Voluntary Debt Exchanges in Sovereign Debt Markets[*]

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PRELIMINARY AND INCOMPLETE

Abstract

We show that in some recent sovereign debt restructurings the sovereign both reduced its debt burden and made all of its payments while the market value of debt claims rose for the holders of the restructured debt. In these observations, both the government and its creditors could mutually benefit from such restructurings; hence, we refer to these episodes as "voluntary" debt exchanges. We present a quantitative model in which these voluntary debt exchanges can occur because the debt level takes values above that which maximizes the market value of debt claims. In contrast to most previous studies on debt overhang, in our model opportunities for voluntary exchanges arise because a debt reduction implies a decline of the sovereign default risk. This is observed in the absence of any effect of debt reductions on future output levels. Although voluntary exchanges are Pareto improving at the time of the restructuring, we show that eliminating the possibility of conducting a voluntary exchange may improve welfare from an ex-ante perspective. Our results highlight a cost of initiatives that facilitate debt restructurings.

JEL classification: F34, F41.

Keywords: Sovereign Default, Debt Restructuring, Voluntary Debt Exchanges, Long-term Debt, Endogenous Borrowing Constraints.

1 Introduction

In sovereign debt restructurings the government swaps previously issued debt for new debt. This paper studies restructurings that occur in the absence of missed debt payments and, at the time of the debt exchange, benefit both the government and the holders of restructured debt. We refer to these episodes as "voluntary" debt exchanges, indicating that there is room for the government and its creditors to agree on the exchange.[1] We first argue that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. We also show that a sovereign default model à la Eaton and Gersovitz (1981) can account for voluntary exchanges. Finally, we show that within our framework, and in spite of benefiting all parties involved at the time of the restructuring, voluntary debt exchanges are not optimal for the government from an ex-ante perspective: eliminating the possibility of conducting these restructurings may improve welfare.

Formally, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. The government is benevolent, issues long-term debt in international markets, and cannot commit to repay its debt. Sovereign debt is held by risk-neutral foreign investors. We extend this baseline model of sovereign default by allowing for voluntary debt exchanges. We assume that the cost of debt exchanges is stochastic and may be either low (zero) or high. The high cost is assumed to be high enough to prevent the government from launching a voluntary exchange. This assumption intends to capture difficulties in the implementation of voluntary exchanges. At the beginning of each period, the debt exchange cost is realized together with the endowment shock. When the cost is low, the government chooses whether to conduct a voluntary debt exchange. If the government conducts a voluntary exchange, it brings its debt to the level that maximizes the market value of debt, generating capital gains from holdings of the restructured debt. If the government does not conduct a voluntary exchange, it decides whether

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[1]Our definition of voluntary exchange should not be interpreted as indicating that the participation of all creditors in the exchange was strictly voluntary. First, it is difficult to determine the extent to which governments coerced bondholders into participating in a debt exchange. Furthermore, even when a debt restructuring is beneficial for creditors as a group, individual creditors could benefit from free-riding on the participation of other creditors. For a discussion of collective action problems associated with debt restructuring see Wright (2011) and the references therein.

to declare an outright default. An outright default is followed by a stochastic period of exclusion from capital markets and lower income levels while the default period lasts.

We show that our model can account for the frequency of voluntary debt exchanges suggested by the findings in Cruces and Trebesch (2013), while still accounting for other features of the data highlighted in the sovereign debt literature. Using a calibration for an archetypical emerging economy, opportunities for voluntary debt exchanges arise in 19 percent of the simulation periods. That is, in 19 percent of the simulation periods the initial debt level is higher than the debt level that maximizes the market value of debt claims. Even though those periods arise after negative income shocks, an outright default does not need to be imminent—i.e., the government would not have defaulted if had not been able to conduct a voluntary exchange.

Why does a debt relief lead to capital gains for the holders of restructured debt? In our model this occurs because lower debt levels reduce future default risk and thus increase the market value of bond holders' debt portfolio. Figure 1 shows the market value of a debt portfolio as a function of the debt stock. If the state of the economy is represented by a point like A, a marginal decline of the debt level is more than compensated by an increase in bond prices, producing an increase in the market value of the lenders' debt portfolio. In this case, both the government and its creditors could benefit from a debt restructuring that reduces debt (for example, to point B).

Among voluntary debt exchanges that prevented an outright default in the period of the exchange, the average debt reduction is 24 percent, and the average capital gain from holdings of the restructured debt is 155 percent. The sovereign enjoys an average welfare gain at the times of the exchange equivalent to permanent consumption increase of 0.7 percent. Among voluntary debt exchanges conducted in periods in which the government would have chosen to stay current on its debt, the average haircut is 5 percent, the average capital gain is 3 percent, and the sovereign enjoys an average welfare gain in those periods equivalent to permanent consumption increase of 0.3 percent.

In spite of the Pareto gains that result at the moment of a voluntary exchange, eliminating the possibility of conducting a voluntary exchange may improve welfare from an ex-ante perspective—resulting in welfare gains equivalent to a permanent consumption increase of up to 0.9 percent. The anticipation of future voluntary exchanges leads lenders to offer lower prices for bonds issued
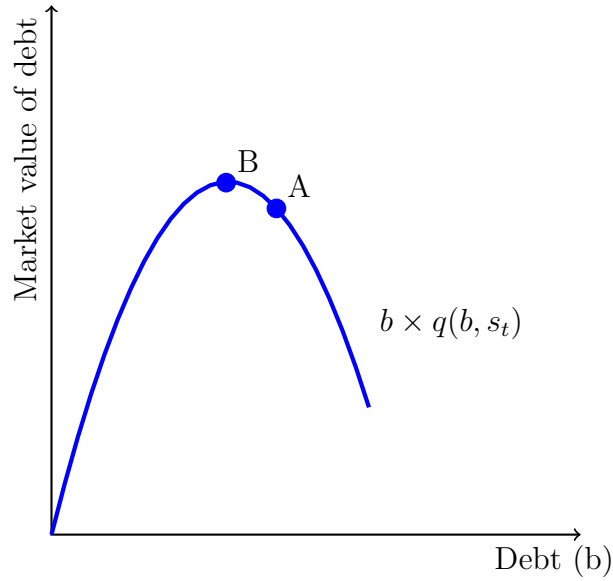
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Figure 1: The market value of debt claims in period t is represented by $b \times q(b, s_t)$, where b denotes the number of bonds and $q(b, s_t)$ denote the price of each bond. The price may depend on other state variables represented by s_t. The market value curve may be a decreasing function of the debt level b because $q(b, s_t)$ is a decreasing function of the debt level b. Thus, for a state characterized by point A, a reduction of the government debt burden to a point like B produces capital gains from holdings of the restructured debt.

by the government, which limits the future resources the government can bring forward when it issues debt. The government's decisions to conduct voluntary debt exchanges are not optimal from an ex-ante perspective. This finding highlights a cost of initiatives to facilitate sovereign debt restructurings.

1.1 Related literature

The possibility that a sovereign debtor and its creditors can jointly benefit from a debt reduction is discuss by Krugman (1988a), Krugman (1988b), Sachs (1989), Froot (1989), among others. They present "debt overhang" models with exogenous debt levels and show that debt relief generates Pareto gains when future debt obligations are high enough to lower aggregate investment. By increasing aggregate investment, a debt relief increases the resources available for creditors to

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confiscate in the case of a default.[2] In contrast, we present a model in which a debt relief has no effect on future available resources but it weakens incentives to default, producing capital gains for debt holders. Our findings also complement their results by showing that (i) opportunities for Pareto improving debt reliefs arise endogenously in a calibrated model with empirically plausible implications, and (ii) governments may want to prevent debt reliefs from an ex-ante perspective, even when these reliefs generate Pareto gains at the time of the debt restructuring.

Aguiar et al. (2009) study a model with endogenous debt levels in which a government that cannot commit to future actions may accumulate excessive debt claims causing a debt overhang effect on investment. The optimal allocation under no commitment lies on the constrained efficient frontier, which rules out the possibility of mutually beneficial debt reductions once the sovereign is in a situation of debt overhang. In contrast, we focus on debt reliefs that produce Pareto Gains at the time of the restructuring even though they may not be optimal from an ex-ante perspective.

In the recent quantitative papers on sovereign default that followed Aguiar and Gopinath (2006) and Arellano (2008) the possibility to engage in mutually beneficial debt relief is ruled out by assumption. We extend the baseline model by allowing for this possibility which enables us to account for some of the heterogeneity among debt restructurings. We show that the model can account for voluntary debt exchanges while still accounting for other features of the data emphasized in the sovereign default literature.

The rest of the article proceeds as follows. Section 2 argues that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. Section 3 introduces the model. Section 4 discusses the calibration. Section 5 presents the results. Section 6 concludes.

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[2]It should be noted that creditors have recently faced difficulties when intending to confiscate assets of a defaulting country (see, for instance, Panizza et al. (2009) and Hatchondo and Martinez (2011)). In many countries (including the U.S.), there are legal procedures that creditors may follow once individuals or corporations renege on their debt. Creditors rely on the enforcement of domestic courts to get—partially—repaid. However, it is difficult for courts to enforce government payments.

2 Recent voluntary debt exchanges

This Section argues that some recent sovereign debt restructurings fit within our definition of a voluntary debt exchange. Recall that we define a voluntary debt exchange as a debt restructuring such that (i) the government does not miss any debt payment, (ii) there is a decline in the government's debt burden, and (iii) there are capital gains from participating in the restructuring.

Governments often restructure their debt without missing debt payments. For instance, Cruces and Trebesch (2013) study 180 sovereign debt restructurings between 1970 and 2010 and show that in 77 of these 180 restructurings the government did not miss debt payments.

Most sovereign debt restructurings are characterized by debt reductions. Measuring the implied debt reductions requires assumptions that allow for the comparison of debt instruments with different characteristics. One measure of debt reduction is the "haircut". There are different haircut measures. Recently, the one proposed by Sturzenegger and Zettelmeyer (2005) has been widely accepted and used (see for example Cruces and Trebesch (2013)):

$$Haircut = 1 - \frac{\text{Present value of debt obtained in the restructuring}}{\text{Present value of debt surrendered in the restructuring}}, \tag{1}$$

where both present values are computed using the yields of the debt received at the moment of the restructuring. Using this measure, Cruces and Trebesch (2013) find that only 6 of the 180 restructurings they study present negative haircuts. This indicates that governments benefited from a debt reduction in the remaining 174 cases.[3]

We find that some recent sovereign debt restructurings that occurred in the absence of missed debt payment were characterized by a lower debt burden for the government and a higher value market value of debt claims for holders of restructured debt. Table 1 presents creditors' capital gains from holding restructured debt in four recent episodes.[4] We compute capital gains using

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[3]It should be noted that debt reductions are only a partial measure of the benefits accrued to sovereign debtors. Debt restructurings typically benefit debtor governments by smoothing and/or extending the maturity profile of debt payments.

[4]Secondary market bond prices make it easier to compute the market values of debt for bond debt restructurings than for bank debt restructurings. Cruces and Trebesch (2013) show that out of the 180 debt restructurings that took place between 1970 and 2010, 162 were bank debt restructurings and only 18 were bond debt restructurings. Out of the 18 bond debt restructurings, 9 occurred without the government missing debt payments. These are the recent restructurings in Belize, Dominica, Dominican Republic, Grenada, Kenya, Moldova, Pakistan, Ukraine,

bond prices to determined the value of the debt portfolio of creditors who participated in the exchange, before and after the restructurings.

$$\text{Capital gain} = \frac{\sum_{i=J+1}^{J+I} q_i(T) \times B_i}{(1+r)^{T-t} \sum_{j=1}^{J} q_j(T-t) \times B_j} - 1, \tag{2}$$

where T denotes the period for which prices of bonds obtained in the exchange are available, t denotes the period for which we compute the market value of the portfolio surrendered in the exchange, $q_i(t)$ denotes the unit price of bonds of type i in period t, and B_i denotes the number of outstanding bonds of type i. Equation 2 refers to a case in which the debt portfolio $(B_1, ..., B_J)$ is exchanged for the debt portfolio $(B_{J+1}, ..., B_{I+J})$. We are interested in the effect of debt reductions on the market value of the creditors' debt portfolio. We compute the market value of the portfolio surrendered in the exchange at different dates in an attempt to control for the effect of the anticipation of the restructuring on the market value of the debt. If the success of the exchange is perfectly anticipated, at the time of the exchange, the value of the portfolio surrendered by lenders would not be lower than the value of the portfolio they received in the exchange. Similarly, close to the time of the exchange, capital gains may tend to measure the effect of surprises about the success of the exchange. Table 1 reports capital gains up to six months before the exchange announcement.

Figure 2 presents the market value of the surrendered debt portfolios even further back from the exchange. A common pattern in the Figure is that the value of the portfolio surrendered in the exchange tend to appreciate in the weeks preceding the exchange, and the market value of portfolio received by lenders was higher than the one of the portfolio they surrendered.[5] Details on the construction of Table 1 and Figure 2 are presented in the Appendix.

Table 1 also shows that the four episodes under study were characterized by debt reductions, as measured by positive haircuts. Notice that the haircut measure (1) could be approximated by the opposite of the capital gain measure (2) for $t = 0$ and for bond prices of surrendered bonds that are imputed based on the yields implied by bond prices of post-exchange debt instruments.

and Uruguay. Table 1 presents data for the four of these restructurings for which we found bond price data.

[5]For instance, Uruguay launched its debt exchange on April 10th, 2003. The closing date for the exchange was May 29th 2003. During that time, the price of the bond included in Figure 2 jumped from 0.49 to 0.61 per unit of face value.

	Dominican Rep.	Pakistan	Ukraine	Uruguay
Exchange announcement	3/30/2005	8/15/1999	2/4/2000	4/10/2003
Launch of the exchange	4/20/2005	11/15/1999	2/14/2000	4/10/2003
Deadline for participating (DP)	5/16/2005	12/15/1999	3/15/2000	5/29/2003
First date with data for all bond prices	5/18/2005	n.a.	03/30/2000	09/23/2003
Capital gains with price of surrendered portfolio at				
First date with data for all bond prices	0.08	n.a.	-0.02	0.05
Deadline for participating	0.08	0.07	0.14	0.11
Launch of the exchange	0.04	0.06	0.18	0.13
Exchange announcement	0.06	0.02	0.25	0.34
One month before exchange announcement	0.06	0.01	0.31	0.26
Six months before exchange announcement	0.34	0.21	0.09	0.30
Acceptance rate	0.94	0.99	0.99	0.95
Haircut	0.05	0.15	0.18	0.10

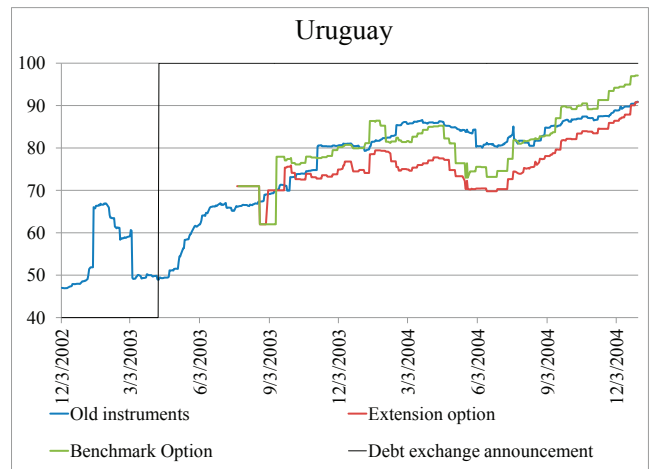Table 1: Capital gains from holdings of restructured debt in recent sovereign debt restructurings.
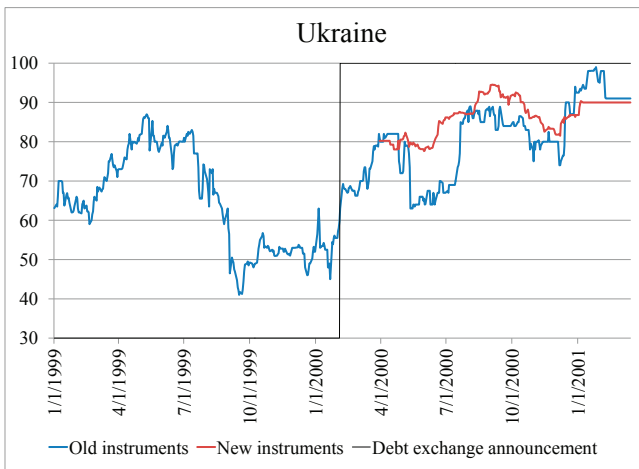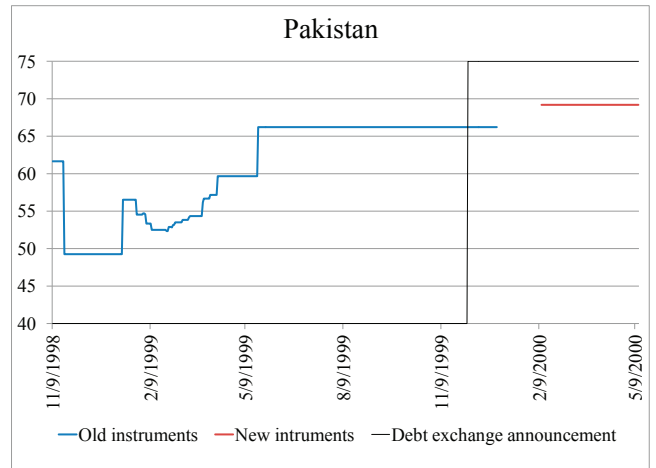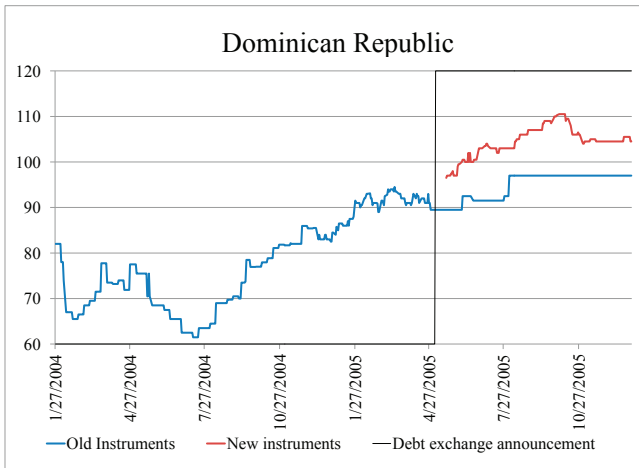
Figure 2: Price of the surrendered and obtained debt portfolios around recent debt restructurings.

Thus, the fact that both haircuts and capital gains were positive in these episodes indicate that the implied yields in sovereign bond crises decreased after the restructurings. This could be due to a decline in country risk implied by the restructuring, which is consistent with our theory of voluntary debt exchanges.

3 Model

There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t, where

$$\log(y_t) = (1 - \rho)\,\mu + \rho\log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right),$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u\left(c\right) = \frac{c^{(1-\gamma)} - 1}{1 - \gamma}.$$

As in Hatchondo and Martinez (2009), we assume that a bond issued in period t consists of a perpetuity with geometrically declining coupon obligations. In particular, a bond issued in period t promises to pay one unit of the good in period $t + 1$ and $(1 - \delta)^{s-1}$ units in period $t + s$, with $s \geq 2$.

The government cannot commit to repay its debt. We refer to events in which the government reneges on its debt as outright defaults. As in previous studies, the cost of an outright default does not depend on the size of the default. Thus, when the government defaults, it does so on

all current and future debt obligations.[6] Following previous studies, we also assume that the recovery rate for outright defaults is zero.

An outright default triggers exclusion from credit markets for a stochastic number of periods. Income is given by $y - \phi^d(y)$ in every period in which the government is excluded from credit markets. As in Arellano (2008), we assume that it is proportionally more costly to default in good times. This is a property of the endogenous default cost derived by Mendoza and Yue (2012). As in Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode $\phi^d(y) = d_0 y + d_1 y^2$. They show that this property allows the equilibrium default model to match the behavior of the spread in the data.

Lenders are risk neutral and discount future payoffs at the rate r. Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition.

Our departure from the baseline setup is to allow for voluntary debt exchanges. We present a stylized model of voluntary exchanges. We intend to capture the difficulties in implementing a voluntary restructuring by assuming that the cost of debt exchanges is i.i.d. and may take a low (zero) or a high value. The high cost is assumed to be sufficiently elevated to make it optimal for the government to not launch a voluntary exchange.

When the cost of a voluntary exchange is zero, the government chooses whether to conduct the exchange. If the government conducts a voluntary exchange, it reduces its debt level to one that maximizes the market value of the restructured debt. In terms of Figure 1, if the government conducts a debt exchange in a period characterized by point A, it lowers its debt level to the one depicted by point B.

Notice that our voluntary exchange assumptions are such that both the government and its creditor (as a group) benefit at the moment of the exchange in equilibrium. The government only chooses to conduct a voluntary exchange when the exchange results in a reduction of its debt level. In terms of Figure 1, a voluntary exchange would take the debt level to the one depicted

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[6]This is consistent with the behavior of defaulting governments in reality. Sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current (see IMF (2002)).

by point B and, therefore, the government only chooses to launch and exchange when its initial debt level is higher than B. Since the post-exchange debt level must maximize the market value of the debt stock, lenders always gain from participating in the exchange (the higher bond price more than compensates the debt reduction granted to the government).

As mentioned in the introduction, while lenders benefit as a group with a voluntary exchange, individual lenders would gain from not participating in the restructuring (and thus enjoying higher post-restructuring bond prices without suffering a debt reduction). Such collective action problems (Wright (2011)) may prevent restructurings but are also sometimes overcome with collective action clauses, minimum participation thresholds, and exit consents(see ?). We interpret a low exchange cost shock as represented circumstances in which collective action problems can be overcome. We calibrate the probability of such shock targeting the frequency of voluntary exchanges, and we study the effects of changing this probability.[7]

The government cannot commit to future outright default, exchange, and borrowing decisions. Thus, one may interpret this environment as a game in which the government making the exchange, default, and borrowing decisions in period t is a player who takes as given the exchange, default and borrowing strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium exchange, default and borrowing strategies depend only on payoff-relevant state variables. Krusell and Smith (2003) discuss the possibility of multiple Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy. That is, we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium policy functions as an approximation of the stationary equilibrium policy functions.

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[7]Since in our model voluntary exchanges are only conducted when there is no cost for the sovereign, our results provide an upper bound of the benefits derived from debt exchanges.

3.1 Recursive formulation

Let b denote the number of outstanding coupon claims at the beginning of the current period. Let $\theta \in \{L, H\}$ denote the exchange cost shock, where L (H) denotes the low (high) value of the shock. Let V denote the value function of a government that is not currently in default. This function satisfies the following functional equations:

$$V(b, y, L) = \max \left\{V^E(y), V^P(b, y), V^D(y)\right\}, \tag{3}$$

and

$$V(b, y, H) = \max \left\{V^P(b, y), V^D(y)\right\}, \tag{4}$$

where V^E denotes the continuation value when the government launches a debt exchange, V^P denotes the continuation value when the government pays its debt obligations (and does not launch an exchange), and V^D denotes the continuation value when the government declares an outright default.

The value function of paying current debt obligations and not launching an exchange is represented by the following functional equation:

$$V^P(b, y) = \max_{b' \geq 0, c} \left\{u(c) + \beta \mathbb{E}_{(y', \theta')|y} V(b', y', \theta')\right\}, \tag{5}$$

subject to

$$c = y - b + q(b', y) \left[b' - (1 - \delta)b\right],$$

where $b' - (1 - \delta)b$ represents current debt issuances, and q denotes the price of a bond at the end of a period. Let $\hat{b}^P$ denote the government's saving rule when it stays current on its debt. The value function when debt obligations are defaulted on satisfies the following functional equation:

$$V^D(y) = u(y - \phi(y)) + \beta \mathbb{E}_{(y', \theta')|y} \left[(1 - \psi)V^D(y') + \psi V(0, y', \theta')\right], \tag{6}$$

where ψ denotes the probability of regaining access to capital markets. Let $\hat{d}$ denote the government's default strategy. The function $\hat{d}$ takes a value of 1 when the government declares an outright default and takes a value of 0 when it stays current on its debt.

Finally, the value function when the government launches a debt exchange satisfies the following functional equation:

$$V^E(y) = u\left(y - \hat{b}^E(y) + [\hat{b}^P(\hat{b}^E(y), y) - (1 - \delta)\,\hat{b}^E(y)]q(\hat{b}^P(\hat{b}^E(y), y), y)\right) + \beta \mathbb{E}_{(y',\theta')|y} V(\hat{b}^P(\hat{b}^E(y), y), y', \theta'),$$

(7)

where

$$\hat{b}^E(y) = \underset{b}{\operatorname{argmax}}\left\{MV(b, y)\right\}$$

(8)

denotes the function that describes the post-exchange coupon obligations, and

$$MV(b, y) = \left[1 - \hat{d}(b, y)\right] b \left[1 + (1 - \delta)\,q\left(\hat{b}^P(b, y), y\right)\right]$$

(9)

is the market value of the lenders' debt portfolio at the time of a voluntary exchange. The function $\hat{b}$ denotes the optimal saving function, which satisfies $\hat{b} = (1 - \hat{d})\hat{b}^P$.

Notice that $MV(b, y) = 0$ for all levels of b that would lead the government to default. Therefore, MV is never maximized by a debt level that would lead to a default. Thus, the government's optimization problem after a debt exchange is equivalent to the optimization problem of a government that enters the period with $\hat{b}^E(y)$ and chooses to pay its debt (without an exchange). This allows us to use the equilibrium borrowing rule $\hat{b}^P$ to solve for the value function $V^E(y)$.

Let $\hat{e}$ denote government's exchange strategy. The function $\hat{e}$ takes a value of 1 when the government chooses to conduct a debt exchange. We assume that the high cost of an exchange is such that $\hat{e}(b, y, H) = 0$ for all (b, y).

The assumption that bond holders price bonds in competitive markets implies that

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\begin{aligned}
q(b', y)(1 + r) &= \mathbb{E}_{(y',\theta')|y}\left\{\hat{e}(b', y', \theta')\frac{\hat{b}^E(y)}{b'}\left[1 + (1 - \delta)\,q(\hat{b}(\hat{b}^E(y'), y'), y')\right]\right. \\
&\quad \left. + [1 - \hat{e}(b', y', \theta')]\left[1 - \hat{d}(b', y', \theta')\right]\left[1 + (1 - \delta)\,q(\hat{b}^P(b', y'), y')\right]\right\}, \quad (10)
\end{aligned}
$$

where $\frac{\hat{b}^E(y)}{b'} \le 1$ denotes the number of bonds received in the exchange per bond surrendered. Notice that the model equivalent of the definition of haircut presented in the introduction is given by $1 - \frac{\hat{b}^E(y)}{b'}$.

A *Markov Perfect Equilibrium* is characterized by

1. a set of value functions V, V^E, V^P, V^D, and

2. rules for voluntary exchanges $\hat{e}$, default $\hat{d}$, and borrowing $\hat{b}^P$,

3. and a bond price function q,

such that:

i. given a bond price function q, the policy functions $\hat{d}$, $\hat{e}$, and $\hat{b}^P$ and the value functions V, V^E, V^P, and V^D solve the Bellman equations (3), (4), (7), (5), and (6).

ii. given policy rules $\hat{d}$, $\hat{e}$, and $\hat{b}^P$,the bond price function q satisfies condition (10).

4   Calibration

Table 2 presents the benchmark values given to all the parameters in the model. A period in the model refers to a quarter. The coefficient of relative risk aversion is set equal to 2, the risk-free interest rate is set equal to 1 percent, and the discount factor is set equal to 0.975. These are standard values in quantitative business cycle and sovereign default studies. The average duration of sovereign default events is three years ($\psi^d = 0.083$), in line with the duration estimated by Dias and Richmond (2007).

   We use Mexico as reference for the calibration. The reason is that Mexico is an archetypical emerging economy and is a common case study in previous work on emerging economies (see, for example, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006)) The parameter values that govern the income process are estimated using GDI data from the first quarter of 1980 to the fourth quarter of 2011. We set $\delta = 3.3$ percent. With this value, bonds have an average duration of 5 years in the simulations, which is roughly the average debt duration observed in Mexico according to Cruces et al. (2002).[8]

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[8]We use the Maculae definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+r^*}{\delta+r^*}$, where r^* denotes the constant per-period yield delivered by the bond. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of foreign sovereign debt in emerging economies—in 2000—of 4.77 years, with a standard deviation of 1.52.

Risk aversion	γ	2
Risk-free rate	r	0.01
Discount factor	β	0.975
Probability of reentry after default	ψ	0.083
Probability of high exchange shock	π	0.02
Income autocorrelation coefficient	ρ	0.94
Standard deviation of innovations	σ_ϵ	1.5%
Mean log income	μ	$(\text{-}1/2)\sigma_\epsilon^2$
Debt duration	δ	0.033
Income cost of defaulting	d_0	-1.01683
Income cost of defaulting	d_1	1.18961

Table 2: Benchmark parameter values.

We use as a calibration target the share of voluntary debt exchanges to default episodes. Debt restructurings that fit within our definition of voluntary exchange are likely to be considered sovereign defaults. For instance, this is often the case for the four restructurings described in Section 2 (for example, Cruces and Trebesch (2013) describe these episodes as default). Credit-rating agencies consider a "technical" default an episode in which the sovereign makes a restructuring offer that contains terms less favorable than the original debt, leaving considerable ambiguity on how to determine whether terms offered to investors were favorable. The haircut definition presented in Section 2 as a measure of debt relief is often considered a measure of investor losses (in fact, the objective of Sturzenegger and Zettelmeyer (2005) when they introduce this definition is to measure investors' losses). This interpretation of the haircut would be consistent with considering all voluntary exchanges as sovereign defaults, as we do for interpreting simulation results.

We calibrate the values of d_0, d_1, and π (the probability of a high exchange cost) targeting the average levels of spread and debt between 1995 (do to data availability) and 2011, and a

	Targets	Simulations
Mean Debt-to-GDI	44	41
Mean r_s	3.40	3.34
Voluntary debt exchanges / defaults	0.33	0.36

Table 3: Calibration targets. Moments for the simulations correspond to the mean value of each moment in 375 simulation samples.

share of voluntary debt exchanges to default episodes of 33 percent. Cruces and Trebesch (2013) report that 43 percent of the default episodes they study occur before the government defaults on debt payments. Since we do not know whether these default episodes implied capital gains for bond holders, 43 percent is an upper bound for the share of voluntary debt exchanges to default episodes.

Table 3 presents the calibration targets and the corresponding moments in the simulations. The recursive problem is solved using value function iteration. The approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough to make the maximum deviation between the value and bond price functions in the first and second period small enough. We solve for the optimal debt level in each state by searching over a grid of debt levels and then using the best point on that grid as an initial guess in a nonlinear optimization routine. The value functions V^E, V^P, and V^D, and the bond price function q are approximated using linear interpolation over y and cubic spline interpolation over debt levels.[9] We use 50 grid points for both debt and income. Expectations are calculated using 75 quadrature points for the income shock.

In order to compute the sovereign spread implicit in a bond price, we first compute the yield i an investor would earn if it holds the bond to maturity (forever in the case of our bonds) and no default is declared. This yield satisfies

$$q_t^M = \frac{1}{(1+i)} + \sum_{j=1}^{\infty} \frac{(1-\delta)^j}{(1+i)^{j+1}}.$$

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[9]Hatchondo et al. (2010) discuss the advantages of using interpolation and solving for the equilibrium of a finite-horizon economy.

The sovereign spread is the difference between the yield i and the risk-free rate r. We report the annualized spread

$$r_t^s = \left(\frac{1+i}{1+r} \right)^4 - 1.$$

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, i.e., $b_t^M (1+r)(\delta + r)^{-1}$, where $\delta = 1$ for one-period bonds. We report debt levels as a percentage of annualized income.

5   Results

We first show that the model simulations reproduce features of the data. We then discuss the opportunities for voluntary debt exchanges in the simulations, the gains at the moment of a voluntary exchange, and ex-ante gains from avoiding voluntary debt exchanges.

5.1   Non-targeted simulation moments

Table 4 reports non-targeted simulation moments. The table shows that allowing for voluntary exchanges does not damage the default model's ability to account for key features of business cycles statistics in emerging economies (Aguiar and Gopinath (2006), Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006)). Simulations feature a volatile and countercyclical spread that leads to more borrowing in good times compared with bad times, as reflected by a volatile and countercyclical trade balance.

5.2   Voluntary debt exchanges

Opportunities for voluntary debt exchanges arise in 19 out of 100 of the simulation periods. That is, in 19 percent of the simulation periods the initial debt level b is higher than the debt level that maximizes the market value of debt, $\hat{b}^E(y)$.

There are two reasons why the initial debt level b may be higher than $\hat{b}^E(y)$. First, the government may choose to end a period with a debt level that has a negative marginal effect on the market value of debt claims outstanding at the end of the period. Assuming differentiability,

	Mexico	Simulations
$\sigma(r_s)$	1.6	1.3
$\rho(r_s, y)$	-0.5	-0.7
$\sigma(tb)$	1.4	0.7
$\rho(tb, y)$	0.7	-0.8
$\rho(r_s, tb)$	0.8	0.8

Table 4: Non-targeted simulation moments. The standard deviation of x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $\rho(x,z)$. Moments for the simulations correspond to the mean value of each moment in 375 simulation samples.

the first order condition for debt accumulation for states in which the government repays its debt and does not launch an exchange is characterized by:

$$\frac{d\left(q\left(b',y\right)b'\right)}{db'} = \frac{-\mathbb{E}_{(y',\theta')|y}V_1(b',y',\theta')}{u'(c)} + q_1\left(b',y'\right)\left(1-\delta\right)b \tag{11}$$

The left hand side consists of the marginal effect of the last bond issued in the period on the market value of debt claims outstanding at the end of the period. In our numerical solution, the first term of the right hand side is always positive while the second term is negative. When the government issues debt, it is concerned about the market value of the debt claims that are issued in the current period, not on the value of debt claims issued in past periods (see Hatchondo et al. (2011)). We find however that periods with a negative marginal effect of debt on the market value of debt claims at the end of the period are rare in the simulations. They are only observed in 4 percent of the simulation periods. Furthermore, b' is typically very close to b in these periods. Figure 3 presents an example of these situations in the simulations. At the optimal debt end-of-period debt level, the market value of the end-of-period debt is decreasing in debt. Notice that this would not happen in a model with one-period bonds (where the issuance proceeds coincide with the value of the debt stock).

In addition, the initial debt level may be higher than $\hat{b}^E(y)$ because of a sufficiently negative income shock. Even when the government chooses $\hat{b}^P(b,y) < \hat{b}^E(y)$, it may still be that next period $\hat{b}^P(b,y) > \hat{b}^E(y')$. The left panel of Figure 4 depicts the market value of debt claims for
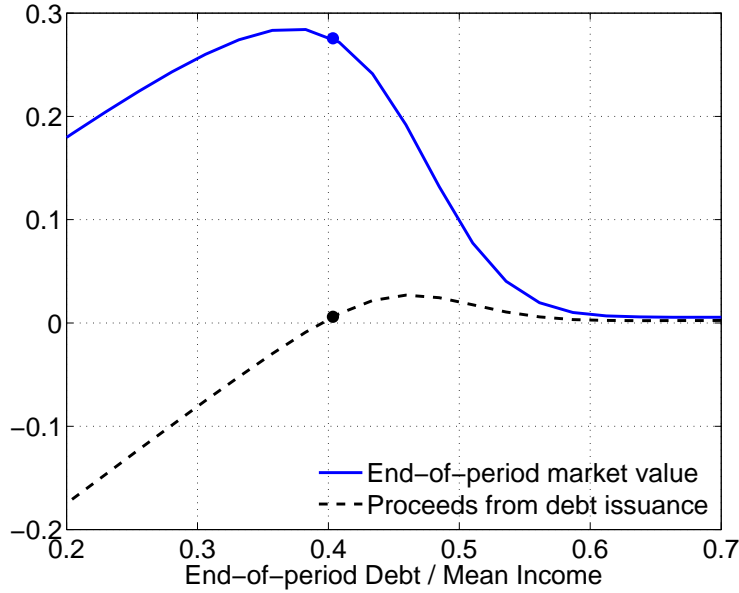
Figure 3: The end-of-period market value of debt claims is defined as $q(b', y)b'$. The proceeds of debt issuances consist of $q(b', y)(b' - (1 - \delta)b)$. The graph assumes that the state is characterized by a debt level of 41 % of mean income and that the current income realization is one standard deviation below the mean. The solid dots represent the optimal decision of a sovereign that cannot exchange debt in the period.

two income levels. The Figure shows that declines in income shift the market value curve to the origin and reduce the debt level that maximizes the value of debt. This occurs because, as is standard in models of sovereign default with pro-cyclical spreads, decreases in current income anticipate higher incentives to default in subsequent periods. This is captured by the shape of the bond price menu available to the government when it issues debt (illustrated in the right panel of Figure 4).

Nearly all outright defaults in the simulations would have been avoided with a voluntary exchange. Figure 5 shows that the government only prefers an outright default over a voluntary exchange for extremely low income levels, for which the income cost of the outright default is low. In our stylized model, the values of a voluntary exchange and an outright default presented in equation (6) and (7) are both independent from the debt level. Thus, the choice between a voluntary exchange and an outright default does not depend on the debt level.

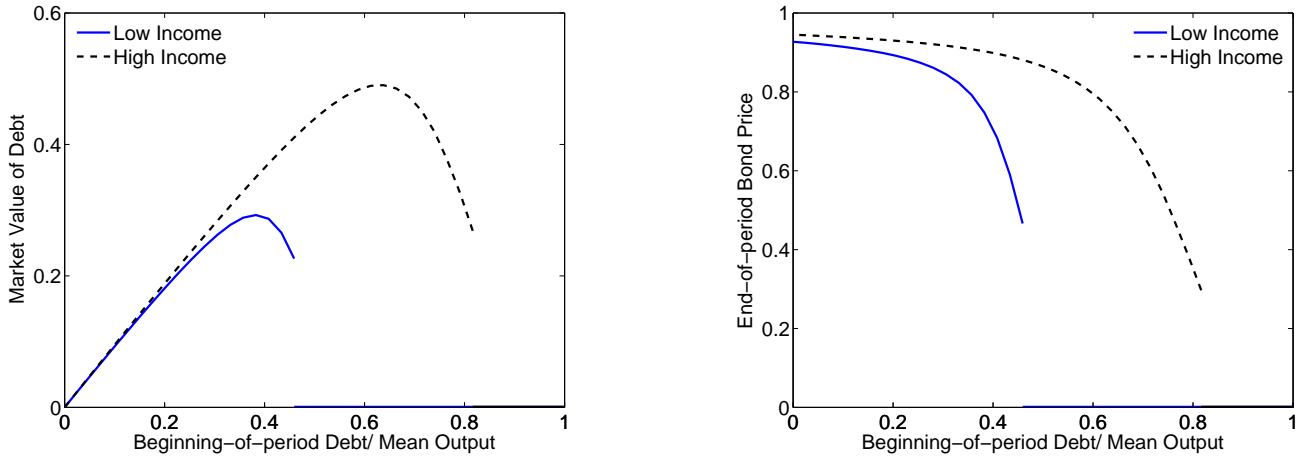In addition, Figure 5 shows that for some state realizations the government would choose to

Figure 4: The left panel plots $MV(b, y)$ for an income realization one standard deviation below the unconditional mean (Low income) and for an income realization one standard deviation above the unconditional mean (High income). The right panel plots the end-of-period price when the cost of an exchange is high: $(1 - \hat{d}(b, y, H))q(\hat{b}^P(b, y), y)$ for the same two income levels.

conduct a voluntary exchange even if the impossibility of an exchange would not have triggered an outright default. Since states that would trigger an outright default are unlikely, only 4 percent of the voluntary debt exchanges in the simulations prevented an outright default in the period of the exchange.

5.3 Gains at the moment of a voluntary debt exchange

Table 5 reports the average capital gain, haircut, and welfare gain for voluntary exchanges. Consistently with our definition of voluntary exchanges, theses episodes produce gains for both lenders (as indicated by positive capital gains) and the government (as indicated by positive haircuts and welfare gains). We measure welfare gains as the constant proportional change in consumption that would leave domestic consumers indifferent between not having the option to conduct a voluntary exchange ($\theta = H$) and having this option ($\theta = L$):

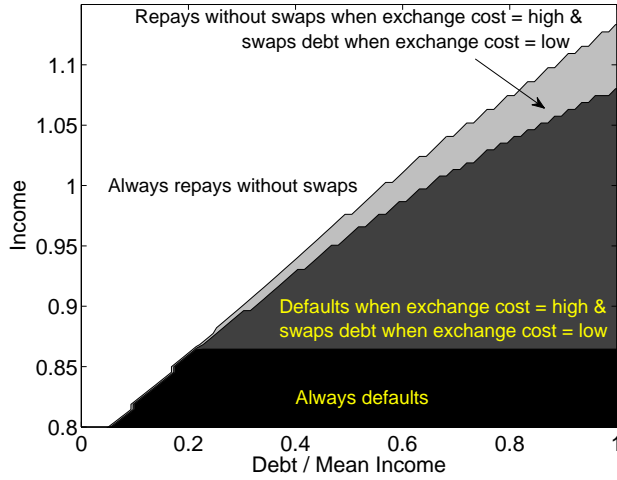$$\left(\frac{V(b, y, L)}{V(b, y, H)} \right)^{\frac{1}{1-\gamma}} - 1.$$

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Figure 5: Equilibrium voluntary exchange and outright default decisions. For each debt level, the figure presents the maximum income level for which the government would choose to default. Thus, the combinations of debt and income levels for which the government would choose a voluntary exchange are between the voluntary exchange and outright default income thresholds.

	Exchanges that prevent an outright default	Other exchanges
Average capital gain	155%	3%
Average haircut	24%	5%
Average welfare gain	0.7%	0.3%

Table 5: Gains from a voluntary debt exchange. Capital gains for exchanges that prevent an outright default are computed using the market value of the debt stock the quarter before the exchange.

5.4 Ex-ante gains from avoiding voluntary debt exchanges

The government decisions to conduct a voluntary debt exchange are not optimal from an ex-ante perspective (evaluated at the mean debt level observed in the simulations). This is illustrated in Figure 6. Thus, even though voluntary exchanges are Pareto improving at the time of the restructuring (Table 5), eliminating the possibility of conducting a voluntary exchange improves ex-ante welfare.

The negative effect of debt exchanges on welfare derives from the deterioration in the terms at which the government can borrow form international markets. Given that creditors are forward
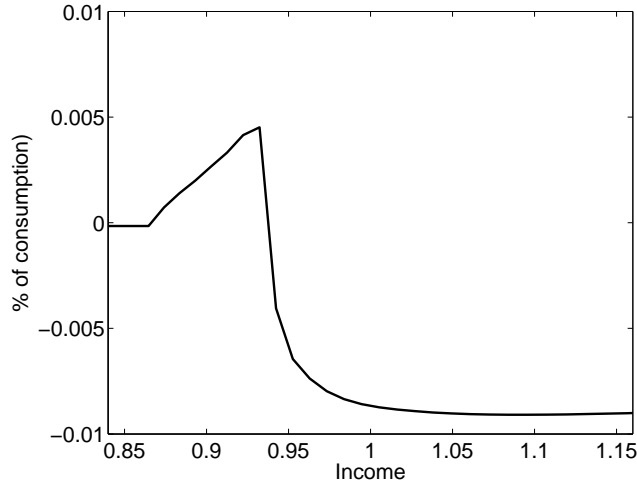
Figure 6: Welfare gains from eliminating the possibility of voluntary exchanges. Welfare gain is evaluated at a debt level equal to the mean debt observed in the simulations and before current exchange cost realization.

looking and forecast that in some future states their debt claims are going to be partially written off, this limits the intertemporal reallocation of resources for the sovereign government, i.e., the government cannot bring as many resources forward as it can do in an economy without exchanges.

To illustrate that point, we simulate the economy with and without exchange opportunities to create $1,000$ samples of 100 periods each. In all of them the government is assumed to start with the mean level of debt (41% of mean annual income) and the mean income level. Eliminating voluntary debt exchanges reduces the average sovereign spread by 40 basis points (from 3.44% to 2.93%) and the volatility of the spread by 17% (from 2.29% to 1.89%). These declines reflect the lower frequency of outright defaults (they reduce from 2.45 to 2.27 times every 100 years) and voluntary debt exchanges (they drop from 1.54 per 100 years to zero). These results are summarized in Table 6.[10]

Our findings are consistent with governments' reluctance to issue easier to restructure debt.

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[10]We also report results for simulations where the economy starts with the mean level of debt and with a level of income one standard deviation above the mean.

	$y_{initial} = y_{mean}$		$y_{initial} = y_{mean} + \sigma_y$	
	Benchmark	No exchanges	Benchmark	No exchanges
Mean Debt-to-GDP	37.72	37.91	38.77	39.01
Mean r_s	3.44	3.02	3.33	2.93
$\sigma(r_s)$	2.29	1.89	2.21	1.83
Outright defaults per 100 years	2.45	2.27	2.19	2.05
Voluntary debt exchanges per 100 years	1.54	0	1.44	0

Table 6: Simulations results obtained assuming that the government cannot conduct voluntary exchanges. Initial Debt-to-GDP level= 41%.

6 Conclusions

This paper discusses the possibility of voluntary debt exchanges defined as debt restructurings that occur before an outright default and produce both debt reductions and capital gains for holders of restructured debt. In contrast with standard debt overhang arguments, the paper presents a theory where Pareto gains are possible without any effect of the debt reduction in production. These gains arise only because of the decline of sovereign risk implied by the debt reduction. In this theory, voluntary exchanges are possible after negative income shocks, but do not require an outright default to be imminent. Most voluntary exchanges happen in periods in which the government would not default. The paper also shows that in spite of the Pareto improvement at the time of voluntary exchanges, the government may prefer an environment in which conducting these exchanges is more difficult.

Overall, the paper presents an attempt to enrich the understanding of gains from renegotiation between creditors and debtors but also highlight a time inconsistency problem. The discussion of these issues would certainly benefits from a more thorough analysis of the possibility of Pareto gains in past sovereign debt restructuring experiences, and richer theories that, for instance, endogenize renegotiations both after outright defaults and in voluntary debt exchanges.[11]

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[11]Arguably, mechanisims that facilitate voluntary debt exchanges may also facilitate the restructurings that follow after an outright default. We abstract from that possibility. Yue (2010) and Benjamin and Wright (2008)

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A   Appendix

A.1   Calculation of capital gains for debt restructurings

We next describe the calculation of capital gains presented in Figure 2 and Table 1. For each restructuring, we report the average capital gain of for all bonds for which we found the necessary price data. Capital gains were calculated for each restructured bond. The weight of each restructured bond in the average is given by the outstanding nominal debt of each bond at the moment of the exchange divided by the total outstanding nominal debt. When comparing bond prices from different dates, bond prices are discounted using the underlying discount rate at the moment of each restructuring, as reported by Trebesch and Cruces (2013). The conditions of each restructuring were obtained from Bloomberg and Zettelmeyer (2007). The announcement date of the debt exchange is not necessarily the formal proposal. It may reflect informal meetings that created expectations of a debt exchange. We present next a brief description of the four exchanges studied in this paper.

A.2   Dominican Republic

Negotiations with the Paris Club in April 2004 allowed the Dominican government to reschedule its bilateral debt service due in 2004, but required the government to seek comparability of treatment with its private creditors. After some delay, the Dominican Senate approved on March 30, 2005, the authorization for the executive branch to proceed with the exchange offer. From early on, consultations took place with investors. We computed capital gains for debt representing 55 percent of the outstanding face value.

A.3   Pakistan

Pakistan negotiated a Paris Club restructuring in January of 1999 which required the country to seek comparable debt relief from private creditors, and in particular, to restructure its international bonds. By July, the government had signed a rescheduling with commercial banks. On November 15, Pakistan launched a bond exchange, ahead of a Paris club deadline that required it to show "progress" in negotiations with bondholders by the end of 1999. The exchange involved

swapping three bonds. All three were to be exchanged for a new bond. There was no nominal haircut. In fact, holders of the two bonds with the shorter average life received slightly more in nominal terms than under the original instruments. The new bond was also expected to be more liquid than the old bonds. We computed capital gains for debt representing 75 percent of the outstanding face value.

A.4 Ukraine

In years 1998 and 1999, a succession of selective restructurings with specific creditors was completed in order to bridge mounting liquidity needs. Although restructurings of 1998-99 provided some immediate cash flow relief, they also created large payments obligations for 2000 and 2001. On February 4, 2000, Ukraine launched a comprehensive exchange offer involving all outstanding commercial bonds. Creditors could choose between two 7-year coupon amortization bonds denominated either in Euros or U.S. dollars, to be issued under English law. The exchange offer established a minimum participation threshold of 85 percent among the holders of bonds maturing in 2000 and 2001. We computed capital gains for debt representing 29 percent of the outstanding face value.

A.5 Uruguay

Uruguay restructured its external debt in 2003 without asking creditors to accept a reduction on the principal. The exchange targeted all traded debt (about half of total sovereign debt) and offered most bondholders a choice between two options. Uruguay used a voluntary mechanism. A combination of the use of exit consents to change the non-payment terms of the old bonds and regulatory incentives contributed to the high acceptance rate, 93%, constituted by 98% of local security holders and approximately 85% of international security holders. We computed capital gains for debt representing 51 percent of the outstanding face value.