# The Factor Structure in Equity Option Prices<sup>\*</sup>

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#### Abstract

Principal component analysis of equity options on Dow-Jones firms reveals a strong factor structure. The first principal component explains 77% of the variation in the equity volatility level, 49% of the variation in the equity option skew and 57% of the implied volatility term structure across equities. Furthermore, the first principal component has a 91% correlation with S&P500 index option volatility, a 42% correlation with the index option skew, and a 74% correlation with the index option term structure. Based on these findings we develop an equity option valuation model that captures the cross-sectional market factor structure as well as stochastic volatility through time. The model assumes a Heston (1993) style stochastic volatility model for the market return but additionally allows for stochastic idiosyncratic volatility for each firm. The model delivers theoretical predictions consistent with the empirical findings in Duan and Wei (2009). We provide a tractable approach for estimating the model on index and equity option data. The model provides a good fit to a large panel of options on stocks in the Dow-Jones index.

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# 1 Introduction

While factor models are standard for understanding equity prices, classic equity option valuation models make no attempt at modeling a factor structure in the underlying equity prices. Typically, a stochastic process is assumed for each underlying equity price and the option is priced on this stochastic process ignoring any links the underlying equity price may have with other equity prices through common factors. Seminal papers in this vein include Black and Scholes (1973), Wiggins (1987), Hull and White (1987), and Heston (1993).

When considering a single stock option, ignoring an underlying equity factor structure may be relatively harmless. However, in portfolio applications it is crucial to understand links between the underlying stocks: Risk managers need to understand the total exposure to the underlying risk factors in a portfolio of stocks and stock options. Equity portfolio managers who use equity options to hedge large downside moves in individual stocks need to know their overall market exposure. Dispersion traders who sell (expensive) index options and buy (cheaper) equity options to hedge need to understand the market exposure of individual equity options. See for example, Driessen, Maenhout and Vilkov (2009).

Our empirical analysis of about four hundred thousand index quotes and more than two million equity option quotes reveals a very strong factor structure. We study three characteristics of option prices: short-term implied volatility (IV) levels, the slope of IV curves across option moneyness, and the slope of IV curves across option maturity.

First, we construct daily time series of short-term at-the-money implied volatility (IV) on the stocks in the Dow Jones Industrials Average and extract their principal components. The first common component explains roughly 77% of the cross-sectional variation in IV levels and the common component has an 91% correlation with the short-term implied volatility constructed from S&P500 index options. Short term equity option IV appears to have a very strong common factor.

Second, a principal component analysis of equity option IV moneyness (the option "skew") reveals a significant common component as well. Roughly 49% of the variation in the skew across equities is captured by the first principal component. Furthermore, this common component has a correlation of 42% with the skew of market index options.

Third, when looking for a common component in the term structure of equity IV, we find that 57% of the variation is explained by the first principal component. This component has a correlation of 74% with the term slope of the option IV from S&P500 index options.

We use the findings from the principal component analysis as guidance to develop a structural model of equity option prices that incorporates a market factor structure. In line with well-known empirical facts in the literature on index options (see for example Bakshi, Cao and Chen, 1997; Heston and Nandi, 2000; Bates, 2000, and Jones, 2003), our model allows for mean-reverting stochastic volatility and correlated shocks to return and volatility. Motivated by our principal component analysis, we allow for idiosyncratic shocks to equity prices which also have mean-reverting stochastic volatility and a separate leverage effect. Individual equity returns are linked to the market index using a standard linear factor model with a constant factor loading. Our model belongs to the affine class which enables us to derive closed-form option pricing formulas. The model can be extended to allow for market-wide and idiosyncratic jumps. Pan (2002), Broadie, Chernov and Johannes (2007), and Bates (2008) among others have argued for the importance of modeling jumps in index options.

We develop a convenient approach to estimating the model from option data. When estimating the model on the firms in the Dow we find that it provides a good fit to observed equity option prices.

Market betas are central to both asset pricing and corporate finance. Multiple applications require estimates of beta such as cost of capital estimation, performance evaluation, portfolio selection, and abnormal return measurement. While it is not the focus of this paper, our model provides option-implied estimates of beta, which is a topic of recent interest, studied by for example Chang, Christoffersen, Jacobs, and Vainberg (2012), and Buss and Vilkov (2012).

Our paper is related to the recent empirical literature on equity options including Dennis and Mayhew (2002), who investigate the ability of firm characteristics to explain the variation in riskneutral skewness. Recent empirical work on equity option returns includes Goyal and Saretto (2009), Vasquez (2011), and Jones and Wang (2012). Bakshi and Kapadia (2003) investigate the volatility risk premium for equity options. Bakshi, Kapadia, and Madan (2003) derive a skew law for individual stocks decomposing individual return skewness into a systematic and idiosyncratic component and find that individual firms display much less (negative) option-implied skewness than the market index. Bakshi, Cao and Zhong (2012) investigate the performance of jump models for equity option valuation. Engle and Figlewski (2012) develop time series models of implied volatilities and study their correlation dynamics. Perhaps most relevant for our work, Duan and Wei (2009) demonstrate empirically that systematic risk matters for the observed prices of equity options on the firm's stock.

Our paper is also related to recent theoretical advances. Mo and Wu (2007) develop an international CAPM model which has features similar to our model. Elkamhi and Ornthanalai (2010) develop a bivariate discrete-time GARCH model to extract the market jump risk premia implicit in individual equity option prices. Finally, Serban, Lehoczky, and Seppi (2008) develop a non-affine model to investigate the relative pricing of index and equity options.

The reminder of the paper is organized as follows. In Section 2 we describe the data set and

present the principal components analysis. In Section 3 we develop the theoretical model. Section 4 highlights a number of implications of the model. In Section 5 we estimate the model and investigate its fit to observed index and equity option prices. In Section 6 we explore the model's implications for expected option return and for portfolio risk management. Section 7 concludes.

# 2 Common Factors in Equity Option Prices

In this section we first introduce the data set used in our study. We then look for evidence of commonality in three crucial features of the cross-section of equity options: Implied volatility levels, moneyness slopes (or skews), and volatility term structures. We rely on a principal component analysis (PCA) of the firm-specific levels of short-term at-the-money implied volatility (IV), the slope of IV with respect to option moneyness, and the slope of IV with respect to option maturity. The results from this model-free investigation help identify desirable features of a structural model of equity option prices.

#### 2.1 Data

We rely on end-of-day option data from OptionMetrics starting on January 2, 1996 and ending on October 29, 2010, which was the time span available at the time of writing. We use the S&P500 index to proxy for the market factor. For our sample of individual equities we choose the firms in the Dow Jones Industrial Average index. Of the 30 firms in the index we excluded Kraft Foods for which OptionMetrics only has data from 2001. We filter out bid-ask option pairs with missing quotes or zero bids, and options that violate standard arbitrage restrictions. For each option maturity, interest rates are estimated by linear interpolation using zero coupon Treasury yields. Dividends are obtained from OptionMetrics and are assumed to be known during the life of each option.

Our study focuses on medium-term options, i.e. options having more than 20 days and less than 365 days to maturity (DTM). Following Bakshi, Cao and Chen (1997), we use mid-quotes (average bid-ask spread) in all computations, and eliminate options with moneyness (S/K) less than 0.9 and greater than 1.1. We also filter out quotes smaller than 3/8, with implied volatility greater than 150%, and for which the present value of dividends are larger than 4% of the stock price.

Table 1 presents the number of option contracts, the number of calls and puts, the average daysto-maturity, and the average implied volatility. The S&P500 index has by far the greatest number of option contracts. We have a total of 393, 429 index option quotes and 2, 370, 951 equity option quotes across the 29 firms. The average implied volatility for the market is 20.51% during the study period. Cisco has the highest average implied volatility (40.78%) while Johnson & Johnson has the smallest (22.90%). Table 1 also shows that our data set is balanced with respect to the number of OTM calls and puts retained.

Table 2 reports the average, minimum, and maximum implied volatility, as well as the average option vega. Note that apart from Cisco the average implied volatilities of OTM puts are always higher than the average implied volatilities of OTM calls.

Figure 1 plots the daily average short-term (9 < DTM < 60) at-the-money (0.95 < S/K < 1.05)implied volatility (IV) for six firms (black lines) as well as for the S&P500 index (grey lines). Figure 1 shows that the variation in the short-term at-the-money (ATM) equity volatility for each firm is highly related to the S&P500 index

#### 2.2 Methodology

We want to assess the extent to which the time-varying volatilities of equities share one or more common components. In order to gauge the degree of commonality in risk-neutral volatilities, we need daily estimates of the level and slope of the implied volatility curve, and slope of the term structure of implied volatility for all firms and the index. For each day t we run the following regression for firm j

$$IV_{j,l,t} = a_{j,t} + b_{j,t} \cdot \left(S_t^j / K_{j,l}\right) + c_{j,t} \cdot (DTM_{j,l}) + \epsilon_{j,l,t}$$
(2.1)

where l denotes the contracts available for firm j on day t and t-1. We include the previous day's data in the sample to ensure that we have a sufficient number of contracts for each firm to estimate the regression coefficients reliably. The regressors are standardized each day by subtracting the mean and dividing by the standard deviation. We run the same regression on the index option IVs. We interpret  $a_{j,t}$  as a measure of the level of implied volatilities of firm j on day t. Similarly,  $b_{j,t}$  captures the slope of implied volatility curve while  $c_{j,t}$  proxies for the slope of the term structure of implied volatility.

Once the regression coefficients have been estimated on each day and for each firm we run a PCA analysis on each of the matrices  $\{a_{j,t}\}$ ,  $\{b_{j,t}\}$ , and  $\{c_{j,t}\}$ . Tables 3-5 contain the results from the PCA analysis and Figure 2-4 plot the first principal component as well as the corresponding index option coefficients,  $a_{I,t}$ ,  $b_{I,t}$ , and  $c_{I,t}$ .

#### 2.2.1 Common Factors in the Levels of Implied Equity Volatility

Table 3 contains the results for implied volatility levels. We report the loading of each equity IV on the first three components. At the bottom of the table we show the average, min and max

loading across firms for each component. We also report the total variation captured as well as the correlation of each component with the S&P500 IV. The results in Table 3 are quite striking. The first component captures 77% of the total cross-sectional variation in short-term IV and it has a 91% correlation with the S&P500 index IV. This suggests that the equity IVs have a very strong common component highly correlated with index option IVs. Note that the loadings on the first component are positive for all 29 firms illustrating the pervasive nature of the common factor.

The top panel of Figure 2 shows the time series of short-term IV for index options. The bottom panel plots the time series of the first PCA component of equity IV. The strong relationship between the two series is readily apparent.

The second PCA component in Table 3 explains 13% of the total variation and the third component explains 3%. The average loadings on these two components are close to zero and the loadings take on a wide range of positive and negative values. The sizable second PCA component and the wide range of loadings show the need for a second source of firm specific variation in equity volatility.

#### 2.2.2 Common Factors in the Moneyness Slope

Table 4 contains the results for IV moneyness slopes. The moneyness slopes contain a significant degree of co-movement. The first principal component explains 49% of cross-sectional variation in the moneyness slope. The second and third components explain 8% and 5% respectively. The first component has positive loadings on all 29 firms where as the second and third components have positive and negative loadings across firms, and average loadings very close to zero.

Table 4 also shows that the first principal component has a 42% correlation with the moneyness slope of S&P500 implied volatility. The second and third components have correlations of 14% and 32% respectively. Equity moneyness slope dynamics clearly seem driven to a non-trivial extent by the market moneyness slope.

Figure 3 plots the S&P500 index IV moneyness slope in the top panel and the first principal component from the equity moneyness slopes in the bottom panel. The relationship between the first principal component and the market slope coefficient is clearly not as strong as for the volatility level in Figure 2.

#### 2.2.3 Common Factors in the Term Structure Slope

Table 5 contains the results for IV term structure slopes. The variation in the term slope captured by the first principal component is 57%, which is lower than for spot volatility (Table 3) but higher than for the moneyness slope (Table 4). The loadings on the first component are positive for all 29 firms. The correlation between the first component and the term slope of S&P500 index option IV is 74%, which is again higher than for the moneyness slope in Table 4 but lower than for the variance level in Table 3. The second and third components capture 9% and 5% of the variation respectively and the wide range of loadings on this factor suggest a scope for firm-specific variation in the IV term structure for equity options.

Figure 4 plots the S&P500 index IV term structure slope in the top panel and the first principal component from the equity term slopes in the bottom panel. Most of the spikes in S&P500 maturity slopes are clearly evident in the first principal component as well. Comparing Figures 2 and 4 we see that the term structure slope is close to zero when volatility is low and strongly negative when volatility is high.

We conclude that while the market volatility term structure captures a substantial share of the variation in equity volatility term structures, the scope for a persistent firm-specific volatility factor seems clear.

#### 2.3 Other Stylized Facts in Equity Option Prices

The literature on equity options has documented a number of important stylized facts that are complementary to our findings above.

Dennis and Mayhew (2002) find that option-implied skewness tends to be more negative for stocks with larger betas. Bakshi, Kapadia and Madan (2003) show that the market index volatility smile is on average more negatively sloped than individual smiles. They also show that the more negatively skewed the risk-neutral distribution, the steeper the volatility smile. Finally, they find that the risk-neutral equity distributions are on average less skewed to the left than index distributions.

Duan and Wei (2009) find that the level of implied equity volatility is related to the systematic risk of the firm and that the slope of the implied volatility curve is related to systematic risk as well. Driessen, Maenhout and Vilkov (2009) find a large negative index variance risk premium, but find no evidence of a negative risk premium on individual variance risk.

We next outline a structural equity option modeling approach that is able to capture these well-know stylized facts, as well as the results from the PCA analysis outlined above.

# 3 Equity Option Valuation using a Single-Factor Structure

We model an equity market consisting of n firms driven by a single market factor,  $I_t$ . The individual stock prices are denoted by  $S_t^j$ , for j = 1, 2, ..., n. Investors also have access to a risk-free bond

which pays a return of r.

The market factor evolves according to the process

$$\frac{dI_t}{I_t} = (r + \mu_I)dt + \sigma_{I,t}dW_t^{(I,1)}$$
(3.1)

where  $\mu_I$  is the instantaneous market risk premium and where volatility is stochastic and follows the standard square root process

$$d\sigma_{I,t}^2 = \kappa_I (\theta_I - \sigma_{I,t}^2) dt + \delta_I \sigma_{I,t} dW_t^{(I,2)}$$
(3.2)

As in Heston (1993),  $\theta_I$  denotes the long-run variance,  $\kappa_I$  captures the speed of mean reversion of  $\sigma_{I,t}^2$  to  $\theta_I$  and  $\delta_I$  measures volatility of volatility. The innovations to the market factor return and volatility are correlated with coefficient  $\rho_I$ . Conventional estimates of  $\rho_I$  are negative and large capturing the so-called leverage effect in aggregate market returns.

Individual equity prices are driven by the market factor as well as an idiosyncratic term which also has stochastic volatility

$$\frac{dS_t^j}{S_t^j} - rdt = \alpha_j dt + \beta_j \left(\frac{dI_t}{I_t} - rdt\right) + \sigma_{j,t} dW_t^{(j,1)}$$
(3.3)

$$d\sigma_{j,t}^2 = \kappa_j (\theta_j - \sigma_{j,t}^2) dt + \delta_j \sigma_{j,t} dW_t^{(j,2)}$$
(3.4)

where  $\alpha_j$  denotes the excess return and  $\beta_i$  is the market beta of firm j.

The innovations to idiosyncratic return and volatility are correlated with coefficient  $\rho_j$ . As suggested by the skew laws derived in Bakshi, Kapadia, and Madan (2003), asymmetry of the idiosyncratic return component is required to explain the differences in the price structure of individual equity versus market index options.

Note that our model of the equity market has a total of 2(n+1) innovations.

#### **3.1** Risk Neutral Distribution

In order to use our model of the equity market to value derivatives we need to postulate a change of measure from the physical (P-measure) distribution developed above to the risk-neutral (Qmeasure) distribution. Following the literature, we assume a change-of-measure of the exponential form

$$\frac{dQ}{dP}(t) = \exp\left(-\int_{0}^{t} \gamma_{u} dW_{u} - \frac{1}{2} \int_{0}^{t} \gamma_{u}' d\left\langle W, W'\right\rangle_{u} \gamma_{u}\right)$$
(3.5)

where  $W_u$  is a vector containing the 2(n+1) innovations and  $\gamma_u$  is a vector of market prices of risk. The exact form of  $W_u$  and  $\gamma_u$  are given in Appendix A.

In the spirit of Cox, Ingersoll, and Ross (1985) and Heston (1993) among others, we assume a price of market variance risk of the form  $\lambda_I \sigma_{I,t}$ . We also assume that idiosyncratic variance risk is not priced. These assumptions yield the following result.

**Proposition 1** Given the change-of-measure in (3.5) the Q-process of the market factor is given by

$$\frac{dI_t}{I_t} = rdt + \sigma_{I,t} d\tilde{W}_t^{(I,1)}$$
(3.6)

$$d\sigma_{I,t}^2 = \tilde{\kappa}_I \left( \tilde{\theta}_I - \sigma_{I,t}^2 \right) dt + \delta_I \sigma_{I,t} d\tilde{W}_t^{(I,2)}$$
(3.7)

where 
$$\tilde{\kappa}_I = \kappa_I + \delta_I \lambda_I$$
, and  $\tilde{\theta}_I = \frac{\kappa_I \theta_I}{\tilde{\kappa}_I}$  (3.8)

and the Q-processes of the individual equities are given by

$$\frac{dS_t^j}{S_t^j} = rdt + \beta_j \left(\frac{dI_t}{I_t} - rdt\right) + \sigma_{j,t} d\tilde{W}_t^{(j,1)}$$
(3.9)

$$d\sigma_{j,t}^2 = \kappa_j \left(\theta_j - \sigma_{j,t}^2\right) dt + \delta_j \sigma_{j,t} d\tilde{W}_t^{(j,2)}$$
(3.10)

where  $d\tilde{W}_t$  denotes the risk-neutral version of  $dW_t$ 

#### **Proof.** See Appendix A.

These propositions provide several insights. Note that the market factor structure is preserved under Q. Consequently, the market beta is the same under the risk-neutral and physical distributions. This is consistent with Serban, Lehoczky, and Seppi (2008), who document that the risk-neutral and objective betas are economically and statistically close for most stocks.

It is also important to note that in our modeling framework, higher moments (variance, skewness, and kurtosis) and their premiums, as defined by the difference between the level of the moment under Q with the level under P, are all affected by the drift adjustment in the variance processes. We will discuss this further below.

#### **3.2** Closed-Form Option Valuation

Our model has been cast in an affine framework which implies that the characteristic function for the log market value and the log equity price can both be derived analytically. The market index characteristic function will be exactly identical to that in Heston (1993). Consider now individual equity options. We need the following proposition:

**Proposition 2** The risk-neutral conditional characteristic function  $\tilde{\phi}_{t,T}^{j}(u)$  for the equity price,  $S_{T}^{j}$ , is given by

$$\widetilde{\phi}_{t,T}^{j}(u) \equiv E_{t}^{Q} \left[ \exp\left(iu\ln\left(S_{T}^{j}\right)\right) \right]$$

$$= \left(S_{t}^{j}\right)^{iu} \exp\left(iur\left(T-t\right) - A^{I}\left(\Lambda^{S},u\right) - B^{I}\left(\Lambda^{S},u\right)\sigma_{I,t}^{2} - A^{j}\left(\Lambda^{S},u\right) - B^{j}\left(\Lambda^{S},u\right)\sigma_{j,t}^{2} \right)$$
(3.11)

where the expressions for  $\Lambda^{S}$ ,  $A^{I}(\Lambda^{S}, u)$ ,  $B^{I}(\Lambda^{S}, u)$ ,  $A^{j}(\Lambda^{S}, u)$ , and  $B^{j}(\Lambda^{S}, u)$  are provided in Appendix B.

#### **Proof.** See Appendix B.

Given the log spot price characteristic function under Q, the price of a European equity call option with strike price K and maturity T - t is

$$C_t^j(K, T-t) = S_t^j \Pi_1^j - K e^{-r(T-t)} \Pi_2^j$$
(3.12)

where the risk-neutral probabilities  $\Pi_1^j$  and  $\Pi_2^j$  are defined by

$$\Pi_{1}^{j} = \frac{1}{2} + \frac{e^{-r(T-t)}}{\pi S_{t}^{j}} \int_{0}^{\infty} \operatorname{Re}\left[\frac{e^{-iu \ln K} \tilde{\phi}_{t,T}^{j}(u-i)}{iu}\right] du$$
(3.13)

$$\Pi_2^j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \operatorname{Re}\left[\frac{e^{-iu\ln K} \tilde{\phi}_{t,T}^j(u)}{iu}\right] du$$
(3.14)

While these integrals must be evaluated numerically, they are well-behaved and can be computed quickly.

## 4 Model Predictions

In this section we derive a number of important implications from our model and assess if the model captures the stylized facts observed in equity option prices in Section 2. For convenience we will assume that beta is positive for each firm below. This is not required by the model but it simplifies certain expressions.

#### 4.1 Equity Option Volatility Level

Duan and Wei (2009) show empirically that firms having a higher systematic risk tend to have a higher level of risk-neutral variance. We now investigate if our model is consistent with this empirical finding.

First, define total spot variance for firm j at time t

$$V_{j,t} \equiv \beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2$$

and define the expectations under P and Q of the corresponding integrated variance by

$$E_t^P[V_{j,t:T}] \equiv E_t^P\left[\int_t^T V_{j,s}ds\right] \quad \text{and} \quad E_t^Q[V_{j,t:T}] \equiv E_t^Q\left[\int_t^T V_{j,s}ds\right]$$

By splitting up the P-expectation between the integrated market variance and the idiosyncratic one, we have

$$E_t^P[V_{j,t:T}] = \beta_j^2 E_t^P[\sigma_{I,t:T}^2] + E_t^P[\sigma_{j,t:T}^2]$$

where  $\sigma_{I,t:T}^2$ , and  $\sigma_{j,t:T}^2$  correspond to the integrated variances from t to T.

Given our model, the expectation of the integrated total variance for equity j under Q is

$$E_t^Q[V_{j,t:T}] = \beta_j^2 E_t^Q[\sigma_{I,t:T}^2] + E_t^Q[\sigma_{j,t:T}^2] = \beta_j^2 E_t^Q[\sigma_{I,t:T}^2] + E_t^P[\sigma_{j,t:T}^2]$$

Note that the second equation holds as long as idiosyncratic risk is not priced (i.e.  $E_t^P[\sigma_{j,t:T}^2] = E_t^Q[\sigma_{j,t:T}^2]$ ).

For any two firms having the same level of expected total variance under the *P*-measure  $(E_t^P[V_{1,t:T}] = E_t^P[V_{2,t:T}])$  we have

$$E_t^P[\sigma_{1,t:T}^2] - E_t^P[\sigma_{2,t:T}^2] = -(\beta_1^2 - \beta_2^2)E_t^P[\sigma_{1,t:T}^2]$$

As a result:

$$\begin{split} E_t^Q[V_{1,t:T}] - E_t^Q[V_{2,t:T}] &= (\beta_1^2 - \beta_2^2) E_t^Q[\sigma_{I,t:T}^2] + \left( E_t^Q[\sigma_{1,t:T}^2] - E_t^Q[\sigma_{2,t:T}^2] \right) \\ &= (\beta_1^2 - \beta_2^2) E_t^Q[\sigma_{I,t:T}^2] + \left( E_t^P[\sigma_{1,t:T}^2] - E_t^P[\sigma_{2,t:T}^2] \right) \\ &= (\beta_1^2 - \beta_2^2) \left( E_t^Q[\sigma_{I,t:T}^2] - E_t^P[\sigma_{I,t:T}^2] \right) \end{split}$$

When the market variance premium is negative, we have  $\tilde{\theta}_I > \theta_I$  which implies that  $E_t^Q[\sigma_{I,t:T}^2] >$ 

 $E_t^P[\sigma_{I,t:T}^2]$ . We therefore have that

$$\beta_1 > \beta_2 \Leftrightarrow E^Q_t[V_{1,t:T}] > E^Q_t[V_{2,t:T}]$$

We conclude that our model is consistent with the finding in Duan and Wei (2009) that firms with high betas tend to have a high level of risk-neutral variance.

#### 4.2 Equity Option Skews

To understand the slope of individual equity option implied volatility curves we need to understand the way beta influences the skewness of the risk-neutral equity return distribution. The next proposition is key to comprehend how beta, systematic risk, and the market index skewness impact the total skewness of equity.

**Proposition 3** The conditional total skewness of the integrated returns of firm "j" under P, noted  $TSk_i^P$ , is given by

$$TSk_{j,t:T}^{P} \equiv Sk^{P} \left( \int_{t}^{T} \frac{dS_{u}^{j}}{S_{u}^{j}} \right) = Sk_{I}^{P} \cdot \left( A_{j,t:T}^{P} \right)^{3/2} + Sk_{j}^{P} \cdot \left( 1 - A_{j,t:T}^{P} \right)^{3/2}$$
(4.1)

Respectively, the total skewness of the integrated returns of firm "j" under Q, noted  $TSk_{j}^{Q}$ , is

$$TSk_{j,t:T}^{Q} \equiv Sk^{Q} \left( \int_{t}^{T} \frac{dS_{u}^{j}}{S_{u}^{j}} \right) = Sk_{I}^{Q} \cdot \left( A_{j,t:T}^{Q} \right)^{3/2} + Sk_{j}^{Q} \cdot \left( 1 - A_{j,t:T}^{Q} \right)^{3/2}$$
(4.2)

where

$$A_{j,t:T}^{P} \equiv \frac{E_{t}^{P}[\beta_{j}^{2}\sigma_{I,t:T}^{2}]}{E_{t}^{P}[V_{j,t:T}]} \quad and \quad A_{j,t:T}^{Q} \equiv \frac{E_{t}^{Q}[\beta_{j}^{2}\sigma_{I,t:T}^{2}]}{E_{t}^{Q}[V_{j,t:T}]}$$

are the proportion of systematic risk of firm j under P and Q, and  $Sk_I = Sk\left(\int_t^T \frac{dI_s}{I_s}\right)$  and  $Sk_j = Sk\left(\int_t^T \sigma_{j,s} dW_s^{(j,1)}\right)$  are the market and idiosyncratic skewness computed under the measure considered.

#### **Proof.** See Appendix C.

The previous expressions show that  $\beta_j$  matters to determine firm "j" conditional total skewness. From a risk-neutral perspective, we see from (4.2) that  $\beta_j$  affects the slope of equity implied volatility curve through  $TSk_{j,t:T}^Q$  by influencing the level of systematic risk proportion  $A_{j,t:T}^Q$ . A higher  $A_{j,t:T}^Q$ mechanically implies a higher loading on the market risk-neutral skewness  $Sk_I^Q$ . Consider two firms having the same quantity of expected total variance under Q and with  $\beta_1 > \beta_2$ , which implies  $A_{1,t:T}^Q > A_{2,t:T}^Q$ . As a result, firm 1 has a greater loading on index risk-neutral skewness than firm 2. In an economy where the index *Q*-distribution is more negatively skewed than idiosyncratic equity distribution<sup>1</sup>, we have the following cross-sectional prediction: higher-beta firms will have more negatively skewed *Q*-distributions. Note that this prediction is in line with the cross-sectional empirical findings of Duan and Wei (2009) and Dennis and Mayhew (2002).

The equation (4.2) also shows how the dynamic of market risk-neutral skewness will affect changes in the slope of individual equity implied volatility curve. Whenever idiosyncratic risk is not priced, we have

$$A_{j,t:T}^Q > A_{j,t:T}^P \Leftrightarrow E_t^Q[\sigma_{I,t:T}^2] > E_t^P[\sigma_{I,t:T}^2]$$

$$\tag{4.3}$$

Given a negative market variance risk premium (i.e.  $E_t^Q[\sigma_{I,t:T}^2] > E_t^P[\sigma_{I,t:T}^2]$ ), the previous expression suggests that equity return moments are more affected by changes in systematic skewness under the Q measure than under the P measure. Consequently, the market variance premium can potentially explain the co-movements in the equity implied volatility slopes found for individual equity options in Section 2.

Using the model the skewness premium of the individual equity  $S_t^j$  takes the following form when  $Sk_j^P = Sk_j^Q = 0$ 

$$TSk_{j,t:T}^{Q} - TSk_{j,t:T}^{P} = \left(A_{j,t:T}^{P}\right)^{3/2} \left[Sk_{I}^{Q} \cdot \left(\frac{A_{j,t:T}^{Q}}{A_{j,t:T}^{P}}\right)^{3/2} - Sk_{I}^{P}\right]$$
(4.4)

Recall that when the market variance risk premium is negative, we have  $A_{j,t:T}^Q > A_{j,t:T}^P$ . Combining this with a negative market skewness premium (i.e.  $Sk_I^Q < Sk_I^P$ ), it implies that the expression in bracket is negative. Given that beta increases the proportion of systematic risk,  $A_{j,t:T}$ , controlling for total physical variance, high-beta firms should have lower skewness premiums.

Figure 5 plots the implied Black-Scholes volatility from model option prices. Each line has a different beta but the same amount of unconditional total equity variance  $V_j = \beta_j^2 \theta_I + \theta_j = 0.1$ . We set the current spot variance to  $\sigma_{I,t}^2 = 0.01$  and  $V_{j,t} = 0.05$ , and define the idiosyncratic variance as the residual  $\sigma_{j,t}^2 = V_{j,t} - \beta_j^2 \sigma_{I,t}^2$ . The market index parameters are  $\kappa_I = 5$ ,  $\theta_I = 0.04$ ,  $\delta_I = 0.5$ ,  $\rho_I = -0.8$ , and the individual equity parameters are  $\kappa_j = 1$ ,  $\delta_j = 0.4$ , and  $\rho_j = 0$ . The risk-free rate is 4% per year and option maturity is 3 months. Figure 5 shows that beta has a substantial impact on the moneyness slope of equity IV even when keeping the total variance constant: The higher the beta, the larger the moneyness slope.

<sup>&</sup>lt;sup>1</sup>This statement is in line with the empirical findings of Bakshi, Kapadia, and Madan (2003).

#### 4.3 The Equity Volatility Term-Structure

We next investigate the model's implication for the term structure of equity volatility. The role of the market beta turns out to be crucial.

Our model implies the following two-component term-structure of equity variance

$$E_t^Q[V_{j,t:T}] = \left(\beta_j^2 \tilde{\theta}_I + \theta_j\right) + \beta_j^2 \left(\sigma_{I,t}^2 - \tilde{\theta}_I\right) e^{-\tilde{\kappa}_I(T-t)} + \left(\sigma_{j,t}^2 - \theta_j\right) e^{-\kappa_j(T-t)}$$
(4.5)

This expression shows how the market variance term-structure affects the variance term-structure for the individual equity. Given different systematic and idiosyncratic mean reverting speeds ( $\tilde{\kappa}_I \neq \kappa_j$ ), we see that  $\beta_j$  has important implications on the term-structure of volatilities. When the idiosyncratic variance process is more persistent ( $\tilde{\kappa}_I > \kappa_j$ ), higher values of beta imply a faster reversion toward the unconditional total variance ( $\tilde{V}_j = \beta_j^2 \tilde{\theta}_I + \theta_j$ ). As a result, when the market variance process is less persistent than the idiosyncratic variance, in the cross-section, firms with higher betas are likely to have steeper volatility term-structures. In other words, higher beta firms are expected to have a greater positive (resp. negative) slope when the market variance termstructure is upward (resp. downward) sloping.

Figure 6 plots the implied Black-Scholes volatility from model prices against option maturity. Each line has a different beta but the same amount of unconditional total equity variance  $V_j = \beta_j^2 \theta_I + \theta_j = 0.1$ . We set the current spot variance to  $\sigma_{I,t}^2 = 0.01$  and  $V_{j,t} = 0.05$ , and define the idiosyncratic variance as the residual  $\sigma_{j,t}^2 = V_{j,t} - \beta_j^2 \sigma_{I,t}^2$ . The parameter values are as in Figure 5. Figure 6 shows that beta has a non-trivial effect on the IV term structure: The higher the beta, the steeper the term structure when the term structure is upward sloping.

In summary, our model suggests that-ceteris paribus-firms with higher betas should have higher levels of volatility, larger moneyness slopes, and higher absolute maturity slopes. We now estimate the model in order to assess if these patterns are indeed observed in the option data.

## 5 Estimation and Fit

In this section, we first describe our estimation methodology and then we report on parameter estimates and model fit. Finally, we relate our estimated betas to patterns in observed equity option IVs.

#### 5.1 Estimation Methodology

Several approaches have been proposed in the literature for estimating stochastic volatility models. Jacquier, Polson, and Rossi (1994) use Markov Chain Monte Carlo in a discrete time set-up. Pan (2002) uses GMM to estimate the objective and risk neutral parameters from returns and option price. Serban, Lehoczky, and Seppi's (2008) estimation strategy is based on simulated maximum likelihood using EM and particle filter methods.

Another approach treats the latent variables as a parameter to be estimated and thus avoids filtering of the latent volatility factor. Such a strategy has been adopted by Bates (2000) and Santa-Clara and Yan (2010) among others. We follow this strand of literature.

Recall that in our model two vectors of latent variables  $\{\sigma_{I,t}^2, \sigma_{j,t}^2\}$  and two sets of structural parameters  $\{\Theta_I, \Theta_j\}$  need to be estimated where  $\Theta_I \equiv \{\tilde{\kappa}_I, \tilde{\theta}_I, \delta_I, \rho_I\}$  and  $\Theta_j \equiv \{\kappa_j, \theta_j, \delta_j, \rho_j, \beta_j\}$ . Our methodology involves two main steps.

In the first step, we estimate the market index dynamic  $\{\Theta_I, \sigma_{I,t}^2\}$  based on S&P500 option prices alone. In the second step, we take the market index dynamic as given, and we estimate the firm-specific dynamics  $\{\Theta_j, \sigma_{j,t}^2\}$  for each firm conditional on estimates of  $\{\Theta_I, \sigma_{I,t}^2\}$  and use equity options for firm *j* only. This step-wise estimation procedure—while not fully efficient in the econometric sense–enables us to estimate our model for 29 equities while ensuring that the same dynamic is imposed for the market-wide index for each of the 29 firms.

Each of the two steps contains an iterative procedure which we now describe in some detail.

#### Step 1: Market Index Volatility and Parameter Estimation

Given a set of starting values,  $\Theta_I^0$ , for the index structural parameters, we first estimate the spot market variance each day by sequentially solving

$$\hat{\sigma}_{I,t}^2 = \arg\min_{\sigma_{I,t}^2} \sum_{m=1}^{N_{I,t}} (C_{I,t,m} - C_m(\Theta_I^0, \sigma_{I,t}^2))^2 / Vega_{I,t,m}^2, \text{ for } t = 1, 2, ...T$$
(5.1)

where  $C_{I,t,m}$  is the market price of index option contract m quoted at t,  $C_m(\Theta_I, \sigma_{I,t}^2)$  is the model index option price,  $N_{I,t}$  is the number of index contracts available on day t, and  $Vega_{I,t,m}$  is the Black-Scholes sensitivity of the option price with respect to volatility evaluated at the implied volatility. These vega-weighted dollar price errors are a good approximation to implied volatility errors and they are much more quickly computed.<sup>2</sup>

Once the set of T market spot variances have be obtained we solve for the set of market para-

<sup>&</sup>lt;sup>2</sup>This approximation has been used in Carr and Wu (2007) and Trolle and Schwartz (2009) among others.

meters as follows

$$\hat{\Theta}_{I} = \arg\min_{\Theta_{I}} \sum_{m,t}^{N_{I}} (C_{I,t,m} - C_{m}(\Theta_{I}, \hat{\sigma}_{I,t}^{2}))^{2} / Vega_{I,t,m}^{2},$$
(5.2)

where  $N_I \equiv \sum_{t=1}^{T} N_{I,t}$  represents the total number of index option contracts available.

We iterate between (5.1) and (5.2) until the improvement in fit is negligible which typically requires 5-10 iterations.

#### Step 2: Equity Volatility and Parameter Estimation

Given an initial value  $\Theta_j^0$  and using the estimated  $\hat{\sigma}_{I,t}^2$  and  $\hat{\Theta}_I$  we can estimate the spot equity variance each day by sequentially solving

$$\hat{\sigma}_{j,t}^2 = \arg\min_{\sigma_{j,t}^2} \sum_{m=1}^{N_{j,t}} (C_{j,t,m} - C_m(\Theta_j^0, \hat{\Theta}_I, \hat{\sigma}_{I,t}^2, \sigma_{j,t}^2))^2 / Vega_{j,t,m}^2, \text{ for } t = 1, 2, ...T$$
(5.3)

where  $C_{j,t,m}$  is the price of equity option m for firm j quoted at t,  $C_m(\Theta_j, \Theta_I, \sigma_{I,t}^2, \sigma_{j,t}^2)$  is the model equity option price,  $N_{j,t}$  is the number of equity contracts available on day t, and  $Vega_{j,t,m}$  is the Black-Scholes Vega of the equity option.

Once the set of T market spot variances have be obtained we solve for the set of market parameters as follows

$$\hat{\Theta}_j = \arg\min_{\Theta_j} \sum_{m,t}^{N_j} (C_{j,t,m} - C_m(\Theta_j, \hat{\Theta}_I, \hat{\sigma}_{I,t}^2, \hat{\sigma}_{j,t}^2)) / Vega_{j,t,m}^2$$
(5.4)

where  $N_j \equiv \sum_{t=1}^{T} N_{j,t}$  is the total number of contract available for security j.

We again iterate between (5.3) and (5.4) until the improvement in fit is negligible. We have confirmed that this estimating technique has good finite sample properties in a Monte Carlo study which is available from the authors upon request.

## 5.2 Parameter Estimates

This section presents results from the market index and 29 equity option model estimations for the 1996-2010 period. For equity options we use contracts on each trading day. For index options we estimate the structural parameters in (5.2) on Wednesday data only because the computational burden is exorbitantly large if all trading days are used.

Our S&P500 index options are European, but our individual equity options are American style.

As a result, their prices are influenced by early exercise premiums. To circumvent possible biases in our implied volatility estimates due to the presence of both early exercise premia and dividends, we eliminate in-the-money (ITM) options for which the early exercise premium matters most.<sup>3</sup>

Table 6 reports estimates of the structural parameters that describe the dynamic of the systematic variance, the beta of each firm, as well as the idiosyncratic variance dynamics. The top row shows estimates for the S&P500 index.

The unconditional market index variance  $\tilde{\theta}_I = 0.0542$  corresponds to 23% volatility per year. The average of the index spot volatility path,  $\frac{1}{T} \sum_{t=1}^{T} \sigma_{I,t}^2$  during our sample is 21.74%. The difference between these two numbers provides a rough estimate of the volatility risk premium. The idiosyncratic  $\theta_i$  estimates range from 0.0093 for General Electric to 0.0887 for Cisco.

In Table 6 the speed of mean-reversion parameter for the market index variance  $\tilde{\kappa}_I = 1.24$  corresponds to a daily variance persistence of 1 - 1.24/365 = 0.9966. The idiosyncratic  $\kappa_j$  range from 0.53 for Chevron to 1.53 for 3M showing that idiosyncratic volatility is highly persistent as well. Interestingly, 3M, Hewlett-Packard and IBM are the only three firms of our cross-section having an idiosyncratic variance process less persistent than the market variance.

As typically found in the literature,  $\rho_I = -0.860$  is strongly negative capturing the so-called leverage effect in the market index. The idiosyncratic  $\rho_j$  are also strongly negative ranging from -0.978 for Microsoft to -0.482 for Disney. The equity option data clearly require additional option skewness from the idiosyncratic volatility component.

The estimates of beta are reasonable and vary from 0.70 for Johnson & Johnson to 1.30 for JP Morgan. The average beta across the 29 firms is 0.99.

The average total spot volatility (ATSV) for firm j is computed as

$$\text{ATSV} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} V_{j,t}} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \left(\beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2\right)}$$

Comparing the beta column with the ATSV column in Table 6 shows that ATSV is generally higher when beta is high.

The final column of Table 6 reports for each firm the systematic risk ratio (SSR) computed from the spot variances as follows

$$SSR = \frac{\sum_{t=1}^{T} \beta_j^2 \sigma_{I,t}^2}{\sum_{t=1}^{T} \left(\beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2\right)}$$

 $<sup>^{3}</sup>$ Table 2 in Bakshi, Kapadia, and Madan (2003) shows that for OTM calls and puts, the difference between Black-Scholes implied volatilities and American option implied volatilities (early excercise premia) are negligible. Elkamhi and Ornthanalai (2010) get a similar result. See also Duan and Wei (2009).

Table 6 shows that the systematic risk ratio varies from close to 30% for Cisco and Hewlett-Packard to above 66% for Chevron. The systematic risk ratio varies is on average 45%, which shows that the estimated variance factor structure in the model is strong. Comparing the beta column with the SSR column in Table 6 shows that there is no apparent linear relationship between beta and SSR: Different firms with beta close to 1 can have radically different SSR and, vice versa, firms with very different betas can have roughly similar SSRs. We conclude that SSR is generally high, suggesting a strong factor structure in model spot volatility which mirrors the model-free factor structure found in the IVs in Figure 2. But it is not necessarily the case that firms with high beta have a high SSR. This of course indicates a key role for the idiosyncratic variance dynamic in our model.

#### 5.3 Model Fit

We measure model fit using the vega root mean squared error (RMSE) defined from the optimization criteria function as

Vega RMSE 
$$\equiv \sqrt{\frac{1}{N} \sum_{m,t}^{N} (C_{m,t} - C_{m,t}(\Theta))^2 / Vega_{m,t}^2}$$

We also report the implied volatility RMSE defined as

IVRMSE 
$$\equiv \sqrt{\frac{1}{N} \sum_{m,t}^{N} (IV_{m,t} - IV(C_{m,t}(\Theta)))^2}$$

where  $IV_{m,t}$  denotes market IV for option m on day t and  $IV(C_{m,t}(\Theta))$  denotes model IV. We use Black-Scholes to compute IV for both model and market prices.

Table 7 reports model fit for the market index and for each of the 29 firms. We report results for all contracts together as well as separately for out-of-the-money (OTM) calls and puts, and for short and long term at-the-money (ATM) contracts. We also report the IVRMSE divided by the average market IV in order to assess relative IV fit. Several interesting findings emerge from Table 7.

- First, the Vega RMSE approximates the IVMRSE closely for the index and for all firms. This suggests that using Vega RMSE in estimation does not bias the IVRMSE results.
- Second, the average IVRMSE across firms is 1.78% and the relative IV (IVRMSE / Average IV) is 6% on average. The fit does not vary much around these averages. The fit of the model is thus quite good across firms.

- Third, the best pricing performance for equity options is obtained for Coca Cola with an IVRMSE of 1.41%. The worst fit is for Bank of America where the IVRMSE is 2.40%. In terms of relative IVRMSE the best fit is for Intel at 4.14% and the worst is for AT&T at 8.15%.
- Fourth, the average IVRMSE fit across firms for OTM calls is 1.74% and for OTM puts it is 1.82%. Using this metric the model fits OTM calls and puts roughly equally well.
- Fifth, the average IVRMSE fit across firms for short-term ATM options is 1.67% and for long-term ATM options it is 1.59%. The model thus fits short-term and long-term ATM options equally well on average.

Figure 7 reports for each firm the average fit over time for different moneyness categories. Moneyness on the horizontal axis is measured by S/K so that OTM calls (and ITM puts) are shown on the left and ITM calls (and OTM puts) are shown on the right side. Figure 7 averages over OTM as well as ITM options although to facilitate computations only the former were used in estimation.

Figure 7.A reports on the first fifteen firms and Figure 7.B reports on the last 14 firms as well as the index. Note that in order to properly see the different patterns across firms, the vertical axis scale differs in each subplot, but the range of implied volatility values is kept fixed at 10% across firms to facilitate comparisons.

Figure 7 shows that the smiles computed using market prices (solid black lines) vary considerably across firms, both in terms of level and shape. Consider for example Cisco in Figure 7.A (third row, second column) which has a relatively steep smirk and high levels of IV versus Bank of America (top row, third column) which has a more symmetric smile and lower levels of IV. The model (dashed grey lines) fits the different IV moneyness shapes remarkably well. The IV errors by moneyness are small in general and no dramatic outliers are apparent.

If any systematic error is apparent it may be that the model tends to underprice the extreme OTM calls (and thus ITM puts) in the left side of the graphs. The bias is small, however, relative to the overall level of IV. The small bias could be driven by an insufficient adjustment for the early exercise premium which affects mostly ITM puts. The bottom right panel in Figure 7.B confirms the finding in Bakshi, Kapadia and Madan (2003) that market index display much more (negative) skewness than do individual equities. The bottom right panel also shows that additional negative skewness in the model is required to fit the relatively expensive OTM puts trading on the market index. Generating market skewness at short maturities can be achieved by return jump models (Bates, 2000). Interestingly, while the Heston (1993) model is unable to adequately capture OTM

index put option IV levels, our model is able to fit OTM equity put options quite well. Nevertheless, allowing expanding the index model we use to allow for jumps is a worthy topic for future study.

In Figure 8 we report for each firm the average (over time) implied volatility as a function of time to maturity (in years). We split the data set into two groups: Days where the IV term structure is upward-sloping (grey lines) and days where it is downward sloping (black lines). We report the average IVs observed in the market (solid lines) as well as the average IV from our fitted model prices (dashed lines). The downward sloping black lines use the left-hand axis and the upward sloping grey lines use the right-hand axis. In order to facilitate comparison between model and market IVs the level of IVs differ between the left and right axis and they differ across firms. In order to facilitate comparison between term structures the difference between the minimum and maximum on each axis is fixed at 10% across all firms.

Figure 8 shows that the term structure of IV differs considerably across firms. Some firms such as Alcoa and American Express have quite flat downward sloping term structures whereas other firms such as General Electric and Hewlett-Packard have much steeper term structures. Generally, across firms, the downward sloping black lines appear to be steeper than the upward sloping grey lines. This pattern is matched well by the model. Figure 8 does not reveal any systematic model biases in the term structure of IVs.

We conclude from Table 7 and Figures 7 and 8 and the model fits the observed equity option data quite well. Encouraged by this finding, we next analyze in some detail how our estimated betas are related to observed patterns in equity option IVs.

#### 5.4 Estimated Equity Betas and Observed Equity Option IVs

The three central cross-sectional predictions of our model, as discussed in Section 4, are as follows:

- 1. Firms with higher betas have higher risk-neutral variance.
- 2. Firms with higher betas have larger moneyness slopes. This is equivalent to stating that firms with higher betas are characterized by more negative skewness.
- 3. Firms with higher betas have steeper positive volatility term structures when the term structure is upward sloping, and steeper negative volatility term structures when the term structure is downward sloping.

We now document how these theoretical model implications are manifested in the estimates for the 29 Dow-Jones firms. Consider first the level of option implied volatility. In the top panel of Figure 9, we scatter plot the time-averaged intercepts from the implied volatility regression in (2.1),  $\frac{1}{T}\sum_{t=1}^{T} a_{j,t}$  against the beta estimate from Table 6 for each firm j. We then run a regression on the 29 points in the scatter and assess the significance and fit. The slope has a t-statistic of 6.40 and the regression fit  $(R^2)$  is quite high at 60%. The regression line shows the positive relationship across firms between our estimated betas and the average implied volatility observed in the market prices of equity options.

In the middle panel of Figure 9 we scatter plot the moneyness slope coefficients from the IV regression in (2.1),  $\frac{1}{T} \sum_{t=1}^{T} b_{j,t}$  against the beta estimate from Table 6 for each firm j. In the moneyness slope regression, the sensitivity to beta has a t-statistic of 1.34 and an  $R^2$  of 6%. Clearly the moneyness scatter is noisy and has several outliers including Alcoa which has a beta above one but also the lowest moneyness slope in the sample. Nevertheless, Panel B shows that an increase in our beta estimate is associated with an increase in the slope of the moneyness smile in observed equity IVs. The two firms with the highest betas (JP Morgan and American Express) have very high moneyness slopes suggesting that the association may be nonlinear which is partly causing the relative poor fit of the linear regression.

Finally, in the bottom panel of Figure 9 we scatter plot the absolute of the term structure slope coefficients from (2.1), that is,  $\frac{1}{T} \sum_{t=1}^{T} c_{j,t}$  against the beta estimate from Table 6 for each firm. In the term slope regression, the sensitivity to beta has a t-statistic of 5.24 and the  $R^2$  is quite high at 50%. Panel C shows that an increase in our beta estimate is associated with an increase in the absolute slope of the term structure in observed equity IVs: Firms with high betas will tend to have a term structure of implied volatility curve that decays more quickly to the unconditional level of volatility than will a firm with a low beta.

We conclude that our estimates of beta when contrasted with the observed market IVs confirm the three main model predictions from Section 4.

## 6 Further Model Implications: Option Risk and Return

In this section we explore some additional implications of our model. We first study the model's implications for equity option risk management by computing the most important option price sensitivity measures. We also derive the expected return on options as implied by our model.

### 6.1 Equity Option Risk Management

In classic equity option valuation models, partial derivatives are used to assess the sensitivity of the option price to the underlying stock price (delta) and equity volatility (vega). In our model the equity option price additionally is exposed to changes in the market level and market variance. Portfolio managers with diversified equity option holdings need to know the sensitivity of the equity option price to these market level variables in order to properly manage risk. The following proposition provides the model's implications for the sensitivity to the market level and market volatility.

**Proposition 4** For a derivative contract  $f^j$  written on the stock price,  $S_t^j$ , the sensitivity of  $f^j$  with respect to market value,  $I_t$  (market delta) is given by:

$$\frac{\partial f^j}{\partial I_t} = \frac{\partial f^j}{\partial S^j_t} \frac{S^j_t}{I_t} \beta_j$$

The sensitivity of  $f^{j}$  with respect to market variance (market vega) takes the form:

$$\frac{\partial f^j}{\partial \sigma_{I,t}^2} = \frac{\partial f^j}{\partial V_{j,t}} \beta_j^2$$

**Proof.** See Appendix D.  $\blacksquare$ 

This proposition shows that the beta of the firm in a straightforward way provides the link between the usual stock price delta  $\frac{\partial f^j}{\partial S_t}$  and the market delta,  $\frac{\partial f^j}{\partial I_t}$ , as well as the link between the usual equity vega,  $\frac{\partial f^j}{\partial V_{j,t}}$ , and the market vega  $\frac{\partial f^j}{\partial \sigma_{I_t}^2}$ .

The result in the proposition will allow market participants with portfolios of equity options on different firms—as well as equity options—to measure and manage their total exposure to the market index level and to the market index volatility. It will also allow for investors engaged in dispersion trading—where index options are sold and equity options bought—to measure and manage their overall exposure to market risk and market volatility risk.

#### 6.2 Equity Option Expected Returns

So far our model implications have focused on option prices. In certain applications, such as option portfolio management, option returns are of interest as well. Our final proposition provides an expression for the expected (P-measure) equity option return as a function of the expected market return.<sup>4</sup>

**Proposition 5** For a derivative  $f^j$  written on the stock price,  $S_t^j$ , the expected excess return on the

<sup>&</sup>lt;sup>4</sup>Recent empirical work on equity and index option returns includes Broadie, Chernov and Johannes (2009), Goyal and Saretto (2009), Constantinides, Czerwonko, Jackwerth, and Perrakis (2011), Vasquez (2011), and Jones and Wang (2012).

derivative contract is given by

$$\frac{1}{dt}E_t^P\left[df^j/f^j - rdt\right] = \frac{\partial f^j}{\partial I_t}\frac{I_t}{f^j}\mu_I = \frac{\partial f^j}{\partial S_t^j}\frac{S_t^j}{f^j}\beta_j\mu_I$$

where  $\frac{\partial f^{j}}{\partial I_{t}}$  and  $\frac{\partial f^{j}}{\partial \sigma_{I,t}^{2}}$  are from the Proposition 4. **Proof.** See Appendix E.

This proposition reveals that the beta of the stock provides a simple link between the expected return on the market index and the expected return on the equity option via the delta of the option. Our model thus allows the investor to decompose the excess return on the option into two parts: The delta of the equity option as well as the beta of the stock. Equity options provide investors which two sources of leverage: First, the beta with the market, and second, the elasticity of the option price with respect to changes in the stock price.

## 7 Summary and Conclusions

Principal Component Analysis reveals a strong factor structure in equity option prices. The first common component explains roughly 77% of the cross-sectional variation in IV and the common component has an 91% correlation with the short-term implied volatility constructed from S&P500 index options. Roughly 49% of the variation in the equity skew is captured by the first principal component. This common component has a correlation of 42% with the skew of market index options. When looking for a common component in the term structure of equity IV we find that 57% of the variation is explained by the first principal component. This component has a correlation of 74% with the term slope of the option IV from S&P500 index options.

Motivated by the findings from the principal component analysis, we develop a structural model of equity option prices that incorporates a market factor. Our model allows for mean-reverting stochastic volatility and correlated shocks to return and volatility. Motivated by our principal components analysis we allow for idiosyncratic shocks to equity prices which also have mean-reverting stochastic volatility and a separate leverage effect. Individual equity returns are linked to the market index using a standard linear factor model with a constant beta factor loading. We derive closed-form option pricing formulas as well as results for option hedging and option expected returns.

We have also developed a convenient estimation method for estimation and filtering based on option prices. When estimating the model on the firms in the Dow we find that it provides a good fit to observed equity option prices. Moreover, we show that our estimates strongly confirm the three main cross-sectional model implications.

Several issues are left for future research. First, it would be interesting to study the empirical implications of our models for option returns. Second, it may be useful to allow for two stochastic volatility factors in the market price process as done for example in Bates (2000). Third, allowing for jumps in the market price is relevant (Bollerslev and Todorov, 2011). Fourth, combining option information with that in high-frequency returns (Patton and Verardo, 2012; Hansen, Lunde, and Voev, 2012) for beta estimation could be interesting. Finally, using the model to construct option-implied betas for use in cross-sectional equity pricing (Conrad, Dittmar and Ghysels, 2013) could be of substantial interest.

# Appendix

This appendix collects proofs of the propositions.

## A. Proof of Proposition 1

The proof has two steps. First, we identify the process of the market prices of risk  $\gamma_t$  (step 1). In a second step, we risk-neutralize the variance processes using the result obtained in step 1.

**Step 1:** We derive the market prices of risk for the market Index  $(I_t)$ . A similar argument can be easily extended to individual equities. We first define the stochastic exponential  $\xi(\cdot)$ :

$$\xi\left(\int_{0}^{t}\omega_{u}^{'}dW_{u}\right) \equiv \exp\left(\int_{0}^{t}\omega_{u}dW_{u} - \frac{1}{2}\int_{0}^{t}\omega_{u}^{'}d\left\langle W,W^{'}\right\rangle_{u}\omega_{u}\right)$$
(7.1)

where  $\omega_u$  is a 2(n+1) vector adapted to the Brownian filtration. Given the dynamics assumed and using  $\xi(\cdot)$ , we have

$$\frac{I_t}{I_s} = \xi \left( \int_s^t \sigma_{I,u} dW_u^{(I,1)} \right) \exp((r + \mu_I)\tau)$$
(7.2)

where  $\tau \equiv t - s$ . Moreover, we can re-write the change of measure<sup>5</sup>

$$\frac{dQ}{dP}(t) = \xi \left( -\int_{0}^{t} \gamma'_{u} dW_{u} \right)$$
(7.3)

<sup>5</sup>We define  $\gamma_u \equiv \left[\gamma_u^{(1,1)}, \gamma_u^{(1,2)}, ..., \gamma_u^{(I,1)}, \gamma_u^{(I,2)}\right]'$  and  $W_u \equiv \left[W_u^{(1,1)}, W_u^{(1,2)}, ..., W_u^{(I,1)}, W_u^{(I,2)}\right]'$ .

In our set-up, the absence of arbitrage implies the following equilibrium condition

$$E_s^P \left[ \frac{I_t}{I_s} \frac{dQ}{dP}(t) \\ \frac{dQ}{dP}(s) \\ \exp(-r\tau) \right] = 1 \iff E_s^P \left[ \frac{I_t}{I_0} \frac{dQ}{dP}(t) \exp(-rt) \right] = \frac{Is}{I_0} \frac{dQ}{dP}(s) \exp(-rs)$$

Therefore, no-arbitrage requires that the process  $\{M(t)\}_{t\geq 0} \equiv \{\frac{I_t}{I_0} \frac{dQ}{dP}(t) \exp(-rt)\}_{t\geq 0}$  is a *P*-martingale. Using the previous notation, we can write

$$M(t) = \frac{I_t}{I_0} \exp(-rt)\xi \left(-\int_0^t \gamma'_u dW_u\right)$$

Given that  $\xi(X_t)\xi(Y_t) = \xi(X_t + Y_t)\exp(\langle X, Y \rangle_t)$ , we decompose  $\frac{dQ}{dP}(t)$  as follows

$$\frac{dQ}{dP}(t) = \prod_{k \in N} \xi \left( -\int_{0}^{t} \gamma_{u}^{(k,1)} dW_{u}^{(k,1)} \right) \xi \left( -\int_{0}^{t} \gamma_{u}^{(k,2)} dW_{u}^{(k,2)} \right) \exp \left( -\rho_{k} \int_{0}^{t} \gamma_{u}^{(k,1)} \gamma_{u}^{(k,2)} du \right)$$

Define  $\frac{dQ}{dP}(t)^{\perp I_t}$  as the orthogonal part of the Radon-Nikodym derivative with respect to  $I_t$ 

$$\frac{dQ}{dP}(t)^{\perp I_t} \equiv \prod_{k \in N \setminus \{I\}} \xi \left( -\int_0^t \gamma_u^{(k,1)} dW_u^{(k,1)} \right) \xi \left( -\int_0^t \gamma_u^{(k,2)} dW_u^{(k,2)} \right) \exp\left( -\rho_k \int_0^t \gamma_u^{(k,1)} \gamma_u^{(k,2)} du \right)$$

Note that  $\frac{dQ}{dP}(t)^{\perp I_t}$  is itself a *P*-martingale. We can now write  $\frac{dQ}{dP}(t)$  as

$$\frac{dQ}{dP}(t) = \xi \left( -\int_{0}^{t} \gamma_{u}^{(I,1)} dW_{u}^{(I,1)} \right) \xi \left( -\int_{0}^{t} \gamma_{u}^{(I,2)} dW_{u}^{(I,2)} \right) \exp \left( -\rho_{I} \int_{0}^{t} \gamma_{u}^{(I,1)} \gamma_{u}^{(I,2)} du \right) \frac{dQ}{dP}(t)^{\perp I_{t}}$$

Using previous notation, M(t) can be re-written

$$M(t) = F(t)\frac{dQ}{dP}(t)^{\perp I_t}$$

where

$$F(t) \equiv \exp(\mu_I t) \xi \left(\int_0^t \sigma_{I,u} dW_u^{(I,1)}\right) \xi \left(-\int_0^t \gamma_u^{(I,1)} dW_u^{(I,1)}\right) \xi \left(-\int_0^t \gamma_u^{(I,2)} dW_u^{(I,2)}\right) \exp\left(-\rho_I \int_0^t \gamma_u^{(I,1)} \gamma_u^{(I,2)} du\right)$$

For M(t) to be a *P*-martingale, it must be that F(t) is itself a *P*-martingale. Given the above property of stochastic exponentials

$$\xi\left(\int_{0}^{t} \sigma_{I,u} dW_{u}^{(I,1)}\right) \xi\left(-\int_{0}^{t} \gamma_{u}^{(I,1)} dW_{u}^{(I,1)}\right) = \xi\left(\int_{0}^{t} \left(\sigma_{I,u} - \gamma_{u}^{(I,1)}\right) dW_{u}^{(I,1)}\right) \exp\left(-\int_{0}^{t} \left(\sigma_{I,u} \gamma_{u}^{(I,1)}\right) du\right)$$

and

$$\xi \left( \int_{0}^{t} \left( \sigma_{I,u} - \gamma_{u}^{(I,1)} \right) dW_{u}^{(I,1)} \right) \xi \left( -\int_{0}^{t} \gamma_{u}^{(I,2)} dW_{u}^{(I,2)} \right) = \xi \left( \int_{0}^{t} \left\{ \left( \sigma_{I,u} - \gamma_{u}^{(I,1)} \right) dW_{u}^{(I,1)} - \gamma_{u}^{(I,2)} dW_{u}^{(I,2)} \right\} \right) ..$$

$$\exp \left( -\rho_{I} \int_{0}^{t} \gamma_{u}^{(I,2)} \left( \sigma_{I,u} - \gamma_{u}^{(I,1)} \right) du \right)$$

As a result, F(t) can be expressed as

$$F(t) = \xi \left( \int_{0}^{t} \left\{ \left( \sigma_{I,u} - \gamma_{u}^{(I,1)} \right) dW_{u}^{(I,1)} - \gamma_{u}^{(I,2)} dW_{u}^{(I,2)} \right\} \right) \exp \left( \int_{0}^{t} \left\{ \mu_{I} - \sigma_{I,u} \left( \gamma_{u}^{(I,1)} + \rho_{I} \gamma_{u}^{(I,2)} \right) \right\} du \right)$$

Therefore, using the previous results, a sufficient condition for M(t) to be a P-martingale is that

$$\mu_I - \sigma_{I,t} \left( \gamma_t^{(I,1)} + \rho_I \gamma_t^{(I,2)} \right) = 0 \quad dP \otimes dt \quad a.s.$$

$$(7.4)$$

The economy is incomplete and there exist a multitude of combinations  $\{\gamma_t^{(I,1)}, \gamma_t^{(I,2)}\}_{t\geq 0}$  satisfying the previous restriction and the Novikov condition.<sup>6</sup> In order to obtain a closed-form solution for the characteristic function of  $\{\sigma_{I,t}\}_{t\geq 0}$ , the literature has imposed the market price of variance risk  $(W_t^{(I,2)})$  to be proportional to  $\sigma_{I,t}$ , so to that

$$\gamma_t^{(I,2)} + \rho_I \gamma_t^{(I,1)} = \lambda_I \sigma_{I,t} \tag{7.5}$$

The two last equations uniquely define  $\gamma_t^{(I,1)}$  and  $\gamma_t^{(I,2)}$ . A similar argument can be used for individual equity with the restriction:  $\gamma_t^{(j,2)} + \rho_j \gamma_t^{(j,1)} = 0$ . Combining results, the Girsanov theorem and the properties of Lévy processes imply that the vector  $\tilde{W}_t$  of Q-Brownian motions satisfy the following dynamic

$$d\tilde{W}_t = dW_t + d\left\langle W, W'\right\rangle_t \gamma_t \tag{7.6}$$

where the prices of systematic risk are

$$\gamma_t^{(I,1)} = \frac{\mu_I - \rho_I \lambda_I \sigma_{I,t}^2}{\sigma_{I,t} (1 - \rho_I^2)} \text{ and } \gamma_t^{(I,2)} = \frac{\lambda_I \sigma_{I,t}^2 - \rho_I \mu_I}{\sigma_{I,t} (1 - \rho_I^2)}$$

<sup>&</sup>lt;sup>6</sup>We further assume the (sufficient) Novikov criterion  $E^P\left[\exp\left(\frac{1}{2}\int_0^t \gamma'_u d\left\langle W, W'\right\rangle_u \gamma_u\right)\right] < \infty P - a.s.$  for all t implying that  $\frac{dQ}{dP}(t)$  is uniformly integrable which ensures the equivalence of the two laws:  $P \sim Q$ , see Protter (1990) Chapter 8.

and the prices of idiosyncratic risk are

$$\gamma_t^{(j,1)} = \frac{\alpha_j}{\sigma_{j,t}(1-\rho_j^2)} \quad \text{and} \quad \gamma_t^{(j,2)} = -\frac{\rho_j \alpha_j}{\sigma_{j,t}(1-\rho_j^2)}$$

**Step 2:** In the following, we present the index return and variance process risk-neutralization. The same argument applies in regards of individual equities. The index *P*-dynamic is

$$\frac{dI_t}{I_t} = (r + \mu_I)dt + \sigma_{I,t}dW_t^{(I,1)}$$

Making use of the expressions defining the Q-Brownian motions, we can re-write this SDE as

$$\frac{dI_t}{I_t} = (r + \mu_I)dt + \sigma_{I,t} \left( d\tilde{W}_t^{(I,1)} - \left( \gamma_t^{(I,1)} + \rho_I \gamma_t^{(I,2)} \right) \right)$$

where  $\gamma_t^{(I,1)}$  and  $\gamma_t^{(I,2)}$  are defined in step 1. As a result,

$$\Rightarrow \frac{dI_t}{I_t} = rdt + \left(\mu_I - \sigma_{I,t}\left(\gamma_t^{(I,1)} + \rho_I\gamma_t^{(I,2)}\right)\right)dt + \sigma_{I,t}d\tilde{W}_t^{(I,1)}$$

and since

$$\gamma_t^{(I,1)} + \rho_I \gamma_t^{(I,2)} = \frac{\mu_I}{\sigma_{I,t}}$$

we have

$$\frac{dI_t}{I_t} = rdt + \sigma_{I,t} d\tilde{W}_t^{(I,1)}$$

The *P*-dynamic for the systematic variance process is

$$d\sigma_{I,t}^2 = \kappa_I (\theta_I - \sigma_{I,t}^2) dt + \delta_I \sigma_{I,t} dW_t^{(I,2)}$$

By the Girsanov theorem  $d\tilde{W}_t^{(I,2)} = (\gamma_t^{(I,2)} + \rho_I \gamma_t^{(I,1)})dt + dW_t^{(I,2)}$  and therefore:

$$d\sigma_{I,t}^2 = \kappa_I (\theta_I - \sigma_{I,t}^2) dt + \delta_I \sigma_{I,t} \left( d\tilde{W}_t^{(I,2)} - \left( \gamma_t^{(I,2)} + \rho_I \gamma_t^{(I,1)} \right) \right)$$

or given

$$\gamma_t^{(I,2)} + \rho_I \gamma_t^{(I,1)} = \lambda_I \sigma_{I,t}$$

we get

$$d\sigma_{I,t}^2 = \tilde{\kappa}_I (\tilde{\theta}_I - \sigma_{I,t}^2) dt + \delta_I \sigma_{I,t} d\tilde{W}_t^{(I,2)}$$

where

$$\tilde{\kappa}_I = \kappa_I + \lambda_I \delta_I$$
 and  $\tilde{\theta}_I = \frac{\kappa_I \theta_I}{\tilde{\kappa}_I}$ 

### B. Proof of Proposition 2

In the following, we focus on the derivation of the equity call option price. Note that the model of Heston (1993) can be obtained by setting  $\beta_j = 0$ ,  $S_t^j = I_t$ , and all the idiosyncratic variance parameters equal to their market variance counterparts. For ease of notation, we define  $\mathcal{L}_{t,T}^k \equiv \int_t^T \sigma_{k,u}^2 du$  and  $W_{\mathcal{L}_{t,T}^k}^m \equiv \int_t^T \sigma_{k,u} d\tilde{W}_u^{(k,m)}$  for  $m \in \{1,2\}$  and  $k \in N$ . Given the Q-processes, one can apply Ito's lemma to obtain the following expression for the individual equity log-returns under Q

$$\ln\left(\frac{S_T^j}{S_t^j}\right) = r\left(T-t\right) - \frac{1}{2}\left(\mathcal{L}_{t,T}^j + \beta_j^2 \mathcal{L}_{t,T}^I\right) + W_{\mathcal{L}_{t,T}^j}^1 + \beta_j W_{\mathcal{L}_{t,T}^I}^1 \tag{7.7}$$

We begin by establishing the required notation.  $\tilde{\phi}_{t,T}^{LR}(.)$  represents the conditional characteristic function of the risk-neutral process followed by the log-returns  $\ln\left(\frac{S_T^j}{S_t^j}\right)$ . Moreover, we define  $\Lambda^S \equiv \{\tilde{\kappa}_I, \tilde{\theta}_I, \sigma_I, \rho_I, \kappa_j, \theta_j, \sigma_j, \rho_j, \beta_j, r, T-t\}$ , and

$$\tilde{\phi}_{t,T}^{LR}\left(\Lambda^{S}, u, \sigma_{I,t}^{2}, \sigma_{j,t}^{2}\right) \equiv E_{t}^{Q} \left[\exp\left(iu \ln\left(\frac{S_{T}^{j}}{S_{t}^{j}}\right)\right)\right]$$

Using (7.7), one may write

$$\tilde{\phi}_{t,T}^{LR}\left(\Lambda^{S}, u, \sigma_{I,t}^{2}, \sigma_{j,t}^{2}\right) = E_{t}^{Q}\left[\exp\left(iu\left(r(T-t) - \frac{1}{2}\left(\mathcal{L}_{t,T}^{j} + \beta_{j}^{2}\mathcal{L}_{t,T}^{I}\right) + W_{\mathcal{L}_{t,T}^{j}}^{1} + \beta_{j}W_{\mathcal{L}_{t,T}^{I}}^{1}\right)\right)\right]$$

making use of the stochastic exponential  $\xi(\cdot)$  as define in Appendix A, we get

$$\tilde{\phi}_{t,T}^{LR}\left(\Lambda^{S}, u, \sigma_{I,t}^{2}, \sigma_{j,t}^{2}\right) = \exp(iur(T-t))E_{t}^{Q}\left[\xi(iu\beta_{j}W_{\mathcal{L}_{t,T}^{I}}^{1})\exp\left(-g_{1}(u)\mathcal{L}_{t,T}^{I}\right)\xi\left(iuW_{\mathcal{L}_{t,T}^{j}}^{2}\right)\exp\left(-g_{2}(u)\mathcal{L}_{t,T}^{j}\right)\right]$$
$$= \exp(iur(T-t))E_{t}^{Q}\left[\xi(iu\beta_{i}W_{\mathcal{L}_{t,T}^{I}}^{1})\exp\left(-g_{1}(u)\mathcal{L}_{t,T}^{I}\right)\right]E_{t}^{Q}\left[\xi\left(iuW_{\mathcal{L}_{t,T}^{j}}^{1}\right)\exp\left(-g_{2}(u)\mathcal{L}_{t,T}^{j}\right)\right]$$

where  $g_1(u) = \frac{iu}{2}\beta_j^2(1-iu)$  and  $g_2(u) = \frac{iu}{2}(1-iu)$ . Following Carr and Wu (2004) or Detemple and Rindisbacher (2010), we define the following change of measure

$$C(t) \equiv E_t^Q \left[ \frac{dC}{dQ} \right] = \xi \left( i u \beta_j W_{\mathcal{L}_t^I}^1 \right) \xi \left( i u W_{\mathcal{L}_t^j}^1 \right)$$

where

$$\xi\left(iu\eta W_{\mathcal{L}_{t}^{k}}^{1}\right) = \exp\left(iu\eta W_{\mathcal{L}_{t}^{k}}^{1} - \frac{(iu\eta)^{2}}{2}\left\langle W_{\mathcal{L}^{k}}^{1}, W_{\mathcal{L}^{k}}^{1}\right\rangle_{t}\right) = \exp\left(iu\eta W_{\mathcal{L}_{t}^{k}}^{1} - \frac{1}{2}(iu\eta)^{2}\mathcal{L}_{t}^{k}\right)$$

for  $\eta \in \mathbb{R}$  and  $k \in \{I, j\}$ . Note that C(t) is a *Q*-martingale with respect to the filtration generated by  $\left\{ \left( W_{\mathcal{L}_{t}^{I}}^{1}, \mathcal{L}_{t}^{I} \right), \left( W_{\mathcal{L}_{t}^{j}}^{1}, \mathcal{L}_{t}^{j} \right) \right\}_{t \ge 0}$  and is defined on the complex plane. The *C*-measure allows us to write

$$\tilde{\phi}_{t,T}^{LR}\left(\Lambda^{S}, u, \sigma_{I,t}^{2}, \sigma_{j,t}^{2}\right) = \exp(iur(T-t))E_{t}^{Q}\left[\frac{C(T)}{C(t)}\xi\left(iu\beta_{i}W_{\mathcal{L}_{t,T}^{I}}^{1}\right)\exp\left(-g_{1}(u)\mathcal{L}_{t,T}^{I}\right)\right] \dots$$
$$E_{t}^{Q}\left[\frac{C(T)}{C(t)}\xi\left(iuW_{\mathcal{L}_{t,T}^{j}}^{1}\right)\exp\left(-g_{2}(u)\mathcal{L}_{t,T}^{j}\right)\right]$$

$$\Rightarrow \tilde{\phi}_{t,T}^{LR} \left( \Lambda^S, u, \sigma_{I,t}^2, \sigma_{j,t}^2 \right) = \exp(iur(T-t)) E_t^C \left[ \exp(-g_1(u)\mathcal{L}_{t,T}^I) \right] E_t^C \left[ \exp\left(-g_2(u)\mathcal{L}_{t,T}^j\right) \right]$$
(7.8)

Given an extension of the Girsanov-Meyer theorem to the complex plane, under the C-measure we have

$$dW_t^{C,(I,2)} = d\tilde{W}_t^{(I,2)} - (iu\rho_I\beta_j\sigma_{I,t})dt$$
$$dW_t^{C,(j,2)} = d\tilde{W}_t^{(j,2)} - (iu\rho_j\sigma_{j,t})dt$$

As a result, for all  $k \in N$ 

$$d\sigma_{k,t}^2 = \kappa_k^C (\theta_k^C - \sigma_{k,t}^2) dt + \delta_k \sigma_{k,t} dW_t^{C,(k,2)}$$
(7.9)

where

$$\kappa_I^C = \tilde{\kappa}_I - iu\rho_I\beta_j\delta_I, \quad \theta_I^C = \frac{\tilde{\kappa}_I\tilde{\theta}_I}{\kappa_I^C}, \quad \kappa_j^C = \kappa_j - iu\rho_j\delta_j, \text{ and } \theta_j^C = \frac{\kappa_j\theta_j}{\kappa_j^C}$$

We make use of the closed-form for the moment generating function of  $E_t^C[\exp(-g(u)\mathcal{L}_{t,T})]$  to obtain an expression for  $\tilde{\phi}_{t,T}^{LR}(\cdot)$ :

$$\tilde{\phi}_{t,T}^{LR}\left(\Lambda^{S}, u, \sigma_{I,t}^{2}, \sigma_{j,t}^{2}\right) = \exp\left(iur(T-t) - A^{I}(\Lambda^{S}, u) - B^{I}(\Lambda^{S}, u)\sigma_{I,t}^{2} - A^{j}(\Lambda^{S}, u) - B^{j}(\Lambda^{S}, u)\sigma_{j,t}^{2}\right)$$

$$\tag{7.10}$$

with

$$B^{I}(\Lambda^{S}, u) = \frac{2g_{1}(u)(1 - e^{-\Psi^{I}(\Lambda^{S}, u)(T-t)})}{2\Psi^{I}(\Lambda^{S}, u) - (\Psi^{I}(\Lambda^{S}, u) - \kappa_{I}^{C})(1 - e^{-\Psi^{I}(\Lambda^{S}, u)(T-t)})}$$
(7.11)

$$A^{I}(\Lambda, u) = \frac{\tilde{\kappa}_{I}\tilde{\theta}_{I}}{\delta_{I}^{2}} \left\{ 2\ln\left(1 - \frac{\left(\Psi^{I}(\Lambda^{S}, u) - \kappa_{I}^{C}\right)}{2\Psi^{I}(\Lambda^{S}, u)} \left(1 - e^{-\Psi^{I}(\Lambda^{S}, u)(T-t)}\right) + \left(\Psi^{I}(\Lambda^{S}, u) - \kappa_{I}^{C}\right)(T-t) \right\}$$
(7.12)

and

$$B^{j}(\Lambda^{S}, u) = \frac{2g_{2}(u)(1 - e^{-\Psi^{j}(\Lambda^{S}, u)(T-t)})}{2\Psi^{j}(\Lambda^{S}, u) - (\Psi^{j}(\Lambda^{S}, u) - \kappa_{j}^{C})(1 - e^{-\Psi^{j}(\Lambda^{S}, u)(T-t)})}$$
(7.13)

$$A^{j}(\Lambda^{S}, u) = \frac{\kappa_{j}\theta_{j}}{\delta_{j}^{2}} \left\{ 2\ln\left(1 - \frac{\left(\Psi^{j}(\Lambda^{S}, u) - \kappa_{j}^{C}\right)}{2\Psi^{j}(\Lambda^{S}, u)}\left(1 - e^{-\Psi^{j}(\Lambda^{S}, u)(T-t)}\right)\right) + \left(\Psi^{j}(\Lambda^{S}, u) - \kappa_{j}^{C}\right)(T-t) \right\}$$
(7.14)

and where

$$\Psi^{I}(\Lambda^{S}, u) = \sqrt{(\kappa_{I}^{C})^{2} + 2\delta_{I}^{2}g_{1}(u)} \text{ and } \Psi^{j}(\Lambda^{S}, u) = \sqrt{(\kappa_{j}^{C})^{2} + 2\delta_{j}^{2}g_{2}(u)}$$

with

$$g_1(u) = \frac{iu}{2}\beta_j^2(1-iu)$$
 and  $g_2(u) = \frac{iu}{2}(1-iu)$ 

Using the fact that  $\tilde{\phi}_{t,T}^{j}(u) = e^{iu\ln(S_{t}^{j})}\tilde{\phi}_{t,T}^{LR}(u)$ , the previous equations can be used to compute the price of a call written on  $S_{t}^{j}$ .

## C. Proof of Proposition 3

The following argument is derived under the P measure; however, a similar argument can be developed under the risk-neutral measure. Given the definition of skewness, the total (conditional) skewness of the integrated returns of firm "j" is

$$Sk^{P}\left(\int_{t}^{T}\frac{dS_{u}^{j}}{S_{u}^{j}}\right) = E_{t}^{P}\left[\left(\int_{t}^{T}\frac{dS_{u}^{j}}{S_{u}^{j}} - E_{t}^{P}\left[\int_{t}^{T}\frac{dS_{u}^{j}}{S_{u}^{j}}\right]\right)^{3}\right] / \left(E_{t}^{P}\left[\left(\int_{t}^{T}\frac{dS_{u}^{j}}{S_{u}^{j}} - E_{t}^{P}\left[\int_{t}^{T}\frac{dS_{u}^{j}}{S_{u}^{j}}\right]\right)^{2}\right]\right)^{3/2}$$
(7.15)

Given that

$$\int_t^T \frac{dS_u^j}{S_u^j} - E_t^P \left[ \int_t^T \frac{dS_u^j}{S_u^j} \right] = \int_t^T \beta_j \sigma_{I,s} dW_s^{(I,1)} + \int_t^T \sigma_{j,s} dW_s^{(j,1)}$$

equation (7.15) can be simplified to

$$Sk^{P}\left(\int_{t}^{T} \frac{dS_{u}^{j}}{S_{u}^{j}}\right) = \frac{E_{t}^{P}\left[\left(\int_{t}^{T} \beta_{j} \sigma_{I,s} dW_{s}^{(I,1)} + \int_{t}^{T} \sigma_{j,s} dW_{s}^{(j,1)}\right)^{3}\right]}{\left(E_{t}^{P}\left[\left(\int_{t}^{T} \beta_{j} \sigma_{I,s} dW_{s}^{(I,1)} + \int_{t}^{T} \sigma_{j,s} dW\right)^{2}\right]\right)^{3/2}}$$

By the properties of the Ito's integrals and the independence of  $W^{(I,1)}$  and  $W^{(j,1)}$ , we have

$$E_{t}^{P}\left[\left(\int_{t}^{T}\beta_{j}\sigma_{I,s}dW_{s}^{(I,1)}+\int_{t}^{T}\sigma_{j,s}dW\right)^{2}\right]=E_{t}^{P}\left[\beta_{j}^{2}\sigma_{I,t:T}^{2}\right]+E_{t}^{P}\left[\sigma_{j,t:T}^{2}\right]=E_{t}^{P}[V_{j,t:T}]$$

and

$$E_{t}^{P}\left[\left(\int_{t}^{T}\beta_{j}\sigma_{I,s}dW_{s}^{(I,1)} + \int_{t}^{T}\sigma_{j,s}dW_{s}^{(j,1)}\right)^{3}\right] = E_{t}^{P}\left[\left(\int_{t}^{T}\beta_{j}\sigma_{I,s}dW_{s}^{(I,1)}\right)^{3}\right] + E_{t}^{P}\left[\left(\int_{t}^{T}\sigma_{j,s}dW_{s}^{(j,1)}\right)^{3}\right]$$

Consequently, the total (conditional) skewness of the integrated returns of firm "j" takes the form

$$Sk^{P}\left(\int_{t}^{T} \frac{dS_{u}^{j}}{S_{u}^{j}}\right) = \frac{E_{t}^{P}\left[\left(\int_{t}^{T} \beta_{j} \sigma_{I,s} dW_{s}^{(I,1)}\right)^{3}\right]}{\left(E_{t}^{P}[V_{j,t:T}]\right)^{3/2}} + \frac{E_{t}^{P}\left[\left(\int_{t}^{T} \sigma_{j,s} dW_{s}^{(j,1)}\right)^{3}\right]}{\left(E_{t}^{P}[V_{j,t:T}]\right)^{3/2}} \\ = \frac{E_{t}^{P}\left[\left(\int_{t}^{T} \beta_{j} \sigma_{I,s} dW_{s}^{(I,1)}\right)^{3}\right]}{\left(E_{t}^{P}[\sigma_{I,t:T}^{2}]\right)^{3/2}} \cdot \left(\frac{E_{t}^{P}[\beta_{j}^{2}\sigma_{I,t:T}^{2}]}{E_{t}^{P}[V_{j,t:T}]}\right)^{3/2} \cdot sign(\beta_{j}) \\ + \frac{E_{t}^{P}\left[\left(\int_{t}^{T} \sigma_{j,s} dW_{s}^{(j,1)}\right)^{3}\right]}{\left(E_{t}^{P}[\sigma_{j,t:T}^{2}]\right)^{3/2}} \cdot \left(\frac{E_{t}^{P}[\sigma_{j,t:T}^{2}]}{E_{t}^{P}[V_{j,t:T}]}\right)^{3/2}$$

Defining  $A_{j,t:T}^P \equiv E_t^P[\beta_j^2 \sigma_{I,t:T}^2] / E_t^P[V_{j,t:T}]$  and given the definition of skewness, we obtain

$$Sk^{P}\left(\int_{t}^{T} \frac{dS_{u}^{j}}{S_{u}^{j}}\right) = Sk_{I}^{P} \cdot \left(A_{j,t:T}^{P}\right)^{3/2} + Sk_{j}^{P} \cdot \left(1 - A_{j,t:T}^{P}\right)^{3/2}$$
(7.16)

which completes the proof.

## D. Proof of Proposition 4

Within our model, the index price  $(I_T)$  takes the form

$$\frac{I_t}{I_0} = B(t)k(\sigma_{I,0:t}^2) \exp(X_{0,t})$$

where

$$B(t) \equiv \exp(rt), \ k(a) \equiv \exp\left(-\frac{a}{2}\right), \ \text{and} \ X_{0,t} \equiv \int_0^t \sigma_{I,u} dW_u^{(I,1)}$$

Taking the derivative of the index price  $I_t$  with respect to  $\beta_j X_{0,t}$  gives

$$\frac{\partial I_t}{\partial \beta_j X_{0,t}} = \frac{I_t}{\beta_j}$$

Moreover, within our model the equity price is given by

$$\frac{S_t^j}{S_0^j} = B(t)k(V_{j,0:t})\exp(\beta_j X_{0,t})\exp\left(\int_0^t \sigma_{j,u} dW_u^{(j,1)}\right)$$

Therefore,

$$\frac{\partial S_t^j}{\partial \beta_j X_{0,t}} = S_t^j$$

Note that chain rule implies

$$\frac{\partial S_t^j}{\partial I_t} = \frac{\partial S_t^j}{\partial \beta_j X_{0,t}} \frac{\partial \beta_j X_{0,t}}{\partial I_t}$$

We know from Malliavin calculus that  $I_t$  is a differentiable function with respect to  $\beta_j X_{0,t}$  (see Di Nunno, Øksendal, Proske, 2008) with a non-zero derivative. As a result, the inverse function theorem implies that

$$\frac{\partial \beta_j X_{0,t}}{\partial I_t} = \frac{1}{\frac{\partial I_t}{\partial \beta_j X_{0,t}}} \Rightarrow \frac{\partial \beta_j X_{0,t}}{\partial I_t} = \frac{\beta_j}{I_t}$$

Therefore,

$$\frac{\partial S_t^j}{\partial I_t} = \frac{S_t^j}{I_t} \beta_j \Rightarrow \frac{\partial f^j}{\partial I_t} = \frac{\partial f^j}{\partial S_t^j} \frac{\partial S_t^j}{\partial I_t} = \frac{\partial f^j}{\partial S_t^j} \frac{S_t^j}{I_t} \beta_j$$

For the option vega, we have

$$\frac{\partial f^{j}}{\partial \sigma_{I,t}^{2}} = \frac{\partial f^{j}}{\partial V_{j,t}} \frac{\partial V_{j,t}}{\partial \sigma_{I,t}^{2}} = \frac{\partial f^{j}}{\partial V_{j,t}} \frac{\partial (\beta_{j}^{2} \sigma_{I,t}^{2} + \sigma_{j,t}^{2})}{\partial \sigma_{I,t}^{2}} = \frac{\partial f^{j}}{\partial V_{j,t}} \beta_{j}^{2}$$

## E. Proof of Proposition 5

Ito's lemma implies that

$$df^{j} = f^{j}_{t}dt + \frac{1}{2}f^{j}_{SS}d\left\langle S^{j}, S^{j}\right\rangle_{t} + f^{j}_{SV_{j}}d\left\langle S^{j}, V_{j}\right\rangle_{t} \\ + \frac{1}{2}f^{j}_{V_{j}V_{j}}d\left\langle V_{j}, V_{j}\right\rangle_{t} + f^{j}_{S}d\tilde{S}^{j}_{t} + f^{j}_{V_{j}}dV_{j,t}$$

The Feynman-Kac formula gives

$$rf^{j} = f_{t}^{j} + f_{S}^{j} \frac{E_{t}^{Q} \left[ dS_{t}^{j} \right]}{dt} + f_{V_{j}}^{j} \frac{E_{t}^{Q} \left[ dV_{j,t} \right]}{dt} + \frac{1}{2} f_{SS}^{j} \frac{d \left\langle S^{j}, S^{j} \right\rangle_{t}}{dt} + f_{SV_{j}}^{j} \frac{d \left\langle S^{j}, V_{j} \right\rangle_{t}}{dt} + \frac{1}{2} f_{V_{j}V_{j}}^{j} \frac{d \left\langle V_{j}, V_{j} \right\rangle_{t}}{dt}$$

where

$$\begin{aligned} \frac{E_t^Q[dS_t^j]}{dt} &= rS_t^j \\ \frac{E_t^Q[dV_{j,t}]}{dt} &= \beta_j^2 \tilde{\kappa}_I (\tilde{\theta}_I - \sigma_{I,t}^2) + \tilde{\kappa}_j (\tilde{\theta}_j - \sigma_{j,t}^2) \\ \frac{d\langle S^j, S^j \rangle_t}{dt} &= \beta_j^2 \sigma_{I,t}^2 + \sigma_{j,t}^2 = V_{j,t} \\ \frac{d\langle S^j, V_j \rangle_t}{dt} &= \left(\beta_j^3 \rho_I \delta_I \sigma_{I,t}^2 + \rho_j \delta_j \sigma_{j,t}^2\right) S_t^j \\ \frac{d\langle V_j, V_j \rangle_t}{dt} &= \left(\beta_j^4 \delta_I^2 \sigma_{I,t}^2 + \delta_j^2 \sigma_{j,t}^2\right) \end{aligned}$$

Using the previous expressions, the dynamic of  $df^j$  can therefore be written

$$df^{j} = \left\{ rf^{j} - f^{j}_{S}rS_{t} - f^{j}_{V_{j}}\beta^{2}_{j}\tilde{\kappa}_{I}(\tilde{\theta}_{I} - \sigma^{2}_{I,t}) + \tilde{\kappa}_{j}(\tilde{\theta}_{j} - \sigma^{2}_{j,t}) \right\} dt$$
$$+ f^{j}_{S}dS^{j}_{t} + f^{j}_{V_{j}}dV_{j,t}$$

Moreover, under the objective probability measure (P), we have the following equalities

$$\frac{\frac{E_t^P[dS_t^j]}{dt}}{\frac{E_t^P[dV_{j,t}]}{dt}} = (r + \beta_j \mu_I) S_t^j$$

$$\frac{E_t^P[dV_{j,t}]}{dt} = \beta_j^2 \kappa_I \left(\theta_I - \sigma_{I,t}^2\right) + \kappa_j (\theta_j - \sigma_{j,t}^2)$$

Consequently,

$$\frac{1}{dt}E_t^P \left[\frac{df^j}{f^j} - rdt\right] = \frac{f_S^j}{f^j}E_t^P \left[dS_t^j - rS_t^jdt\right] + \frac{f_{V_j}^j}{f^j}\beta_j^2 E_t^P \left[d\sigma_{I,t}^2 - \tilde{\kappa}_I(\tilde{\theta}_I - \sigma_{I,t}^2)dt\right] + \frac{f_{V_j}^j}{f^j}E_t^P \left[d\sigma_{j,t}^2 - \tilde{\kappa}_j(\tilde{\theta}_j - \sigma_{j,t}^2)dt\right]$$

which simplifies to

$$\frac{1}{dt}E_t^P\left[\frac{df^j}{f^j} - rdt\right] = f_S^j \frac{S_t^j}{f^j} \beta_j \mu_I + f_{V_j}^j \frac{\beta_j^2}{f^j} (\tilde{\kappa}_I \tilde{\theta}_I - \kappa_I \theta_I) + f_{V_j}^j \frac{1}{f^j} (\tilde{\kappa}_j \tilde{\theta}_j - \kappa_j \theta_j)$$
(7.17)

In the previous expression, Q is the risk-neutral distribution defined by 3.5. Consistent with Appendix A, we risk-neutralize the market variance process such that  $\tilde{\kappa}_I \tilde{\theta}_I = \kappa_I \theta_I$  while the idiosyncratic risk is assumed to not be priced (i.e.  $\tilde{\theta}_j = \theta_j$  and  $\tilde{\kappa}_j = \kappa_j$ ). Consequently, we obtain

$$\frac{1}{dt}E_t^P\left[\frac{df^j}{f^E} - rdt\right] = f_S^j \frac{S_t^j}{f^j} \beta_j \mu_I = f_I^j \frac{I_t}{f^j} \mu_I$$

where the second equation makes use of the result in Proposition 4.

Note that part of the literature (see Broadie, Chernov, and Johannes, 2009), risk-neutralizes the variance process such that  $\tilde{\kappa}_I = \kappa_I$ . Given such a restriction, 7.17 would become

$$\frac{1}{dt}E^P\left[\frac{df^j}{f^E} - rdt\right] = f_S^j \frac{S_t^j}{f^j} \beta_j \mu_I + f_{V_j}^j \frac{\beta_j^2}{f^j} \kappa_I(\tilde{\theta}_I - \theta_I)$$

Note that the market variance risk-premium is equal to

$$E^{P}[d\sigma_{I,t}^{2}] - E^{Q}\left[d\sigma_{I,t}^{2}\right] = \lambda_{I}\delta_{I}\sigma_{I,t}^{2}dt$$

Therefore, we have

$$\Rightarrow \kappa_I(\tilde{\theta}_I - \theta_I) = \lambda_I \delta_I \sigma_{I,t}^2$$

Combining the previous results, we finally obtain

$$\frac{1}{dt}E^P\left[\frac{df^j}{f^j} - rdt\right] = f_S^j \frac{S_t^j}{f^j} \beta_j \mu_I + f_{V_j}^j \frac{\beta_j^2}{f^j} \lambda_I \delta_I \sigma_{I,t}^2.$$

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Figure 1: Short-Term At-the-money Implied Volatility. Six Equities and the S&P500 Index

Notes to Figure: We plot the time series of implied volatility for six equities (black) and S&P500 index (grey). On each day we use contracts with between 9 and 60 days to maturity and a moneyness (S/K) between 0.95 and 1.05. For every trading day and every security, we average the available implied volatilities to obtain an estimate of short-term at-the-money implied volatility.





Notes to Figure: The top panel plots implied volatility from short-term at-the-money (ATM) S&P500 index options. The bottom panel plots the first principal component of implied short-term ATM implied volatility from options on 29 equities in the Dow.



## Figure 3: Implied Volatility Moneyness Slopes: S&P500 Index and Equity First Principal Component

Notes to Figure: The top panel plots over time the slope of implied volatility with respect to moneyness from short-term S&P500 index options. The bottom panel plots the first principal component of the implied volatility moneyness slopes from options on 29 equities in the Dow.



## Figure 4: Implied Volatility Term Structure Slopes: S&P500 Index and Equity First Principal Component

Notes to Figure: The top panel plots the slope of the implied volatility term structure from S&P500 index options. The bottom panel plots the first principal component of the implied volatility term structure from options on 29 equities in the Dow.



Figure 5: Beta and Implied Volatility Across Moneyness. 3-month Equity Options

Notes to Figure: We plot implied Black-Scholes volatility from model prices. Each line has a different beta but the same amount of unconditional total equity variance  $V_j = \beta_j^2 \theta_I + \theta_j = 0.1$ . We set the current spot variance to  $\sigma_{I,t}^2 = 0.01$  and  $V_{j,t} = 0.05$ , and define the idiosyncratic variance as the residual  $\sigma_{j,t}^2 = V_{j,t} - \beta_j^2 \sigma_{I,t}^2$ . The market index parameters are  $\kappa_I = 5$ ,  $\theta_I = 0.04$ ,  $\delta_I = 0.5$ ,  $\rho_I = -0.8$ , and the individual equity parameters are  $\kappa_j = 1$ ,  $\delta_j = 0.4$ , and  $\rho_j = 0$ . The risk-free rate is 4% per year and option maturity is 3 months.





Notes to Figure: We plot implied Black-Scholes volatility from model prices. Each line has a different beta but the same amount of unconditional *total* equity variance  $V_j = \beta_j^2 \theta_I + \theta_j = 0.1$ . We set the current spot variance to  $\sigma_{I,t}^2 = 0.01$  and  $V_{j,t} = 0.05$ , and define the idiosyncratic variance as the residual  $\sigma_{j,t}^2 = V_{j,t} - \beta_j^2 \sigma_{I,t}^2$ . The market index parameters are  $\kappa_I = 5$ ,  $\theta_I = 0.04$ ,  $\delta_I = 0.5$ ,  $\rho_I = -0.8$ , and the individual equity parameters are  $\kappa_j = 1$ ,  $\delta_j = 0.4$ , and  $\rho_j = 0$ . The risk-free rate is 4% per year.



Figure 7.A: Average Market and Model Implied Volatility Smile

Notes to Figure: We plot the average (across time) implied volatility against moneyness for 15 firms. The solid black line denotes market IVs and the dashed grey denotes model IVs.



Figure 7.B: Average Market and Model Implied Volatility Smile (Continued)

Notes to Figure: We plot the average (across time) implied volatility against moneyness for 14 firms and the index. The solid black line denotes market IVs and the dashed grey denotes model IVs.



Figure 8.A: Market and Model Term Structures of At-The-Money Implied Volatility

Notes to Figure: The solid black line (left axis) shows the average market IV for days with downward sloping term structures and vice versa for the grey line (right axis). The dotted lines show the average model IVs. Moneyness (S/K) is between 0.95 and 1.05.



Figure 8.B: Market and Model Term Structures of At-The-Money Implied Volatility (Continued)

Notes to Figure: The solid black line (left axis) shows the average market IV for days with downward sloping term structures and vice versa for the grey line (right axis). The dotted lines show the average model IVs. Moneyness (S/K) is between 0.95 and 1.05.





Notes to Figure: We plot cross-sectional regressions of the average implied volatility (IV) levels from Figure 2 (top panel), average moneyness slopes from Figure 3 (middle panel), and average absolute term-structure slope from Figure 4 (bottom panel) against the estimated betas from Table 6.

<u>Company</u>	<u>Ticker</u>	Total Number of Quotes			Average	Average
		All	<u>Puts</u>	<u>Calls</u>	DTM	IV
S&P500 Index	SPX	393,429	199,756	193,673	91	20.51%
Alcoa	AA	60,937	30,683	30,254	116	36.42%
American Express	AXP	95,246	47,937	47,309	120	33.65%
Bank of America	BAC	85,129	43,159	41,970	128	32.46%
Boeing	BA	91,446	45,964	45,482	127	31.28%
Caterpillar	CAT	94,211	47,238	46,973	123	33.03%
JP Morgan	JPM	100,235	50,627	49,608	125	34.07%
Chevron	CVX	91,143	46,069	45,074	130	25.23%
Cisco	CSCO	65,032	32,737	32,295	123	40.78%
AT&T	Т	54,186	28,032	26,154	114	28.54%
Coca Cola	KO	84,738	43,114	41,624	130	24.19%
Disney	DIS	66,060	33,536	32,524	120	30.83%
Dupont	DD	81,191	41,216	39,975	121	28.26%
Exxon Mobil	XOM	82,362	41,681	40,681	125	24.54%
General Electric	GE	89,313	45,227	44,086	130	28.94%
Hewlett-Packard	HPQ	89,046	44,680	44,366	125	37.39%
Home Depot	HD	81,683	41,386	40,297	127	32.32%
Intel	INTC	75,533	38,081	37,452	123	37.76%
IBM	IBM	110,620	55,612	55,008	123	28.94%
Johnson & Johnson	JNJ	71,789	36,553	35,236	131	22.90%
McDonald's	MCD	80,828	40,899	39,929	126	27.46%
Merck	MRK	87,223	44,219	43,004	122	28.43%
Microsoft	MSFT	90,038	45,376	44,662	126	32.07%
3M	MMM	90,625	45,717	44,908	125	25.37%
Pfizer	PFE	79,480	40,450	39,030	128	29.37%
Procter & Gamble	PG	86,648	43,961	42,687	129	23.53%
Travellers	TRV	43,767	22,225	21,542	119	28.42%
United Technologies	UTX	86,063	43,213	42,850	126	27.70%
Verizon	VZ	67,948	34,995	32,953	118	27.43%
Walmart	WMT	88,431	44,624	43,807	130	27.87%
Average		81,757	41,352	40,405	124	29.97%

### **Table 1: Companies, Tickers and Option Contracts**

Note to Table: For each firm, we report the total number of options quotes, and the number of puts and calls quotes over the sample period 1996-2010. DTM refers to the average number of days-to-maturity in the option sample. Finally, IV denotes the average implied volatility in the sample.

	Out-of-the-money Call Options			<u>Ou</u>	Out-of-the-money Put Options			
<u>Ticker</u>	<u>Avg IV</u>	<u>max(IV)</u>	min(IV)	Avg Vega	<u>Avg IV</u>	<u>max(IV)</u>	min(IV)	<u>Avg Vega</u>
SPX	19.6%	82.6%	5.4%	172.00	21.5%	83.5%	5.1%	181.58
AA	35.1%	143.4%	12.4%	8.12	36.7%	142.6%	17.6%	8.07
AXP	34.1%	149.7%	9.3%	12.40	35.3%	143.3%	11.8%	12.38
BAC	29.8%	149.9%	5.1%	11.31	32.5%	149.8%	9.9%	11.44
BA	30.4%	92.3%	11.2%	12.69	31.9%	92.9%	14.8%	12.67
CAT	32.1%	105.7%	14.8%	12.56	34.4%	113.6%	17.1%	12.61
JPM	34.0%	149.4%	6.7%	11.06	36.0%	148.9%	11.6%	11.03
CVX	23.9%	98.0%	7.3%	15.68	27.2%	100.1%	11.6%	15.85
CSCO	41.3%	109.1%	16.1%	9.34	41.2%	111.4%	15.4%	9.19
Т	27.1%	100.4%	7.2%	7.05	30.5%	89.7%	9.6%	7.10
KO	22.9%	69.5%	5.2%	10.79	24.9%	70.5%	9.0%	10.85
DIS	30.3%	102.2%	6.7%	8.06	31.1%	105.1%	14.1%	7.94
DD	26.6%	92.2%	7.1%	9.82	29.7%	94.2%	12.6%	9.80
XOM	23.4%	89.1%	5.8%	13.26	25.8%	97.2%	8.2%	13.31
GE	28.8%	147.2%	6.1%	11.51	30.6%	145.1%	7.0%	11.46
HPQ	36.3%	112.6%	11.6%	11.28	37.1%	93.7%	13.9%	11.19
HD	30.9%	98.5%	8.7%	8.55	32.0%	106.4%	11.8%	8.45
INTC	38.6%	92.6%	10.4%	12.06	39.2%	90.6%	15.8%	11.91
IBM	29.2%	87.9%	7.5%	22.08	30.1%	87.5%	12.0%	22.06
JNJ	22.7%	71.3%	5.1%	13.93	24.6%	76.9%	8.7%	13.91
MCD	26.3%	90.5%	7.0%	9.44	28.0%	74.0%	11.0%	9.45
MRK	27.4%	84.7%	10.2%	12.48	29.8%	93.7%	11.7%	12.25
MSFT	33.1%	91.5%	8.3%	13.55	33.6%	93.8%	10.6%	13.32
MMM	24.6%	83.2%	7.4%	17.95	26.7%	84.0%	11.7%	18.04
PFE	29.5%	122.7%	7.5%	10.20	31.3%	74.9%	13.2%	10.01
PG	23.1%	72.2%	5.5%	15.44	24.8%	72.8%	9.2%	15.43
TRV	26.9%	144.5%	6.8%	9.42	29.3%	113.4%	13.1%	9.45
UTX	26.5%	90.3%	8.2%	15.59	28.5%	87.7%	12.3%	15.59
VZ	25.3%	90.9%	6.6%	8.94	29.0%	90.3%	10.6%	8.90
WMT	27.5%	70.6%	10.3%	10.39	28.5%	71.1%	10.5%	10.39
Average	29.2%	103.5%	8.4%	11.89	31.0%	100.5%	11.9%	11.86

#### Table 2: Summary Statistics on Implied Volatility. 1996-2010

Note: For each firm, we report the average, max, and min of implied volatility. We use Black-Scholes to compute implied volatility (IV) for index and equity OTM calls, and we use binomial trees with 200 steps for OTM equity puts. Option vega is computed using Black-Scholes.

Company	1st Component	2nd Component	3rd Component
Alcoa	15.94%	2.03%	-3.27%
American Express	18.64%	-11.47%	-38.33%
Bank of America	13.96%	-12.21%	17.16%
Boeing	16.82%	4.37%	3.29%
Caterpillar	11.05%	5.71%	-16.51%
JP Morgan	22.73%	11.71%	10.81%
Chevron	11.06%	-14.96%	10.02%
Cisco	10.52%	-10.99%	-19.08%
AT&T	19.83%	-17.64%	16.85%
Coca Cola	19.71%	5.32%	-37.52%
Disney	16.22%	-5.08%	-8.90%
Dupont	20.22%	-24.74%	-2.46%
Exxon Mobil	17.37%	-18.92%	14.54%
General Electric	12.74%	5.17%	-3.83%
Hewlett-Packard	15.97%	-18.33%	4.55%
Home Depot	28.45%	23.67%	6.34%
Intel	16.46%	13.14%	2.25%
IBM	11.63%	10.68%	-15.26%
Johnson & Johnson	13.50%	-2.26%	1.65%
McDonald's	12.68%	-4.94%	13.26%
Merck	27.54%	13.73%	-8.70%
Microsoft	20.09%	-4.49%	-15.39%
3M	13.34%	-10.25%	17.10%
Pfizer	16.60%	-10.72%	-31.93%
Procter & Gamble	18.17%	-29.66%	4.79%
Travellers	18.45%	-4.96%	-16.26%
United Technologies	32.51%	48.09%	44.50%
Verizon	21.91%	33.68%	-17.21%
Walmart	22.27%	-42.74%	26.29%
Average	17.81%	-2.31%	-1.42%
Min	10.52%	-42.74%	-38.33%
Max	32.51%	48.09%	44.50%
Variation Captured	77.18%	12.54%	2.60%
Correlation with S&P500 Short-			
Term Implied Volatility	90.99%	18.88%	-3.77%

# Table 3: Principal Component Analysis of Short-Term Implied Equity Volatility. Component Loadings and Properties

Note to Table: This table present the loading of each individual company short term IV on the first three principal components. The estimates are obtained by regressing each individual equity short-term ATM implied volatility proxy on the components obtained from the PCA. We also report the Average, Min and Max of component loadings across firms. Finally we report the total cross sectional variation captured by each of the first three components as well as their correlation with the S&P500 short-term IV.

<u>Company</u>	1st Component	2nd Component	3rd Component
Alcoa	16.65%	-5.25%	-0.48%
American Express	20.23%	0.97%	11.69%
Bank of America	14.37%	12.08%	-2.79%
Boeing	21.65%	-2.60%	-1.50%
Caterpillar	18.70%	4.03%	5.61%
JP Morgan	18.54%	35.83%	-0.38%
Chevron	17.35%	18.07%	-25.79%
Cisco	16.77%	-10.17%	15.16%
AT&T	16.62%	18.01%	-2.49%
Coca Cola	17.83%	-46.43%	43.74%
Disney	22.70%	-19.13%	0.27%
Dupont	14.95%	12.40%	-2.45%
Exxon Mobil	20.12%	10.96%	-6.05%
General Electric	16.10%	6.91%	-4.98%
Hewlett-Packard	20.76%	0.91%	-8.29%
Home Depot	22.74%	-5.49%	13.79%
Intel	21.38%	-10.87%	3.43%
IBM	21.02%	-7.33%	8.34%
Johnson & Johnson	22.19%	-3.73%	-10.99%
McDonald's	16.11%	11.05%	-28.18%
Merck	22.45%	-8.78%	5.36%
Microsoft	18.69%	-10.37%	2.72%
3M	24.19%	8.91%	-13.98%
Pfizer	19.54%	1.59%	-9.66%
Procter & Gamble	11.02%	-2.23%	28.74%
Travellers	14.28%	-0.75%	24.32%
United Technologies	20.37%	-4.21%	-15.17%
Verizon	13.44%	-10.80%	-37.09%
Walmart	4.98%	64.62%	48.35%
Average	18.13%	2.01%	1.42%
Min	4.98%	-46.43%	-37.09%
Max	24.19%	64.62%	48.35%
Variation Captured	49.37%	7.81%	5.18%
Correlation with S&P500			
Moneyness Slope	41.91%	14.43%	31.56%

# Table 4: Principal Component Analysis of Equity IV Moneyness Slope.Component Loadings and Properties

Note to Table: For the first three principal components of implied volatility (IV) moneyness slope we report the loadings of each firm as well as the average, min and max loading across firms. The time series of moneyness slopes are obtained by regressing equity IV on moneyness for each firm on each day. We also report the total cross sectional variation captured by each of the first three components as well as their correlation with the S&P500 moneyness slope.

<u>Company</u>	1st Component	2nd Component	3rd Component
Alcoa	19.39%	-5.13%	-2.11%
American Express	19.26%	-10.78%	-17.93%
Bank of America	13.02%	-5.91%	-7.60%
Boeing	15.52%	6.64%	-14.32%
Caterpillar	14.55%	0.15%	-23.64%
JP Morgan	16.71%	15.03%	23.64%
Chevron	11.86%	-4.88%	-16.01%
Cisco	12.23%	-4.90%	-17.69%
AT&T	21.86%	-17.35%	4.73%
Coca Cola	19.22%	6.33%	-25.05%
Disney	18.03%	3.47%	-17.29%
Dupont	20.00%	-13.30%	17.03%
Exxon Mobil	21.22%	-15.33%	6.49%
General Electric	16.04%	-1.51%	-14.69%
Hewlett-Packard	13.20%	-3.77%	-15.62%
Home Depot	25.71%	27.29%	6.95%
Intel	18.18%	11.32%	-12.41%
IBM	13.72%	1.89%	-25.94%
Johnson & Johnson	15.98%	-1.76%	-2.11%
McDonald's	12.18%	0.07%	-14.92%
Merck	24.57%	32.51%	15.59%
Microsoft	18.82%	-8.37%	-7.85%
3M	13.62%	-6.28%	-7.58%
Pfizer	15.62%	-3.26%	-15.54%
Procter & Gamble	23.30%	-41.14%	7.09%
Travellers	20.16%	1.09%	-2.64%
United Technologies	23.43%	49.06%	36.20%
Verizon	18.00%	30.19%	-2.95%
Walmart	28.55%	-40.68%	53.86%
Average	18.07%	0.02%	-3.18%
Min	11.86%	-41.14%	-25.94%
Max	28.55%	49.06%	53.86%
Variation Captured	56.61%	9.24%	5.20%
Correlation with S&P500			
Term Structure Slope	74.26%	16.27%	-5.81%

# Table 5: Principal Component Analysis of Equity IV Term Structure Slope. Component Loadings and Properties

Note to Table: For the first three principal components of implied volatility (IV) term structure slope we report the loadings of each firm as well as the average, min and max loading across firms. The time series of tem structure slopes are obtained by regressing equity IV on maturity for each firm on each day. We also report the total cross sectional variation captured by each of the first three components as well as their correlation with the S&P500 term structure slope.

						Average Total	Systematic
<u>Ticker</u>	<u>Kappa</u>	<u>Theta</u>	<u>Delta</u>	<u>Rho</u>	Beta	<u>Spot Volatility</u>	<u>Risk Ratio</u>
SPX	1.24	0.0542	0.366	-0.860		21.74%	
AA	0.98	0.0207	0.202	-0.659	1.17	41.06%	38.28%
AXP	0.73	0.0229	0.182	-0.801	1.23	39.16%	46.62%
BAC	0.76	0.0147	0.150	-0.775	1.06	40.76%	32.07%
BA	0.99	0.0508	0.317	-0.747	1.00	33.31%	42.37%
CAT	1.10	0.0329	0.269	-0.814	1.16	35.43%	50.84%
JPM	0.80	0.0184	0.172	-0.914	1.30	38.97%	52.27%
CVX	0.53	0.0357	0.195	-0.829	0.98	26.25%	66.13%
CSCO	0.97	0.0887	0.414	-0.943	1.13	44.95%	29.89%
Т	0.75	0.0350	0.229	-0.909	0.98	30.36%	48.87%
KO	1.00	0.0340	0.260	-0.824	0.76	25.61%	41.43%
DIS	1.04	0.0319	0.257	-0.482	1.06	33.48%	47.41%
DD	1.06	0.0341	0.268	-0.907	0.98	30.20%	49.32%
XOM	1.18	0.0180	0.206	-0.800	0.98	25.79%	68.31%
GE	0.62	0.0093	0.107	-0.843	1.16	32.78%	58.78%
HPQ	1.44	0.0660	0.436	-0.697	1.03	41.11%	29.91%
HD	1.01	0.0264	0.231	-0.808	1.16	35.40%	50.93%
INTC	1.16	0.0533	0.350	-0.738	1.13	41.60%	34.58%
IBM	1.46	0.0235	0.262	-0.691	0.95	32.15%	41.60%
JNJ	1.05	0.0356	0.273	-0.822	0.70	23.81%	41.39%
MCD	1.31	0.0518	0.369	-0.774	0.79	28.54%	36.31%
MRK	1.71	0.0443	0.390	-0.846	0.93	29.95%	45.50%
MSFT	1.02	0.0240	0.217	-0.978	1.08	35.03%	44.60%
MMM	1.53	0.0248	0.275	-0.798	0.90	26.98%	52.71%
PFE	0.88	0.0601	0.326	-0.848	0.89	30.41%	40.28%
PG	0.95	0.0341	0.255	-0.787	0.77	24.55%	46.58%
TRV	0.84	0.0331	0.236	-0.823	0.90	31.38%	39.28%
UTX	1.22	0.0351	0.293	-0.764	0.91	29.74%	44.08%
VZ	1.04	0.0497	0.322	-0.876	0.90	29.00%	45.40%
WMT	0.59	0.0550	0.254	-0.727	0.82	29.38%	36.75%
Average	1.02	0.0367	0.27	-0.80	0.99	32.66%	44.91%

#### **Table 6: Model Parameters and Properties. Index and Equity Options**

Note to Table: We use option data from 1996 to 2010 to estimate risk-neutral parameter values for the market index as well as the 29 individual equities. The individual equity parameters are estimated taking the market index parameter values as given. The last two columns report the average spot volatility through the sample and the proportion of total variance accounted for by the systematic market risk factor.

_		All Contracts	S	Calls	Puts	Short Term	Long Term
	Vega		IVRMSE /	OTM	OTM	ATM	ATM
Ticker	<u>RMSE</u>	<b>IVRMSE</b>	Average IV	<b>IVRMSE</b>	<b>IVRMSE</b>	<b>IVRMSE</b>	<b>IVRMSE</b>
SPX	1.90%	2.01%	9.79%	1.87%	2.14%	1.52%	1.91%
AA	1.82%	1.82%	5.01%	1.80%	1.85%	1.67%	1.70%
AXP	1.90%	1.91%	5.67%	1.86%	1.96%	1.59%	1.76%
BAC	2.39%	2.40%	7.40%	2.37%	2.43%	2.19%	1.95%
BA	1.65%	1.66%	5.30%	1.58%	1.73%	1.55%	1.47%
CAT	1.86%	1.87%	5.66%	1.82%	1.92%	1.78%	1.56%
JPM	2.36%	2.37%	6.95%	2.25%	2.49%	2.08%	2.01%
CVX	1.81%	1.81%	7.18%	1.82%	1.80%	1.85%	1.64%
CSCO	1.91%	1.91%	4.70%	1.93%	1.90%	1.75%	1.92%
Т	2.31%	2.33%	8.15%	2.31%	2.35%	2.29%	2.09%
KO	1.41%	1.41%	5.83%	1.38%	1.44%	1.37%	1.27%
DIS	1.57%	1.57%	5.11%	1.46%	1.69%	1.43%	1.42%
DD	1.90%	1.91%	6.76%	1.80%	2.02%	1.84%	1.73%
XOM	1.55%	1.55%	6.33%	1.59%	1.51%	1.49%	1.54%
GE	2.02%	2.03%	7.01%	1.92%	2.14%	1.96%	1.51%
HPQ	1.66%	1.66%	4.45%	1.64%	1.69%	1.62%	1.47%
HD	1.69%	1.69%	5.24%	1.65%	1.74%	1.59%	1.53%
INTC	1.56%	1.56%	4.14%	1.56%	1.57%	1.47%	1.49%
IBM	1.63%	1.63%	5.63%	1.58%	1.69%	1.45%	1.57%
JNJ	1.48%	1.49%	6.52%	1.46%	1.53%	1.35%	1.21%
MCD	1.64%	1.65%	6.01%	1.62%	1.68%	1.64%	1.30%
MRK	1.70%	1.70%	5.99%	1.67%	1.74%	1.75%	1.54%
MSFT	1.73%	1.72%	5.38%	1.69%	1.76%	1.59%	1.75%
MMM	1.54%	1.54%	6.09%	1.49%	1.60%	1.41%	1.39%
PFE	1.60%	1.60%	5.46%	1.59%	1.62%	1.52%	1.36%
PG	1.60%	1.60%	6.82%	1.54%	1.67%	1.38%	1.39%
TRV	1.96%	1.96%	6.91%	1.96%	1.97%	1.76%	1.93%
UTX	1.57%	1.57%	5.69%	1.54%	1.62%	1.54%	1.51%
VZ	2.05%	2.07%	7.55%	2.03%	2.11%	2.05%	1.87%
WMT	1.45%	1.45%	5.22%	1.42%	1.49%	1.38%	1.16%
Average	1.77%	1.78%	6.00%	1.74%	1.82%	1.67%	1.59%

#### **Table 7: Model Fit. Index and Equity Options**

Note to Table: For the S&P500 index and for each firm we compute the implied volatility root mean squared error (IVRMSE) along with the vega-based approximation used in estimation and IVRMSE divided by the average market IV from Table 1. We also report IVRMSE for out-of-the-money (OTM) call and put options separately. Finally, we report IVRMSE for at-the-money (ATM) short term and long term options. At the money is defined by 0.975<S/K<1.025 and short (and long) term are defined as less than (more than) six months to maturity.