}(\xi^1 = 1, \xi^2 = 1) &= 1 - 2p_1 - p_2.
\end{align*}
\]

Consider a situation in which the intermediaries choose qualities \( q^1 \) and \( q^2 \), respectively, and let \( f^n(\xi^1, \xi^2) \) denote the binary variable that takes on value 0 if intermediary \( n \in \{1, 2\} \) is insolvent in state \((\xi^1, \xi^2)\), and 1 otherwise. Define

\[
\begin{align*}
z^1(\xi^1, \xi^2| q^1, q^2, \pi) &= (1 - \pi)f^1(\xi^1, \xi^2)\widehat{CF}_P^1(\xi^1, q^1) + \pi f^2(\xi^1, \xi^2)\widehat{CF}_P^2(\xi^2, q^2), \\
z^2(\xi^1, \xi^2| q^1, q^2, \pi) &= \pi f^1(\xi^1, \xi^2)\widehat{CF}_P^1(\xi^1, q^1) + (1 - \pi)f^2(\xi^1, \xi^2)\widehat{CF}_P^2(\xi^2, q^2).
\end{align*}
\]

Because of insolvency costs, it follows that

\[
\widehat{CF}^n(\xi^1, \xi^2| q^1, q^2, \pi) = f^n(\xi^1, \xi^2)z^n(\xi^1, \xi^2| q^1, q^2, \pi),
\]

and

\[
f^n(\xi^1, \xi^2) = \frac{z^n(\xi^1, \xi^2| q^1, q^2)}{s^n}.
\]

A realization of cash flows and insolvency that satisfies (7-10) in each state is said to be an outcome of the clearing mechanism. The time-0 value of an intermediary is then

\[
V^n(q^1, q^2| \pi) = \sum_{x_1, x_2 \in \{0, 1\}} \widehat{CF}^n(x_1, x_2| q^1, q^2, \pi)\mathbb{P}(\xi^1 = x_1, \xi^2 = x_2) - qcs^n.
\]

Equations (7-10) provide the adaptation of the clearing system in Eisenberg and Noe

\footnote{Note here that the subscripts of \( p \) refer to the number of projects that yield low realizations, \( n \) to which investment, \( n \), is considered (which is not needed since we assume symmetric probabilities).}
(2001) to our setting. This specification is almost identical to theirs (state by state), except for the important difference that insolvency is costly in our setting, represented by $d > 0$.

As a consequence of insolvency costs, there may be multiple solutions to the clearing mechanism (7,9,10) that lead to different net cash flows to intermediaries. This is because the insolvency of one intermediary can trigger the insolvency of another in a self-generating circular fashion. Along similar lines as Elliott et al. (2014), who also introduce solvency costs in their clearing mechanism, we focus on the unique outcome that minimizes the number of insolvencies. As noted in their study, since insolvencies are complements, all intermediaries, as well as society, agree that their number should be minimized. Briefly, their method initially assumes no insolvencies and then calculates which nodes become insolvent iteratively, taking into account that the insolvency of one node may trigger that of another.

We use a similar algorithm as Elliott et al. (2014) to define the outcome of the clearing mechanism, which will be the one we focus on henceforth. The algorithm also leads to a natural shock-propagation mechanism, through which insolvencies spread step-by-step, triggering others. With two intermediaries, the propagation mechanism is of course simple: either the insolvency of one intermediary triggers that of the other, or it does not. In the general case an iterative algorithm is needed, which we define in Appendix A. We stress that although our clearing mechanism is similar to that in earlier work, what distinguishes or work is our focus on the endogenous development and coexistence of heterogeneous financial norms (represented by the $q$’s), and how these norms are influenced by and influence the equilibrium network structure.

It is worthwhile discussing why it may be beneficial for agents to interact. Clearly, the contracts offer a simple form of risk-sharing. Agents are risk neutral, but the cost of insolvency introduces a motive for avoiding low outcomes that trigger solvency costs, effectively introducing risk aversion. By sharing risks, the negative effects of a low realization for the two agents can be limited.

It is also clear that the incentive for an agent to invest in high quality is affected by the interaction with other agents. One may conjecture several potential effects, depending on the economic environment. In some circumstances, an agent’s incentive to invest in quality may decrease compared with the case of no interaction, since the benefits of quality are shared whereas the costs are not. Under other circumstances, the incentive to invest in quality may actually increase, because risk sharing allows insolvency to be avoided for high-quality intermediaries that interact, although it cannot be avoided when they act in isolation. Thus our stylized contracting environment promises to allow for rich equilibrium behavior.
3.3.1 Equilibrium

Intermediaries can either act in isolation (corresponding to $\pi = 0$) or share risks ($\pi = 1/2$). For a risk-sharing outcome to be an equilibrium, both agents must have correct beliefs about the quality decisions made by their counterparties. We make the standard assumptions that each agent may unilaterally decide to sever a link to the other agent, and that bilaterally the two agents can decide to add a link between themselves. For an outcome with risk sharing to be an equilibrium, it follows that neither agent can be made better off by acting in isolation. For an isolated outcome to be an equilibrium, it cannot be that both agents are better off by sharing risk.

We model the mechanism by a strategic game with the sequence of events described in Figure 2, where we have formulated the game in the general $N$ agent case. At $t = -2$, given

![Figure 2: Sequence of events in network formation game with endogenous financial norms.](image)

that $\pi = 1/2$, each agent may unilaterally decide to sever the link to the other agent and switch to $\pi = 0$, leading to the isolated outcome. If, on the other hand, $\pi = 0$, each agent can propose to switch to $\pi = 1/2$, in which case the other agent has the option to accept or decline at $t = -1$. Then, after the resulting network is determined, agents choose quality and outcomes are realized. Note that we implicitly assume that the actual quality decision is not contractible.\footnote{In our stylized model, the quality decision can of course be inferred from the realization of project cash flows. The issue could be avoided by assuming a small positive probability for $\Delta R = 0$ in case of a low realization. For simplicity, we assume that contracts are restricted to being linear in realized project cash flows.}
An equilibrium is now described by \((q^1, q^2)\), and \(\pi\), such that neither agent has an incentive to sever the link (in the case \(\pi = 1/2\)), and it is not the case that both agents have an incentive to form a link (in the case \(\pi = 0\)). Moreover, the agents’ beliefs about the other agent’s actions, both in the case when \(\pi\) remains the same and in the case when it switches because of actions at \(t = -2\) and \(t = -1\), need to be correct.

For the action \(q \in \{0, 1\}\), we let \(\neg q\) denote the complementary action (\(\neg q = 1 - q\)). It follows that the three numbers, \(q^1, q^2\) and \(\pi > 0\), describe an equilibrium with risk sharing if:

\[
\begin{align*}
V^1(q^1, q^2|\pi) &\geq V^1(-q^1, q^2|\pi), \\
V^2(q^1, q^2|\pi) &\geq V^2(q^1, -q^2|\pi), \\
V^1(q^1, q^2|\pi) &\geq V^1, \\
V^2(q^1, q^2|\pi) &\geq V^2.
\end{align*}
\]

The first two conditions ensure that it is incentive-compatible for both agents to choose the suggested investment strategies given that they share risks, whereas the latter two state that risk sharing dominates acting in isolation for both agents.

Interesting dynamics arise already in this network with only two intermediaries, as seen in the following example. We choose parameter values \(R_H = 1.2, R_L = 0.1, \Delta R = 0.5, c = 0.05, p_1 = 0.1, p_2 = 0.05, \pi = 0.5\), and vary the insolvency threshold, \(d\). Since the setting is symmetric, it follows that \(V^1 = V^2 = V_I\), and \(V^1(q, \neg q) = V^2(\neg q, q)\), reducing the number of incentive constraints that need to be checked. The resulting value functions are shown in Figure 3.

There are five different regions with qualitatively different equilibrium behavior. In the first region, \(0 < d < 0.35\), the unique equilibrium is the one where both agents invest in quality, \((q^1, q^2) = (1, 1)\), and there is no risk-sharing, leading to values \(V_I\) for both intermediaries. No intermediary ever becomes insolvent in this case (since \(R_L + \Delta R > d\)). The outcome where both agents invest and share risk would lead to the same values, but cannot be an equilibrium because each agent would deviate and choose to avoid investments in this case, given that the other agent invests. For example, if intermediary 1 does not invest, but intermediary 2 does, agent 1 reaches \(V^1(0, 1) > V^1(1, 1)\) by avoiding the cost of investment but still capturing the benefits of not becoming insolvent after a low realization. Therefore, \(V^1(1, 1)\) cannot be sustained in equilibrium. Now, \(V^1(0, 1)\) can of course not be an equilibrium either, since under this arrangement intermediary 2 is on the (blue) \(V^1(1, 0)\) line, which is inferior to \(V_I\). So only the isolated outcome survives as an equilibrium.

In the second region, \(0.35 \leq d < 0.6\), there are two equilibria, both with investments,
Figure 3: Value functions in economy with two intermediaries, as a function of solvency threshold, $d$. The following value functions are shown: $V_I$ (circles, dotted black line), $V^1(1, 1)$ (squares, magenta), $V^1(0, 1)$ (stars, red), $V^1(1, 0)$ (pluses, blue), and $V^1(0, 0)$ (crosses, green).

$(q^1, q^2) = (1, 1)$, and the same value for both intermediaries, $V_I$. In addition to the isolated outcome, the outcome with risk-sharing and investments in quality by both agents is now an equilibrium. The reason is that the solvency threshold has now become so high that agent 1 has an incentive to invest in the risk-sharing outcome when agent 2 invests, to avoid insolvency which otherwise occurs if both $\xi^1 = 0$, and $\xi^2 = 0$.

The third region is $0.6 \leq d < 0.65$, in which the unique equilibrium is for intermediaries to share risk and not invest, $(q^1, q^2) = (0, 0)$, leading to value $V^1(0, 0)$ for both agents. Indeed, this strategy dominates the value under isolation, $V_I$, which for $d \geq 0.6$ entails the strategy of not investing in quality since in that region insolvency occurs even when such investments are made (this is the reason for the discontinuity in $V_I$ at $d = 0.6$). Note that $V^1(0, 0)$ is dominated by $V^1(1, 1)$ and $V^1(0, 1)$, though neither can constitute an equilibrium. The outcome $V^1(1, 1)$ is not sustainable, since it is better for either agent to switch to not investing in quality, as it is for agent 2 under $V^1(0, 1)$. So the only equilibrium is the one with risk-sharing.

When $0.65 \leq d < 0.9$, i.e., in the fourth region, $V^1(0, 0)$ decreases substantially compared with the third region, because for such high levels of the default threshold, both intermediaries become insolvent if there is one low realization, whereas two low realizations were needed in the third region. This makes $V^1(0, 0)$ inferior to the isolated outcome, $V_I$ (in which
both agents choose not to invest since $d$ is so high), because of a contagion effect. When risks are shared, a low realization for one intermediary not only causes that intermediary to become insolvent but also triggers the insolvent of the other intermediary. Thus, the only remaining equilibrium is now $V^1(1, 1)$, i.e., for agents to share risk and for both to invest in quality, $(q^1, q^2) = (1, 1)$.

Finally, when $d \geq 0.9$, the isolated equilibrium without quality investments, $(q^1, q^2) = (0, 0)$, is the only remaining equilibrium, since any risk sharing equilibrium will lead to contagion.

We note that equilibrium quality choice is non-monotone in $d$, in contrast to the isolated case in which $q$ is naturally non-increasing in $d$ (see (1)). With interaction between intermediaries, the unique quality choice in the third region, $0.6 \leq d < 0.65$, is $(q^1, q^2) = (0, 0)$, whereas in the fourth region, $0.65 \leq d < 0.9$, $(q^1, q^2) = (1, 1)$ in equilibrium, as the prospects for high-quality investments increases with $d$ in parts of the domain in this case. Then, for $d > 0.9$, investments in quality again become inferior, leading to $(q^1, q^2) = (0, 0)$.

We also note that all equilibrium outcomes have $q^1 = q^2$. This is natural for the isolated equilibria, but also occurs for the risk-sharing equilibria. It suggests that the “financial norm”—defined as the quality it chooses—depends on the financial norms of the intermediary with which it interacts. We wish to explore this effect in more complex financial networks. In such networks, not only will the actions of neighbors of an intermediary matter but so will the actions of the neighbors of those intermediaries, their neighbors’ neighbors, etc.

### 3.4 General networks of intermediaries

We now introduce the general network model with $N \geq 2$ intermediaries, represented by the graph $\mathcal{G} = (\mathcal{N}, E), \mathcal{N} = \{1, \ldots, N\}$. The relation $E \subset \mathcal{N} \times \mathcal{N}$ describes which intermediaries are connected in the network. Specifically, the edge $e = (n, n') \in E$, if and only if there is a connection (edge, link) between intermediary $n$ and $n'$. No intermediary is connected to itself, $(n, n) \notin E$ for all $n$, i.e., $E$ is irreflexive. We define the transpose of the link $(n, n')^T = (n, n')$, and assume that connections are bidirectional, i.e., $e \in E \iff e^T \in E$. The operation $E + e = E \cup \{e, e^T\}$, augments the link $e$ (and its transpose) to the network, whereas $E - e = E \setminus \{e, e^T\}$ severs the link if it exists. The number of neighbors of node $n$ is $Z_n(E) = |\{(n, n') \in E\}|$.

Intermediaries will in general have different scale and number of neighbors, and therefore choose to share different amounts of risk amongst themselves. Similar to the case with two intermediaries, we choose a simple sharing rule, represented by the sharing matrix $\widehat{\Pi} \in \mathbb{R}^{N \times N}_+$, where $0 \leq (\widehat{\Pi})_{nn'} \leq 1$ is the amount of risk that is swapped between intermediary $n$ and
with the summing up constraint that \( \hat{\Pi} s = s \), where \( s = (s^1, \ldots, s^N)' \). It will be more convenient to characterize the fraction of risk that agent \( n' \) shares with agent \( n \), which is represented by the matrix \( \Pi = \hat{\Pi}\Lambda_{s}^{-1} \). Here, we have used the notation that for a general vector, \( v \in \mathbb{R}^N \), we define the diagonal matrix \( \Lambda_v = \text{diag}(v) \). We also use the notation that \( \delta_n \) represents a vector of zeros, except for the \( n \)th element which is 1. In our previous example with two intermediaries of unit scale, \( s = (1, 1)' \) and the sharing matrix is

\[
\Pi = \begin{bmatrix}
1 - \pi & \pi \\
\pi & 1 - \pi
\end{bmatrix}.
\]

The network represents a restriction on which sharing rules are feasible. As we will discuss, this restriction can be self-imposed by intermediaries in equilibrium, who may chose not to interact even if they may, or it could arise exogenously. Specifically, for a sharing rule to be feasible it must be that every off-diagonal element in the sharing matrix that is strictly positive is associated with a pair of agents who are linked, \( \hat{\Pi}_{nn'} > 0 \Rightarrow (n, n') \in E \).

We choose to study a specifically simple class of sharing rules that ensure that all weights are nonnegative and that each intermediary keeps some of its own project risk, namely

\[
(\hat{\Pi})_{nn'} = \min \left\{ \frac{s^n}{1 + Z_n(E)}, \frac{s^{n'}}{1 + Z_{n'}(E)} \right\}, \quad n \neq n',
\]

and

\[
(\hat{\Pi})_{nn} = s^n - \sum_{n' \neq n} \hat{\Pi}_{nn'}.
\]

We write \( \Pi(E) \) when stressing the underlying network from which the sharing rules is constructed.

The joint quality decision of all agents is represented the vector \( q = (q^1, \ldots, q^N) \in \{0, 1\}^N \). In the general case, the cost of investing in quality may vary across intermediaries, represented by the vector \( c = (c^1, \ldots, c^N) \in \mathbb{R}_+^N \). The state realization is represented by the vector \( \xi = (\xi^1, \ldots, \xi^N) \in \{0, 1\}^N \). We will work with a limited state space, assuming that \( \xi \in \Omega \subset \{0, 1\}^N \) where \( \Omega \) is a strict subset of \( \{0, 1\}^N \), and mainly focus on three such sets: The first set is \( \Omega^1 = \{ \xi \in \{0, 1\}^N : \xi^1 \geq N - 1 \} \), with \( \mathbb{P}(\xi = 1 - \delta_n) = p_1 \), \( 1 \leq n \leq N \), and associated probability space \( \mathbb{P} : \Omega^1 \to [0, 1] \). Here, \( \mathbf{1} = (1, \ldots, 1) \) is an \( N \)-vector of ones. For this set, either zero or one low realization occurs, and the probability for a low realization is the same for all intermediaries. The second state space is \( \Omega^2 = \{ \xi \in \{0, 1\}^N : \xi^1 \geq N - 2 \} \), for which no more than two realizations may be low, with full symmetry across intermediaries,
so that
\[ P(\xi = 1 - \delta_n) = p_1, \quad 1 \leq n \leq N, \]
\[ P(\xi = 1 - \delta_n - \delta_{n'}) = p_2, \quad 1 \leq n < n' \leq N, \]
\[ P(\xi = 1) = 1 - Np_1 - \frac{N(N - 1)}{2}p_2. \]

Finally, the third state pace is \( \Omega^A = \{0, 1\} \), for which either no or all realizations are low, the latter case with probability \( P(\xi = 0) = p_A \).

Solvency is represented by a vector \( f \in \{0, 1\}^N \), and realized cash flows to agents by the random vector \( \widetilde{CF} = (\widetilde{CF}_1, \ldots, \widetilde{CF}_N)' \in \mathbb{R}_+^N \). The realized project cash-flows are then represented by the vector \( \widetilde{CF}_p = (\widetilde{CF}_1, \ldots, \widetilde{CF}_N)' \), where
\[ \widetilde{CF}_p^n(\xi, q) = R_H\xi^n + (R_L + q^n\Delta R)(1 - \xi^n), \quad n = 1, \ldots, N. \]

The general network version of the clearing system that calculates a mapping \( \widetilde{CF}_1(\xi, q) = \mathcal{CM}[\widetilde{CF}_p(\xi, q)] \), is described by the iterative algorithm described in detail in Appendix A. The outcome of the algorithm is the solution that minimizes the number of insolvencies, or equivalently maximizes the total cash-flows to agents. In vector form, this can be written as
\[ \widetilde{CF}_1 = \mathcal{CM}[\widetilde{CF}_p] = \max_f (A_f \Pi A_f) \times \widetilde{CF}_p, \quad \text{s.t.} \quad (13) \]

\[ f = X (\Lambda_s^{-1} \times \widetilde{CF}_1). \quad (14) \]

Here, \( X \) operates element-wise in (14), \( X(v) = (X(v^1), X(v^2), \ldots, X(v^N))' \). The net cash flows to the intermediaries are then given by the vector
\[ w(\xi|q, E) = \widetilde{CF}_1(\xi) - c\Lambda_s q. \quad (15) \]

This is the general network version of equations (7-10). The \( t = 0 \) value vector of intermediaries, given quality investments, \( q \), and network \( E \) is then given by
\[ V(q|E) = \sum_{\xi \in \Omega} w(\xi|q, E)P(\xi). \quad (16) \]

A variation of the algorithm assumes that insolvencies only propagate up until a fixed number of steps, \( \bar{m} \), terminating the algorithm when \( m \) reaches \( \bar{m} \) (see the description in Appendix A). We write \( \widetilde{CF}_1 = \mathcal{CM}[\widetilde{CF}_p|\bar{m}] \) in this case, and the standard version of the clearing mechanism is then \( \mathcal{CM}[\widetilde{CF}_p] = \mathcal{CM}[\widetilde{CF}_p|\infty] \).
3.4.1 Equilibrium

Let $E^*$ denote the complete network in which all agents are connected. We assume that there is a maximum possible network, $\bar{E} \subset E^*$, such that only links that belong to $\bar{E}$ may exist in the sharing network. This restriction on feasible networks could, for example, represent environments in which it is impossible for some agents to credibly commit to deliver upon a contract written with some other agents, due to low relationship capital, limited contract enforcement across jurisdictions, etc. A network, $E$, is feasible if $E \subset \bar{E}$. If all agents who may be linked actually choose to be linked in equilibrium, i.e., if $E = \bar{E}$, we say that the equilibrium network is maximal. It may also be the case that $E$ is a strict subnetwork of $\bar{E}$, just as was the case in the economy with two intermediaries where $\bar{E} = E^*$, but $E = \emptyset$ for some parameter values because agents chose the isolated outcome in equilibrium.

To define equilibrium, we build upon the pairwise stability concept of Jackson and Wolinsky (1996). We require equilibrium in this multistage game to be subgame perfect. The game and equilibrium requirements are explained in detail in the appendix. Here we provide a summary. The sequence of events is as in Figure 2. Consider a candidate equilibrium, represented by a network $E$ and quality choices $q$. Each agent, $n$, has the opportunity to accept the sharing rule, $\Pi(E)$, as is, by neither severing nor proposing new links at $t = 0$. But, in line with the pairwise stability concept, any agent $n$ can also unilaterally decide to sever a link with one neighbor, $n'$, leading to the sharing network $E' = E - (n,n')$, and corresponding sharing rule $\Pi(E')$. Also, any agent can propose an augmentation of another link $(n,n') \in \bar{E} \setminus E$, which if agent $n'$ accepts leads to the sharing network $E'' = E + (n,n')$ with sharing rule $\Pi(E'')$. Finally, we assume that each agent can unilaterally choose the isolated outcome, $V^n_I$, by severing its links to all other agents.

The possibility to unilaterally sever all links in a sharing network—although not technically part of the standard definition of pairwise stability—is natural, in line with there being a participation constraint that no intermediary can be forced to violate. It provides a slight extension of the strategy space.

The severance, proposal, and acceptance/rejection of links occur at $t = -2$ and $t = -1$. The agents then decide whether to invest in quality or not at $t = 0$, each agent choosing $q^n \in \{0,1\}$. A pair $(q,E)$, where $E \subset \bar{E}$, is now defined to be an equilibrium, if agents given network structure $E$ choose investment strategy $q$, if no agent given beliefs about other agent’s actions—under the current network structure as well as under all other feasible network structures in $\bar{E}$—has an incentive to either propose new links or sever links, and if every agent’s beliefs about other agents actions under network $E$ as well as under all alternative network formations are correct.
4 Analysis of equilibrium

We wish to understand how the quality choices—the financial norms—of agents affect and are affected by equilibrium network structure. To gain some intuition, we first study a specific example with $N = 8$ intermediaries, all of which have scale equal to unity, $s = 1$, with the shock structure $\Omega^2$. The maximal network, $\bar{E}$, which is also an equilibrium network is shown in panel A of Figure 4. We say that the equilibrium network is maximal. As shown in panel A, for low $\Delta R$ the equilibrium outcome is for all intermediaries to be of low quality. This is simply because investing is quality does not increase the payoff in low states sufficiently to avoid insolvency for any intermediary for such low $\Delta R$.

For $\Delta R = 0.12$, shown in Panel B, by investing in quality, intermediaries can now avoid insolvency. This is what intermediaries 6-8 do in equilibrium. However, intermediaries 1-5 cannot sustain an equilibrium in which they also invest in quality, since it is too tempting for them to free-ride on the investments by others. This is because they have so many neighbors that their investment decision is not pivotal for avoiding insolvency. For $\Delta R = 0.15$, shown in Panel C, it is an equilibrium strategy also for 1 and 5 to invest in quality. This is because they can now avoid insolvency for several cases in which two shocks hit the network, in which case they are pivotal. This is less of a carrot for agents 2-4 though, who still cannot sustain quality investments in equilibrium. Finally, when $\Delta R = 0.2$, nodes 2-4 can also be made to invest in quality in equilibrium, since they are now also pivotal in avoiding insolvency after two shocks in a sufficient number of states. Thus, as shown in Panel B and C, heterogeneous financial norms may coexist in equilibrium, and intermediaries with the same quality tend to be linked (or at least to be close in the network) in these situation.

We wish to explore this relationship between network structure and quality choice for a larger class of networks. We simulate 1,000 networks for which equilibrium exists, each with $N = 9$ nodes. We use the Erdős-Rényi random graph generation model, in which the probability that there is a link between any two agents is i.i.d., with the probability 0.25 for a link between any two nodes, and we also randomly vary $c$ across intermediaries, $c^n \sim U(0, 0.025)$. For computational reasons, we focus on networks in which equilibria are maximal, $E = \bar{E}$.

Table 1 shows summary statistics for nodes that are of high quality compared with those of low quality in equilibrium. We see that there are on average more high-quality nodes in equilibrium with these parameters. Also, the average cost of investing in quality for high-quality nodes is lower than for low-quality nodes. More interestingly, the average number of neighbors of high-quality nodes is higher, and the average quality of neighbors of high-quality nodes is higher than of low-quality nodes. All these differences are statistically significant.
Figure 4: Equilibrium outcomes for different $\Delta R$. Low quality nodes are marked in black, whereas high quality nodes are yellow. In panel A (upper left corner), $\Delta R = 0.05$, and all nodes are of low quality. In panel B (upper right corner), $\Delta R = 0.12$, and nodes 6–8 are of high quality. In panel C (lower left corner), $\Delta R = 0.15$, and nodes 1, 5–8 are of high quality. In panel D, $\Delta R = 0.2$ and all nodes are of high quality. Parameters: $c = 0.0025$, $R_H = 1.2$, $R_L = 0.1$, $d = 0.68$, $p_1 = 0.0625$, $p_2 = x$, $\bar{E} = E$. Shock structure $\Omega^2$, and scale $s = 1$. 
Table 1: Summary statistics of high- and low-quality intermediaries. Number of simulations: 1,000. Parameters: $R_H = 1.1$, $R_L = 0.2$, $\Delta R = 0.3$, $d = 0.75$, $p_1 = 4/90$, $p_2 = 1/90$, $c \sim U(0, 0.025)$. Shock structure $\Omega^2$, an scale $s = 1$.

The last result is especially important, since it shows that the financial norms that arise in the network are indeed closely related to network positions, i.e., that different clusters exist in which nodes have different norms. Another way of measuring whether such clusters exist is to partition each network into a high-quality and a low-quality component, and study whether the number of links between these two clusters is lower than what would be if quality were randomly generated across nodes. Specifically, consider a network with a total of $K = |\{(n, n') \in E\}|$ links, and a partition of the nodes into two clusters: $\mathcal{N} = \mathcal{N}^A \cup \mathcal{N}^B$, of size $N^A = |\mathcal{N}^A|$ and $N^B = |\mathcal{N}^B| = N - N^A$, respectively, and the number of links between the two components: $M = |\{(n, n') \in E : n \in \mathcal{N}^A, n' \in \mathcal{N}^B\}|$. In the terminology of graphs, $M$ is the size of the cut-set, and is lower the more disjoint the two clusters are. The number of links one would expect between the two clusters, if links were randomly generated, would be $W = \frac{1}{N(N-1)}N^A N^B K$, so if the average $M$ in the simulations is significantly lower than the average $W$, this provides further evidence that financial norms are clustered. Indeed, the average $M$ in our simulations is 12.2, substantially lower than the average $W$ which is 14.2, corroborating the presence of endogenous financial norms.

### 5 Computation of Equilibrium

Although the equilibrium calculations are straightforward in economies with networks up to about 15 nodes, they become computationally infeasible for real-world large-scale networks—potentially with thousands of intermediaries. In addition, in practice some or all of the model parameters ($d$, $p$, $R_L$, and $R_H$) may be unobservable, and therefore have to be estimated from observed dynamics. We introduce a numerical method that addresses these two issues jointly, by approximating an equilibrium that optimally matches observed insolvency dynamics, and that can be applied to large-scale problems.

Specifically, we assume that $w$, $E$, and $c$ are observable, whereas the parameter values $R_L$, $R_H$, $d$, $p$, and $\Delta R$, and the quality vector $q \in \{0, 1\}^N \overset{\text{def}}{=} \mathcal{D}$ are not. We assume that the shock structure is $\Omega^A$, and use the clearing mechanism $\mathcal{CM}[\mathcal{CF}_P|2]$, allowing for two steps of shock propagation.
The value of project cash-flows, given a bad state realization in $\Omega^A$, are:

$$\overline{CF}_P(0, q) = \Lambda_s(R_L 1 + \Delta R \times q),$$

which, via the clearing mechanism, leads to the function $w^n(\xi|q, E), n = 1, \ldots, N$.

Now, assume that we conjecture that the equilibrium quality vector is $q$. We can calculate the best response set for agents (at time 0), given the actions of other agents, $F(q) = \mathcal{F}(q|R_L, R_H, p, \Delta R, d, c) = \{\hat{q} : \hat{q}^n = \arg \max_{x \in \{0,1\}} V^n(x, \hat{q}^{-n})\}$. For $q$ to be consistent with equilibrium, it must be that $q \in F(q)$. We define the equilibrium set $Q \subset D$, as

$$(q) = \{q : q \in F(q)\}.$$

Assume that $v^n = w^n + \epsilon, n = 1, \ldots, N$, are observed after a shock, where the measurement errors, $\epsilon^n$ are i.i.d. An estimate of the unobservable quality vector, $q$, is then given by solving the least squares problem:

$$\min_{\Delta R, R_L, R_H, p, d, q \in \mathcal{Q}(R_L, R_H, p, \Delta R, d, c)} ||v - CM[\overline{CF}_P(0, q|2)] + \Lambda_s \Lambda_c q||.$$

(17)

Here, the mean-square ($L^2$) norm or the least absolute deviations ($L^1$) norm, for example, could be used.

The program (17) is an integer optimization problem over a nontrivial domain $\mathcal{Q}$. A simplification is given by replacing the constraint $q \in \mathcal{Q}$ with the penalty function $||q - F(q)||$, leading to the relaxed problem:

$$\min_{\Delta R, R_L, R_H, p, d, q \in D} ||v - CM[\overline{CF}_P(0, q|2)] + \Lambda_s \Lambda_c q|| + A||q - F(q)|\Delta R, R_L, R_H, p, d, c)||].$$

(18)

The properties of the solutions to the relaxed problem (18) vary with the parameter $A$. For large $A$, the equilibrium constraint becomes relatively more important, and for sufficiently large $A$ the solution will be the same as the solution to (17). For $A = 0$, no equilibrium constraint is imposed. The advantage of choosing a small $A$ is that a closer match to $v$ will be found. The disadvantage is that the equilibrium constraints imposed by requiring agents to act consistently are weakened. An immediate effect of choosing a low $A$ is that the calculated $q$ may become very volatile across nodes for small $A$.

In our equilibrium computations, we use the formulation (18), with the least absolute deviations norm and parameter value $A = 1$. An advantage with using the least absolute deviations norm is that the the second error term in (18) has the natural interpretation of being the number of agents whose strategies are not consistent with equilibrium.
6 Empirical identification of the mortgage network

A key component of our theoretical model is that links are defined through the passing of risky “projects” from one node to another. The natural interpretation in mortgage markets is that these projects represent loans. An alternative application of our model would be to study ownership links, as done in XX and YY, focusing on the liability side of the balance sheet. We believe that our approach, focusing on the loan side, is especially constructive to understand the mortgage market in recent years, given the important role of loan foreclosures in the crisis, and in line with Nier et al. (2008).

6.1 Tracking mortgages from originator to securitizer

The available data sources that account for the firm-level composition of the residential-mortgage origination market define individual mortgage originators either as entities that underwrite and fund mortgage originations (the definition used by the Home Mortgage Disclosure Act (HMDA) surveys) or alternatively through the identification of the legal creditor of record as shown in the local land recording facilities, the assessor’s (or equivalent) offices. We use the latter source of data and definition of originator in this paper. Both HMDA and the assessor data allow originators to be classified by type, such as federal commercial banks, S&Ls/state banks, finance companies, credit unions, and mortgage companies. This classification of individual mortgage originators into lender type makes an analysis of the network structure of the industry tractable. It also corresponds well to the dueling regulatory and functional structure of the industry.

A significant challenge in network analysis of the U.S. mortgage market is the fact that there is no unique mortgage identifier that can be used to track individual mortgages through the supply chain. Thus, identifying the supply chain network from the address of the houses that collateralize the loans, the identity of the legally recorded loan originator, the pools or special purpose entities (SPE) in which the loans are securitized, the holding companies that exercise the control rights to structure the SPEs, to underwrite the bonds, and often to retain the equity positions of the SPEs requires complicated merging protocols between disparate data sets. Another challenge is measuring the life-of-loan performance of individual loans in the mortgage network. To address these limitations, we create a new data set that merges together three different data sets. These data sets are: 1) the Dataquick Historical Transaction data that records the legal originator of record, the recording date and the loan principal but has no other information about the mortgages such as the performance of the loan or who it was sold to; 2) the newly updated ABSNet loan- and pool-level origination and transaction data that includes detailed information about the individual loans in each
pool and their performance over time but less information about the ownership of the shelf and the initial originator of record; 3) the prospectuses for all the pools in the ABSNet data that provide the legal names of all the agents involved in the securitization and the legal name of the SPE which we obtained from the Securities and Exchange Commission (SEC) website.

The Dataquick Transaction data provides extensive coverage of lien records for all of the U.S. including the name of the legally recorded originator of the mortgage and a taxonomy of originator types (e.g., bank, finance company, credit union, home builder, mortgage company, savings and loan institution, Mortgage Electronic Registration System (MERS)\(^\ddagger\)). The Dataquick assessor’s data also includes information on the loan amount; whether it is a first, second, or third lien position; the mortgage recording date; and the value of the house price at purchase (if the mortgage is a purchase-money mortgage). There are 103 million individual first-lien records, including every state in the U.S. from 1996–2013, in the Dataquick Transaction Data.

An important advantage of the Dataquick originator taxonomy is that it allows us to separately identify mortgage-company subsidiaries, finance companies, and the regulated bank or savings-and-loan retail lender within large bank or thrift holding companies such as Wells Fargo and Countrywide. The taxonomy also allows us to unravel the different functional entities within large mortgage companies such as New Century (a mortgage REIT with many subsidiaries). This taxonomy is important because it makes the empirical network analysis tractable while accurately representing the primary competitive differences among the firms.

Since our interest is in securitized loan networks, we merge the Dataquick lien data with loan-level data obtained from ABSNet. The ABSNet data set includes detailed information about private-label securitized mortgages, including the initial loan balance, loan contract features, the loan zip code, the origination date, and the identifiers for the special purpose entity (SPE) in which the loan is securitized. ABSNet records a total of 13,453,796 first-lien loan originations between 2002 and 2007. We successfully merge 9,099,280 of these loan records with the Dataquick Historical Transaction data, giving us an originator of record and a lender type for each merged loan. For the remaining unmerged Dataquick loans, we found that Dataquick had a lender name for 3,370,730 of these loans and we used the Dataquick taxonomy to assign a lender-type to each. We were unable to identify a lender name for

\(^\ddagger\)As discussed in Hunt, Stanton, and Wallace (2012), the MERS system was widely used to record liens. Interestingly, in our data the assessor lien records indicate whether it was a MERS-recorded lien, but in addition gives the legal name of the origination lien holder. For a relatively small number of mortgages, the name of the actual lien holder name is missing, so that MERS appears as the lien holder. We chose not to delete these observations if we knew the remaining identities in the supply chain.
1,083,796 loans, so these observations were discarded.

We then downloaded all the prospectuses for all the residential mortgage-backed security deals from the SEC website to obtain the full deal name for each SPE in the ABSNet data from each prospectus. We then hand-searched the SEC deal names to obtain the name of the shelf registration for the SPE and the identity of the holding company that had made the shelf registration. We successfully linked 12,370,000 loans to their county of origin, lender of record, lender type, securitization shelf, and the holding company that controlled the shelf registration. For each loan, we track the month-by-month payment performance from the origination date until December 2013 (the end of our performance data). We thus identify all the ex post foreclosure outcomes for each loan over our performance period.


In our model of intermediary networks the functional and geographic location of agents matters. We therefore first develop a network representation technology that accounts for spatial competition among lender-types to originate mortgages for borrowers (houses) within proximate geographic areas. As discussed previously, there are seven first-lien originator types in our taxonomy: banks, credit unions, finance companies, home builders, mortgage companies, savings and loan institutions, and MERS. Table 2 shows that there was a significant increase in first-lien private-label securitized lending from 2002 until its peak in 2005, followed by a decline until 2007. Mortgage companies accounted for more than half of the first-lien originations over the period. The next largest lender types were the banks (regulated by the Office of the Comptroller of the Currency (OCC) and the Federal Deposit Insurance Corporation (FDIC)) and the savings and loan institutions (regulated by the Office of Thrift Supervision). Both of these lender types were engaged primarily in retail lending (again these categories do not include the mortgage company and financial company subsidiaries of these institutions). The home builders with mortgage subsidiaries, the credit unions, and the MERS originators accounted for a much smaller market share of overall private-label securitized first lien mortgage origination.

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13Our representation also accounts for competition to purchase originated mortgages across the securitization SPEs, and competition among the holding companies that ultimately control the securitization channels.

14Private-label securitization accounted for 21.26% of all mortgage origination in the U.S. in 2005 (see Inside Mortgage Finance, January 12, 2007).

15The “mortgage company” lender type includes the mortgage company subsidiaries of large commercial banks such as Wells Fargo Mortgage Company, Inc., or Countrywide Home Loans, Inc., that may or may not have been consolidated on the regulated holding-company balance sheets, and whose lending activities were regulated separately from the lending activities of the retail entities within the same holding companies (see Engel and McCoy, 2011). These entities also had different cost structures due to their organization as wholesale lenders that usually did not involve the ownership of actual bricks and mortar origination facilities.
Table 2: Private-label securitized residential mortgage originations by lender type and year of origination.

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banks</td>
<td>106,253</td>
<td>238,703</td>
<td>434,382</td>
<td>613,001</td>
<td>555,028</td>
<td>206,931</td>
<td>2,154,298</td>
</tr>
<tr>
<td>Credit Unions</td>
<td>9,567</td>
<td>19,999</td>
<td>34,531</td>
<td>44,313</td>
<td>36,928</td>
<td>11,140</td>
<td>156,478</td>
</tr>
<tr>
<td>Finance Companies</td>
<td>10,987</td>
<td>15,099</td>
<td>11,885</td>
<td>11,032</td>
<td>2,408</td>
<td>148</td>
<td>51,559</td>
</tr>
<tr>
<td>Home Builders</td>
<td>88,205</td>
<td>154,516</td>
<td>301,570</td>
<td>324,607</td>
<td>222,300</td>
<td>70,925</td>
<td>709,123</td>
</tr>
<tr>
<td>Mortgage Companies</td>
<td>574,680</td>
<td>1,086,534</td>
<td>1,628,435</td>
<td>1,955,751</td>
<td>1,644,587</td>
<td>463,350</td>
<td>7,353,337</td>
</tr>
<tr>
<td>Savings and Loans</td>
<td>107,799</td>
<td>175,563</td>
<td>322,968</td>
<td>409,257</td>
<td>368,464</td>
<td>101,315</td>
<td>1,485,366</td>
</tr>
<tr>
<td>MERS</td>
<td>808</td>
<td>657</td>
<td>121</td>
<td>112</td>
<td>1,018</td>
<td>182</td>
<td>2,898</td>
</tr>
<tr>
<td>Totals</td>
<td>898,299</td>
<td>1,691,071</td>
<td>2,733,892</td>
<td>3,358,073</td>
<td>2,830,733</td>
<td>853,991</td>
<td>12,370,000</td>
</tr>
</tbody>
</table>

In our networks, we assign each loan to the lender-type and county pairing for the observed originator of record for the loan and for the location of the house collateralizing the loan. There are 1,848 counties in our data, and hence the potential for 12,936 pairings of lender-type and county. Each of these pairings is then linked with the observed securitization shelf for each of the loans from the lender-type and county pairing. An important characteristics of the observed mortgage origination networks is that single county/lender-type nodes are securitized through more than one shelf (a one to many mapping). There are a total of 190 pool securitization shelves and 77 separate holding companies in our data.

Figure 5 shows the mortgage network for years 2002, 2005, 2006 and 2007, each panel showing the network of originations for a different year. The upper rim of the network represents the lender-type and county pairs for mortgages originated in each year. The second tier of nodes represents the shelves in which the mortgages were securitized, and the lowest tier of nodes represents the holding companies that control the shelves. As is clear from the networks presented in Figure 5, the private-label securitized mortgage market was highly inter-linked, and the volume of origination (the density of the origination linkages in the upper tier) was significantly higher in 2005 than in 2002, although the number of holding companies controlling the securitization flow remained quite small and concentrated. Performance is measured by the percentage of loans along a given edge that have foreclosed by December, 2013, and an edge is colored red if more than 30% of the loans tracked by the edge have foreclosed by 2013.

Comparing panels a) and b), the ex post performance of the 2002 and 2005 vintages of origination differed in the intensity and the geographic location of the foreclosures. Among

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16 Although not shown in the graph, Stanton et al. (2014) discuss another important mechanism that is coordinating the actions of the originators. This mechanism is the warehouse funding and the asset backed commercial paper facilities that provided short-term funding capital to the originators throughout the mortgage market. These funding entities were also largely controlled by the same holding companies that controlled the securitization shelves.
Figure 5: Networks for lender-type and county pairs for 2002, 2005, 2006 and 2007 vintage originations and subsequent foreclosure as of December, 30, 2013.
the 2002 vintage loans, only a highly localized set of lender-types and locations experienced significant levels of foreclosure by 2013. Although not shown, mortgage companies accounted for more than 60% of mortgage foreclosures for the 2002 vintage loans. In addition, though again not shown explicitly in Figure 5, among the ten worst mortgage foreclosure counties for the 2002 vintage mortgages four were Midwestern counties (Wayne, Cook, Cuyahoga and Marion); four were Southern and Southwestern counties (Shelby, Harris, Dallas and Tarrant); and only two were Western counties (Los Angeles and Maricopa). In the top forty worst 2002 vintage counties, only three were in California and six were in Florida. In contrast the 2005 vintage mortgages, as shown in the right-hand panel of Figure 5, began to experience much more widespread incidence of lender-type and county pairings that generated edges with more than 30% of foreclosure as the foreclosure incidence expands from the previous locations and lender-types to include close-by neighbors. Now the top forty worst counties included fifteen California counties and eight Florida counties, and the scale of foreclosures in the prior ten worst counties grew substantially. Mortgage companies again accounted for just less than 60% of all foreclosures among the 2007 vintage of originations. As a result, the network shows a growing concentration of red and a blush of red throughout about one half of the edges.

Panels c) and d) of Figure 5 show the network for originations in 2006 and 2007, respectively. The lender volumes look similar to those of 2005. However, the degree of foreclosures has increased dramatically and has propagated through substantial portions of the overall private-label first lien mortgage market. The initial lender-type and geographic concentration areas are still apparent in both networks. However, the scale of the foreclosure volume associated with these vintages has contaminated large swaths of lender-type and county pairings as well as a large fraction of the shelves and the holding companies. Only an admirable few originators, holding companies, and shelves appear to have withstood the temptation to originate mortgages that, ex post, experienced a high incidence of foreclosure either due to inferior underwriting strategies, reliance on risky contracts, and/or bad luck associated with the economic performance of the geographic locations.

As further evidence on the abysmal performance of a highly localized set of mortgage originators within the first lien private-label securitized mortgage market, Figure 6 shows the percentage of the mortgage origination by vintage for each lender that subsequently foreclosed by 2013. These originators are the top ten by ranking for the highest overall number of foreclosures for at least four of the six years of the sample. As shown, Argent Mortgage Company had a staggering foreclosure rate of more than 80% of all the mortgages that it originated in 2006. Residential Funding Company had nearly a 20% foreclosure rate for the loans that it securitized starting with the 2002 vintage and its foreclosure rate
rose to more than 50% for the 2006 and 2007 vintages. Option One Mortgage Corporation exhibited a similarly elevated foreclosure rate over the entire period. All three of these originators were mortgage companies. Wells Fargo Bank, N.A., significantly underperformed Wells Fargo Home Loans, Inc., (a mortgage company within the Wells Fargo Bank holding company) over nearly every vintage of origination. The foreclosure rate for Wells Fargo Bank originated mortgages in the 2006 vintage had an ex post foreclosure rate of 35%, whereas the mortgage company foreclosure rate for the same vintage was less than 30%. Loans originated by Bank of America exhibited foreclosure rates very similar to those of Wells Fargo Bank. Countrywide Home Loans, Inc. (a mortgage company), and Washington Mutual, FSB (a savings and loan institution), exhibited nearly identical foreclosure rates for every vintage of origination, reaching a high of 45% of total origination for the 2006 vintage.

![Figure 6: Percentage of mortgages originated that subsequently foreclosed by 2013 for the 10 mortgage originators with the highest volume of mortgage foreclosures.](image)

Figure 6: Percentage of mortgages originated that subsequently foreclosed by 2013 for the 10 mortgage originators with the highest volume of mortgage foreclosures.

Figure 7 reports the percentage of first lien private-label securitized mortgages that foreclosed by 2013 for the 10 holding companies with the worst performance record for four of the seven years of the sample. From 2004 onward, UBS Financial Services, ACC Capital Holdings, and Lehman Brothers Holdings, Inc had the worst foreclosure record for mortgages that were acquired and securitized in their shelves. More than 50% of the 2006 vintages loans
that were securitized by UBS and ACC had foreclosed by December of 2013. More than 50% of the 2007 vintage loans that were securitized by Lehman Brothers had foreclosed by December of 2013. Among the holding companies, the foreclosure rates for the 2006 and 2007 vintage mortgages that they securitized were around 45%. The one exception was Countrywide Financial Corp that exhibited a decrease from 40% to 35% in the foreclosure rate for the 2007 vintage mortgages as compared to that of the 2006 vintages. Overall, for these top worst holding companies, there was a very large fourfold increase in mortgage foreclosure rates from the 2004 to the 2006 and 2007 vintages of mortgage securitization. These holding companies all appear in the network representations with substantial amounts of red among the edges that report to them.

![Graph showing foreclosure rates for 10 shelf holding companies.](image)

Figure 7: Percentage of first lien private-label securitized mortgages that subsequently foreclosed by 2013 for the 10 shelf holding companies with the worst performance record.

Overall, our network representations of the first lien securitized mortgage market in the U.S indicate that the market is highly interlinked and very concentrated at the level of the securitization funding outlets. Ex post foreclosure outcomes were initially highly localized among a relatively small number of lender-types and counties and then with later vintages the seeds of future foreclosure outcomes were propagated to the closest competing firms and localities. Once these propagation mechanisms reached the large California and Florida counties, that have traditionally represented a large fraction of the U.S. mortgage market, the negative outcomes for mortgage origination appeared to saturate the entire market. These
trends appear to have been initially instigated by a relatively few, large firms, however, through the trading mechanisms in this market and due to the nature of competition in such a network structure, the market appears to have tilted to an outcome with a preponderance of poorly designed and badly localized mortgage origination.

6.3 Mortgage Origination Costs

In addition to information on the flow of mortgages through the private-label supply chain, we also need information on the costs of mortgage origination for the county/lender-types that are shown at the periphery nodes in the graphs above. To obtain these costs we collect data from two sources. For the banks, thrifts, and credit unions that primarily engaged in retail lending, we obtain data from Bankrate.com\(^\text{17}\) that carries out an annual survey of about 306 institutions that are primarily banks and thrifts in the U.S. The costs reported by Bankrate.com include the lender fees, third-party fees, and government fees that are obtained from their lender surveys. Bankrate.com reports the survey results as averages of total origination costs by state.\(^\text{18}\)

For the wholesale lenders, we use the loan-level origination costs, including all of the cost categories that were reported by Bankrate.com and these data were obtained from the New Century origination data set.\(^\text{19}\) Since New Century was one of the largest mortgage companies operating in the U.S. over this period, we have loan-level origination cost data for all states in the U.S. and these costs have a comparable composition to those of Bankrate.com. We assume that the New Century costs are a good proxy for the other mortgage companies with which they competed, including the mortgage companies that were part of bank holding companies. We then assign the origination costs to county/lender-type pairs according to the state in which the county is located and according to the type of lender.

Figure 8 presents the average U.S. retail and wholesale origination costs for mortgages from 2003 through 2007. These costs are reported per dollar of mortgage principal. As shown in the figure, despite the fact that retail lenders are originating from bricks and mortar sites with their own employees, on average, their costs are consistently lower than those of the mortgage companies. Overall, the costs of origination fell from their peak in 2004, although the wholesale lenders appear to have maintained the costs from 2005 through 2007. Thus

\(^{17}\)http://www.bankrate.com/

\(^{18}\)The Lender fees include the: points in dollars, administration fee; commitment fee; document preparation; funding fee; lender fee; processing fees; tax service fee; underwriting fee; and wire transfer fee. The third-party fees include the: appraisal fee; the attorney or settlement fees; credit report; flood certification; pest & other inspection fees; postage/courier fees; survey; title insurance; and title work (title search, plat drawing, name search). The government fees include the: recording fee; city/county/state tax stamps and intangible taxes.

\(^{19}\)These data were made available from the bankruptcy trustee of New Century Financial Corporation.
Figure 8: Average U.S. retail and wholesale origination costs and reported as the cost per dollar of mortgage principal.
in 2006, the average cost of originating a $250,000 mortgage by retail lenders was $3,025, whereas the average cost of originating the same mortgage by wholesale lenders was $3,625. The same loan balance in 2006 would have cost $2,713 in Missouri and $3,887 in New York, if it was originated by retail lenders.

7 Model Application to the 2006 U.S. Private-label Mortgage Data

We now apply the computational algorithm of Section 4 for the 2006 mortgage data. First, we reconfigure Figure 5c above, using the approach in Stanton, Walden, and Wallace (2013) (SWW) in which the high and low foreclosure segments of the network are plotted separately. Figure 9 presents the reconfigured network with a low foreclosure part (left panel) and a high foreclosure part (right panel). Because of the different mappings used between nodes, we have put the county/lender-type nodes closest to the center of the figure, in contrast to SWW (and Figure 5c), where these nodes are shown in the periphery. Otherwise, the methodologies used to create the graphs are the same. As is clear from Figure 9, the high foreclosure segment of the network constitutes a concentrated part of the graph.

We estimate the cost vector, $c$, for each county/lender-type node using the origination cost for each loan, as discussed above. For each node we compute the average of all costs of nodes flowing through that node. As a proxy for $v_n$, we use $1 - r^n$, where $r^n$ is the observed foreclosure rate for loans flowing through a node. The network $E$ is created by forming a link between any two nodes between which a loan flows. Using the algorithm of Section 4, we calculate the quality vector, $q$, and parameter estimates, $R_L$, $R_H$, $\Delta R$, and $p$.

In total, there are 515 low-quality nodes in the estimated network. In Figure 10, we show the estimated low-quality part of the network. Black nodes are those of high quality, whereas red nodes are of low quality. We also draw the links that exist between any pair of nodes of low quality. The low-quality part of the network constitutes a distinct subnetwork. For example, the number of links between the nodes in 2006, $M$, and the number of links one would expect between nodes, if the partition of the network was random, $W$ (introduced in Section 4), are 41,062 and 30,216, respectively. Thus, $M$ is substantially lower than what would be under a random partition of the network.

We use the estimated equilibrium network to calculate the distribution of the number of insolvencies, as shown in Figure 11, under the shock structure $\Omega^2$. Recall that if a shock hits, it hits either one or two nodes. For the vast majority of such shocks, the cumulative propagation is fairly limited: With 99.97% chance, at most 20 nodes become insolvent.
Figure 9: High (left) and low (right) foreclosure part of network. The outer circle of nodes represents holding companies (63 in total in 2006), sorted clockwise in increasing order of quality of loans. The middle circle represents shelf companies (142 in total in 2006) with the same ordering. The inner circle (only present in the right panel) represents county-type nodes (3,681 in total in 2006). The cutoff is made so that links with a higher than 80% foreclosure rate are shown in the right panel, whereas links between holding and shelf companies with a lower foreclosure rate are shown in the left panel. Year: 2006.
Figure 10: Red nodes in the figure represent those that are estimated as being of low quality, using the algorithm described in Section 4. Links between pairs of nodes of low quality are also shown. Equilibrium parameter estimates: $R_L = 0.54$, $d = 0.663$, $\Delta R = 0.1220$, $p = 0.42$. There are 3,371 high-quality and 515 low-quality nodes in the network. Year: 2006.
However, there are a few shocks whose cumulative effects reach a threshold and then, in line with a phase transition mechanism, affect the whole network. In this case, with 0.03% chance, about two thirds of the nodes become insolvent.

![Distribution of insolvent nodes.](image)

Figure 11: Distribution of number of insolvent nodes. The probability is 99.97% that at most 20 nodes become insolvent. There is a 0.03% chance that 2,731-2,733 nodes become insolvent. Year: 2006.

Figure 12 shows an example of a limited shock, initially hitting a holding company, and ultimately spreading to 17 other nodes. Figure 13, in contrast, shows a shock that propagates throughout the network. It initially hits one holding company and one shelf company. It then “spreads” step by step. Each step here represents a new round of insolvencies in the clearing mechanism, when the amount of capital goes below the insolvency threshold, $d$, for a new set of nodes, bringing them down and in turn further affecting their counterparties in the next step. The shock is still fairly limited after 4 steps, but once step 6 is reached, its effects are global. It ultimately causes almost all the shelf companies, as well as most county-type nodes, to become insolvent, whereas the holding companies show higher resilience (according to the assumptions of the model), as shown in step 8 beyond which no further insolvencies occur.

Overall, our application of the computational algorithm of Section 4 using the realized network structure and foreclosure outcomes of the 2006 mortgage data, generates predicted equilibrium quality metrics, $q$, that appear to be concentrated among an inner core of county/lender-type nodes, their securitization shelves, and the holding companies for those shelves. Using this implied equilibrium quality structure, our analysis suggests that in most states of the world the network is quite resilient to shocks, since the full effect of a shock is limited to a relatively limited number of nodes. However, the model computations also
Figure 12: Example of minor shock. In total 18 nodes become insolvent after a shock initially hits the holding company in the lower left part of the figure. Year: 2006.
Figure 13: Example of major shock. In total 2,732 nodes become insolvent after an initial shock that jointly hits one holding and one shelf company. The shock propagates step-by-step, causing new nodes to become insolvent, until ultimately (after 8 steps) most shelf and county-type nodes are insolvent. The holding companies are more resilient. Note that links are not shown in step 8, for expositional reasons. Year: 2006.
suggest that the network is prone, with positive probability, to a cascading outcome in which almost all the shelf companies, as well as most county/lender-types become insolvent. This outcome with cascading insolvencies may be used as a definition of systemic vulnerability of the financial market, being caused by an ex ante small shock that propagates and is amplified within the system, eventually affecting the whole market.

Our estimates of the inherent vulnerability of the 2006 network may thus provide a potentially powerful ex ante indicator of the systemic risks of such networks. Specifically, the computational algorithm may potentially be applied to an observed network structure together with ex ante predictions of mortgage foreclosure, solving for the implied unobservable quality measures of intermediaries, and subsequently stress testing the system with various shocks.

8 Conclusions

We develop a theoretical model of a network of intermediaries, which we apply to the U.S. mortgage supply chain. In our model, heterogeneous financial norms and systemic vulnerabilities arise endogenously. We show that the optimal behavior of each intermediary, in terms of its attitude toward risk, the quality of the projects that it undertakes, and the intermediaries it chooses to interact with, is influenced by the behavior of its prospective counterparties. These network effects, together with the intrinsic differences between intermediaries, jointly determine financial health, quality, and systemic vulnerability, at the aggregate level as well as for individual intermediaries. Our model therefore allows us to evaluate the relative importance of network effects for intermediary behavior. It also makes predictions about the conditions under which a network is systemically vulnerable to shocks and about how such systemic vulnerabilities can be detected empirically.

We apply our model to the mortgage-origination and securitization network of financial intermediaries, using a large data set of more than twelve million mortgages originated and securitized through the private-label supply chain from 2002–2007. We show that annual origination supply chains operate in highly interlinked networks and that the ex post foreclosure rates of loans are quite concentrated in certain parts of the empirical network. We then apply the model’s computational algorithm to the observed 2006 network of county/lender-types, securitization shelves, and holding companies., tracking the ex-post foreclosure performance of each loan in the network and compare the evolution of credit risk with the model’s predictions under different stress scenarios. We find that credit risk evolves in a concentrated manner among highly linked nodes, defined by the geography of the network and the interactions between originator and counterparty nodes. This suggests that network
effects are indeed of vital importance for understanding the U.S. mortgage supply chain.
A Clearing Algorithm for general network model

Algorithm 1 (Clearing mechanism).

1. Set iteration \( m = 0 \) and the initial insolvency vector \( f_1 = 1 \).
2. Repeat:
   1. Set \( m = m + 1 \).
   2. Calculate \( z_m = \Lambda f_m \Pi \Lambda f_m \tilde{C}F_P \).
   3. Calculate \( f_{m+1} = X \left( \frac{z_m^n}{s^n} \right), \) \( n = 1, \ldots, N \).
3. Until \( f_{m+1} = f_m \).
4. Calculate the \( t = 1 \) cash flow as \( \tilde{C}F_1^n = \Lambda f_m z_m^n \).

The iteration over \( M \) can be viewed as showing the gradual propagation of insolvencies, where \( f_M - f_{M-1} \) show the insolvencies that are triggered in step \( M \), by the insolvencies that occurred in step \( M - 1 \).

We can write the algorithm using returns, defining \( R_1 = \Lambda_s^{-1} \tilde{C}F_P \), and \( \Gamma = \Lambda_s^{-1} \Pi \). The algorithm then becomes even simpler:

Algorithm 2 (Clearing mechanism on return form).

1. Set iteration \( m = 0 \) and the initial insolvency vector \( f_1 = 1 \).
2. Repeat:
   1. Set \( m = m + 1 \).
   2. Calculate \( z_m = \Lambda f_m \Gamma \Lambda f_m R_m \).
   3. Calculate \( f_{m+1} = X(z_m) \).
3. Until \( f_{m+1} = f_m \).
4. Calculate the \( t = 1 \) cash flow as \( \tilde{C}F_1^n = \Lambda_s R_m \).

It is straightforward to show that the two algorithms are equivalent.

B Network formation game

We describe the network formation game between the \( N \) intermediaries/agents. The sequence of events are as in Figure 2. We use subgame perfect, pairwise stable Nash as the equilibrium concept. We make one extension of the pairwise stability concept in the definition of agents’ action space. Specifically, agents are allowed to unilaterally decide to become completely isolated by severing links to all agents they are connected to. In contrast, with the standard definition of pairwise stability agents are only allowed to sever exactly one link, or propose
the addition of one link. The assumption that agents can choose to become isolated can thus be viewed as a network participation constraint.

The sequence of events is as follows: At \( t = -2 \)—the proposal/severance stage of the game—there is a given initial network, \( E \subset \bar{E} \). Recall that \( \bar{E} \) here is the maximal network in the economy, which arises if all possible links are present. Each agent, \( n = 1, \ldots, N \), simultaneously chooses from the following mutually exclusive set of actions:

1. Sever links to all other agents and become completely isolated.
2. Sever exactly one existing link to another agent, \((n,n') \in E\).
3. Propose the formation of a new link to (exactly) one other agent, \((n,n') \in \bar{E} \setminus E\).
4. Do nothing.

In contrast to actions 1. and 2., which are unilateral, agent \( n' \) needs to agree for the links to actually be added to the network under action 3.

The set of networks that can potentially arise from this process is denoted by \( \mathcal{E} \). We note that \( E' \subset \bar{E} \) for all \( E' \in \mathcal{E} \). The set of actual proposals for addition of links, generated by actions 3., is denoted by \( L^A \). The set of links that are actually severed, generated by actions 1. and 2., is \( L^S \). The total set of potential link modifications is \( \mathcal{L} = \{(L^A, L^S)\} \).

At \( t = -1 \)—the acceptance/decline stage—\( L^A \) and \( L^S \) are revealed to all agents, who then simultaneously choose whether to accept or decline proposed links. Formally, for each proposed link, \( \ell = (n,n') \in L^A \), agent \( n' \) chooses an action \( a_\ell \in \{D,A\} \) (representing the actions of Declining or Accepting the proposed link). The total set of actions is then \( A = \{a_\ell : \ell \in L_A\} \in \{D,A\}^{L_A} \), which for each \( n' \) can be decomposed into \( A^{n'} \cup A^{-n'} \), where \( A^{n'} = \{(n,n') \in L_A\} \) represents the actions taken by agent \( n' \), and \( A^{-n'} = A \setminus A^{n'} \) the actions taken by all other agents. Altogether, \( E, L_A, L_D, \) and \( A \) then determines the resulting network, \( E' \), after the first two stages of the game, at \( t = 0 \).

At \( t = 0 \)—the quality choice stage—each agent, \( n = 1, \ldots, N \), simultaneously chooses the quality \( q^n \in \{0,1\} \). The joint quality actions of all agents are summarized in the action vector \( q \in \{0,1\}^N \).

At \( t = 1 \), shocks, \( \xi \), are realized, leading to realized net cash flows \( w^n(\xi|q,E') \) as defined by equations (13-14). The value of intermediary \( n \) at \( t = 0 \) is thus

\[
V^n(q|E') = \sum_{\xi \in \Omega} w^n(\xi|E', q) \mathcal{P}(\xi), \quad n = 1, \ldots, N. \tag{19}
\]

**Equilibrium**

An equilibrium to the network formation game is an initial network and quality strategies, together with a set of beliefs about agent actions for other feasible network structures, such
that no agent has an incentive to add or sever links, given that no other agents do so, and agents' have consistent beliefs about each others' behavior on and off the equilibrium path.

Specifically, the action-network pair \((q,E)\), together with \(t = -1\) acceptance strategies: 
\[ A: \mathcal{L} \to \{D,A\}^L, \] 
and \(t = 0\) quality strategies 
\[ Q: \mathcal{E} \to \{0,1\}^N \] constitute an equilibrium, if

1. At \(t = 0\), strategies are consistent in that for each \(E' \in \mathcal{E}\), and \(q = Q(E')\),
\[ q^n \in \arg\max_{x \in \{0,1\}} V^n((x,q^{-n})|E), \]
for all \(n = 1, \ldots, N\), i.e., it is optimal for each agent, \(n\), to choose strategy \(q^n\), given that the other agents choose \(q^{-n}\).

2. At \(t = -1\), strategies are consistent in that for each \((L_A,L_D) \in \mathcal{L}\), \(A^n(L_A,L_D)\) is the optimal action for each agent, \(n\), given that the other agents choose \(A^{-n}(L_A,L_D)\). Optimal here, means that the action maximizes \(V^n\) at \(t = 0\).

3. At \(t = -2\), the strategy \(L = \emptyset\) is consistent, i.e., for each agent \(n\), given that no other agent severs links or proposes additional links, it is optimal for agent \(n\) not to do so either, given the value such actions would lead to at \(t = -1\).

An equilibrium is said to be maximal if \(E = \bar{E}\).

It follows that the following conditions are necessary and sufficient for there to exists acceptance and quality strategies such that \((q,E)\) is an equilibrium:

1. For all \(n\), \(V^n(q|E) \geq V^n((-q^n,q^{-n})|E)\).
2. For all \(n\), \(V^n(q|E) \geq V^n\).
3. For all \((n,n') \in E\), \(\exists q' \in \{L,H\}^N\) such that
   - \(V^n(q|E) \geq V^n(q'|E - (n,n'))\),
   - For all \(n''\), \(V^n''(q'|E - (n,n')) \geq V^n''((-q''|E - (n,n'))|E - (n,n'))\),
4. For all \((n,n') \in E\), \(\exists q' \in \{L,H\}^N\) such that
   - \(V^n(q|E) \geq V^n(q'|E + (n,n'))\) or \(V^n'(q|E) \geq V^n'(q'|E + (n,n'))\),
   - For all \(n''\), \(V^n''(q'|E + (n,n')) \geq V^n''((-q''|E - (n,n'))|E + (n,n'))\).

The first condition ensures that each agent makes the optimal quality choice at \(t = 0\), given that no change to the network is made. The second condition ensures that it is not optimal for any agent to sever all links and become isolated. The third condition ensures that there are consistent beliefs about future actions, such that no agent has an incentive to sever a link at \(t = -2\). The fourth condition ensures that there are consistent beliefs about future actions, so that no two agents can be made jointly better off by adding a link.
References


