The Risk Premia Embedded in Option Panels

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40 Years after the Black-Scholes-Merton Model

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Motivation

The Option Panel can identify both

- the pricing of volatility and jump risks,
- and the state variables (factors) driving their dynamics.

No-arbitrage and structural models: tight connection b/w option surface and underlying:

- option surface spanned by one or two factors,
- which are part of the volatility of the underlying asset,
- and determine uniquely the dynamics of the market risk premia.
Motivation

However, option surface has richer dynamics

- the term structure of the VIX index has nontrivial dynamics with changing slope,
- volatility smirk variation not entirely spanned by volatility level

WE LET THE OPTION DATA SPEAK

- we estimate a general risk-neutral model that
  - successfully captures dynamics of option surface,
  - utilizing only no-arbitrage restrictions from underlying asset data,
  - but without any restrictions coming from the $\mathbb{P}$ dynamics.
- show pricing of jump risk cannot be completely tied to the market volatility
- show option panel has significant incremental predictive power for risk premia.
Outline

- Estimation Method
- The Risk-Neutral Asset Pricing Model
- Linking Option Dynamics with the Underlying Asset
- Connections with Structural Models
Equity Index Options

Pricing the OTM SPX Options under the Risk-Neutral Distribution,

\[ O_{t,k,\tau} = \begin{cases} 
\mathbb{E}_t^Q \left[ e^{-\int_{t+\tau}^t r_s \, ds} (X_{t+\tau} - K)^+ \right], & \text{if } K > F_{t,t+\tau}, \\
\mathbb{E}_t^Q \left[ e^{-\int_{t+\tau}^t r_s \, ds} (K - X_{t+\tau})^+ \right], & \text{if } K \leq F_{t,t+\tau}.
\end{cases} \]

- \( \tau \) is time-to-maturity,
- \( F_{t,t+\tau} \) is futures price,
- \( K \) is strike price; \( k = \ln(K/F_{t,t+\tau}) \),
- \( r_t \) is risk-free rate,

Option Prices Translated to Black-Scholes Implied Volatilities: \( \kappa_{t,k,\tau} \).
Parametric Estimation of Risk-Neutral Dynamics

We assume parametric model for the risk-neutral dynamics of $X$:

- the parameter vector of the model is $\theta$,
- the state vector determining the dynamics is $S_t$,
- model-implied BSIV is $\kappa(k, \tau, S_t; \theta)$.

Allowing for Observation Error, the Observed BSIV are,

$$\hat{\kappa}_{t,k,\tau} = \kappa_{t,k,\tau} + \epsilon_{t,k,\tau}$$
Parametric Estimation of Risk-Neutral Dynamics

We Estimate the State Vector and Parameter Vector via,

\[
\left( \left\{ \hat{S}^n_t \right\}_{t=1,...,T}, \hat{\theta}^n \right) = \arg\min_{\{Z_t\}_{t=1,...,T}, \theta \in \Theta} \sum_{t=1}^{T} \left\{ \frac{1}{N_t} \sum_{j=1}^{N_t} \left( \frac{\hat{\kappa}_{t,k,\tau} - \kappa(k_j, \tau_j, Z_t, \theta)}{\hat{V}_t^{(n,n)}} \right)^2 \hat{V}_t^{(n,mn)} \right. \\
+ \left. \lambda_n \frac{\left( \hat{V}_t^{(n,mn)} - \xi(Z_t) \right)^2}{\hat{V}_t^{(n,n)}} \right\},
\]

\(\hat{V}_t^{(n,mn)}\) Nonparametric Estimator of Volatility from HF Data,

\(\lambda_n\) is penalty weight \((\lambda_n \to 0)\).

\(\xi(\cdot)\) is Model-Based Map from State Vector to Spot Volatility.
Parametric Estimation of Risk-Neutral Dynamics

Our Nonparametric HF Volatility Estimator is Truncated Realized Volatility,

\[
\hat{V}_t^{(n,m_n)} = \frac{n}{m_n} \sum_{i=tn-m_n+1}^{tn} (\Delta_i^n f)^2 1\{|\Delta_i^n f| \leq \alpha_t \cdot n^{-\varpi}\}, \quad \Delta_i^n f = f_{i/n} - f_{(i-1)/n},
\]

where \( \alpha_t > 0, \ \varpi \in (0, 1/2) \), and \( m_n \) deterministic sequence with \( m_n/n \to 0 \).
Observation Scheme

Underlying price process

Option cross section

Maturity

Moneyness

$t_1$

$t_2$

$t_3$
Skew and Term Structure Scatter Plots

One−Factor Gaussian Jump Model

Data

One−Factor Exponential Jump Model
Skew and Term Structure Scatter Plots

Two-Factor Exponential Jump Model

Data

Three-Factor Exponential Jump Model
The Risk-Neutral Asset Price Model

\[
\frac{dX_t}{X_{t-}} = (r_t - \delta_t) dt + \sqrt{V_{1,t}} dW_{1,t}^Q + \sqrt{V_{2,t}} dW_{2,t}^Q + \int_{\mathbb{R}^2} (e^x - 1) \tilde{\mu}^Q(dt, dx, dy),
\]

\[
dV_{1,t} = \kappa_1 (\bar{v}_1 - V_{1,t}) dt + \sigma_1 \sqrt{V_{1,t}} dB_{1,t}^Q + \mu_1 \int_{\mathbb{R}^2} x^2 1_{\{x<0\}} \mu(dt, dx, dy),
\]

\[
dV_{2,t} = \kappa_2 (\bar{v}_2 - V_{2,t}) dt + \sigma_2 \sqrt{V_{2,t}} dB_{2,t}^Q,
\]

\[
dU_t = -\kappa_3 U_t dt + \mu_u \int_{\mathbb{R}^2} \left[ (1 - \rho_3) x^2 1_{\{x<0\}} + \rho_3 y^2 \right] \mu(dt, dx, dy).
\]

Jump Compensator (under $Q$):

\[
\left\{ \begin{array}{c}
(c^{-1}_{x<0} \lambda_- e^{-\lambda_- |x|} + c^+_{x>0} \lambda_+ e^{-\lambda_+ x}) 1_{\{y=0\}} + c^{-1}_{x=0, y<0} \lambda_- e^{-\lambda_- |y|} \\
(c^- + c^+ V_{1,t-} + c^- V_{2,t-} + c^- U_{t-}, c^+ = c^+_0 + c^+_1 V_{1,t-} + c^+_2 V_{2,t-} + c^+_3 U_{t-})
\end{array} \right\} dx \otimes dy,
\]

\[
c^- = c^-_0 + c^-_1 V_{1,t-} + c^-_2 V_{2,t-} + c^-_3 U_{t-}, \quad c^+ = c^+_0 + c^+_1 V_{1,t-} + c^+_2 V_{2,t-} + c^+_3 U_{t-}.
\]
The Risk-Neutral Asset Price Model


• Negative Price Jumps have Additional Source of Time Variation via $U$.

• $U$ connected with volatility by Common Jumps and Jump Intensities.

WE DECOUPLE TAIL RISK FROM VOLATILITY
Option Surface Sensitivity to State Vector

Solid Line: Implied Volatility with State Variables at Sample Means.
Dashed Line: $V_1$ Shift; Dotted Line: $V_2$ Shift; x-x-x Line: $U$ Shift.
### Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>-0.913</td>
<td>0.028</td>
<td>$\sigma_2$</td>
<td>0.110</td>
<td>0.006</td>
<td>$c^-_2$</td>
<td>0.913</td>
<td>3.730</td>
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<td>$\bar{v}_1$</td>
<td>0.007</td>
<td>0.000</td>
<td>$\mu_u$</td>
<td>1.756</td>
<td>0.647</td>
<td>$c^+_2$</td>
<td>14.269</td>
<td>4.986</td>
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<tr>
<td>$\kappa_1$</td>
<td>8.325</td>
<td>0.167</td>
<td>$\kappa_3$</td>
<td>0.522</td>
<td>0.080</td>
<td>$c^-_3$</td>
<td>19.836</td>
<td>5.460</td>
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<tr>
<td>$\sigma_1$</td>
<td>0.323</td>
<td>0.014</td>
<td>$\rho_3$</td>
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<td>0.614</td>
<td>$\lambda_-$</td>
<td>21.157</td>
<td>0.240</td>
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<tr>
<td>$\rho_2$</td>
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<td>0.036</td>
<td>$c^+_0$</td>
<td>0.723</td>
<td>0.079</td>
<td>$\lambda_+$</td>
<td>48.365</td>
<td>2.053</td>
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<tr>
<td>$\bar{v}_2$</td>
<td>0.016</td>
<td>0.001</td>
<td>$c^-_1$</td>
<td>34.592</td>
<td>1.931</td>
<td>$\mu_-$</td>
<td>11.602</td>
<td>0.262</td>
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<tr>
<td>$\kappa_2$</td>
<td>0.480</td>
<td>0.033</td>
<td>$c^+_1$</td>
<td>88.178</td>
<td>14.711</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Mean Negative Price Jump $\approx -4.7\%$, Mean Positive Price Jump $\approx 2.1\%$

2. Frequency of Negative Jumps 3.4%, Positive Jumps 2.6% per Year.

3. Left Jump Tail twice as Fat as Right Jump Tail; $c^{\pm}$ coefficients differ significantly.

4. $U$ very persistent, Accounts for 40% of Variation in Left Tail.
Variance and Negative Risk-Neutral Jump Intensity
Formal Diagnostic Tests

We compute the following Z-scores

\[ Z_{t, \tau^*, K} = \frac{\sum_{j: k_j \in K} \left( \bar{K}_{t, k_j, \tau^*} - \kappa(k_j, \tau^*, \hat{S}_t^n, \hat{\theta}^n) \right)}{\sqrt{\text{Avar} \left( \sum_{j: k_j \in K} \left( \bar{K}_{t, k_j, \tau^*} - \kappa(k_j, \tau^*, \hat{S}_t^n, \hat{\theta}^n) \right) \right)}} \xrightarrow{\mathcal{L}} N(0, 1), \]

where \( \tau^* \) is particular maturity and \( K \) is part of the moneyness space.
Formal Diagnostic Tests: One-Factor Gaussian Jump Model

Z-score: short maturity OTM Put options

Z-score: short maturity ATM options

Z-score: short maturity OTM Call options

Z-score: long maturity OTM Put options

Z-score: long maturity ATM options

Z-score: long maturity OTM Call options
Formal Diagnostic Tests: The Three Factor Model
Ratio Model-Implied Negative Jump to Total Return Variation

One-Factor Gaussian Jump Model

One-Factor Double-Exponential Jump Model

Two-Factor Double-Exponential Jump Model

Three-Factor Double-Exponential Jump Model

0 0.25 0.5 0.75

$U$ drives Risks and/or Risk premia?
Variance Risk = Quadratic Variation over $[t, t + \tau]$:

$$QV_{t,t+\tau} = QV^c_{t,t+\tau} + QV^j_{t,t+\tau},$$

$$QV^c_{t,t+\tau} = \int_t^{t+\tau} (V_{1,s} + V_{2,s}) \, ds; \quad \text{Variance Risk of Continuous Part of } X.$$

$$QV^j_{t,t+\tau} = \int_t^{t+\tau} \int_{\mathbb{R}^2} x^2 \mu(ds, dx, dy), \quad \text{Variance Risk of Jump Part of } X.$$
Linking Information in Option Panel with Underlying Asset

Measures of Jump Risk:

\[ LT^K_{t,t+\tau} \equiv \int_t^{t+\tau} \int_{\mathbb{R}^2} 1\{x \leq -K\} \mu(ds, dx, dy), \]
\[ RT^K_{t,t+\tau} \equiv \int_t^{t+\tau} \int_{\mathbb{R}^2} 1\{x \geq K\} \mu(ds, dx, dy). \]

We have,

\[ LT^K_{t,t+\tau} = \int_t^{t+\tau} \int_{\mathbb{R}^2} 1\{x \leq -K\} \nu_s^P(dx, dy) \, ds + \epsilon^L_{t,t+\tau}, \quad \mathbb{E}_t^P(\epsilon^L_{t,t+\tau}) = 0, \]
\[ RT^K_{t,t+\tau} = \int_t^{t+\tau} \int_{\mathbb{R}^2} 1\{x \geq K\} \nu_s^P(dx, dy) \, ds + \epsilon^R_{t,t+\tau}, \quad \mathbb{E}_t^P(\epsilon^R_{t,t+\tau}) = 0. \]


**Linking Information in Option Panel with Underlying Asset**

Measures of Risk Premia:

\[
\frac{1}{\tau} \log \left( \frac{X_{t+\tau}}{X_t} \right) - \frac{1}{\tau} \int_t^{t+\tau} \left( r_s - \delta_s - q^P_s \right) ds = \text{ERP}_t^\tau + \epsilon_t^{E_{t,t+\tau}}, \quad \mathbb{E}^P_t \left[ \epsilon_t^{E_{t,t+\tau}} \right] = 0,
\]

\[
\hat{\text{VRP}}_t^\tau = \frac{1}{\tau} \left[ \hat{Q}V_{t,t+\tau} - \mathbb{E}_t^Q (QV_{t,t+\tau}) \right] = \text{VRP}_t^\tau + \epsilon_t^{V_{t,t+\tau}}, \quad \mathbb{E}^P_t \left[ \epsilon_t^{V_{t,t+\tau}} \right] = 0,
\]

- \( \mathbb{E}_t^Q [QV_{t,t+\tau}] \) may be obtained in “Model-Free” fashion from VIX Index.
- ERP = Equity Risk Premium.
- VRP = Variance Risk Premium.
Predictive Regressions for Returns and Risk Premia

We Explore the \textbf{Predictive Regressions},
\begin{equation}
y_t = \alpha_0 + \alpha_1 V_{1,t} + \alpha_2 V_{2,t} + \alpha_3 U_t + \epsilon_t, \quad \text{with } y_t \text{ representing:}
\end{equation}

Actual Price Jump Risk:

\begin{itemize}
  \item \( y_t = \hat{LT}_{t,t+\tau}^K \), or \( \hat{RT}_{t,t+\tau}^K \).
\end{itemize}

Actual (Diffusive) Variance Risk:

\begin{itemize}
  \item \( y_t = \hat{QV}_{t,t+\tau}^c \).
\end{itemize}

Size of Risk Premiums:

\begin{itemize}
  \item \( y_t = \log \left( \frac{X_{t+\tau}}{X_t} \right) - \frac{1}{\tau} \int_t^{t+\tau} (r_s - \delta_s - q_s^P) \, ds \) \ or \ \( \hat{\text{VRP}}_{t}^\tau \).
\end{itemize}
Empirical Evidence on Option Factors and Risk (Premia)

Predictor variables: $V_1$ (dashed-dotted), $V_2$ (dashed) and $\tilde{U}$ (solid), where $\tilde{U}$ is Residual from Linear Projection of $U$ on $V_1$ and $V_2$. Dashed lines in $R^2$ Plots are from Constrained Regressions including only $V_1$ and $V_2$. 
Empirical Evidence on Option Factors and Risk (Premia)

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Structural Model Implications for Option Factors and Risk (Premia)

Is Our Evidence Consistent with Standard Structural Models of Risk and Risk Premia?

Consider affine setting + representative agent with Epstein-Zin preferences:

- In this setting asset prices are affine functions of structural shocks
- The jump part of the equilibrium-based pricing kernel is

\[ E \left( \int_0^t \int_{\mathbb{R}^n} (Y - 1) \tilde{\mu}^P (ds, dx) \right), \]

with \( Y \) time-invariant and non-random

\[ \frac{\nu^Q(ds, dx)}{\nu^P(ds, dx)} \] TIME-INARIANT (no room for \( U! \))
Empirical Evidence on Option Factors and Risk (Premia)

Predictive Regressions implied by Dreschler & Yaron (2011) Structural Model. The predictive variables are the conditional mean of consumption growth (solid line), stochastic volatility of consumption growth (dashed-dotted line) and central tendency of stochastic volatility (dashed line).
Structural Model Implications for Option Factors and Risk (Premia)

Structural models tie risks and risk premia too closely together (relative to data).

Can Structural Models Loosen Link between Drivers of Risk and Risk Premia?

1. Time-Varying Risk Aversion?

   - jumps in consumption growth + habit persistence,
   - but we need to de-couple habit persistence from volatility.

2. Imperfect Information, Confidence Risk, Ambiguity Aversion?

   - ambiguity about jump risk component of model,
   - but need to limit effect of the ambiguity aversion on predictability of jump risk.
Conclusion

Used more Elaborate Option Pricing Model than Explored Hitherto.

Extract Three Option-Implied Factors, with One Controlling Left Jump Tail.

Third Factor Improves Fit to Option Surface Significantly.

Jump Tail Factor has No Impact on Actual Volatility and Jump Dynamics.

Jump Tail Factor is Critical for Equity and Variance Risk Premium.

New Factor cannot be Retrieved from Dynamics of Underlying Asset.

New Factor not readily Associated with Fundamental Risk Factor.

Suggests we Need Time-Varying Risk Aversion or Ambiguity Aversion.

Further Improvements in Option Fit feasible, but may Lose Affine Pricing.