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Cash-in-the-market pricing and optimal resolution of bank failures

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Abstract

As the number of bank failures increases, the set of assets available for acquisition by the surviving banks enlarges but the total amount of available liquidity within the surviving banks falls. This results in 'cash-in-the-market' pricing for liquidation of banking assets. At a sufficiently large number of bank failures, and in turn, at a sufficiently low level of asset prices, there are too many banks to liquidate and inefficient users of assets who are liquidity-endowed may end up owning the liquidated assets. In order to avoid this allocation inefficiency, it may be *ex-post* optimal for the regulator to bail out some failed banks. We show however that there exists a policy that involves liquidity assistance to surviving banks in the purchase of failed banks and that is equivalent to the bailout policy from an *ex-post* standpoint. Crucially, the liquidity provision policy gives banks incentives to differentiate, rather than to herd, makes aggregate banking crises less likely, and, thereby dominates the bailout policy from an *ex-ante* standpoint.

Summary

The idea that rescuing troubled banks can create incentives for excessive risk-taking is widely spread. However, empirical evidence suggests that regulatory actions taken in response to banking problems vary significantly. In many episodes, regulatory actions appear to depend on whether the problems arise from idiosyncratic reasons specific to particular institutions or from aggregate reasons with potential threats to the whole system. When faced with individual bank failures, authorities usually seek a private sector resolution, whereas government involvement is an important feature of the resolution process during financial crises that affect a significant portion of the banking industry, that is, during crises that are systemic in nature. We argue in this paper that this difference in regulatory actions arises from the fact that resolution options open for an isolated failure of a single institution are different from those available when facing a systemic failure. When only a few banks fail, these banks can be acquired by the surviving banks. However, when the crisis is systemic, that is, for a large number of failures, the liquidity of surviving banks may not be enough for them to acquire all failed banks at the full price. This may lead to the price of failed banks' assets being determined by the available liquidity in the market, resulting in 'cash-in-the-market' pricing of failed banks' assets. Furthermore, during systemic crises, it is more likely that investors outside the banking sector, who are liquidity endowed but potentially not the most efficient users of these assets, end up purchasing some failed banks' assets, leading to social welfare losses associated with the misallocation of banking assets.

Thus, when the banking crisis is systemic in nature, there are 'too many (banks) to liquidate' and bailing out some of the failed banks by the authorities may be optimal in order to avoid allocation inefficiencies. However, this bailout policy may be suboptimal, because it may induce banks to herd by lending to similar industries or betting on common risks such as interest and mortgage rates, in order to increase the likelihood of being bailed out. This in turn increases the likelihood of experiencing systemic banking crises in the first place. We show in this paper that other regulatory options such as the provision of liquidity to surviving banks to be used in acquiring failed banks' assets can mitigate this problem. We show that this policy is equivalent to the bailout policy, but gives banks incentives to differentiate, rather than to herd.

In this paper, we formalise these ideas in a framework wherein the optimal bank failure resolution policies and the cash-in-the-market pricing are endogenously derived. We consider a variety of

resolution policies that broadly cover the entire spectrum of policies employed by regulators to resolve bank failures. In particular, failed banks may be closed, in which case their assets are sold to surviving banks and outsiders at market-clearing prices, or failed banks may be bailed out, in which case their owners are allowed to continue operating the banks. The regulator may also provide liquidity to surviving banks to be used in acquiring failed banks' assets. We show that by virtue of assisting surviving banks in acquiring more failed banks, the liquidity provision policy increases the anticipated surplus for banks in states with cash-in-the-market prices. In turn, this mitigates banks' incentives to herd.

1 Introduction

The widespread belief that rescuing troubled banks can create moral hazard and can give banks incentives to take excessive risk dates back to Bagehot (1873): 'Any aid to a present bad bank is the surest mode of preventing the establishment of a good bank.' However, empirical evidence suggests that regulatory actions taken in response to banking problems vary significantly. In many episodes, regulatory actions appear to depend on whether the problems arise from idiosyncratic reasons specific to particular institutions or from aggregate reasons with potential threats to the whole system, as documented in Santomero and Hoffman (1998) and Kasa and Spiegel (1999). Hoggarth, Reidhill and Sinclair (2004) also study resolution policies adopted in 33 banking crises over the world during 1977–2002. They document that when faced with individual bank failures authorities have usually sought a private sector resolution where the losses have been passed onto existing shareholders, managers and sometimes uninsured creditors, but not to taxpayers. However, government involvement has been an important feature of the resolution process during systemic crises: at early stages, liquidity support from central banks and blanket government guarantees have been granted, usually at a cost to the budget; bank liquidations have been very rare and creditors have rarely made losses.

We argue in this paper that this difference in regulatory actions arises from the fact that resolution options open for an isolated failure of a single institution are different from those available when facing a systemic failure. When only a few banks fail, these banks can be acquired by the surviving banks. However, regulators cannot commit not to intervene when the crisis is systemic. In particular, for a large number of failures, the liquidity of surviving banks enables them to acquire all failed banks only at fire-sale prices.[(1)] The resulting 'cash-in-the-market' pricing, as in Allen and Gale (1994, 1998), makes it more likely that investors outside the banking sector, who are liquidity endowed but potentially not the most efficient users of these assets, will end up purchasing some failed banks' assets.

Thus, when the banking crisis is systemic in nature, there are 'too many (banks) to liquidate' and bailing out some of the failed banks may be optimal in order to avoid allocation inefficiencies. However, this *ex-post* optimal bailout policy may be suboptimal from an *ex-ante* standpoint, for

(1) This effect is akin to the industry-equilibrium hypothesis of Shleifer and Vishny (1992) who argue that when industry peers of a firm in distress are financially constrained, the peers may not be able to pay a price for assets of the distressed firm that equals the value of these assets to them.

example, because it may induce banks to herd by lending to similar industries or betting on common risks such as interest and mortgage rates, in order to increase the likelihood of being bailed out. This in turn increases the *ex-ante* likelihood of experiencing systemic banking crises. Can other regulatory options such as the provision of liquidity mitigate this time-inconsistency problem? We show in this paper that the answer is yes. We propose a novel liquidity provision policy that involves assisting surviving banks in their purchase of failed banks. We show that this policy is equivalent *ex post* to the bailout policy, but gives banks incentives to differentiate *ex ante*, rather than to herd.

We formalise these ideas in a framework wherein the *ex-ante* and the *ex-post* optimal regulatory policies and the cash-in-the-market pricing are endogenously derived. We consider a two-period model with n banks, a regulator, and outside investors who could purchase banking assets were they to be liquidated. Examining a setting with an arbitrary number of banks enables us to explore the richness of the cash-in-the-market price function. The regulator adopts policies to resolve bank failures with the objective of maximising the total output generated by the banking sector net of any costs associated with the adopted policies.

We consider the following regulatory policies: failed banks may be closed, in which case their assets are sold to surviving banks and outsiders at market-clearing prices, or failed banks may be bailed out, in which case their owners are allowed to continue operating the banks. The regulator may also provide liquidity to the various players – failed banks, surviving banks, and outsiders. For simplicity, we assume that deposits are fully insured (at least in the first period). The immediacy of funds required for deposit insurance, net of the proceeds received from liquidating the failed banks and net of liquidity provided to the system during failure resolution, entails fiscal costs for the regulator. The regulatory policies are rationally anticipated by banks and depositors.

Three central assumptions drive our results: (i) banks have access to limited liquidity – in particular, we assume that surviving banks only have their first-period profits to acquire the failed banks' assets (we relax this assumption later and allow pledgeability of future cash flows), (ii) banks are more efficient users of banking assets than outsiders as long as bank owners take good projects,[2] and (iii) there is a possibility of moral hazard in that bank owners derive private benefits from bad projects; hence, banks take good projects only if bank owners retain a large enough share in bank profits.[3]

If the return from the first-period investment of the bank is low, then the bank is in default. The surviving banks, if any, use their first-period profits to purchase failed banks' assets. Up to a critical number of bank failures, liquidity with the surviving banks is enough to purchase all the banking assets at their 'fundamental' price: surviving banks compete with each other and their surplus is eroded to zero. Beyond this critical number of failures, additional assets cannot be absorbed by the available liquidity of surviving banks at the fundamental price. Thus, the market-clearing price declines with each additional failure. Under assumption (ii), outsiders are not as efficient users of banking assets as the surviving banks. Hence, they do not enter the market unless prices fall sufficiently. In other words, there is limited market participation. If the price declines sufficiently, then the liquidity-endowed outsiders enter the market and purchase some of the assets. This gives rise to an allocation inefficiency.

The regulator decides whether to allow such a private sector resolution, that is, to let the surviving banks and/or outsiders purchase failed banks' assets, or to intervene. We first consider intervention in the form of bailing out some or all of the failed banks. In a bailout, a failed bank is not liquidated and its existing owners are allowed to continue operating the bank. Bailouts are thus associated with an opportunity cost for the regulator: the regulator incurs a higher fiscal cost to pay off the insured deposits since no proceeds are collected through asset sales. Since bailouts are costly, the regulator does not intervene as long as failed banks are sold to the surviving banks. However, when asset prices decline sufficiently, it is optimal for the regulator to prevent sales to outsiders by bailing out banks until the fiscal cost of a bailout exceeds the misallocation cost from liquidating the marginal bank.

If outsider funds are limited, then once the number of failures is sufficiently large, even the participation of outsiders in the market for asset sales is not enough to sustain the price at the

(2) James (1991) studies the losses from bank failures in the United States during the period 1985 through mid-year 1988, and documents that 'there is significant going concern value that is preserved if the failed bank is sold to another bank (a 'live bank' transaction) but is lost if the failed bank is liquidated by the Federal Deposit Insurance Corporation (FDIC)'.

(3) For detailed evidence on large inside ownership of banks around the world and its role in alleviating moral hazard problems, see Caprio, Laeven and Levine (2005).

threshold value of outsiders and there is a further decline in the price as the number of failures increase. In this range, the entire liquidity in the market, that is, all funds with the surviving banks and the outsiders, are collected through the sale of failed banks. If the regulator decides to bail out a bank in these cases, the proceeds from the asset sale are not affected and bailouts do not entail any additional fiscal costs. Thus, the regulator bails out more failed banks and prices are sustained at the threshold level for outsiders. Crucially, the states where the number of failures is high are always associated with welfare losses – either fiscal costs through bailouts or misallocation costs through liquidations to outsiders.

We show that there exists a liquidity provision policy that is *ex-post* equivalent in welfare terms to this bailout policy. It is trivially suboptimal to provide liquidity to a bailed-out bank and it is also suboptimal to provide liquidity to outsiders due to their relative lack of expertise. However, liquidity provision of a specific form to the surviving banks can do as well as bailouts. Intuitively, bailouts become *ex-post* optimal when the price of failed banking assets falls sufficiently to elicit participation of outsiders in asset sales. The lack of sufficient liquidity with efficient users – the surviving banks – is the primary cause for this price drop. Assisting the surviving banks in the purchase of assets by providing them with sufficient liquidity can ensure that the market-clearing price for asset sales never falls enough to draw outsiders into the market. Effectively, the regulator can enable the surviving banks to buy failed banks at lower prices than the outsiders (that is, price discriminate) and thereby eliminate the allocation inefficiency. We show that the minimum amount of liquidity provision required to just keep the outsiders out of the market for asset sales entails the same fiscal cost as the *ex-post* optimal bailout policy.

Next, we compare these two policies from an *ex-ante* standpoint in a setting where banks differentiate or herd taking account of the costs and benefits of doing so. Specifically, we assume that each bank invests either in a common industry or in a bank-specific industry. This decision affects the correlation of bank returns and in turn the likelihood that banks fail together. *Ex ante*, the regulator wishes to implement a low correlation between banks' investments in order to minimise the likelihood that many banks fail, and simultaneously implement resolution policies that are *ex-post* optimal.

Employing the bailout policy, the regulator can implement such a welfare-maximising outcome only if it can commit to sufficiently diluting the share of bank owners when they are bailed out. By so doing, the regulator can make bailout subsidies small enough that banks have incentives to differentiate in order to capture the surplus from buying assets at cash-in-the-market prices. However, assumption (iii) implies that such a dilution may not always be feasible. If the moral hazard due to private benefits is sufficiently high, then excessive dilution of a bank's equity leads bank owners to choose bad projects and this generates continuation values that are worse than liquidation values. In this case, the only credible mechanism through which the regulator can implement low correlation is committing to liquidate a sufficiently large number of failed banks. In general, this is *ex-post* inefficient and thus lacks commitment. We show that this time-inconsistency problem leads to inefficient herding by banks when the likelihood of bank failure is sufficiently high.

From an *ex-ante* standpoint, the liquidity provision policy differs from the bailout policy in a crucial manner. By virtue of assisting surviving banks in acquiring more failed banks, the liquidity provision policy increases the anticipated surplus for banks in states with cash-in-the-market prices. In turn, this strengthens incentives of banks to differentiate from each other. Formally, relative to the bailout policy, the liquidity provision policy can implement a low correlation between banks' investments for a wider parameter range, in terms of the severity of the moral hazard problem due to private benefits. To summarise, an important policy implication of our analysis is that a specific form of the lender-of-last-resort (LOLR) policy – one that assists surviving banks rather than the troubled ones – can ameliorate the commitment problem in optimal resolution of bank failures.(4)

Our paper is related to the banking literature that has focused on optimal bank closure policies. Mailath and Mester (1994) and Freixas (1999) discuss the time-inconsistency of closure policy in a single-bank model. Penati and Protopapadakis (1988) assume that the regulator provides insurance to uninsured depositors when the number of banking failures is large, and illustrate that this leads banks to invest inefficiently in common markets so as to attract deposits at a lower cost. Mitchell (1997) considers an argument along the lines of the 'signal-jamming' model of Rajan

(4) In an extension contained in the unabridged version of the paper, we allow banks to issue equity to outsiders as claims against a fraction of their future profits. The price for shares of surviving banks follows an interesting pattern. When the number of failures is large, cash-in-the-market pricing results in the price of failed banks' assets falling below the threshold value of outsiders. Since purchasing failed banks' assets at such prices becomes profitable for outsiders, in equilibrium they must be compensated for purchasing shares in surviving banks. As a result, share price of surviving banks *also* falls below their fundamental value. Thus, limited funds within the whole system and the resulting cash-in-the-market pricing affects not only the price of failed banks' assets but also the price of shares of surviving banks.

(1994) to show that if the regulator bails out banks when they fail together, then banks coordinate on disclosing their losses and delay classifying bad loans by rolling them over. Perotti and Suarez (2002) consider a dynamic model where selling failed banks to surviving banks (reducing competition) increases the charter value of surviving banks and gives banks *ex-ante* incentives to stay solvent. However, in contrast to their paper, we examine the effect of closure policies on interbank correlation. The liquidity provision policy we propose is similar in spirit to the competition policy considered by Perotti and Suarez. In our paper, the liquidity provision policy raises anticipated surplus for banks from surviving and gives them incentives to differentiate. In Perotti and Suarez, reduced competition increases anticipated surplus from surviving and alleviates excessive risk-taking.

While direct exposure to common risks analysed in this paper is one reason banks may fail together, interbank linkages may also lead to joint failures since these linkages may lead to the problems of one bank being transmitted to other banks in a contagion-type phenomenon, as analysed by Rochet and Tirole (1996), Allen and Gale (2000) and Kahn and Santos (2005). In a different contagion model, Diamond and Rajan (2005) show that a bank failure can cause aggregate liquidity shortages and regulatory intervention may be optimal. The focus of their paper is on demonstrating that liquidity and solvency interact in a complex manner and are difficult to distinguish *ex post*. They do not analyse implications of such effects for the *ex-ante* investment choices of banks.

Finally, some of the ideas presented in this paper are related to the analysis in Acharya (2001) and Acharya and Yorulmazer (2007). In our opinion, the strongest differentiating point of the current paper is its modelling generality in allowing for an arbitrary number of banks and endogenously deriving the cash-in-the-market pricing as well as the *ex-ante* and the *ex-post* optimal bailout policies. This lends the model an element of richness that we have exploited to provide an in-depth normative analysis of various options available to regulators to resolve and restructure failed banks. Acharya and Yorulmazer (2007) considers a two-bank model, which focuses more on the positive analysis: it compares the too-big-to-fail problem with the too-many-to-fail problem and shows that herding incentives are stronger for small banks than for large banks, but it does not consider the liquidity provision role of the central bank in assisting purchases of failed banks by surviving banks – a novel policy implication of analysis in this paper.

The remainder of the paper is structured as follows. Section 2 and Section 3 present the model and the analysis. Section 5 considers extensions of the benchmark model. Section 6 concludes. Proofs not contained in the text are contained in the appendix.

2 Benchmark model

The benchmark model is outlined in Figure 1. We consider an economy with three dates – $t = 0, 1, 2$, n banks, bank owners, depositors, outside investors, and a regulator. Each bank can borrow from a continuum of depositors of measure 1. Bank owners, as well as depositors, are risk-neutral, and obtain a time-additive utility u_t where u_t is the expected wealth at time t. Depositors receive a unit of endowment at $t = 0$ and $t = 1$. Depositors also have access to a reservation investment opportunity that gives them a utility of 1 per unit of investment. In each period, that is at date $t = 0$ and $t = 1$, depositors choose to invest their good in this reservation opportunity or in their bank.

Deposits take the form of a simple debt contract with maturity of one period. In particular, the promised deposit rate is not contingent on investment decisions of the bank or on realised returns. In order to keep the model simple and yet capture the fact that there are limits to equity financing due to associated costs (for example, due to asymmetric information as in Myers and Majluf (1984), we do not consider any bank financing other than deposits. We relax this assumption partly in an extension in the unabridged version of the paper.

Banks require one unit of wealth to invest in a risky technology. The risky technology is to be thought of as a portfolio of loans to firms in the corporate sector. The performance of the corporate sector determines its random output at date $t + 1$. We assume that all firms a bank has lent to can either repay fully the borrowed bank loans or they default on these loans. In case of a default, we assume for simplicity that there is no repayment.

Suppose R_t is the promised return on a bank loan at time t. We denote the random repayment on this loan as $\widetilde{R}_t$, $\widetilde{R}_t \in \{0, R_t\}$. The probability that the return from these loans is high in period t is α_t:

$$\widetilde{R}_t = \begin{cases} R_t & \text{with probability} \quad \alpha_t, \\ 0 & \text{with probability} \quad 1 - \alpha_t \end{cases} \tag{1}$$

We assume that the returns in the two periods are independent but allow the probability, as well as the level of the high return, to be different in the two periods. This helps isolate their effect on our results.

There is a potential for moral hazard at the level of an individual bank. If the bank chooses a bad project, then when the return is high, it cannot generate R_t but only $(R_t - \overline{\Delta})$ and its owners enjoy a non-pecuniary benefit of $B < \overline{\Delta}$. Therefore, for the bank owners to choose the good project, appropriate incentives have to be provided by giving them a minimum share of the bank's profits. We denote the share of bank owners as θ. If r_t is the cost of borrowing deposits, then the incentive-compatibility constraint is:

$$\alpha_t \left[\theta(R_t - r_t)\right] \geqslant \alpha_t \left[\theta((R_t - \overline{\Delta}) - r_t) + B\right] \qquad (IC) \qquad \textbf{(2)}$$

We have assumed that the bank is able to pay the promised return of r_t when the investment had the high return irrespective of whether the project is good or bad. The left-hand side of the (*IC*) constraint is the expected profit for the bank from the good project when it has a share of θ of the profit. On the right-hand side, we have the expected profit from the bad project when bank owners have a share of θ, plus the non-pecuniary benefit of choosing the bad project. Using this constraint, we can show that bank owners need a minimum share of $\overline{\theta} = \frac{B}{\overline{\Delta}}$ to choose the good project.[5] We assume that at $t = 0$, the entire share of the bank profits belongs to the bank owners, and therefore, there is no moral hazard to start with.

In addition to banks and depositors, there are risk-neutral outside investors who have limited funds amounting to w to purchase banking assets were these assets to be sold. Outsiders do not have the skills to generate the full value from banking assets. In particular, outsiders are inefficient users of banking assets relative to the bank owners, provided bank owners operate good projects. This can be considered a metaphor for some form of expertise or 'learning-by-doing' effect for making and administering loans. It is also a simple way of introducing barriers to entry in the banking sector. To capture this formally, we assume that outsiders cannot generate R_t in the high state but only $(R_t - \Delta)$. Thus, when the banking assets are sold to outsiders, there may be a social welfare loss due to a misallocation of the assets. We also assume that $\overline{\Delta} > \Delta$ so that outsiders can generate more than what the banks can generate from bad projects.

(5) See Hart and Moore (1994) and Holmstrom and Tirole (1998) for models with similar incentive-compatibility constraints.

The notion that outsiders may not be able to use the banking assets as efficiently as the existing bank owners is akin to the notion of *asset-specificity*, first introduced in the corporate finance literature by Williamson (1988) and Shleifer and Vishny (1992). In summary, this literature suggests that firms, whose assets tend to be *specific*, that is, whose assets cannot be readily redeployed by firms outside of the industry, are likely to experience lower liquidation values because they may suffer from 'fire-sale' discounts in cash auctions for asset sales, especially when firms within an industry get simultaneously into financial or economic distress.[(6)] In the evidence of such specificity for banks and financial institutions, James (1991) shows that the liquidation value of a bank is typically lower than its market value as an ongoing concern. In particular, his empirical analysis of the determinants of the losses from bank failures reveals a significant difference in the value of assets that are liquidated and similar assets that are assumed by acquiring banks.

Finally, there is a regulator who employs policy instruments such as sale of failed banks' assets through auctions, bailouts and liquidity provision with the objective of maximising the total output generated by the banking sector net of any costs associated with these policy options. These policies are assumed to be rationally anticipated by banks and depositors. Below we describe these policies informally. The formal description follows in the model analysis.

We assume that deposits are fully insured in the first period. The provision of immediate funds to pay off failed deposits, net of any proceeds from the sale of failed banks' assets, entails fiscal costs for the regulator (assumed to be exogenous to the model). The fiscal costs of providing funds to the banking sector with immediacy can be linked to a variety of sources, most notably, (i) distortionary effects of tax increases required to fund deposit insurance and bailouts; and, (ii) the likely effect of huge government deficits on the country's exchange rate, manifested in the fact that banking crises and currency crises have often occurred as 'twins' in many countries (especially in emerging market countries). Ultimately, the fiscal cost we have in mind is one of immediacy: government expenditures and inflows during the regular course of events are smooth, relative to the potentially rapid growth of off balance sheet contingent liabilities such as

(6) There is strong empirical support for this idea in the corporate finance literature, as shown, for example, by Pulvino (1998) for the airline industry, and by Acharya, Bharath, and Srinivasan (2003) for the entire universe of defaulted firms in the US over the period 1981 to 1999 (see also Berger, Ofek and Swary (1996) and Stromberg (2000)).

deposit-insurance funds, costs of bank bailouts, costs of liquidity provision, etc.[7]

Note that the second period is the last period in our model and there is no further investment opportunity. As a result, our analysis is not affected by whether deposits are insured for the second investment or not.

If the bank return from the first-period investment is high, then the bank operates one more period and makes the second-period investment. If the return is low, then the bank is in default. We assume that the surviving banks (if any) use their first-period profits (and the liquidity provided by the regulator, if any) to purchase failed banks' assets. The regulator decides whether to let the surviving banks and/or outsiders purchase failed banks' assets or to intervene and bail out some or all of the failed banks, and whether to assist the surviving banks in purchasing assets by providing them liquidity at zero cost.[8] In particular, the regulator simply transfers liquidity without it being returned back by the banks in the future. When a bank is bailed out, the regulator may dilute the equity share of bank owners. Proceeds from the sale of failed banks reduce the costs of providing deposit insurance. Hence, bailouts are associated with an opportunity cost for the regulator. Similarly, provision of liquidity also increases the costs of providing deposit insurance. These costs are also a part of the regulator's objective function.

Depending on the first-period returns, some of the banks (say k out of n) fail. Since banks are identical at $t = 0$, we denote the possible states at $t = 1$ with k, the number of bank failures.

3 Analysis from an *ex-post* standpoint

We analyse the model proceeding backwards from the second period to the first period.

The promised deposit rate at $t = 0, 1$ is denoted by r_t. We assume throughout that $R_t > r_t$ for $t = 0, 1$.

(7) See, for example, the discussion on fiscal costs associated with banking crises in Calomiris (1998). Hoggarth, Reis and Saporta (2002) find that the cumulative output losses have amounted to a whopping 15%-20% annual GDP in the banking crises of the past 25 years. Caprio and Klingebiel (1996) argue that the bailout of the thrift industry cost $180 billion (3.2% of GDP) in the US in the late 1980s. They also document that the estimated cost of bailouts were 16.8% for Spain, 6.4% for Sweden and 8% for Finland. Honohan and Klingebiel (2000) find that countries spent 12.8% of their GDP to clean up their banking systems whereas Claessens, Djankov and Klingebiel (1999) set the cost at 15%-50% of GDP.
(8) Note that given the relative expertise of banks compared to outsiders, it is never optimal to provide liquidity to outsiders. Also, there is no benefit of providing liquidity to a bailed-out bank.

The surviving banks operate for another period at $t = 1$. Since returns from each period's investments are assumed to be independent, the probability of having the high return for each bank is equal to α_1. As this is the last period there is no further investment opportunity. The expected pay-off to the bank from its second-period investment, $E(\pi_2)$, is thus

$$E(\pi_2) = \alpha_1[R_1 - r_1] \tag{3}$$

Note that this pay-off is independent of interbank correlation.

For a bank to continue operating for another period, it needs to pay its old depositors r_0 and it needs an additional one unit of wealth for the second investment. A failed bank cannot generate the needed funds, $(1 + r_0)$, from its depositors at $t = 1$: its depositors are endowed with only one unit of wealth at $t = 1$. Thus, the bank is in default. An important possibility is that the surviving banks and/or outsiders may purchase the assets of failed banks. Next, we investigate sales of failed banks' assets and the resulting asset prices when there are no bailouts or liquidity provision.

3.1 Asset sales and liquidation values

In examining the purchase of failed banks' assets, several interesting issues arise. First, surviving banks and outsiders may compete with each other if there are enough resources with them to acquire all failed banks' assets. Second, unless the game for asset acquisition is specified with reasonable restrictions, an abundance of equilibria arises. To keep the analysis tractable and, at the same time, reasonable, we make the following assumptions:

(i) The regulator pools all failed banks' assets and auctions these assets to the surviving banks and the outsiders. Assets of a failed bank can be acquired partially, and when a portion of these assets are acquired, the purchasing bank can also access the same portion ('branches') of the failed bank's depositors. In essence, the bank can be sold in parts. This assumption of partial bank sales is motivated by the literature on share auctions (Wilson (1979)) and simplifies the analysis substantially. When only a part of the total failed banks' assets are sold and the remaining are bailed out, the assets to be sold are chosen randomly.

(ii) Denoting the surviving banks as $i \in \{1, 2, \ldots, (n - k)\}$ and the outsiders as $i = 0$, each surviving bank and outsiders submit a schedule $y_i(p)$ for the amount of assets they are willing to purchase as a function of the price p at which a unit of the banking asset (inclusive of associated deposits) is being auctioned, where $y_i(p) \in [0, k]$.

(iii) We assume that surviving banks cannot raise additional financing from the markets, an assumption we relax in an extension in the unabridged version of the paper. Hence, the resources available with each surviving bank for purchasing failed banks' assets, denoted by l, equal the first-period profits, that is, $l = (R_0 - r_0)$. In Section 3.3, we allow the regulator to provide surviving banks some liquidity to be used for the asset purchase.

(iv) The regulator cannot price-discriminate in the auction. Later on, in Section 3.3, we relax this assumption and let the regulator provide liquidity to surviving banks to be used for asset purchase, which can be shown to be equivalent to some sort of price discrimination.

(v) The regulator determines the auction price p so as to maximise the expected output of the banking sector, subject to the natural constraint that portions allocated to surviving banks and outsiders add up at most to the number of failed banks, that is, $\sum_{i=0}^{n-k} y_i(p) \leq k$. Given the allocation inefficiency of selling assets to outsiders, it turns out that if the surviving banks and the outsiders pay the same price for the failed banks' assets, the regulator allocates the maximum amount he can to the surviving banks.

(vi) We focus on the symmetric outcome where all surviving banks submit the same schedule, that is, $y_i(p) = y(p)$ for all $i \in \{1, 2, \ldots, (n-k)\}$.

First, we derive the demand schedule for surviving banks. Let $\overline{p} = [\alpha_1(R_1 - r_1)] = E(\pi_2)$, which is the expected profit for a surviving bank from the risky asset in the second period. The expected profit of a surviving bank from the asset purchase can be calculated as:

$$y(p)[\overline{p} - p] \tag{4}$$

The surviving bank wishes to maximise these profits subject to the resource constraint

$$y(p) \cdot p \leq l \tag{5}$$

Hence, for $p < \overline{p}$, surviving banks are willing to purchase the maximum amount of failed banks' assets using their resources. Thus, for $p < \overline{p}$, optimal demand schedule for surviving banks is

$$y(p) = \frac{l}{p} \tag{6}$$

For $p > \overline{p}$, the demand is $y(p) = 0$, and for $p = \overline{p}$, $y(p)$ is indeterminate. In words, as long as purchasing bank assets is profitable, a surviving bank wishes to use up all its resources to purchase failed banks' assets. For a formal proof of this result that takes into account the correlation structure of assets, see the appendix.

We can derive the demand schedule for outsiders in a similar way. Note that outsiders can generate only $(R_1 - \Delta)$ in the high state. Let $\underline{p} = [\alpha_1 ((R_1 - \Delta) - r_1)] = \left[\overline{p} - \alpha_1 \Delta\right]$, the expected profit for the outsiders from the risky asset in the second period.

For $p < \underline{p}$, outsiders are willing to supply all their funds for the asset purchase. Thus, for $p < \underline{p}$, outsiders' optimal demand schedule is

$$y_0(p) = \frac{w}{p} \tag{7}$$

For $p > \underline{p}$, the demand is $y_0(p) = 0$, and for $p = \underline{p}$, $y_0(p)$ is indeterminate. Thus, for $p > \underline{p}$, there is limited participation in the market for banking assets.

Next, we analyse how the regulator optimally allocates the failed banks' assets and the price function that results.

We know that in the absence of financial constraints, the efficient outcome is to sell the failed banks' assets to surviving banks. However, the surviving banks may not be able to pay the threshold price of $\underline{p}$ for all failed banks' assets. If prices fall further, these assets become profitable for the outsiders and they participate in the auction.

The regulator cannot set $p > \overline{p}$ since in this case we have $y(p) = y_0(p) = 0$. If $p \leqslant \overline{p}$, and the number of failed banks is sufficiently small, the surviving banks have enough funds to pay the full price for all the failed banks' assets. More specifically, for $k \leq \underline{k}$, where

$$\underline{k} = \text{floor}\left(\frac{nl}{l + \overline{p}}\right) \tag{8}$$

and floor(z) is the largest integer smaller than or equal to z, the regulator sets the auction price at $p^* = \overline{p}$. At this price, surviving banks are indifferent between any quantity of assets purchased. Hence, the regulator can allocate a share $y(p^*) = \frac{k}{(n-k)}$ to each surviving bank.

For moderate values of k, surviving banks cannot pay the full price for all failed banks' assets but can still pay at least the threshold value of $\underline{p}$, below which outsiders have a positive demand.

Formally, for $k \in \{\underline{k}+1, \ldots, \overline{k}\}$, where

$$\overline{k} = \text{floor}\left(\frac{nl}{l+\underline{p}}\right) \tag{9}$$

the regulator sets the price at $p^* = \left(\frac{(n-k)l}{k}\right)$, and again, all banking assets are acquired by the surviving banks. Note that, in this region, surviving banks use all available funds and the price falls as the number of failures increases. This effect is basically the cash-in-the-market pricing as in Allen and Gale (1994, 1998) and is also akin to the industry-equilibrium hypothesis of Shleifer and Vishny (1992) who argue that when industry peers of a firm in distress are financially constrained, the peers may not be able to pay a price for assets of the distressed firm that equals the value of these assets to them.

For $k > \overline{k}$, the surviving banks cannot pay the threshold price of $\underline{p}$ for all failed banks and profitable options emerge for outsiders. At this point, outsiders have a positive demand and are willing to supply their funds for asset purchase. With the injection of outsider funds, prices can be sustained at $\underline{p}$ until some critical number of failures $\overline{\overline{k}} \geqslant \overline{k}$. However, for $k > \overline{\overline{k}}$, even the injection of outsiders' funds is not enough to sustain the price at $\underline{p}$.

Formally, for $k \in \{\overline{k}+1, \ldots, \overline{\overline{k}}\}$, where

$$\overline{\overline{k}} = \text{floor}\left(\frac{nl+w}{l+\underline{p}}\right) \tag{10}$$

the regulator sets the price at $\underline{p}$. At this price, outsiders are indifferent between any quantity of assets purchased. Hence, the regulator can allocate a share of $y(\underline{p}) = \left(\frac{l}{\underline{p}}\right)$ to each surviving bank and the rest, $y_0(\underline{p}) = \left(k - \frac{(n-k)l}{\underline{p}}\right)$, to outsiders.

And beyond this point, that is, $k > \overline{\overline{k}}$, the price is again strictly decreasing in k and is given by

$$p^*(k) = \frac{(n-k)\,l + w}{k} \tag{11}$$

and $y(p^*) = \left(\frac{l}{p^*}\right)$ and $y_0(p^*) = \left(\frac{w}{p^*}\right)$.

The resulting price function is downward sloping in the number of failed banks k in two separate regions. In the first downward-sloping region, outsiders have not yet entered the market ($k \in \{\underline{k}+1, \ldots, \overline{k}\}$) and there is cash-in-the-market pricing given the limited liquidity of surviving banks. In the second downward-sloping region, even the liquidity of outsiders is not enough to sustain the price at $\underline{p}$, their highest valuation of banking assets. Thus, there is cash-in-the-market pricing in this region given the limited liquidity of the *entire* set of market players bidding for failed assets, that is, of surviving banks as well as outsiders. This price function is formally stated in the following proposition and is illustrated in Figure 2.

Proposition 1 In the absence of bailout and liquidity provision policies, the price of failed banks' assets as a function of the number of failed banks is as follows:

$$p^*(k) = \begin{cases} \overline{p} & \text{for} \quad k \leqslant \underline{k} \\ \frac{(n-k)l}{k} & \text{for} \quad k \in \{\underline{k}+1, \ldots, \overline{k}\} \\ \underline{p} & \text{for} \quad k \in \{\overline{k}+1, \ldots, \overline{\overline{k}}\} \\ \frac{[(n-k)l]+w}{k} & \text{for} \quad k > \overline{\overline{k}} \end{cases} \quad \textbf{(12)}$$

Note that by assumption (iii), surviving banks cannot raise additional financing from outsiders. In an extension in the unabridged version of the paper, we relax this assumption and show that our results are robust to allowing surviving banks to issuing equity to outsiders. In this case, we have two markets: one for assets of failed banks and one for shares of surviving banks. When the number of failures is large, that is, when $k > \overline{\overline{k}}$, the price of failed banks' assets falls below the threshold value of outsiders, $\underline{p}$. Since purchasing failed banks' assets at such prices becomes profitable for outsiders, in equilibrium they must be compensated for purchasing shares in surviving banks. As a result, share price of surviving banks also falls below their fundamental value, $\overline{p}$ (see Figures 8 and 9). In other words, surviving banks can raise equity financing only at discounts. The resulting dilution limits their ability to raise such financing and purchase more of failed banks' assets. Thus, limited funds within the whole system and the resulting cash-in-the-market pricing affects not only the price of failed banks' assets but also the price of shares of surviving banks. Hence, for simplicity, in the rest of the paper, we focus on the case where banks cannot generate funds against their future returns.

3.2 Bailouts

To summarise the result from the previous analysis, when the number of bank failures is sufficiently small, $k \leq \overline{k}$, all failed banks' assets are resolved through a purchase by surviving banks. Since this allocation entails no welfare losses, the regulator does not have any incentive to intervene *ex post*. In contrast, if $k > \overline{k}$, then some of these assets are purchased by outsiders who are not the most efficient users. Hence, it may be optimal for the regulator to bail out some banks (we analyse the liquidity provision policy later). In particular, the regulator compares the misallocation cost resulting from asset sales to outsiders with the cost of bailing out failed banks. Since the misallocation cost is constant at $(\alpha_1 \Delta)$ per unit of failed banks' assets, the regulator bails out failed banks as long as the marginal cost of a bailout is less than this misallocation cost.

Note that for a failed bank to continue operating, it needs a total of $(r_0 + 1)$ units. Since available deposits for a bank amount to only one unit (the $t = 1$ endowment of its depositors), the bank cannot operate unless the regulator injects r_0 at $t = 1$. Under our assumption of full deposit insurance, the regulator does inject r_0 at $t = 1$, so that a bailout is equivalent to the regulator granting permission to a failed bank's owners to operate one more period.

In order to analyse the regulator's decision to bail out or liquidate failed banks, we make the following assumptions:

(i) The regulator incurs a fiscal cost of $f(c)$ when it injects c units of funds into the banking sector. We assume this cost function is strictly increasing and convex (possibly linear): $f' > 0$ and $f'' \geq 0$. We do not model this cost for which we have in mind fiscal and opportunity costs to the regulator from providing funds with immediacy to the banking sector (see footnote 7).

(ii) If the regulator decides not to bail out a failed bank, the existing depositors are paid back r_0 through deposit insurance and the failed bank's assets are sold at the market-clearing price. Thus, when the regulator bails out b of the k failed banks, the fiscal cost incurred is $f(kr_0 - (k - b) \cdot p^*(k - b))$ as proceeds from sale of the remaining $(k - b)$ banks are $[(k - b) \cdot p^*(k - b)]$.

The crucial difference between bailouts and asset sales from an *ex-post* standpoint is that proceeds from asset sales lower the fiscal cost from immediate provision of deposit insurance, whereas bailouts produce no such proceeds. In other words, bailouts entail an opportunity cost to the regulator in fiscal terms.

(iii) The regulator can take an equity share in the bailed-out bank(s). Let β be the share the regulator takes in a bailed-out bank. If the bailed-out bank has a high return from the second investment (which has a probability of α_1), then the regulator gets back $\beta(R_1 - r_1)$ at $t = 2$. *Ex post*, such dilution of a bailed-out bank's equity is merely a transfer from the bank owners to the regulator. However, as argued before, if the regulator takes a share greater than $(1 - \overline{\theta})$, then the bank owners are left with a share of less than $\overline{\theta}$, the critical share below which the bank chooses the bad project. Since sales to outsiders generate a higher pay-off compared to that from a bailed-out bank that chooses a bad project ($\overline{\Delta} > \Delta$), the regulator never takes a share greater than $(1 - \overline{\theta})$. As the value of this stake will be realised in the future, it is assumed not to affect the cost of providing deposit insurance with immediacy.

We characterise the optimal bailout policy under these assumptions. The regulator's objective is to maximise the total expected output of the banking sector net of any bailout or liquidation costs. As argued before, the regulator never intervenes when $k \leqslant \overline{k}$.

When $k \in \{\overline{k} + 1, \ldots, \overline{\overline{k}}\}$, the price for failed banks' assets is $\underline{p}$ and the marginal cost of bailing out the b^{th} bank is

$$g(k, b) = f(kr_0 - (k - b)\underline{p}) - f(kr_0 - (k - b + 1)\underline{p}) \qquad \textbf{(13)}$$

This marginal cost is (at least weakly) increasing in b.[(9)] Hence, there is a maximum number of banks, denoted by $\overline{b}(k)$, up to which the bailout costs are smaller than misallocation costs. Formally, $\overline{b}(k)$ satisfies the following conditions:

$$g(k, \overline{b}) \leqslant \alpha_1 \Delta < g(k, \overline{b} + 1) \qquad \textbf{(14)}$$

The maximum number of banks that can be acquired by the surviving banks is $(n - k)y(\underline{p})$, where $y(\underline{p}) = \frac{l}{\underline{p}}$. Thus, the regulator bails out $b^*(k) = \min\left\{\overline{b}(k), [k - (n - k)y(\underline{p})]\right\}$ banks.

Finally, when $k > \overline{\overline{k}}$, all the funds of the surviving banks and the outsiders, a total of $[(n - k)l + w]$, are collected through the sale of the failed banks' assets. Hence, if the regulator decides to bail out a bank, the proceeds from the asset sale are not affected and bailouts do not incur any (opportunity) fiscal cost. Thus, the regulator first bails out $\widehat{b}(k)$ failed banks where $\widehat{b}(k)$ is the maximum number of bailouts such that the regulator can collect all the available liquidity in the system, $[(n - k)l + w]$, by selling the remaining $\left[k - \widehat{b}(k)\right]$ banks. Note that when the

(9) In particular,

$$\frac{\partial g}{\partial b} = \underline{p}\left[f'(kr_0 - (k - b)\underline{p}) - f'(kr_0 - (k - b + 1)\underline{p})\right].$$

Note that, $\left(kr_0 - (k - b)\underline{p}\right) > \left(kr_0 - (k - b + 1)\underline{p}\right)$, and since $f'' \geq 0$, we have $\frac{\partial g}{\partial b} \geq 0$.

regulator bails out $\widehat{b}(k)$ failed banks, the price for the remaining $\left[k-\widehat{b}(k)\right]$ failed banks' assets reaches $\underline{p}$. Formally, we have

$$(n-k)l+w=(k-\widehat{b}(k))\underline{p} \tag{15}$$

which gives us

$$\widehat{b}(k)=\left\lceil k-\frac{(n-k)l+w}{\underline{p}}\right\rceil \tag{16}$$

From this point on, the outsider funds are sufficient to sustain the price for asset sales of remaining $\left[k-\widehat{b}(k)\right]$ banks at the level $\underline{p}$ and an additional bailout decreases the proceeds from asset sales by $\underline{p}$. Hence, the regulator's decision to bail out additional banks is similar to that for the case where $k \in \{\overline{k}+1,\ldots,\overline{\overline{k}}\}$. In particular, the regulator bails out a total of $[\widehat{b}(k)+\overline{\overline{b}}(k)]$ banks, until the marginal bailout cost starts exceeding the misallocation cost. Thus, $\overline{\overline{b}}(k)$ is given by the condition:

$$h(k,\overline{\overline{b}}) \leqslant \alpha_1\Delta < h(k,\overline{\overline{b}}+1) \tag{17}$$

where $h(k,b)$ is defined as:

$$h(k,b)=f(kr_0-((k-\widehat{b}(k))-\overline{\overline{b}})\underline{p})-f(kr_0-((k-\widehat{b}(k))-\overline{\overline{b}}+1)\underline{p}) \tag{18}$$

Note that the marginal bailout cost $h(k,b)$ reflects the fact that with $[\widehat{b}(k)+\overline{\overline{b}}]$ bailouts, the regulator receives proceeds from asset sales amounting to $\left[(k-\widehat{b}(k))-\overline{\overline{b}})\underline{p}\right]$.

To summarise, the regulator's optimal bailout policy $b^*(k)$ is such that

$$b^*(k)=\begin{cases}\min\left\{\overline{b}(k),[k-(n-k)y(\underline{p})]\right\} & \text{for} \quad k \in \{\overline{k}+1,\ldots,\overline{\overline{k}}\}\\ \min\left\{\widehat{b}(k)+\overline{\overline{b}}(k),[k-(n-k)y(\underline{p})]\right\} & \text{for} \quad k>\overline{\overline{k}}\end{cases} \tag{19}$$

Banks are chosen randomly between the three options of being sold to surviving banks, bailed out, or liquidated to outsiders, and the regulator takes a share of β in all bailed-out banks.

We state this closure/bailout policy formally in a proposition:

Proposition 2 Under the *ex-post* optimal bailout policy,

(i) When $k \leqslant \overline{k}$, surviving banks purchase all failed banks' assets and the regulator does not intervene.

(ii) When $k > \overline{k}$, the regulator bails out $b^*(k)$ of the k failed banks, where $b^*(k)$ is defined by conditions ***(14)**, **(17)** and **(19)**. The banks to be bailed out are chosen randomly with equal*

probability.

The optimal bailout policy has the intuitive property that in states with a large number of bank failures, there are 'too many (banks) to liquidate' and the regulator is forced to bail out some of the failed banks. In particular, irrespective of the fiscal cost function, it is always optimal to bail out up to $\widehat{b}(k)$ banks in the region of high bank failures, $k > \overline{\overline{k}}$, where there is cash-in-the-market pricing due to limited liquidity of the *entire* market for banking assets. Bailouts in this region entail no opportunity costs for the regulator but help avoid misallocation costs.[10]

With bailouts, the price never falls below the reservation price of outsiders, $\underline{p}$. The resulting price function is illustrated in Figure 3. It differs from Figure 2, the no bailout case, in that there is only one downward-sloping region in Figure 3 compared to Figure 2.

Note that the bailout policy $b^*(k)$ is not always monotone increasing in k over the entire range. This is because the marginal cost of bailout, $g(k, b)$, is strictly increasing in k if the fiscal cost function is strictly convex: with more bank failures, the fiscal cost of deposit insurance is higher, and given the convexity of this cost function, incurring the opportunity cost of bailouts becomes more severe. In other words, $\overline{b}(k)$ is decreasing in k when the cost function is convex. A similar argument applies to the marginal cost $h(k, b)$ and the $\overline{\overline{b}}(k)$. The general behaviour of the bailout policy $b^*(k)$ is illustrated in Figure 4. Nevertheless, note that $b^*(k)$ has the property that bailouts occur only when bank failures are sufficiently large in number ($k > \overline{k}$), and, if large enough to reach the second cash-in-the-market price region ($k > \overline{\overline{k}}$), then the opportunity cost of bailouts becomes zero (up to bailouts of $\widehat{b}(k)$ banks).

In order to derive and exploit a closed-form expression for the optimal bailout policy, it is useful to consider a linear cost function: $f(c) = Fc$, $F > 0$. One analytical advantage of the linear cost function is that the bailout policy $b^*(k)$ is now always monotone increasing. Specifically, we obtain that $b^*(k) = \left[k - \frac{(n-k)l}{\underline{p}}\right]$, for $k > \overline{k}$, if the cost parameter F is sufficiently small, and $b^*(k) = \widehat{b}(k)$, for only $k > \overline{\overline{k}}$, otherwise. In words, when the fiscal cost parameter is small, outsiders are kept entirely out of the market for bank sales: surviving banks acquire as many failed

(10) In our model, the level of asset prices have no 'mark-to-market' effect on collateral values and future liquidity of surviving banks. In a richer setting with such a collateral channel for the amplification of fire-sale prices, bailouts would be optimal *ex post* not just to avoid the allocation inefficiency but also to prevent precipitous declines in the market prices of banking assets.

banks as they can and the remaining (if any) failed banks are bailed out. When the fiscal cost parameter is large, outsiders participate in asset sales until the number of bank failures is high enough that the second cash-in-the-market region is reached. At this point, all incremental banks that have failed are bailed out. The resulting bailout policy is given by the following corollary and is illustrated in Figure 5.

Corollary 3 With a linear fiscal cost function $f(c) = Fc$, the regulator bails out $b^*(k)$ of the k failed banks when $k > \overline{k}$, where $b^*(k)$ is defined as follows:

(i) When $F \leq \frac{\alpha_1 \Delta}{\underline{p}}$,

$$b^*(k) = \left\lceil k - \frac{(n-k)l}{\underline{p}} \right\rceil \tag{20}$$

(ii) When $F > \frac{\alpha_1 \Delta}{\underline{p}}$,

$$b^*(k) = \begin{cases} 0 & \text{for} \quad k \in \{\overline{k}+1, \ldots, \overline{\overline{k}}\} \\ \widehat{b}(k) & \text{for} \quad k > \overline{\overline{k}} \end{cases} \tag{21}$$

where $\widehat{b}(k)$ is given by equation **(16)**.

Note that with the linear fiscal cost function the number of bailed-out banks increases linearly with a slope of $\left(1 + \frac{l}{\underline{p}}\right)$ in the number of failed banks k. Hence, for an additional bank failure, the number of bailed-out banks increases by more than one. The reason for this is that as more banks fail, not only the failed banks' assets to be sold increase but also the available funds within the surviving banks decrease. As a result, an additional bank failure increases the number of failed banks' assets that cannot be acquired by surviving banks by $\left(1 + \frac{l}{\underline{p}}\right)$. By implication, as more banks fail, the probability of being bailed out $\left(\frac{b^*(k)}{k}\right)$ increases as well.[11] In other words, as more banks fail, not only the number of bailed out banks but also the likelihood of being bailed out for failed banks increase. We interpret this result as a stronger form of the too-many-to-fail problem.

3.3 Liquidity provision

As argued, the main reason for bailouts is to prevent allocation inefficiency resulting from sales to outsiders. However, the regulator can employ alternative policies to prevent misallocation of

(11) We have $\frac{\partial(b^*(k)/k)}{\partial k} = \frac{l}{\underline{p}} > 0$.

banking assets. We consider a policy of assisting the surviving banks wherein the regulator provides liquidity to these banks without it being returned back in the future to the regulator. Since surviving banks are efficient users of failed banks' assets, this policy can also prevent misallocation of banking assets if surviving banks end up with enough funds to acquire all failed banks' assets. We compare the liquidity provision policy and the *ex-post* optimal bailout policy characterised in Proposition 2 in terms of the level of social welfare from an *ex-post* standpoint. In particular, we show that a specific form of liquidity provision policy and the *ex-post* optimal bailout policy result in the same level of *ex-post* social welfare.

There are two cases to consider: first, when $k \in \{\overline{k}+1, ..., \overline{\overline{k}}\}$, and second when $k > \overline{\overline{k}}$.

Note that for $k \in \{\overline{k}+1, \ldots, \overline{\overline{k}}\}$, the injection of outsider funds of w ensures that the price stays at $\underline{p}$ (see Figure 2). Thus, bailing out a bank adds $\underline{p}$ to the total funds needed by the regulator to resolve bank failures. Alternatively, the regulator can provide $\underline{p}$ units of funds to surviving banks to assist their purchase of the failed banks. Since banks are efficient users of these assets, this form of liquidity provision prevents misallocation of banking assets. Hence, both liquidity provision and bailout policies increase the funds needed by the regulator by $\underline{p}$ and both policies prevent the misallocation cost of $(\alpha_1 \Delta)$. Thus, from an *ex-post* point of view, for $k \in \{\overline{k}+1, \ldots, \overline{\overline{k}}\}$, bailouts and liquidity provision to surviving banks result in the same level of social welfare.

Formally, when the regulator employs the *ex-post* optimal bailout policy characterised in Proposition 2, for $k \in \{\overline{k}+1, \ldots, \overline{\overline{k}}\}$, the regulator bails out $b^*(k)$ banks, where $b^*(k)$ is given in equation **(14)**. The proceeds from the asset sales equal $\left[(k - b^*(k))\, \underline{p}\right]$. And outsiders acquire $\left[k - \frac{(n-k)l}{\underline{p}} - b^*(k)\right]$ units of failed banks' assets. Hence, we can write the social welfare cost as:

$$C_b(k) = f\left(kr_0 - \left[(k - b^*(k))\, \underline{p}\right]\right) + \left[k - \frac{(n-k)l}{\underline{p}} - b^*(k)\right](\alpha_1 \Delta) \qquad \textbf{(22)}$$

The regulator can achieve the same level of *ex-post* social welfare using the following liquidity provision policy. Suppose the regulator provides $[b^*(k)\underline{p}]$ units of liquidity to surviving banks so that surviving banks can altogether acquire $\left[\frac{(n-k)l}{\underline{p}} + b^*(k)\right]$ units of failed banks' assets. Then, under the liquidity provision policy, each surviving bank receives

$$\widehat{l}(k) = \frac{b^*(k)\underline{p}}{(n-k)} \qquad \textbf{(23)}$$

units of liquidity. In this case, all failed banks' assets are sold at a price of $\underline{p}$, resulting in total proceeds of $k\underline{p}$. Outsiders acquire $\left[k - \frac{(n-k)l}{\underline{p}} - b^*(k)\right]$ units of failed banks' assets so that we can write the social welfare cost as:

$$C_l(k) = f\left(\left[kr_0 + b^*(k)\underline{p}\right] - k\underline{p}\right) + \left[k - \frac{(n-k)l}{\underline{p}} - b^*(k)\right](\alpha_1 \Delta) \qquad \textbf{(24)}$$

which equals $C_b(k)$ in equation **(22)**, the welfare cost under the bailout policy.

For the second case, recall from the discussion of bailouts that for $k > \overline{\overline{k}}$, bailing out up to $\widehat{b}(k)$ banks, where $\widehat{b}(k)$ is given in equation **(16)**, does not incur any (opportunity) fiscal cost. Thus, when $k > \overline{\overline{k}}$, the regulator first bails out $\widehat{b}(k)$ failed banks. At this point, the price for the remaining $\left[k - \widehat{b}(k)\right]$ failed banks' assets reaches $\underline{p}$ and an additional bailout decreases the proceeds from the asset sales by $\underline{p}$.

We show that for $k > \overline{\overline{k}}$, just like the bailout policy, a specific form of liquidity provision does not incur any (opportunity) fiscal cost. Under the liquidity provision policy, the regulator can provide $\left[\widehat{b}(k)\underline{p}\right]$ units of liquidity to surviving banks. With these additional funds, surviving banks have a total liquidity of $\left[(n-k)l + \widehat{b}(k)\underline{p}\right]$ units, and with the outsider funds of w, this is just enough to keep the price for all failed banks' assets at $\underline{p}$. Now, surviving banks can acquire the $\widehat{b}(k)$ failed banks that were bailed out under the bailout policy. The regulator collects back $\left(k\underline{p}\right)$ units from the sale of failed banks' assets. Thus, the funds needed by the regulator equal $\left[\left(kr_0 + \widehat{b}(k)\underline{p}\right) - k\underline{p}\right]$, which is identical to the funds needed under the bailout policy.

Once $\widehat{b}(k)$ banks have been sold, the total amount of liquidity in the whole system is sufficient to sustain the price for the sale of remaining $\left[k - \widehat{b}(k)\right]$ banks at the level $\underline{p}$, whereby an additional bailout or liquidity provision decreases the proceeds from asset sales by $\underline{p}$. Hence the regulator's decision is similar to that for the case where $k \in \{\overline{k}+1, ..., \overline{\overline{k}}\}$. Therefore, as argued above, we can show that the liquidity provision policy and the *ex-post* optimal bailout policy result in the same level of *ex-post* social welfare.

To summarise, consider a liquidity provision policy of transferring $\widehat{l}(k)$ units of liquidity to each surviving bank for assistance in the asset purchase, where

$$\widehat{l}(k) = \begin{cases} 0 & \text{for} \quad k \leqslant \overline{k} \\ \frac{b^*(k)\underline{p}}{(n-k)} & \text{for} \quad k \in \{\overline{k}+1, \ldots, n-1\} \\ 0 & \text{for} \quad k = n \end{cases} \qquad \textbf{(25)}$$

and, where $b^*(k)$ is the *ex-post* optimal bailout policy characterised in equation **(19)**. Then, we obtain the following formal proposition.[(12)]

Proposition 4 *Ex-post optimal bailout policy given in Proposition 2 and the liquidity provision policy characterised in equation* **(25)** *result in the same level of ex-post social welfare.*

4 Analysis from an *ex-ante* standpoint

We already showed that from an *ex-post* point of view, the regulator is indifferent between the bailout and liquidity provision policies. In this section, we analyse the *ex-ante* optimality of these policies. In particular, we allow banks to choose the level of correlation in their investments, analyse the incentives different regulatory policies create for the choice of interbank correlation, and compare the resulting social welfare from an *ex-ante* point of view. For the remainder of the paper, we use the linear fiscal cost function $f(c) = Fc$.

4.1 Correlation of bank returns

In this section, we allow banks to choose the level of correlation in their investments. At $t = 0$, banks borrow deposits and then choose the composition of loans in their respective portfolios. This choice determines the level of correlation between the returns from their respective investments. We refer to this correlation as 'interbank correlation'.

We suppose that there is a common industry that all banks can access and there are n other industries, one for each bank, such that only bank i can access industry i (region, set of customers, etc). To focus on the effect of interbank correlation, we assume that the returns from the common

(12) Note that, under the liquidity provision policy, for $k > \overline{k}$, surviving banks acquire $\left[\frac{(n-k)l}{\underline{p}} + b^*(k)\right]$ units of failed banks' assets using their funds of $((n-k)l)$. Hence, they pay an 'effective' price of $p_e^*(k)$ for each unit of asset they acquire, where

$$p_e^*(k) = \begin{cases} \overline{p} & \text{for} \quad k \leqslant \underline{k} \\ \frac{(n-k)l}{k} & \text{for} \quad k \in \{\underline{k}+1, \ldots, \overline{k}\} \\ \frac{[(n-k)l]\underline{p}}{\left[(n-k)l+(b^*(k))\underline{p}\right]} & \text{for} \quad k \in \{\overline{k}+1, \ldots, n-1\} \\ \underline{p} & \text{for} \quad k = n \end{cases} \qquad \textbf{(26)}$$

Thus, the liquidity provision policy characterised in equation **(25)** can also be replicated by a policy of price discrimination where the regulator charges the price $p_e^*(k)$ given in equation **(26)** to surviving banks and a price $\underline{p}$ for outsiders.

and the n-specific industries have the same return structure and they are independent. That is, the return from industry i, denoted by $\widetilde{R}_{it}$, is given as:

$$\widetilde{R}_{it} = \begin{cases} R_{it} & \text{with probability} \quad \alpha_t \\ 0 & \text{with probability} \quad 1 - \alpha_t \end{cases} \tag{27}$$

where $i \in \{1, 2, ..., n\}$ denotes bank-specific industries and $i = c$ denotes the common industry. Banks choose whether to invest a unit of wealth in the bank-specific industry or in the common industry, that is, $x_i \in \{0, 1\}$.[(13)] The vector of choices $(x_1, ..., x_n)$ determines the joint probability distribution of bank returns.

If banks in equilibrium choose to lend to firms in the common industry, then they are assumed to be perfectly correlated, that is, the correlation of banks' returns is $\rho = 1$. However, if they choose different industries, then their returns are independent, that is, $\rho = 0$. Note that the individual probability of each bank succeeding or failing (α_t and $(1 - \alpha_t)$ at time t, respectively) is independent of the interbank correlation. We focus on the joint choice of banks and hence denote x_i's simply as x.

This gives us the following probabilities for the number of bank failures at $t = 1$. When $x = 0$, banks invest in independent industries and we obtain a Binomial distribution for the number of bank failures:

$$\Pr(k) = C(n, k)\, \alpha_0^{n-k}\, (1 - \alpha_0)^k \quad \text{for } k \in \{0, 1, ...n\} \tag{28}$$

where $C(n, k)$ is the number of combinations of k objects from a total of n.

When $x = 1$, banks invest in the common industry and we obtain:

$$\Pr(k) = \begin{cases} \alpha_0 & \text{for} \quad k = 0 \\ 0 & \text{for} \quad k \in \{1, ..., n - 1\} \\ 1 - \alpha_0 & \text{for} \quad k = n \end{cases} \tag{29}$$

Next, we analyse banks' choice of correlation in their investments under different policies the regulator employs.

(13) The case where banks invest both in the common asset as well as in the bank-specific asset gives rise to qualitatively similar results, but is technically quite involved. The two-bank version of this more realistic modelling of bank investments is contained in the Addendum to Acharya and Yorulmazer (2007).

4.2 Interbank correlation under bailout policy

We consider in this section the *ex-post* optimal bailout policy. First, we derive banks' expected profits when they invest in idiosyncratic industries and the common industry, that is, when the interbank correlation ρ equals 0 and 1, respectively. We show that the level of correlation chosen by banks depends on the expected profit banks make from the purchase of failed banks' assets when they survive and the expected subsidy they receive through bailouts when they fail. In Proposition 5, we formally state the conditions under which banks choose to invest in the common industry.

In the first period all banks are identical. Hence, we consider a representative bank. Formally, the objective of each bank is to choose the level of interbank correlation ρ at date 0 that maximises

$$E(\pi_1(\rho)) + E(\pi_2(\rho)) \tag{30}$$

where discounting has been ignored since it does not qualitatively affect the results. The expected pay-off to the bank at date 0 from its first-period investment, $E(\pi_1)$, is

$$E(\pi_1) = \alpha_0(R_0 - r_0) \tag{31}$$

which does not depend on the level of interbank correlation. Hence, banks only take into account the second-period profits when choosing ρ.

Note that when banks invest in the same industry, if the return is low, then all banks fail together and the regulator bails out (randomly picked) $b^*(n)$ of them taking an equity stake of β in the bailed-out banks. Thus, the expected profit from the second-period investment is given as:

$$E(\pi_2(1)) = \alpha_0\, E(\pi_2) + (1-\alpha_0)\left(\frac{b^*(n)}{n}\right)(1-\beta)E(\pi_2) \tag{32}$$

When banks invest in idiosyncratic industries, profitable opportunities from asset purchases may arise. Using the price function in Figure 3, we can see that when only few banks fail, $k \leq \underline{k}$, surviving banks pay full price for the acquired assets. Thus, banks profit from asset purchase only in states that exhibit cash-in-the-market pricing, that is, when $k > \underline{k}$. For $k \in \{\underline{k}+1, \ldots, \overline{k}\}$, each surviving bank captures a surplus from asset purchase that equals

$$
\begin{aligned}
& y(p^*) \cdot \left[\overline{p} - p^*\right] && \textbf{(33)} \\
= & \frac{k}{(n-k)} \cdot \left[\overline{p} - \frac{(n-k)l}{k}\right] && \textbf{(34)}
\end{aligned}
$$

Note that for $k \leq \overline{k}$, all failed banks are purchased by surviving banks and bank owners of failed banks have no continuation pay-offs.

For $k > \overline{k}$, the regulator bails out some of the failed banks and prices are sustained at $\underline{p}$. As a result, failed banks receive a bailout subsidy equal to

$$\phi(k) = \frac{b^*(k)}{k} \cdot (1-\beta)E(\pi_2) \quad \textbf{(35)}$$

In this region, the surplus per unit of banking asset purchased is $(\overline{p} - \underline{p}) = \alpha_1 \Delta$, and each surviving bank captures a surplus equal to $\left[\frac{l}{\underline{p}} \cdot (\alpha_1 \Delta)\right]$. However, surviving banks do not receive any *additional* benefit as k increases since for $k > \overline{k}$, they always use all their funds l to purchase failed banks' assets at the same price $\underline{p}$.

Given this analysis, we can calculate $E(\pi_2(0))$, which has returns from investments, asset purchase and bailouts for different states in the second period (see equation **(A-3)** in the appendix). Comparing this to $E(\pi_2(1))$ in equation **(32)**, we obtain the following result:

Proposition 5 Under the *ex-post* optimal bailout policy (characterised in Corollary 3), there exists a critical threshold $\beta^* \in (0, 1)$ such that:

(i) If the continuation moral hazard is severe, that is, $(1 - \overline{\theta}) < \beta^*$, *then the regulator takes an equity stake of* β *in the bailed-out bank(s), such that,* $\beta \leq (1 - \overline{\theta}) < \beta^*$, *and banks invest in the common industry.*

(ii) If $(1 - \overline{\theta}) \geqslant \beta^*$, *then the regulator takes an equity stake* β, *such that* $\beta^* \leqslant \beta \leqslant (1 - \overline{\theta})$, *and banks invest in different industries.*

Proof: *See appendix.*

Note that this result is in contrast to the case with no bailouts. Without bailouts, failed banks receive no subsidy from the regulator, whereas surviving banks capture some rents through asset purchase at discounted prices. As a result, without bailouts, banks always choose the low correlation. To summarise, while bailouts lower the *ex-post* allocation inefficiency, they may give banks incentives to herd, since bailouts occur only when a sufficiently large number of banks have failed. The presence of continuation moral hazard implies that the regulator will not always be able to dilute the equity stake of bailed-out banks up to a level that induces banks *ex ante* to invest in different industries.

It can be shown that with the linear fiscal cost function, the optimal bailout policy (Corollary 3) implies that $\beta^* < 1$ always. Furthermore, as the level of outsider funds w decreases, the second cash-in-the-market region becomes larger ($\overline{\overline{k}}$ decreases). Thus, the region over which $\widehat{b}(k)$ banks are always bailed out increases (again, see Corollary 3). This increases the bailout subsidy for banks in states with a large number of bank failures and aggravates bank incentives to herd. Formally, β^* is decreasing in w: as w decreases, the regulator must hold a larger share in bailed-out banks to induce a low *ex-ante* correlation among banks. In turn, herding incentives prevail over a larger range of the continuation moral-hazard parameter $\overline{\theta}$.

This leads to the intuitively appealing conclusion that in economies where banking crises are likely to be economy wide in nature, that is, accompanied by lower aggregate wealth, herding incentives of banks are likely to be more pronounced. Furthermore, from the standpoint of positive analysis, our model suggests that the too-many-to-fail (or too-many-to-liquidate) problem and the induced herding are likely to be more prevalent in banking systems where the governance of banks is poor: in other words, where agency problems (for example, fraud by bank owners) are more severe so that banks are required in equilibrium to hold greater equity stakes (high $\overline{\theta}$) for incentive reasons.

4.3 Interbank correlation with liquidity provision

While *ex post* the regulator is indifferent between bailout and the liquidity provision policies, these two policies may create different *ex-ante* incentives for banks. In particular, while bailouts induce herding incentives, liquidity provision can strengthen incentives for differentiation. Here, we analyse banks' *ex-ante* incentives under the liquidity provision policy.

The expected profit for banks when they invest in different industries can be calculated as in the case of bailout policy. We do not repeat the derivation in the text but highlight the important differences. See equation **(A-7)** in the appendix for the whole expression.[14]

On the one hand, when the regulator follows the liquidity provision policy, surviving banks can acquire a larger amount of failed banks' assets. In particular, for $k > \overline{k}$, they can also acquire the $b^*(k)$ bailed-out banks, that is, each surviving bank acquires $\left(\frac{b^*(k)}{n-k}\right)$ additional units of banking assets using $(n-k)l$ of their funds. This gives them an extra surplus of

$$\gamma\left(k\right)=\left(\frac{b^*(k)}{n-k}\right)\overline{p} \tag{36}$$

from the asset purchase, relative to the case with bailouts.

However, under the liquidity provision policy, there are no bailouts (unless all banks fail), whereas under the *ex-post* optimal bailout policy, a failed bank is bailed out with probability $\left(\frac{b^*(k)}{k}\right)$, giving rise to a bailout subsidy for failed banks that is given by $\phi(k)$, for $k > \overline{k}$ (see equation **(35)**).

The following proposition, in whose derivation the surplus $\gamma\left(k\right)$ plays a crucial role, characterises banks' choice of correlation under the liquidity provision policy.

Proposition 6 Under the liquidity provision policy characterised in equation **(25)**, there exists a critical threshold β_l^* such that:

(i) If the continuation moral hazard is severe, that is, $(1-\overline{\theta}) < \beta_l^*$, *then the regulator takes an equity stake of* β *in the bailed-out bank(s), such that,* $\beta \leq (1-\overline{\theta}) < \beta_l^*$, *and banks invest in the common industry.*

(ii) If $(1-\overline{\theta}) \geqslant \beta_l^*$, *then the regulator takes an equity stake* β, *such that* $\beta_l^* \leqslant \beta \leqslant (1-\overline{\theta})$, *and banks invest in different industries.*

Proof: *See appendix.*

With the linear fiscal cost function, under the liquidity provision policy, we have $\beta_l^* < 1$ always.

(14) Note that when all banks fail, that is, $k = n$, there is no surviving bank to acquire failed banks' assets and the regulator follows the *ex-post* optimal bailout policy. As a result, the expected profit for banks when they invest in the common industry ($x = 1$) is the same under the liquidity provision and bailout policies, and is given in equation **(32)**.

However, the outsiders' funds w have a different effect on banks' herding incentives when the regulator employs the liquidity provision policy compared to the case with bailouts. Intuitively, for efficient allocation of assets, it is desirable to have as much liquidity within the surviving banks as possible relative to the liquidity with outsiders. Recall that as the level of outsider funds w decreases, the second cash-in-the-market region becomes larger ($\overline{\overline{k}}$ decreases). Thus, the region over which the regulator provides liquidity to surviving banks widens. This increases the expected surplus surviving banks make from asset purchases and mitigates bank incentives to herd. Formally, β_l^* is decreasing in w: as w decreases, $E(\gamma)$ increases and the regulator can induce a low *ex-ante* correlation among banks by holding a smaller share in bailed-out banks. In turn, herding incentives can be mitigated over a larger range of the continuation moral-hazard parameter $\overline{\theta}$.

4.4 Welfare analysis

Having derived the response of banks to different regulatory policies, in terms of the choice of interbank correlation, we now analyse *ex-ante* welfare under this choice. As before, we start with analysing *ex-ante* welfare under the bailout policy. To this end, we first derive the total expected output generated by the banking industry when the interbank correlation, ρ, equals 0 or 1. We show that the interbank correlation affects welfare through the misallocation and fiscal costs that vary with the number of bank failures. Next, we state in Proposition 7 sufficient conditions under which the expected total output is maximised when banks operate in different industries. Finally, we show in Proposition 8 that the *ex-post* optimal bailout policy is not optimal from an *ex-ante* standpoint, and illustrate in Proposition 9 that the liquidity provision policy can mitigate this time-inconsistency problem.

Let $E(\Pi_t(\rho))$ be the expected output generated by the banking sector at date t, net of liquidation and/or bailout costs. If banks invest in the same industry at date 0, then with probability α_0, all n banks have the high return so that $E(\Pi_1(1)) = n\alpha_0 R_0$. However, if they invest in different industries, then with probability $Pr(k) = C(n,k)\,\alpha_0^{(n-k)}(1-\alpha_0)^k$, k banks have the low return while the remaining $(n-k)$ have the high return. Thus,

$$E(\Pi_1(0)) = R_0 \sum_{k=0}^{n}(n-k)\Pr(k) = R_0\left[n - \sum_{k=0}^{n} k\ \Pr(k)\right] \qquad \textbf{(37)}$$

Note that $\left[\sum_{k=0}^{n} k\ \Pr(k)\right]$ is the expected number of failed banks and is equal to $[(1-\alpha_0)n]$ for the Binomial distribution, so that, $E(\Pi_1(0)) = E(\Pi_1(1)) = n\alpha_0 R_0$. Thus, total expected output in the first period is independent of the choice of interbank correlation.

In the second period, the number of banks that continue operating depends on the outcome of the first-period investments and the regulator's action.

If banks invest in the same industry, then with probability α_0, they all succeed and continue operating in the second period; with probability $(1-\alpha_0)$, they all fail, $b^*(n)$ of them are bailed out by the regulator at a fiscal cost of $f(nr_0 - (n - b^*(n))\underline{p})$, and the remaining $(n - b^*(n))$ failed banks' assets are sold to outsiders resulting in a misallocation cost of $[(n - b^*(n))\,(\alpha_1\Delta)]$. Thus,

$$\begin{aligned} E(\Pi_2(1)) &= \alpha_0\, n\,(\alpha_1 R_1) + (1-\alpha_0)\Big[n\,(\alpha_1 R_1) - f(nr_0 - (n - b^*(n))\underline{p}) - (n - b^*(n))\,(\alpha_1\Delta)\Big] \\ &= n\,(\alpha_1 R_1) - (1-\alpha_0)\Big[f(nr_0 - (n - b^*(n))\underline{p}) + (n - b^*(n))\,(\alpha_1\Delta)\Big] \qquad \textbf{(38)} \end{aligned}$$

Next, consider the case where banks invest in different industries. From Proposition 2, we know that for $k > \bar{k}$, $b^*(k)$ banks are bailed out, $(n-k)$ surviving banks buy as many units of banking assets as possible with their available resources of $((n-k)l)$ units, and the remaining $\left[k - \left(\frac{(n-k)l}{\underline{p}}\right) - b^*(k)\right]$ banks are sold to outsiders. This gives us

$$\begin{aligned} E(\Pi_2(0)) &= n\,(\alpha_1 R_1) - \sum_{k=1}^{\bar{k}} \Pr(k)\, f(kr_0 - k\cdot p^*(k)) - \sum_{k=\bar{k}+1}^{n} \Pr(k)\, f(kr_0 - (k - b^*(k))\underline{p}) - \\ &\quad - (\alpha_1\Delta)\cdot \sum_{k=\bar{k}+1}^{n} \Pr(k)\left[k - \left(\frac{(n-k)l}{\underline{p}} + b^*(k)\right)\right] \qquad \textbf{(39)} \end{aligned}$$

The last term represents the expected misallocation cost. The second and third terms represent the fiscal cost for small and large number of bank failures, respectively.

From a welfare point of view, the optimal level of correlation is $\rho = 0$ when $E(\Pi_2(0)) > E(\Pi_2(1))$. Hence, we compare $E(\Pi_2(1))$ in equation **(38)** and $E(\Pi_2(0))$ in equation **(39)** term by term.

First, we compare the misallocation cost in these two cases. Note that the number of failed banks' assets that are purchased by outsiders (weakly) increases as the number of failures increase. Therefore the misallocation cost is maximum when all n banks fail. When banks choose the high correlation, the probability that they all fail is $(1 - \alpha_0)$, while, when they choose the low correlation, there is a positive misallocation cost with probability of at most $\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\right]$. Thus, $(1 - \alpha_0) \geqslant \left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\right]$ is a sufficient condition for the expected misallocation cost to be higher when banks' returns are perfectly correlated.

Next, the expected fiscal cost of deposit insurance net of proceeds from asset sales is strictly greater if banks choose the high correlation. To see this, recall that the expected number of bank failures is equal to $[(1 - \alpha_0)n]$ for both the high and the low correlation cases. Hence, the expected amount of funds needed for deposit insurance is $[(1 - \alpha_0)nr_0]$ in both cases. However, when banks choose the high correlation, if all banks fail, then the regulator collects only $\underline{p}$ per unit of failed banking assets from outsiders. In contrast, if banks choose the low correlation, then up to an intermediate number of failures, $\bar{k}$, surviving banks are able to pay a price that is higher than $\underline{p}$. Thus, expected proceeds from asset sales are greater when banks choose the low correlation. Thus, $(1 - \alpha_0) \geqslant \left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\right]$ is in fact a sufficient condition for low correlation to dominate high correlation from an overall welfare perspective. For example, if $\bar{k} \geqslant n/2$ (a condition that simplifies to $l \geq \underline{p}$), then $\alpha_0 < 1/2$ is sufficient to obtain that the socially optimal level of correlation is $\rho = 0$.

Hence, we obtain the following formal result.

Proposition 7 For all α_0 such that $\alpha_0 < \left(\sum_{k=0}^{\bar{k}} \Pr(k)\right)$, the expected total output of the banking sector at date 0 (net of any anticipated costs of liquidations and bailouts) is maximised when banks operate in different industries, that is, when $\rho = 0$.

Proof: *See appendix.*

We showed that when the likelihood of bank failures is sufficiently high, social welfare is maximised when banks invest in different industries. Thus, the regulator may wish to implement 'hard' closure policies, such as liquidating a sufficient number of banks in order to create incentives for banks to choose the low correlation.[15] These policies may however not be *ex-post* optimal. For example, conditional upon reaching states where the number of bank failures is large, liquidation of banks may not be credible if bailout costs are smaller than liquidation costs. Another way the regulator can induce low correlation among banks is by diluting the equity share of bailed-out banks (see Proposition 5). However, this may also lack commitment *ex post*: if the minimum dilution required to induce low correlation is sufficiently large, then such dilution may have adverse consequences for continuation moral hazard and banks may choose bad projects.

We formalise this trade-off below. In particular, we characterise the *ex-ante* optimal bailout policy assuming that the regulator can commit to *ex-post* implementation of this policy. We also examine when the *ex-ante* optimal policy is not subgame perfect and thus time-inconsistent.

If the regulator commits not to bail out more than a threshold number of banks, then banks may choose to invest in different industries. Thus, we focus on a bailout policy $\widetilde{b}^*(k) = \min\{b^*(k), \widetilde{b}\}$, where $\widetilde{b}$ is the maximum number of bank bailouts that implements the low correlation and $b^*(k)$ is the *ex-post* optimal bailout policy (see Figure 6). Note that a policy that never bails out banks ($\widetilde{b} = 0$) always implements the low correlation. Hence, $\widetilde{b}$ is well defined. Since it is optimal to bail out more than $\widetilde{b}$ banks *ex post*, it suffices to concentrate the analysis on $\widetilde{b}^*(k)$ among the set of bailout strategies that implements the low correlation.[16]

Note that it is optimal for the regulator to commit to not bailing out more than $\widetilde{b}$ banks if and only if the expected output generated by the banking industry is higher when banks choose the low correlation than the case where the regulator follows the *ex-post* optimal bailout policy and banks choose the high correlation. The trade-off is simple: *ex post*, the regulator cares only about continuation pay-offs in states where $k > \overline{k}$, whereas *ex ante*, the regulator is willing to give up some of these pay-offs in order to induce better incentives for banks to be less correlated. This gives us the following proposition on the time-inconsistency of *ex-ante* optimal policy.

Proposition 8 For all α_0 such that $\alpha_0 \leqslant \left[1 - \left(\lambda\left(\frac{p}{\overline{p}^*(n)}\right)\sum_{k=\overline{k}+1}^{n}\Pr(k)\right)\right]$, where

(15) We assume that the regulator cannot write contracts that 'force' banks to adopt specific investment choices, that is, the regulator cannot impose regulation that is explicitly contingent on interbank correlation.
(16) This analysis is interesting only when the *ex-post* optimal level of bailouts, namely $b^*(k)$, is large enough to induce banks to choose the high correlation. In other cases, we would not observe time-inconsistency in the regulatory actions. More specifically, in these other cases, it is possible for the regulator to implement a low correlation among banks at date 0 without affecting the *ex-post* optimal closure policy.

$\lambda = \max\left\{\frac{\alpha_1 \Delta}{F\underline{p}}, 1\right\}$, if the regulator can credibly commit to a bailout strategy *ex ante*, then it is optimal to commit to the strategy $\widetilde{b}^*(k) = \min\{b^*(k), \widetilde{b}\}$, where $\widetilde{b}$ is the maximum number of bank bailouts that implements the low correlation and $b^*(k)$ is defined in equation **(20)**.

Proof: *See appendix.*

The result in Proposition 8 is quite intuitive. For some values of α_0, low correlation may be the socially optimal level of correlation. For those values, the regulator may need to commit to a strategy of excessive liquidation to give banks the incentives to choose the low correlation. This may however be too costly in terms of expected misallocation costs. Our analysis shows that a condition for this time-inconsistency to arise is that the likelihood of bank failures, $(1 - \alpha_0)$, be sufficiently high.

It is possible that the regulator can mitigate this time-inconsistency problem using the liquidity provision policy instead of the bailout policy since with additional liquidity, surviving banks' surplus from asset purchases increases. At the same time, the liquidity provision policy characterised in equation **(25)** achieves the same level of *ex-post* social welfare as the *ex-post* optimal bailout policy. Thus, to mitigate herding incentives, the regulator may not have to resort to excessive liquidation in states where banking crises are systemic. To illustrate this, we show that the regulator can implement low correlation for a wider range of parameter values using the liquidity provision policy compared to the *ex-post* optimal bailout policy.

Recall that under the liquidity provision policy, surviving banks receive the additional benefit of $\gamma(k)$ in equation **(36)**. However, under this policy, there are no bailouts whereas under the *ex-post* optimal bailout policy, a failed bank receives an expected bailout subsidy of $\phi(k)$ given in equation **(35)**. Hence, if $\gamma(k) > \phi(k)$ for all $k > \overline{k}$, then it is sufficient to show that the expected gain for banks from the liquidity provision policy through asset purchases overweighs the expected loss of bailout subsidy. If this condition holds, then the regulator can implement low correlation by taking a smaller stake in the bailed-out banks under the liquidity provision policy, that is, $\beta_l^* \leqslant \beta^*$. The following proposition states a sufficient condition for this result to hold.

Proposition 9 For $l \geqslant \underline{p}$, we have $\beta_l^* \leqslant \beta^*$.

Proof: *See appendix.*

First, note that this is only a sufficient condition which guarantees that $\gamma(k) > \phi(k)$ for all $k > \overline{k}$. The herding incentives are in fact mitigated under the liquidity provision policy for a larger range of parameter values. Second, the condition $l \geqslant \underline{p}$ also simplifies to $\overline{k} \geqslant n/2$, a condition under which it is socially optimal to have low interbank correlation for all $\alpha < 1/2$. Furthermore, while both $\gamma(k)$ and $\phi(k)$ increase as more banks fail, $\gamma(k)$ increases faster than $\phi(k)$. Hence, $\gamma(\overline{k}) > \phi(\overline{k})$, is sufficient for $\gamma(k) > \phi(k)$, for all $k > \overline{k}$. And the condition $l \geqslant \underline{p}$ simply guarantees that.

To summarise, assisting surviving banks by giving them liquidity to purchase failed banks arises as an *ex-post* optimal regulatory policy that dominates the bailout policy from an *ex-ante* incentive standpoint.

5 Empirical support and robustness

In this section, we discuss how our paper relates to the resolution of bank failures in practice and also provide empirical support for the assumptions and results of our model.

5.1 Resolution of bank failures

5.1.1 General overview

One of our main results is that in states where the number of bank failures is small, the regulator should optimally let the surviving bank(s) acquire failed banks, whereas in multiple bank failures, it may sometimes be optimal for the regulator to intervene. In particular, the regulator uses a variety of policy options to resolve bank failures including bailouts, sales to surviving banks, with and without government assistance (in the form of liquidity provision and price discrimination), and, finally, sales to outsiders. These policies broadly cover the spectrum of options regulators employ to resolve bank failures. See the report by the Basel Committee on Banking Supervision (BCBS (2002)) for a detailed discussion of the resolution policies.

Regulators use a range of policy options in practice. On the one end, banks are kept open through an injection of capital, which would correspond to a bailout in our paper. On the other end, troubled banks are sometimes closed and liquidated, which in our paper would correspond to a sale to outsiders. Between these two extremes, regulators employ other options such as sale of failed bank's assets to other banks with or without government assistance.

To minimise the fiscal cost of resolution regulators usually seek a private sector resolution first, where the troubled bank is sold to another healthy institution without any government assistance. These arrangements are typically called 'merger and acquisition' (M&A) agreements. In our model, the regulator employs this policy when he decides to sell failed banks' assets to surviving banks without any liquidity provision or price discrimination in the auction. Note that for moderate number of failures, that is, for $k \leqslant \overline{k}$, the regulator employs this policy to resolve all bank failures in our model.

However, during systemic crises ($k > \overline{k}$), because of the limited liquidity of surviving banks, the regulator may have to resort to bailouts and support sales by some sort of government assistance to attract potential buyers. Specifically, the regulator employs the resolution policy of providing liquidity to surviving banks to be used for the asset purchase. This policy was shown to be equivalent to some form of price-discrimination policy where the regulator charges a lower price to surviving banks compared to the price it charges outsiders. Both of these policies are in effect sale of failed banks' assets to surviving banks with some government assistance. In practice, regulators often employ similar policies, which are broadly classified as 'purchase and assumption' (P&A) agreements. Such P&A agreements are generally enhanced by government guarantees to attract buyers. For example, if the whole bank is purchased the acquirer may receive a government payment covering the difference between the market value of assets and liabilities. If only some deposits are purchased, the acquirer may be given the option of purchasing any of the others and get their pick of bank assets. Furthermore, P&A agreements typically include some form of a put option, whereby the government promises to buy back the assets at a stated percentage of value within a specified time frame, and often also include a contractual profit or loss-sharing agreement.

5.1.2 State-contingent closure rules

In the introduction, we provided some empirical evidence on the fact that regulatory actions taken in response to banking problems appear to depend on the performance of the whole banking system and the overall economy itself. Below we provide some further evidence.

Santomero and Hoffman (1998) document the existence of state-contingent regulatory actions. They argue that liquidation of banking assets has not been a preferred strategy during systemic crises since fire-sale prices, large bid-ask spreads and the virtual lack of bids are common elements of a mass liquidation. Furthermore, during systemic crises, it has been costly for regulators to close down a significant portion of the banking system due to issues relating to investment disruption and consumer confidence. Kasa and Spiegel (1999) use bank closure data for the United States for the period 1992 through 1997 and provide similar evidence. They compare an *absolute* closure rule, which closes banks when their asset/liability ratios fall below a given threshold with a *relative* closure rule, which closes banks when this ratio falls sufficiently below the industry average. A direct implication of the relative closure rule is forbearance when the banking industry as a whole performs poorly. They find strong evidence that US bank closures were based on relative performance during this period.

In addition, Brown and Dinc (2006) analyse failures among large banks in 21 major emerging markets in the 1990s and show that the government decision to close or take over a failing bank depends on the financial health of other banks in that country. In particular, intervention is delayed if other banks in that country are also weak. They show that this too-many-to-fail effect is robust to controlling for bank-level characteristics, macroeconomic factors, political factors such as electoral cycle and potential IMF pressure, as well as worldwide time-specific factors.

More broadly on the resolution policies used by the regulators, Goodhart and Schoenmaker (1995) provide a cross-country survey of 104 bank failures in 24 different countries during the 1980s and early 1990s. They show that liquidation of failed banks has not been the rule but the exception. While 31 of the 104 banks were liquidated, 22 were bailed out using a rescue package and 49 were taken over by other banks. Hawkins and Turner (1999) document evidence on the resolution methods used during the banking crises in the 1980s and 1990s for 23 countries (see Table 12 on page 40). They show that in 20 of these countries, government capital injection was used as one of the resolution policies. Furthermore, in 20 of these countries, domestic bank mergers were employed, while troubled banks were taken over by foreign banks in nine countries.

Lindgren *et al* (1999) show that in all five crisis-stricken countries in Asia in the late 1990s, public funds were used to recapitalise institutions and weak institutions were merged or taken over by other institutions (Table 6, page 31). Finally, Hoelscher *et al* (2006) provide evidence from the recent banking crises in twelve countries. They show that using public funds for the resolution of banking failures, mergers and purchase and assumption agreements were used as resolution policies in Argentina, Finland, Indonesia, Korea, Malaysia, Sweden, Thailand, Turkey and Venezuela.

Finally, in evidence for government assistance during the sale of failed institutions, Santomero and Hoffman (1998) document that during the resolution of Savings and Loans crises, the Federal Savings and Loan Insurance Corporation (FSLIC) tried to merge failed thrifts with a stronger thrift, a process that involved non-competitive bidding since it implied restrictions on non-thrift institutions (a form of price discrimination) to participate in the auctions. Furthermore, they document that FSLIC used incentives such as future payments of capital losses, yield maintenance guarantees and tax benefits to make it attractive for potential buyers to acquire troubled institutions.

5.2 Evidence on herding

We modelled herding by banks as lending to similar firms or sectors of the economy. In practice, there are countervailing effects such as competition in loan margins which make herding less attractive. Though we do not allow for competition in lending in our model, note that there is nevertheless an effect that counteracts herding incentives: when banks survive, they can acquire failed banks at discounted prices. Indeed, our model has precise implications for when we should expect herding incentives to dominate. In particular, given the time-inconsistency of bailout policy, herding incentives of banks are stronger when the likelihood of bank defaults is high (Proposition 8). Thus, the question of whether banks herd or not, and in which states, is ultimately an empirical one. Hence, we provide some evidence supportive of our modelling approach and results. We also discuss that banks correlate their portfolios in several other ways that are less likely to erode profit margins, and present evidence supporting this claim.

In evidence that studies correlation of different banks' assets through the correlation of their equity returns, Luengnaruemitchai and Wilcox (2004) find that during the period 1976 to 2001, banks in the United States chose market and asset betas that clustered together more when banking sector was troubled, in terms of banks having low capital ratios. They find a lower standard deviation of bank betas in such times. Their interpretation of this finding is one of herding by banks to seek 'safety in similarity': '[Banks] more tightly mimicked each other during troubled times'. In troubled times, banks would be concerned more about regulatory bailouts.

A number of studies have analysed herding by employing evidence from the overexposure of banks to emerging market economies before the debt crisis of 1982–84. Guttentag and Herring (1984), for example, discuss three potential explanations. While their first two explanations are related to bounded rationality of banks, the only rational explanation they consider is that too-many-to-fail guarantees created incentives for banks to get overexposed to risks in these countries. They suggest that deposit insurance, existence of lender-of-last-resort facilities as well as official support for debtor countries the IMF gave banks the impression that they would be protected against risks. They also suggest that by herding and keeping concentrations in line with each other, banks made sure that any problem that occurred would be a system-wide problem, not just the problem of an individual institution. Banks reasoned that, this, in turn, would make it harder for the regulatory authorities to blame or discriminate against individual institutions and would induce governments to take action to prevent the adverse consequences of a system-wide banking crisis.

While directly investing in similar industries is one way for banks to increase the correlation of their returns, there are also other approaches. First, banks could bet on systematic risk factors such as interest rate risk through choosing from a range of products such as mortgages and interest rate derivatives. That is, banks could specialise within a class of risk exposures to achieve a trade-off between incentives to correlate and to differentiate. Second, banks can achieve high levels of default correlation through interbank lending since this leads to the problems of one bank being transmitted to other banks in a contagion-type phenomenon, and indirectly increases the likelihood of bailout of the problem bank.

Another interesting alternative for banks to lend to similar set of customers and get exposed to similar risks without erosion of profit margins is to participate in syndicated loans to common set of borrowers (as in the debt crises of 1980s for less-developed countries and the extension of telecom loans in the late 1990s). Through syndication, banks can ensure that they are more likely to receive regulatory subsidies when the loans perform poorly, affecting all syndicate members. Adams (1991) argues that before the emerging market debt crisis, banks comforted themselves by herding, thinking that as long as all banks made similar loans, any crisis would be system-wide and would force governments to bail out those countries in trouble.(17) She also argues that syndicated loans acted as an important vehicle for herding and hundreds of billions of dollars in loans were syndicated between 1970 and 1982. On a similar point, Jain and Gupta (1987) also discuss the role of syndicated loans for bank herding during the emerging market debt crisis.

5.3 Discussion: separation of ownership and control

Perhaps an important limitation of our modelling approach is the assumption of uniformity of ownership and control. In particular, we have assumed that when failed banks' assets are acquired by outsiders, they are also managed by outsiders who are not as good as existing managers in creating value from these assets. This assumption also allows us to focus on the investment choices of owners (also the effective managers in our model) at date 0.

What would be the effect of allowing for separation of ownership and control in our model? In general, this would allow for the possibility that outside owners can control the failed banks' assets but employ the original managers to run at least some of these assets. In order for our results to go through, we need to assume that an entirely frictionless transfer of ownership is not possible. A straightforward way of introducing such a friction would be to assume that outside owners are fragmented and uncoordinated (in our model, they are competitive) and acquire the sold assets piecemeal rather than as ongoing concerns. For instance, a bank could be sold piecemeal whereby its branches in different states, its commercial and investment banking operations, its retail versus wholesale businesses, and perhaps even different slices of its loan portfolio, are allocated to different participants in the auction. The resulting separate entities would not then all be managed by the single manager or CEO of the original whole bank. Note that, empirically, piecemeal liquidations do result in lower bank values than do sales of banks as ongoing concerns (James (1991)).

(17) According to Pedro-Pablo Kuczynski, a former World Bank official, Peruvian cabinet minister, and later an investment banker with First Boston Corporation, 'banks preferred to lend to the public sector, not for ideological reasons but because government guarantees eliminated commercial risk'.

In other words, sales to outsiders would result in loss of ongoing concern value for the bank as a whole, and also result in partial or full loss of control of the bank for the existing management. In contrast, sales to insiders in our model preserve bank value but always result in change of management: the management of acquiring banks has the expertise to run assets of the failed bank as effectively as the old management. Finally, the bailout of a failed bank by the regulator is a change in ownership to a *single* outsider, namely the regulator. This preserves the bank value and also results in full retention of control by the existing management of the failed bank. Given these outcomes, bailouts are more attractive for existing management than are sales to insiders or outsiders. Since bailouts occur in states where many banks fail together and reduce the incidence of sales to outsiders, bank management would have *ex-ante* incentives to herd as in our benchmark model, in particular, when the likelihood of adverse shocks to assets is high. In contrast, old owners are wiped out regardless of whether their bank is sold to outsiders or insiders, or if it is bailed out.

We would like to acknowledge that this qualitative argument presents one set of assumptions under which the separation of ownership and control would continue to deliver similar results as the one we obtained in this paper. Our goal in abstracting from this separation has been to formulate a parsimonious yet rich framework for analysing limited expertise, cash-in-the-market pricing, and induced regulatory policies. Embedding managerial agency problems fully in such a framework remains an important topic for future research.

6 Conclusion

This paper analysed optimal resolution policies during banking crises of varying adversity. As the number of failed banks increases, total funding capacity of surviving banks decreases. In turn, surviving banks may not be able to buy all failed banks at the fundamental price of their assets, as in the industry-equilibrium hypothesis of Shleifer and Vishny (1992) and in the cash-in-the-market pricing in Allen and Gale (1994, 1998). Thus, some failed banks would have to be liquidated to investors outside the banking sector, resulting in a loss of continuation values. In order to avoid this allocation inefficiency from there being ‘too many (banks) to liquidate’, the regulator may find it *ex-post* optimal to bail out banks; in contrast, if only some banks fail, then these banks can be acquired by the surviving banks and no regulatory intervention is required. However, this *ex-post* optimal bailout policy induces banks to herd so that they are more likely to be bailed out.

Thus, the genesis of inefficient systemic risk potentially lies in the *ex-post* optimal bailout policy of the regulator.

We show that this time-inconsistency problem can be mitigated if *ex post* the regulator follows a policy of providing liquidity to surviving banks to assist the purchase of failed banks. This also constitutes an important policy implication of our paper. The liquidity provision policy we recommend is akin to the lender-of-last-resort policy but different in that it involves assisting safe banks for efficient resolution of bank failures rather than assisting the failed banks themselves.

The framework developed in this paper is flexible and tractable to provide a foundation for examining several other aspects of financial crises and their resolution. For example, surviving banks are financially constrained in our model and use their first-period profits for asset purchases. It would be interesting to extend our model to endogenise banks' choice of available funds for the asset purchase, in particular, by allowing banks to choose *ex ante* a portfolio of liquid and illiquid assets. Such an extension can shed light on the relative levels of socially and privately optimal liquidity position of banks.

Appendix

A.1 Proofs:

Proof for profitability of asset purchase: Note that asset purchase by a surviving bank is possible only when banks invest in different industries.

At price p, a surviving bank can purchase $\left(\frac{l}{p}\right)$ units of failed banks' assets. Thus in its investment portfolio, the bank will have a total of $\left(\frac{l}{p}+1\right)$ units of banking assets. Let x_j be the units of assets of the j^{th} bank in the bank's portfolio after asset purchase and let $x = \sum_j x_j$. Note that even though there is no deposit insurance in the second period, depositors always receive their reservation value of 1, in expected terms. Let $\widetilde{r}_1$ be the random return depositors receive from the bank, where $E(\widetilde{r}_1) = 1$. We can write the bank's expected profit as

$$E(\pi_2) = E\left[\sum_j x_j \widetilde{R}_j - p(x-1) - x\widetilde{r}_1\right] = E\left[\sum_j x_j \widetilde{R}_j\right] - x - p(x-1) \qquad \textbf{(A-1)}$$

By independence of $\widetilde{R}_j$, we can write

$$\begin{aligned} E(\pi_2) &= \sum_j x_j E(\widetilde{R}_j) - x - p(x-1) = \sum_j x_j(\alpha_1 R_1) - x - p(x-1) \\ &= x(\alpha_1 R_1 - 1) - p(x-1) = (x-1)(\alpha_1 R_1 - 1 - p) + \alpha_1 R_1 - 1 \qquad \textbf{(A-2)} \end{aligned}$$

Note that if the bank does not purchase the assets, the expected profit is $(\alpha_1 R_1 - 1)$. Thus, for $p < \alpha_1 R_1 - 1$, the bank uses all its available funds for the asset purchase.(18) $\diamondsuit$

Proof of Proposition 5: For $k \leqslant \bar{k}$, no failed bank is bailed out, all failed banks' assets are

(18) Note that this proof assumes that there is no deposit insurance at $t = 2$, an assumption that does not affect any of our other results. With insurance at $t = 2$, acquiring uncorrelated assets can coinsure depositors, lowering the value of the deposit-insurance option for bank owners (this happens when α_1 is sufficiently small). This implies that the market for banking assets will clear only at lower prices compared to the case with no deposit insurance. This will make the effect of cash-in-the-market pricing even stronger, in particular, it will make bailouts more likely. Note, however, that if there was a fairly priced deposit insurance premium collected at the beginning of the second period but after the asset purchases have taken place, then the effective cost of borrowing deposits remains the same for banks. In this case, the current proof for the profitability of the asset purchase still holds.

purchased by surviving banks (capturing a surplus of $[(\frac{k}{n-k})E(\pi_2) - l]$), and a failed bank's expected second-period profit is equal to zero.

For $k > \bar{k}$, $b^*(k)$ banks are bailed out, where $b^*(k)$ is given in Corollary 3. The expected continuation pay-off for a bailed-out bank is $(1-\beta)E(\pi_2)$, where β is the regulator's share, and each surviving bank captures a surplus of $\left[\frac{l}{\underline{p}} \cdot (\alpha_1 \Delta)\right]$.

If banks choose $x = 0$, their returns are independent and we have a Binomial distribution for the number of failures, k. We can write banks' expected profit $E(\pi_2(0))$ as:

$$\begin{aligned} = \; & \alpha_0 \left[E(\pi_2) + \sum_{j=\underline{k}+1}^{\bar{k}} \left[P(j) \left[\left(\frac{j}{n-j} \right) E(\pi_2) - l \right] \right] \right] + \\ & \alpha_0 \left[\sum_{j=\bar{k}+1}^{n-1} \left[P(j) \left(\frac{l\,(\alpha_1 \Delta)}{\underline{p}} \right) \right] \right] + (1-\alpha_0) \left[\sum_{j=\bar{k}}^{n-1} \left[P(j) \left(\frac{b^*(j+1)}{j+1} \right) (1-\beta) E(\pi_2) \right] \right] \end{aligned} \qquad \textbf{(A-3)}$$

where j is the number of failures among the remaining $(n-1)$ banks and $P(j)$ is the corresponding probability for this event, when we exclude the bank that we calculate the expected profit for. For the Binomial distribution, we have that $P(j) = C(n-1, j)\, \alpha_0^{n-1-j}\, (1-\alpha_0)^j$.

If banks choose $x = 1$, they are perfectly correlated and their expected return is:

$$E(\pi_2(1)) = \alpha_0 E(\pi_2) + (1-\alpha_0) \left(\frac{b^*(n)}{n} \right) (1-\beta) E(\pi_2) \qquad \textbf{(A-4)}$$

Next, we derive the critical equity share, denoted by β^*, which if taken by the regulator in each bailed-out bank, then banks would choose not to invest in the common industry.

Note that, banks choose $x = 1$ if and only if $E(\pi_2(1)) - E(\pi_2(0)) > 0$. Let

$d(\beta) = [E(\pi_2(1)) - E(\pi_2(0))]$, where

$$d(\beta) = (1-\alpha_0)(1-\beta)E(\pi_2)\left[\frac{b^*(n)}{n} - \sum_{j=\overline{k}}^{n-1}\left[P(j)\left(\frac{b^*(j+1)}{j+1}\right)\right]\right] - \alpha_0\left[\sum_{j=\underline{k}+1}^{\overline{k}}\left[P(j)\left[\left(\frac{j}{n-j}\right) E(\pi_2) - l\right]\right] + \sum_{j=\overline{k}+1}^{n-1}\left[P(j)\left(\frac{l(\alpha_1\Delta)}{\underline{p}}\right)\right]\right] \quad \textbf{(A-5)}$$

Note that only the first term depends on β. Also, note that $\left(\frac{b^*(k)}{k}\right)$ is increasing in k. Hence, $\left(\frac{b^*(n)}{n}\right) > \left(\frac{b^*(k)}{k}\right)$ for all $k \in \{0, ..., n-1\}$ and we have $\frac{\partial d}{\partial \beta} < 0$. Thus, there exists a unique β^* such that $[E(\pi_2(1)) - E(\pi_2(0))] > 0$ if and only if $\beta < \beta^*$, where

$$\beta^* = 1 - \left[\frac{\alpha_0\left(\sum_{j=\underline{k}+1}^{\overline{k}}\left[P(j)\left[\left(\frac{j}{n-j}\right) E(\pi_2) - l\right]\right] + \sum_{j=\overline{k}+1}^{n-1}\left[P(j)\left(\frac{l(\alpha_1\Delta)}{\underline{p}}\right)\right]\right)}{(1-\alpha_0)E(\pi_2)\left[\frac{b^*(n)}{n} - \sum_{j=\overline{k}}^{n-1}\left[P(j)\left(\frac{b^*(j+1)}{j+1}\right)\right]\right]}\right] \quad \textbf{(A-6)}$$

For $\beta^* \leqslant 0$, banks always choose $x = 0$. For $\beta^* > 0$, banks choose $x = 1$ when $\beta < \beta^*$, otherwise, they choose $x = 0$.

Note however that the regulator will *ex post* never take a share β that exceeds $(1 - \overline{\theta})$: it is better to liquidate the bank than to have it run inefficiently by bank owners due to poor provision of incentives. Thus, if $(1 - \overline{\theta}) < \beta^*$, then the regulator's share β is smaller than β^* in the subgame perfect equilibrium, and banks choose $x = 1$. If $(1 - \overline{\theta}) \geq \beta^*$, then the regulator takes a share β between β^* and $(1 - \overline{\theta})$ and this induces banks to choose $x = 0$.

Also, note that with the linear cost function, $\left(\frac{b^*(k)}{k}\right)$ is increasing in k, whereby the denominator in the last term in equation **(A-5)** is positive, and, in turn, $\beta^* < 1$ always. $\diamondsuit$

Proof of Proposition 6: Note that under the liquidity provision policy characterised in equation **(25)**, for $k > \overline{k}$, each surviving bank can acquire $\left(\frac{l}{\underline{p}} + \frac{b^*(k)}{(n-k)}\right)$ units of failed banks' assets, where $b^*(k)$ is given by the *ex-post* optimal bailout policy. Hence, if banks choose $x = 0$, we can write each bank's expected profit $E(\pi_2^l(0))$ as:

$$= \alpha_0 \left[E(\pi_2) + \sum_{j=\underline{k}+1}^{\bar{k}} \left[P(j) \left[\left(\frac{j}{(n-j)} \right) \overline{p} - l \right] \right] \right] +$$

$$\alpha_0 \left[\sum_{j=\bar{k}+1}^{n-1} \left[P(j) \left[\left(\frac{l}{\underline{p}} + \frac{b^*(j)}{(n-j)} \right) \overline{p} - l \right] \right] \right] + (1-\alpha_0)^n \left(\frac{b^*(n)}{n} \right) (1-\beta) E(\pi_2) \quad \textbf{(A-7)}$$

where j is the number of failures among the remaining $(n-1)$ banks and $P(j)$ is the corresponding probability for this event, when we exclude the bank that we calculate the expected profit for.

If banks choose $x = 1$, they are perfectly correlated and their expected return is:

$$E(\pi_2^l(1)) = \alpha_0 E(\pi_2) + (1-\alpha_0) \left(\frac{b^*(n)}{n} \right) (1-\beta) E(\pi_2) \qquad \textbf{(A-8)}$$

Note that, banks choose $x = 1$ if and only if $E(\pi_2(1)) > E(\pi_2(0))$. Let $d_l(\beta) = [E(\pi_2(1)) - E(\pi_2(0))]$, where

$$d_l(\beta) = (1-\alpha_0)(1-(1-\alpha_0)^{n-1}) \left(\frac{b^*(n)}{n} \right) (1-\beta) E(\pi_2)$$

$$-\alpha_0 \left[\sum_{j=\underline{k}+1}^{\bar{k}} \left[P(j) \left[\left(\frac{j}{(n-j)} \right) \overline{p} - l \right] \right] + \sum_{j=\bar{k}+1}^{n-1} \left[P(j) \left[\left(\frac{l}{\underline{p}} + \frac{b^*(j)}{(n-j)} \right) \overline{p} - l \right] \right] \right] \quad \textbf{(A-9)}$$

Note that only the first term depends on β and we have $\frac{\partial d_l}{\partial \beta} < 0$. Thus, $E(\pi_2(1)) - E(\pi_2(0)) > 0$ if and only if $\beta < \beta_l^*$, where

$$\beta_l^* = 1 - \left[\frac{\alpha_0 \left[\sum_{j=\underline{k}+1}^{\bar{k}} \left[P(j) \left[\left(\frac{j}{(n-j)} \right) E(\pi_2) - l \right] \right] + \sum_{j=\bar{k}+1}^{n-1} \left[P(j) \left[\left(\frac{l}{\underline{p}} + \frac{b^*(j)}{(n-j)} \right) \overline{p} - l \right] \right] \right]}{(1-\alpha_0)(1-(1-\alpha_0)^{n-1}) \left(\frac{b^*(n)}{n} \right) E(\pi_2)} \right] \quad \textbf{(A-10)}$$

For $\beta_l^* > 0$, banks choose $x = 1$ when $\beta < \beta_l^*$, otherwise, they choose $x = 0$.

Note however that the regulator will *ex post* never take a share β that exceeds $(1-\overline{\theta})$: it is better to liquidate the bank than to have it run inefficiently by bank owners due to poor provision of incentives. Thus, if $(1-\overline{\theta}) < \beta_l^*$, then the regulator's share β is smaller than β_l^* in the subgame perfect equilibrium, and banks choose $x = 1$. If $(1-\overline{\theta}) \geq \beta_l^*$, then the regulator takes a share β between β_l^* and $(1-\overline{\theta})$ and this induces banks to choose $x = 0$.

Note that the denominator in the last term in the above expression for β_l^* is positive, and, in turn, $\beta_l^* < 1$ always. For $\beta_l^* \leqslant 0$, banks always choose $x = 0$.

Proof of Proposition 7: The socially optimal level of correlation is $\rho = 0$ when $E(\Pi_2(0)) > E(\Pi_2(1))$. With the linear fiscal cost function $f(c) = Fc$, we have

$$E(\Pi_2(1)) = n\,(\alpha_1 R_1) - \underbrace{(1-\alpha_0)\,[nFr_0]}_{\text{expected cost of insurance}} + F\underbrace{(1-\alpha_0)\left[(n-b^*(n))\underline{p}\right]}_{\text{expected recovery from asset sales}} + \underbrace{(1-\alpha_0)\left[(n-b^*(n))\,(\alpha_1\Delta)\right]}_{\text{expected misallocation cost}} \quad \textbf{(A-11)}$$

and

$$\begin{aligned} E(\Pi_2(0)) \;&=\; n\,(\alpha_1 R_1) - \underbrace{\sum_{k=1}^{n} \Pr(k)\,k\,(Fr_0)}_{\text{expected cost of insurance}} - \underbrace{\left[\sum_{k=\overline{k}+1}^{n} \Pr(k)\left(k - \frac{(n-k)l}{\underline{p}} - b^*(k)\right)\right][\alpha_1\Delta]}_{\text{expected misallocation cost}} \\ &\quad + F\left[\underbrace{\sum_{k=1}^{\overline{k}} \Pr(k)\,(k\cdot p^*(k)) + \sum_{k=\overline{k}+1}^{n} \Pr(k)\,((k-b^*(k))\underline{p})}_{\text{expected recovery from asset sales}}\right] \end{aligned} \quad \textbf{(A-12)}$$

For $E(\Pi_2(0))$, we can write the expected cost of insurance as $(1-\alpha_0)nFr_0$. Note that this term is common in $E(\Pi_2(1))$ and $E(\Pi_2(0))$.

Note that $p^*(k) > \underline{p}$ for $k = 1, ..., \overline{k}$ and we can show that the expected recovery from asset sales when $\rho = 0$ is greater than

$$\underline{p}\left[\sum_{k=1}^{n} \Pr(k)\,k - \sum_{k=\overline{k}+1}^{n} \Pr(k)\,b^*(k)\right] \quad \textbf{(A-13)}$$

If $(1-\alpha_0) \geqslant \left[\sum_{k=\overline{k}+1}^{n} \Pr(k)\right]$, then the above expression is greater than the expected funds recovered from asset sales when $\rho = 1$, which is equal to $\left((1-\alpha_0)\,(n-b^*(n))\,\underline{p}\right)$.

Note that using equations **(20)** and **(21)**, we can show that the number of failed banks' assets that are sold to outsider (weakly) increases in k. Thus, $\left[k - \frac{(n-k)l}{\underline{p}} - b^*(k)\right] \leqslant (n-b^*(n))$ for all $k = \overline{k}+1, ..., n-1$, and these two expressions are equal when $k = n$. Again, if

$(1-\alpha_0) \geqslant \left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\right]$, the expected cost of misallocation of failed banks' assets is greater when $\rho = 1$.

Thus, if $(1-\alpha_0) \geqslant \left[\sum_{k=\widehat{k}+1}^{n} \Pr(k)\right]$, which can also be written as

$$\alpha_0 \leqslant \left[1 - \sum_{k=\bar{k}+1}^{n} \Pr(k)\right] = \sum_{k=0}^{\bar{k}} \Pr(k) \qquad \textbf{(A-14)}$$

then $E(\Pi_2(0)) > E(\Pi_2(1))$ and the socially optimal level of correlation is $\rho = 0$. $\diamondsuit$

Proof of Proposition 8: We already know that when the regulator never bails out banks, banks choose the low correlation. Now, let $\widetilde{b}$ be the maximum number of bailouts, that is, the bailout strategy $\widetilde{b}^*(k) = \min\{\widetilde{b}^*(k), \widetilde{b}\}$, that induces the low correlation.

We find a sufficient condition under which, if the regulator can credibly commit to a strategy *ex ante*, he chooses not to bail out more than $\widetilde{b}$ failed banks.

Since the regulator bails out fewer banks under the strategy $\widetilde{b}^*(k)$ compared to the *ex-post* optimal strategy $b^*(k)$, the price of failed banks' assets could fall below $\underline{p}$ for sufficiently large number of failures. We denote the price function when the regulator follows strategy $\widetilde{b}^*(k)$ as $\widetilde{p}^*(k)$. Formally, for $k \in \{1, ..., \bar{k}\}$, we have $\widetilde{p}^*(k) = p^*(k)$, and, for $k > \bar{k}$, $\widetilde{p}^*(k)$ can take values less than $\underline{p}$.

If the regulator commits not to bail out more than $\widetilde{b}$ banks, banks choose the low correlation and the resulting expected output is given as:

$$\begin{aligned} E(\widetilde{\Pi}_2(0)) \;=\; & n\,(\alpha_1 R_1) - \underbrace{\sum_{k=1}^{n} \Pr(k)\, kFr_0}_{\text{expected insurance cost}} - \underbrace{\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\left[k - \frac{(n-k)l}{\widetilde{p}^*(k)} - \widetilde{b}^*(k)\right]\right][\alpha_1 \Delta]}_{\text{expected misallocation cost}} \\ & + F\left[\underbrace{\sum_{k=1}^{\bar{k}} \Pr(k)\,(k \cdot p^*(k)) + \sum_{k=\bar{k}+1}^{n} \Pr(k)\,\left(k - \widetilde{b}^*(k)\right)\,\widetilde{p}^*(k))}_{\text{expected recovery from asset sales}}\right] \qquad \textbf{(A-15)} \end{aligned}$$

When the regulator chooses the *ex-post* optimal number of bailouts $b^*(k)$, banks choose the high correlation and we obtain:

$$E(\Pi_2(1)) = n\,(\alpha_1 R_1) - (1-\alpha_0)\left[f(nr_0 - (n - b^*(n))\,\underline{p}) + (n - b^*(n))\,(\alpha_1 \Delta)\right] \quad \textbf{(A-16)}$$

The socially optimal level of correlation is $\rho = 0$ when $E(\widetilde{\Pi}_2(0)) \geqslant E(\Pi_2(1))$.

For $E(\widetilde{\Pi}_2(0))$, we can write the expected cost of insurance as $[(1-\alpha_0)nFr_0]$, which is common in $E(\Pi_2(1))$ and $E(\widetilde{\Pi}_2(0))$.

We first prove the case for $F \leqslant \frac{\alpha_1 \Delta}{\underline{p}}$, where the *ex-post* optimal bailout strategy is given in equation **(20)** in Corollary 3. In this case, we have

$$E(\Pi_2(1)) = n\,(\alpha_1 R_1) - (1-\alpha_0)Fnr_0 \quad \textbf{(A-17)}$$

Thus, a sufficient condition to have $E(\widetilde{\Pi}_2(0)) > E(\Pi_2(1))$ is

$$\sum_{k=\overline{k}+1}^{n} \Pr(k)\left(k - \widetilde{b}^*(k)\right)\widetilde{p}^*(k)\left[F - \frac{\alpha_1 \Delta}{\widetilde{p}^*(k)}\right] + (\alpha_1\Delta)\sum_{k=\overline{k}+1}^{n}\Pr(k)\left[\frac{(n-k)l}{\widetilde{p}^*(k)}\right] + F\sum_{k=1}^{\overline{k}}\Pr(k)\,(k\cdot p^*(k)) \geqslant 0 \quad \textbf{(A-18)}$$

Note that $\underline{p} \geqslant \widetilde{p}^*(k)$ for $k \in \{\overline{k}+1, ..., n\}$. Thus, $\left[\frac{(n-k)l}{\widetilde{p}^*(k)}\right] \geqslant \left[\frac{(n-k)l}{\underline{p}}\right]$. Note that $\left[F - \frac{\alpha_1\Delta}{\widetilde{p}^*(k)}\right] \leqslant 0$. Also, we have $\left(k - \widetilde{b}^*(k)\right) \leqslant k$, so that condition **(A-18)** is automatically satisfied when

$$\sum_{k=\overline{k}+1}^{n} \Pr(k)\,k\underline{p}\left[F - \frac{\alpha_1 \Delta}{\widetilde{p}^*(k)}\right] + (\alpha_1\Delta)\sum_{k=\overline{k}+1}^{n}\Pr(k)\left[\frac{(n-k)l}{\underline{p}}\right] + F\sum_{k=1}^{\overline{k}}\Pr(k)\,(k\cdot p^*(k)) \geqslant 0 \quad \textbf{(A-19)}$$

Since $\widetilde{p}^*(k) \geqslant \underline{p}$ for $k \in \{1, ..., \overline{k}\}$, we can show that condition **(A-19)** is automatically satisfied when

$$(1-\alpha_0)nF\underline{p} + (\alpha_1\Delta)\sum_{k=\overline{k}+1}^{n}\Pr(k)\underbrace{\left[\frac{(n-k)l}{\underline{p}} - \frac{k\underline{p}}{\widetilde{p}^*(k)}\right]}_{\geqslant -\frac{n\underline{p}}{\widetilde{p}^*(n)}\ (\text{since } \widetilde{p}^*(n) \leqslant \widetilde{p}^*(k))} \geqslant 0 \quad \textbf{(A-20)}$$

Thus, it is sufficient to have

$$(1-\alpha_0)nF\underline{p} \geqslant (\alpha_1\Delta)\frac{n\underline{p}}{\widetilde{p}^*(n)}\sum_{k=\overline{k}+1}^{n}\Pr(k) \quad \textbf{(A-21)}$$

which can be written as $\alpha_0 \leqslant 1 - \left[\lambda\left(\frac{\underline{p}}{\widetilde{p}^*(n)}\right)\sum_{k=\overline{k}+1}^{n}\Pr(k)\right]$, where $\lambda = \left(\frac{\alpha_1\Delta}{F\underline{p}}\right)$.

Next, we prove the case for $F > \frac{\alpha_1\Delta}{\underline{p}}$. As in the previous case, the term $[nE(\pi_2) - (1-\alpha_0)Fnr_0]$ is common in $E(\Pi_2(1))$ and $E(\widetilde{\Pi}_2(0))$, thus we omit these terms. Using the *ex-post* optimal bailout strategy in equation **(21)**, we get

$$E(\Pi_2(1)) = (1-\alpha_0)w\left(F - \frac{\alpha_1\Delta}{\underline{p}}\right) \qquad \textbf{(A-22)}$$

We also have

$$\begin{aligned} E(\widetilde{\Pi}_2(0)) &= -[\alpha_1\Delta]\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\ \left[k - \frac{(n-k)l}{\widetilde{p}^*(k)} - \widetilde{b}^*(k)\right]\right] \\ &\quad +F\left[\sum_{k=1}^{\bar{k}} \Pr(k)\ (k\cdot p^*(k)) + \sum_{k=\bar{k}+1}^{n} \Pr(k)\ \left(k-\widetilde{b}^*(k)\right)\ \widetilde{p}^*(k))\right] \qquad \textbf{(A-23)} \end{aligned}$$

Note that $p^*(k) \geqslant \widetilde{p}^*(k) \geqslant \widetilde{p}^*(n) = \frac{w}{n-\widetilde{b}}$, for all $k \in \{1, ..., n\}$. Using this, we can get a sufficient condition as:

$$\begin{aligned} &(1-\alpha_0)w\left(F - \frac{\alpha_1\Delta}{\underline{p}}\right) \\ < \quad &-[\alpha_1\Delta]\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\ \underbrace{\left[k - \frac{(n-k)l}{\widetilde{p}^*(k)}\right]}_{\leqslant n}\right] + (1-\alpha_0)w\left(\frac{nF}{\left(n-\widetilde{b}\right)}\right) \\ &-\sum_{k=\bar{k}+1}^{n} \Pr(k)\ \widetilde{b}^*(k)\ \left[F\widetilde{p}^*(k) - \alpha_1\Delta\right] \qquad \textbf{(A-24)} \end{aligned}$$

which can be written as

$$(1-\alpha_0)w\left(\frac{F\widetilde{b}}{\left(n-\widetilde{b}\right)} + \frac{\alpha_1\Delta}{\underline{p}}\right) > [\alpha_1\Delta]\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\ n\right] + \sum_{k=\bar{k}+1}^{n} \Pr(k)\ \widetilde{b}^*(k)\ \left[F\widetilde{p}^*(k) - \alpha_1\Delta\right] \qquad \textbf{(A-25)}$$

since $\widetilde{b} < n$.

Since $\widetilde{p}^*(k) \leqslant \underline{p}$ for all $k \in \{k+1, ..., n\}$, we have:

$$(1-\alpha_0)w\left(\frac{F\widetilde{b}}{\left(n-\widetilde{b}\right)} + \frac{\alpha_1\Delta}{\underline{p}}\right) > [\alpha_1\Delta]\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\ n\right] + \sum_{k=\bar{k}+1}^{n} \Pr(k)\ \widetilde{b}^*(k)\ \left[F\underline{p} - \alpha_1\Delta\right] \qquad \textbf{(A-26)}$$

Also, $\widetilde{b}^*(k) \leqslant \widetilde{b}$, for all $k \in \{1, ..., n\}$. And, since $F > \frac{\alpha_1 \Delta}{\underline{p}}$, it is sufficient to show

$$(1-\alpha_0)w\left(\frac{F\widetilde{b}}{(n-\widetilde{b})} + \frac{\alpha_1 \Delta}{\underline{p}}\right) > \left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\right]\left[\left(n-\widetilde{b}\right)(\alpha_1 \Delta) + \widetilde{b}F\underline{p}\right] \qquad \textbf{(A-27)}$$

which can be written as

$$(1-\alpha_0) > \frac{\underline{p}\left(n-\widetilde{b}\right)\left[\left(n-\widetilde{b}\right)(\alpha_1 \Delta) + \widetilde{b}F\underline{p}\right]}{w\left[\left(n-\widetilde{b}\right)(\alpha_1 \Delta) + \widetilde{b}F\underline{p}\right]}\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\right] = \frac{\underline{p}}{\widetilde{p}^*(n)}\left[\sum_{k=\bar{k}+1}^{n} \Pr(k)\right] \qquad \textbf{(A-28)}$$

Hence, for $F > \frac{\alpha_1 \Delta}{\underline{p}}$, a sufficient condition to obtain $E(\widetilde{\Pi}_2(0)) \geqslant E(\Pi_2(1))$ is

$$\alpha_0 < 1 - \left[\frac{\underline{p}}{\widetilde{p}^*(n)}\left(\sum_{k=\bar{k}+1}^{n} \Pr(k)\right)\right] \qquad \textbf{(A-29)}$$

Thus, if the regulator can credibly commit to a strategy *ex ante*, then for $\alpha_0 < 1 - \left[\lambda\left(\frac{\underline{p}}{\widetilde{p}^*(n)}\right)\sum_{k=\bar{k}+1}^{n} \Pr(k)\right]$, where $\lambda = \max\left\{\frac{\alpha_1 \Delta}{F\underline{p}}, 1\right\}$, he chooses not to bail out more than $\widetilde{b}$ failed banks, that induces banks to choose the low correlation *ex ante*. However, we showed earlier that this strategy is time-inconsistent. ♢

Proof of Proposition 9: Recall from the proofs of Propositions 5 and 6 that

$$\begin{aligned} d(\beta) &= [E(\pi_2(1)) - E(\pi_2(0))] \text{ and} \\ d_l(\beta) &= \left[E(\pi_2^l(1)) - E(\pi_2^l(0))\right] \end{aligned}$$

Let $D(\beta) = d(\beta) - d_l(\beta)$. Note that under both the bailout and liquidity provision policies, the expected return for banks from investing in the common industry is the same, that is, $E(\pi_2^l(1)) = E(\pi_2(1))$. Hence, we have

$$D(\beta) = E(\pi_2^l(0)) - E(\pi_2(0))$$

Recall that $\frac{\partial d}{\partial \beta} < 0$ and $\frac{\partial d_l}{\partial \beta} < 0$. Hence, $D(\beta) \geqslant 0$ for all β, implies that $\beta_l^* \leqslant \beta^*$.

Next, we find a sufficient condition under which $D(\beta) \geqslant 0$.

We have $D(\beta) = E(\pi_2^l(0)) - E(\pi_2(0))$ given as:

$$= \alpha_0 \left[\sum_{j=\bar{k}+1}^{n-1} \left[P(j) \left[\left(\frac{b^*(j)}{n-j} \right) \overline{p} \right] \right] \right] - (1-\alpha_0) \left[\sum_{j=\bar{k}}^{n-2} \left[P(j) \left(\frac{b^*(j+1)}{j+1} \right) (1-\beta)\overline{p} \right] \right] \tag{A-30}$$

where j is the number of failures among the remaining $(n-1)$ banks and $P(j)$ is the corresponding probability, when we exclude the bank that we calculate the expected profit for. Note that we can write this as:

$$= \sum_{k=\bar{k}+1}^{n-1} \left[\Pr(k) \left[\underbrace{\left(\frac{b^*(k)}{n-k} \right) \overline{p}}_{=\gamma(k)} \right] \right] - \left[\sum_{j=\bar{k}+1}^{n-1} \left[\Pr(k) \left[\underbrace{\left(\frac{b^*(k)}{k} \right) (1-\beta)\overline{p}}_{\phi(k)} \right] \right] \right] \tag{A-31}$$

$$= \sum_{k=\bar{k}+1}^{n-1} \left[\Pr(k) \left[(\gamma(k) - \phi(k)) \right] \right] \tag{A-32}$$

where $\Pr(k)$ is the probability of having k failed banks out of the n banks. The benefit from the liquidity provision policy, which is denoted by $\gamma(k)$, arises from the extra units of failed banking assets a surviving bank can purchase. The cost of the liquidity provision policy, denoted by $\phi(k)$, is basically the forgone bailout subsidy.

Hence, a sufficient condition for $\left[E(\pi_2^l(0)) - E(\pi_2(0))\right] \geqslant 0$ is $\left[\gamma(k) - \phi(k)\right] \geqslant 0$ for all $k \in \{\bar{k}+1, ..., n-1\}$. We have

$$\begin{aligned} \gamma(k) - \phi(k) &= \left[\left(\frac{b^*(k)}{(n-k)} \right) - \left(\frac{b^*(k)}{k} \right) (1-\beta) \right] \overline{p} && \text{(A-33)} \\ &= b^*(k)\overline{p} \left[\left(\frac{k-(n-k)(1-\beta)}{(n-k)k} \right) \right] && \text{(A-34)} \end{aligned}$$

Note that $\left[\gamma(k) - \phi(k)\right]$ is increasing in k. Hence, if $\left[\gamma(\bar{k}+1) - \phi(\bar{k}+1)\right] \geqslant 0$, then $\left[\gamma(k) - \phi(k)\right] \geqslant 0$ for all $k \in \{\bar{k}+1, ..., n-1\}$. Thus,

$$(\bar{k}+1) - (n-(\bar{k}+1))(1-\beta) \geqslant 0 \tag{A-35}$$

is a sufficient condition for $\left[E(\pi_2^l(0)) - E(\pi_2(0))\right] \geqslant 0$.

Also, note that $(\bar{k}+1) \geqslant \left(\frac{nl}{l+\underline{p}}\right)$ from the expression for $\bar{k}$ from equation **(9)**. And note that the expression in equation **(A-35)** is increasing in the number of failed banks so that the inequality in equation **(A-35)** is satisfied for $(\bar{k}+1)$ if it is satisfied for $\left(\frac{nl}{l+\underline{p}}\right)$. Hence, we can show that the condition in equation **(A-35)** is satisfied when

$$l-(1-\beta)\underline{p} \geqslant 0 \qquad \textbf{(A-36)}$$

Hence, for $(1-\beta) \leqslant \left(l/\underline{p}\right)$, we have $\gamma(k) \geqslant \phi(k)$, for all $k > \bar{k}$. A sufficient condition for this to hold for all $\beta \in [0, 1]$ is $l \geqslant \underline{p}$. This guarantees that the expected profit from choosing idiosyncratic industries is higher when the regulator uses the liquidity provision policy instead of the *ex-post* optimal bailout policy. Hence, for $l \geqslant \underline{p}$, the regulator can induce banks to choose the low correlation for a wider range of parameter values when he uses the liquidity provision policy, that is, $\beta_l^* \leqslant \beta^*$. $\diamondsuit$

Figure 1: Timeline of the model

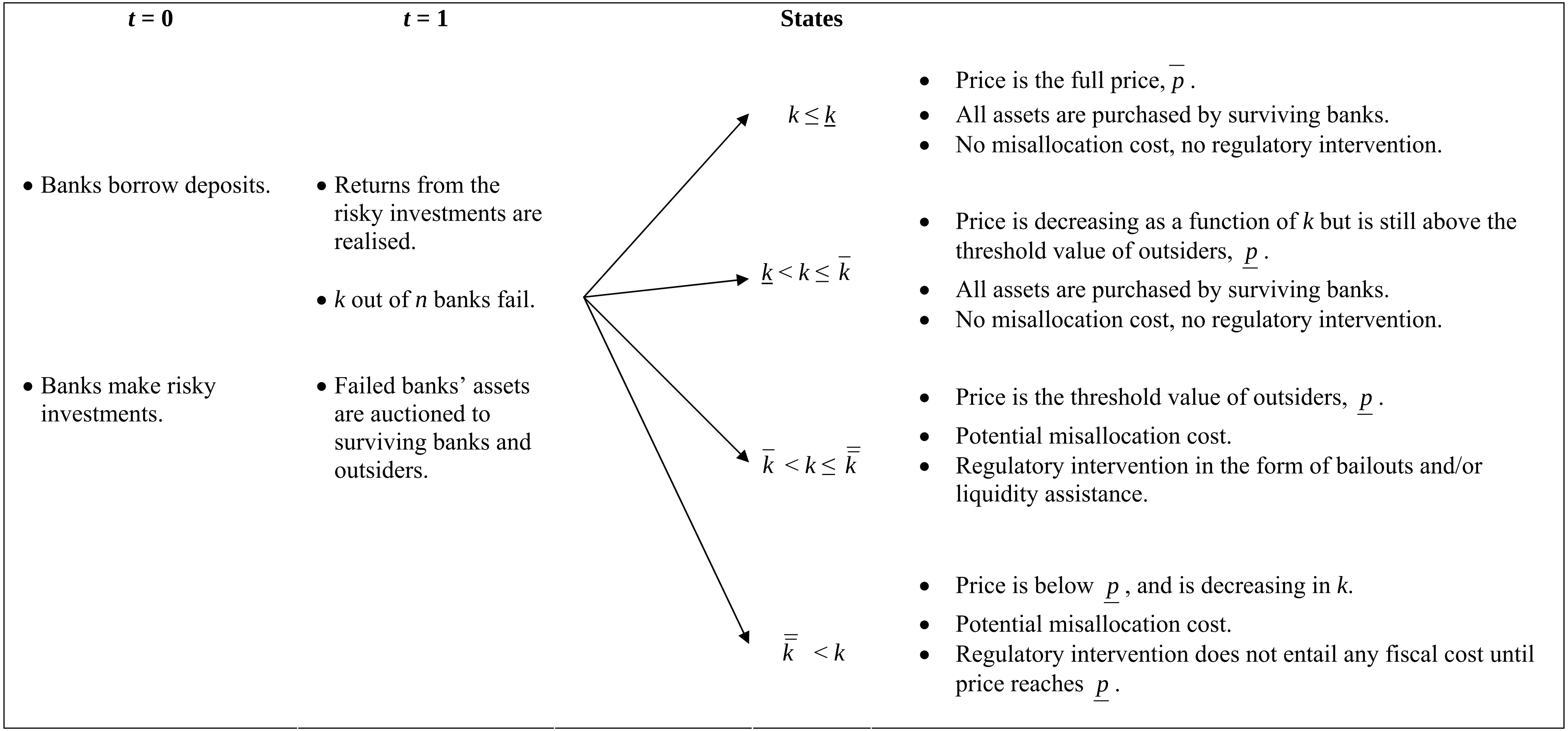


Figure 2: Price without bailouts and liquidity provision

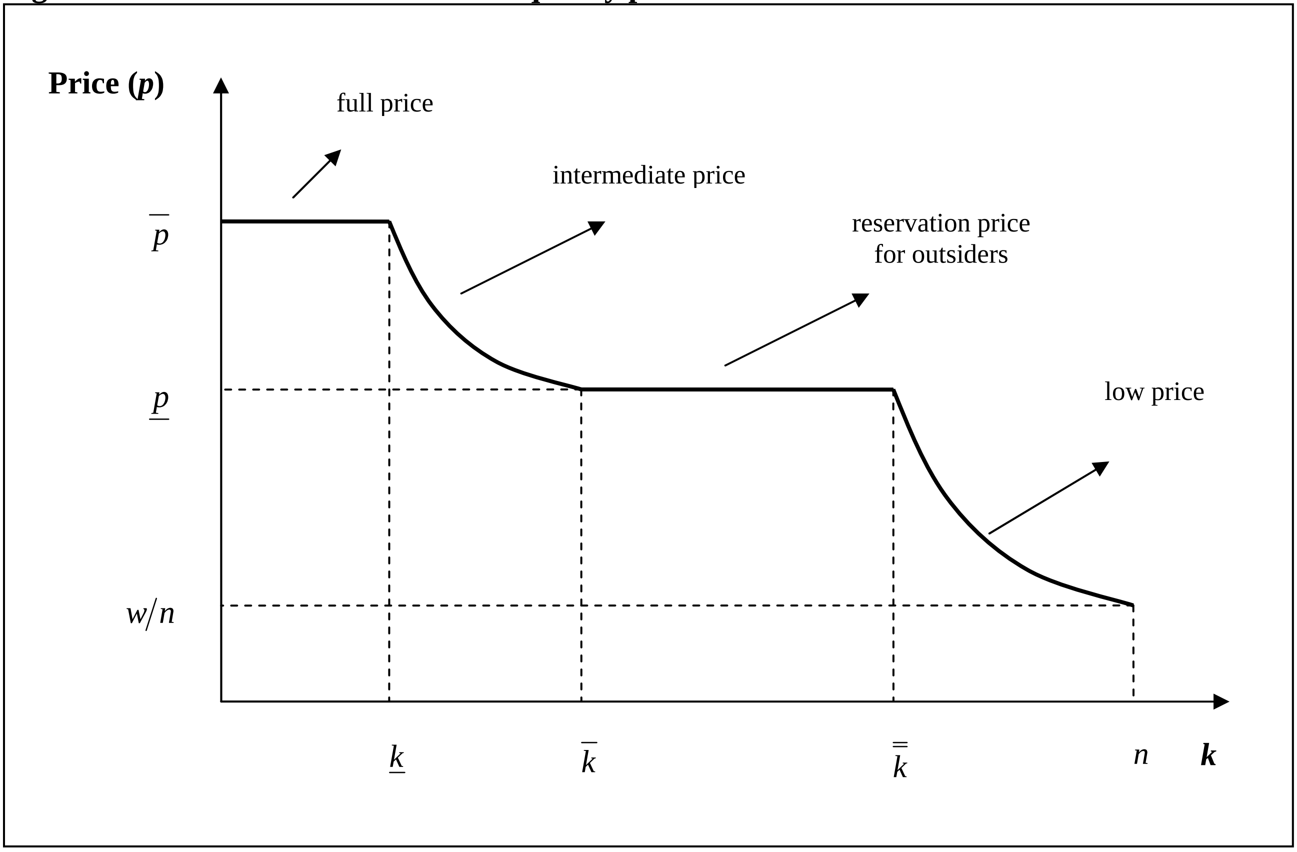


Figure 3: Price with bailouts and/or liquidity provision

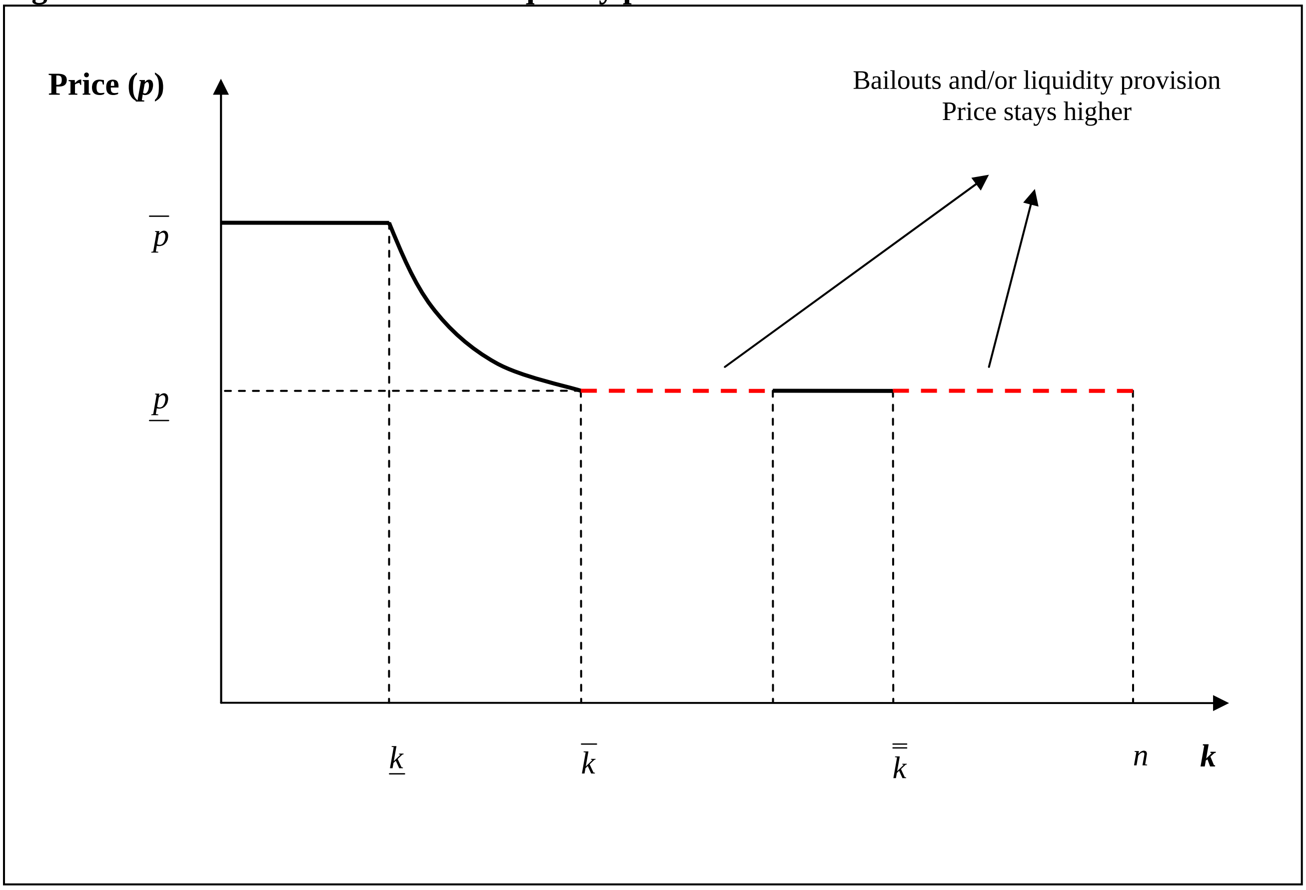


Figure 4: Number of bailouts as a function of number of failures with convex fiscal cost

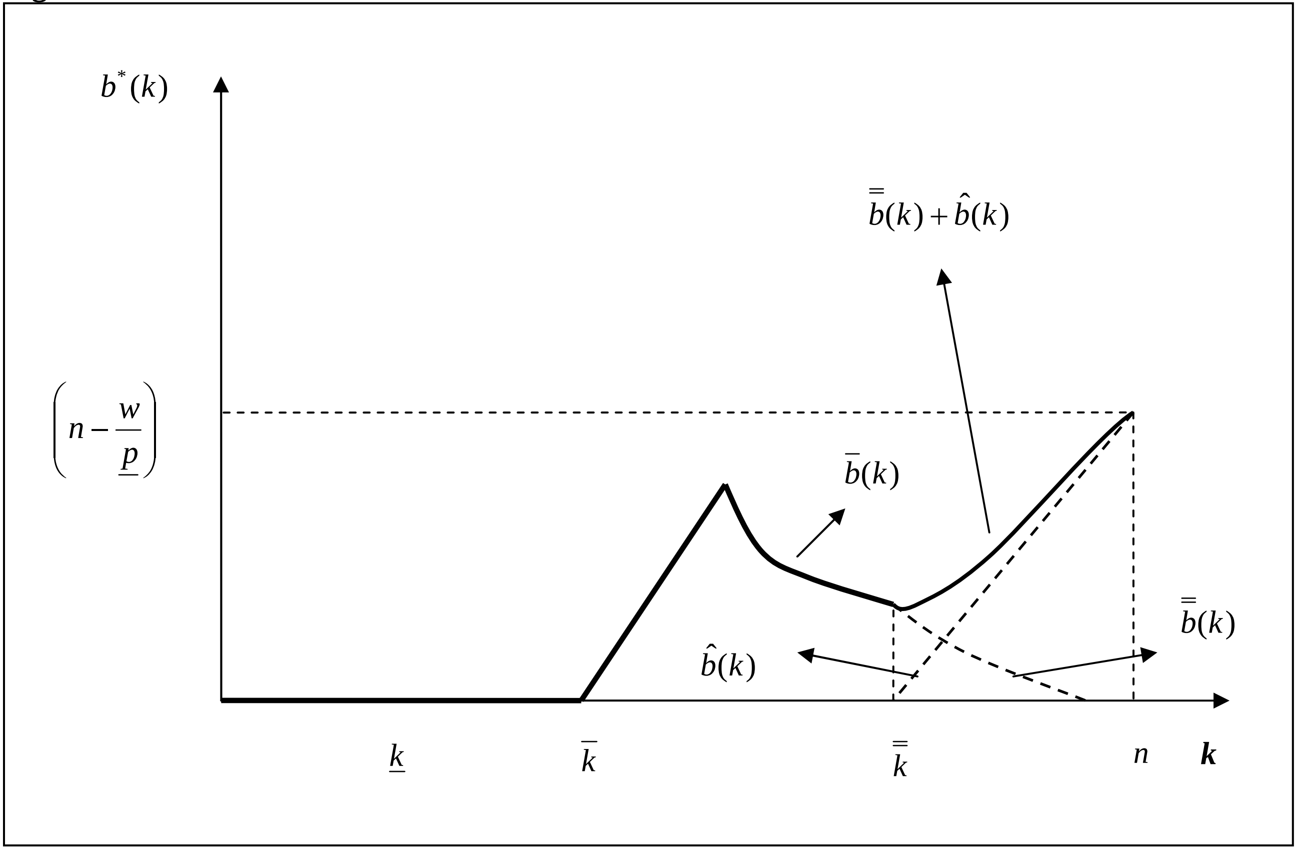


Figure 5: Number of bailouts as a function of number of failures with linear fiscal cost

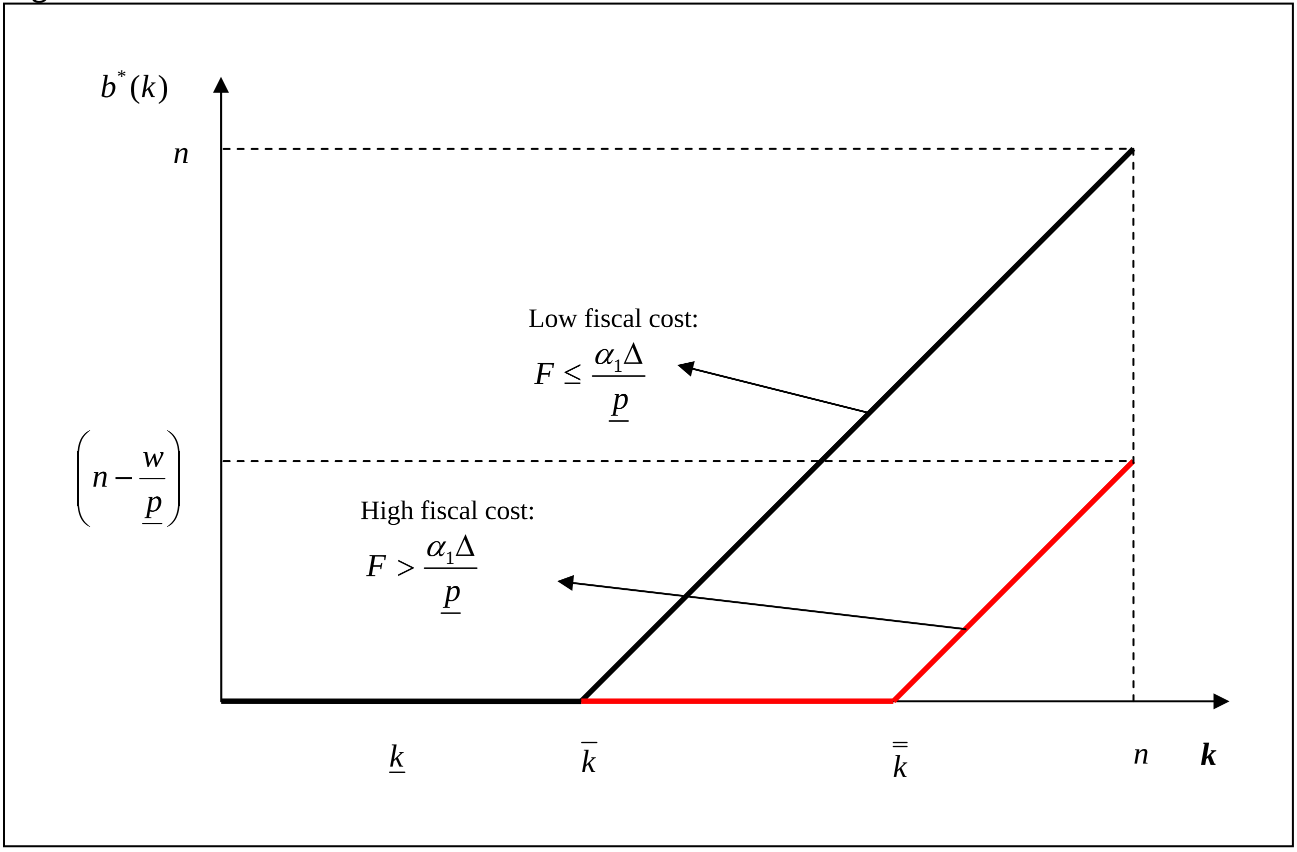


Figure 6: Bailout strategy that implements low correlation

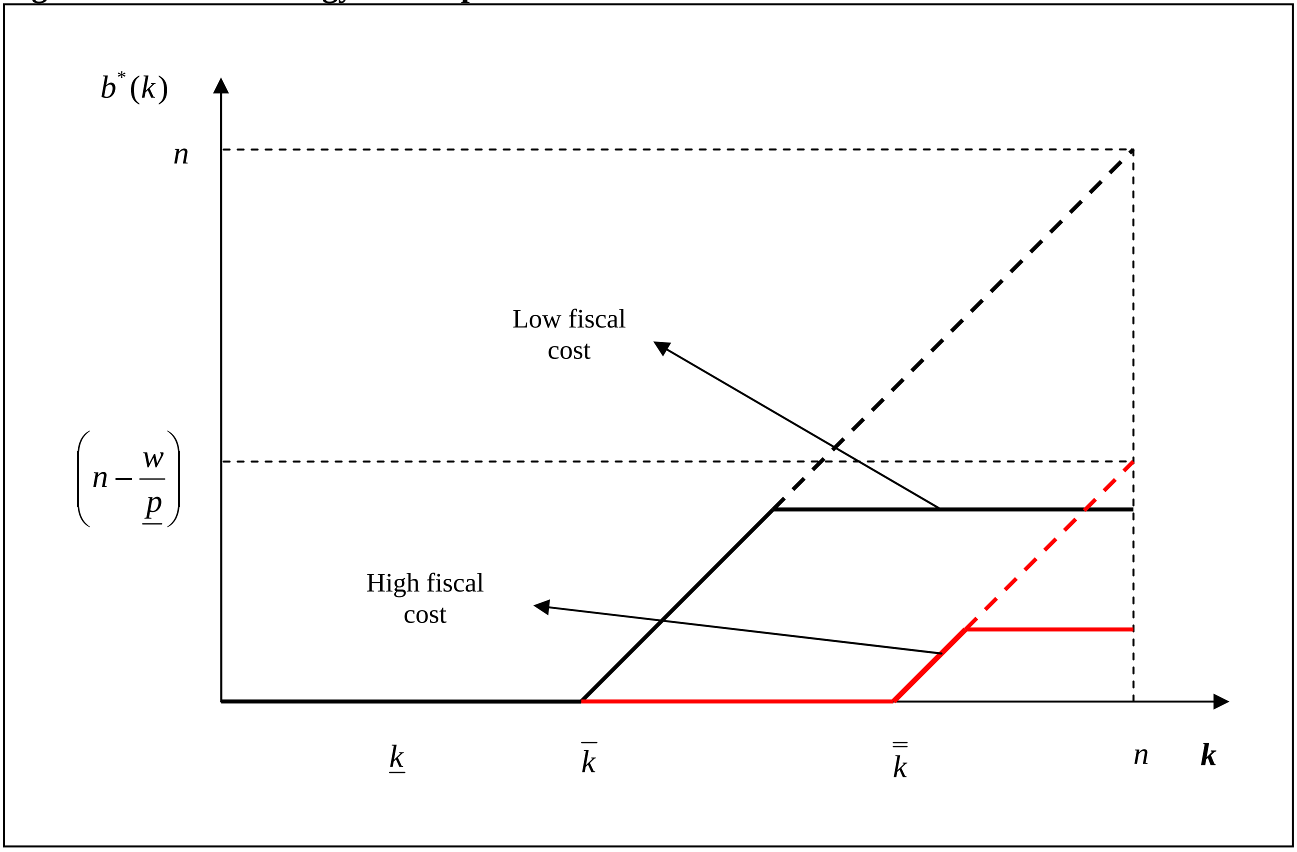
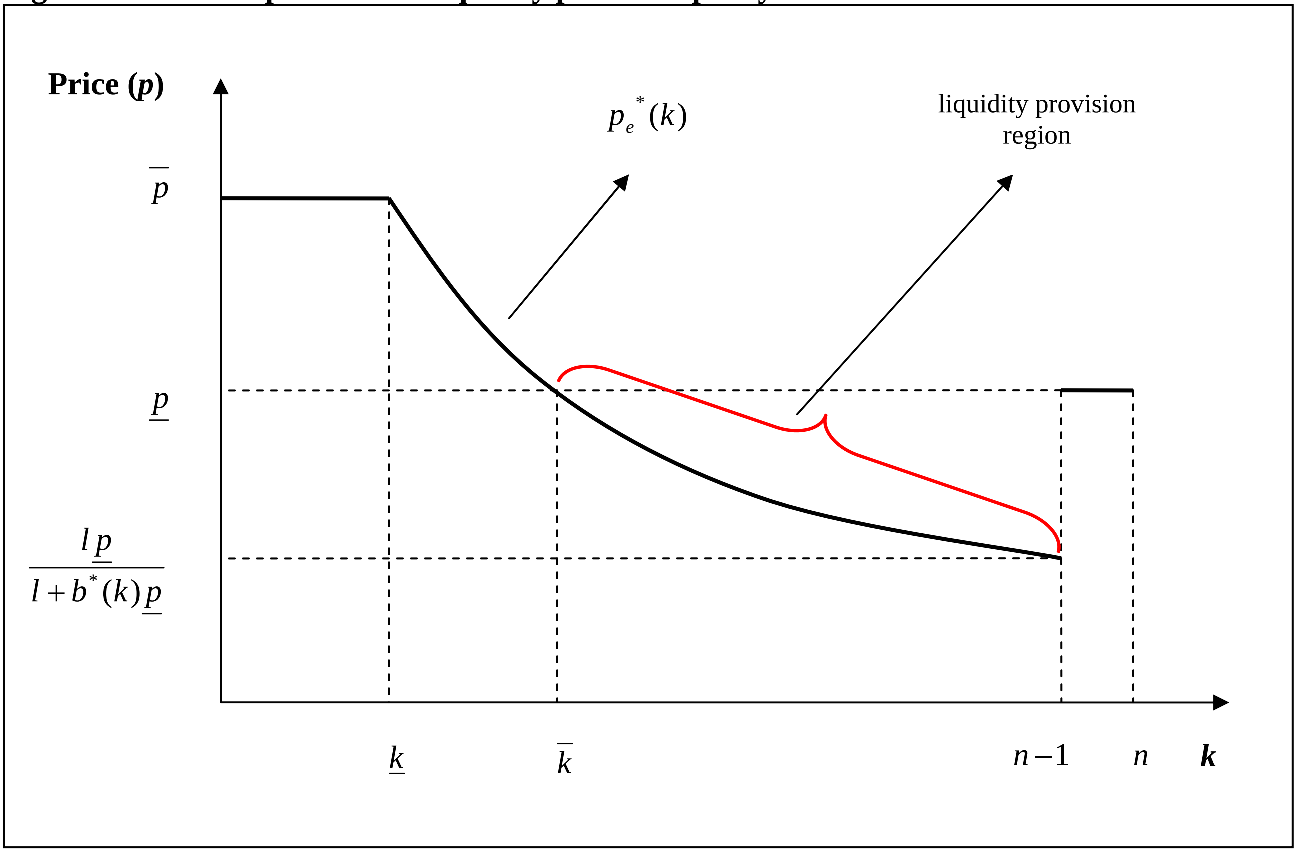


Figure 7: Effective price under liquidity provision policy



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