Risk Neutral Densities
Heston (EURUSD)
Heston + Residuals (EURUSD)
Petrobras possible prices in 1 month

Petrobras risk neutral density

Market
Log-Normal
Extrapolate by conditioning
Arbitrage Portfolio
Interest rates evolution
Fed Funds evolution
Historical Volatilities by Strike

• Usual historical volatility is a single number
• Break-Even volatility is an average of squared returns, weighted by the Gammas, which depend on the strike
Averaging over Multiple Paths
Optimal Investment

Input:
- Risk neutral measure $Q$
- Subjective measure $P$ (view)
- Utility function $U$
- Initial budget

$\Rightarrow$ Marginal utility proportional to Radon-Nikodym density:

$$U'(g(X_T)) = \lambda \frac{dQ}{dP}(X_T)$$

Output:
- Optimal investment:

$$g(X_T) = (U')^{-1}\left(\lambda \frac{dQ}{dP}(X_T)\right)$$
Subjective Density and Optimal Payoff
Express View on the Market
Implied View

Input:
• Risk neutral measure \( Q \)
• Utility function \( U \)
• Optimal investment \( g \)

Output:
• Subjective measure \( P \) (view)

\[
\frac{dP}{dQ}(X_T) = \frac{\lambda}{U'(g(X_T))}
\]
Implied View

Alternative Interpretation: complete model case

\[ g(X_T) = f(X_0) + \int_0^T \Delta_x f(X_t) dx_t \]

Collection of conditional holdings of the underlying ➔ Collection of conditional drift

Bruno Dupire, Bloomberg
Vanilla Call
Asian Call

Asian Call (SO = 100, K = 100, T = 1), Bachelier model (Vol = 20), Utility Fcn: \( \log(x+0.001) \)
Butterfly
One Touch

One Touch (S₀ = 100, K = 120, T=1), Bachelier model (Vol = 20), Utility Fcn: $\log(w+0.001)$
Slalom