Identifying Taylor Rules in macro-finance models

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Identifying the Taylor rule

- Long-term goal
  - Integrate models of bond pricing and monetary policy
- Open question
  - Can we identify the monetary policy parameter(s)?
  - Can we distinguish systematic policy from shocks to it?

Identifying the Taylor rule

- Common views
  - Macro: not identified
  - Finance: can't extract policy component from affine model
- What we do
  - Describe conditions for identification
  - Revisit earlier work on dynamic rational expectations models

Outline

- Two examples of the problem
- Rational expectations solutions and identification
- More complex models
- What if you don't see the state?

Setup

- State

$$x_{t+1} = Ax_t + Cw_{t+1}$$

- Shocks

$$s_{it} = d_i^\top x_t$$

- Identification issue: we observe state x, but not shock s_i

Cochrane's example

- Model

$$
\begin{aligned}
i_t &= r + E_t\pi_{t+1} && \text{(Fisher equation)} \\
i_t &= r + \tau\pi_t + s_t && \text{(Taylor rule)}
\end{aligned}
$$

Cochrane's example

- Model

$$\begin{aligned} i_t &= r + E_t\pi_{t+1} && \text{(Fisher equation)} \\ i_t &= r + \tau\pi_t + s_t && \text{(Taylor rule)} \end{aligned}$$

- Expectational difference equation

$$E_t\pi_{t+1} = \tau\pi_t + s_t$$

- Solution: $\pi_t = b^\top x_t$ with $b^\top = -d^\top(\tau I - A)^{-1}$
- Identification problem: any τ works for some d

$$b^\top A = \tau b^\top + d^\top$$

Affine example

- Model

$$\begin{aligned} m^{\$}_{t+1} &= -\delta^\top x_t - \lambda^\top\lambda/2 + \lambda^\top w_{t+1} && \text{((log) Pricing kernel)} \\ i_t &= -\log E_t \exp(m^{\$}_{t+1}) \;=\; \delta^\top x_t && \text{(Interest rate)} \end{aligned}$$

Affine example

- Model

$$
\begin{aligned}
m^{\$}_{t+1} &= -\delta^\top x_t - \lambda^\top \lambda/2 + \lambda^\top w_{t+1} && \text{((log) Pricing kernel)} \\
i_t &= -\log E_t \exp(m^{\$}_{t+1}) \;=\; \delta^\top x_t && \text{(Interest rate)}
\end{aligned}
$$

- Expectational difference equation

$$
E_t \pi_{t+1} \quad = \quad \tau \pi_t + s_t
$$

- Is interest rate equation a Taylor rule or the Fisher equation?

$$
\delta = \tau b^\top + d^\top \text{ or } \delta = b^\top A
$$

Questions

- Would an extra shock help?

$$
\begin{aligned}
i_t &= E_t\pi_{t+1} + s_{1t} && \text{(Fisher equation)} \\
i_t &= \tau\pi_t + s_{2t} && \text{(Taylor rule)}
\end{aligned}
$$

  - If shocks are independent, can use s_{1t} as an instrument

- Would long rates help?
  - May span state, but we see state anyway

A representative-agent model

- Equations

$$
\begin{aligned}
i_t &= -\log E_t \exp(m_{t+1} - \pi_{t+1}) && \text{(Euler/Fisher equation)} \\
m_t &= -\rho - \alpha g_t && \text{((log) Pricing kernel)} \\
g_t &= g + s_{1t} && \text{(Consumption growth)} \\
i_t &= \tau\pi_t + s_{2t} && \text{(Taylor rule)}
\end{aligned}
$$

- Guess $i_t = a^\top x_t$, $\pi_t = b^\top x_t$
- Solution: $b^\top \ = \ (\alpha d_1^\top - d_2^\top)(\tau I - A)^{-1}$, $a^\top = \tau b^\top + d_2^\top$
- Estimate a and b from OLS of i_t and π_t on x_t
- We have n knowns a and $n+1$ unknowns (τ, d_2)
- One exclusion restriction is sufficient to identify τ e.g., independence of shocks

An exponential-affine model

- Equations

$$
\begin{aligned}
i_t &= -\log E_t \exp(m_{t+1} - \pi_{t+1}) && \text{(Euler/Fisher equation)} \\
m_{t+1} &= -\rho - s_{1t} + \lambda^\top w_{t+1} && \text{((log) Pricing kernel)} \\
i_t &= \tau \pi_t + s_{2t} && \text{(Taylor rule)}
\end{aligned}
$$

- Guess $i_t = a^\top x_t$, $\pi_t = b^\top x_t$
- Solution: $b^\top \;=\; (\delta^\top - d^\top)(\tau I - A)^{-1}$, $a^\top = \tau b^\top + d^\top$
- Estimate a and b from OLS of i_t and π_t on x_t
- We have n knowns a and $n+1$ unknowns (τ, d)
- One exclusion restriction is sufficient to identify τ, e.g., $d_{2i} = 0$.

Observations

- OLS of i_t on π_t does not recover τ
- One restriction on MP shock suffices to identify the Taylor rule
- It does not matter whether they are shocks in the Euler equation or not
- This success reflects what Hansen and Sargent call the hallmark of rational expectations models: that cross-equation restrictions connect the parameters in one equation to those in the others
- What about more elaborate models?

A Phillips curve

- Equations

$$
\begin{aligned}
i_t &= -\log E_t \exp(m_{t+1} - \pi_{t+1}) && \text{(Euler equation)} \\
m_t &= -\rho - \alpha g_t && \text{(Pricing kernel)} \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa g_t + s_{1t} && \text{(Phillips curve)} \\
i_t &= \tau_1 \pi_t + \tau_2 g_t + s_{2t} && \text{(Taylor rule)}
\end{aligned}
$$

- Guess $i_t = a^\top x_t$, $\pi_t = b^\top x_t$ and $g_t = c^\top x_t$
- Solution:

$$
\begin{aligned}
b^\top &= \beta A b^\top + \kappa c^\top + d_1^\top \\
a^\top &= \tau_1 b^\top + \tau_2 c^\top + d_2^\top
\end{aligned}
$$

- Estimate a, b and c from OLS of i_t, π_t and g_t on x_t
- In the 2nd eq-n we have n knowns a and $n+2$ unknowns (τ_1, τ_2, d_2)
- Two exclusion restrictions is sufficient to identify τ_1 (and τ_2)

Unobserved state

- Until now we have assumed observed state
  - This assumption allowed us to run the OLS critical for identification
- What changes when the state has to be estimated?
  - Nothing
- Have to discuss identification and estimation of the state

Identifying the state

- The state can be "rotated", $\hat{x}_t = Tx_t$, without affecting implications for observed variables
- The new state is:

$$\hat{x}_{t+1} \quad = \quad TAT^{-1}\hat{x}_t + TCw_{t+1} \quad = \quad \hat{A}\hat{x}_t + \hat{C}w_{t+1}.$$

- The interest rate is $i_t = r + a^\top T^{-1}\hat{x}_t = r + \hat{a}^\top \hat{x}_t$
- Similarly, $\hat{b}^\top = b^\top T^{-1}$
- The Taylor rule implies

$$\hat{a}^\top \quad = \quad \tau\hat{b}^\top + \hat{d}_2^\top,$$

- Identification of τ is exactly the same as before
- Have to impose restrictions to fix a state: $C = I$ and lower-triangular A is a common choice

Transformation-invariant restrictions

- The impact on restrictions: $d_2^\top e = 0$ translates into $\hat{d}_2^\top \hat{e} = 0$ with $\hat{e} = Te$.
- If T is unknown, can we deduce $\hat{e}$?

Transformation-invariant restrictions

- The impact on restrictions: $d_2^\top e = 0$ translates into $\hat{d}_2^\top \hat{e} = 0$ with $\hat{e} = Te$.
- If T is unknown, can we deduce $\hat{e}$?
- Ex. 1: The TR shock is uncorrelated with the other (Euler equation) shock – standard in the New-Keynesian literature

$$\hat{d}_2^\top E(\hat{x}\hat{x}^\top)\hat{d}_1 = (d_2^\top T^{-1})(TV_x T^\top)(d_1^\top T^{-1})^\top = d_2^\top V_x d_1$$

- Ex. 2: In the optimal monetary policy setting all variables are affected by s_{1t}, thus $s_{2t} = ks_{1t}$
  - The implied restriction is $d_2^\top - kd_1^\top = 0$
  - In terms of the transformed state:

$$\hat{d}_2^\top - k\hat{d}_1^\top = d_2^\top T^{-1} - kd_1^\top T^{-1} = (d_2^\top - kd_1^\top)T^{-1} = 0$$

Estimating the state

- Introduce a measurement equation:

$$y_t \quad = \quad Gx_t + Hv_t.$$

- The Kalman filter, $x_{t|t} = E(x_t|y^t)$, recovers all states when (A, G) is observable:

$$rank \begin{bmatrix} G \\ GA \\ \vdots \\ GA^{n-1} \end{bmatrix} = n$$

- As a result, $i_t = a^\top x_{t|t} + a^\top \varepsilon_t$, $\varepsilon_t \perp y^t$
- All the earlier logic applies still by replacing x_t with $x_{t|t}$

Term Structure and Survey Forecasts

- Financial and survey data are rich sources of information about the unobserved state
- In our models forward rates f_t^h and forecasts $F_t(\cdot)$ for horizon h are

$$\begin{aligned} f_t^h &= a^\top A^h x_t + v_t \\ F_t(\pi_{t+h}) &= E_t(\pi_{t+h}) + v_t = b^\top A^h x_t + v_t \\ F_t(g_{t+h}) &= E_t(g_{t+h}) + v_t = c^\top A^h x_t + v_t \end{aligned}$$

- Usually x_t is low-dimensional and $dim(y_t)$ is large. In these cases, researchers attach measurement errors to all observables y_t and use the Kalman filter to estimate the state.

Conclusion

- Brute-force identification of the Taylor rule is impossible
- Rational expectations framework brings information from the whole system to bear on the TR coefficients
- We offer a constructive approach towards identification
- In general, one needs exclusion restrictions on MP shocks to identify TR; typical models impose more than what's required