The Risk Premia Embedded in Option Panels

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Motivation

The Option Panel can identify both

- the pricing of volatility and jump risks,
- and the state variables (factors) driving their dynamics.

No-arbitrage and structural models: tight connection b/w option surface and underlying:

- option surface spanned by one or two factors,
- which are part of the volatility of the underlying asset,
- and determine uniquely the dynamics of the market risk premia.

Motivation

However, option surface has richer dynamics

- the term structure of the VIX index has nontrivial dynamics with changing slope,
- volatility smirk variation not entirely spanned by volatility level

WE LET THE OPTION DATA SPEAK

- we estimate a general risk-neutral model that
  - successfully captures dynamics of option surface,
  - utilizing only no-arbitrage restrictions from underlying asset data,
  - but without any restrictions coming from the $\mathbb{P}$ dynamics.
- show pricing of jump risk cannot be completely tied to the market volatility
- show option panel has significant incremental predictive power for risk premia.

Outline

- Estimation Method
- The Risk-Neutral Asset Pricing Model
- Linking Option Dynamics with the Underlying Asset
- Connections with Structural Models

Equity Index Options

Pricing the OTM SPX Options under the Risk-Neutral Distribution,

$$O_{t,k,\tau} \;=\; \begin{cases} \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} r_s\,ds}\,(X_{t+\tau}-K)^+ \right], & \text{if } \; K > F_{t,t+\tau}\,, \\ \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} r_s\,ds}\,(K-X_{t+\tau})^+ \right], & \text{if } \; K \le F_{t,t+\tau}\,. \end{cases}$$

- $\boldsymbol{\tau}$ is time-to-maturity,
- $\mathbf{F}_{\mathbf{t},\mathbf{t}+\tau}$ is futures price,
- $\boldsymbol{K}$ is strike price; $\boldsymbol{k} = \ln(K/F_{t,t+\tau})$,
- $\mathbf{r}_{\mathrm{t}}$ is risk-free rate,

Option Prices Translated to Black-Scholes Implied Volatilities: $\boldsymbol{\kappa}_{t,k,\tau}$.

Parametric Estimation of Risk-Neutral Dynamics

We assume parametric model for the risk-neutral dynamics of X:

- the parameter vector of the model is $\boldsymbol{\theta}$,
- the state vector determining the dynamics is $\mathbf{S_t}$,
- model-implied BSIV is $\boldsymbol{\kappa}\,(k, \tau, \mathbf{S_t}\,; \theta)$.

Allowing for Observation Error, the Observed BSIV are,

$$\hat{\boldsymbol{\kappa}}_{t,k,\tau} = \kappa_{t,k,\tau} + \epsilon_{t,k,\tau}$$

Parametric Estimation of Risk-Neutral Dynamics

We **Estimate** the **State Vector** and **Parameter Vector** via,

$$\left(\{\widehat{\mathbf{S}}_t^n\}_{t=1,\dots,T},\widehat{\theta}^n\right) = \underset{\{\mathbf{Z}_t\}_{t=1,\dots,T},\,\theta\in\Theta}{\operatorname{argmin}} \sum_{t=1}^{T}\left\{\frac{1}{N_t}\sum_{j=1}^{N_t}\frac{(\widehat{\kappa}_{t,k,\tau}-\kappa(k_j,\tau_j,\mathbf{Z}_t,\theta))^2}{\hat{V}_t^{(n,n)}} + \lambda_n\frac{\left(\widehat{V}_t^{(n,m_n)}-\xi(\mathbf{Z}_t)\right)^2}{\widehat{V}_t^{(n,n)}}\right\},$$

$\widehat{V}_t^{(n,m_n)}$ Nonparametric Estimator of Volatility from HF Data,

λ_n is penalty weight ($\lambda_n \to 0$).

$\xi(\cdot)$ is Model-Based Map from State Vector to Spot Volatility.

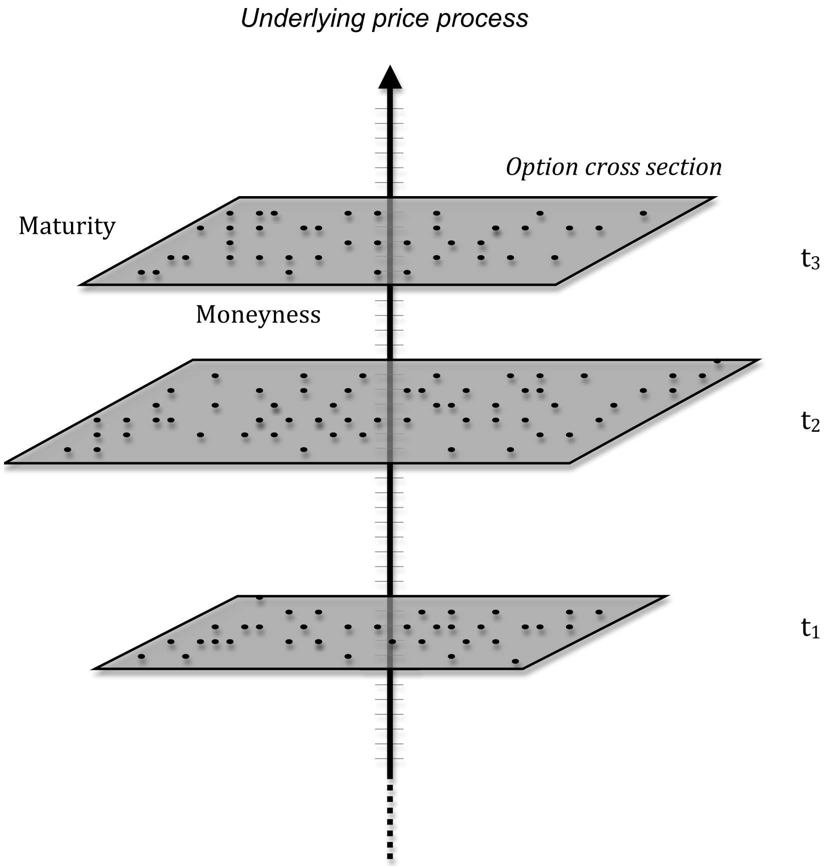
Parametric Estimation of Risk-Neutral Dynamics

Our Nonparametric HF Volatility Estimator is Truncated Realized Volatility,

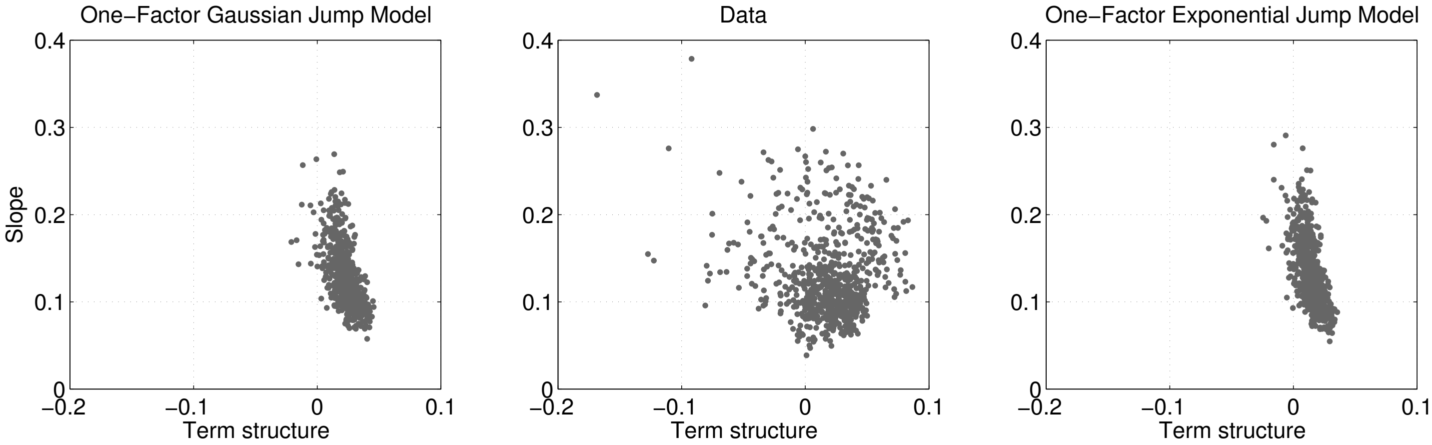
$$\widehat{V}_t^{(n,m_n)} = \frac{n}{m_n} \sum_{i=tn-m_n+1}^{tn} (\Delta_i^n f)^2 \, 1_{\{|\Delta_i^n f| \le \alpha_t \cdot n^{-\varpi}\}}, \qquad \Delta_i^n f = f_{\frac{i}{n}} - f_{\frac{i-1}{n}},$$

where $\alpha_t > 0$, $\varpi \in (0, 1/2)$, and m_n deterministic sequence with $m_n/n \to 0$.

Observation Scheme



Skew and Term Structure Scatter Plots



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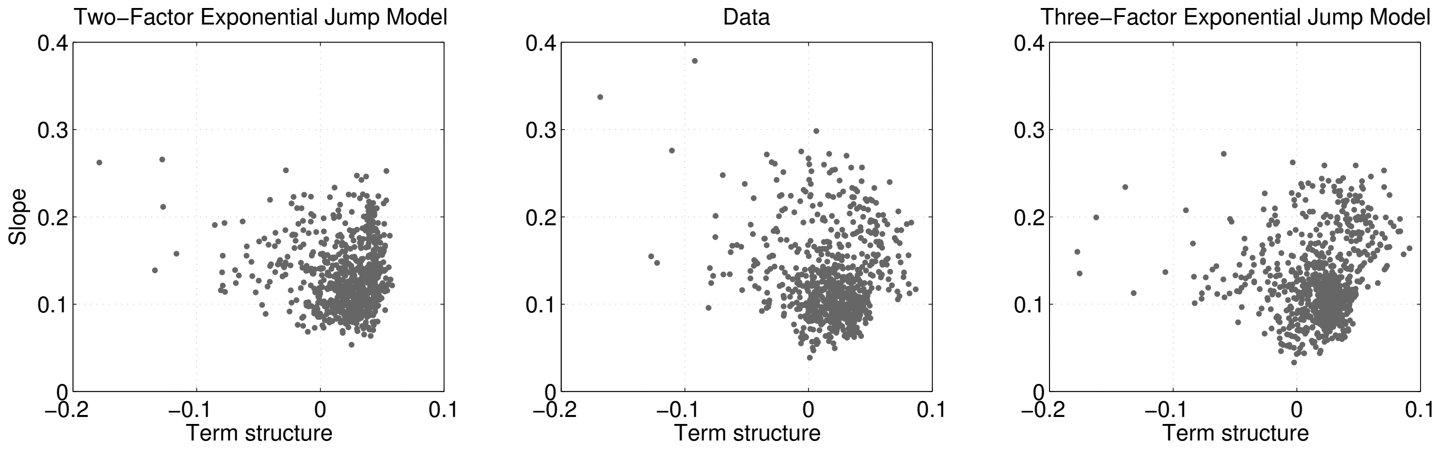


The Risk-Neutral Asset Price Model

$$\begin{aligned}
\frac{dX_t}{X_{t-}} &= (r_t - \delta_t)\,dt + \sqrt{V_{1,t}}\,dW^{\mathbb{Q}}_{1,t} + \sqrt{V_{2,t}}\,dW^{\mathbb{Q}}_{2,t} + \int_{\mathbb{R}^2} (e^x - 1)\,\widetilde{\mu}^{\mathbb{Q}}(dt,dx,dy),\\
dV_{1,t} &= \kappa_1\,(\overline{v}_1 - V_{1,t})\,dt + \sigma_1\sqrt{V_{1,t}}\,dB^{\mathbb{Q}}_{1,t} + \mu_1 \int_{\mathbb{R}^2} x^2\,1_{\{x<0\}}\,\mu(dt,dx,dy),\\
dV_{2,t} &= \kappa_2\,(\overline{v}_2 - V_{2,t})\,dt + \sigma_2\sqrt{V_{2,t}}\,dB^{\mathbb{Q}}_{2,t},\\
dU_t &= -\kappa_3\,U_t\,dt + \mu_u \int_{\mathbb{R}^2} \Big[(1-\rho_3)\,x^2\,1_{\{x<0\}} + \rho_3\,y^2\Big]\,\mu(dt,dx,dy).
\end{aligned}$$

Jump Compensator (under $\mathbb{Q}$):

$$\Big\{\Big(c^- 1_{\{x<0\}}\lambda_- e^{-\lambda_-|x|} + c^+ 1_{\{x>0\}}\lambda_+ e^{-\lambda_+ x}\Big)\,1_{\{y=0\}} + c^- 1_{\{x=0,\ y<0\}}\lambda_- e^{-\lambda_-|y|}\Big\}\ dx\otimes dy,$$

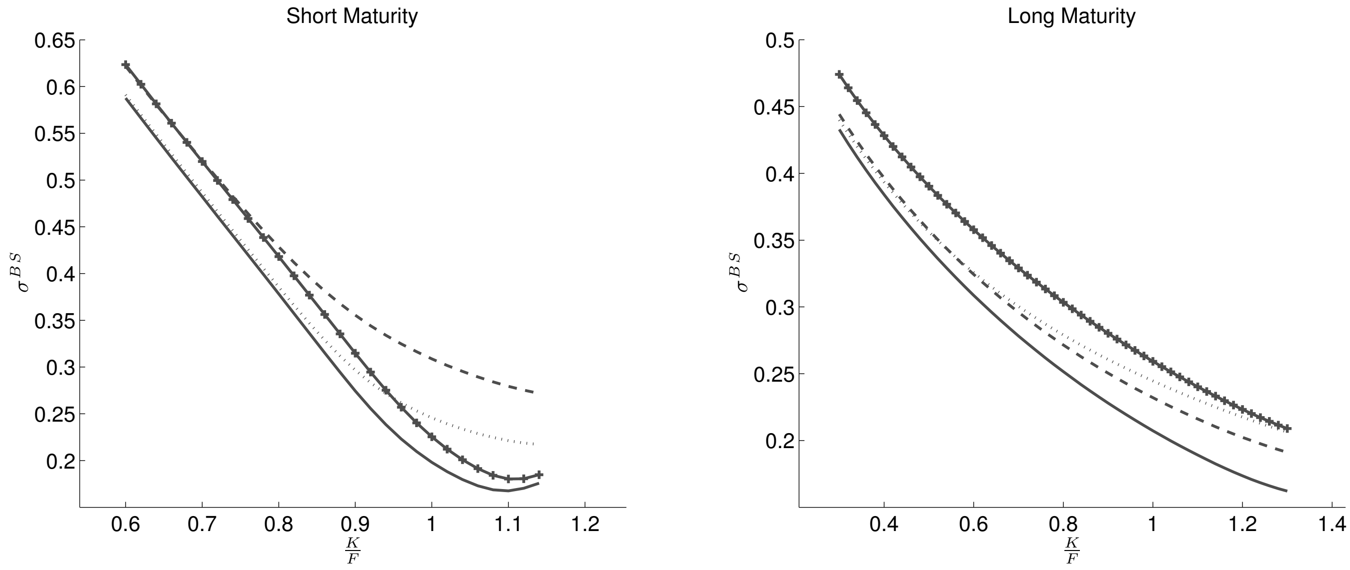
$$c^- = c_0^- + c_1^- V_{1,t-} + c_2^- V_{2,t-} + c_3^- U_{t-}, \qquad c^+ = c_0^+ + c_1^+ V_{1,t-} + c_2^+ V_{2,t-} + c_3^+ U_{t-}\,.$$

The Risk-Neutral Asset Price Model

- Price-Volatility Co-Jumps: Squared Jump Size enters Vol Dynamics (GARCH).
- Negative Price Jumps have Additional Source of Time Variation via U.
- U connected with volatility by Common Jumps and Jump Intensities.

WE DECOUPLE TAIL RISK FROM VOLATILITY

Option Surface Sensitivity to State Vector



Solid Line: Implied Volatility with State Variables at Sample Means.

Dashed Line: V_1 Shift; Dotted Line: V_2 Shift; x-x-x Line: U Shift.

Parameter Estimates

Parameter	Estimate	Std.	Parameter	Estimate	Std.	Parameter	Estimate	Std.
ρ_1	-0.913	0.028	σ_2	0.110	0.006	c_2^-	0.913	3.730
$\overline{v}_1$	0.007	0.000	μ_u	1.756	0.647	c_2^+	14.269	4.986
κ_1	8.325	0.167	κ_3	0.522	0.080	c_3^-	19.836	5.460
σ_1	0.323	0.014	ρ_3	0.117	0.614	λ_-	21.157	0.240
ρ_2	-0.945	0.036	c_0^+	0.723	0.079	λ_+	48.365	2.053
$\overline{v}_2$	0.016	0.001	c_1^-	34.592	1.931	μ_-	11.602	0.262
κ_2	0.480	0.033	c_1^+	88.178	14.711			

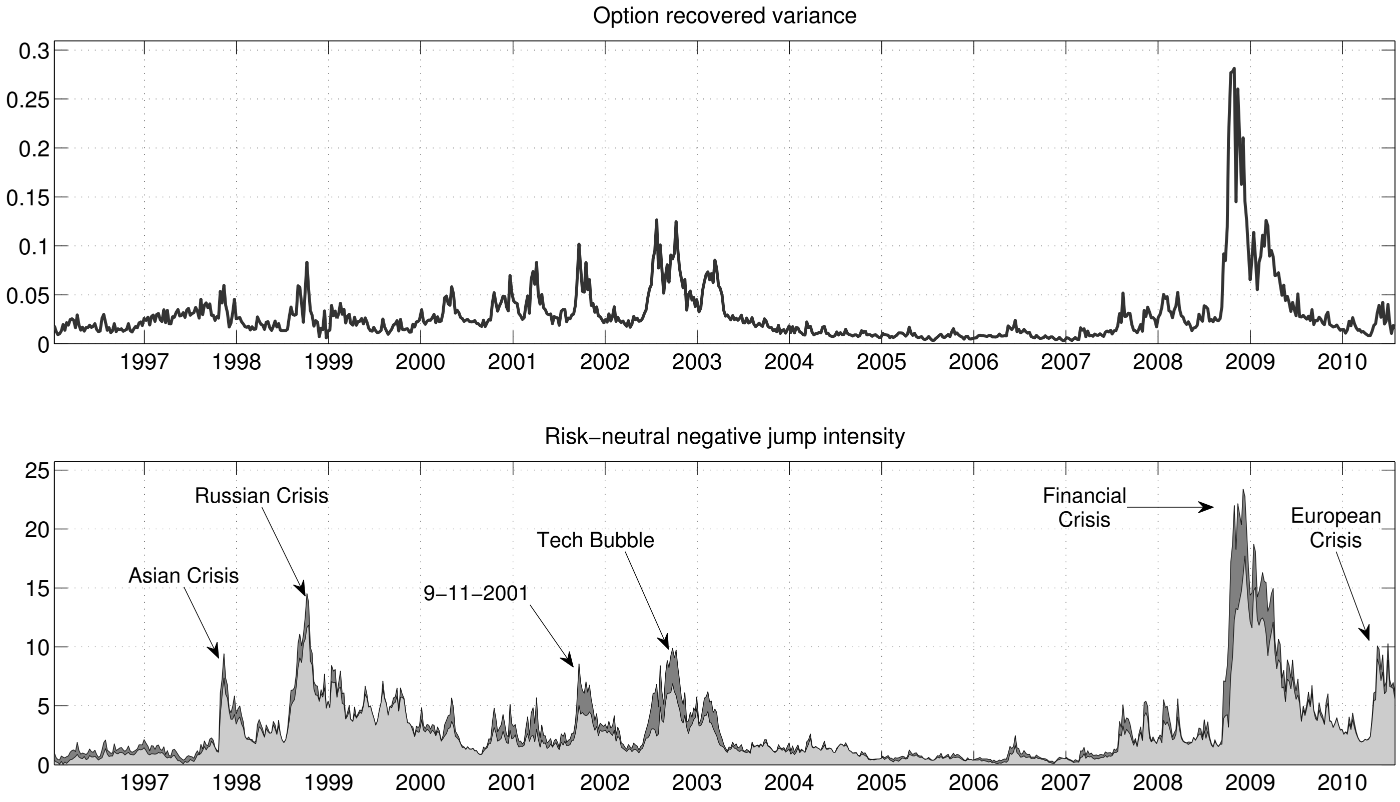
1. Mean Negative Price Jump ≈ -4.7%, Mean Positive Price Jump ≈ 2.1%

2. Frequency of Negative Jumps 3.4%, Positive Jumps 2.6% per Year.

3. Left Jump Tail twice as Fat as Right Jump Tail; $c^{\pm}$ coefficients differ significantly.

4. U very persistent, Accounts for 40% of Variation in Left Tail.

Variance and Negative Risk-Neutral Jump Intensity



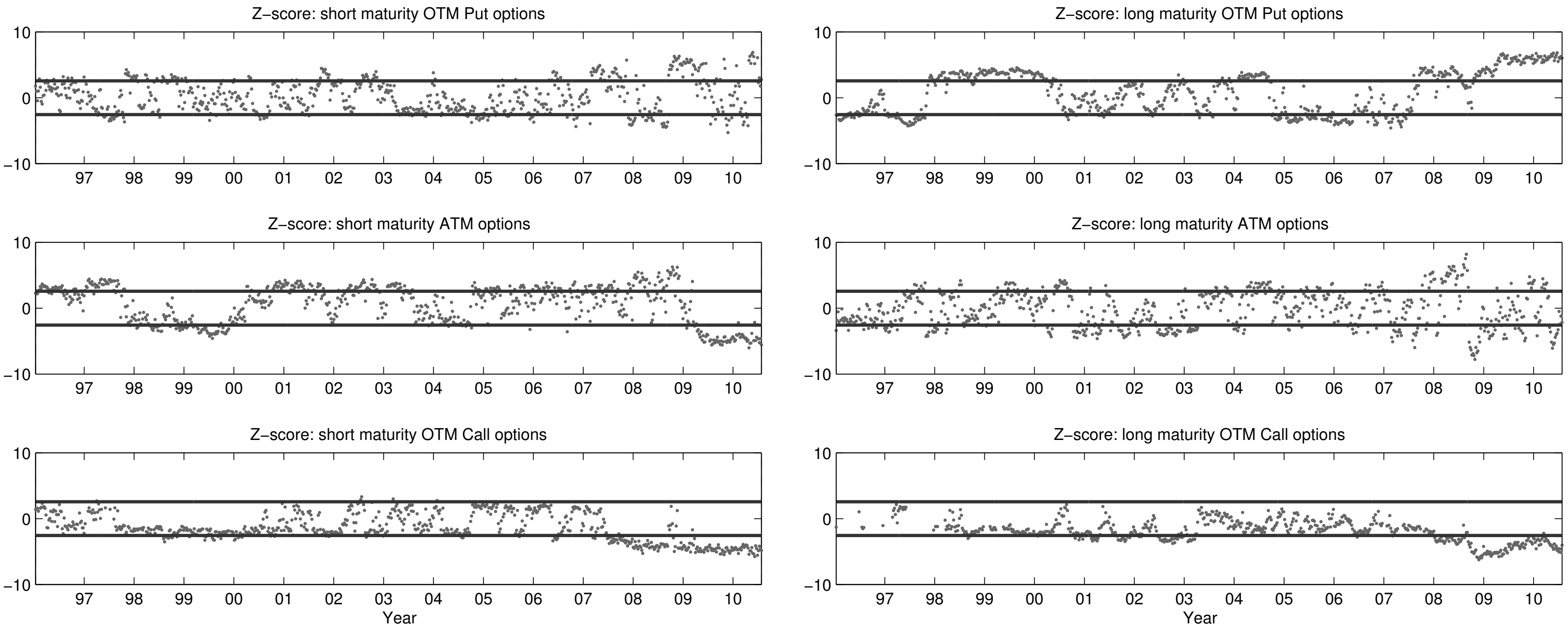
Formal Diagnostic Tests

We compute the following Z-scores

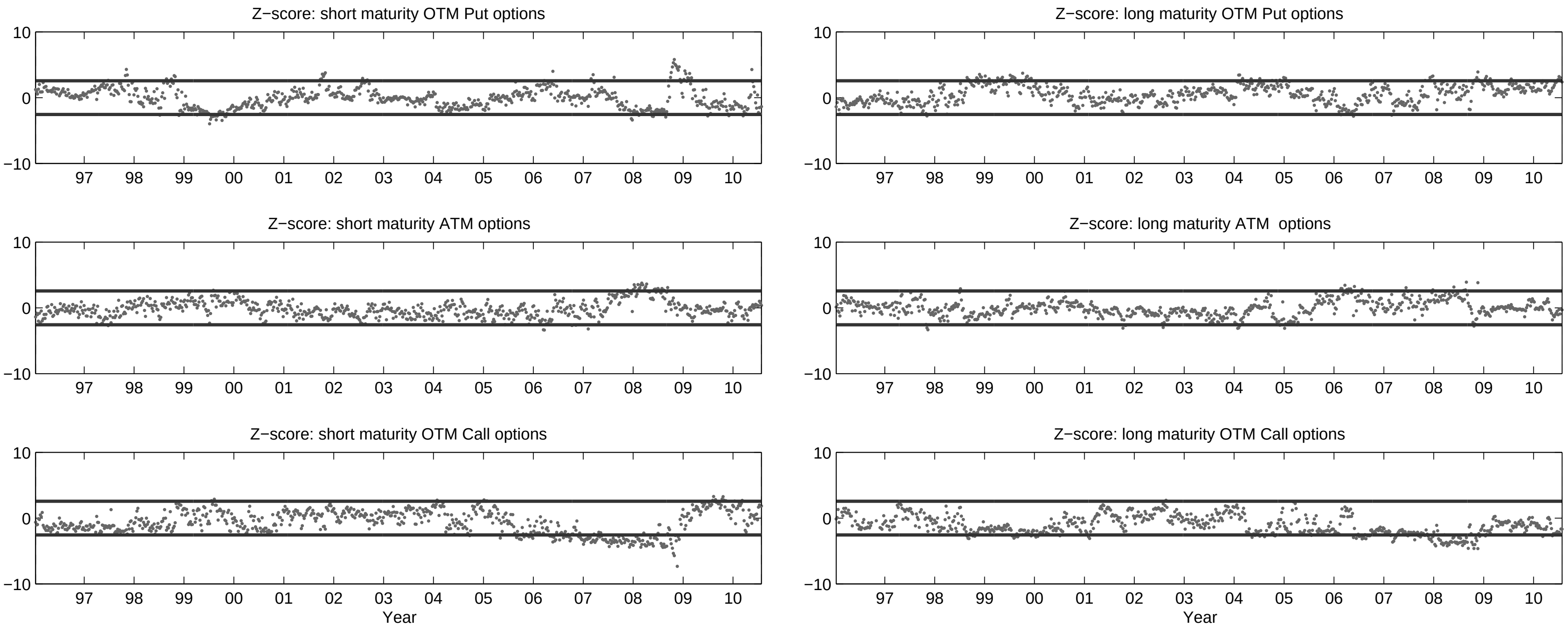
$$Z_{t,\tau^*,\mathcal{K}} = \frac{\sum\limits_{j:k_j\in\mathcal{K}} \left(\overline{\kappa}_{t,k_j,\tau^*} - \kappa(k_j,\tau^*,\widehat{\mathbf{S}}_t^n,\widehat{\theta}^n)\right)}{\sqrt{\widehat{\mathrm{Avar}}\left(\sum_{j:k_j\in\mathcal{K}}\left(\overline{\kappa}_{t,k_j,\tau^*} - \kappa(k_j,\tau^*,\widehat{\mathbf{S}}_t^n,\widehat{\theta}^n)\right)\right)}} \xrightarrow{\mathcal{L}} N(0,1),$$

where τ^* is particular maturity and $\mathcal{K}$ is part of the moneyness space.

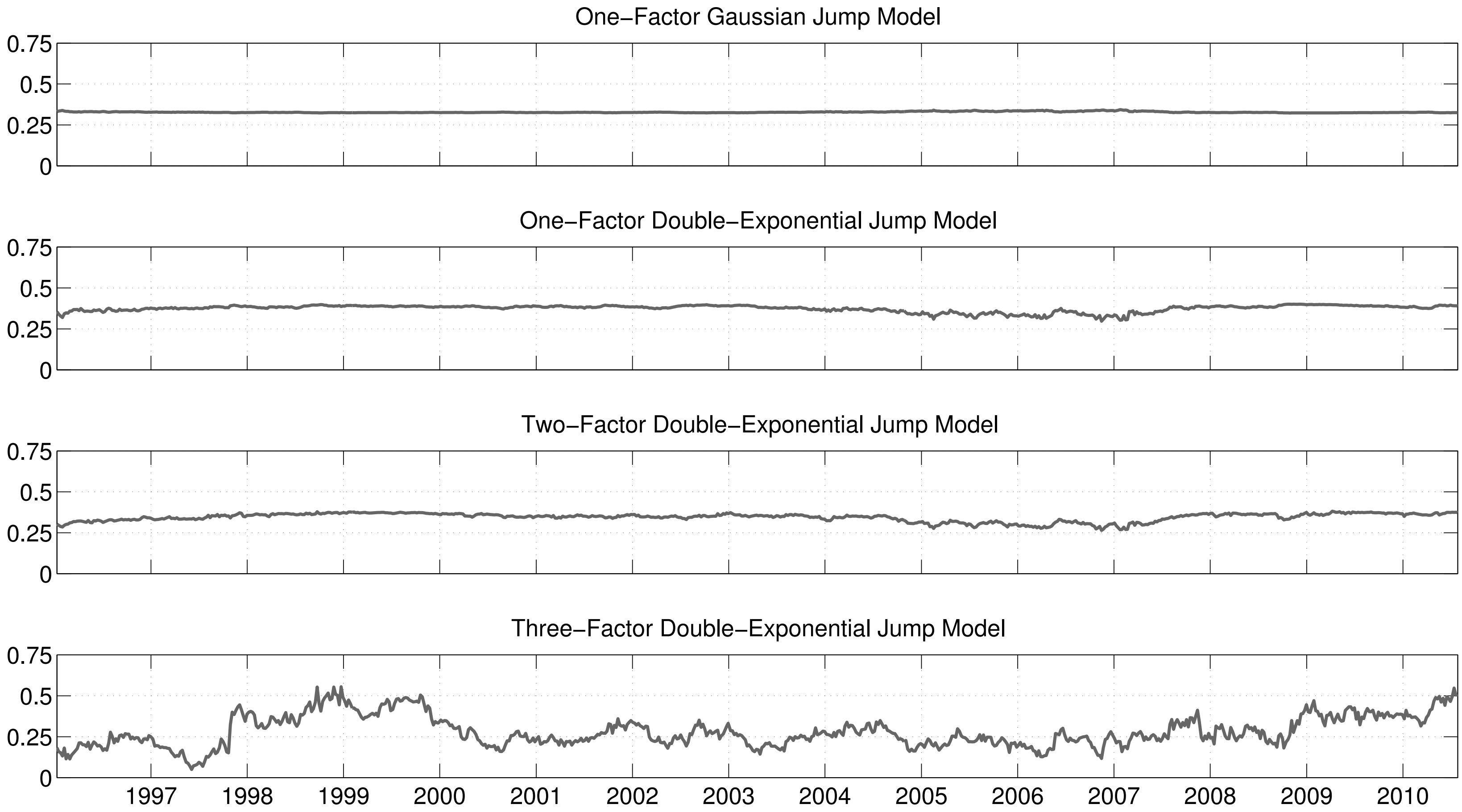
Formal Diagnostic Tests: One-Factor Gaussian Jump Model



Formal Diagnostic Tests: The Three Factor Model



Ratio Model-Implied Negative Jump to Total Return Variation



U drives Risks and/or Risk premia?

Linking Information in Option Panel with Underlying Asset

Variance Risk = Quadratic Variation over $[t, t+\tau]$:

$$QV_{t,t+\tau} = QV^{c}_{t,t+\tau} + QV^{j}_{t,t+\tau}\,,$$

$$QV^{c}_{t,t+\tau} = \int_t^{t+\tau} (V_{1,s} + V_{2,s})\, ds; \quad \text{Variance Risk of Continuous Part of } X.$$

$$QV^{j}_{t,t+\tau} = \int_t^{t+\tau} \int_{\mathbb{R}^2} x^2\, \mu(ds, dx, dy), \quad \text{Variance Risk of Jump Part of } X.$$

Linking Information in Option Panel with Underlying Asset

Measures of Jump Risk:

$$
\begin{aligned}
LT^{K}_{t,t+\tau} &\equiv \int_t^{t+\tau}\int_{\mathbb{R}^2} 1_{\{x\le -K\}}\,\mu(ds,dx,dy),\\
RT^{K}_{t,t+\tau} &\equiv \int_t^{t+\tau}\int_{\mathbb{R}^2} 1_{\{x\ge K\}}\,\mu(ds,dx,dy).
\end{aligned}
$$

We have,

$$
\begin{aligned}
LT^{K}_{t,t+\tau} &= \int_t^{t+\tau}\int_{\mathbb{R}^2} 1_{\{x\le -K\}}\,\nu^{\mathbb{P}}_s(dx,dy)\,ds \;+\; \epsilon^{L}_{t,t+\tau}, \qquad \mathbb{E}^{\mathbb{P}}_t(\epsilon^{L}_{t,t+\tau}) = 0,\\
RT^{K}_{t,t+\tau} &= \int_t^{t+\tau}\int_{\mathbb{R}^2} 1_{\{x\ge K\}}\,\nu^{\mathbb{P}}_s(dx,dy)\,ds \;+\; \epsilon^{R}_{t,t+\tau}, \qquad \mathbb{E}^{\mathbb{P}}_t(\epsilon^{R}_{t,t+\tau}) = 0.
\end{aligned}
$$

Linking Information in Option Panel with Underlying Asset

Measures of Risk Premia:

$$\frac{1}{\tau}\log\left(\frac{X_{t+\tau}}{X_t}\right) - \frac{1}{\tau}\int_t^{t+\tau}(r_s - \delta_s - q_s^{\mathbb{P}})\,ds \;=\; \mathrm{ERP}_t^{\tau} + \epsilon_{t,t+\tau}^{E}, \qquad \mathbb{E}_t^{\mathbb{P}}\left[\epsilon_{t,t+\tau}^{E}\right] \;=\; 0,$$

$$\widehat{\mathrm{VRP}}_t^{\tau} \;=\; \frac{1}{\tau}\left[\widehat{QV}_{t,t+\tau} \;-\; \mathbb{E}_t^{\mathbb{Q}}\left(QV_{t,t+\tau}\right)\right] \;=\; \mathrm{VRP}_t^{\tau} + \epsilon_{t,t+\tau}^{V}, \qquad \mathbb{E}_t^{\mathbb{P}}\left[\epsilon_{t,t+\tau}^{V}\right] \;=\; 0,$$

- $\mathbb{E}_t^{\mathbb{Q}}\left[QV_{t,t+\tau}\right]$ may be obtained in "Model-Free" fashion from VIX Index.
- ERP $=$ Equity Risk Premium.
- VRP $=$ Variance Risk Premium.

Predictive Regressions for Returns and Risk Premia

We Explore the **Predictive Regressions**,

$$y_t \;=\; \alpha_0 \,+\, \alpha_1\, V_{1,t} \,+\, \alpha_2\, V_{2,t} \,+\, \alpha_3\, U_t \,+\, \epsilon_t\,, \quad \text{with } y_t \text{ representing:}$$

Actual Price Jump Risk:

- $y_t \;=\; \widehat{LT}^{K}_{t,t+\tau}$, or $\widehat{RT}^{K}_{t,t+\tau}$.

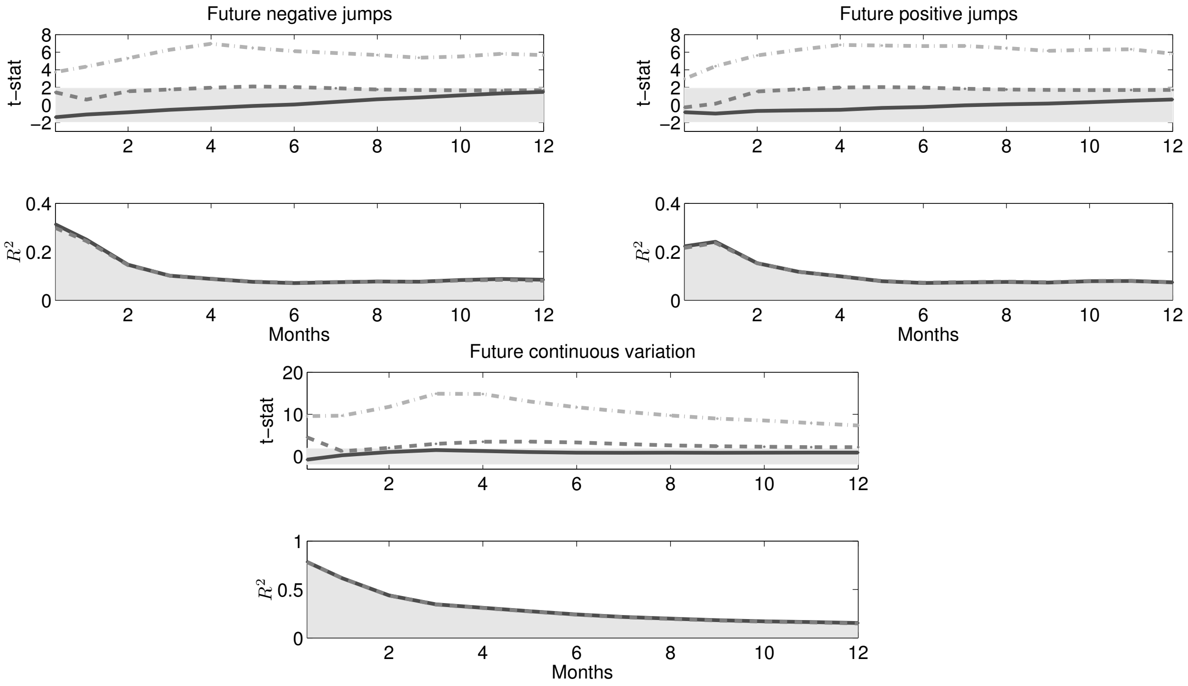
Actual (Diffusive) Variance Risk:

- $y_t \;=\; \widehat{QV}^{c}_{t,t+\tau}$.

Size of Risk Premiums:

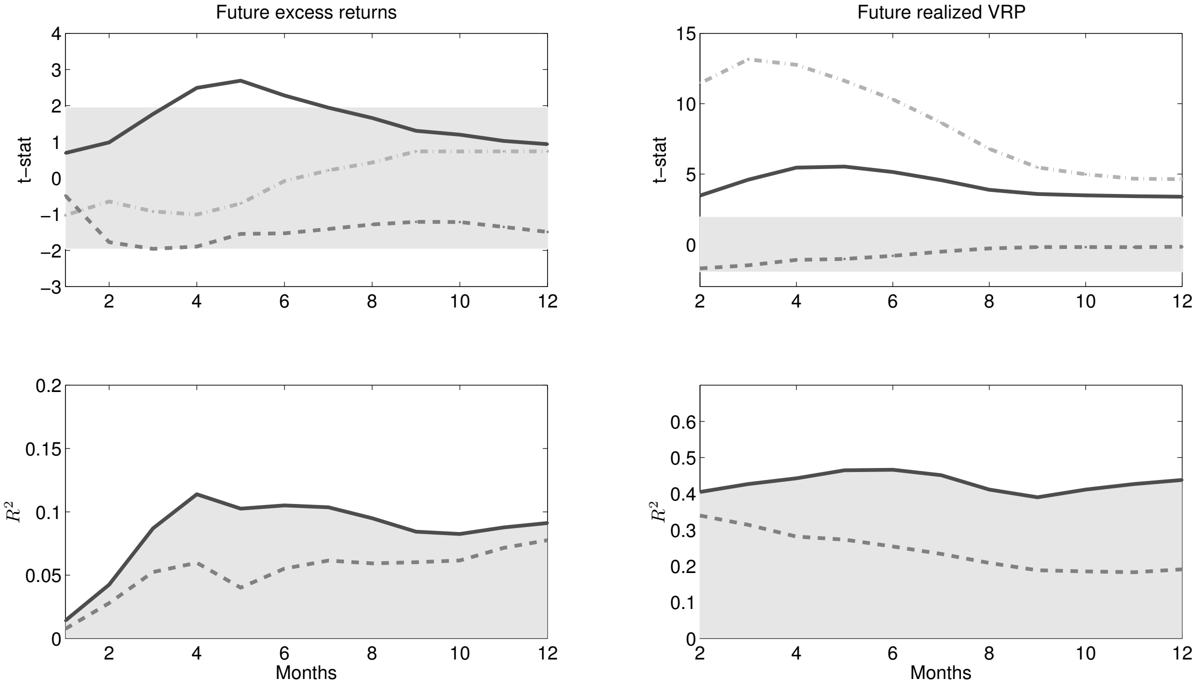
- $y_t \;=\; \log\left(\frac{X_{t+\tau}}{X_t}\right) - \frac{1}{\tau}\int_t^{t+\tau}(r_s - \delta_s - q_s^{\mathbb{P}})ds$ or $\widehat{\mathrm{VRP}}^{\tau}_{t}$.

Empirical Evidence on Option Factors and Risk (Premia)



Predictor variables: V_1 (dashed-dotted), V_2 (dashed) and $\widetilde{U}$ (solid), where $\widetilde{U}$ is Residual from Linear Projection of U on V_1 and V_2. Dashed lines in R^2 Plots are from Constrained Regressions including only V_1 and V_2.

Empirical Evidence on Option Factors and Risk (Premia)



Predictor variables: V_1 (dashed-dotted), V_2 (dashed) and $\widetilde{U}$ (solid), where $\widetilde{U}$ is Residual from Linear Projection of U on V_1 and V_2. Dashed lines in R^2 Plots are from Constrained Regressions including only V_1 and V_2.

Structural Model Implications for Option Factors and Risk (Premia)

Is Our Evidence Consistent with Standard Structural Models of Risk and Risk Premia?

Consider affine setting + representative agent with Epstein-Zin preferences:

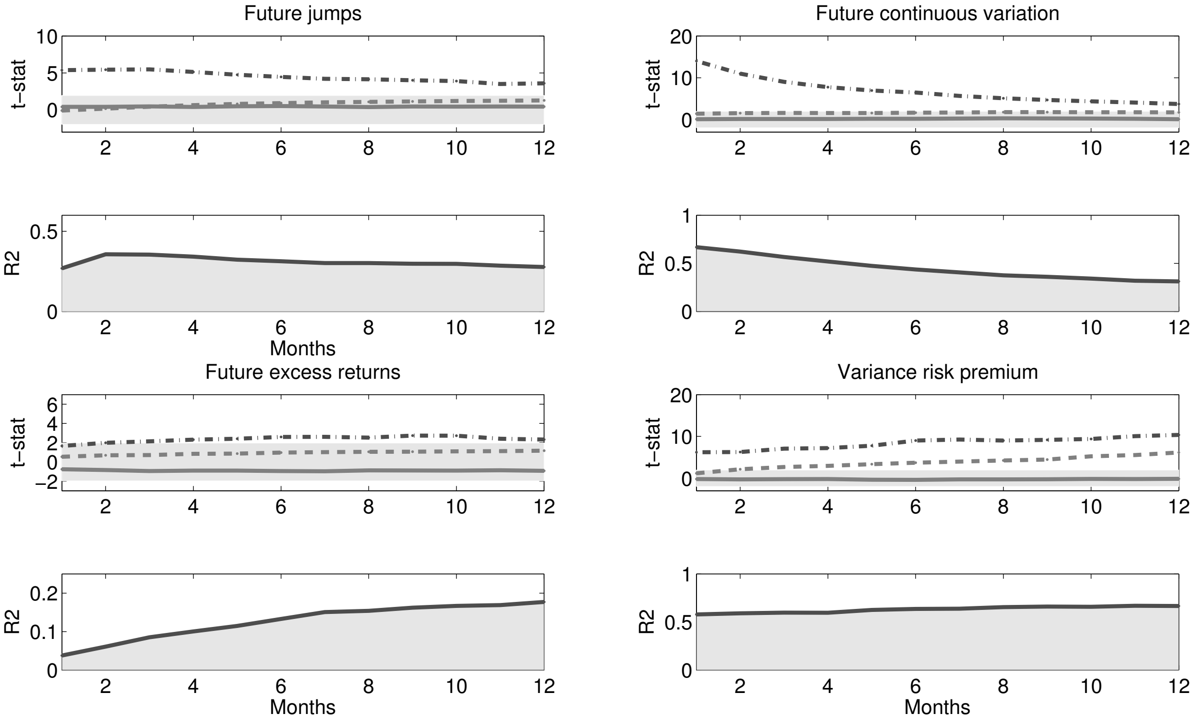
- In this setting asset prices are affine functions of structural shocks
- The jump part of the equilibrium-based pricing kernel is

$$\mathcal{E}\left(\int_0^t \int_{\mathbb{R}^n} (Y-1)\widetilde{\mu}^{\mathbb{P}}(ds, d\mathbf{x})\right),$$

  with Y time-invariant and non-random

$\frac{\nu^{\mathbb{Q}}(ds,d\mathbf{x})}{\nu^{\mathbb{P}}(ds,d\mathbf{x})}$ TIME-INVARIANT (no room for U!)

Empirical Evidence on Option Factors and Risk (Premia)



Predictive Regressions implied by Dreschler & Yaron (2011) Structural Model. The predictive variables are the conditional mean of consumption growth (solid line), stochastic volatility of consumption growth (dashed-dotted line) and central tendency of stochastic volatility (dashed line).

Structural Model Implications for Option Factors and Risk (Premia)

Structural models tie risks and risk premia too closely together (relative to data).

Can Structural Models Loosen Link between Drivers of Risk and Risk Premia?

1. Time-Varying Risk Aversion?

- jumps in consumption growth + habit persistence,
- but we need to de-couple habit persistence from volatility.

2. Imperfect Information, Confidence Risk, Ambiguity Aversion?

- ambiguity about jump risk component of model,
- but need to limit effect of the ambiguity aversion on predictability of jump risk.

Conclusion

Used more Elaborate Option Pricing Model than Explored Hitherto.

Extract Three Option-Implied Factors, with One Controlling Left Jump Tail.

Third Factor Improves Fit to Option Surface Significantly.

Jump Tail Factor has No Impact on Actual Volatility and Jump Dynamics.

Jump Tail Factor is Critical for Equity and Variance Risk Premium.

New Factor cannot be Retrieved from Dynamics of Underlying Asset.

New Factor not readily Associated with Fundamental Risk Factor.

Suggests we Need Time-Varying Risk Aversion or Ambiguity Aversion.

Further Improvements in Option Fit feasible, but may Lose Affine Pricing.