Independent and Affiliated Analysts: Disciplining and Herding*

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Abstract

The paper investigates strategic interactions between an independent analyst and an affiliated analyst when the analysts' information acquisition and the timing of their recommendations are endogenous. Compared to the independent analyst, the affiliated analyst has superior information but faces a conflict of interest. I show that the independent analyst's recommendation, albeit endogenously less informative than the affiliated analyst's, disciplines the affiliated analyst's biased forecasting behavior. Meanwhile, the independent analyst sometimes herds with the affiliated analyst in order to improve forecast accuracy. Contrary to conventional wisdom, I show that herding with the affiliated analyst may actually motivate the independent analyst to acquire more information upfront, reinforce his ability to discipline the affiliated analyst, and benefit investors.

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1 Introduction

This paper investigates strategic interactions between an independent analyst and an affiliated analyst when the analysts' information acquisition and the timing of their recommendations are endogenous. In the paper, I differentiate affiliated analysts from independent analysts by two features: (a) the affiliated analyst faces a conflict of interest and (b) he has superior information compared to the independent analyst.

These two features have been widely noted by regulatory bodies, practitioners, and researchers. For example, the Global Analyst Research Settlement (Global Settlement) between the United States' regulators and the nation's top investment firms directly addresses conflicts of interest between research and investment banking businesses. An example of superior information affiliated analysts receive is confidential, non-public information they obtain in the due diligence process as an underwriter in an Initial Public Offering (IPO). The inappropriate release of such confidential information in a restricted period prior to Facebook's IPO was the focus of the Commonwealth of Massachusetts' case against Citigroup.[1] Empirically, Lin and McNichols (1998), Barber et al. (2007), and Mola and Guidolin (2009) document evidence suggesting that affiliated analysts face conflicts of interest when issuing stock recommendations, while Jacob et al. (2008) and Chen and Martin (2011) document evidence suggesting that analysts receive superior information because of their affiliations with the company.

On one hand, researchers argue that independent analysts' incentives are more aligned with investors and find the existence of independent analysts disciplines affiliated analysts' biased forecasting behavior (e.g., Gu and Xue, 2008). Consistent with the disciplining argument, the Global Settlement requires investment banks to acquire and distribute three independent research reports along with their own reports for every company they cover. On the other hand, since independent analysts' information is inferior, it is reasonable to suspect they have incentives to herd with affiliated analysts, given the well-documented herding behavior among financial analysts (e.g., Welch, 2000; Hirshleifer and Teoh, 2003).

If independent analysts herd with affiliated analysts, to what extent is their disciplining

[1] http://www.sec.state.ma.us/sct/current/sctciti/Citi_Consent.pdf

role compromised? Casual intuition suggests that herding would jeopardize the ability to discipline, which is consistent with the prevailing view in academic research that analysts' herding behavior discourages information production and is undesirable from the investor's perspective. In the Abstract of *Herding Behavior among Financial Analysts: A Literature Review*, Van Campenhout and Verhestraeten (2010) write:

> Analysts' forecasts are often used as an information source by other investors, and therefore deviations from optimal forecasts are troublesome. Herding, which refers to imitation behavior as a consequence of individual considerations, can lead to such suboptimal forecasts and is therefore widely studied.

Contrary to conventional wisdom, this paper shows that the independent analyst's disciplining role and herding behavior may reinforce each other. I show that if the independent analyst's informational disadvantage is large, herding with the affiliated analyst actually motivates the independent analyst to acquire more information upfront, reinforces his disciplining role, and ultimately benefits the investor.

The model has three players: an affiliated analyst, an independent analyst, and an investor. Each analyst acquires a private signal about an underlying, risky asset (the firm) and publicly issues a stock recommendation at a time that is strategically chosen. When choosing the timing of their recommendations, both analysts face a trade-off between the accuracy and timeliness of their recommendations.[2] Compared to the independent analyst, the affiliated analyst is assumed to face a conflict of interest but has superior information. To model the affiliated analyst's conflict of interest, I assume he receives an additional reward (independent of the reward for timeliness and accuracy) if the investor is convinced to buy the stock. To model the independent analyst's informational disadvantage, I assume the signal he endogenously acquires is less precise than the affiliated analyst's signal due to exogenous higher information acquisition costs. The precision of the analysts' information is interpreted as a firm-wide choice (e.g., hiring a star analyst or devoting more resources to a specific industry) and is assumed

[2] See Schipper (1991) and Gul and Lundholm (1995) on the tradeoff between accuracy and timeliness of recommendations.

to be publicly observed.

Due to his conflict of interest, the affiliated analyst has an incentive to over-report a bad signal in order to induce the investor to buy the stock. The model shows that the independent analyst disciplines the affiliated analyst's biased forecasting behavior in equilibrium. Intuitively, since the independent analyst's recommendation provides information to the investor, the extent the affiliated analyst can misreport his signal without being ignored by the investor is bounded by the quality of the independent analyst's recommendation.

The independent analyst's herding behavior also arises in equilibrium. Since the analysts' recommendations can be either favorable or unfavorable, the only reason for the independent analyst to delay his recommendation is to herd with the affiliated analyst. In equilibrium, the affiliated analyst's unfavorable recommendation is more informative than his favorable recommendation, so the independent analyst's expected benefit from waiting is higher if his private signal is good. The endogenous benefit of waiting, together with an exogenous cost of waiting, leads to a *conditional herding* equilibrium under which the independent analyst reports a bad signal immediately but waits and herds with the affiliated analyst upon observing a good signal.

Surprisingly, conditional herding causes the independent analyst to acquire more information and play a greater disciplining role than if he were prohibited from herding. The reason is that herding introduces an indirect benefit to information acquisition. By acquiring better information and reporting a bad signal right away, the independent analyst motivates the affiliated analyst to truthfully reveal a bad signal more often – this is the disciplining role. The affiliated analyst's more accurate reporting means that the independent analyst who receives a good signal too will be more accurate, since he herds with the affiliated analyst. This indirect benefit of information acquisition derived from herding motivates the independent analyst to acquire better information upfront. That is, there is an induced complementarity between the independent analyst's ex-post herding and ex-ante information acquisition.

Empirical Implications

First, the model predicts a positive association between independent analysts' degree of herding[3] with affiliated analysts and the informativeness of affiliated analysts' recommendations for firms with high information acquisition costs. The predicted association is negative for firms with low information acquisition costs.

Second, the model predicts that the dispersion between affiliated and independent analysts' recommendations decreases over time. Moreover, the decrease of dispersion is driven by independent analysts' recommendations converging to affiliated analysts' recommendations but not vice versa. The prediction of a shrinking dispersion is consistent with O'Brien et al. (2005) and Bradshaw et al. (2006) who found that affiliated analysts' recommendations are more optimistic than independent analysts' only in the first several months surrounding public offerings, while there is no difference afterwards.[4]

Regulatory Implications

First, the model shows affiliated analysts can be disciplined by independent analysts even when the latter's recommendations are less informative and involve herding behavior. The result that the independent analyst herding with the affiliated analyst may actually benefit the investor is relevant in the light of the Jumpstart Our Business Startups Act (JOBS Act). The JOBS Act permits affiliated analysts to publish research reports with respect to an emerging growth company any time after its IPO.[5] The paper suggests that making it possible for independent analysts to herd with affiliated analysts right after the IPO may increase independent analysts' disciplining role and benefit investors.[6]

Second, the paper points out that regulations mitigating affiliated analysts' conflicts of interest such as the Global Settlement can hurt the investor in some cases. The reason is that such regulations may crowd out independent analysts' incentives to acquire information. The result offers a rationale for the evidence in Kadan et al. (2009) who found the overall

[3] Welch (2000) proposes a methodology for estimating the degree of herding.

[4] O'Brien et al. (2005) write "we choose public offerings as a starting point because the financing event allows us to distinguish affiliated from unaffiliated analysts."

[5] "Jumpstart Our Business Startups Act – Frequently Asked Questions About Research Analysts and Underwriters". http://www.sec.gov/divisions/marketreg/tmjobsact-researchanalystsfaq.htm

[6] Before the JOBS Act, affiliated analysts were restricted by the federal securities laws from issuing forward looking statements during the "quiet period," which extends from the time a company files a registration statement with the Securities and Exchange Commission until (for firms listing on a major market) 40 calendar days following an IPO's first day of public trading.

informativeness of recommendations has declined following the Global Settlement and related regulations.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 lays out the model, and Section 4 characterizes the equilibrium. Section 5 delivers the main point of the paper by illustrating how herding behavior motivates the independent analyst to acquire better information and enhances his disciplining role over the affiliated analyst in equilibrium. Section 6 develops more detailed empirical and regulatory implications. Section 7 discusses the robustness of the main results. Section 8 concludes the paper. Appendix A specifies results deferred from the main text, and Appendix B presents all proofs.

2 Related Literature

The paper is related to the herding literature. Banerjee (1992) and Bikhchandani, Hirshldifer and Welch (1992) are two seminal papers showing that agents may rationally ignore their own information and herd with their predecessor's action for statistical reasons. Scharfstein and Stein (1990) and Trueman (1994) develop models where herding is driven by the agents' reputation concern. Arya and Mittendorf (2005) show the manager may purposely disclose proprietary information in order to direct herding from outside information providers. While the classical herding literature assumes that agents act in an exogenous sequential order, the sequence of actions is endogenous in my model. Existing herding models show that herding behavior and the loss of information are inherently linked, while my paper finds a setting where herding behavior leads to more information acquisition ex-ante and more information being revealed ex-post.

The endogenous timing of analysts' actions was first studied by Gul and Lundholm (1995), who model the trade-off between the accuracy and the timeliness of a forecast. Guttman (2010) gives conditions under which the time of the two analysts' forecasts cluster together or separate apart. In Gul and Lundholm (1995) and Guttman (2010), the analysts have homogeneous incentives. By modeling two analysts with heterogeneous incentives, my paper captures some institutional differences between affiliated and independent analysts and generates results that cannot be derived from earlier work.

Prior research has studied information acquisition in settings with a single analyst. Fischer and Stocken (2010) study a cheap-talk model and draw the conclusion that the analyst's information acquisition depends on the precision of public information. While the public information is provided by a non-strategic party in Fischer and Stocken (2010), both information providers behave strategically in my model. Langberg and Sivaramakrishnan (2010) endogenizes the analyst's information acquisition in a voluntary disclosure model similar to Dye (1985) and show the analyst's feedback can induce less voluntary disclosure from the manager.[7] My model contributes to this literature by developing an induced complementarity between the independent analyst's ex-post herding and ex-ante information acquisition.

The independent analyst's disciplining role in my model shares features of the disciplinary role of accounting information. Among others, Dye (1983), Liang (2000), and Arya et al. (2004) show that accounting information disciplines other softer sources of information in principal-agent contracting settings. Like accounting information which is usually considered to be less informative and less timely than other information sources such as managers' voluntary disclosures, the independent analyst's recommendation in my model is also (endogenously) less informative and less timely than the affiliated analyst's recommendation.

Empirically, Gu and Xue (2008) document independent analysts' disciplining role: affiliated analysts' forecasts become more accurate and less biased when independent analysts are following the same firms than when they are not. They also document that independent analysts' forecasts are less accurate than affiliated analysts forecast ex-post. Both findings are consistent with predictions of my model. In addition, Gu and Xue (2008) argue their results suggest that independent analysts are better than affiliated analysts in representing ex-ante market expectations, which is in line with the model's assumption that the independent analyst's incentive is more aligned with the investor.

The model's assumption that analysts face a trade-off between accuracy and timeliness

[7] Taking the analyst's information acquisition as given, Arya and Mittendorf (2007) and Mittendorf and Zhang (2005) also model interactions between an analyst and a manager.

of their recommendations is motivated by empirical evidence. Schipper (1991) discusses the tradeoff between timeliness and accuracy of analysts' forecasts. Cooper et al. (2001) document that analysts forecasting earlier have greater impact on stock prices than following analysts, and Loh and Stulz (2011) find similar results in the context of analysts' recommendation revisions. Regarding the incentive to issue accurate forecasts, Mikhail et al. (1999), Hong and Kubik (2003), Jackson (2005), and Groysberg et al. (2011) document evidence that analysts are rewarded for issuing accurate forecasts through higher payments, promising future careers, better reputations, and/or less turnover.

3 Model Setup

The model considers an economy consisting of an underlying, risky asset (the firm) and three players: an affiliated analyst, an independent analyst, and an investor. Whether an analyst is affiliated or independent is commonly known, and I will specify their differences later. The value of the firm is modeled as a random state variable ω whose prior distribution is also commonly known. Each analyst acquires a private signal about the value of the firm and then publicly issues a stock recommendation at a time that is strategically chosen. After observing both analysts' recommendations, the investor updates her belief about the value of the firm and makes an investment decision.

3.1 Endogenous Private Information Acquisition

The value of the firm is modeled as a state variable $\omega \in \{H, L\}$ with the common prior belief that both states are equally likely. At $t = 0$, the beginning of the game, the independent analyst (indicated by the superscript I) acquires his private signal $y^I \in Y^I = \{g, b\}$ about the underlying state ω at cost $c(p)$, where $p \in [\frac{1}{2}, 1]$ is the precision of y^I and is defined as follows

$$p = \Pr(y^I = g|\omega = H) = \Pr(y^I = b|\omega = L) \tag{1}$$

The cost of information acquisition $c(p)$ increases in the precision p of the signal in a convex manner and is assumed to be

$$c(p) = e \times (p - \frac{1}{2})^2 \tag{2}$$

where e is a positive constant commonly known, and a greater e means acquiring information becomes more costly. The cost of not acquiring any information is zero, i.e., $c(p = \frac{1}{2}) = 0$.

At the same time, the affiliated analyst (indicated by the superscript A) is endowed with a private signal $y^A \in Y^A = \{g, b\}$ whose precision $p^A \in [\frac{1}{2}, 1]$ is defined analogously as in (1). Assuming the affiliated analyst costlessly receives his signal with a fixed precision p^A is a simplification not crucial to the model. It is enough to assume the cost of information acquisition is sufficiently lower for the affiliated analyst so that he acquires more precise information in equilibrium.[8]

Conditional on the realization of the state ω, the signals received by the two analysts are independent. That is,

$$\Pr(y^A, y^I|\omega) = \Pr(y^A|\omega)\Pr(y^I|\omega), \forall p \tag{3}$$

From each analyst's perspective, the conditional independence assumption says the other analyst's private signal is more likely to be the same as his own signal than to be different.

The paper assumes the precision (not the realization) of the analysts' signals, p^A and p, is observable. One can interpret the precision as the *firm-wide* research quality. In practice, it takes time and effort for the research firm to increase its information precision, such as setting up a larger research group for the industry, hiring a star analyst, or becoming part of the managers' network. These actions and investments have to be made up front and are, to a substantial extent, observable to the market.

[8] I obtain qualitatively similar results by assuming both analysts simultaneously acquire information, and the affiliated analyst's information acquisition cost is $c(p^A) = \frac{e}{2} \times (p^A - \frac{1}{2})^2$. I am unable to obtain closed-form solutions for one of the key cutoff conditions under this alternative setup.

3.2 Endogenous Timing of Public Recommendations

After observing their private signals at $t = 0$, both analysts simultaneously choose either to issue a stock recommendation immediately at $t = 1$ or to defer the recommendation to $t = 2$. While deferring a recommendation is costly (which will be made precise shortly), doing so may be worthwhile as recommendations issued at $t = 1$ (if any) are observable and provide additional information to the analyst who waits until $t = 2$ to issue his recommendation.

Since each analyst issues only one recommendation in the model, a specific analyst can issue a recommendation at $t = 2$ if and only if he was silent earlier at $t = 1$. To be concrete, denote r_t^I as the recommendation issued by the independent analyst at time $t \in \{1, 2\}$ and R_t^I as his action space at t. Then we have

$$r_1^I \in R_1^I = \{\widehat{H}, \widehat{L}, \emptyset\} \tag{4}$$

where $r_1^I = \emptyset$ means keeping silent at $t = 1$, and

$$r_2^I \in R_2^I = \begin{cases} \{\widehat{H}, \widehat{L}\} \text{ if } r_1^I = \emptyset \\ \emptyset \text{ if } r_1^I \in \{\widehat{H}, \widehat{L}\} \end{cases} \tag{5}$$

The affiliated analyst's action space R_1^A (and R_2^A) is defined analogously as R_1^I (and R_2^I).

The analyst's small message space $\{\widehat{H}, \widehat{L}\}$ is less restrictive than might be thought initially: Kadan et al. (2009) document that most leading investment banks adopted a three-tier recommendation system similar to (Buy, Hold, Sell) after the Global Settlement and related regulations were implemented in 2002.[9]

[9] A small message space is also assumed in most herding models (e.g., Scharfstein and Stein, 1990; Banerjee, 1992; Trueman 1994).

3.3 Analyst and Investor Payoffs

The independent analyst maximizes his payoff function U^I by choosing both what and when to recommend:

$$U^I = Accurate + \delta \times Timely - c(p) \tag{6}$$

where $Accurate$ and $Timely$ have values of either zero or one and $c(p)$ is the cost of information acquisition defined in (2). $Accurate = 1$ if his recommendation r^A is consistent with the realization of the state ω, and 0 otherwise. $Timely = 1$ if the independent analyst makes a *non-null* recommendation ($r_1^I \in \{\widehat{H}, \widehat{L}\}$) early at $t = 1$, and $Timely = 0$ if he defers his recommendation to $t = 2$. The positive constant δ is the reward for issuing a timely recommendation and can be equivalently understood as the cost of deferring a recommendation to $t = 2$.

U^I captures the analyst's trade-off between the timeliness and accuracy of his recommendation, first discussed by Schipper (1991) and supported by subsequent empirical findings (e.g., Cooper et al., 2001; Loh and Stulz, 2011; Hong and Kubik, 2003; Jackson, 2005).[10]

The affiliated analyst maximizes his payoff function U^A by choosing both what and when to recommend:

$$U^A = Accurate + \delta \times Timely + \alpha \times Buy \tag{7}$$

where $Accurate$ and $Timely$ are defined the same way as in the independent analyst's payoff (6). $Buy = 1$ if the investor eventually chooses to "Buy" after observing both recommendations, and 0 otherwise.

$\alpha \times Buy$ in U^A captures the affiliated analyst's *conflict of interest*, and the positive constant α measures the degree of the conflict of interest. Due to his conflict of interest, the affiliated analyst has an incentive to misreport his bad signal in order to induce the investor to buy. Among others, Dugar and Nathan (1995), Lin and McNichols (1998), Michaely and Womack (1999), and Mola and Guidolin (2009) document evidence suggesting affiliated analysts face

[10] The timeliness is also noted by practitioners. In an interview with the Wall Street Journal, an analyst said, "it is better to be first than to be out there saying something that looks like you're following everyone else." (*Small Time, in Big Demand*. The Wall Street Journal, June-05-2012.)

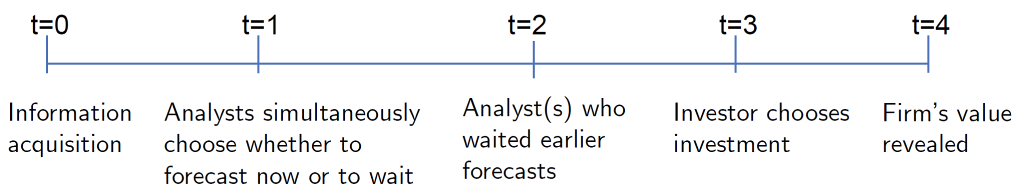
conflicts of interest and tend to issue optimistic recommendations.

The investor makes her investment decision $d \in \{Buy, NotBuy\}$ at $t = 3$ after observing both analysts' recommendations, including the timing of the recommendations. The investor's payoff U^{Inv} is determined by her investment decision as well as the realization of the value of the firm.[11]

$$U^{Inv} = \begin{cases} 1 \text{ if } d = Buy \text{ and } \omega = H \\ -1 \text{ if } d = Buy \text{ and } \omega = L \\ 0 \text{ if } d = NotBuy \end{cases} \tag{8}$$

Figure 1 summarizes the timeline of the game.

Figure 1: Timeline



3.4 Two Central Frictions: Incentives and Information

Central to the model are strategic interactions caused by two frictions: (a) the affiliated analyst's conflict of interest and (b) the independent analyst's informational disadvantage. These two frictions differentiate the affiliated analyst from the independent analyst in the model.

To introduce the independent analyst's informational disadvantage, it is helpful to analyze a benchmark case in which the independent analyst is the only analyst in the economy. In the benchmark case, the independent analyst forecasts at $t = 1$ and independently in the sense that $r^I = \widehat{H}$ if and only if $y^I = g$. Denoting p^* as the optimal precision chosen by the independent analyst in the benchmark case, then p^* solves the following non-strategic optimization problem

$$p^* = \underset{p \in [\frac{1}{2}, 1]}{\arg\max}\ p - e \times (p - \frac{1}{2})^2 \tag{9}$$

[11]The paper does not model the market microstructure, specifically the supply of the share and the endogenous pricing function. Instead, the paper focuses on the strategic interactions between the two analysts and the information production in equilibrium.

Solving the program, we obtain $p^* = \frac{1+e}{2e}$. To capture the independent analyst's informational disadvantage, I assume $p^* < p^A$, which is equivalent to the following assumption on the parameters of the model

$$e > \frac{1}{2p^A - 1} \tag{10}$$

As will be shown later, the assumption $e > \frac{1}{2p^A-1}$ is a sufficient condition under which the signal the independent analyst acquires is less precise than the affiliated analyst's signal in equilibrium. The assumption is supported by empirical evidence such as Jacob et al. (2008) who found affiliated analysts receive superior information compared to the information independent analysts receive.

The affiliated analyst's conflict of interest is captured by the term $\alpha \times Buy$ in his payoff function (7), and α measures the degree of the conflict of interest. To avoid trivial analyses, I assume the conflict of interest is neither too weak nor too strong, that is

$$2p^A - 1 + \delta = \underline{\alpha} \leq \alpha \leq \overline{\alpha} = \frac{2p^A - 1}{1 - p^A + p^*(2p^A - 1)} \tag{11}$$

If the affiliated analyst's conflict of interest is too weak ($\alpha < \underline{\alpha}$), he can perfectly reveal his private signal through his recommendation. If the affiliated analyst's conflict of interest is too strong ($\alpha > \overline{\alpha}$), he cannot credibly communicate his private signal at all. I characterize equilibria for $\alpha < \underline{\alpha}$ and $\alpha > \overline{\alpha}$ in Appendix A for completeness.

4 Equilibrium Analysis

This paper's equilibrium concept is Perfect Bayesian Equilibrium.[12] What makes the analysis challenging is the endogenous order of the analysts' actions as it complicates the possible history of the game and therefore players' strategies.[13] I present the analysis in two steps: I

[12] A profile of strategies and system of beliefs (σ, μ) is a Perfect Bayesian Equilibrium of the extensive form game with incomplete information if it satisfies two properties: (i) the strategy profie σ is sequentially rational given the belief μ and (ii) the belief μ is derived from strategy profile σ by Bayes Rule for any information set H such that $\Pr(H|\sigma) > 0$.

[13] For example, when issuing a recommendation early at $t = 1$, the analyst is not sure whether it will be observed by the other analyst when making recommendations.

first analyze a benchmark case (in Subsection 4.1) where only the independent analyst can choose the timing of his recommendation and then allow both analysts to choose the timing of their recommendations (in Subsection 4.2). The reason to analyze the benchmark case is twofold. First, it is the simplest setting in which the independent analyst's disciplining role and herding behavior arise endogenously, and therefore represents a simpler model in which key tensions of the game can be illustrated. Second, the equilibrium characterized in the benchmark case carries over to the more general game both qualitatively and quantitatively.

4.1 Endogenous Timing of Independent Analyst's Recommendation

For the moment, suppose the affiliated analyst issues his recommendation at $t = 1$ and focus on the independent analyst's strategy. The analysis also illustrates the steps used in solving the more general game in Subsection 4.2.

4.1.1 Properties simplifying the equilibrium analysis

Before solving the game using backward induction, I specify some properties (necessary conditions of the two analysts' strategies) of the equilibrium. These properties, which hold in the general game where both analysts can choose the timing of their recommendations, narrow the search for an equilibrium to a smaller family of strategies.

While the affiliated analyst can bias his recommendation in both directions, the following lemma tells us that focusing on over-reporting is without loss of generality.

Lemma 1 *The affiliated analyst never under-reports his good signal in equilibrium, i.e.,* $\Pr(r^A = \widehat{L}|y^A = g) = 0$.

Proof. All proofs are in Appendix B.

The following lemma narrows the search of the independent analyst's forecasting strategy in equilibrium.

Lemma 2 *If the independent analyst keeps silent at* $t = 1$ *in equilibrium, it must be that he herds with the affiliated analyst's recommendation* r_1^A *at* $t = 2$ *for any* $r_1^A \neq \emptyset$*.*

The lemma establishes a *perfect correlation* between waiting at $t = 1$ and herding behavior at $t = 2$ in equilibrium. The intuition is as follows: the independent analyst will not receive any informational gain from waiting (to observe r^A) unless his final recommendation is different from what he would have recommend if he did not wait, i.e., $r^I(y^I, r^A) \neq r^I(y^I)$. In the language of voting theory, information about the affiliated analyst's signal is valuable to the independent analyst only when it is *pivotal*.[14] Two conditions are necessary for the independent analyst who receives y^I to benefit from waiting to observe the affiliated analyst's recommendation r^A: r^A disagrees with his own signal y^I, and the independent analyst herds with r^A in the sense that $r_2^I = r^A$. Since waiting is costly, it must be accompanied by a subsequent herding in equilibrium. This intuition leads to the following proposition.

Proposition 1 *(Endogenous Benefit of Waiting) In equilibrium, the independent analyst's expected gain from waiting to observe r^A is at least weakly higher if he receives a good signal than if he receives a bad signal.*

The proposition opens the gate for endogenous timing of the independent analyst's recommendation: since the independent analyst's benefit of waiting depends on the realization of his private signal while the cost of waiting δ is exogenous, independent analysts observing different signals may choose to forecast at different times in equilibrium.

The intuition for Proposition 1 is as follows. We know from Lemma 2 that the independent analyst does not benefit from waiting unless he subsequently herds with the affiliated analyst's recommendation indicating a *difference* in the two analysts' signals. Therefore upon observing $y^I = b$ (or $y^I = g$), the independent analyst's informational gain from waiting can be measured by the informativeness of the affiliated analyst's favorable recommendation $\widehat{H}$ (or unfavorable recommendation $\widehat{L}$). Given his incentive to over-report the bad signal, the affiliated analyst's unfavorable recommendation is more informative than his favorable recommendation in equilibrium, which implies the independent analyst's informational gain from waiting is higher if

[14] The argument does not depend on the analyst's signal space being binary; it applies even if one introduces any continuous signal for the analysts. Instead, the analysts' small message space is critical to the argument. Herding would have not been in equilibrium if the analysts had a continuous message space.

he observes a good signal than a bad signal.[15]

4.1.2 Equilibrium

The game is solved by backward induction. Taking the independent analyst's precision choice $p \geq \frac{1}{2}$ at $t = 0$ as given, the following lemma characterizes the unique subgame equilibrium.

Lemma 3 *When only the independent analyst can choose the timing of his recommendation, the unique subgame equilibrium following a given $p \geq \frac{1}{2}$ is*

(i) ***Independent Forecasting Equilibrium*** *if $\delta \geq \frac{(p^A-p)(2p-1)}{p^A+p-1}$, in which the independent analyst forecasts independently at $t = 1$, or*

(ii) ***Conditional Herding Equilibrium*** *if $\delta < \frac{(p^A-p)(2p-1)}{p^A+p-1}$, in which the independent analyst upon observing a bad signal forecasts $\widehat{L}$ at $t = 1$, but upon observing a good signal waits and subsequently herds with the affiliated analyst's recommendation at $t = 2$.*

In both cases, the affiliated analyst over-reports his bad signal with probability $\beta = \frac{p^A-p}{p^A+p-1}$. The investor bases her investment decision on the affiliated analyst's recommendation unless $r^A = \widehat{H}$ but $r^I = \widehat{L}$, in which case she does not buy with probability $\frac{\alpha-(2p^A-1)}{\alpha(1-p^A-p+2p^Ap)}$.

The result is simple: given the initial precision choice p, the subgame equilibrium depends on the value of the exogenous cost of deferring recommendations to $t = 2$. If deferring his recommendation is extremely costly ($\delta \geq \frac{(p^A-p)(2p-1)}{p^A+p-1}$), the independent analyst forecasts early (and thus independently) regardless of the realization of his signal. If waiting becomes less expensive, the independent analyst waits and herds with the affiliated analyst's recommendation after observing a good signal, since the informational gain from waiting is higher in this case (Proposition 1).

It is worth noting that while Lemma 3 is derived as a mixed strategy equilibrium, the results do not hinge on the randomization of mixed strategies. I show in Section 7 that the main results of the paper are preserved in a richer game in which the equilibrium is in pure strategies.

[15] Rigorously, the probability that r^A disagrees with y^I is lower if $y^I = g$. However, as shown in the proof, the potential benefit of changing a recommendation upon disagreement more than offsets the lower probability of that disagreement.

The following proposition endogenizes the independent analyst's precision choice at $t = 0$ and specifies the overall equilibrium of the benchmark considered in this Subsection.

Proposition 2 *When only the independent analyst can choose the timing of his recommendation, the unique Perfect Bayesian Equilibrium is*

(i) Independent Forecasting Equilibrium if $\delta \geq \Pi$, in which the precision $p = p^*$.

(ii) Conditional Herding Equilibrium if $\delta < \Pi$, in which the precision $p = p^{ch}$.

The players' strategies in each equilibrium are specified in Lemma 3, $\Pi = \frac{4p^A p^{ch} - p^A - p^{ch}}{p^A + p^{ch} - 1} - \frac{e^2(1-2p^{ch})^2 + 2e + 1}{2e}$, $p^* = \frac{1+e}{2e}$, *and $p^{ch} \in (\frac{1}{2}, p^A)$ is the unique real root to the cubic function*[16]

$$2(p^A + p^{ch} - 1)^2(e - 2ep^{ch}) + (2p^A - 1)^2 = 0 \tag{12}$$

The condition on δ in Proposition 2 ensures that (a) the precision p specified in the proposition is ex-ante optimal when the independent analyst chooses it, and (b) the equilibrium is sequentially rational (thus satisfies the conditions in Lemma 3) for the specified p.

4.2 Endogenous Timing of Both Analysts' Recommendations

Allowing both analysts to choose the timing of their recommendations substantially increases the possible history of the game and therefore leads to a much larger strategy space for each player. However as shown in the lemma below, the equilibrium characterized in the benchmark (studied in Subsection 4.1) continues to be an equilibrium of the general game.

Lemma 4 *For $\delta \geq \Pi$ ($\delta < \Pi$), the Independent Forecasting Equilibrium (Conditional Herding Equilibrium) characterized in Proposition 2 is an equilibrium of the general game in which the timing of both analysts' recommendations is endogenous.*

The proof in Appendix B also shows that the equilibrium survives standard equilibrium refinements, particularly the Cho-Kreps's Intuitive Criterion and the (more demanding) Universal Divinity Criterion developed by Banks and Sobel.[17]

[16] The cubic function has a unique real root and two non-real complex conjugate roots.

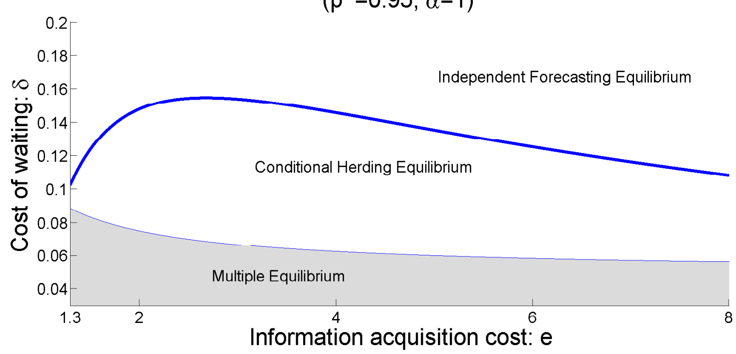
[17] I adopt the definition 11.6 in Fudenberg and Tirole (1991). Denote $D(t, T, m)$ as the set of the investor's mixed-strategy best responses to an out-of-equilibrium message m and beliefs concentrated on the support of

When the timing of both analysts' recommendations is endogenous, the possible history of the game increases and therefore the players' strategy spaces grow exponentially. To maintain tractability, I confine attention to equilibria where the affiliated analyst's waiting decision is in pure strategies. Equilibria with this property are summarized in the following lemma, and they are equilibria even if one allows for arbitrary mixed strategies.[18]

Lemma 5 *In addition to the equilibrium characterized in Lemma 4, another equilibrium emerges for small δ in which the affiliated analyst forecasts at $t = 2$ while the independent analyst forecasts at $t = 1$. Details of the additional equilibrium are specified in Appendix A.*

Figure 2 illustrates the lemma and shows the equilibrium (or equilibria) obtained for different values of the parameters. The shaded area in Figure 2 shows that both the Conditional Herding Equilibrium and the additional equilibrium characterized in Lemma 5 are equilibria of the game for small δ.

Figure 2: A numerical example
(p^A=0.95, α=1)



In the additional equilibrium characterized in Lemma 5, the affiliated analyst issues his recommendation later than the independent analyst. The independent analyst chooses his non-strategic precision p^* and forecasts early at $t = 1$ because he correctly conjectures that

the affiliated analyst's type space $T = \{g, b\}$ that makes a type-t affiliated analyst strictly prefer sending out m to his equilibrium message. Similarly denote $D^0(t, T, m)$ as the set of mixed best responses that makes type-t exactly indifferent. In my context, an equilibrium survives the Universal Divinity (or D2) criterion if and only if for *all* the out-of-equilibrium messages m, the equilibrium assigns zero probability to the type-message pair (t, m) if there exists another type t' such that $D(t, T, m) \cup D^0(t, T, m) \subset D(t', T, m)$.

[18] See Theorem 3.1 in Fudenberg and Tirole (1991): In a game of perfect recall, mixed strategies and behavior strategies (mixed strategies of extensive-form games) are equivalent. Then the claim is true by the definition of a Nash Equilibrium.

the affiliated analyst always forecasts at $t = 2$. The affiliated analyst issues $\widehat{H}$ if his own signal is good or the independent analyst issues $\widehat{H}$. If the affiliated analyst receives a bad signal and the independent analysts issues $\widehat{L}$, what the affiliated analyst issues depends on α: he issues $\widehat{L}$ if α is small, while he randomizes between $\widehat{L}$ and $\widehat{H}$ if α is large. The additional equilibrium fails the Universal Divinity Criterion for the small α case. In addition, while the paper assumes the cost of waiting δ is identical for both analysts, the affiliated analyst with more precise information may have a higher cost of waiting than the independent analyst, which would also rule out the additional equilibrium in which the affiliated analyst forecasts later than the independent analyst.[19] Throughout the remainder, I confine attention to the Conditional Herding Equilibrium and the Independent Forecasting Equilibrium (recall that one or the other of the two, but not both, exists for a given set of parameters).

The independent analyst's endogenous herding decisions in the Conditional Herding Equilibrium has a subtle effect on his ex-ante information acquisition p^{ch}. As will be shown in Section 5, herding with the affiliated analyst in equilibrium can motivate the independent analyst to acquire more information than he would acquire without herding ($p^{ch} > p^*$) and reinforce his ability to discipline the affiliated analyst's biased forecasting behavior.

5 Herding Reinforces Disciplining

This section delivers the main point of the paper. The independent analyst's disciplinary role over the affiliated analyst's forecasting strategy is important to understand the result and is formalized in the following lemma.

Lemma 6 *(The Independent Analyst's Disciplinary Role) In equilibrium, the affiliated analyst will over-report his bad signal less often if the independent analyst acquires better information.*

[19] Gul and Lundholm (1995) make a similar assumption and Zhang (1997) develops a model in which players with more precise signals choose to take actions earlier because of the information leakage. Nevertheless, the multiple equilibria problem can be seen as a limitation of this study and of signaling models in general. Using equilibrium refinements to narrow the set of equilibrium is itself controversial, because of the strong assumptions the refinements make. An alternative approach is to accept all equilibria as equally plausible. In my model, this would mean accepting that either the affiliated or the independent analyst might forecast first. Since the Conditional Herding Equilibrium is the one that best captures the notion of disciplining (which is the focus of the paper) and seems consistent with observed analyst behavior, I focus on that equilibrium.

Formally, we have $\frac{d}{dp}\beta < 0$ *in equilibrium, where* $\beta \doteq \Pr(r^A = \widehat{H} | y^A = b)$.

Intuitively, as the independent analyst acquires more information, the investor puts more weight on the independent analyst's recommendation when making her investment decision, which means relatively less weight is given to the affiliated analyst's recommendation. Less attention from the investor makes the affiliated analyst *endogenously* care more about being accurate since the only reason he may over-report a bad signal is to convince the investor to buy the stock. In other words, the *endogenous weight* the affiliated analyst puts on the accuracy of his recommendation increases if the independent analyst acquires better information upfront. The independent analyst's disciplining effect is consistent with Gu and Xue (2008) who find that the affiliated analysts' recommendations become more accurate and less biased when independent analysts are following the same firms than when they are not.

Lemma 6 shows that the effectiveness of the independent analyst's disciplining role depends on how much information he acquires ex-ante, while the ex-post herding *per se* is irrelevant. Therefore instead of asking how the independent analyst's herding behavior affects his disciplining role, the real question is how the herding behavior affects the independent analyst's ex-ante information acquisition (and thus the ability to discipline the affiliated analyst). The next Proposition shows that the independent analyst's ex-post herding behavior will motivate better information acquisition ex-ante (and therefore reinforces the disciplining role) if his informational disadvantage is large.

Proposition 3 *(Herding Motivates Better Information Acquisition) The independent analyst acquires more precise information in the Conditional Herding Equilibrium than in the Independent Forecasting Equilibrium if and only if his informational disadvantage is large. Formally,* $p^{ch} > p^* \Leftrightarrow e > \frac{1}{(\sqrt{2}-1)(2p^A-1)}$.

Why does the independent analyst spend more effort acquiring private information, knowing that he will discard that information ex-post half of the time (whenever $y^I = g$) and herds with the affiliated analyst? Analyzing the marginal benefit of information acquisition from the independent analyst's perspective provides the answer. In the Conditional Herding

Equilibrium, the marginal benefit is

$$\underbrace{\frac{1}{2}\times 1}_{\text{Direct benefit}} \quad + \quad \underbrace{\frac{1}{2}\{\beta(p) + \overbrace{(p^A - p)}^{\text{benefit of herding}} \overbrace{\frac{d}{dp}[-\beta(p)]}^{\text{discipline}}\}}_{\text{Indirect benefit associated with disciplining}} \tag{13}$$

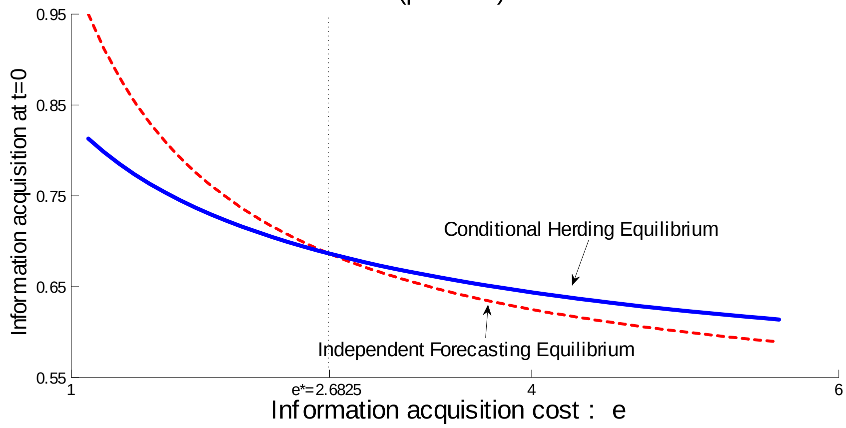
where $\beta(p)$ is the equilibrium probability that the affiliated analyst over-reports his bad signal.

The first term corresponds to the independent analyst observing a bad signal, in which case he will forecast $r^I = \widehat{L}$ immediately. In this case, acquiring better information mechanically increases the likelihood of his recommendation being accurate at a marginal rate of 1.

The second term corresponds to the independent analyst observing a good signal, in which case he will wait and herd with the affiliated analyst at $t = 2$. In this case, information acquisition has an indirect benefit. As the independent analyst acquires more information, the affiliated analyst faces more stringent discipline and his best response is to truthfully report an unfavorable recommendation $r^A = \widehat{L}$ more often (i.e., $\frac{d}{dp}[-\beta(p)] > 0$). The response by the affiliated analyst in turn implies that the independent analyst observing a good signal is more likely to enjoy a precision jump of $(p^A - p)$ by herding with the affiliated analyst's (more precise) unfavorable recommendation $\widehat{L}$. It is the very ex-post herding behavior that allows the independent analyst to benefit from the discipline effect he provides and motivates him to acquire more information than he would have acquired were he forced to forecast independently.

In the Independent Forecasting Equilibrium, the marginal benefit of information acquisition comes solely from the direct benefit, discussed in the first term of equation (13). Therefore, the independent analyst acquires more information in the Conditional Herding Equilibrium if and only if the indirect benefit via herding dominates the direct benefit. As the precision choice p decreases in the information acquisition cost e, the condition $e > \frac{1}{(\sqrt{2}-1)(2p^A-1)}$ in Proposition 3 simply puts a lower bound on the potential precision jump $p^A - p$, above which the indirect benefit outweighs the direct benefit. To illustrate Proposition 3, Figure 3 compares the independent analyst's information acquisition p^* and p^{ch} as a function of the information acquisition cost parameter e, in which $p^A = 0.95$.

Figure 3: Herding motivates better information acquisition if e>e*
(p^A=0.95)



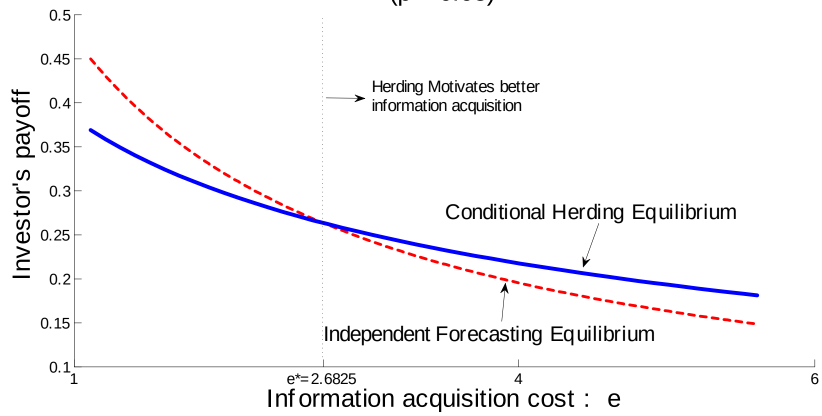
What is the effect of the independent analyst's herding behavior on the investor's payoff? The answer is not clear at this point: while the independent analyst may acquire better information in the Conditional Herding Equilibrium (Proposition 3), he sometimes discards that information and herds with the affiliated analyst who by assumption faces a conflict of interest. The following proposition summarizes the result.

Proposition 4 *(Herding Benefits the Investor) Forcing the independent analyst to forecast independently would make the investor weakly worse-off if and only if the independent analyst's informational disadvantage is large, i.e.,* $e > \frac{1}{(\sqrt{2}-1)(2p^A-1)}$.

The result confirms the idea that herding per se does not affect the independent analyst's disciplining role. Given the affiliated analyst's incentive to over-report a bad signal, the independent analyst's recommendation disciplines the affiliated analyst only when it is unfavorable ($r^I = \widehat{L}$). In equilibrium, the independent analyst reports his bad signal immediately while he herds only if his private signal is good, which does not compromise his ability to discipline the affiliated analyst. As shown in Proposition 3, if the independent analyst's informational disadvantage is large, his herding strategy motivates better information acquisition and, therefore, reinforces the disciplining benefit enjoyed by the investor.

Figure 4 compares the investor's utility in the Independent Forecasting Equilibrium (the dotted line) and the Conditional Herding Equilibrium as a function of e. Forcing the independent analyst to forecast independently implements the Independent Forecasting Equilibrium, however doing so weakly decreases the investor's payoff for $e > 2.6825$ as otherwise the equilibrium would be the Conditional Herding Equilibrium if the cost of waiting is not too large.

Figure 4: Herding benefits the investor if e>e*
(p^A=0.95)



6 Empirical and Regulatory Implications

From Lemma 6 and Proposition 4, we know that the affiliated analyst's recommendation will become more informative if the independent analyst acquires better information, which is in line with the disciplining story documented by Gu and Xue (2008). Since the model derives the necessary and sufficient condition for the independent analyst's herding behavior to motivate better information acquisition (Proposition 3), it generates predictions about the association between the independent analyst's herding behavior and the informativeness of the affiliated analyst's recommendations. Moreover, depending on the characteristics of the firm, the sign of the association is different.

Corollary 1 *The model predicts a positive association between independent analysts' degree of herding and the informativeness of affiliated analysts' recommendations for firms with high information acquisition costs, while the predicted association is negative for firms with low information acquisition costs.*

This is a sharp prediction that can be used to test my model. Welch (2000) proposes a methodology for estimating the degree of herding and proxies for other variables are common in the existing literature.

The following prediction is about the dynamics of the dispersion of the analysts' recommendations over time.

Corollary 2 *The model predicts that the dispersion between independent and affiliated analysts' recommendations decreases over time even if no new information is released.*

Traditional wisdom attributes the decrease in the dispersion of analysts' recommendations to the arrival of *new* information, which decreases the uncertainty analysts face and leads to similar opinions. The model offers an alternative and more endogenous explanation. Instead of relying on exogenous "*new*" information available from outside, the decrease of dispersion in my model is caused by how "*old*" information is gradually comprehended and used over time inside the analyst market.

The corollary explains O'Brien et al. (2005) and Bradshaw et al. (2006) who find that affiliated analysts' recommendations are more optimistic than independent analysts' recommendations only in the first several months surrounding IPOs and SEOs, while there is no difference between the two recommendations made later. According to the model, only the independent analysts who observe bad signals choose to issue recommendations early, which explains the affiliated analysts' optimism at the beginning. We do not expect any difference later on because independent analysts who choose to wait will herd with affiliated analysts' recommendations. In addition, since Proposition 4 shows the optimality of the independent analyst's herding behavior from the investor's point of view, my model suggests that the empirical evidence documented above may actually come from the equilibrium (conditional herding equilibrium) that is favorable to investors. To the best of my knowledge, this prediction has not been tested outside public offering settings.

The next corollary addresses a potential, undesirable consequence of regulations mitigating the affiliated analyst's conflict of interest.

Corollary 3 *Regulations mitigating the affiliated analyst's conflict of interest such as the Global Settlement do not necessarily benefit the investor.*

In the model, a smaller α captures the effect of regulations mitigating the affiliated analysts's conflict of interest. While it is easy to show $\frac{d}{d\alpha}U^{Inv} = 0$ in equilibrium (which is driven by the mixed strategies), the idea that lowering the affiliated analyst's bias does not necessarily benefit the investor is more general. In Section 7, I modify the base model so that only pure strategy equilibria exist and show that lowering the affiliated analyst's conflict of interest could *strictly decrease* the investor's payoff. The reason is that the independent analyst tends to put more trust in the affiliated analyst as the latter's conflict of interest becomes less severe. It could be that a smaller α completely wipes out the independent analyst's incentive to acquire information ex-ante and therefore the affiliated analyst faces no disciplining, in which case the investor is worse off.[20]

7 Robustness of Main Results

Due to simplifications made for tractability, the affiliated analyst and the investor play mixed strategies in the base model (see Lemma 3). I show in this section that the main results of the base model are preserved in a game where only pure strategy equilibria exist. To ease exposition, I restrict the affiliated analyst to issuing his recommendation at $t = 1$ (as in Subsection 4.1). The goal is to demonstrate that results of the paper are not driven by mixed strategies.

7.1 Modified setup

Three modifications are made to the base model. While the investor is assumed to be risk neutral with a binary action space $\{Buy, NotBuy\}$ in the base model, she is now assumed to be risk averse (CARA utility) and has a continuous action space. The investor is endowed with e amount of "dollars" which can be invested between a risk-free asset and a risky asset (the firm). The return of the risk-free asset is normalized to be zero and the return of the risky asset is $\omega \in \{H = 1, L = -1\}$ with the common prior belief that both states are equally likely.

[20] The reasoning for the second part of Corollary 3 is similar to Fischer and Stocken (2010) who find more precise public information may completely crowd out an analyst's information acquisition.

Both assets pay out at the end of the game, and the time value of money is ignored for clean notation. A portfolio consisting of A units of the risk-free asset and B units of the risky asset costs the investor $A + B * m$ dollars to form and will generate wealth w to the investor at the end of the game

$$w = A + B * m * (1 + \omega) \tag{14}$$

where m is the price of the risky asset when the portfolio is formed.[21] The investor maximizes the following utility function

$$U^{INV} = -e^{-\rho * w} \tag{15}$$

where $\rho > 0$ is the coefficient of absolute risk aversion. The model does not allow short selling of the risky asset and therefore $B \geq 0$.

The reward the affiliated analyst receives for inducing the investor's buy action is modified to be proportional to the units of the risky asset the investor buys. If the investor buys B units of the risky asset, the affiliated analyst's payoff function is

$$U^A = Accurate + \alpha \times B \tag{16}$$

which is a natural extension of $U^A = Accurate + \alpha \times Buy$ used in the base model as (16) incorporates the fact that the risk averse investor will buy different numbers of shares of the risky asset in response to different recommendations.

Finally, instead of having a binary support $\{b, g\}$ in the base model, the affiliated analyst's private signal y^A is now assumed to have a continuous support:

$$y^A = \omega + \tau \tag{17}$$

where $\omega \in \{H = 1, L = -1\}$ is the return of the risky asset and the noise term τ is normally

[21] The paper does not model the supply of the share and therefore m is taken as given.

distributed

$$\tau \sim N(0,1) \tag{18}$$

and the variance of τ is normalized to 1 without loss of generality.[22]

7.2 Equilibrium analysis

As the investor is risk-averse, her holdings of the firm vary continuously with her posterior assessment of the firm. Intuitively, the investor will hold more of the risky asset if her posterior assessment is more optimistic, which is verified by the following lemma.

Lemma 7 *In equilibrium, the investor buys B units of the risky asset at a given price m:*

$$B = \begin{cases} \frac{\log(\frac{q_H}{1-q_H})}{2*\rho*m} & \textit{if } q_H \geq \frac{1}{2} \\ 0 & \textit{otherwise} \end{cases} \tag{19}$$

and $\frac{dB}{dq_H} \geq 0$, where $q_H = \Pr(\omega = H|r^A, r^I)$ is derived using Bayesian Rule given the prior distribution of ω and both analysts' equilibrium strategies.

The following lemma states the properties of the independent analyst's strategy in equilibrium. As in the base model (see Proposition 1), the independent analyst observing a good signal is more likely to wait and herd with the affiliated analyst, which opens the gate for the endogenous timing of the independent analyst's recommendation.

Lemma 8 *In equilibrium, the independent analyst will herd with r^A if he keeps silent at $t = 1$, and the gain from waiting is higher if he observes a good signal than if he observes a bad signal.*

The following lemma shows that the affiliated analyst follows an intuitive switching strategy in equilibrium.

[22]The probability density function $\varphi(y^A|\omega)$ satisfies the monotonic likelihood ratio property (MLRP) in the sense that $\frac{\varphi(y^A|\omega=H)}{\varphi(y^A|\omega=L)}$ increases in y^A.

Lemma 9 *In equilibrium, the affiliated analyst's strategy is characterized by a unique cut-off point $s < 0$ such that he forecasts $\widehat{H}$ if and only if the realization of his signal is greater than s. Formally, $r^A = \widehat{H} \Leftrightarrow y^A > s$.*

With all players' equilibrium strategies in place, we are ready to present the equilibrium.

Proposition 5 *The modified game only has pure strategy equilibria, and the equilibrium takes one of the following forms*

(1) ***Independent Forecasting Equilibrium*** *where the independent analyst forecasts independently at $t = 1$.*

(2) ***Conditional Herding Equilibrium*** *where the independent analyst forecasts independently at $t = 1$ if and only if his signal is bad while otherwise he waits and herds with the affiliated analyst at $t = 2$.*

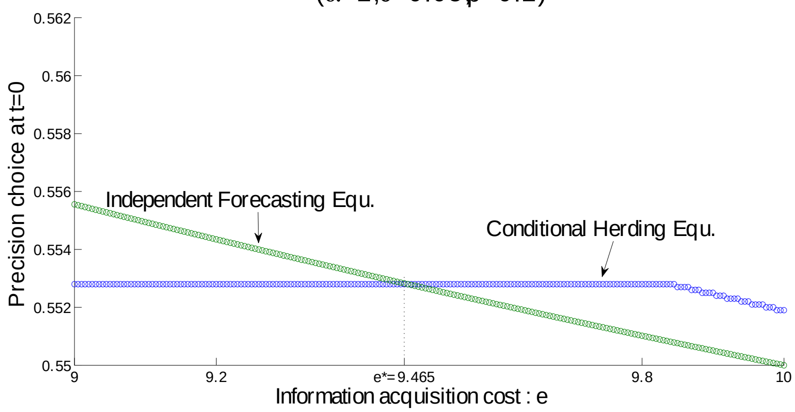
(3) ***No Information Acquisition Equilibrium*** *where the independent analyst does not acquire private information and always herds with the affiliated analyst's recommendation at $t = 2$.*

In any equilibrium, the investor's investment strategy is defined in Lemma 7 and the affiliated analyst follows a switching strategy described in Lemma 9.

As in the base model, the endogenous benefit of waiting leads to a conditional herding equilibrium, under which the independent analyst reports his bad signal immediately while he waits and herds with the affiliated analyst otherwise.

The main result of the base model, that herding with the affiliated analyst motivates the independent analyst to acquire more information (Proposition 3) and ultimately benefits the investor (Proposition 4), arises in the modified game as well. Figure 5 plots the precision chosen by the independent analyst in the Conditional Herding Equilibrium and the Independent Forecasting Equilibrium (characterized in Proposition 5) as a function of the information acquisition cost parameter e, in which $\alpha = 2$, $\delta = 0.05$, and $\rho = 0.2$. In this example, the unique equilibrium of the game is the Conditional Herding Equilibrium for all values of e. It is clear that the independent analyst acquires better information in the conditional herding equilibrium than in the Independent Forecasting Equilibrium if his informational disadvantage is large ($e > 9.465$ in Figure 5), which is consistent with Proposition 3. One can also check that the investor is strictly better-off in the Conditional Herding Equilibrium for $e > 9.465$, which is consistent with Proposition 4.

Figure 5: Herding motivates better information acquisition
(α=2,δ=0.05,ρ=0.2)



8 Concluding Remarks

The paper studies how an independent analyst interacts with an affiliated analyst. Inspired by features noted by practitioners and academic researchers, the paper assumes that, compared to the independent analyst, the affiliated analyst faces a conflict of interest but has superior information. Consistent with our intuition and empirical findings, the paper shows that the independent analyst both disciplines and herds with the affiliated analyst. On one hand, the independent analyst's incentive is more aligned with the investor and therefore he disciplines the affiliated analyst's biased forecasting behavior. On the other hand, the independent analyst sometimes defers his recommendation and herds with the affiliated analyst as the latter has more precise information.

While traditional wisdom suggests that disciplining and herding are in conflict with each other, I show that the independent analyst's disciplining role and herding behavior may actually be complements in equilibrium. In particular, if the independent analyst's informational disadvantage is large, herding with the affiliated analyst actually motivates the independent analyst to acquire more information upfront, reinforces his disciplining role, and ultimately benefits the investor. This point and other findings of the paper are intended to improve our understanding of independent analysts' role and offer a rationale for some empirical observations.

The main point that herding can motivate better information acquisition and reinforce disciplining seems likely to apply to settings other than affiliated and independent analysts. For example, mutual fund managers base their portfolio choices on both buy-side and sell-side analysts' forecasts. While sell-side analysts potentially face conflicts of interest such as trade-generating incentives, it has been documented that their forecasts are more precise than buy-side analysts (e.g., Chapman et al., 2008). The paper suggests that buy-side analysts may serve a disciplinary role. Moreover, in order to induce buy-side analysts to acquire more information, fund managers may purposely allow buy-side analysts to herd with sell-side analysts by passing along the latter's forecast to buy-side analysts.

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Appendix A

Equilibrium for $\alpha > \overline{\alpha}$ and $\alpha < \underline{\alpha}$

(i) For $\alpha > \overline{\alpha}$, the game has an equilibrium in which the affiliated analyst issues a fixed recommendation at $t = 1$, that is $r^A \equiv \widehat{L}$ or $r^A \equiv \widehat{H}$. The independent analyst chooses $p^* = \frac{1+e}{2e}$ and forecasts independently at $t = 1$, that is $r^I = \widehat{H} \Leftrightarrow y^A = g$. The investor bases her investment decision on r^I alone unless the affiliated analyst makes an out-of-equilibrium recommendation, in which case the investor does not buy.

(ii) For $\alpha < 2p^A - 1$, the game has an equilibrium in which the affiliated analyst truthfully reports his signal at $t = 1$, i.e., $r^A = \widehat{H} \Leftrightarrow y^A = g$. For $\delta \leq \alpha \leq \underline{\alpha} = 2p^A - 1 + \delta$, the game has an equilibrium in which the affiliated analyst perfectly signals his signal by the timing of his recommendation: he issues $\widehat{H}$ at $t = 2$ upon observing a good signal while otherwise he issues $\widehat{L}$ at $t = 1$. The independent analyst chooses precision $p^* = \frac{1+e}{2e}$ and forecasts independently at $t = 1$ if $\delta > p^A - (p^* - c(p^*))$ while otherwise he acquires no information and herds with r^A at $t = 2$. The investor bases her investment on r^A alone.

Details of the additional equilibria summarized in Lemma 5

For $\delta < \widehat{\delta}$, the affiliated analyst issues his recommendation at $t = 2$ while the independent analyst chooses precision $p^* = \frac{1+e}{2e}$ and forecasts independently at $t = 1$. Particularly,

(i) If $\alpha \leq \frac{p^A + p^* - 1}{1 - p^A + 2p^A p^* - p^*}$ and $\delta < \widehat{\delta}$, the affiliated analyst issues $\widehat{L}$ if and only if both $y^A = b$ and $r_1^I = \widehat{L}$, and the investor buys if and only if the affiliated analyst issues $\widehat{H}$ at $t = 2$.

(ii) If $\alpha > \frac{p^A + p^* - 1}{1 - p^A + 2p^A p^* - p^*}$ and $\delta < \widehat{\delta}$, the affiliated issues $r^A = \widehat{H}$ unless both his signal is bad and the independent analysts issues $\widehat{L}$, in which case the affiliated analyst issues $\widehat{H}$ with probability $\beta = \frac{p_H - p^*}{p_H + p^* - 1}$. The investor bases her investment decision on r^A unless $r^A = \widehat{H}$ but $r^I = \widehat{L}$, in which case she does not buy with probability $1 - \frac{1 - p^* - p_H}{\alpha(p_H + p^* - 1 - 2p_H p^*)}$.

In both cases, the investor will not buy if the affiliated analyst forecasts at $t = 1$ (the out-of-equilibrium path). Constants $\widehat{\delta} = p^* + p^*\alpha + p^A(\alpha - 2p^*\alpha - 1)$.

Appendix B

Notation: p^A (p) is the precision of the affiliated (independent) analyst's signal y^A (y^I); δ is the cost of deferring a recommendation, e is the information acquisition cost parameter, and α measures the affiliated analyst's degree of conflict of interest.

Proof of Lemma 1. Denote $\beta(p) \doteq \Pr(r^A = \widehat{H}|y^A = b, p)$ and $\gamma(p) \doteq \Pr(r^A = \widehat{L}|y^A = g, p)$, I will show that in equilibrium $\gamma(p) = 0$. The argument holds for all p and therefore I will write β and γ for simplicity. Denote $I_{\widehat{H}} = \Pr(\omega = H|r^A = \widehat{H})$ and $I_{\widehat{L}} = \Pr(\omega = L|r^A = \widehat{L})$ as the informativeness of $r^A = \widehat{H}$ and $r^A = \widehat{L}$ respectively. Notice $I_{\widehat{H}}, I_{\widehat{L}} \in [1 - p^A, p^A]$ are well-defined as $\alpha \leq \overline{\alpha}$ guarantees both $r^A = \widehat{L}$ and $r^A = \widehat{H}$ can be observed in equilibrium. It is an important observation that

$$(I_{\widehat{H}} - \frac{1}{2})(I_{\widehat{L}} - \frac{1}{2}) = \frac{(2p^A - 1)^2(\beta + \gamma - 1)^2}{4\,(1 - (\gamma - \beta)^2)} \geq 0$$

and that $I_{\widehat{H}} = \frac{1}{2} \Leftrightarrow I_{\widehat{L}} = \frac{1}{2} \Leftrightarrow \beta + \gamma = 1$.

First, I claim that $I_{\widehat{H}} = \frac{1}{2}$(thus $I_{\widehat{L}} = \frac{1}{2}$) cannot hold in equilibrium. Suppose the opposite is true, then both $r^A = \widehat{H}$ and $r^A = \widehat{H}$ are ignored by the investor, which means that the affiliated analyst is strictly better off by forecasting truthfully, i.e., $r^A = \widehat{H}$ if and only if $y^A = g$. However the truthful reporting strategy implies $I_{\widehat{H}} = I_{\widehat{L}} = p^A$ and contradicts $I_{\widehat{H}} = I_{\widehat{L}} = \frac{1}{2}$. Since $I_{\widehat{H}} = \frac{1}{2}$ cannot be part of an equilibrium, we are left with two possible scenarios: $I_{\widehat{H}}, I_{\widehat{L}} < \frac{1}{2}$ or $I_{\widehat{H}}, I_{\widehat{L}} > \frac{1}{2}$.

Next, I claim that $I_{\widehat{H}}, I_{\widehat{L}} < \frac{1}{2}$ cannot hold in equilibrium. Suppose by contradiction that in equilibrium $I_{\widehat{L}} < \frac{1}{2}$ and $I_{\widehat{H}} < \frac{1}{2}$. Given $y^A = b$, the affiliated analyst's payoff is $p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{L}\right]$ if he forecasts $r^A = \widehat{L}$, and is $1 - p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{H}\right]$ if $r^A = \widehat{H}$. The expectation operator $E[\cdot|y^A, r^A]$ is taken over y^I, taking the independent analyst's strategy and the investor's strategy as given. The independent analyst's payoff function (7) guarantees that in equilibrium his strategy must satisfy $I_{\widehat{H}} < \frac{1}{2} \Rightarrow \Pr(r^I = \widehat{H}|y^I; Time) \leq \Pr(r^I = \widehat{H}|y^I; Time) \forall y^I$. The $Time$ parameter reflects that whether the independent analyst's information set contains r^A depends on the *timing* of his recommendation (or waiting strategy), which is measurable only with respect to y^I and cannot depend on r^A. In addition, the investor's payoff function (8) guarantees that in equilibrium her strategy must be such that $I_{\widehat{H}} < \frac{1}{2} \Rightarrow \Pr(Buy|r^I, r^A = \widehat{H}) \leq \Pr(Buy|r^I, r^A = \widehat{L}) \forall r^I$. Given such properties of the independent analyst's strategy and the investor's strategy, we have $I_{\widehat{H}} < \frac{1}{2} \Rightarrow E\left[Buy|y^A = b, r^A = \widehat{H}\right] \leq E\left[Buy|y^A = b, r^A = \widehat{L}\right]$ in equilibrium, and

$$I_{\widehat{H}} < \frac{1}{2} \Rightarrow p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{L}\right] > 1 - p^A + \alpha * E\left[Buy|y^A = b, r^A = \widehat{H}\right].$$

Therefore $I_{\widehat{H}} < \frac{1}{2}$ implies that forecasting $\widehat{L}$ must be a dominant strategy in equilibrium for the affiliated analyst if $y^A = b$. This implies $\beta(p) = 0$ for $\forall p$ and thus $I_{\widehat{L}} \geq \frac{1}{2}$, a contradiction to the assumption that $I_{\widehat{L}} < \frac{1}{2}$.

Finally we are left with $I_{\widehat{H}}, I_{\widehat{L}} > \frac{1}{2}$ and I claim that $\gamma(p) = 0$ in equilibrium. Following the similar argument developed above, one can show $I_{\widehat{H}} > \frac{1}{2} \Rightarrow E\left[Buy|y^A = g, r^A = \widehat{H}\right] \geq E\left[Buy|y^A = g, r^A = \widehat{L}\right]$ and

$$I_{\widehat{H}} > \frac{1}{2} \Rightarrow p^A + \alpha * E\left[Buy|y^A = g, r^A = \widehat{H}\right] > 1 - p^A + \alpha * E\left[Buy|y^A = g, r^A = \widehat{L}\right].$$

Therefore, in any equilibrium with $I_{\widehat{H}} > \frac{1}{2}$, $r^A = \widehat{H}$ is the affiliated analyst's strict best response upon observing $y^A = g$, and this proves the claim $\gamma(p) = 0$. ■

Proof of Lemma 2. The lemma is surely true if the independent analyst's private signal y^I and the affiliated analyst's recommendation r^A imply the same recommendation, so what is left is the case when y^I and r^A imply different recommendations. Suppose by contradiction that the independent analyst will stick to y^I if r^A implies differently, which means after all he forecasts independently in the sense that $r^I = \widehat{L}$ if and only if $y^I = b$. But there is a profitable deviation for the the independent analyst by simply forecasting independently at $t = 1$ and avoiding the waiting cost δ, a contradiction. ■

Proof of Proposition 1.

Consider the case in which both $r^A = \widehat{H}$ and $r^A = \widehat{L}$ are on the equilibrium path. After observing $y^I \in \{g, b\}$ with precision p, the independent analyst will obtain expected utility $p + \delta - c(p)$ if he forecasts immediately. On the other hand, if he defers his recommendation to $t = 2$ in equilibrium, we know by Lemma 2 that he will herd with r^A at $t = 2$. Let $E\left[Gain|y^I\right]$ be the expected informational gain from deferring a recommendation given y^I, we have

$$\begin{aligned} E\left[Gain|y^I = b\right] &= p(1-\beta)p^A + (1-p)\left[p^A + (1-p^A)\beta\right] - (p+\delta) \\ E\left[Gain|y^I = g\right] &= p\left[p^A + (1-p^A)\beta\right] + (1-p)(1-\beta)p^A - (p+\delta). \end{aligned}$$

where $\beta = \Pr(r^A = \widehat{H}|y^A = b)$ is the probability that the affiliated analyst over-reports a bad signal, and we know from Lemma 1 that we can ignore the under-reporting strategy as long as both $r^A = \widehat{H}$ and $r^A = \widehat{L}$ can be observed in equilibrium. It is easy to check that

$$E\left[Gain|y^I = g\right] - E\left[Gain|y^I = b\right] = (2p-1)\beta \geq 0.$$

If only $r^A = \widehat{H}$ (or $r^A = \widehat{L}$) is reported on the equilibrium path, then r^A is uninformative and the gain from observing it is zero for the independent analyst regardless of his signal y^I.

■

Proof of Lemma 3. First, I show that, in equilibrium, the independent analyst will not defer his recommendation upon observing $y^I = b$. Suppose by contradiction this is not the case. Then the independent analyst will also defer his recommendation upon observing $y^I = g$. The result is the independent analyst will unconditionally defer his recommendation, and (by Lemma 2) herd with the affiliated analyst's recommendation r^A. Knowing this, the affiliated analyst will forecast $r^A = \widehat{H}$ for all y^A, which makes his recommendation completely uninformative and contradicts the assumption that the independent analyst chooses to herd in the first place.

Upon observing $y^I = g$, the independent analyst will either forecast $\widehat{H}$ if he chooses to forecast at $t = 1$, or herd with r^A at $t = 2$ if he chooses to defer (by Lemma 2).

Given the independent analyst's forecasting strategy and $p > \frac{1}{2}$, the affiliated analyst sets $\beta = \Pr(r^A = \widehat{H}|y^A = b) = \frac{p^A - p}{p^A + p - 1}$ so that the investor is indifferent from "Buy" and "Not Buy" upon observing $r^A = \widehat{H}$ but $r^I = \widehat{L}$. The investor, upon observing $r^A = \widehat{H}$ but $r^I = \widehat{L}$, chooses not to buy with probability $\rho = \frac{\alpha-(2p^A-1)}{\alpha(1-p^A-p+2p^A p)}$ so that the affiliated analyst is indifferent between reporting $\widehat{H}$ and $\widehat{L}$ upon observing a bad signal $y^A = b$. One can see that $0 \leq \rho \leq 1$ is guaranteed in the non-trivial region $\underline{\alpha} \leq \alpha \leq \overline{\alpha}$.

Given $y^I = g$, the independent analyst obtains an expected payoff $p+\delta-c(p)$ if he forecasts early, while his expected payoff from waiting (and herding with r^A) is $p\left[p^A + (1-p^A)\beta\right] + (1-p)(1-\beta)p^A - c(p)$. Substituting β from above, simple algebra shows that the independent analyst will forecast his good signal at $t = 1$ if and only if

$$\delta \geq \frac{(p^A - p)(2p-1)}{p^A + p - 1}.$$

Collecting conditions completes the proof. ■

Proof of Proposition 2. Denote p^* (p^{ch}) as the optimal precision the independent analyst chooses if the equilibrium of the overall game is the Independent Forecasting Equilibrium (and the Conditionally herd equilibrium). Then p^* satisfies

$$p^* = \underset{p\in[\frac{1}{2},1]}{\arg\max} p + \delta - e \times (p - \frac{1}{2})^2, \tag{20}$$

which gives $p^* = \frac{1+e}{2e}$. p^{ch} satisfies

$$p^{ch} \in \arg\max_p \frac{1}{2}(p+\delta) + \frac{1}{2}\left[p(p^A + (1-p^A)\beta) + (1-p)(1-\beta)p^A\right] - e(p-\frac{1}{2})^2.$$

p^{ch} is solved from the following f.o.c

$$e - 2e \times p^{ch} + \frac{(2p^A - 1)^2}{2(p^A + p^{ch} - 1)^2} = 0. \tag{21}$$

Since $p^A + p^{ch} - 1 > 0$, p^{ch} can be solved equivalently from the following cubic function

$$2(p^A + p^{ch} - 1)^2(e - 2e \times p^{ch}) + (2p^A - 1)^2 = 0.$$

This equation has a unique real root and two complex conjugate roots for any $p^A > \frac{1}{2}$ (Chapter 10 in Irving, 2003), and it is easy to check the second-order condition is $-2e - \frac{(2p^A-1)^2}{(p^A+p-1)^3} < 0$.

The independent analyst's expected payoff at $t = 0$, after plugging in p^* and p^{ch}, is denoted as U^I_{IF} in the Independent Forecasting Equilibrium and U^I_{CH} in the Conditional Herding Equilibrium. Simple algebra shows that

$$U^I_{CH} > U^I_{IF} \Leftrightarrow \delta < \Pi,$$

where

$$\Pi = \frac{4p^A p^{ch} - p^A - p^{ch}}{p^A + p^{ch} - 1} - \frac{e^2(1-2p^{ch})^2 + 2e + 1}{2e}.$$

Together with the self-fulfilling conditions characterized in Lemma 3, one can see the overall equilibrium is the Conditional Herding Equilibrium if δ is in the following set

$$\left\{\delta : \delta < \min\left(\Pi \ , \frac{(p^A - p^{ch})(2p^{ch}-1)}{p^A + p^{ch} - 1}\right)\right\} = \{\delta : \delta < \Pi\},$$

where the equality is by straightforward algebra $\frac{(p^A-p^{ch})(2p^{ch}-1)}{p^A+p^{ch}-1} - \Pi = \frac{(2ep^{ch}-1)^2}{2e} > 0$.

Analogously, the overall equilibrium is the Independent Forecasting Equilibrium if δ is in the following set

$$\left\{\delta : \delta > \max\left(\Pi \ , \frac{(p^A - p^*)(2p^*-1)}{p^A + p^* - 1}\right)\right\},$$

and the remainder of the proof is to show $\Pi \geq \frac{(p^A-p^*)(2p^*-1)}{p^A+p^*-1}$, which means $\max\left(\Pi \ , \frac{(p^A-p^*)(2p^*-1)}{p^A+p^*-1}\right) = \Pi$ and verifies the proposition.

Proving $\Pi \geq \frac{(p^A-p^*)(2p^*-1)}{p^A+p^*-1}$ follows the graphic investigation of three claims. Claim 1: Both Π and $\delta^* \doteq \frac{(p^A-p^*)(2p^*-1)}{p^A+p^*-1}$ strictly increase in e for $e < e^*$ while strictly decreases in e for $e > e^*$ (where $e^* = \frac{1}{(\sqrt{2}-1)(2p^A-1)}$). Claim 2: Π and δ^* achieve the *same* global maximum value at $e = e^*$, i.e., $\max \Pi = \Pi(e = e^*) = \delta^*(e = e^*) = \max \delta^*$. Claim 3: $\left|\frac{d\delta^*}{de}\right| > \left|\frac{d\Pi}{de}\right|$ holds for $e < e^*$ and $e > e^*$.

Proof of Claim 1: For the δ^* part, first notice that $\delta(p) = \frac{(p^A-p)(2p-1)}{p^A+p-1}$ is strictly concave in p and $\frac{d\delta}{dp} > 0$ if and only if $p < 1 - p^A + \frac{2p^A-1}{\sqrt{2}}$. As $p^* = \frac{1+e}{2e}$ and $\frac{dp^*}{de} < 0$, we know

$p^* < 1 - p^A + \frac{2p^A - 1}{\sqrt{2}}$ if and only if $e > e^* = \frac{1}{(\sqrt{2}-1)(2p^A-1)}$. Finally, since $\delta^* = \frac{(p^A - p^*)(2p^* - 1)}{p^A + p^* - 1} = \delta(p = p^*)$, the claim for δ^* is true by applying the Chain Rule $\frac{d\delta^*}{de} = \frac{d\delta^*}{dp^*}\frac{dp^*}{de}$ and the fact that $\frac{dp^*}{de} < 0$.

For the Π part, rewrite Π as $\Pi(e, p^{ch}(e))$ to emphasize that its second argument p^{ch} is also a function of e. Differentiating $\Pi(e, p^{ch}(e))$ with respect to e,

$$\frac{d}{de}\Pi(e, p^{ch}(e)) = \frac{\partial \Pi(e, p^{ch}(e))}{\partial e} + \frac{\partial \Pi(e, p^{ch}(e))}{\partial p^{ch}}\frac{dp^{ch}}{de}.$$

Notice that

$$\frac{\partial \Pi(e, p^{ch}(e))}{\partial p^{ch}} = 2(e - 2ep^{ch}) + \frac{(2p^A - 1)^2}{(p^A + p^{ch} - 1)^2} = 0.$$

The last equality comes from the fact that p^{ch} is the optimal precision chosen in the conditional herding equilibrium, and by (21) that p^{ch} satisfies

$$e - 2ep^{ch} + \frac{(2p^A - 1)^2}{2(p^A + p^{ch} - 1)^2} = 0.$$

Therefore, $\frac{d}{de}\Pi(e, p^{ch}(e))$ can be simplified as

$$\begin{aligned} \frac{d}{de}\Pi(e, p^{ch}(e)) &= \frac{\partial \Pi(e, p^{ch}(e))}{\partial e} \\ &= \frac{1}{2}\left(\frac{1}{e^2} - (2p^{ch} - 1)^2\right). \end{aligned}$$

Straightforward calculus shows

$$\frac{d\Pi}{de} > 0 \Leftrightarrow p^{ch} < \frac{1+e}{2e}.$$

We know from Proposition 3 that $p^{ch} < \frac{1+e}{2e} = p^*$ if and only if $e < e^*$. Monotonic transformation gives

$$\frac{d\Pi}{de} > 0 \Leftrightarrow e < e^*.$$

which verifies Claim 1.

Proof of Claim 2: As both Π and δ^* are continuous functions, the fact that they achieve their global maximum value at $e = e^*$ is a direct consequence of Claim 1. We know by Proposition 3 that $p^* = p^{ch} = 1 - p^A + \frac{2p^A-1}{\sqrt{2}}$ at $e = e^*$. Substituting p^* and p^{ch} verifies the claim.

Proof of Claim 3: We know from Claim 1 that $\frac{d\Pi}{de}$ can be simplified as $\frac{1}{2}(\frac{1}{e^2} - (2p^{ch}-1)^2)$, and algebra shows that $\frac{d\delta^*}{de} = \frac{1-e(2+e(2p^A-1)^2-4p^A)}{e^2(e-1-2ep^A)^2}$. Simple algebra shows

$$\begin{aligned} dif &= \frac{d\Pi}{de} - \frac{d\delta^*}{de} \\ &= -2p^{ch^2} + 2p^{ch} - \frac{1}{2} + \frac{\frac{3}{2} + \frac{2}{(-1+e-2ep^A)^2} - \frac{4}{1-e+2ep^A}}{e^2}. \end{aligned}$$

Evaluating dif at $e = e^*$ we know $dif(e = e^*) = 0$, where the last equation uses the result from Proposition 3 that $p^{ch} = p^* = 1 - p^A + \frac{2p^A-1}{\sqrt{2}}$ at $e = e^*$. Since dif is quadratic in p^{ch} which is positive by definition, it is easy to verify that

$$dif \leq 0 \Leftrightarrow p^{ch} \geq 1 - p^A + \frac{2p^A - 1}{\sqrt{2}}.$$

Applying the Implicit Function Theorem to the f.o.c defining p^{ch}, we have

$$\frac{dp^{ch}}{de} = \frac{1 - 2p^{ch}}{2e + \frac{(2p^A-1)^2}{(p^A+p^{ch}-1)^3}} < 0.$$

Monotonic transition gives

$$dif \leq 0 \Leftrightarrow e \leq e^*.$$

which, together with Claim 1, verifies the claim.

Finally $p^{ch} \in (\frac{1}{2}, p^A)$ is easy to show by combining the fact $\frac{dp^{ch}}{de} < 0$ and the results of Proposition 3. ■

Proof of Lemma 4. One can verify the lemma by replicating the proof of Proposition 2 (players assigning probability one to $y^A = b$ on the out-of-equilibrium path supports the equilibrium). The remainder shows that the specified out-of-equilibrium belief satisfies both the Intuitive Criterion and the Universal Divinity Criterion.

For the purpose of equilibrium refinement, it is sufficient to check beliefs assigned to the affiliated analyst's out-of-equilibrium messages only.[23] As the affiliated analyst forecasts at $t = 1$ in equilibrium, the game has two out-of-equilibrium messages: $r^A = \widehat{H}$ at $t = 2$ and $r^A = \widehat{L}$ at $t = 2$, which are denoted as $r_2^A = \widehat{H}$ and $r_2^A = \widehat{L}$.

Universal Divinity Criterion: I illustrate the argument for the out-of-equilibrium message $r_2^A = \widehat{H}$ (referred as m for short), and the argument for $r_2^A = \widehat{L}$ is similar.

Some notation is necessary to apply the criterion. Let $BR(\mu, r^I, m)$ be the investor's *pure-strategy* best response to the out-of-equilibrium message m, given the belief μ over the affiliated analyst's type (his signal $y^A \in \{g, b\}$) and the independent analyst's recommendation r^I. Similarly let $MBR(\mu, r^I, m)$ be the set of *mixed-strategy* best response to m, given μ and r^I, that is the set of all probability distributions over $BR(\mu, r^I, m)$.

Then define $D(t, T, m)$ to be the set of the investor's mixed-strategy best responses to the out-of-equilibrium message m and beliefs concentrated on support of affiliated analyst's type space T that makes type $t \in \{g, b\}$ strictly prefer m to his equilibrium payoff.

$$D(t, T, m) = \bigcup_{\{\mu : \mu(T)=1\}} \{x \in MBR(\mu, r^I, m) s.t. u^*(t) < u(m, x, t)\}. \tag{22}$$

where $x \doteq \Pr(Buy|m)$ is the probability that the investor buys the stock upon observing the out-of-equilibrium message m, $u^*(t)$ is the type-t affiliated analyst's payoff on the equilibrium path, $u(m, x, t)$ is type-t's expected payoff by sending out m when the investor reacts to it with x, and $\mu = \Pr(y^A = g|m)$ is the investor's belief that the affiliated analyst is *good* type. Similarly let $D^0(t, T, m)$ be the set of mixed best responses that make type t exactly indifferent. Finally, $D(t, T, m)$ and $D^0(t, T, m)$ are functions of r^I, which I will return to later.

Algebra shows that the *good* type affiliated analyst (with $y^A = g$) prefers the out-of-equilibrium message $r_2^A = \widehat{H}$ to his equilibrium action if $x > \frac{(2p^A-1)\left[\alpha+e2p^A+e(2p^A+2p(1+\alpha-2p^A)-\alpha)\right]+\delta}{(2p^A-1+e)\alpha} \doteq A$, where $p \in \{p^*, p^{ch}\}$ is the independent analyst's precision choice in equilibrium. Likewise,

[23] Since the independent analyst's payoff does not depend on the investor's action, one can assign arbitrary beliefs to his out-of-equilibrium actions and those beliefs will survive both equilibrium refinement criteria used in the paper.

the *bad* type affiliated analyst (with $y^A = b$) prefers sending out $r_1^A = \widehat{H}$ if $x > \frac{2p^A-1+\delta}{\alpha}$. Furthermore, the following is true under the maintained assumption $\alpha > \underline{\alpha}$ (see (11))

$$A > B. \tag{23}$$

Now calculate $MBR(\mu, r^I, m)$, the investor's *mixed-strategy* best response. Notice the investor's best response depends not only on her belief about the affiliated analyst's type, but also on the independent analyst's recommendation. Denote $\mu^*(r^I = \widehat{L})$ as the probability of $y^A = g$ such that the investor is indifferent about buying or not buying upon observing $r^I = \widehat{L}$; and similarly $\mu^*(r^I = \widehat{H})$ as the probability of $y^A = g$ such that the investor is indifferent about buying or not buying upon observing $r^I = \widehat{H}$. Clearly

$$\mu^*(r^I = \widehat{H}) < \mu^*(r^I = \widehat{L}).$$

The set of investor's mixed best response $MBR(\mu, r^I, m)$ is

$$MBR(\mu, r^I, m) = \begin{cases} x = 0 \text{ if } \mu < \mu^*(r^I = \widehat{H}) \\ x \in [0,1] \text{ if } \mu = \mu^*(r^I = \widehat{H}) \text{ and } r^I = \widehat{H} \\ x = 0 \text{ if } \mu = \mu^*(r^I = \widehat{H}) \text{ and } r^I = \widehat{L} \\ x = 0 \text{ if } \mu^*(r^I = \widehat{H}) < \mu < \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{L} \\ x = 1 \text{ if } \mu^*(r^I = \widehat{H}) < \mu < \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{H} \\ x = 1 \text{ if } \mu = \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{H} \\ x \in [0,1] \text{ if } \mu = \mu^*(r^I = \widehat{L}) \text{ and } r^I = \widehat{L} \\ x = 1 \text{ if } \mu > \mu^*(r^I = \widehat{L}). \end{cases}$$

Therefore

$$
\begin{aligned}
D(g,T,m) &= \underset{\{\mu:\mu(T)=1\}}{\cup}\{x > A \cap MBR(\mu, r^I, m)\} \\
&= \begin{cases} x > A \text{ if } r^I = \widehat{H} \\ x > A \text{ if } r^I = \widehat{L} \end{cases} \\
&= x > A.
\end{aligned}
$$

To understand the second equality, note that set $MBR(\mu, r^I, m)$ inside the union operation depends on both μ and r^I while the union is taken only with respect to μ, and therefore the outcome is a function of r^I. The last equality shows that the set $D(good, T, m)$ *degenerates to a deterministic set.*

Similarly, one can show that $D(b,T,m)$ is as follows:

$$
\begin{aligned}
D(b,T,m) &= \underset{\{\mu:\mu(T)=1\}}{\cup}\{x > B \cap MBR(\mu, m)\} \\
&= \begin{cases} x > B \text{ if } r^I = \widehat{H} \\ x > B \text{ if } r^I = \widehat{L} \end{cases} \\
&= x > B.
\end{aligned}
$$

As we know from (23) $A > B$, we have

$$
D(good, T, m) \cup D^0(good, T, m) \subset D(bad, T, m). \tag{24}
$$

According to the Universal Divinity Criterion, (24) means the equilibrium should assign probability zero to type $y^A = g$ upon observing the out-of-equilibrium message m, which is consistent with the strategy specified in Lemma 4.

Intuitive Criterion: On one hand, suppose players assign probability one to $y^A = g$ upon observing any of the two out-of-equilibrium messages $r_2^A = \widehat{H}$ and $r_2^A = \widehat{L}$. Given the proposed belief, it is easy to show that the affiliated analyst observing $y^A = b$ (*bad*-type) is strictly better off by sending out either of the two out-of-equilibrium messages than choosing his equilibrium action. This implies that neither of the out-of-equilibrium messages can be eliminated for the *bad*-type affiliated analyst by equilibrium dominance used in the Intuitive Criterion (Cho and Kreps, 1987 Page 199-202). On the other hand, even if any of the out-of-equilibrium messages can be eliminated for the *good*-type ($y^A = g$) affiliated analyst, the *bad*-type affiliated analyst does not have incentive to send out that message and being identified. ■

Proof of Lemma 5. Recall that the details of the additional equilibrium are stated in Appendix A.

Part (i): $\alpha \leq \frac{p^A+p^*-1}{1-p^A+2p^Ap^*-p^*}$ ensures that it is a strict best response for the affiliated analyst to issue $\widehat{L}$ upon observing both $y^A = b$ and $r_1^I = \widehat{L}$. $\delta \leq \widehat{\delta}$ prevents the affiliated analyst from deviating the equilibrium by issuing recommendations at $t = 1$. It is then easy to verify the equilibrium.

Part (ii): $\beta = \frac{p^A-p^*}{p^A+p^*-1}$ is chosen so that the investor is indifferent between "Buy" and "Not Buy" after observing $\{r^A = \widehat{H} \cap r^I = \widehat{L}\}$. Likewise, $\rho = 1 - \frac{1-p^*-p^A}{\alpha(p^A+p^*-1-2p^Ap^*)}$ is chosen so that affiliated analyst is indifferent between issuing $\widehat{H}$ and $\widehat{L}$ when observing $\{y^A = b, r^I = \widehat{L}\}$, and $0 \leq \rho \leq 1$ requires $\alpha > \frac{p^A+p^*-1}{1-p^A+2p^Ap^*-p^*}$. Substituting β and ρ, one can show $\delta < \widehat{\delta}$ prevents the affiliated analyst from deviating the equilibrium by issuing recommendations at $t = 1$. ■

Proof of Lemma 6. Recall that $\beta = \frac{p^A-p}{p^A+p-1}$ in both the Conditional Herding Equilibrium and the Independent Forecasting Equilibrium. Simple algebra verifies the Lemma. ■

Proof of Proposition 3. From the Proof of Proposition 2 we know $p^* = \frac{1+e}{2e}$ while p^{ch} maximizes

$$U(p) = \frac{1}{2}p + \frac{1}{2}\left[p(p^A + (1-p^A)\beta) + (1-p)(1-\beta)p^A - \delta\right] - e(p - \frac{1}{2})^2$$

and $U'(p)|_{p=p^{ch}} = 0$. Also, it is easy to show

$$U''(p) = -2e - \frac{(2p^A-1)^2}{(p^A+p-1)^3} < 0.$$

Therefore, $U'(p)$ is strictly decreasing with respect to p. Evaluating $U'(p)$ at $p = p^*$, we obtain

$$U'(p)|_{p=p^*} = \frac{2(2ep^A - e)^2}{(2ep^A - e + 1)^2} - 1.$$

Algebra shows

$$U'(p)|_{p=p^*} > 0 = U'(p)|_{p=p^{ch}} \Leftrightarrow e > \frac{1}{(\sqrt{2} - 1)(2p^A - 1)}.$$

Since $U''(p) < 0$, $U'(p)|_{p=p^*} > U'(p)|_{p=p^{ch}}$ implies $p^* < p^{ch}$. Therefore,

$$p^{ch} > p^* \Leftrightarrow e > \frac{1}{(\sqrt{2} - 1)(2p^A - 1)}.$$

This completes the proof. ■

Proof of Proposition 4. Denote p^{eq} as the precision acquired by the independent analyst in equilibrium, which is given in Proposition 2, one can show the investor's equilibrium payoff is

$$U^{Inv}(p^{eq}) = \frac{(2p^A - 1)(2p^{eq} - 1)}{2(p^A + p^{eq} - 1)}$$

and

$$\frac{d}{dp^{eq}} U^{Inv}(p^{eq}) > 0.$$

This inequality, together with Proposition 3, completes the proof. ■

Proof of Lemma 6. $\beta = \frac{p^A - p}{p^A + p - 1}$, and simple algebra verifies the Lemma. ■

Proof of Corollary 1. Direct implication of Proposition 3 ■

Proof of Corollary 2. The dispersion of analysts' recommendation is the ex-ante percentage that two analysts' recommendation are different ($\widehat{H}$ versus $\widehat{L}$) among all the recommendations observed by the investor up to period t. In the Conditional Herding Equilibrium, one can show $Dispersion_1 = \frac{\Pr(r^A = \widehat{H}, y^I = b)}{\Pr(y^I = b)}$ and $Dispersion_2 = \Pr(r^A = \widehat{H}, y^I = b)$, where the subscript represents period t. It is clear that $Dispersion_1 > Dispersion_2$. ■

Proof of Lemma 7. Given the investor's endowment is e, holding y units of risky asset and

x units of risk-free asset will generate wealth w

$$\begin{aligned} w &= (1+\omega) * m * y + x \\ &= (1+\omega) * m * y + e - m * y \\ &= \omega * m * y + e. \end{aligned}$$

The second equality makes uses of the budget constraint $e = m * y + x$. The optimal holding B is

$$B = \arg\max_{y \geq 0} q_H * -e^{-\rho*(e+my)} + (1-q_H) - e^{-\rho*(e-my)},$$

where $q_H = \Pr(\omega = H | r^A, r^I)$ is the posterior probability of $\omega = H$. Solving the program we obtain

$$B = \begin{cases} \frac{\log(\frac{q_H}{1-q_H})}{2*\rho*m} & \text{if } q_H \geq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Simple algebra verifies the lemma. ■

Proof of Lemma 8. The proof of Lemma 2 can be used to prove the first part of the lemma.

To show the second part of the lemma, let us first state a necessary condition for the *affiliated* analyst's strategy to be in equilibrium. Point-wise mappings $\beta(y^A, p) \doteq \Pr(r^A = \widehat{H} | y^A, p)$ and $\gamma(y^A, p) \doteq \Pr(r^A = \widehat{L} | y^A, p)$ for $\forall y^A, \forall p$ characterize the affiliated analyst's strategy, and I will write $\beta(y^A)$ and $\gamma(y^A)$ for short as the argument below holds for all p. The informativeness of r^A is calculated as follows

$$\begin{aligned} I_{\widehat{H}} &= \Pr(\omega = H | r^A = \widehat{H}) = \frac{\int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A}{\int_{-\infty}^{+\infty} \beta(y^A)\varphi_H dy^A + \int_{-\infty}^{+\infty} \beta(y^A)\varphi_L dy^A} \\ I_{\widehat{L}} &= \Pr(\omega = L | r^A = \widehat{L}) = \frac{\int_{-\infty}^{+\infty} \gamma(y^A)\varphi_L dy^A}{\int_{-\infty}^{+\infty} \gamma(y^A)\varphi_L dy^A + \int_{-\infty}^{+\infty} \gamma(y^A)\varphi_H dy^A}. \end{aligned}$$

where φ_H and φ_L are the probability density function of y^A conditional on state $\omega = H$ and L. One can show that

$$(I_{\widehat{H}} - \frac{1}{2})(I_{\widehat{L}} - \frac{1}{2}) \geq 0$$

and $(I_{\widehat{H}}-\frac{1}{2})(I_{\widehat{L}}-\frac{1}{2})=0 \Leftrightarrow \int_{-\infty}^{+\infty}\beta(y^A)\varphi_H dy^A=\int_{-\infty}^{+\infty}\beta(y^A)\varphi_L dy^A \Leftrightarrow I_{\widehat{H}}, I_{\widehat{L}}=\frac{1}{2}$. Arguments developed in Lemma 1 can be used to show that (1) in equilibrium $I_{\widehat{H}}, I_{\widehat{L}}>\frac{1}{2}$, and (2) in equilibrium $r^A=\widehat{H}$ is the affiliated analyst's strict best response for any $y^A\geq 0$ and therefore

$$\beta(y^A)=1, \forall y^A\geq 0 \text{ in equilibrium.} \tag{25}$$

Now turn to the independent analyst's expected payoff by deferring his forecast to $t=2$ after observing a signal y^I with precision p, denoted as $U_{t2}^I(y^I,p)$. We know that waiting implies subsequent herding in equilibrium. Therefore

$$\begin{aligned}
U_{t2}^I(b,p) &= p*\int_{-\infty}^{+\infty}\gamma(y^A)\varphi_L dy^A+(1-p)\int_{-\infty}^{+\infty}\beta(y^A)\varphi_H dy^A \\
U_{t2}^I(g,p) &= p*\int_{-\infty}^{+\infty}\beta(y^A)\varphi_H dy^A+(1-p)\int_{-\infty}^{+\infty}\gamma(y^A)\varphi_L dy^A,
\end{aligned}$$

and

$$\begin{aligned}
U_{t2}^I(g,p)-U_{t2}^I(b,p) &= (2p-1)\left(\int_{-\infty}^{+\infty}\beta(y^A)\varphi_H dy^A-\int_{-\infty}^{+\infty}\gamma(y^A)\varphi_L dy^A\right) \\
&= (2p-1)\left(\int_{-\infty}^{+\infty}\beta(y^A)\varphi_H dy^A+\int_{-\infty}^{+\infty}\beta(y^A)\varphi_L dy^A-1\right) \\
&\geq (2p-1)\left(\int_{0}^{+\infty}\beta(y^A)\varphi_H dy^A+\int_{0}^{+\infty}\beta(y^A)\varphi_L dy^A-1\right) \\
&= (2p-1)\left(\int_{0}^{+\infty}\varphi_H dy^A+\int_{0}^{+\infty}\varphi_L dy^A-1\right) \\
&= 0.
\end{aligned}$$

The inequality is by $\beta(y^A)\geq 0$, the second last equality is by $\beta(y^A)=1, \forall y^A\geq 0$ (see (25)), and the last equality uses the fact that $\int_0^{+\infty}\varphi_H dy^A+\int_0^{+\infty}\varphi_L dy^A=1$ due to the symmetry of ω (i.e., $L=-H$). ■

Proof of Lemma 9. I claim that if (in equilibrium) the affiliated analyst chooses to forecast $\widehat{L}$ after observing his signal $y^A=a$, he will also forecast $\widehat{L}$ for any signal $y^A<a$. Similarly, if (in equilibrium) the affiliated analyst chooses to forecast $\widehat{H}$ after observing his signal $y^A=b$,

then he will also forecast $\widehat{H}$ for any signal $y^A > b$.

Denote $U^A(r^A, y^A)$ as the affiliated analyst's expected utility when his private signal is y^A and he forecasts r^A. In particular,

$$\begin{aligned}
U^A(r^A &= \widehat{H}, y^A) = \Pr(\omega = H|y^A) + \alpha * E[shares(r^A = \widehat{H}, r^I(y^I; Time))|y^A] \\
U^A(r^A &= \widehat{L}, y^A) = \Pr(\omega = L|y^A) + \alpha * E[shares(r^A = \widehat{L}, r^I(y^I; Time))|y^A].
\end{aligned}$$

where $shares(r^A, r^I)$ is the number of risky asset the investor buys after observing r^A and r^I and in equilibrium follows Lemma 7. The expectation operator $E[\cdot|y^A]$ is taken over y^I while taking the independent analyst's strategy $r^I(y^I; Time)$ as given. The $Time$ parameter in $r^I(y^I; Time)$ reflects that whether the independent analyst's information set contains r^A depends on the timing of his recommendation (or waiting strategy) which (for a given p) is measurable only with respect to y^I. Then define

$$\Delta_{\widehat{H} \succ \widehat{L}}(y^A) = U^A(r^A = \widehat{H}, y^A) - U^A(r^A = \widehat{L}, y^A).$$

Collecting terms, one can rewrite $\Delta_{\widehat{H} \succ \widehat{L}}(y^A)$ as

$$\Delta_{\widehat{H} \succ \widehat{L}}(y^A) = Benefit_{\widehat{H} \succ \widehat{L}}(y^A) - Cost_{\widehat{H} \succ \widehat{L}}(y^A),$$

in which

$$Benefit_{\widehat{H} \succ \widehat{L}}(y^A) = \alpha * E[shares(r^A = \widehat{H}, r^I(y^I; Time)) - shares(r^A = \widehat{L}, r^I(y^I; Time))|y^A]$$

and

$$\begin{aligned}
Cost_{\widehat{H} \succ \widehat{L}}(y^A) &= \Pr(\omega = L|y^A) - \Pr(\omega = H|y^A) \\
&= \frac{\varphi_L}{\varphi_H + \varphi_L} - \frac{\varphi_H}{\varphi_H + \varphi_L},
\end{aligned}$$

where φ_H and φ_L are the probability density function of y^A conditional on state $\omega = H$ and

L. From the Monotonic Likelihood Ration Property (MLRP), we know that $\frac{\varphi_H}{\varphi_L}$ increases in y^A and

$$\frac{d}{dy^A} Cost_{\widehat{H} \succ \widehat{L}}(y^A) < 0.$$

Writing out the expectation operation in $Benefit_{\widehat{H} \succ \widehat{L}}(y^A)$, we have

$$\begin{aligned} Benefit_{\widehat{H} \succ \widehat{L}}(y^A) &= \alpha \Pr(y^I = g|y^A) \left[shares(r^A = \widehat{H}, r^I(g; Time)) - shares(r^A = \widehat{L}, r^I(g; Time)) \right] \\ +\alpha \Pr(y^I &= b|y^A) \left[shares(r^A = \widehat{H}, r^I(b; Time)) - shares(r^A = \widehat{L}, r^I(b; Time)) \right]. \end{aligned}$$

Notice that terms in both square brackets above are independent of the *realization* of y^A. Further, I claim that the following holds in equilibrium, regardless of the independent analyst's waiting strategy.

$$\begin{aligned} shares(r^A &= \widehat{H}, r^I(g; Time)) - shares(r^A = \widehat{L}, r^I(g; Time)) \\ &\geq shares(r^A = \widehat{H}, r^I(b; Time)) - shares(r^A = \widehat{L}, r^I(b; Time)). \end{aligned} \tag{26}$$

To see this, we know from Lemma 8 that the independent analyst's waiting strategy can only take three forms: (i) always wait for $\forall y^I$, (ii) wait if and only if $y^I = b$, and (iii) never wait for $\forall y^I$. Since waiting implies herding in equilibrium, it is easy to check the inequality above holds for case (i) and case (ii). In case (iii), the independent analyst forecasts independently and one can calculate in this case

$$\left[shares(\widehat{H}, r^I(g)) - shares(\widehat{L}, r^I(g)) \right] - \left[shares(\widehat{H}, r^I(b)) - shares(\widehat{L}, r^I(b)) \right] = \log(\frac{p}{1-p}) \geq 0.$$

The equality makes uses of $shares(\widehat{L}, \widehat{L}) = 0$ and verifies (26).

MLRP implies $\frac{d}{dy^A} \Pr(y^I = g|y^A) > 0$. Therefore we have

$$\frac{d}{dy^A} Benefit_{\widehat{H} \succ \widehat{L}}(y^A) > 0,$$

which makes uses $\frac{d}{dy^A} \Pr(y^I = b|y^A) = -\frac{d}{dy^A} \Pr(y^I = g|y^A)$.

Since $\Delta_{\widehat{H}\succ\widehat{L}}(y^A) = Benefit_{\widehat{H}\succ\widehat{L}}(y^A) - Cost_{\widehat{H}\succ\widehat{L}}(y^A)$, we have

$$\frac{d}{dy^A}\Delta_{\widehat{H}\succ\widehat{L}}(y^A) > 0.$$

Since $\Delta_{\widehat{H}\succ\widehat{L}}(y^A) > 0$ means $r^A = \widehat{H}$ while $\Delta_{\widehat{H}\succ\widehat{L}}(y^A) < 0$ means $r^A = \widehat{L}$, $\frac{d}{dy^A}\Delta_{\widehat{H}\succ\widehat{L}}(y^A) > 0$ verifies the claim. ■

Proof of Proposition 5. Direct implication of Lemmas 7, 8, and 9. ■