Identifying Taylor Rules in macro-finance models

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Identifying the Taylor rule

- Long-term goal
  - Integrate models of bond pricing and monetary policy

- Open question
  - Can we identify the monetary policy parameter(s)?
  - Can we distinguish systematic policy from shocks to it?
Identifying the Taylor rule

Common views
- Macro: not identified
- Finance: can’t extract policy component from affine model

What we do
- Describe conditions for identification
- Revisit earlier work on dynamic rational expectations models
Outline

- Two examples of the problem
- Rational expectations solutions and identification
- More complex models
- What if you don’t see the state?
Setup

- State

\[ x_{t+1} = Ax_t + Cw_{t+1} \]

- Shocks

\[ s_{it} = d_i^\top x_t \]

- Identification issue: we observe state \( x \), but not shock \( s_i \)
Cochrane’s example

- Model

\[ i_t = r + E_t \pi_{t+1} \quad \text{(Fisher equation)} \]
\[ i_t = r + \tau \pi_t + s_t \quad \text{(Taylor rule)} \]
Cochrane’s example

- **Model**

\[ i_t = r + E_t \pi_{t+1} \]  
(Fisher equation)

\[ i_t = r + \tau \pi_t + s_t \]  
(Taylor rule)

- **Expectational difference equation**

\[ E_t \pi_{t+1} = \tau \pi_t + s_t \]

- **Solution:** \( \pi_t = b^\top x_t \) with \( b^\top = -d^\top (\tau I - A)^{-1} \)

- **Identification problem:** any \( \tau \) works for some \( d \)

\[ b^\top A = \tau b^\top + d^\top \]
Affine example

- Model

\[
\begin{align*}
    m_{t+1}^\$ & = -\delta^\top x_t - \lambda^\top \lambda / 2 + \lambda^\top w_{t+1} & \text{((log) Pricing kernel)} \\
    i_t & = -\log E_t \exp(m_{t+1}^\$) = \delta^\top x_t & \text{(Interest rate)}
\end{align*}
\]
Affine example

Model

\[ m_{t+1}^s = -\delta^\top x_t - \lambda^\top \lambda/2 + \lambda^\top w_{t+1} \]  
\[ i_t = -\log E_t \exp(m_{t+1}^s) = \delta^\top x_t \]  
((log) Pricing kernel)

(Interest rate)

Expectational difference equation

\[ E_t \pi_{t+1} = \tau \pi_t + s_t \]

Is interest rate equation a Taylor rule or the Fisher equation?

\[ \delta = \tau b^\top + d^\top \]  
or  
\[ \delta = b^\top A \]
Questions

Would an extra shock help?

\[ i_t = E_t \pi_{t+1} + s_{1t} \]  
\[ i_t = \tau \pi_t + s_{2t} \]  

(Fisher equation)  
(Taylor rule)

If shocks are independent, can use \( s_{1t} \) as an instrument

Would long rates help?

May span state, but we see state anyway
A representative-agent model

- **Equations**

  \[
  i_t = -\log E_t \exp(m_{t+1} - \pi_{t+1}) \quad \text{(Euler/Fisher equation)}
  \]
  
  \[
  m_t = -\rho - \alpha g_t \quad \text{((log) Pricing kernel)}
  \]
  
  \[
  g_t = g + s_{1t} \quad \text{(Consumption growth)}
  \]
  
  \[
  i_t = \tau \pi_t + s_{2t} \quad \text{(Taylor rule)}
  \]

- **Guess** \(i_t = a^\top x_t, \pi_t = b^\top x_t\)

- **Solution**: \(b^\top = (\alpha d_1^\top - d_2^\top)(\tau I - A)^{-1}, a^\top = \tau b^\top + d_2^\top\)

- **Estimate** \(a\) and \(b\) from OLS of \(i_t\) and \(\pi_t\) on \(x_t\)

- **We have** \(n\) knowns \(a\) and \(n + 1\) unknowns \((\tau, d_2)\)

- **One exclusion restriction is sufficient to identify** \(\tau\) e.g., independence of shocks
An exponential-affine model

- Equations

\[ i_t = - \log E_t \exp (m_{t+1} - \pi_{t+1}) \]  \hspace{1cm} \text{(Euler/Fisher equation)}

\[ m_{t+1} = -\rho - s_{1t} + \lambda^\top w_{t+1} \]  \hspace{1cm} \text{((log) Pricing kernel)}

\[ i_t = \tau\pi_t + s_{2t} \]  \hspace{1cm} \text{(Taylor rule)}

- Guess \( i_t = a^\top x_t, \pi_t = b^\top x_t \)

- Solution: \( b^\top = (\delta^\top - d^\top)(\tau I - A)^{-1}, a^\top = \tau b^\top + d^\top \)

- Estimate \( a \) and \( b \) from OLS of \( i_t \) and \( \pi_t \) on \( x_t \)

- We have \( n \) knowns \( a \) and \( n + 1 \) unknowns \( (\tau, d) \)

- One exclusion restriction is sufficient to identify \( \tau \), e.g., \( d_{2i} = 0 \).
Observations

- OLS of $i_t$ on $\pi_t$ does not recover $\tau$

- One restriction on MP shock suffices to identify the Taylor rule

- It does not matter whether they are shocks in the Euler equation or not

- This success reflects what Hansen and Sargent call the hallmark of rational expectations models: that cross-equation restrictions connect the parameters in one equation to those in the others

- What about more elaborate models?
A Phillips curve

Equations

\[ i_t = -\log E_t \exp (m_{t+1} - \pi_{t+1}) \]  
(Euler equation)

\[ m_t = -\rho - \alpha g_t \]  
(Pricing kernel)

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa g_t + s_{1t} \]  
(Phillips curve)

\[ i_t = \tau_1 \pi_t + \tau_2 g_t + s_{2t} \]  
(Taylor rule)

Guess \( i_t = a^\top x_t, \pi_t = b^\top x_t \) and \( g_t = c^\top x_t \)

Solution:

\[ b^\top = \beta A b^\top + \kappa c^\top + d_1^\top \]

\[ a^\top = \tau_1 b^\top + \tau_2 c^\top + d_2^\top \]

Estimate \( a, b \) and \( c \) from OLS of \( i_t, \pi_t \) and \( g_t \) on \( x_t \)

In the 2nd eq-n we have \( n \) knowns \( a \) and \( n+2 \) unknowns \( (\tau_1, \tau_2, d_2) \)

Two exclusion restrictions is sufficient to identify \( \tau_1 \) (and \( \tau_2 \))
Until now we have assumed observed state
- This assumption allowed us to run the OLS critical for identification

What changes when the state has to be estimated?
- Nothing

Have to discuss identification and estimation of the state
Identifying the state

- The state can be “rotated”, $\hat{x}_t = Tx_t$, without affecting implications for observed variables.
- The new state is:

$$\hat{x}_{t+1} = TAT^{-1}\hat{x}_t + TCw_{t+1} = \hat{A}\hat{x}_t + \hat{C}w_{t+1}.$$  

- The interest rate is $i_t = r + a^\top T^{-1}\hat{x}_t = r + \hat{a}^\top \hat{x}_t$
- Similarly, $\hat{b}^\top = b^\top T^{-1}$
- The Taylor rule implies

$$\hat{a}^\top = \tau \hat{b}^\top + \hat{d}_2^\top,$$

- Identification of $\tau$ is exactly the same as before.
- Have to impose restrictions to fix a state: $C = I$ and lower-triangular $A$ is a common choice.
The impact on restrictions: $d_2^\top e = 0$ translates into $\hat{d}_2^\top \hat{e} = 0$ with $\hat{e} = T e$.

If $T$ is unknown, can we deduce $\hat{e}$?
Transformation-invariant restrictions

- The impact on restrictions: \( d_2^\top e = 0 \) translates into \( \hat{d}_2^\top \hat{e} = 0 \) with \( \hat{e} = Te \).

- If \( T \) is unknown, can we deduce \( \hat{e} \)?

- Ex. 1: The TR shock is uncorrelated with the other (Euler equation) shock – standard in the New-Keynesian literature

\[
\hat{d}_2^\top E(\hat{x}\hat{x}^\top)\hat{d}_1 = (d_2^\top T^{-1})(TV_x T^\top)(d_1^\top T^{-1})^\top = d_2^\top V_x d_1
\]

- Ex. 2: In the optimal monetary policy setting all variables are affected by \( s_{1t} \), thus \( s_{2t} = ks_{1t} \)
  - The implied restriction is \( d_2^\top - kd_1^\top = 0 \)
  - In terms of the transformed state:

\[
\hat{d}_2^\top - k\hat{d}_1^\top = d_2^\top T^{-1} - kd_1^\top T^{-1} = (d_2^\top - kd_1^\top) T^{-1} = 0
\]
Introduce a measurement equation:

\[ y_t = Gx_t + Hv_t. \]

The Kalman filter, \( x_{t|t} = E(x_t|y^t) \), recovers all states when \((A, G)\) is observable:

\[
\begin{bmatrix}
G \\
GA \\
\vdots \\
GA^{n-1}
\end{bmatrix}
\]

\[ \text{rank} = n \]

As a result, \( i_t = a^\top x_{t|t} + a^\top \varepsilon_t, \varepsilon_t \perp y^t \)

All the earlier logic applies still by replacing \( x_t \) with \( x_{t|t} \)
Financial and survey data are rich sources of information about the unobserved state.

In our models forward rates $f_t^h$ and forecasts $F_t(\cdot)$ for horizon $h$ are

$$f_t^h = a^\top A^h x_t + \nu_t$$

$$F_t(\pi_{t+h}) = E_t(\pi_{t+h}) + \nu_t = b^\top A^h x_t + \nu_t$$

$$F_t(g_{t+h}) = E_t(g_{t+h}) + \nu_t = c^\top A^h x_t + \nu_t$$

Usually $x_t$ is low-dimensional and $\text{dim}(y_t)$ is large. In these cases, researchers attach measurement errors to all observables $y_t$ and use the Kalman filter to estimate the state.
Brute-force identification of the Taylor rule is impossible.

Rational expectations framework brings information from the whole system to bear on the TR coefficients.

We offer a constructive approach towards identification.

In general, one needs exclusion restrictions on MP shocks to identify TR; typical models impose more than what’s required.