Inflation, Demand for Liquidity, and Welfare

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Motivation

- Inflation affects relative prices of holding different types of assets and hence welfare.

- Most previous studies use representative-agent models and aggregate evidence to measure the cost.

  - Dotsey and Ireland (1996), Lucas (2000), among others.

- Heterogeneous behavior and micro evidence can be important.

Motivation

- Recent work on welfare cost of inflation take into account heterogeneity.
  - Welfare cost varies considerably across households: Mulligan and Sala-i-Martin (2000), Doepke and Schneider (2006), Meh and Terajima (2008), Erosa and Ventura (2002), Chiu and Molico (2008)
  - Aggregate welfare effects can differ when heterogeneity is considered
- Not much done in the literature:
  - Money holding for transaction purpose varies with age.
- This is important because
  - Welfare cost of inflation will differ across age groups
  - Potential nonlinear effects of inflation when aggregated

Other literature

- Lucas (2000) points out an importance of using micro data to estimate the gains/costs of inflation.
- Mulligan and Sala-i-Martin (2000) and Attanasio et al. (2002) use micro data to estimate the welfare cost of inflation.
- Dotsey and Ireland (1996) analyze a general equilibrium model of money demand with an intermediation cost of credit transaction technology.
- Erosa and Venture (2002) incorporates heterogeneity over household income.
- Chui and Molico (2010) uses a search model of demand for money.
- Heer and Maussner (2012) analyze the effects of inflation on distributions of both income and wealth.
- Heer et al. (2007) document that the money-age profile is hump-shaped and money is weakly correlated with income and wealth.

What we do

1. Ask welfare implications of inflation by
   - building an OLG model where money and credit are used for transaction; and
   - calibrating model to capture age, cohort and time effects on money-consumption ratios.
2. Document money-consumption ratios, i.e., liquidity demand for money
   - People are very different between ages and between social classes over money holdings and wealth
3. Use data to disentangle age, cohort and time effects

Findings

- Money-consumption ratio is higher for older and poor households.
  - 5 times higher for old households (aged 76-85) relative to that for young (aged 26-35)
  - 2 times higher for poor households relative to that for rich households
- These effects do not disappear once we control for cohort and time effects.
- Age-specific transaction cost captures age profile of money holding.
- Aggregate welfare effects when inflation $\uparrow$ from 1.92% to 10%,
  - Aggregate consumption decreases by 0.83%.
- Distributional effects are summarize as follows,
  - To be added

Data: Two Household Surveys

- Our main data sources are two household surveys (repeated cross-section)
- Canadian Financial Monitor (CFM), 1999-2010, by Ipsos Reid
  - "money" holdings information available for all years
  - consumption information available only for 2008-2010
- Survey of Household Spending (SHS), 1999-2009, by Statistics Canada
  - no information on money holdings
  - consumption information available for all years
- Money: checking account and some savings accounts (for transactions)
- Consumption: durables (excluding housing), non-durables, and service

Data: Combining CFM and SHS

- To separate out age, cohort and time effects, we need data on money-con ratios over a longer period than 2008-2010 from CFM.
- Obtain a 11-year series by combining CFM and SHS, following Bethencourt and Ríos-Rull (2009):
  1. From 2008-2010 CFM, calculate a joint distribution (in quintile) of households over money and consumption.
  2. For each year over 1999-2009, calculate average money holdings of households in each quintile from CFM and average consumption in each quintile from SHS.
  3. Holding fixed the joint distribution from Step (1), assign the average money holdings and consumption in the respective quintile in each year over 1999-2009.
  4. For each year and each consumption quintile, calculate average money-consumption ratios over money quintile using the marginal distribution from Step (1) as weights.

Data: Joint distribution of Money and Consumption

- CFM 2008-2010 contain household-level information regarding money and consumption. Hence, we can construct a joint distribution of households over money and consumption:

Marginal Dist.		$w_{.1}$	$w_{.2}$	$w_{.3}$	$w_{.4}$	$w_{.5}$	
	5th	w_{51}	w_{52}	w_{53}	w_{54}	w_{55}	$w_{5.}$
Money	4th	w_{41}	w_{42}	w_{43}	w_{44}	w_{45}	$w_{4.}$
Quintile	3rh	w_{31}	w_{32}	w_{33}	w_{34}	w_{35}	$w_{3.}$
	2nd	w_{21}	w_{22}	w_{23}	w_{24}	w_{25}	$w_{2.}$
	1st	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	$w_{1.}$
		1st	2nd	3rd Consumption Quintile	4th	5th	Marginal Distribution

Data: Joint distribution of Money and Consumption

- For each year over the 1999-2007 period,
  - CFM has information on money, $\{w_{1.}, ..., w_{5.}\}$, and
  - we can calculate average money holdings in each quintile.

  - SHS has information on consumption, $\{w_{.1}, ..., w_{.5}\}$, and
  - we can calculate average consumption in each quintile.

- Use these information to approximate money-consumption ratios in each consumption quintile.

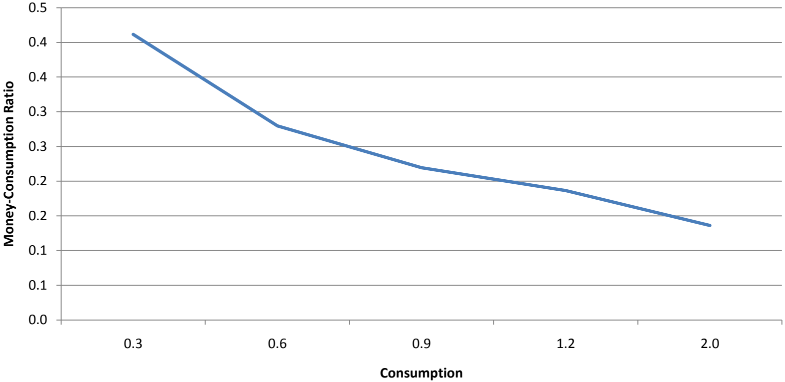
Data: Combining CFM and SHS

- Do this for six age groups:
    - Aged 26-35, 36-45, 46-55, 56-65, 66-75 and 76-85.

Money-Consumption Ratio by Consumption

- Money-consumption ratio declines as consumption increases.

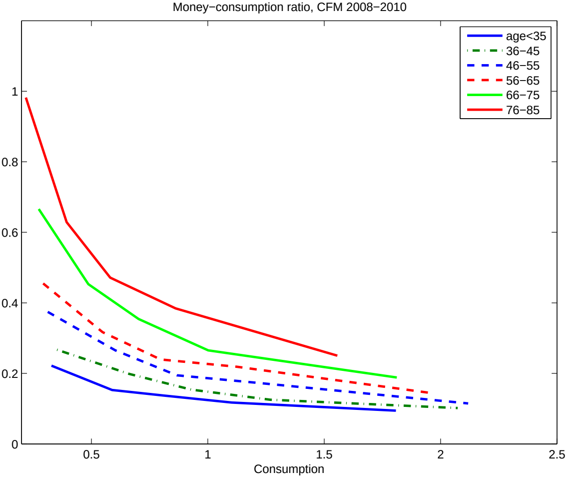
Money-Consumption Ratio by Consumption



Money-Consumption Ratio by Consumption and Age

- Money-consumption ratio rises with household age.

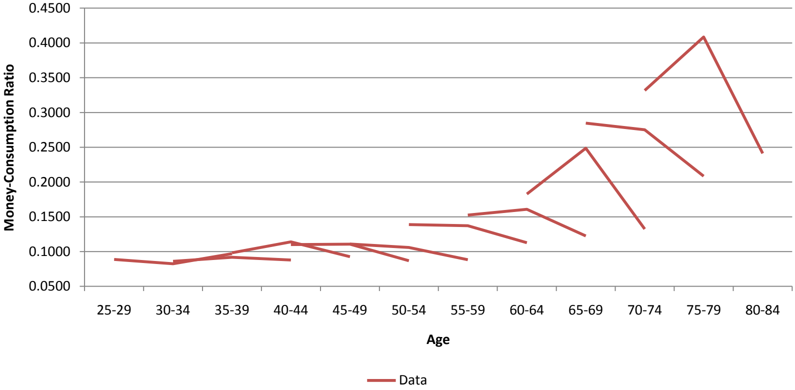
Money-Consumption Ratio by Consumption and Age



Money-Consumption Ratio by Cohort

- Money-consumption ratio declines for newer cohorts.
  - Older cohorts have higher money-consumption ratios given consumption.

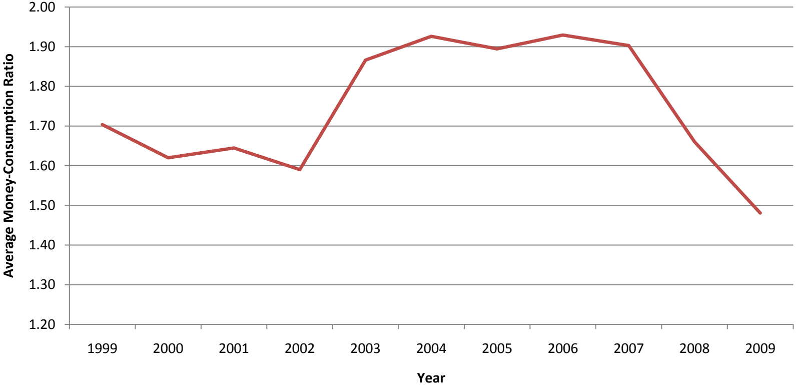
Money-Consumption Ratio by Cohort



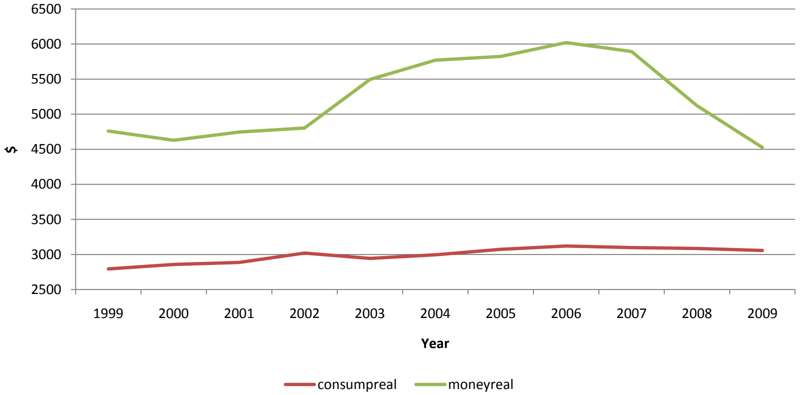
Money-Consumption Ratio Over Time

- Aggregate money-consumption ratios change over time with the macroeconomic environment.

Money-Consumption Ratio Over Time



Money and Consumption Over Time



Empirical Analysis on Money-Consumption Ratios by Age, Cohort and Time

- It is difficult to separate out these three effects.
- Our identification strategy and assumptions are:
  - Three effects are independent
  - Cohort effects are assumed to be exponential with respect to the differences in birth year ($\mu^{\Delta birth\ year/10}$)
  - Time effects are time-specific (λ_{time})
  - Age effects are age-specific (α_{age})
- Estimate μ, λ_t and α_i using annual data on money-consumption ratios from 1999 to 2009, with six 10-year age groups.

Estimation of cohort, time and age effects

- Use the following moment conditions:

$$
\begin{array}{cccc}
\left(\frac{m}{c}\right)_{1,1999} = \alpha_1 & \left(\frac{m}{c}\right)_{1,2000} = \alpha_1 \mu^{\frac{1}{10}} \lambda_{2000} & \cdots & \left(\frac{m}{c}\right)_{1,2009} = \alpha_1 \mu \lambda_{2009} \\
\left(\frac{m}{c}\right)_{2,1999} = \alpha_2 \frac{1}{\mu} & \left(\frac{m}{c}\right)_{2,2000} = \alpha_2 \frac{1}{\mu^{1-\frac{1}{10}}} \lambda_{2000} & \cdots & \left(\frac{m}{c}\right)_{2,2009} = \alpha_2 \lambda_{2009} \\
\vdots & \vdots & & \vdots \\
\left(\frac{m}{c}\right)_{i,1999} = \alpha_i \frac{1}{\mu^{i-1}} & \left(\frac{m}{c}\right)_{i,2000} = \alpha_i \frac{1}{\mu^{i-\frac{1}{10}}} \lambda_{2000} & \cdots & \left(\frac{m}{c}\right)_{i,2009} = \alpha_i \frac{1}{\mu^{i-2}} \lambda_{2009} \\
\vdots & \vdots & & \vdots \\
\left(\frac{m}{c}\right)_{I,1999} = \alpha_I \frac{1}{\mu^{I-1}} & \left(\frac{m}{c}\right)_{I,2000} = \alpha_I \frac{1}{\mu^{I-\frac{1}{10}}} \lambda_{2000} & \cdots & \left(\frac{m}{c}\right)_{J,2009} = \alpha_I \frac{1}{\mu^{I-2}} \lambda_{2009}
\end{array}
$$

- This gives us $I \times 11$ equations and $I + 11$ parameters.

Estimation of cohort, time and age effects

Additional assumptions we make are:

- Cohort effects reduce the demand for money over time: $\mu \in (0, 1)$

Estimation results

α_1	α_2	α_3	α_4	α_5	α_6	μ
0.091	0.109	0.123	0.175	0.297	0.498	0.994
(0.001)	(0.001)	0.002)	(0.002)	(0.003)	(0.007)	(0.864)

λ_{00}	λ_{01}	λ_{02}	λ_{03}	λ_{04}	λ_{05}	λ_{06}	λ_{07}	λ_{08}	λ_{09}
0.96	0.86	0.72	0.96	0.95	0.93	1.02	0.93	0.77	0.67
(0.014)	(0.013)	(0.013)	(0.014)	(0.015)	(0.016)	(0.015)	(0.015)	(0.013)	(0.007)

- $\lambda_{99} \equiv 1$ by normalization
- Use the estimates to calibrate the following model.
- For calibration, the averages over $\lambda_{99} - \lambda_{04}$ and $\lambda_{05} - \lambda_{09}$ are used as the time effects since the model period is 10 years.

We build on Erosa and Ventura (2002)

Seminal work on distribution of welfare cost of inflation:

- An infinitely-lived agent model with costly credit transaction.
- Study distribution of welfare cost over income.
- But abstract from life-cycle effects of inflation which is our focus.

Model

- Build an OLG model
- Consumption can be purchased with money and costly credit
- Agents live for $I = 7$ periods
- Agents differ in income profile ($J = 5$ exogenous income groups)
- Focus on transaction demand for money, and abstract from other roles of money such as hedging for liquidity risks
- Exogenous labour endowments and supply

Household's problem

$$\max_{\{c_{ij}, s_{ij}, m_{i+1,j}, a_{i+1,j}\}} \sum_{i=1}^{I} \beta^{i-1} \frac{c_{ij}^{1-\sigma}}{1-\sigma} \qquad \text{s.t.}$$

$$c_{ij}(1 - s_{ij}) \leq m_{ij};$$

$$c_{ij} + w \cdot \underbrace{\int_0^{s_{ij}} \gamma_j(x)dx}_{\text{transaction cost}} + a_{i+1,j} + (1+\pi)m_{i+1,j} \leq$$

$$[1 + r(1 - \tau_a)]a_{ij} + m_{ij} + (1 - \tau_z)w\, z_{ij};$$

$$a_{1,j} = 0, \;\; m_{1,j} = \underline{m}$$

Government Budget Constraint and Inflation

- Government budget constraint (G–exogenous government spending):

$$G = \pi M/P + \tau_l w L + \tau_a r A$$

- All money is held by households.

$$\sum_{i=1,j=1}^{I,J} \mu_{ij} \; m_{i+1,j} = M/P$$

- There is a constant inflation rate.

$$M_{t+1} = (1+\pi) M_t$$

Transaction technology

$$\gamma_i(x) = \gamma_i \, \eta_{t_i} \cdot \left(\frac{x}{1-x}\right)^{\theta_i}$$

- Fixed cost with respect to consumption and variable with respect to money-credit ratios
- Age effects: γ_i and θ_i
- Cohort effects (new): we assume cohort effects (η_{t_i}) on transaction costs to vary with cohort (indexed by t_i)
- Use data to discipline γ_i, θ_i, η_{t_i}

Calibration strategy

- Household money demand for consumption ($\frac{m_{ij}}{c_{ij}}$) are different in age (i), income (j) and time (t).
- Assume that time effects are driven by macroeconomic parameters such as tax rates, inflation and interest rates.
- Our focus will be on matching money-consumption ratios and consumption from the model to those in the data.
  - Data: $(\frac{m_{ijt}}{c_{ijt}}) = f_{ijt}(\alpha_i, \mu, \lambda_t)$ and c_{ijt}
  - Model: $(\frac{m_{ijt}}{c_{ijt}}) = \frac{1}{1+\left[\widetilde{R}_t c_{ijt}/(w\gamma_i \eta_{t_i})\right]^{1/\theta_i}}$ and c_{ijt}
- Dynamically calibrate along a transition where macroeconomic parameters are changing.
- Use τ_{zt} to balance the government budget.

Dimension of calibration

- Household groups:
  - Age, $I = 7$ (We will not target $i = 1$ HHs for calibration as their portfolio is fixed by assumption.)
  - Income (and consumption class), $J = 5$
  - Total 35 groups (30 groups without $i = 1$ HHs)
- Time periods: 2 periods, 1999 and 2009

Calibration: Data

- 2 periods x 35 household labour income, $\{z_{ijt}^{data}\}_{i=1,j=1,t=1}^{I,J,T}$
- 2 periods x 30 household consumption, $\{c_{ijt}^{data}\}_{i=2,j=1,t=1}^{I,J,T}$
- 2 periods x 30 household money-consumption ratios, $\{\frac{m_{ijt}^{data}}{c_{ijt}^{data}}\}_{i=2,j=1,t=1}^{I,J,T}$
  - Out of these, estimate 6 α_i^{data}'s (age), μ^{data} (cohort) and λ^{data} (time)
- 2 periods x 5 aggregate moments: π_t^{data}, r_t^{data}, $\tilde{R}_t^{\ data}$, τ_{at}^{data} and G_t^{data}

Calibration: List of parameters

- 35 household labour endowments, $\{z_{ij}\}_{i=1,j=1}^{I,J}$
- 30 discount factors, $\{\beta_{ij}\}_{i=2,j=1}^{I,J}$
- 12 age-dependent credit-transaction cost parameters: 6 γ_i's and 6 θ_i's
- 6 cohort-effects parameter, $\{\eta_{t_i}\}_{t_i=t_2}^{t_7}$, t_i is the birth year for $i = 2, ..., 7$
- 10 aggregate parameters: π_t, r_t, $\tilde{R}_t$, τ_{at} and G_t

Calibration WITHOUT solving the model: Parameters and moments

- 35 labour endowments: $\{z_{ij}\}_{i=1,j=1}^{I,J} = \frac{1}{T}\{z_{ijt}^{data}\}_{i=1,j=1,t=1}^{I,J,T}$
- 10 agg. parameters: $\pi_t = \pi_t^{data}$, $r_t = r_t^{data}$, $\tilde{R}_t = \tilde{R}_t^{data}$, $\tau_{at} = \tau_{at}^{data}$, and $G_t = G_t^{data}$ for $t = 1, 2$

Calibration WITH solving the model: Parameters

- 30 discount factors, $\{\beta_{ij}\}_{i=2,j=1}^{I,J}$
- 12 age-dependent credit-transaction cost parameters: 6 γ_i's and 6 θ_i's
- 6 cohort-effects parameter, $\{\eta_{t_i}\}_{t_i=t_2}^{t_7}$, t_i is the birth year for $i = 2, ..., 7$, and set $\eta_{t_1} = \eta_{t_2} = 1$,
- 2 periods of τ_{zt}, w_t

Calibration WITH solving the model: Moments

- 30 household consumption at $t = 1$, $\{c_{ij,t=1}^{data}\}_{i=2,j=1}^{I,J} = \{c_{ij,t=1}\}_{i=2,j=1}^{I,J}$

- 6 age-i average household money-consumption ratios at $t = 1$,

$$\frac{1}{J}\sum_j \{\frac{m_{ij,t=1}^{data}}{c_{ij,t=1}^{data}}\}_{j=1}^{J} = \frac{1}{J}\sum_j \frac{1}{1+\left[\widetilde{R_{t=1}} c_{ij}/(w\gamma_i\eta_{t_i})\right]^{1/\theta_i}}$$

- 6 age-i averaged slope of household money-consumption ratios over consumption at $t = 1$ (and/or $t = 2$),

$$\frac{1}{J}\sum_j \left[\left(\frac{m_{i,j+1,t=1}^{data}}{c_{i,j+1,t=1}^{data}} - \frac{m_{ij,t=1}^{data}}{c_{ij,t=1}^{data}}\right) / \left(c_{i,j+1,t=1}^{data} - c_{ij,t=1}^{data}\right)\right] =$$
$$\frac{1}{J}\sum_j \left(\frac{1}{1+\left[\widetilde{R_{t=1}} c_{i,j+1,t=1}/(w\gamma_i\eta_{t_i})\right]^{1/\theta_i}} - \frac{1}{1+\left[\widetilde{R_{t=1}} c_{ij,t=1}/(w\gamma_i\eta_{t_i})\right]^{1/\theta_i}}\right)$$
$$/\left(c_{i,j+1,t=1} - c_{ij,t=1}\right)$$

Calibration WITH solving the model: Moments

- 6 ratios of averaged money-consumption ratios over income; for $i = 2$ to 7

$$\frac{1}{\mu \cdot \lambda} = \frac{\sum_{j=1}^{J} \frac{1}{1+\left[\widetilde{R_{t=1}} c_{ij,t=1}/(w_{t=1}\gamma_i \eta_{t_i})\right]^{1/\theta_i}}}{\sum_{j=1}^{J} \frac{1}{1+\left[\widetilde{R_{t=2}} c_{ij,t=2}/(w_{t=2}\gamma_i \eta_{t_i+1})\right]^{1/\theta_i}}}$$

- 2 periods of government budget equations,

$$G_t^{data} = \pi_t^{data} M_t + \tau_a^{data} r_t^{data} A_t + \tau_{zt} w_t Z_t^{data}$$

- 2 periods of labour demand: $w_t = f_L(K_t, L_t)$

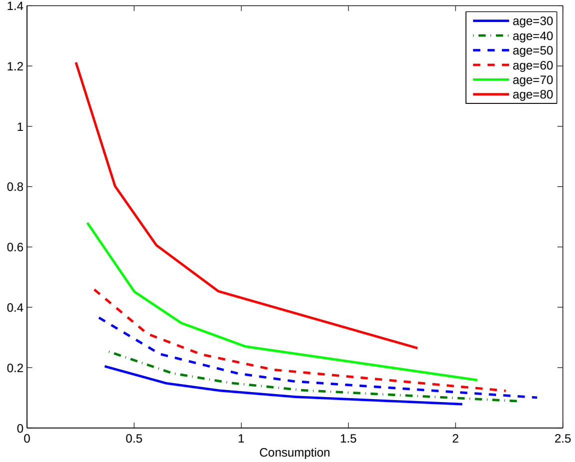
Calibration results

Parameter	Value	Target	Data	Model
γ_2	0.0013	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{2,j}$	0.1457	0.1315
γ_3	0.0020	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{3,j}$	0.1754	0.1595
γ_4	0.0041	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{4,j}$	0.2324	0.2093
γ_5	0.0040	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{5,j}$	0.2888	0.2662
γ_6	0.0060	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{6,j}$	0.4127	0.3817
γ_7	0.0083	$\frac{1}{5}\sum_j \left(\frac{m}{c}\right)_{7,j}$	0.6087	0.6675
θ_2	1.7790	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{2,j}$	-0.1031	-0.0959
θ_3	1.6838	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{3,j}$	-0.1223	-0.1174
θ_4	1.4857	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{4,j}$	-0.1932	-0.1871
θ_5	1.4816	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{5,j}$	-0.2544	-0.2700
θ_6	1.3309	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{6,j}$	-0.4581	-0.4693
θ_7	1.2800	$\frac{1}{4}\sum_j \Delta\left(\frac{m}{c}\right)_{7,j}$	-0.9254	-0.9988

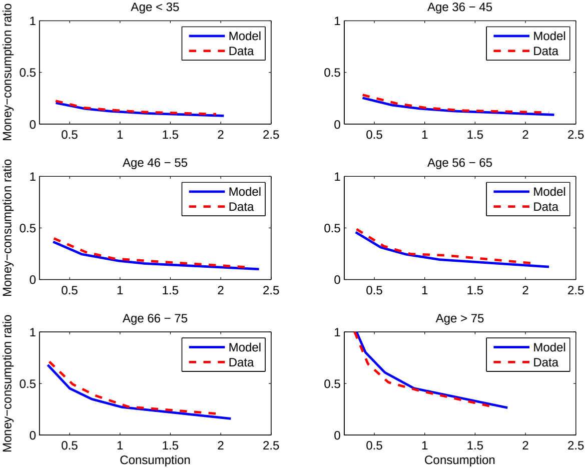
Calibration results

Parameter	Value	Target	Data	Model
η_2	0.6783	$\frac{\lambda_{9904}}{\mu \cdot \lambda_{0509}}$	1.0580	0.9610
η_3	0.4629	$\frac{\lambda_{9904}}{\mu \cdot \lambda_{0509}}$	1.0580	0.9669
η_4	0.3230	$\frac{\lambda_{9904}}{\mu \cdot \lambda_{0509}}$	1.0580	0.9621
η_5	0.2244	$\frac{\lambda_{9904}}{\mu \cdot \lambda_{0509}}$	1.0580	0.9542
η_6	0.1513	$\frac{\lambda_{9904}}{\mu \cdot \lambda_{0509}}$	1.0580	0.9572
η_7	0.1138	$\frac{\lambda_{9904}}{\mu \cdot \lambda_{0509}}$	1.0580	0.9577
$\beta_{i,1}$	0.9552,...,0.7833	$c_{i,1}$	0.3827,...,0.2573	0.3684,...,0.2319
$\beta_{i,2}$	0.9308,...,0.7617	$c_{i,2}$	0.6828,...,0.4632	0.6605,...,0.4171
$\beta_{i,3}$	0.9221,...,0.7628	$c_{i,3}$	0.9368,...,0.6779	0.9133,...,0.6130
$\beta_{i,4}$	0.9175,...,0.7660	$c_{i,4}$	1.2710,...,1.0054	1.2723,...,0.9070
$\beta_{i,5}$	0.9548,...,0.8082	$c_{i,5}$	2.0730,...,1.7970	2.0619,...,1.8499

Calibration results - Money-consumption ratios



Calibration results - Money-consumption ratios



Results – to be added