The Relationship between Systematic Risk Proportion and the Slope of the Implied Volatility Curve

by

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Abstract:

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Abstract:

In this empirical study, we examine one of the fundamental assumptions of the Black-Scholes Option Pricing Theory; that the proportion of systematic risk of total risk has no effect on intrinsic option prices. This hypothesis was first proposed by Jin-Chuan Duan and Jason Wei (2006) and we will use a similar methodology in testing the hypothesis that the slope of implied volatility curve is related to the proportion of systematic risk of the underlying asset. We will use daily option quotes on the components of the S&P100 index in order to explore the relationship between the systematic risk proportion and the slope of the implied volatility curve: specifically, we hope to establish that it has a direct impact in that the higher the systematic risk proportion, the steeper the slope of the implied volatility curve.

1. Introduction

The Black-Scholes Option Pricing Theory (1973) is one of the cornerstones of modern continuous time finance and is used in a multitude of applications. For all of its elegance, there is a huge amount of evidence in academic literature that the Black-Scholes model fails empirically; for instance, options with different strike prices on the same underlying stock give different implied volatilities (we should expect them to be the same) – a phenomenon known as the volatility smile. Another similar pattern is noticed when looking at options on the same underlying security but with different maturities. In this case, the term structure of volatility is not constant but rather flattens with maturity. These two problems with the volatility surface have been widely studied (for example, Dumas, Fleming and Whaley (1998)).

These two problems can be traced to two of the key assumptions of the Black Scholes model: first, the stock price follows a continuous path through time (the geometric Brownian motion assumption) and that the instantaneous volatility of the stock rate of return is nonstochastic.

There are a number of key assumptions behind the Black Scholes model and one of the most debated assumptions is that of assuming the price of the underlying security follows a geometric Brownian motion Wt, in particular with constant drift μ (Expected gain) and volatility σ:

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t$$

Empirically, when used to price options, the volatility assumption is normally obtained through some historic average of total volatility. Under the CAPM theory, risk exists on two dimensions: systematic and unsystematic. If we break down total volatility into its systematic and unsystematic components, we may find that it is easier to estimate the future systematic component of total volatility as it is much harder to make assertions about future unsystematic (firm specific) volatilities than it is about the general market.

Since the Black Scholes paper, there has been much theoretical work done in improving option pricing models by relaxing the assumption about the underlying asset's return distribution: Cox and Ross (1975) put forward a pure jump model and Merton (1976) a mixed diffusion jump model, both of which relaxed the continuity assumption. The constant volatility assumption was relaxed by Cox and Ross (1976) with the constant elasticity of variance diffusion model, Geske (1979) compound option diffusion model and Rubenstein (1985) with the displaced diffusion model. Other models combining the two have since been proposed including Heston (1993). Generally, these can be classed as models including jump-diffusion processes, models with stochastic volatility and local volatility models. These are all important developments, although they can be very hard to implement and may not address all of the shortcomings of the Black-Scholes Option Pricing Model.

There are some other empirical problems that appear to be at odd with the risk neutral pricing assumption of Black-Scholes and cannot easily be dealt with: that the Black-Scholes implied volatilities tend to be higher than the historical volatility, that index options have a more pronounced volatility smile / smirk than individual equity options and that the risk neutral return distribution's negative skewness is more pronounced that that of the physical return distribution (Bakshi and Madan (2006), Dennis and Mayhew (1998)).

So, in testing to see if the proportion of systematic risk is correlated with the level and slope of implied volatility, we hope to gain insight into whether or not the systematic risk proportion as a variable can partially explain some of the empirically observed shortcomings of the Black-Scholes model. There has already been some work in this area: for example, Dennis and Mayhew (2002) established empirically the link between risk-neutral skewness and systematic risk; Bakshi, Madan, Kapadia (2003) demonstrated the empirical pricing differences of individual stock options and index options is related to the differences in risk-neutral skewness and kurtosis. Duan and Wei (2006) argued that implied volatility, risk-neutral skewness and kurtosis are all tied to systematic risk, which is consistent with the local risk-neutral valuation theory of Duan (1995) which tied the option price of assets exhibiting a GARCH-type feature to the underlying asset's risk premium.

Duan and Wei (2006) conducted tests and concluded that systematic risk proportion is related to both the slope and the level of the implied volatility curve and proposed that systematic risk is priced into options. Specifically, as the risk-neutral return distribution is different from the physical return distribution by a risk premium term, the systematic risk proportion may help to explain some of the empirically observed irregularities. This may be intuitive on the surface: for example, equity indices are expected to have a higher amount of systematic risk versus individual stocks which is consistent with their results: when adjusting for different levels of total risk, a higher amount of systematic risk leads to a higher level of implied volatility and a steeper implied volatility curve.

In this study, we hope to verify these results by using a different data set and testing slightly different hypotheses.

Our empirical analysis will begin using option data sourced from OptionMetrics (which contains data on all US listed index and equity options and the corresponding underlying assets - US exchange listed and NASDAQ equities and market indices). The option data is on a daily basis from the period of Jan 1st 2000 to December 31st 2004 and consists of over 14 million options written on the components of the S&P 100 Index. We test one of the null hypotheses in Duan and Wei:

- The slope of the implied volatility curve is not related to the systematic risk proportion.

2. Methodology

As previously mentioned, the Black-Scholes model (1973) states that option prices only depend on the over level of risk of the underlying asset and not on the level of systematic risk or the risk premium. Duan and Wei give the example: *imagine two stocks that are identical in every aspect except for the level of systematic risk or risk premium. The prices of options on these two stocks must be equal if the terms of the options are identical. When these option prices are converted into implied volatilities, they should not be related to systematic risk at all.*

In essence, we want to test the hypothesis:

a. The slope of the implied volatility curve of the options on the j^{th} stock is unrelated to the systematic risk proportion b_j.

The data set consists of over 14 million observations from daily option data on the component stocks of the S&P 100 Index for the period January 1st 2000 until December 31st 2004 (five years worth of data or a total of 260 weeks). This data was sourced from OptionMetrics and included measures of implied volatility, time to expiration and strike. The options are American style and traded on the Chicago Board of Options Exchange. Daily stock prices and returns for the S&P 100 component stocks for the period January 1st 1999 until December 31st 2004 were sourced from the Center for Research in Security Prices ("CRSP") and are used to calculate historical volatilities, betas, and hence the proportion of systematic risk. There are several calculation definitions to introduce before conducting tests of hypotheses. First off, historical volatility $\sigma_{historical}$ is calculated on a daily basis for each stock on a 22 (one month) rolling window off the daily standard deviations of the returns.

Systematic risk is calculated using a beta in a normal CAPM fashion that regresses the returns of the individual components of the S&P100 against the returns of the S&P500 index over a 250 day (one year) rolling window via ordinary least squares regression. Thus for stock j,

$$R_{jt} = \alpha_j + \beta_j R_{mt} + \varepsilon_{jt} \qquad [1]$$

From this, we can calculate

$$\text{Systematic Risk} = \beta^2_j\, \sigma^2_m \qquad [2]$$

$$\text{Hence: Total Risk} = \sigma^2_j \qquad [3]$$

Hence the systematic risk proportion is

$$b_j = \beta^2_j\, \sigma^2_m / \sigma^2_j \qquad [4]$$

To test our hypothesis, we use a similar method to Duan Wei (2006) and perform a two-pass regression on our data set. We first need to obtain time series of estimates for the level and slope of the implied volatility curve. These results are then used to run cross-sectional regressions to test our hypotheses and see if they are related to the systematic risk proportion as defined above. The cross-sectional regression is repeated over several time periods and the time-averaged regression coefficients are then used to determine whether or not we reject our hypotheses.

When calculating the level and slope of the implied volatility curve, we need to define the number of non-overlapping windows. Because of the number of options, we have opted for a one week window. We also define a moneyness function:

$$M = \ln(S/K) / \sigma\sqrt{T} \qquad [5]$$

We split all of the option data into two groups: the ‘left hand side’ with $S/K < 1$ (corresponding to out of the money calls or in the money puts) and those on the ‘right hand side’ with $S/k > 1$ (in the money calls or out of the money puts). We then split these into weekly periods and regress the implied volatility against the moneyness function defined above for the each of the j[th] stock:

$$\sigma_{jk}^{implied} = \alpha_{0j} + \alpha_{1j} (\ln(S_{jk}/K_{jk}) / \sigma_j^{historical}\sqrt{T_{jk}}) \text{ for } k = 1, 2\ldots I_j [6]$$

for 260 weeks. In the above notation, I_j signifies the number of options for the j[th] stock.

Here, α_{0j} represents the level of the implied volatility curve and α_{1j} the slope of the implied volatility curve unadjusted for different levels of systematic risk across time. We will have 261 observations for each α_{0j} and α_{1j} for each of the 100 stocks. It is worth noting that this is a departure from the method of Duan Wei (2006) which tests the following equation over four different moneyness buckets:

$$\sigma_{jk}^{imp} - \sigma_j^{his} = a_{0j} + a_{1j}(y_{jk} - \bar{y}_j) + \varepsilon_{jk}, \qquad k = 1, 2, ..., I_j,$$

The intercept α0j and regression coefficient a1j are measures of the level and the slope of the implied volatility for a particular moneyness bucket, after adjusting for the j-th stock's total risk. By subtracting out some proxy of historic volatility, this allows the regression to make tests about the level of the slope which we have chosen to omit in this study.

For the cross-sectional regressions, we will perform two different regressions for each of the 261 non-overlapping periods using α_{1j} from the first regression as dependent variables for each of the j = 1,2,…, 100. Measures for skewness and kurtosis are calculated off a 250 day window of daily data.

$$\alpha_{1j} = \gamma_0 + \gamma_1 * b_j + \xi_j \qquad [7]$$

$$\alpha_{1j} = \gamma_0 + \gamma_1 * b_j + \gamma_2 * \text{Skew}_j + \gamma_3 * \text{Kurtosis}_j + \xi_j \qquad [8]$$

The time series of these 261 regression coefficients are averaged and a corresponding t-statistic is calculated. Equation [7] represents an unconditional test of

Hypothesis (a) that the slope of the implied volatility curve of the options on the j^{th} stock is unrelated to the systematic risk proportion b_j. We should not reject it if $\gamma_1 = 0$.

Equation [8] represents a conditional test of Hypothesis (a) controlling for the effects of the risk-neutral skewness and kurtosis, and we should obtain $\gamma_1 = 0$ if the systematic risk proportion exerts no effect once the influence of the risk-neutral skewness and kurtosis is considered.

3. Data Analysis

We expect to find that a strong link exists between the slope of the implied volatility and the level of systematic proportion. The most striking feature is that the greater the proportion of systematic risk, the greater the slope of the implied volatility curve.

We expect to reject the first hypothesis. We have already mentioned that a key point of contention in the original Black Scholes model is that the underlying asset's price follows geometric Brownian motion with constant drift and standard deviation, implying that the returns on the underlying security is lognormally distributed. It is a well established fact that the return distribution of the market is leptokurtic (TG Andersen, T Bollerslev, FX Diebold, H Ebens 2000). If we were to include this in the option pricing model (and some have, with random jumps built in to the underlying asset price movement) we would see precisely the same volatility smile. Hence one reason for the volatility smile is the leptokurtic distribution of underlying asset returns. The greater the systematic risk proportion, the greater the exposure the underlying asset has to the market return distribution (and hence exposure to the leptokurtic return distribution) and the greater the effect this leptokurtic distribution will have on the slope of the implied volatility curve.

Another possible explanation lies with the empirically established fact that market returns are negatively skewed (Berd, Engle, Voranov 2005). This empirically observed physical return distribution is in direct conflict with the Black Scholes assumption with the risk neutral distribution; specifically, the physical return distribution differs from the risk neutral distribution by a risk premium term, resulting in the negative skew. Again, a higher amount of systematic risk means that trends in the market returns (specifically, the negative skewness) will be reflected in a more pronounced affect on the implied volatility curve – so that with a greater level of systematic risk, the steeper the slope of the implied volatility curve will be.

Our second hypothesis tests whether or not the same relationship between the systematic risk proportion and the slope of the implied volatility curve still exists even when we control for the effects of negative skewness and leptokurtosis. If we reject this hypothesis, then we can show that systematic risk proportion affects the slope of the implied volatility curve beyond merely increasing the exposure of the curve to observed differences between the physical return distribution and the risk neutral measure assumed by Black Scholes.

Our results firmly reject both hypotheses: clearly, there is a strong relationship between the systematic risk proportion and the slope of the implied volatility curve as proxied by the moneyness function defined previously. Additionally, there are a number of interesting results.

The results testing the slope of the implied volatility and the systematic risk proportion are as follows. We obtained a co-efficient of -0.005523442 with a T-statistic of -0.80773524 for the left side of the curve with S/K < 1 (corresponding to out of the money calls) and a coefficient of 0.01171764 with a T-statistic of 2.380935732 for the section of the curve S/K > 1.

Test 1	**Full Sample**	
	Coefficient	T-statistic
S/K < 1	-0.00552	-0.80774
S/K > 1	0.01172	2.38094

This seems to indicate that there is a link between the amount of systematic risk and the slope of the implied volatility curve on both the left and right sides of the curve but with a stronger link on the right side of the curve which corresponds to in the money calls. These results are exactly as we expect: because of the negative skew of the underlying physical return distribution (and to a lesser degree, the leptokurtic nature of it), we expect systematic risk proportion to have a greater effect on in the money calls. Although we normally we normally think of implied volatility curves for equity options as being downsloping on the left with a turning point on the right, because of the way we have defined our moneyness function (with ln(S/K) rather than the conventional K/S), we expect to have a stronger relationship and a positive coefficient for our right hand side (in the money calls).

Similarly, with the weaker effect for out of the money calls (corresponding to our left hand side of the curve ln(S/K), we have a negative sign as we may expect to see a slightly negative slope, so a higher systematic risk proportion will still increase the slope of the implied volatility curve. The other signs are clearly correct, as systematic risk proportion should always be non negative by definition, as we square the beta in the systematic risk proportion. It seems clear that systematic risk proportion does have an effect on the slope of the implied volatility curve; namely, the higher the systematic risk proportion, the steeper the slope of the implied volatility curve.

We also can examine the results of the second test that adjusts for the effects of skewness and kurtosis of the underlying return distribution. We obtained a co-efficient of -0.011348909 with a T-statistic of -1.674398784 for the left side of the curve and a co-efficient of 0.014941934 with a corresponding T-statistic of 2.858496766 for the right side of the curve.

Test 2	**Full Sample**					
	Coefficient	T-statistic	Kurtosis Coefficient	T-statistic	Skewness Coefficient	T-statistic
S/K < 1	-0.01135	-1.67440	-0.00003	-0.23848	0.00375	2.51896
S/K > 1	0.01494	2.85850	0.00054	3.98238	0.00076	0.79382

This further strengthens our conjecture that there exists a link between systematic risk proportion and the slope of the implied volatility curve - the coefficients on both the left and right hand side of the curve are significantly non-zero both in terms of the size as well as the T-statistic. Once again, the relationship is stronger for in the money calls (our right hand side), but also exists on the left hand side of the curve too.

The most startling result from this test is when controlling for the effects of the kurtosis and skewness of the underlying asset returns, the effect of systematic risk proportion on the slope of the implied volatility curve is even stronger than our first hypothesis results both in terms of the absolute value of the coefficients average and in terms of the T-statistics. Since we attributed part of the relationship between systematic risk proportion and the slope of the implied volatility curve to the skewness and kurtosis of the underlying return, it seems strange that the relationship still exists when controlling for the factors. Even more strange is that the relationship appears stronger with these factors.

There are a number of possible explanations for this. Firstly, the measures of skewness and kurtosis that were calculated based on a year long rolling window of daily returns for the underlying stock. Calculating these measures off daily data (regardless of the window size) may not be the best way. Theoretically, if the stock returns are really independent, identically distributed between each period, if we increase the length of each period, we would expect to see kurtosis and skewness to closer reflect a normal distribution by the central limit theorem. However, this is not the case. Because of a time aggregation effect, as the time period increases, we normally see less kurtosis but greater negative skew (for example, Berd, Engle, Voranov 2005). Hence in order to more accurately account for the effect of skewness and kurtosis of the underlying return distribution on the systematic risk proportion, we may redefine our measures for skewness and kurtosis (and possibly even our volatility measures) to be based off a yearly window of monthly data rather than daily data. If we do this, we should expect to observe greater negative skewness in the underlying return distribution, in which case we should expect the significance of the systematic risk proportion coefficient to be smaller after controlling for kurtosis and skewness.

We must also worry about our proxy of systematic risk proportion. Using the beta derived from a CAPM regression off one year of daily data may not be the best indicator of systematic risk. For example, if the market volatility is $\sigma_m = 0.2$ and there are two stocks, A and B, with $\sigma_A = 0.4$ and $\sigma_B = 0.5$. If the stocks' correlations with the market are $\rho_A = 0.75$ and $\rho_B = 0.60$, then the two stocks will have the same beta, 1.50, yet very different systematic risk proportions, 0.563 versus 0.360. We may use different other models for equity risk such as a three factor French Fama model (in fact, in Duan Wei, they recalculate tests using these risk factors). Instead, we can conduct the following tests which use beta instead of (or in conjunction with) systematic risk proportion:

$$\alpha_{1j} = \gamma_0 + \gamma_1 * \beta_j + \xi_j \qquad [9]$$

$$\alpha_{1j} = \gamma_0 + \gamma_1 * \beta_j + \gamma_2 * b_j + \gamma_3 * \text{Skew}_j + \gamma_4 * \text{Kurtosis}_j + \xi_j \qquad [10]$$

Our results are very similar: when looking at [9], the relationship between beta and the slope of the implied volatility curve we have significant relationships that indicate the higher the beta, the steeper the slope of the implied volatility curve. The only difference is that the effect seems stronger on the left hand side for out of the money calls which is a bit unusual.

Test 1	**Full Sample**	
	Coefficient	T-statistic
S/K < 1	-0.03812	-20.00439
S/K > 1	0.01382	8.91056

When we run regression [10], we observe similar patterns; namely, the same over all observation remains the same (the higher the beta, the steeper the slope of the implied volatility curve is). The unusual observation remains; namely, that after controlling for the skewness, kurtosis, and now beta, we still see a strong relationship between systematic risk proportion and the slope of the implied volatility curve.

Test 2	**Full Sample**		Kurtosis		Skewness	
	Coefficient	T-statistic	Coefficient	T-statistic	Coefficient	T-statistic
S/K < 1	-0.04023	-19.70191	-0.00067	-5.35851	0.00461	3.00874
S/K > 1	0.01531	9.80677	0.00074	5.82256	0.00036	0.38916

Test 3	**Full Sample**		Systematic Risk Proportion		Kurtosis		Skewness	
	Coefficient	T-statistic	Coefficient	T-statistic	Coefficient	T-statistic	Coefficient	T-statistic
S/K < 1	-0.07111	-19.06982	0.14333	15.93996	-0.00011	-0.78993	0.00250	1.84605
S/K > 1	0.03378	12.42330	-0.08530	-11.54820	0.00043	3.21525	0.00056	0.63562

There are a number of further tests we could carry out. For example, we have so far restricted ourselves to slope tests (testing the slope of the implied volatility curve). If we recalibrate our first regression to include a term that allows us to compare the level of the implied volatility curve in addition to the slope, we may be able to gain further insight into the affect of systematic risk proportion on implied volatility curves.

We can test the following hypothesis, similar to our original one:

b. The level of the implied volatility curve of the options on the j^{th} stock is unrelated to the systematic risk proportion b_j; and

We would run the following as our first regression:

$$\sigma_{jk}^{implied} - \sigma_j^{historical} = \alpha_{0j} + \alpha_{1j} (\ln(S_{jk}/K_{jk}) / \sigma_j^{historical}\sqrt{T_{jk}}) \text{ for } k = 1, 2\ldots I_j \qquad [11]$$

Here we subtract the historical volatility (alternatively, we could use the at the money volatility) in order to allow comparisons between different days and stocks, for which the historic volatility changes. We would then run the following tests for α_{0j}:

$$\alpha_{0j} = \gamma_0 + \gamma_1 * b_j + \xi_j \qquad [12]$$

$$\alpha_{0j} = \gamma_0 + \gamma_1 * b_j + \gamma_2 * \text{Skew}_j + \gamma_3 * \text{Kurtosis}_j + \xi_j \qquad [13]$$

It may also be interesting to see how maturity of different options affects the slope of the implied volatility curve. As time increases, we may expect to see more of the negative skewness affect the implied volatility curve. We could set this test up as follows:

$$\alpha_{1j} = \gamma_0 + \gamma_1 * b_j + T*\gamma_2 + \xi_j \qquad [14]$$

$$\alpha_{1j} = \gamma_0 + \gamma_1 * b_j + \gamma_2 * \text{Skew}_j + \gamma_3 * \text{Kurtosis}_j + T*\gamma_4 + \xi_j \qquad [15]$$

Here, T represents a dummy variable so that we can observe the impact of different maturities on the implied volatility curve. In addition, other versions of the second regression can include dummy variables for stock volatility or market volatility in addition to merely the proportion of systematic risk.

It is also important to comment on the nature of this data set vs the data set that used in Duan Wei. They covered the period of January 1st 1991 to December 31st 1995 which was showed a clear upward trend in the general market, with substantial volatility. In our data set, we cover the period January 31st 2000 to December 31st 2004 which is a low volatility period but a wide range of data with both the tech bust and subsequent pick up. Especially with the low market volatility, we may see abnormal amounts of idiosyncratic risk which may distort our results somewhat. Beyond merely timing differences, the scope of the data set is also very different. At first, I decided to broaden the data set to include not only the thirty most heavily traded options (by underlying), but to include all components of the S&P100. In retrospect, this may have been a mistake as not all underlying assets are heavily traded or have very liquid option markets. It is possible to rerun the tests using the thirty most heavily traded issuances – the results are as follows:

Test 1	**30 Largest Volume Sample**	
	Coefficient	T-statistic
	0.00105	0.02740
	0.16467	5.63102

Test 2	**30 Largest Volume Sample**					
			Kurtosis		Skewness	
	Coefficient	T-statistic	Coefficient	T-statistic	Coefficient	T-statistic
S/K < 1	-0.00433	-0.66287	-0.00106	-3.30023	0.00531	3.04015
S/K > 1	0.03823	7.64690	0.00034	1.29958	-0.00736	-5.96332

After running the tests on this smaller sample, we find that the results are more or less in line with our original results across the full sample. The effect is even stronger on the right hand side (for in the money calls) for both tests and we also see the same effect that the relationship is stronger after controlling for the kurtosis and skewness of the underlying return distribution. The only difference lies with the sign of the coefficient for the left hand side for the first test.

4. Conclusion

In conclusion, we have verified the result established in the Duan Wei paper that the level of systematic risk proportion has a direct effect on the slope of the implied volatility curve. This matches with our expectations; for example, we know that empirically, the physical return distribution of the underlying exhibits a number of features that distinguishes it from the risk neutral measure assumed by Black Scholes. Even adjusting for this in our second hypothesis test, we still find that there is a clear link between systematic risk proportion and the slope of the implied volatility curve. This effect remains across different specifications of the systematic risk component. This is consistent with some of the alternative option pricing models that have been developed, as some local volatility models (GARCH option pricing model, Duan 1995) predicts that a higher systematic risk proportion leads to a steeper slope in the implied volatility curve.

This effect is most pronounced on the right hand side of our curve, which corresponds to out of the money calls / in the money puts, which is consistent with our assumptions about the properties of the distribution of the underlying returns.

The effect observed in this study is a lot weaker than the original Duan paper and there is an unusual development in that after controlling for the skewness and kurtosis of the underlying physical return distribution, the effect of systematic risk proportion is even stronger on the slope of the implied volatility curve. Part of this can be explained by our testing procedures, specifically the ways in which skewness and kurtosis measures were calculated.

Hypothesis Test Results

Test 1	Full Sample		30 Largest Volume Sample	
	Coefficient	T-statistic	Coefficient	T-statistic
S/K < 1	-0.00552	-0.80774	0.00105	0.02740
S/K > 1	0.01172	2.38094	0.16467	5.63102

Test 2	Full Sample					
			Kurtosis		Skewness	
	Coefficient	T-statistic	Coefficient	T-statistic	Coefficient	T-statistic
S/K < 1	-0.01135	-1.67440	-0.00003	-0.23848	0.00375	2.51896
S/K > 1	0.01494	2.85850	0.00054	3.98238	0.00076	0.79382

Test 2	30 Largest Volume Sample					
			Kurtosis		Skewness	
	Coefficient	T-statistic	Coefficient	T-statistic	Coefficient	T-statistic
S/K < 1	-0.00433	-0.66287	-0.00106	-3.30023	0.00531	3.04015
S/K > 1	0.03823	7.64690	0.00034	1.29958	-0.00736	-5.96332

Further Test Results

Tests using Beta rather than Systematic Risk Proportion

Test 1 **Full Sample**

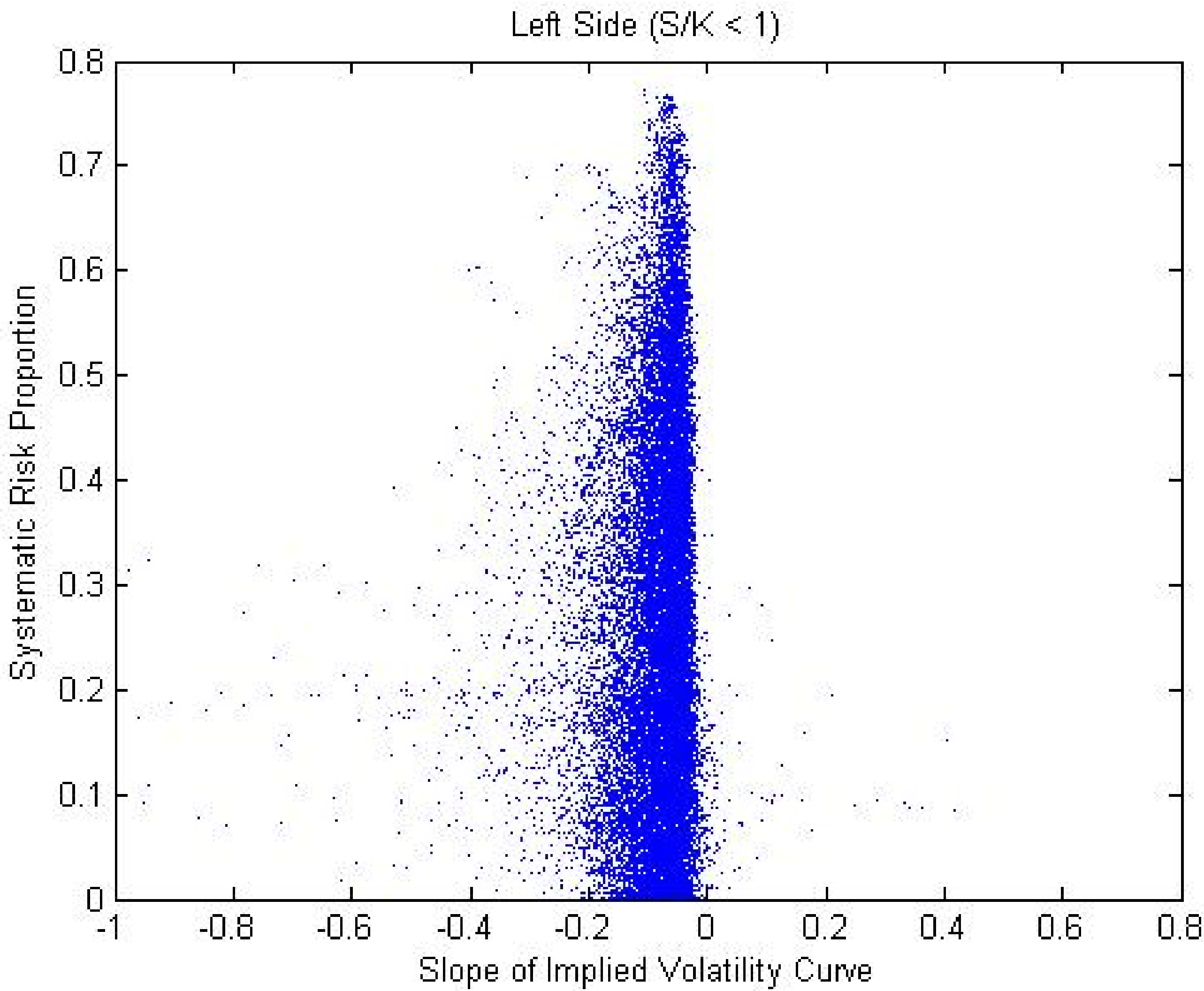
	Coefficient	T-statistic
S/K < 1	-0.03812	-20.00439
S/K > 1	0.01382	8.91056

Test 2 **Full Sample**

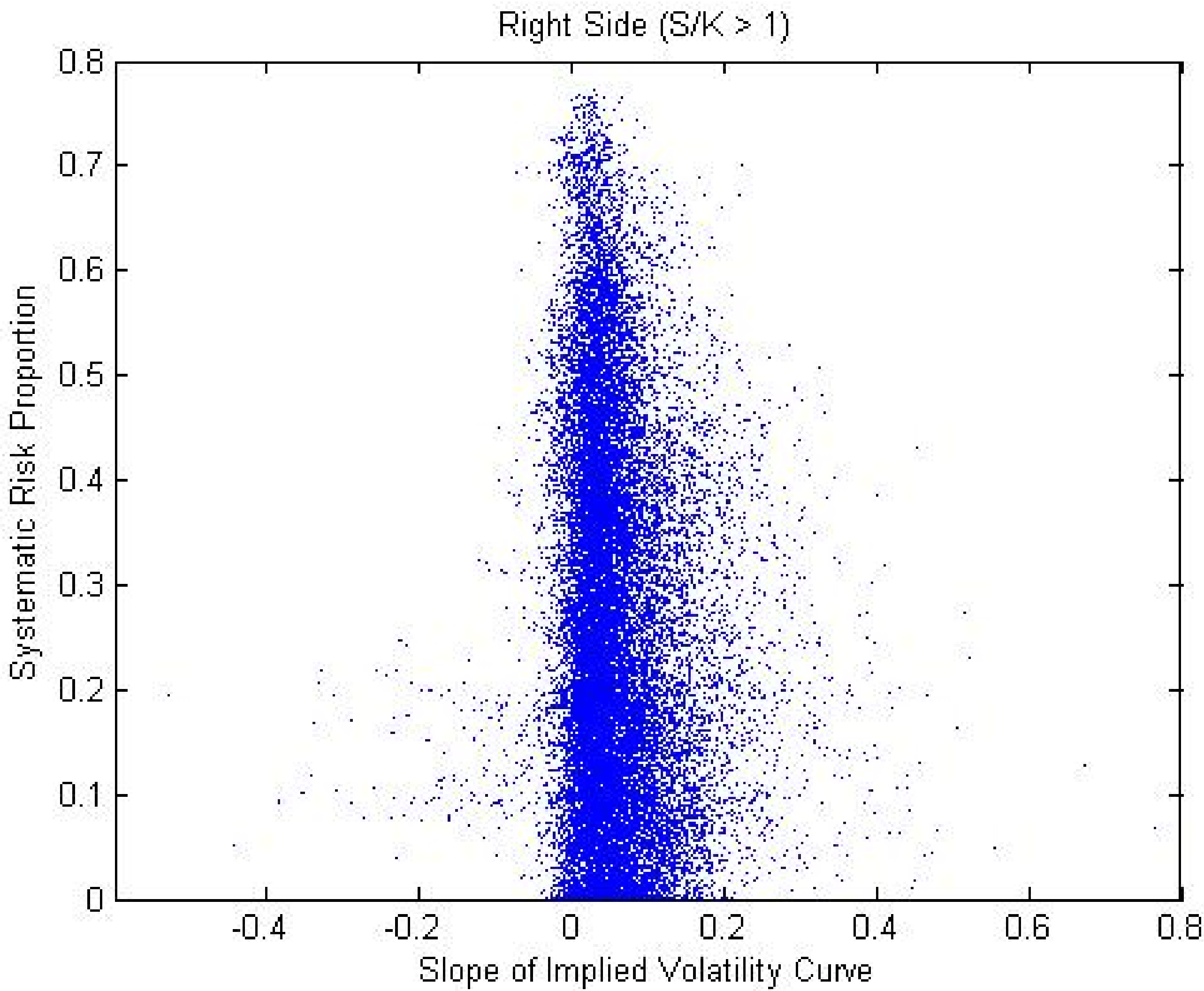
	Coefficient	T-statistic	Kurtosis Coefficient	T-statistic	Skewness Coefficient	T-statistic
S/K < 1	-0.04023	-19.70191	-0.00067	-5.35851	0.00461	3.00874
S/K > 1	0.01531	9.80677	0.00074	5.82256	0.00036	0.38916

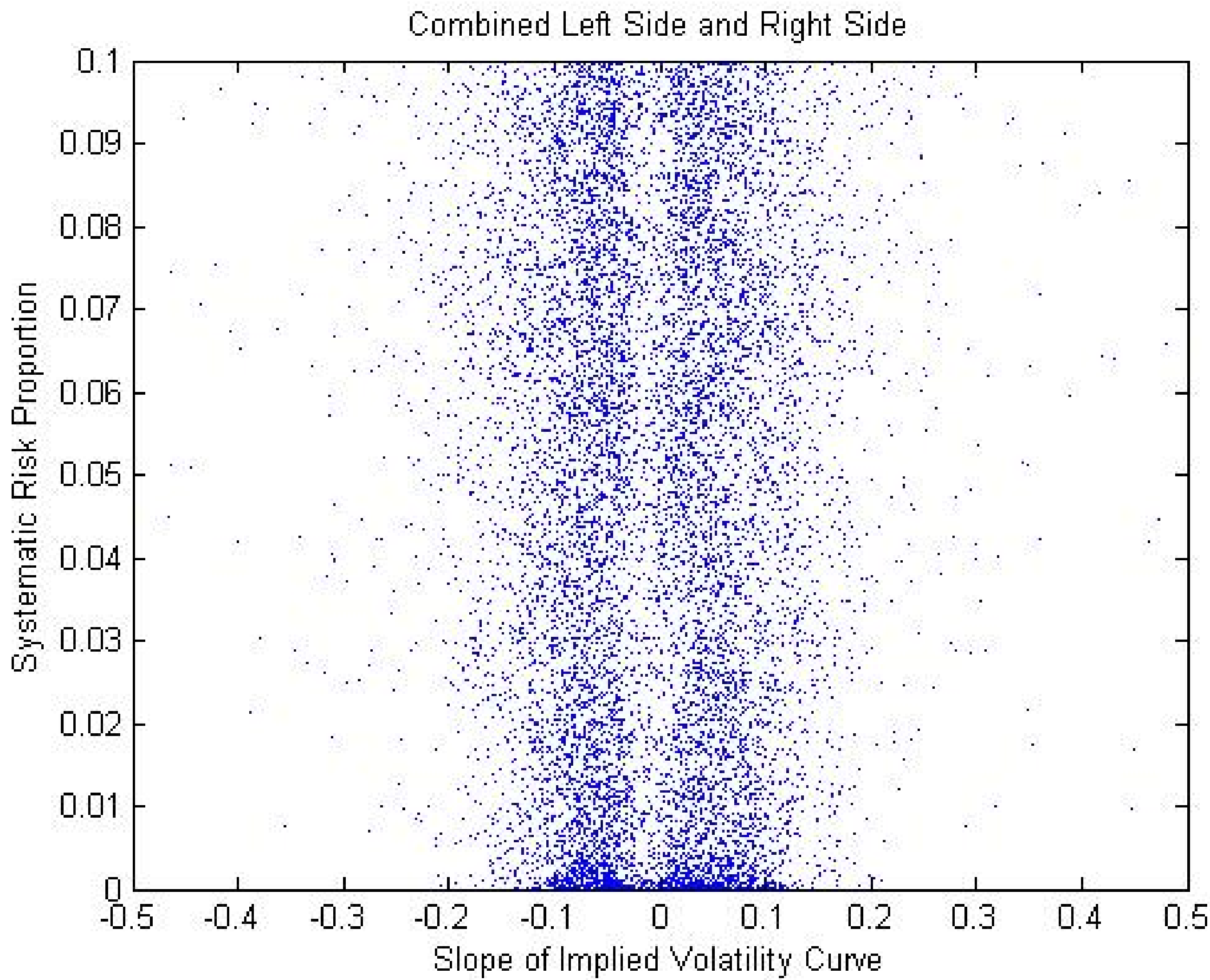
Test 3 **Full Sample**

	Coefficient	T-statistic	Systematic Risk Proportion Coefficient	T-statistic	Kurtosis Coefficient	T-statistic	Skewness Coefficient	T-statistic
S/K < 1	-0.07111	-19.06982	0.14333	15.93996	-0.00011	-0.78993	0.00250	1.84605
S/K > 1	0.03378	12.42330	-0.08530	-11.54820	0.00043	3.21525	0.00056	0.63562

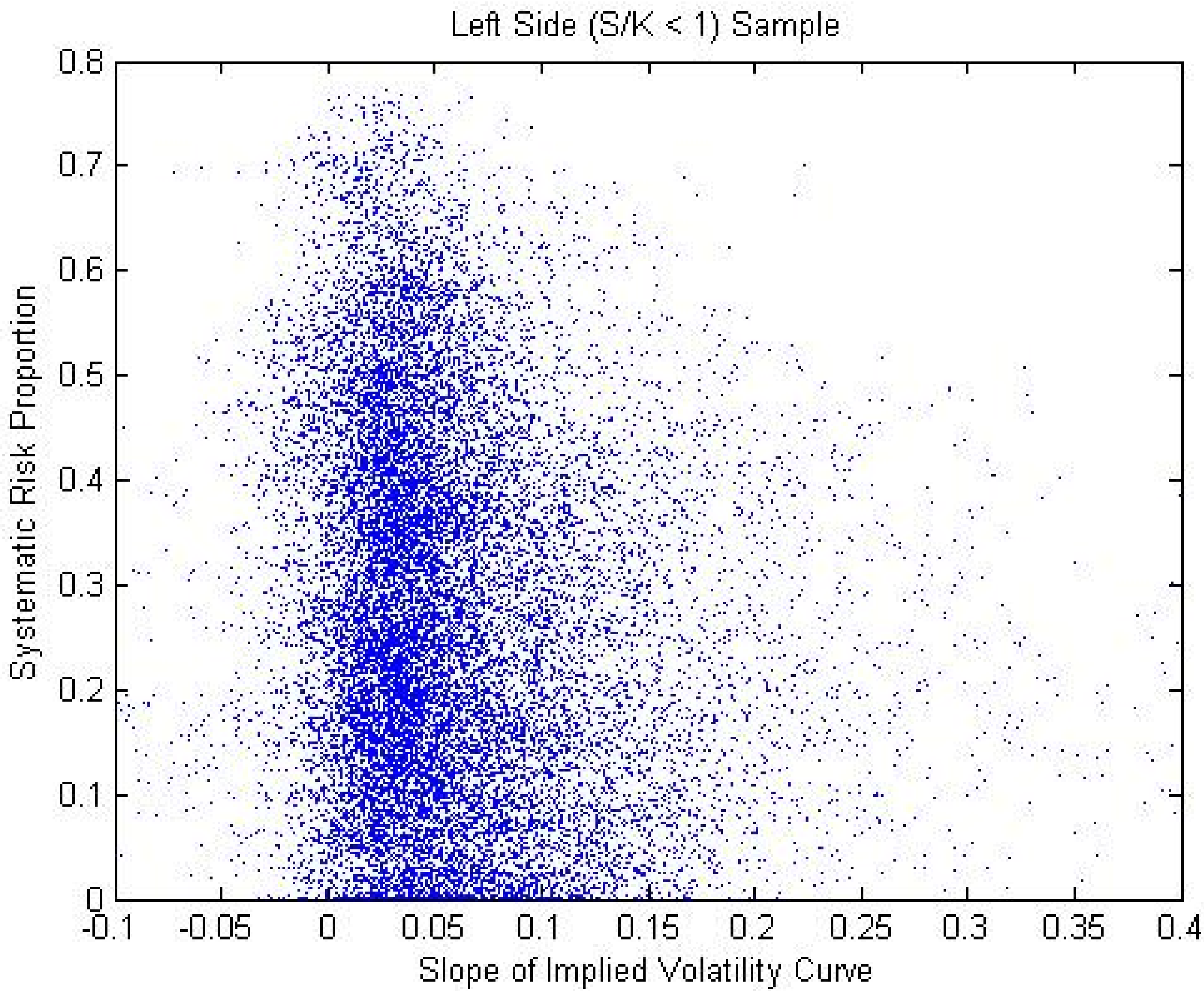


Left Side (S/K < 1)
Systematic Risk Proportion
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Slope of Implied Volatility Curve

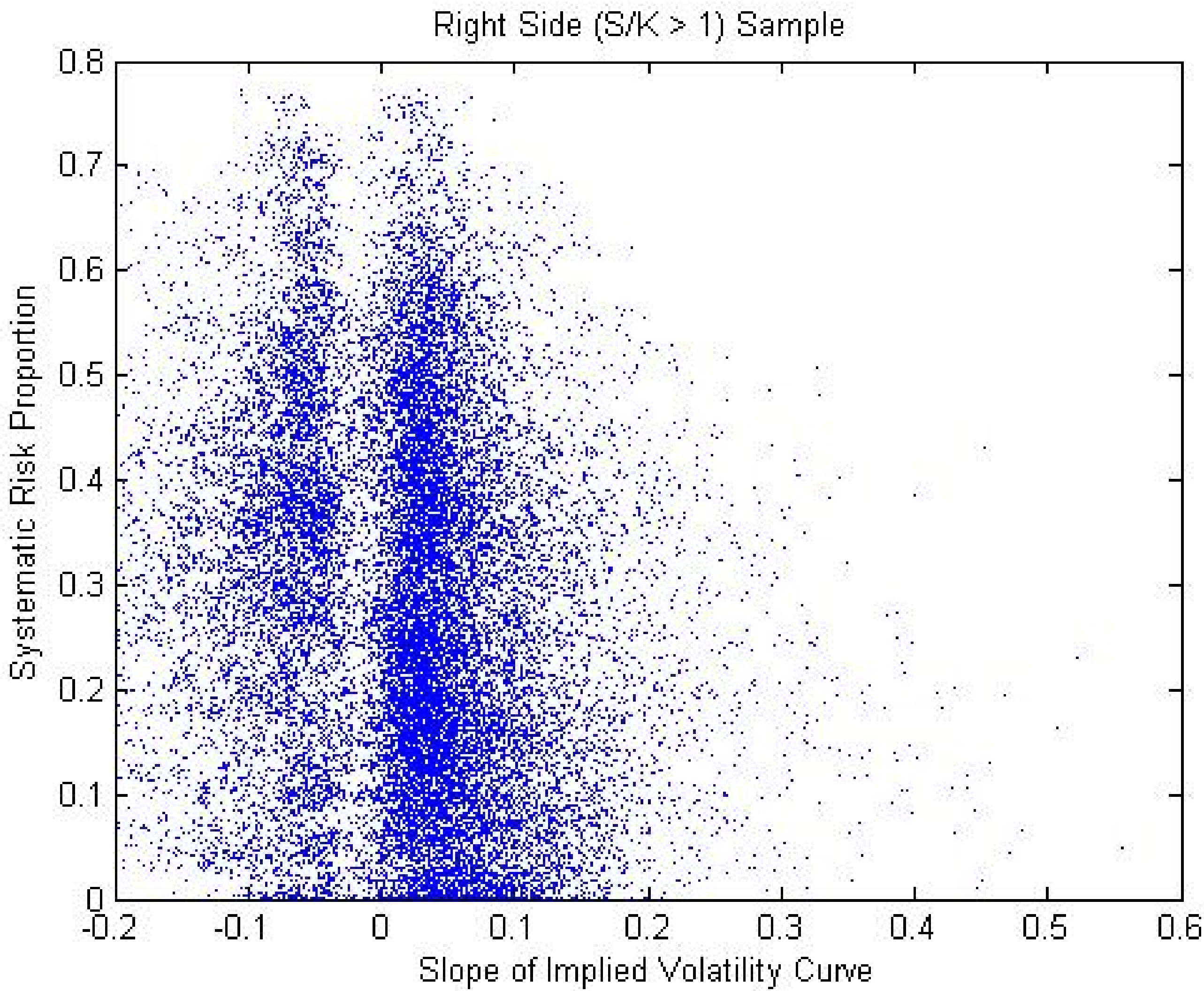

Right Side (S/K > 1)
Systematic Risk Proportion
Slope of Implied Volatility Curve
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.4
-0.2
0
0.2
0.4
0.6
0.8

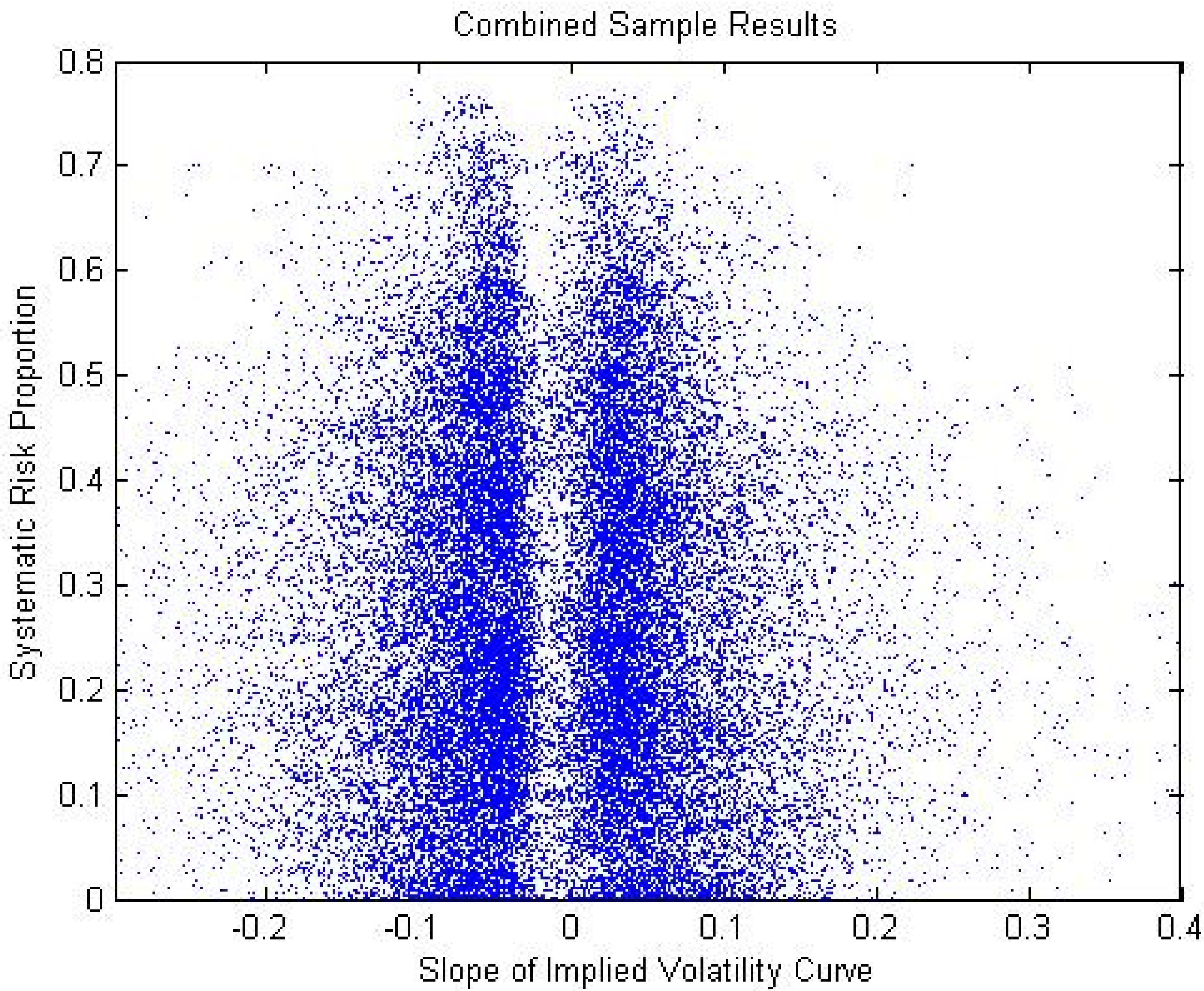


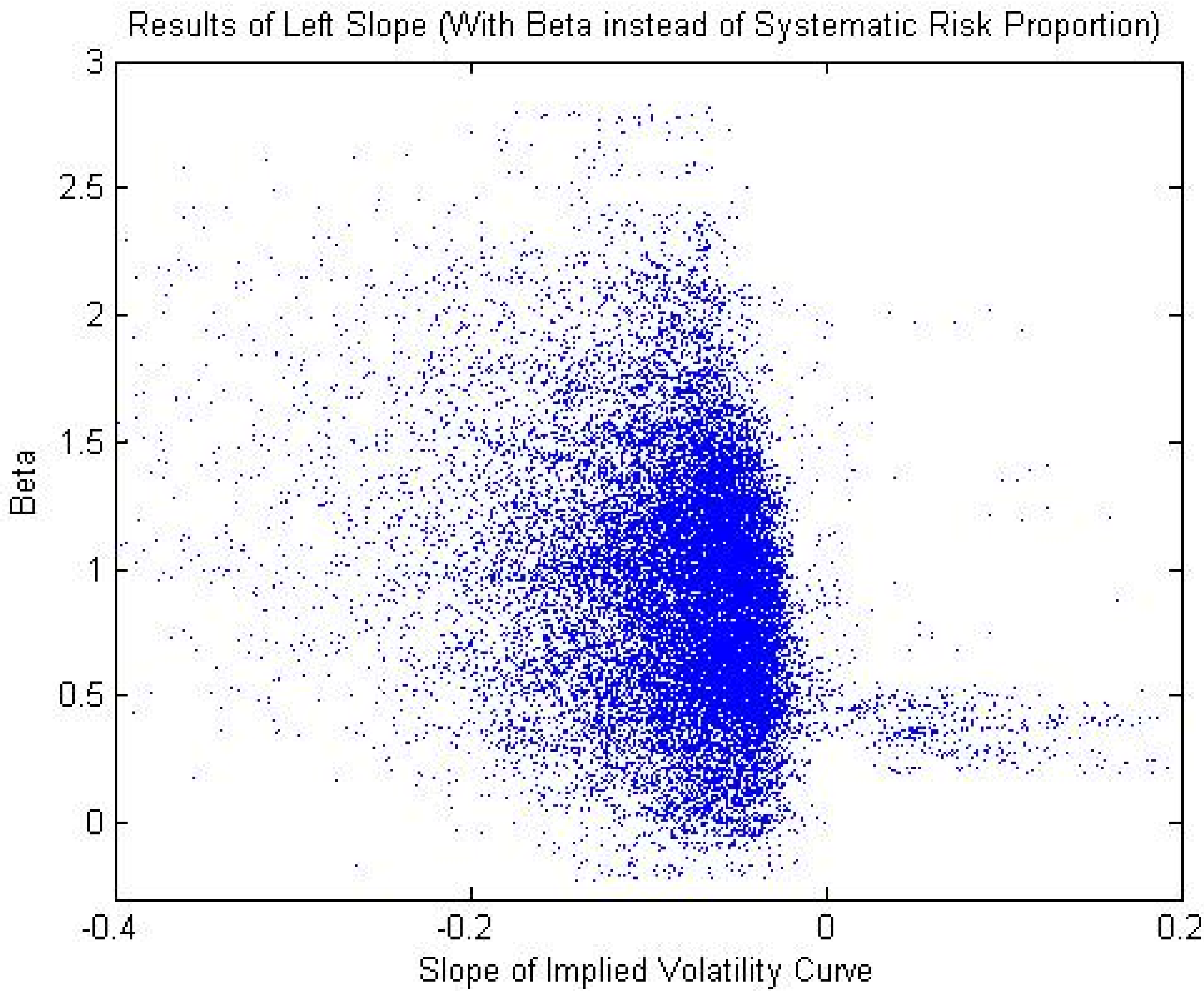
Combined Left Side and Right Side
Systematic Risk Proportion
Slope of Implied Volatility Curve
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

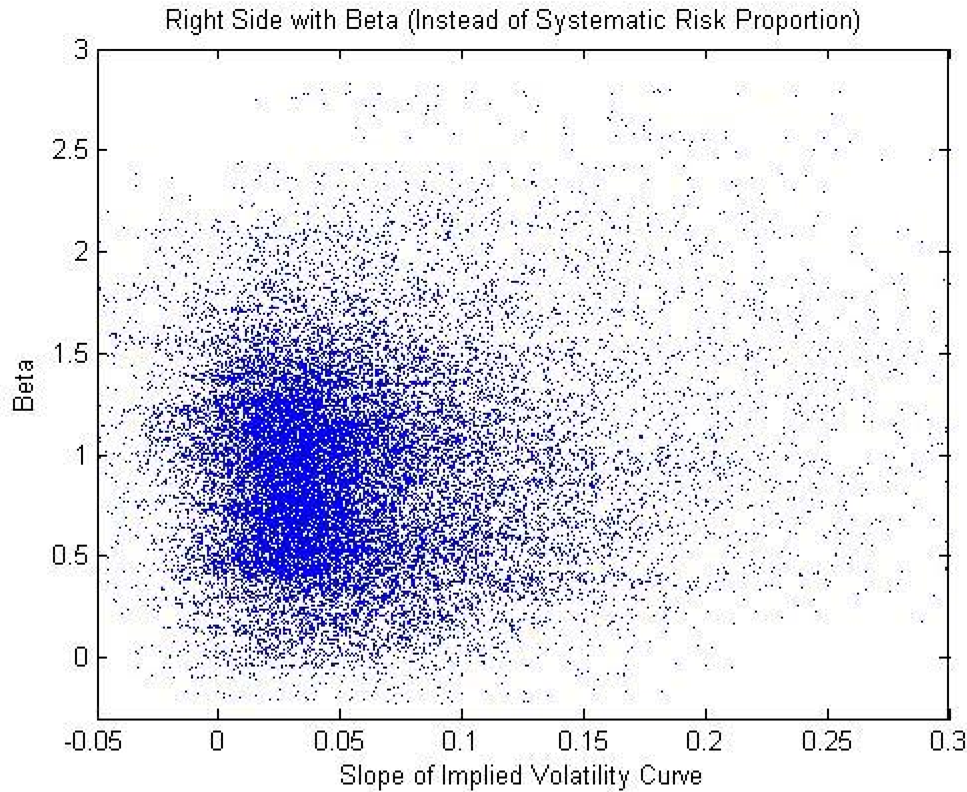


Left Side (S/K < 1) Sample
Systematic Risk Proportion
Slope of Implied Volatility Curve
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4


Right Side (S/K > 1) Sample
Systematic Risk Proportion
Slope of Implied Volatility Curve
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6


Combined Sample Results
Systematic Risk Proportion
Slope of Implied Volatility Curve
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.2
-0.1
0
0.1
0.2
0.3
0.4


Results of Left Slope (With Beta instead of Systematic Risk Proportion)
Beta
Slope of Implied Volatility Curve
3
2.5
2
1.5
1
0.5
0
-0.4
-0.2
0
0.2


Right Side with Beta (Instead of Systematic Risk Proportion)
Beta
3
2.5
2
1.5
1
0.5
0
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
Slope of Implied Volatility Curve

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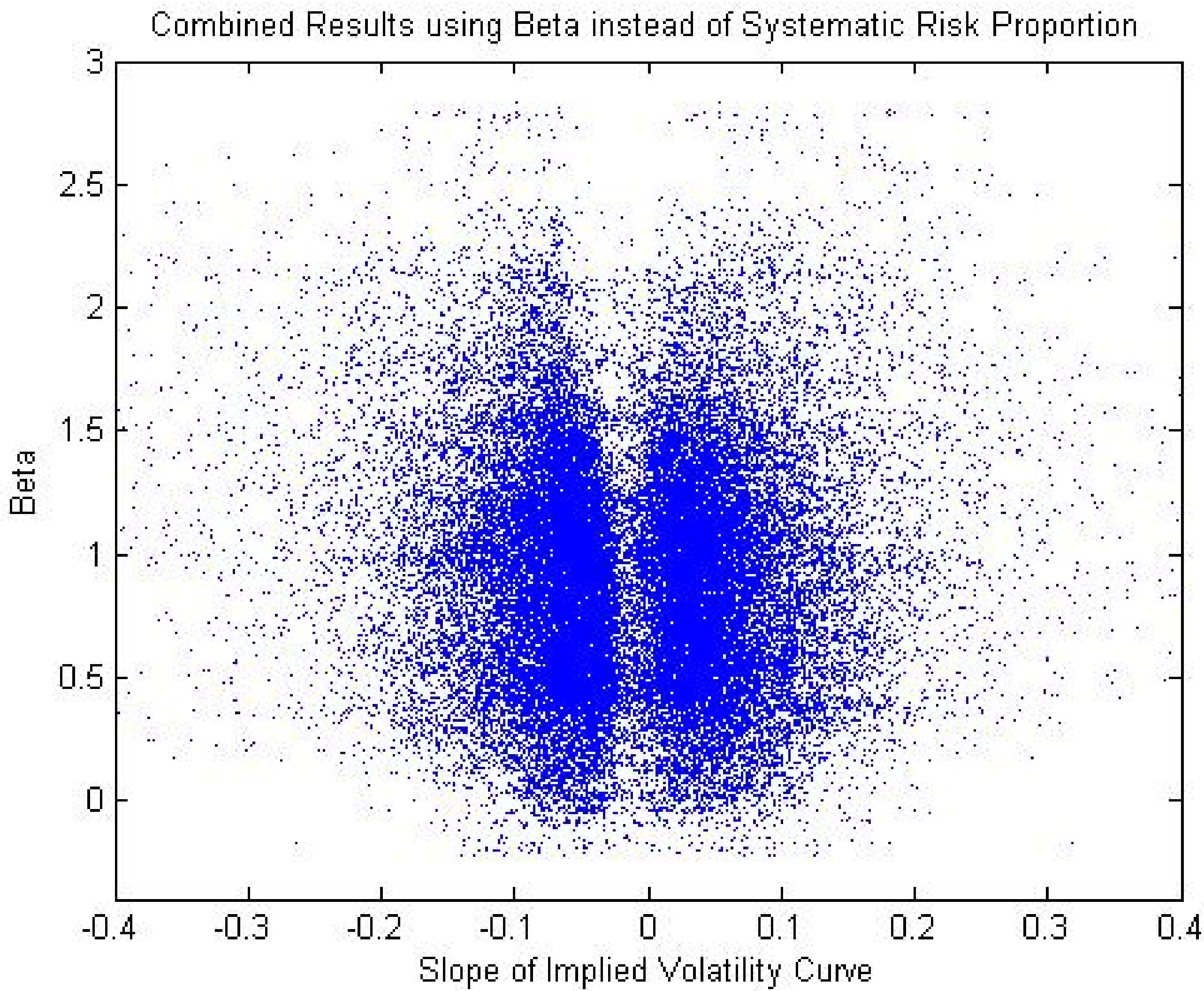

Combined Results using Beta instead of Systematic Risk Proportion
3
2.5
2
1.5
1
0.5
0
Beta
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
Slope of Implied Volatility Curve