Factor Structure in Commodity Futures Returns and Volatility

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Motivation

- "Financialization" of commodity futures markets.
  - Dramatic increase in non-hedger trading activity in commodity futures markets starting around 2004. => Bubbles? ΔPrice discovery? ΔRisk-sharing?
  - *Cheng and Xiong (2013)* survey.
- Debate over factor structure in commodity futures returns
  - *Bakshi, Gao and Rossi (2013)*. Yes
  - *Szymanowska, de Roon, Nijman, and van den Goorbergh (2014)*. Yes
  - *Daskalaki, Kostakis, and Skiadopoulos (2014)*. No
- Availability of high-frequency data of high quality since around 2004
  - We can assess factor structure in realized volatility paralleling recent work in equity markets by *Chen and Petkova (2012)*, and *Kelly et al. (2014)*.
  - We can construct realized beta with the stock market.

Research Questions

1. What are the stylized facts of daily commodity futures (CF) returns and volatility? (next slide)
2. Is there a strong factor structure in CF returns? No
3. Do the realized stock market betas of CFs with the stock market vary significantly over time (here: 2004 – 2014)? Yes
4. Does the ratio of CF volatility explained by the stock market vary significantly over time? Yes
5. Is there a strong factor structure in CF volatility? Yes
6. Is the stock market strongly related to the common component of CF volatility? Yes

Stylized Facts (SFs)

1. Daily realized commodity futures volatility has very high persistence.
2. The log of realized commodity futures volatility is close to normally distributed.
3. Weak evidence of a factor structure in daily commodity futures returns (excl. meats.)
4. The factor structure in daily commodity futures volatility is much stronger than the factor structure in returns.
5. Little evidence of a time-trend in the degree of integration across commodity futures markets during the 2004-2014 period.
6. The strong common factor in commodity volatility is strongly related to stock market volatility.
7. Commodity betas with the stock market were high during 2008-2010 but have since returned to a level close to zero.
8. Commodity futures returns standardized by expected realized volatility are closer to normally distributed than the returns themselves but still display leptokurtosis.

Data and Cleaning

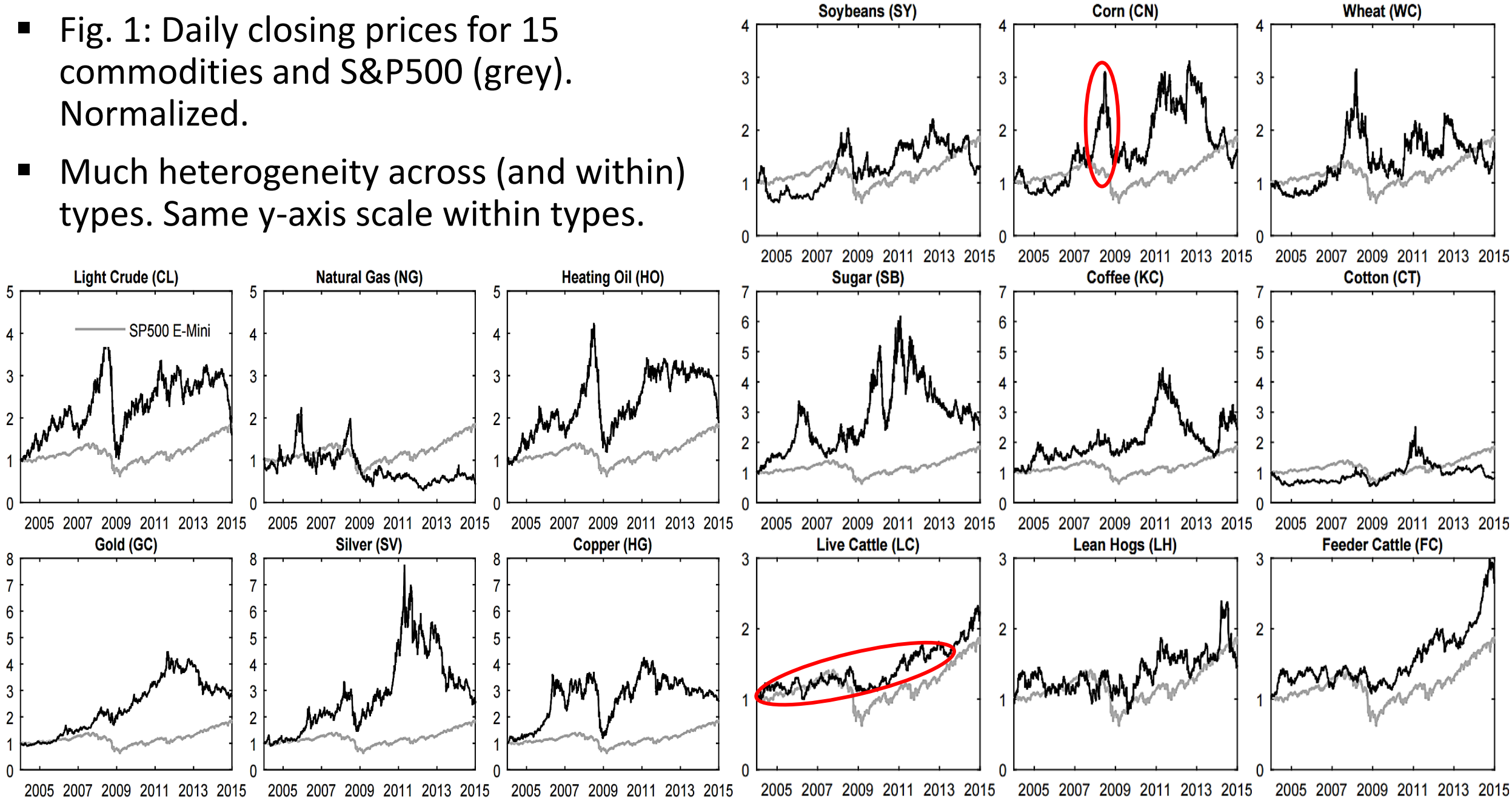
- From TickData we have intraday futures observations on more than 60 different commodities. We analyze more than 750 million trades.
- Focus on the 1/1/2004-31/12/2014 period. “Post-Financialization.”
- We restrict attention to commodities that are traded in either Chicago or New York during the entire period.
- We use *Gorton, Hayashi and Rouwenhorst (2012)* classification:
  1) energy, 2) metals, 3) grains, 4) softs, 5) meats.
- We choose the three most actively traded commodities within each type.
- S&P500 E-Mini futures data to proxy for the stock market.
- TickData’s proprietary algorithmic data filters.
- Algorithm in *Barndorff-Nielsen, Hansen, Lunde and Shephard (2009)*.

Table 1: The 15 Selected Commodities.

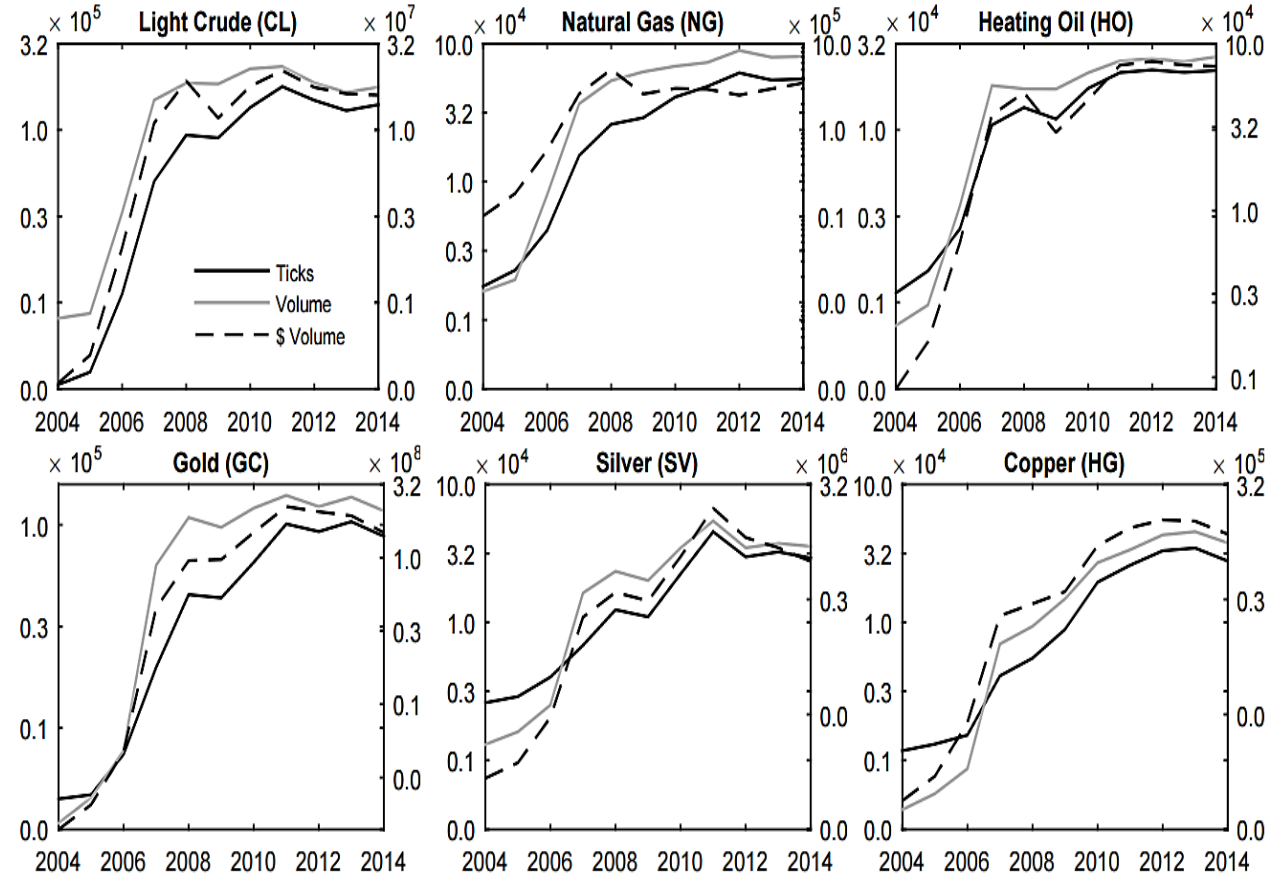
Classification	Commodity	Exchange	Time Zone	Transactions
Energy:	Light Crude (CL)	NYMEX/CME	New York	253,872,929
	Natural Gas (NG)	NYMEX/CME	New York	87,835,845
	Heating Oil (HO)	NYMEX/CME	New York	37,633,186
Metals:	Gold (GC)	COMEX/CME	New York	148,537,170
	Silver (SV)	COMEX/CME	New York	51,209,261
	Copper (HG)	COMEX/CME	New York	42,004,247
Grains:	Soybeans (SY)	CBOT/CME	Chicago	75,497,017
	Corn (CN)	CBOT/CME	Chicago	80,165,009
	Wheat (WC)	CBOT/CME	Chicago	41,042,846
Softs:	Sugar #1 (SB)	ICE	New York	26,361,596
	Coffee "C" (KC)	ICE	New York	12,711,409
	Cotton #2 (CT)	ICE	New York	10,543,301
Livestock:	Live Cattle (LC)	CME	Chicago	13,362,636
	Lean Hogs (LH)	CME	Chicago	11,469,432
	Feeder Cattle (FC)	CME	Chicago	2,440,045

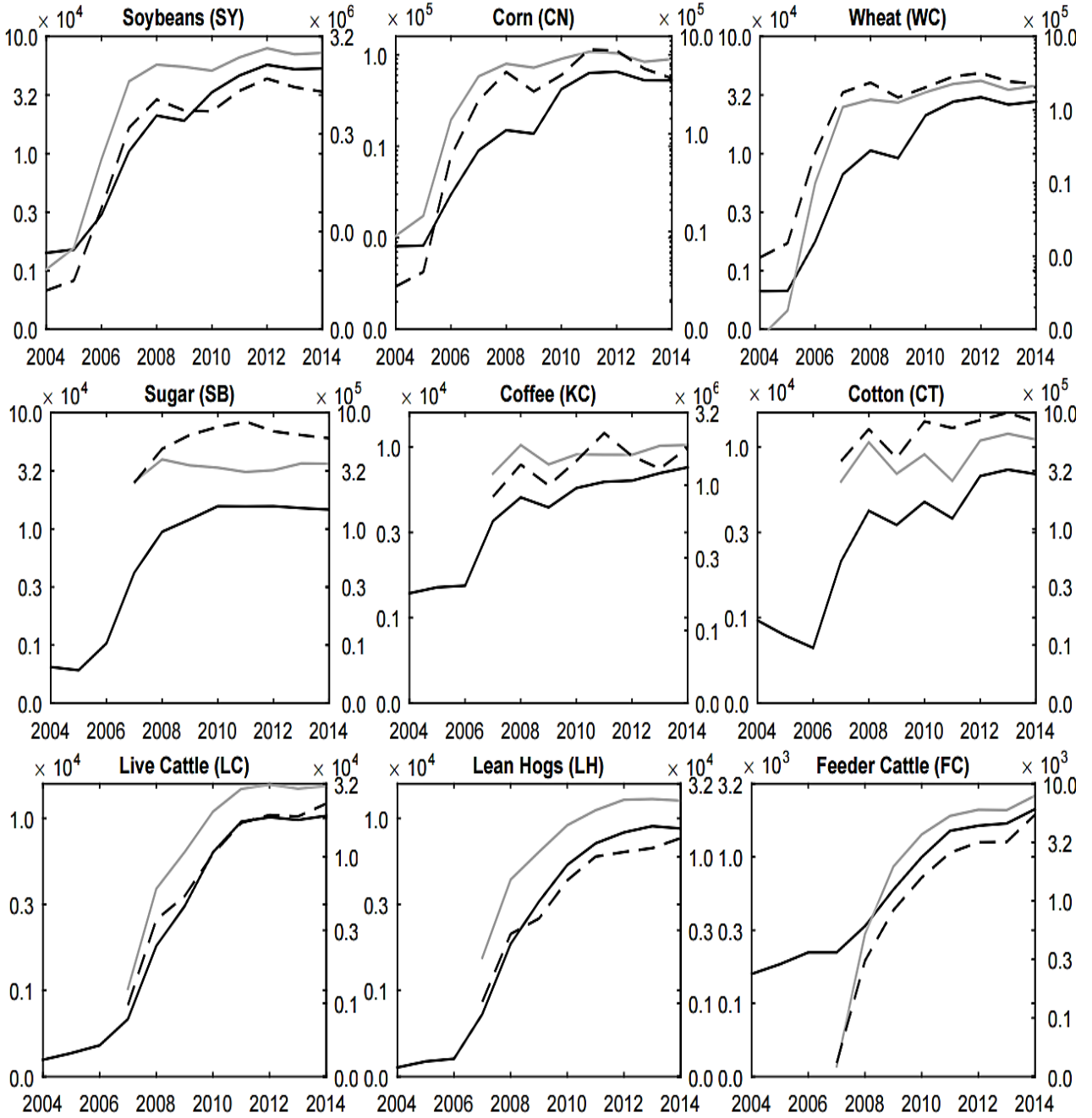
Notes: For each commodity futures contract we report the total number of transactions available during our 2004-2014 sample period.

- Fig. 1: Daily closing prices for 15 commodities and S&P500 (grey). Normalized.
- Much heterogeneity across (and within) types. Same y-axis scale within types.



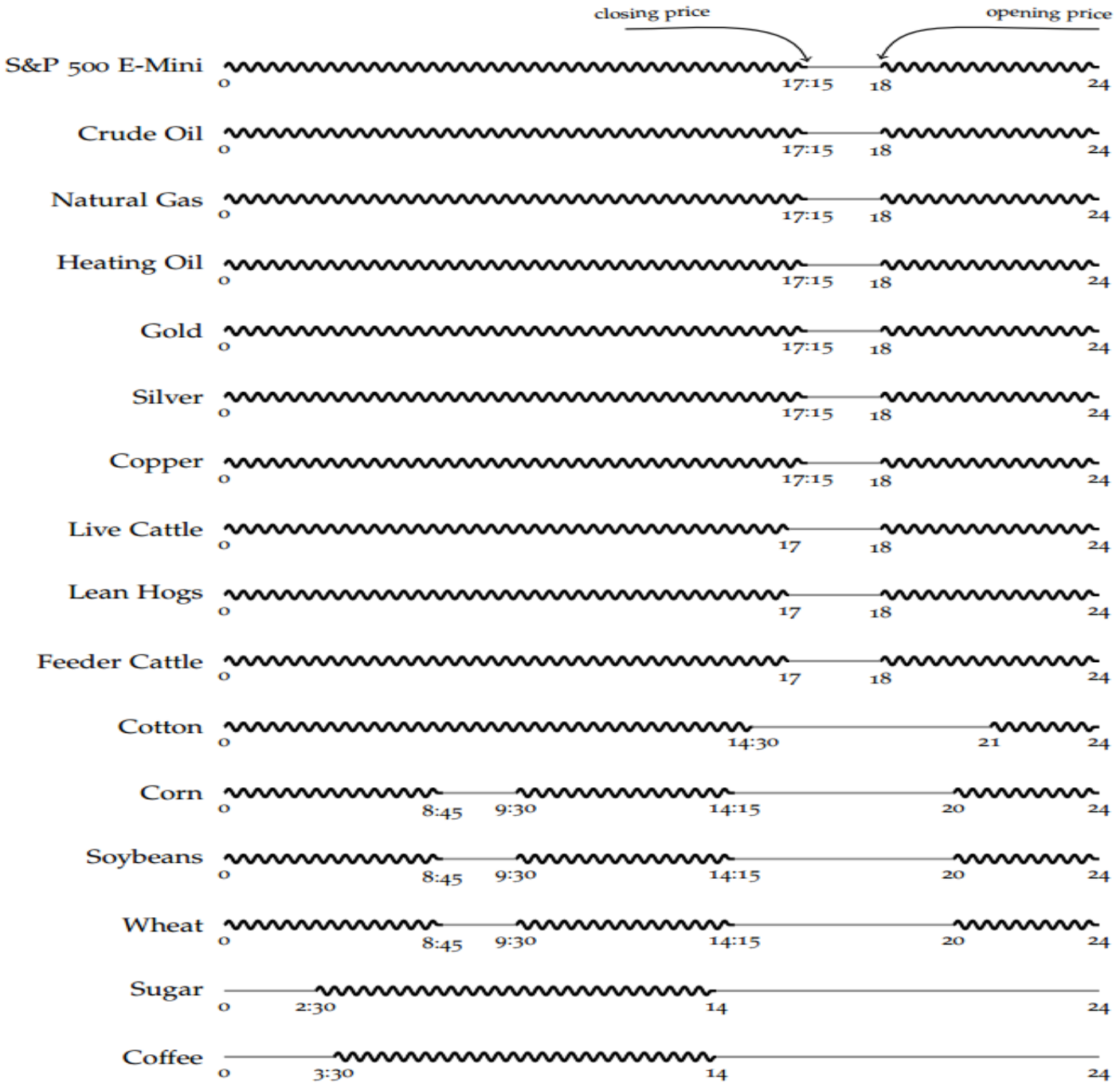
- Fig. 2: Number of ticks (black, left axis), volume (grey, left axis), and dollar volume (dashed, right axis). Log-scales.
- "Financialization" (yearly averages).
- Numbers for "most active" only.





Narrow 6 hour trading window for meats until June 2016

- Fig. 3: Daily trading hours (curly line).
- The widest feasible estimation window is used for Rvol computation.
- For Rcov, the estimation window corresponds to the overlap in trading hours between the commodity and the S&P500 E-mini.



Realized Measures

- 5-minute Realized volatility (RVol) with subsampling on 1-minute grid.
- ARMA(1,1) on log RVol to generate expected log RVol.
- Realized covariance (RCov) with S&P500 using 2-scale estimator again.
- Overnight adjustment using optimal weights on intraday and overnight volatility as in *Hansen and Lunde (2005)*.

Fig. 4: Autocorrelation function of daily log-realized volatility.

SF1: log RVol is extremely persistent.

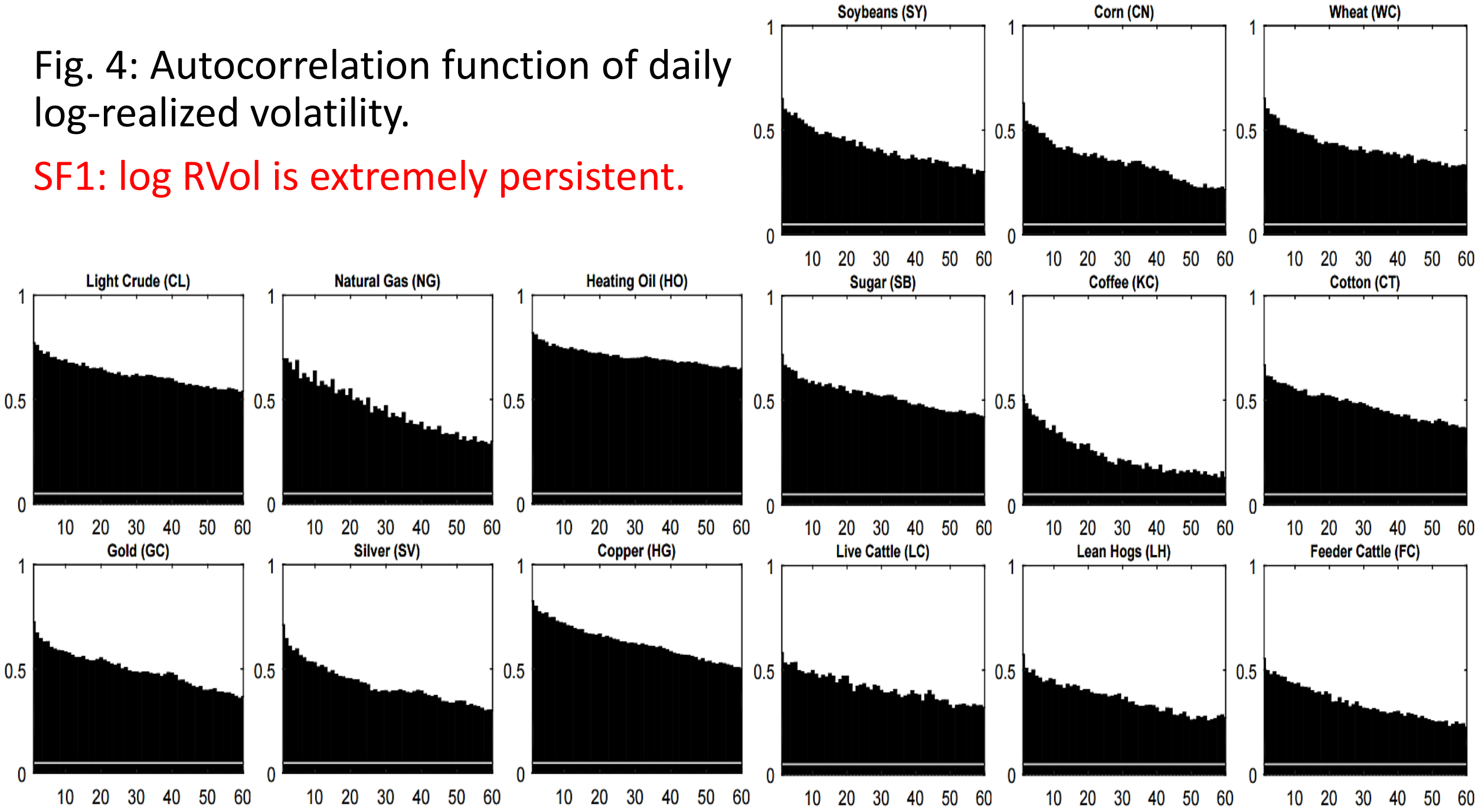
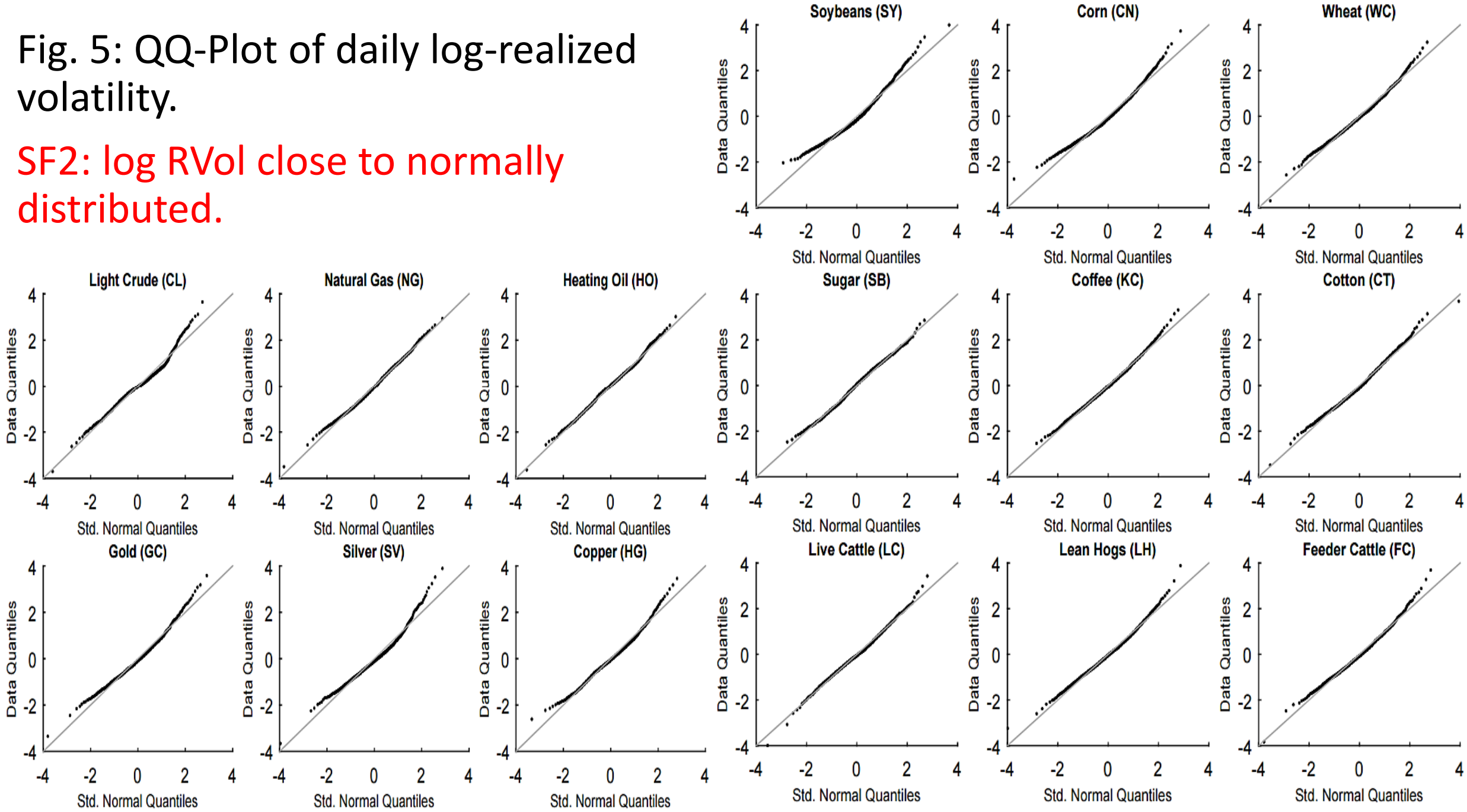


Fig. 5: QQ-Plot of daily log-realized volatility.

SF2: log RVol close to normally distributed.



- Fig. 7: Daily expected log-realized volatility from ARMA(1,1).
- S&P500 in grey.
- More commonality than in returns?

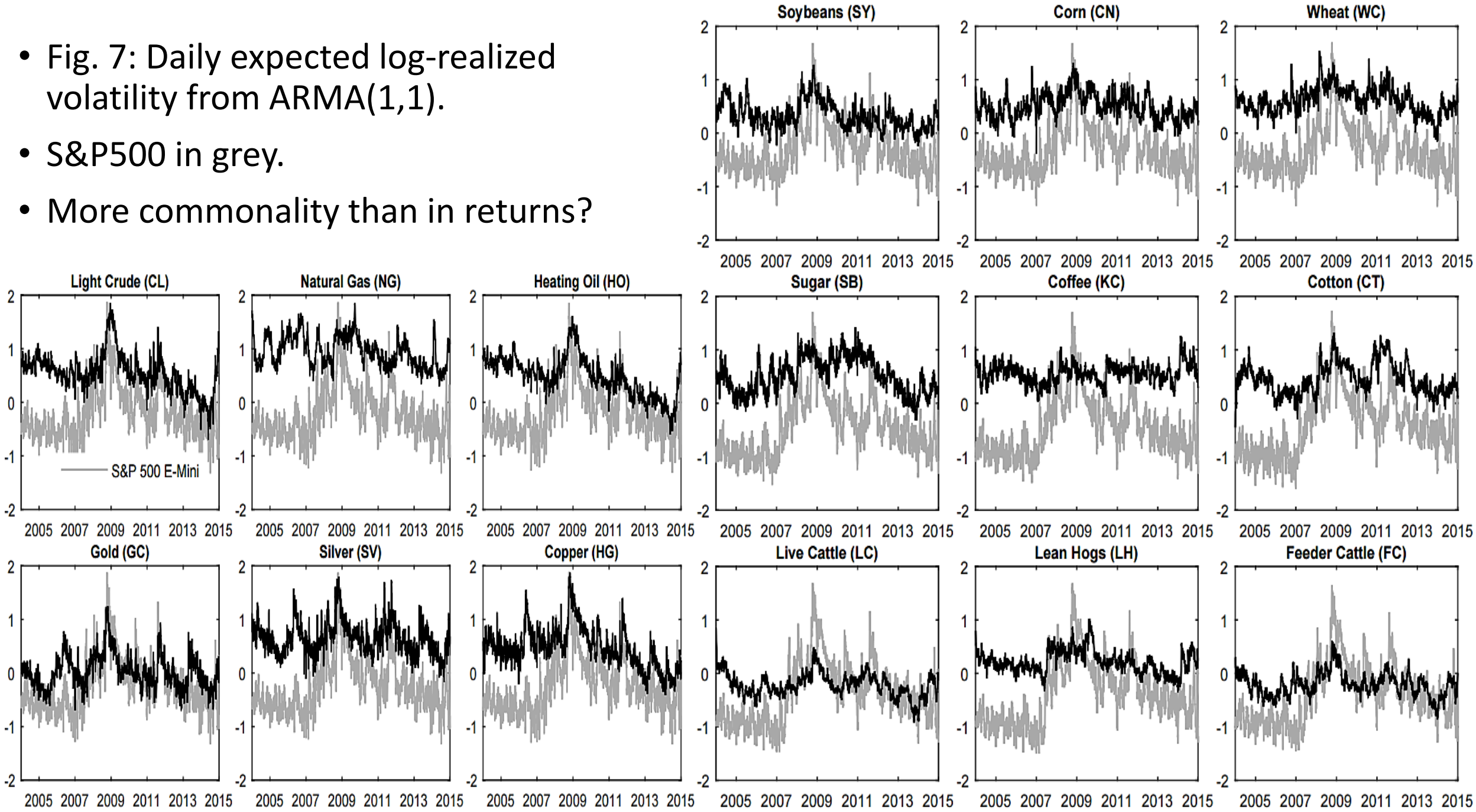


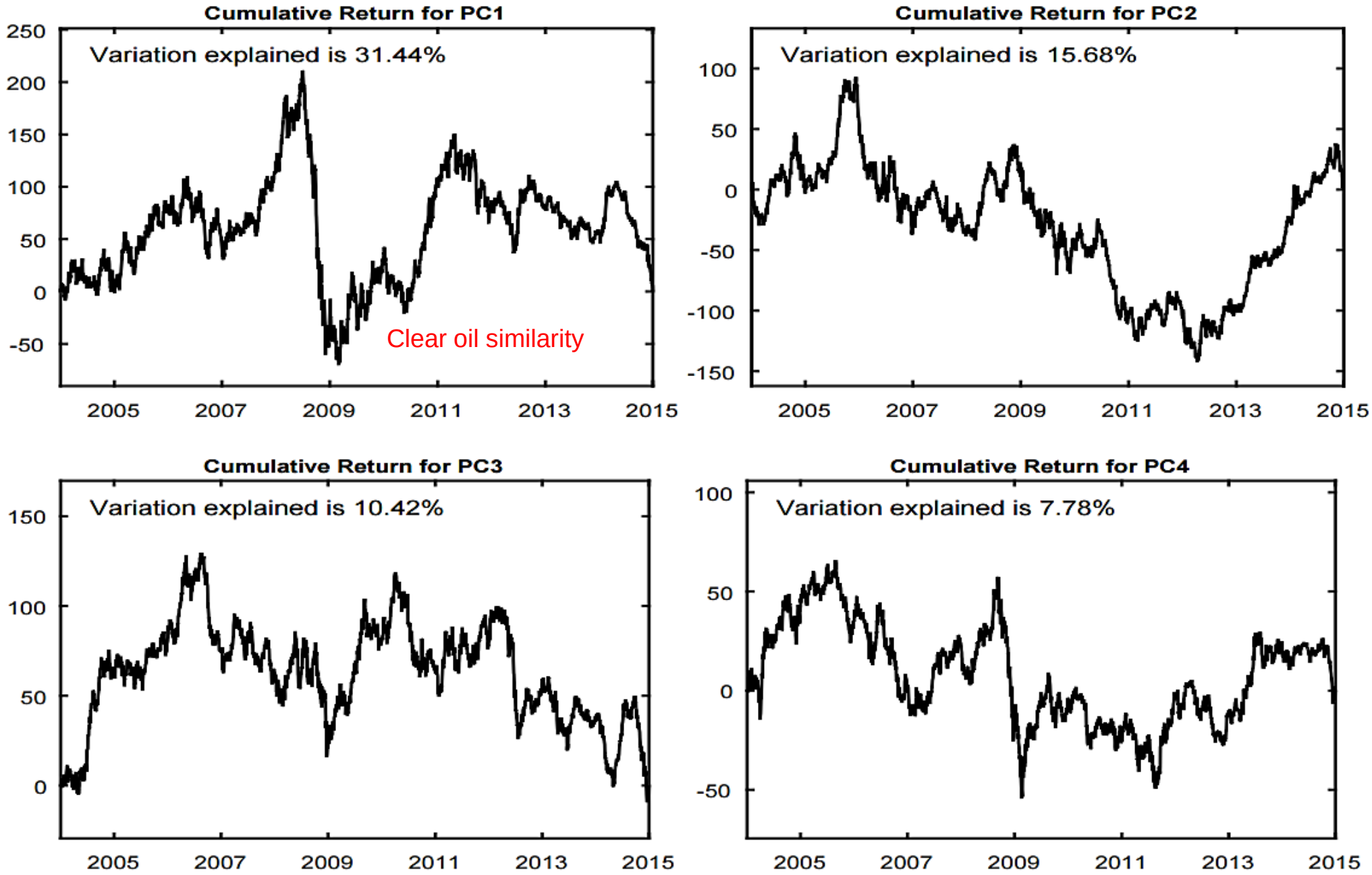
Table 4: Unconditional Correlations for Daily Returns (upper diagonal) and Log Volatility (lower diagonal).

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*	
Light Crude		30.0	86.5	29.6	36.1	46.8	31.7	27.5	25.0	26.2	22.3	23.5	18.9	12.2	13.3	
Natural Gas	45.2		33.7	8.7	11.0	13.1	13.6	13.5	10.4	11.4	9.8	7.7	7.5	1.1	6.1	
Heating Oil	94.1	52.5		27.3	32.4	40.7	30.8	25.5	22.4	23.4	21.2	22.9	13.9	9.7	8.8	
Gold	55.3	32.5	49.1		79.8	38.6	21.1	21.7	18.4	14.7	16.9	16.1	5.4	4.2	−0.1	
Silver	58.1	29.7	50.8	86.9		46.8	28.6	27.4	23.3	20.0	21.8	21.1	12.2	7.5	5.9	
Copper	74.6	42.4	72.5	71.3	70.6		30.5	25.3	23.7	26.1	23.1	25.1	18.7	10.2	12.9	
Soybeans	61.3	23.6	61.5	38.3	42.4	50.3		63.1	49.5	25.6	21.7	31.9	12.3	7.8	−3.1	
Corn	51.3	22.7	44.4	43.8	44.7	57.0	68.6		65.1	26.9	21.5	29.1	15.0	7.5	−11.4	
Wheat	50.3	21.8	43.9	42.7	46.1	54.3	60.2	79.4		25.3	22.6	27.0	14.7	10.1	−4.4	
Sugar	43.4	27.3	40.1	38.7	42.0	46.8	36.5	52.9	64.3		25.7	22.9	12.8	5.0	6.4	
Coffee	3.4	2.9	2.4	−6.4	0.3	−12.4	16.8	0.4	12.5	−0.2		19.2	11.6	7.5	6.8	
Cotton	49.8	6.8	41.8	20.8	39.9	35.2	51.7	48.2	54.5	47.7	24.6		12.2	5.2	5.3	
Live Cattle	59.5	18.4	56.0	38.0	46.9	54.6	52.7	47.3	56.4	49.1	14.2	58.0		32.7	75.7	
Lean Hogs	49.6	37.0	51.2	28.4	29.3	46.8	57.1	46.7	55.0	41.6	16.3	36.5	53.6		26.3	
Feeder Cattle	56.3	11.4	48.1	40.0	43.0	47.0	52.5	51.2	57.2	44.5	5.7	46.1	80.8	44.3		
Average Correlations																*Average*
For Returns	35.3	18.5	33.3	26.8	31.6	32.1	31.0	30.5	28.9	24.8	23.5	24.6	24.2	16.5	16.6	26.5
For E(log RVol)	56.8	31.6	53.9	45.3	48.7	54.1	51.6	50.6	53.2	45.0	12.0	44.1	52.4	46.2	48.5	46.3
Average Correlations																*Average*
With S&P 500 Returns	37.2	6.8	30.2	6.3	20.2	43.5	17.1	16.3	14.9	16.5	15.7	20.5	20.7	7.1	18.6	19.4
With S&P 500 E(log RVol)	63.0	17.1	47.9	62.5	57.1	59.8	44.1	53.8	57.0	55.1	10.5	51.3	43.6	48.6	51.8	48.2

Notes: The sample period is 5 January 2004 to 31 December 2014.

SF3: Weak evidence of facture structure in CF returns. 65% of variation explained.

Figure 7.a: First Principal Component of Log Returns. Cumulative Sum.



SF4: Much stronger evidence of facture structure in CF volatility—even for meats.

79% of the cross-sectional variance explained.

SF6: Correlation with S&P500 vol is 69% for PC1.

Figure 7.b: First Principal Component of Log Realized Volatility.

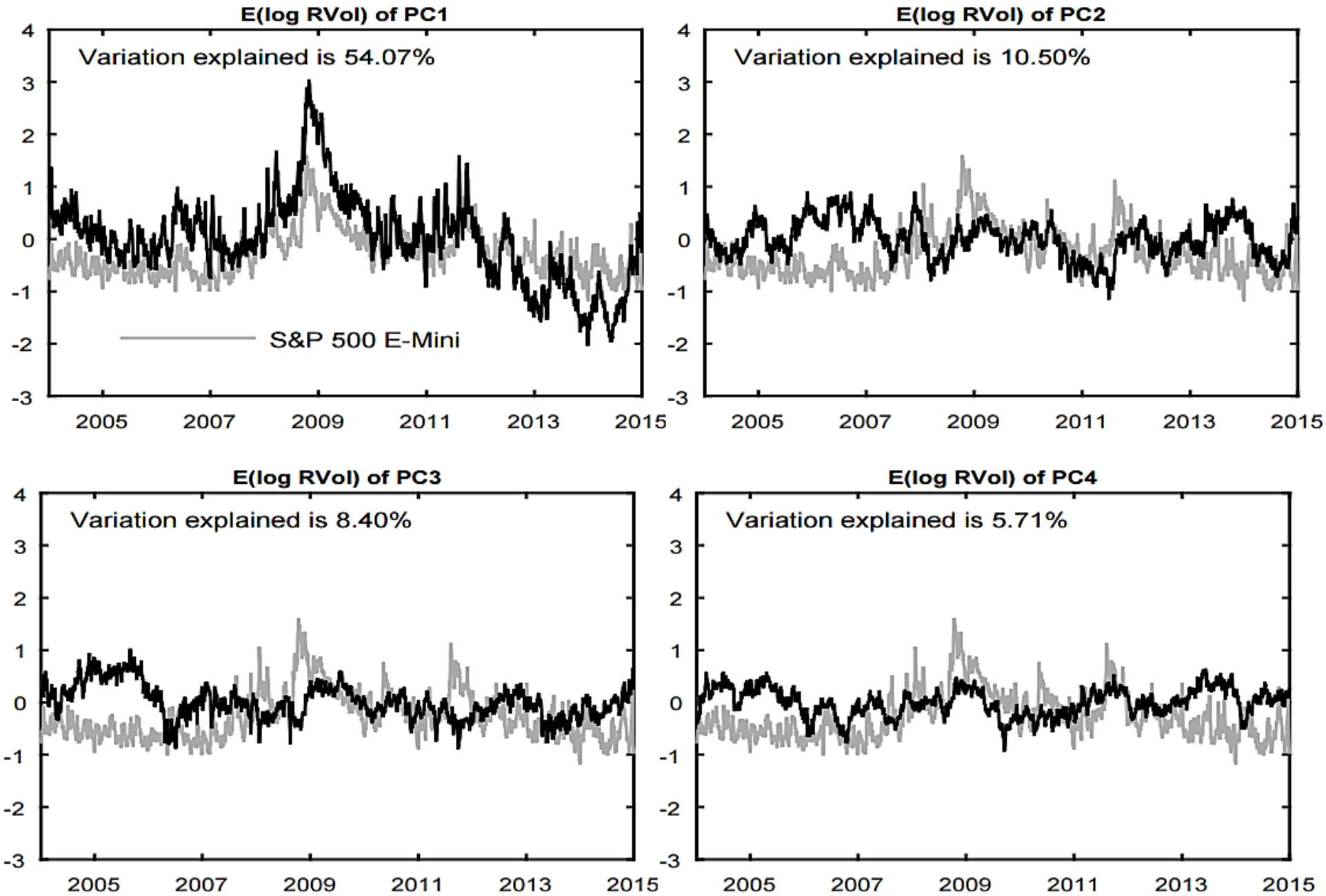


Table 5.a: Regression of Daily Futures Returns on Principal Components.

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
PC1	0.355	0.204	0.317	0.139	0.257	0.255	0.208	0.231	0.224	0.187	0.162	0.155	0.051	0.042	0.021
PC2	−0.026	0.136	−0.060	0.087	0.141	−0.115	−0.128	0.146	0.149	−0.083	−0.077	−0.097	−0.017	−0.027	−0.012
PC3	−0.181	−0.228	−0.169	−0.136	−0.145	0.188	−0.206	0.335	0.333	−0.053	−0.025	−0.062	0.004	0.010	−0.053
PC4	0.075	−0.049	0.088	0.053	−0.002	0.048	0.002	−0.050	−0.029	0.029	0.007	−0.023	0.024	0.028	0.028
R^2	0.442	0.107	0.452	0.320	0.260	0.342	0.394	0.430	0.338	0.149	0.122	0.157	0.060	0.022	0.036

Table 5.b: Regression of Daily Expected Log Realized Volatility on Principal Components.

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
PC1	0.408	0.154	0.389	0.260	0.266	0.409	0.223	0.203	0.210	0.223	0.009	0.201	0.191	0.152	0.176
PC2	0.091	0.153	0.106	0.133	0.095	0.226	−0.155	−0.180	−0.240	−0.164	−0.109	−0.321	−0.176	−0.093	−0.166
PC3	0.331	0.151	0.479	−0.468	−0.363	−0.200	0.111	−0.109	0.167	−0.129	0.072	0.134	0.024	0.103	−0.022
PC4	0.283	−0.250	0.349	0.334	0.238	0.006	0.071	−0.080	0.177	−0.215	0.034	0.139	0.094	−0.109	0.099
R^2	0.843	0.294	0.836	0.705	0.674	0.722	0.526	0.541	0.657	0.405	0.068	0.442	0.566	0.413	0.482

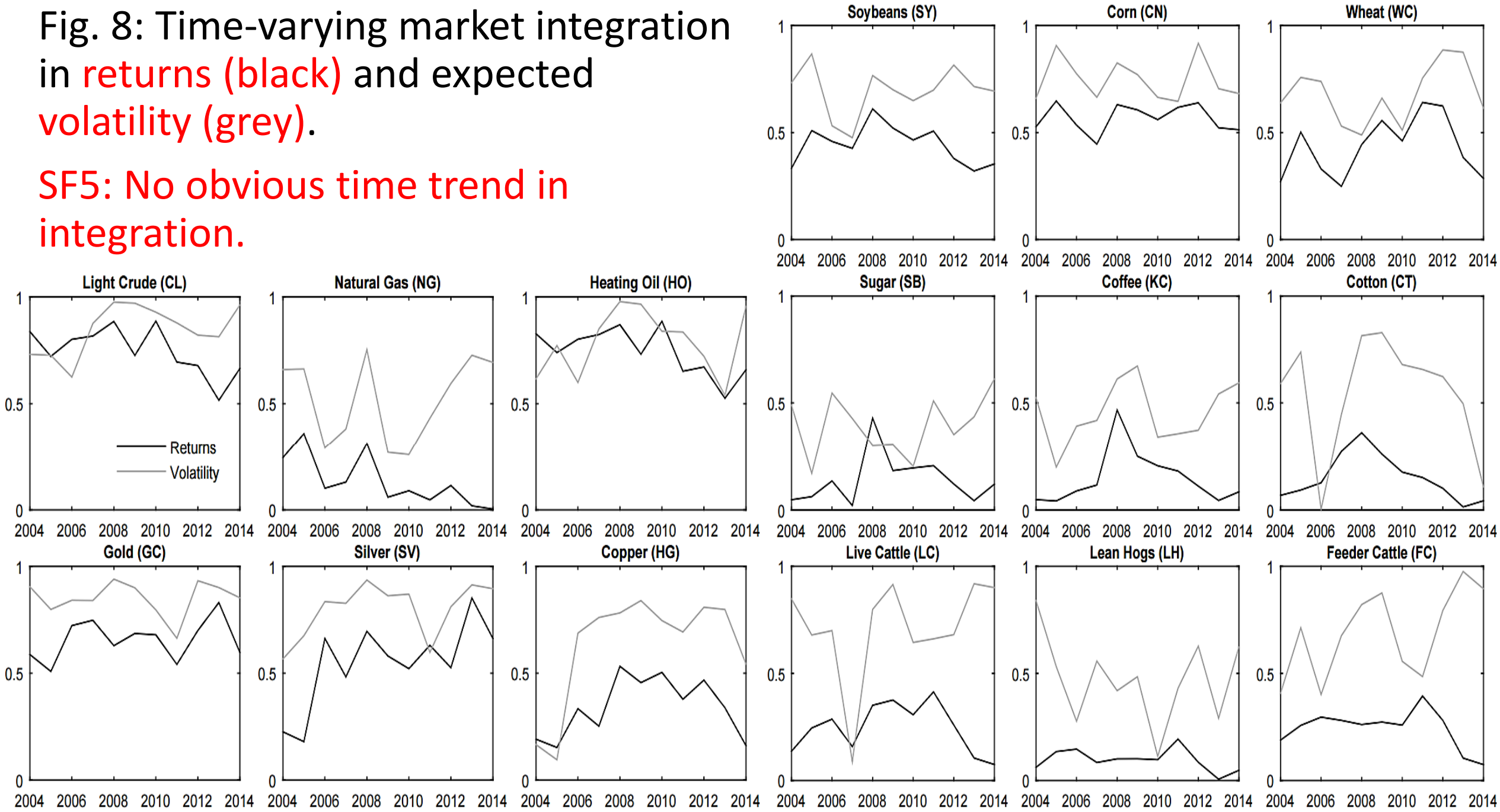
The average R^2 is 55% for volatilities versus 24% for returns.

Time-Varying Integration?

- We follow *Pukthuanthong and Roll (2009)*:
  - Regress the return on each commodity on first 10 PCs each year.
  - Compute the 10 PCs annually using only the remaining 14 commodities.
  - Plot annual adjusted R^2 from regression.
- Results robust to using different number of PCs in the regression.
- We conduct same analysis on expected RVol.

Fig. 8: Time-varying market integration in returns (black) and expected volatility (grey).

SF5: No obvious time trend in integration.



Commodity Futures and the Stock Market

- Regress each commodity PC on S&P500 to obtain a orthogonalized PCs.
- Regress the return for each commodity on S&P500 return and on first four orthogonalized PCs.
- Do the same for realized commodity log volatility with S&P500 log volatility.

Regression of Daily Futures Returns on S&P 500 and the Orthogonalized Principal Components.

Table 6.a: Results for Returns: $r_{Com,t} = \alpha + \beta_{S\&P\,500} r_{S\&P\,500,t} + \beta_{P\hat{C}1} P\hat{C}1 + \beta_{P\hat{C}2} P\hat{C}2 + \beta_{P\hat{C}3} P\hat{C}3 + \beta_{P\hat{C}4} P\hat{C}4 + \epsilon_t$

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
Constant	−0.036	−0.152*	−0.010	0.027	0.017	0.021	0.019	−0.018	−0.046	−0.009	−0.018	−0.029	0.001	−0.035	0.016
S&P 500	0.645*	0.171*	0.478*	0.056	0.351*	0.677*	0.222*	0.245*	0.252*	0.272*	0.249*	0.292*	0.147*	0.074*	0.130*
$P\hat{C}1$	0.314*	0.209*	0.293*	0.147*	0.237*	0.202*	0.206*	0.228*	0.218*	0.175*	0.151*	0.137*	0.038*	0.037*	0.006
$P\hat{C}2$	−0.025	0.149*	−0.055*	0.101*	0.151*	−0.106*	0.154*	0.181*	0.181*	−0.094*	−0.087*	−0.103*	−0.014	−0.027*	−0.019*
$P\hat{C}3$	−0.161*	−0.254*	−0.152*	−0.185*	−0.176*	0.176*	0.191*	0.319*	0.320*	−0.048*	−0.019	−0.061*	−0.004	0.009	−0.041*
$P\hat{C}4$	0.045	−0.035	0.050*	0.045*	−0.003	−0.060*	−0.003	−0.046*	−0.035	0.022	0.006	0.015	0.015	0.026	0.019
R^2	0.452	0.112	0.433	0.379	0.260	0.402	0.394	0.430	0.338	0.149	0.123	0.163	0.072	0.021	0.053

Average R^2 is 25%.

Table 6.b: Results for Volatility: $E\left(\log RVol_{Com,t}\right) = \alpha + \beta_{S\&P\,500} E\left(\log RVol_{S\&P\,500,t}\right) + \beta_{P\hat{C}1} P\hat{C}1 + \beta_{P\hat{C}2} P\hat{C}2 + \beta_{P\hat{C}3} P\hat{C}3 + \beta_{P\hat{C}4} P\hat{C}4 + \epsilon_t$

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
Constant	0.723*	1.017*	0.625*	0.177*	0.786*	0.645*	0.431*	0.613*	0.719*	0.672*	0.604*	0.600*	−0.106*	0.313*	−0.086*
S&P 500	0.514*	0.102*	0.397*	0.440*	0.397*	0.544*	0.258*	0.289*	0.318*	0.343*	0.032	0.343*	0.243*	0.184*	0.243*
$P\hat{C}1$	0.397*	0.224*	0.464*	0.170*	0.201*	0.350*	0.236*	0.174*	0.164*	0.171*	−0.005	0.136*	0.180*	0.155*	0.156*
$P\hat{C}2$	−0.067*	−0.156*	−0.094*	−0.186*	−0.127*	−0.241*	0.195*	0.167*	0.219*	0.146*	0.117*	0.342*	0.181*	0.111*	0.162*
$P\hat{C}3$	−0.359*	−0.075	−0.487*	0.355*	0.281*	0.180*	−0.044	0.173*	0.234*	0.164*	−0.072	−0.031	0.020	−0.053	0.053
$P\hat{C}4$	0.242*	−0.233*	0.262*	0.393*	−0.276*	0.037	0.097	−0.043	0.140*	−0.212*	0.019	0.118*	0.116*	−0.095	0.125*
R^2	0.860	0.310	0.839	0.717	0.636	0.727	0.531	0.542	0.661	0.410	0.075	0.461	0.555	0.398	0.481

Average R^2 is 55%.

Realized beta and Systematic Risk Ratio

- Realized beta is simply:

$$R\beta_{i,t} = \frac{RCov_{i,t}}{RV_{S\&P500,t}}$$

- Systematic risk ratio is defined as:

$$0 \leq SRR_{i,t} := \frac{R\beta_{i,t}^2 \cdot RV_{S\&P500,t}}{RV_{i,t}} \leq 1$$

- Interpret SRR as the commodity *i* variation that is explained by the market's.
- Expected values of both using ARMA(1,1)s.

Fig. 9: Expected daily realized beta with the stock market clearly dynamic.

(75% and 90% bootstrapped confidence bands).

SF7: Several betas were large during and after financial crisis but have declined again.

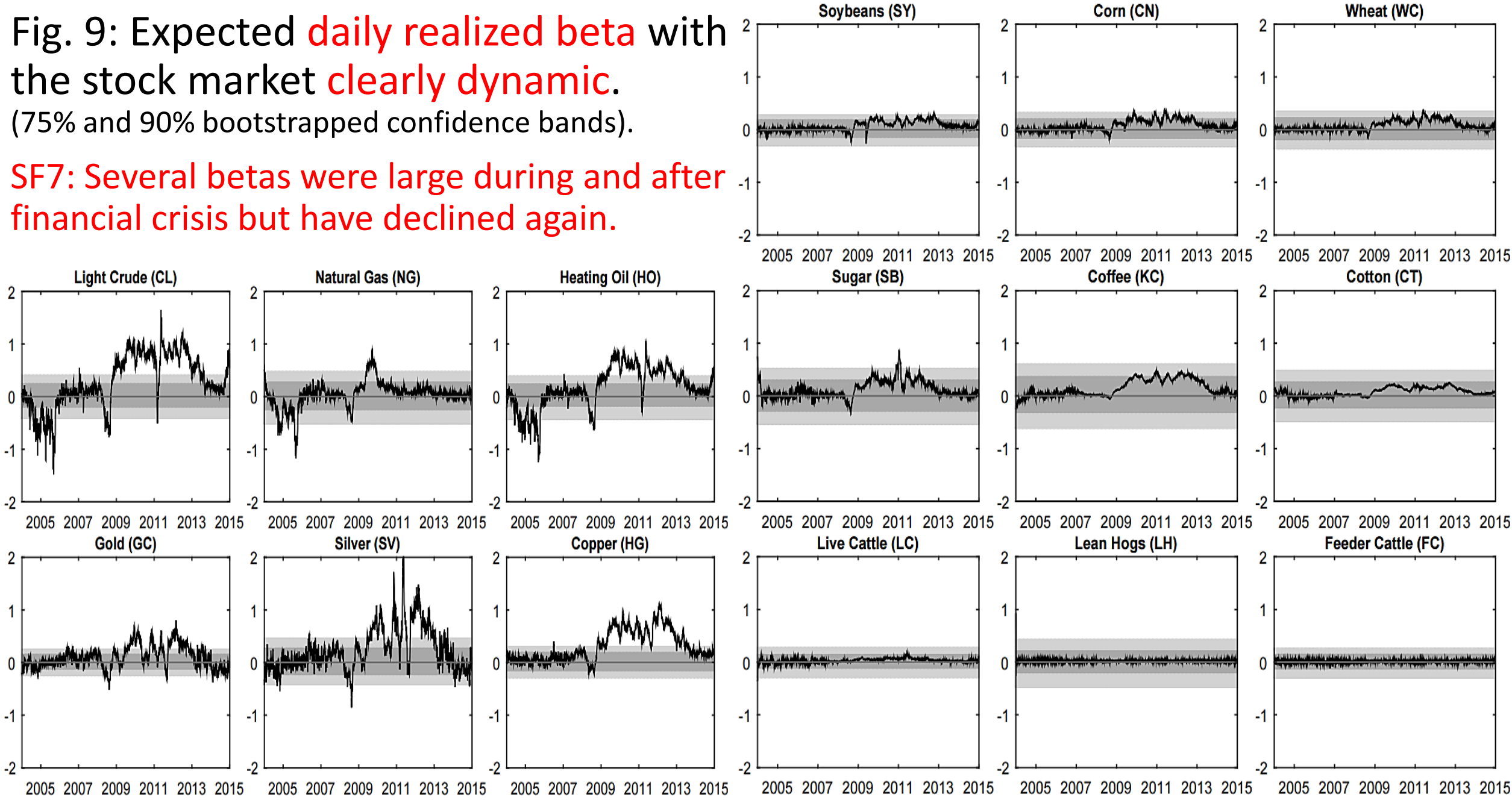


Fig. 10: Expected daily systematic risk ratio with the stock market.

(75% and 90% bootstrapped confidence bands).

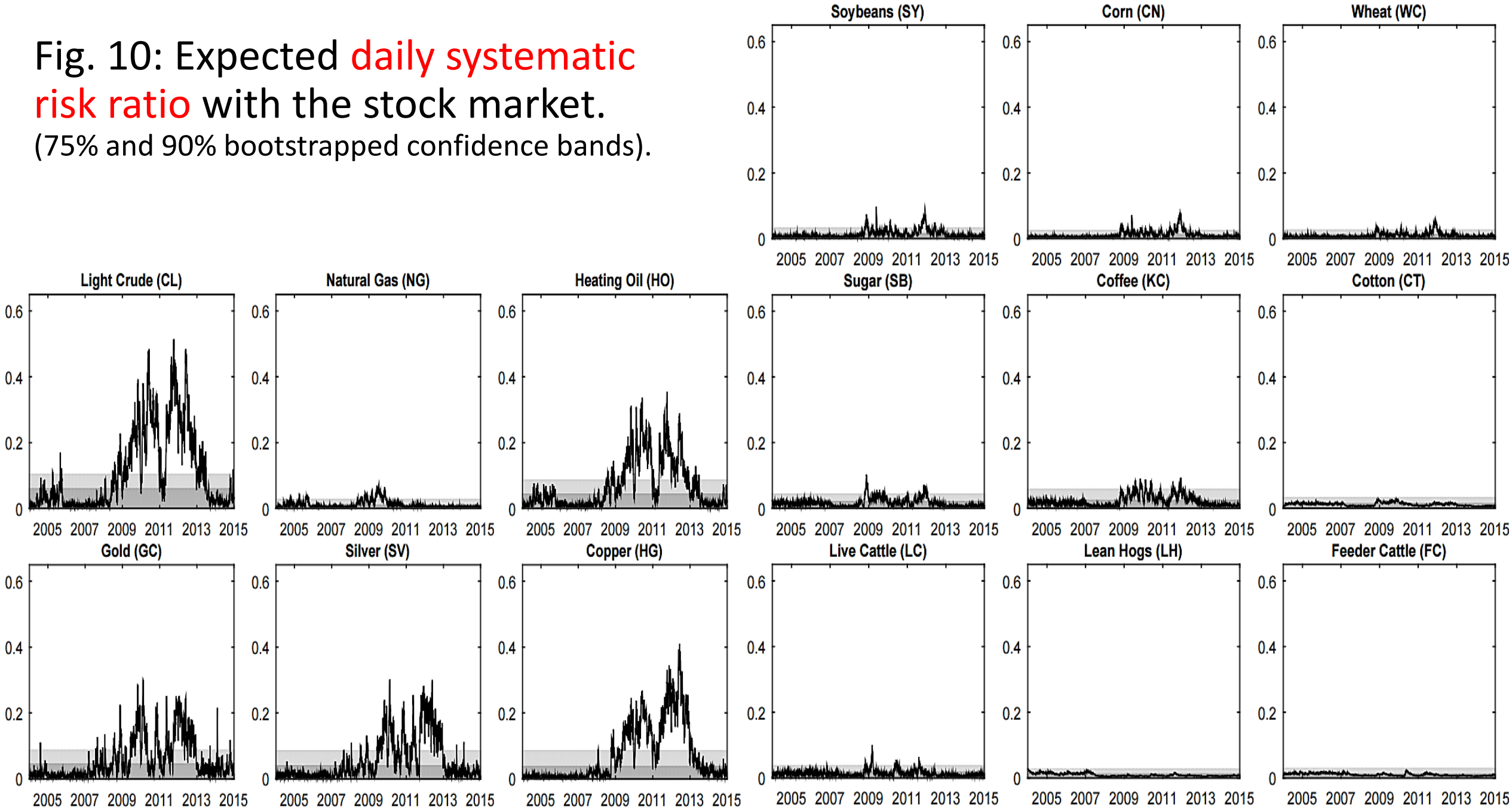


Fig. 11: Threshold correlations between commodity and stock market returns.

(Implied Gaussian threshold correlation in dashed line).

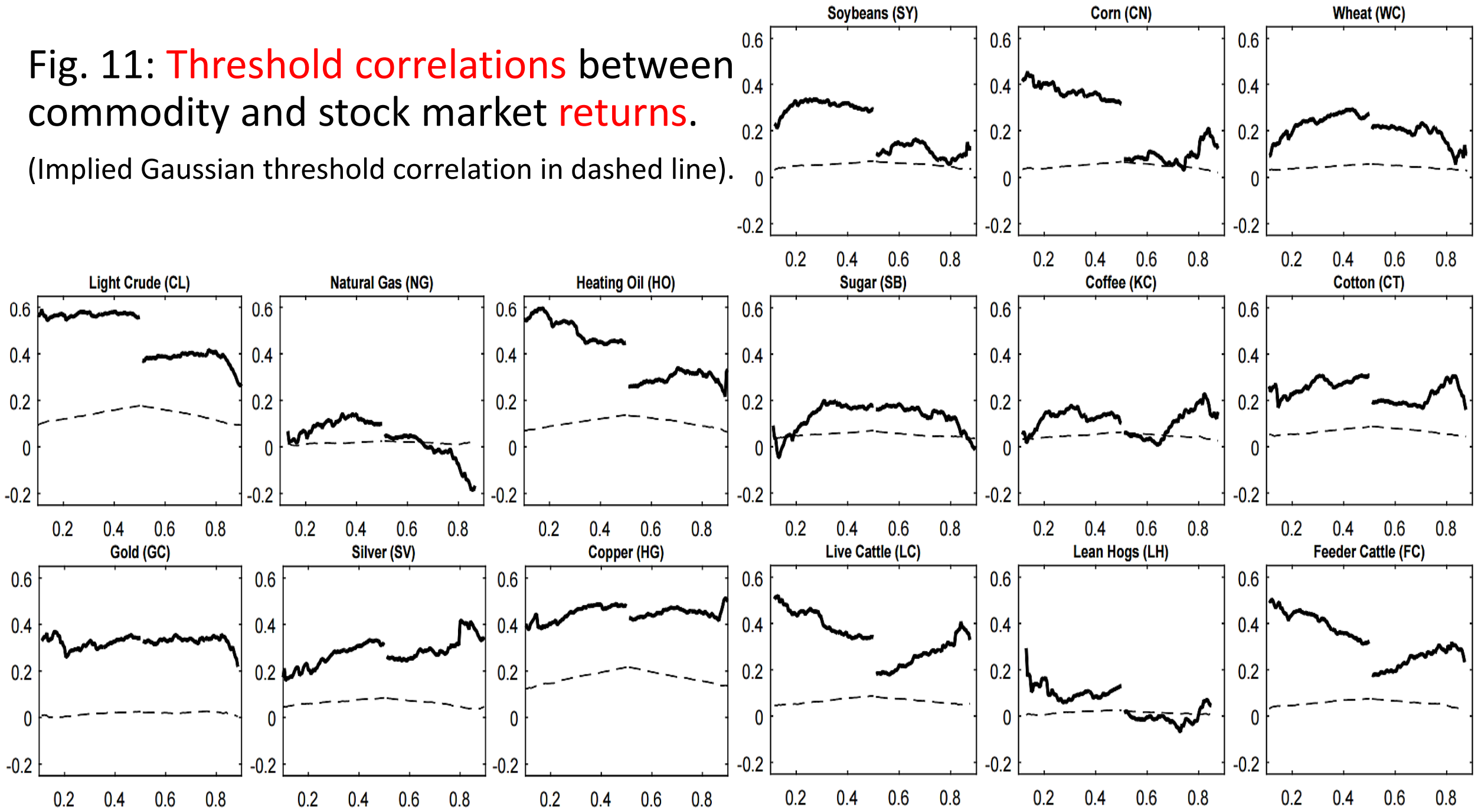
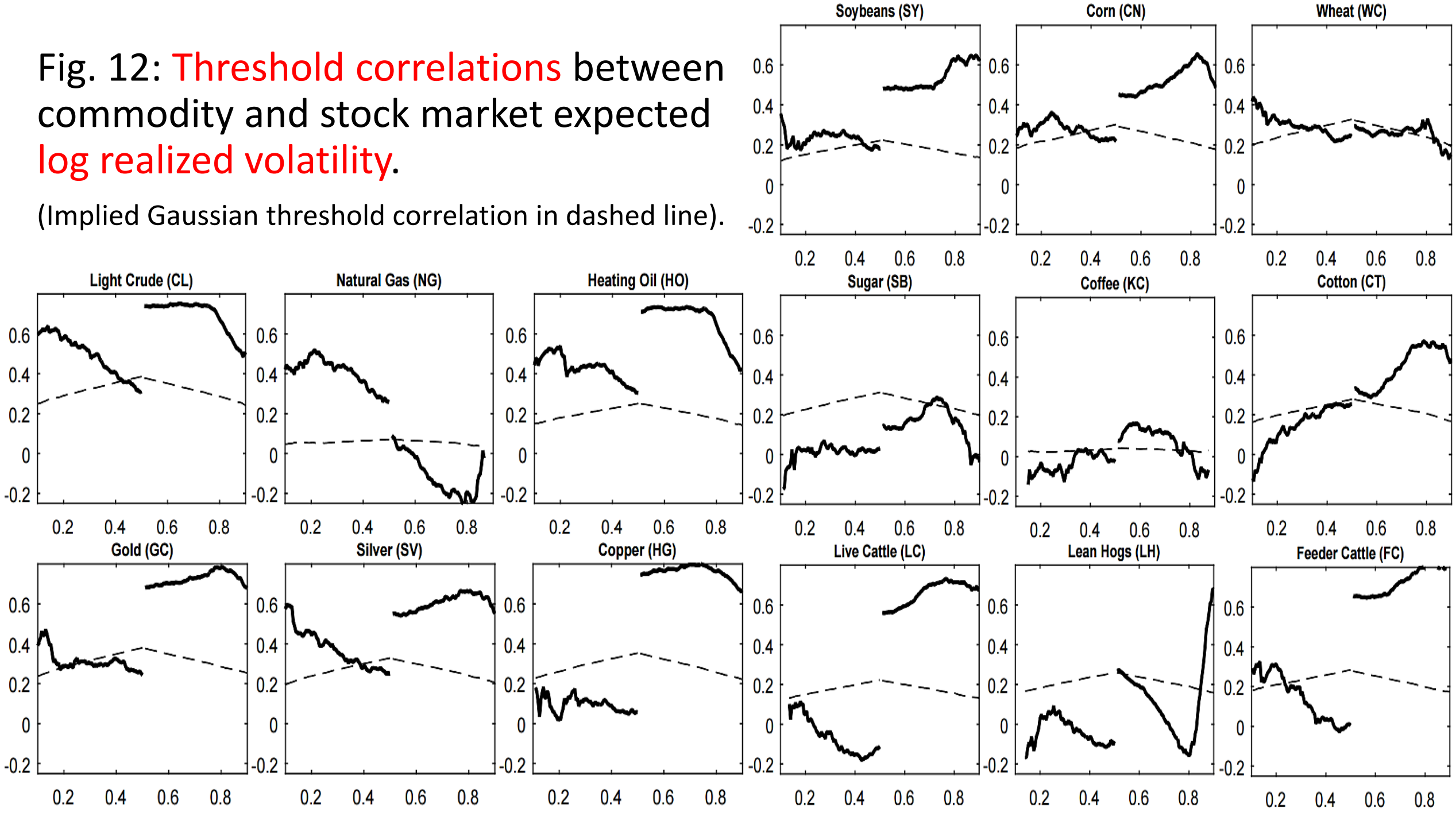


Fig. 12: Threshold correlations between commodity and stock market expected log realized volatility.

(Implied Gaussian threshold correlation in dashed line).

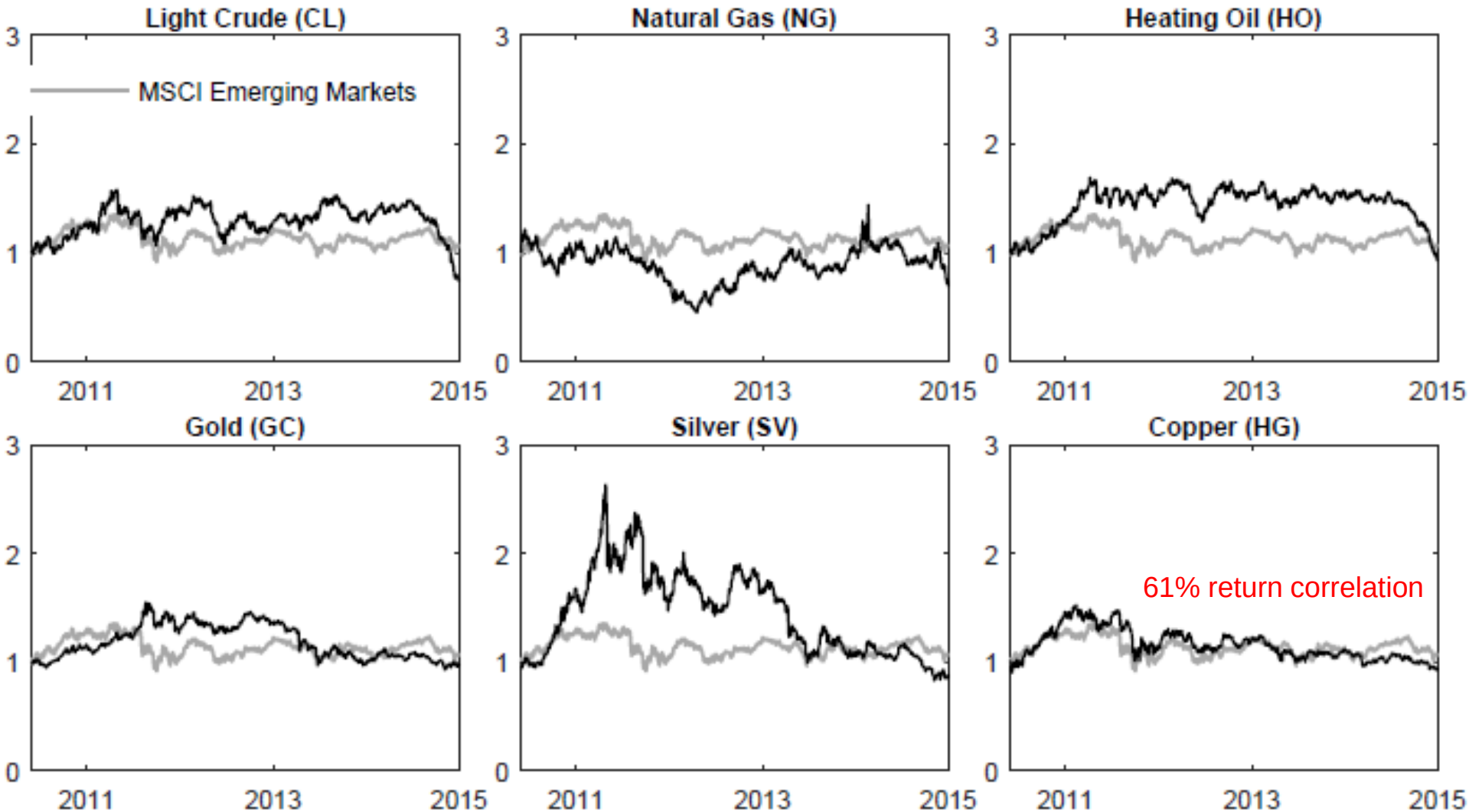


Extensions in Progress

- Study the relationship between emerging stock markets and commodity markets.
- Assessing the impact of time-to-delivery and seasonality on commodity volatility.
- Investigating macro drivers of commodity volatility and beta.
- Checking robustness on sub-periods.

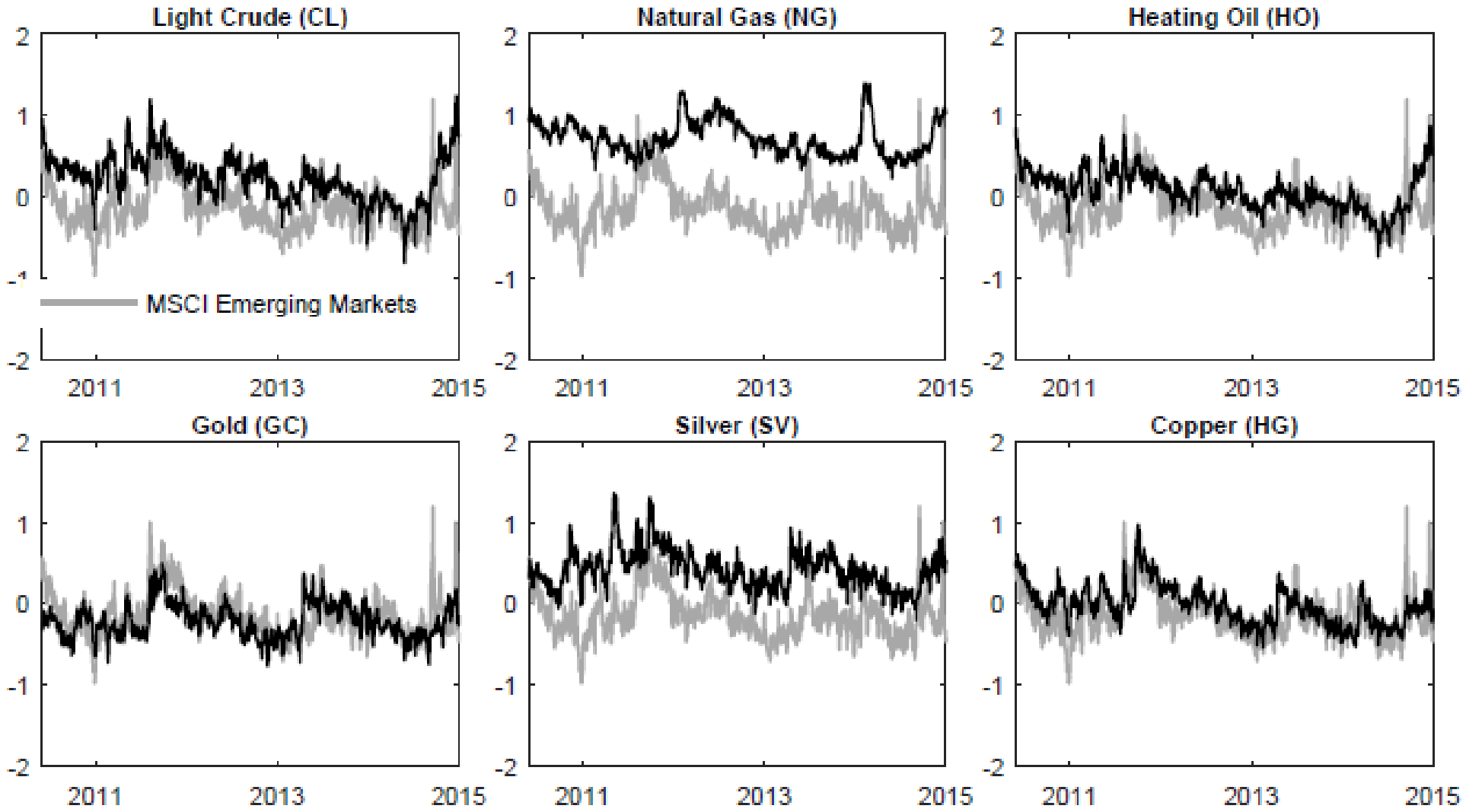
Commodities and MSCI Emerging Markets I

Daily Closing Prices



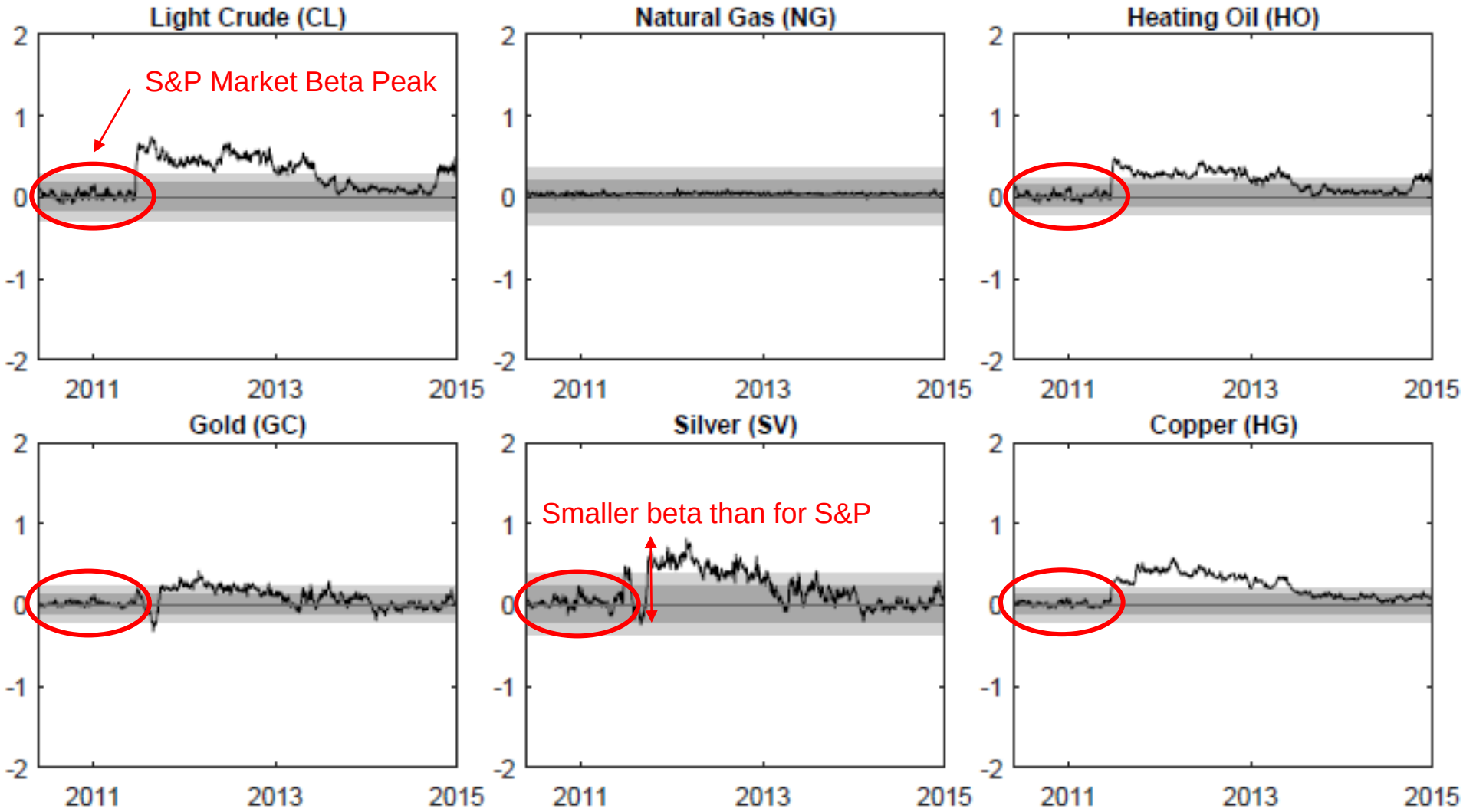
Commodities and MSCI Emerging Markets II

Daily Expected Log Rvol



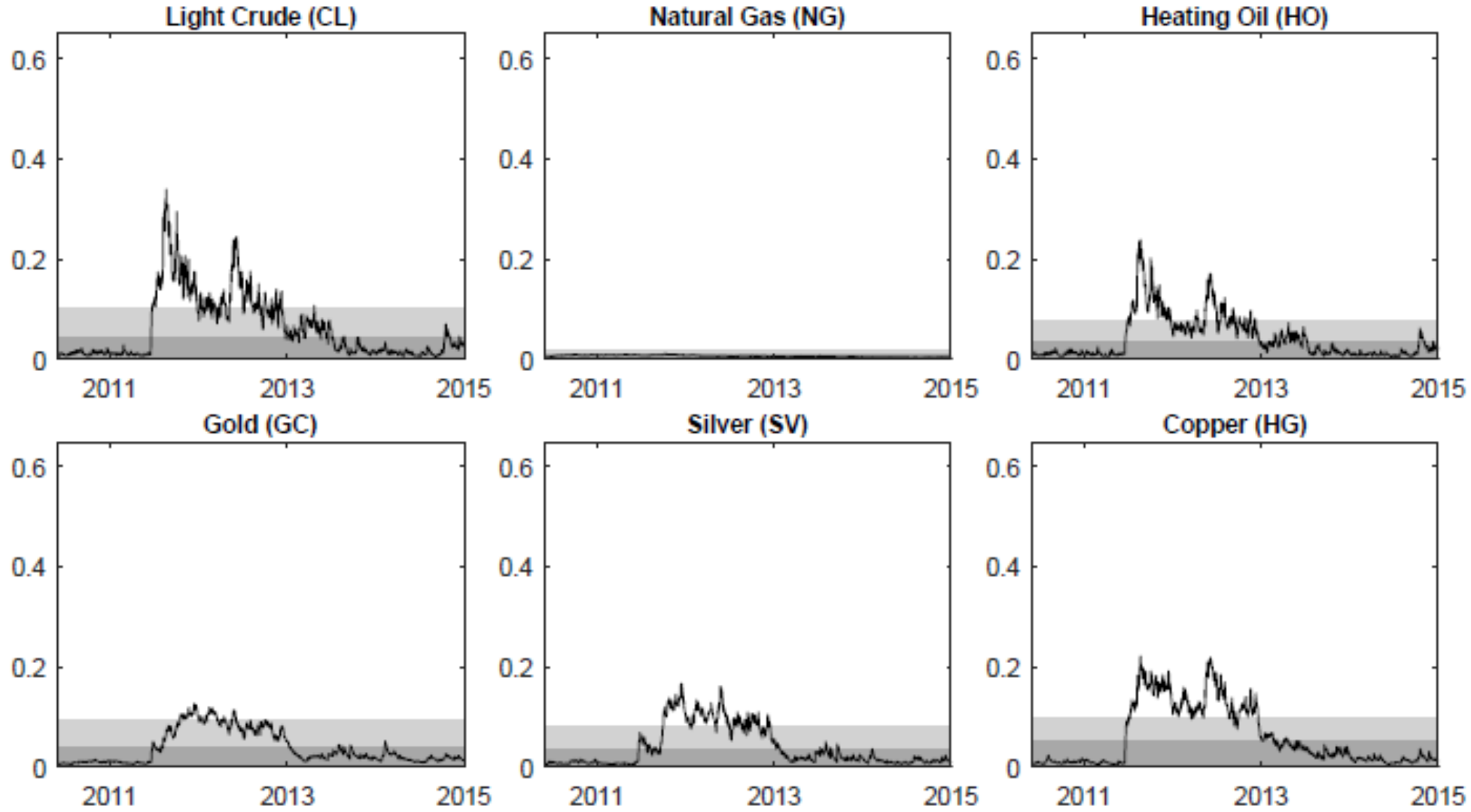
Commodities and MSCI Emerging Markets III

Daily Expected Beta



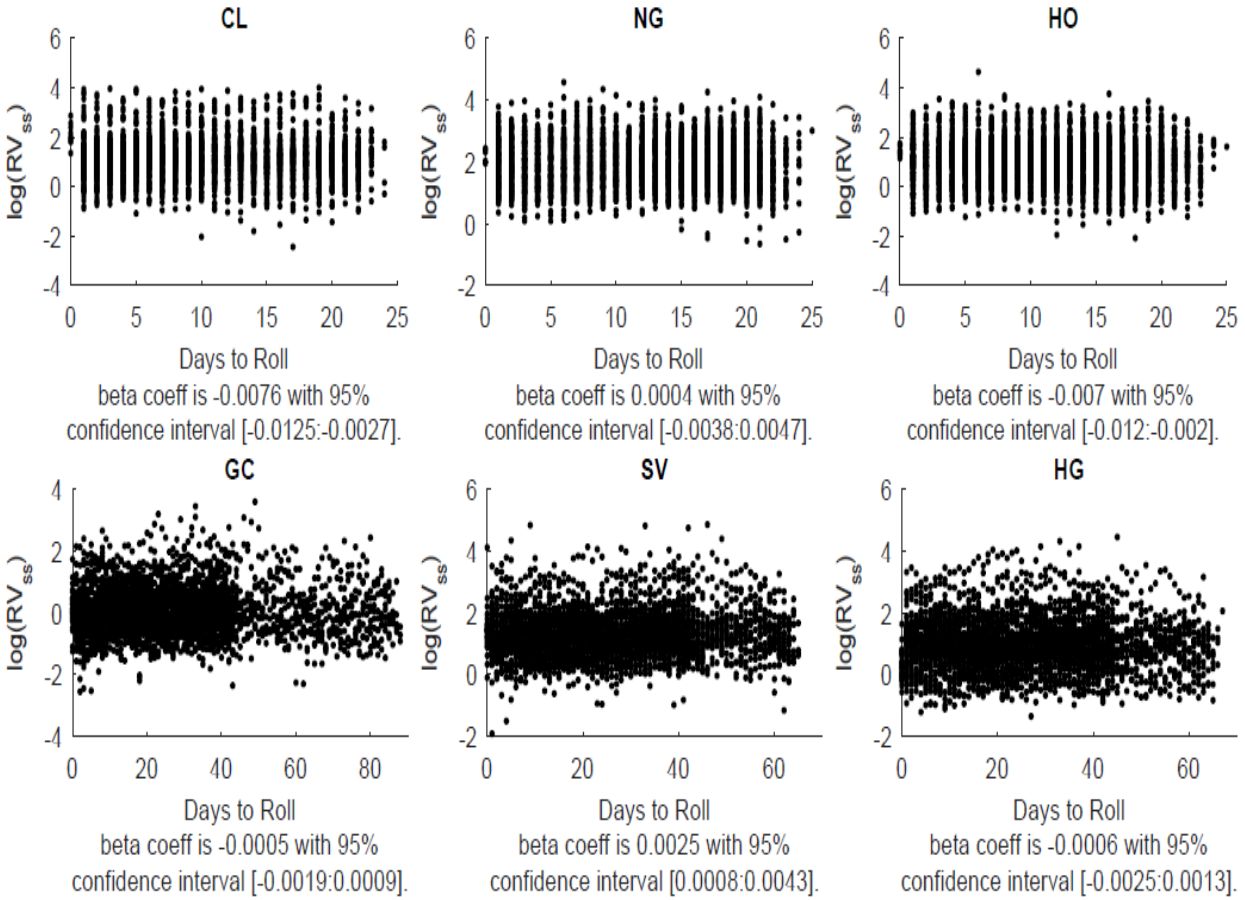
Commodities and MSCI Emerging Markets IV

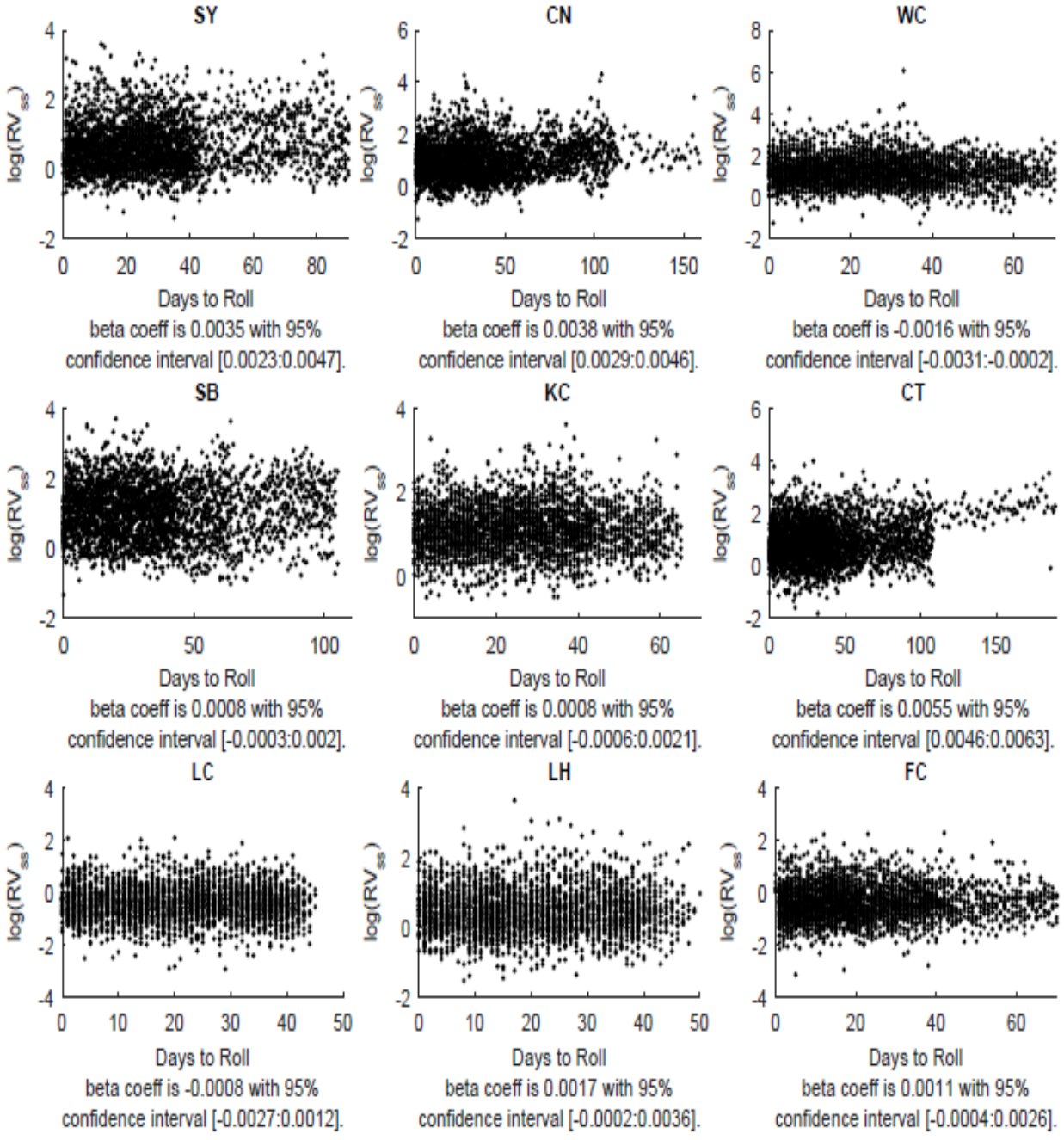
Daily Expected SRR



log(RV) vs. the number of days until next roll.

Volatility does not appear to be systematically different close to the roll-date (no Samuelson).





Summary

1. Daily realized commodity futures volatility has very high persistence.
2. The log of realized commodity futures volatility is close to normally distributed.
3. Weak evidence of a factor structure in daily commodity futures returns (excl. meats.)
4. The factor structure in daily commodity futures volatility is much stronger than the factor structure in returns.
5. Little evidence of a time-trend in the degree of integration across commodity futures markets during the 2004-2014 period.
6. Strong common factor in commodity volatility largely driven by stock market vol.
7. Commodity betas with the stock market were high during 2008-2010 but have since returned to a level close to zero.
8. Commodity futures returns standardized by expected realized volatility are closer to normally distributed than the returns themselves but still display leptokurtosis.

Appendix

Figure 13: QQ Plot of Daily Stock Return Shocks.

SF8: Return shocks still leptokurtotic.

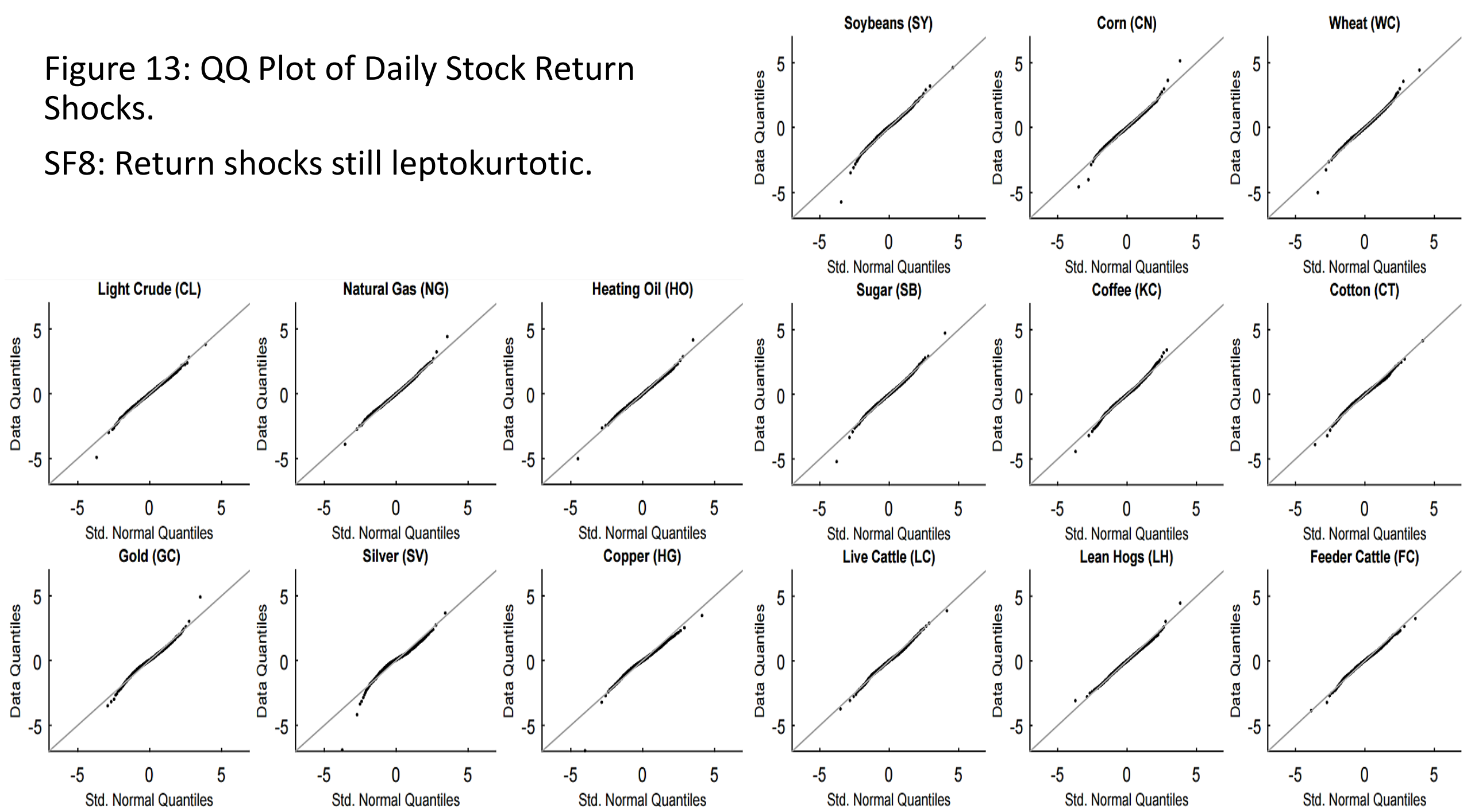


Table 2.a: Sample Statistics for Daily Futures Returns (in percent).

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
Mean	−0.022	−0.144	0.001	0.028	0.024	0.035	0.024	−0.012	−0.041	−0.003	−0.012	−0.022	0.005	−0.035	0.019
Std. Dev.	2.139	3.037	1.945	1.239	2.226	1.952	1.647	1.911	2.105	2.076	2.002	1.783	0.905	1.346	0.880
Min	−12.936	−15.160	−9.436	−8.760	−16.837	−11.709	−7.419	−7.852	−10.017	−11.645	−8.209	−6.676	−3.681	−4.820	−3.897
Max	10.096	15.266	10.591	10.516	14.288	11.570	7.583	8.813	8.673	10.187	12.544	7.507	3.637	5.689	3.263
Skewness	−0.219	0.271	−0.006	−0.376	−1.065	−0.256	−0.229	0.039	0.030	−0.233	0.209	−0.133	0.035	−0.008	−0.158
Kurtosis	5.758	4.888	4.750	8.426	9.633	6.325	4.837	4.563	4.619	5.269	5.004	4.175	4.238	3.798	3.989
$ACF(1)$	−0.059*	−0.067*	−0.021	−0.001	−0.004	−0.076*	−0.015	0.028	−0.016	−0.029	−0.055*	0.039	0.027	0.040	0.083*
$Q(5)$	17.565*	39.675*	4.667	3.807	0.101	25.388*	3.235	2.988	4.351	12.814	17.302*	13.547	9.446	9.230	26.196*
$Q(21)$	48.889*	62.884*	23.748	34.820	30.347	64.897*	25.744	35.244	31.351	31.105	40.367*	34.113	40.817*	42.822*	58.133*
#Obs	2817	2816	2816	2816	2815	2813	2771	2771	2771	2762	2762	2763	2767	2768	2766

Table 2.b: Sample Statistics for Daily Futures Return Shocks.

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
Mean	0.008	0.009	0.001	−0.003	0.001	−0.006	0.007	−0.004	0.004	−0.003	0.001	−0.003	0.000	0.001	−0.002
Std. Dev.	0.949	0.971	0.937	0.988	0.946	0.918	1.002	0.994	0.991	0.982	1.029	0.915	0.964	0.942	0.931
Min	−4.920	−3.903	−5.022	−7.945	−6.902	−6.966	−5.736	−4.567	−5.010	−5.204	−4.423	−3.884	−3.712	−3.078	−3.844
Max	3.814	4.409	4.152	4.908	3.670	3.475	4.606	5.141	4.423	4.733	7.533	4.155	3.875	4.461	3.277
Skewness	−0.181	0.177	0.000	−0.597	−0.744	−0.447	−0.143	0.040	0.124	−0.035	0.227	−0.145	0.010	0.006	−0.181
Kurtosis	3.659	3.510	3.519	6.576	6.258	5.372	4.253	4.683	4.036	4.109	4.867	3.874	3.718	3.491	3.582
$ACF(1)$	−0.047	−0.039	−0.008	0.004	0.034	−0.031	0.003	0.020	−0.003	−0.003	−0.047	0.033	0.022	0.057*	0.079*
$Q(5)$	9.499	14.086	2.568	3.525	3.926	5.468	3.746	3.664	3.844	7.454	12.184	14.764	4.848	9.813	21.589*
$Q(21)$	19.672	30.579	11.406	21.784	24.350	25.060	17.646	31.798	20.201	21.218	30.919	26.453	32.908	36.781	49.206*

Table 3.a: Sample Statistics for Daily Realized Volatility.

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
Mean	1.986	2.876	1.841	1.159	2.162	1.837	1.541	1.827	2.009	1.935	1.895	1.812	0.893	1.383	0.914
Std. Dev.	1.152	1.156	0.877	0.571	1.136	0.988	0.662	0.769	0.883	0.815	0.600	0.826	0.317	0.510	0.361
Min	0.350	0.741	0.351	0.274	0.458	0.519	0.576	0.639	0.525	0.570	0.761	0.403	0.231	0.465	0.233
Max	23.857	18.765	12.371	6.486	19.559	12.013	6.143	8.677	21.215	10.168	6.177	7.461	3.515	6.779	5.513
Skewness	5.068	2.072	2.270	2.676	3.883	2.673	2.040	2.369	5.760	1.479	1.512	1.792	1.796	2.533	2.603
Kurtosis	63.907	17.612	16.005	15.465	34.035	15.240	9.096	13.541	93.590	8.719	7.364	8.254	9.613	17.111	18.887
$ACF(1)$	0.610*	0.643*	0.726*	0.708*	0.636*	0.759*	0.598*	0.530*	0.557*	0.670*	0.496*	0.627*	0.539*	0.501*	0.498*
$Q(5)$	4972*	5326*	7097*	5568*	4127*	7318*	4098*	2692*	2787*	4765*	2504*	4809*	3342*	2653*	2758*
$Q(21)$	18247*	16187*	26456*	18787*	11783*	24957*	12960*	7459*	8116*	14976*	6447*	16806*	11296*	8388*	8389*

Table 3.b: Sample Statistics for Log Daily Realized Volatility.

	Light Crude	*Natural Gas*	*Heating Oil*	*Gold*	*Silver*	*Copper*	*Soy-beans*	*Corn*	*Wheat*	*Sugar*	*Coffee*	*Cotton*	*Live Cattle*	*Lean Hogs*	*Feeder Cattle*
Mean	0.579	0.987	0.515	0.058	0.680	0.501	0.360	0.534	0.633	0.579	0.595	0.508	−0.168	0.270	−0.151
Std. Dev.	0.440	0.368	0.431	0.405	0.400	0.444	0.364	0.358	0.345	0.402	0.293	0.406	0.325	0.319	0.340
Min	−1.051	−0.300	−1.048	−1.296	−0.782	−0.656	−0.551	−0.448	−0.643	−0.563	−0.273	−0.909	−1.466	−0.765	−1.455
Max	3.172	2.932	2.515	1.870	2.973	2.486	1.815	2.161	3.055	2.319	1.821	2.010	1.257	1.914	1.707
Skewness	0.512	0.229	0.138	0.603	0.810	0.491	0.713	0.616	0.629	0.042	0.309	0.356	0.239	0.519	0.510
Kurtosis	4.581	3.054	3.235	3.847	4.658	3.630	3.464	3.665	4.822	2.765	3.413	3.099	3.787	4.072	3.933
$ACF(1)$	0.772*	0.694*	0.819*	0.726*	0.711*	0.825*	0.651*	0.630*	0.652*	0.719*	0.522*	0.667*	0.583*	0.574*	0.556*
$Q(5)$	7746*	6486*	8891*	6164*	5605*	8704*	4937*	4155*	4847*	6084*	2957*	5213*	4074*	3581*	3462*
$Q(21)$	27994*	21152*	33480*	20554*	16880*	30558*	15607*	11909*	14887*	20586*	7656*	18156*	13986*	11696*	11105*