Retirement, Home Production and Labor Supply Elasticities

Richard Rogerson
Johanna Wallenius

October 2013
Background/Motivation

Consider an individual with period utility function:

\[ u(c_t) + \alpha \frac{1}{1 - \frac{1}{\gamma}} (1 - h_{mt} - h_{nt})^{1-\frac{1}{\gamma}} \]

where

\[ c_t = [a g_t^{1-\frac{1}{\eta}} + (1 - a) h_{nt}^{1-\frac{1}{\eta}}]^{\eta/(\eta-1)} \]

\( \gamma \) and \( \eta \) are important joint determinants of labor supply responses for this individual in a variety of contexts. “Consensus” ranges for these two elasticity parameters:

\( \gamma \in [.4, .5] \) and \( \eta \in [1.4, 2.0] \)

Interestingly, the literatures estimating these two preference parameters are effectively distinct.
What We Do

In this paper we derive a relation that links these two elasticities to changes in time allocations and consumption expenditure at retirement.

When we evaluate this expression using typical values for what happens at retirement, we find that these two consensus ranges are strongly inconsistent.

What does this mean?

1. subliterature arguing for higher values of $\gamma$ is correct
2. estimates of $\eta$ are too high
3. 1 and 2
4. our model is misspecified
5. all of the above
Regarding Misspecification...

In recent years many authors have noted many features that have a large effect on estimates of $\gamma$:

- human capital accumulation (Imai and Keane (2004), Wallenius (2011))
- credit constraints for younger workers (Domeij and Floden)
- incomplete markets (Low (2006))
- restrictions on hours (Chang and Kim (2006), Rogerson (2011))
- wage setting (Ham and O’Riley (2013))

Our theoretical derivation will be robust to these elements.
Should Macroeconomists Care About $\gamma$? Evolving Answers

- However, it has long been understood that the value of $\gamma$ estimated from individual panel data for continuously employed individuals is not the appropriate value to import into a stand-in household model.

- Issue: Aggregate labor supply elasticity must reflect adjustment along intensive and extensive margins.
Incorporating this insight requires models that allow for adjustment along the extensive margin.

“Standard” models assumed that all adjustment is along the intensive margin, in which case \( \gamma \) does map into the aggregate elasticity.

At the other extreme, if all adjustment is along the extensive margin then \( \gamma \) is irrelevant for aggregate labor supply elasticities.
In Hansen (1985), when all households are identical, the economy that assumes indivisible labor behaves as if there is a stand-in household with $\gamma = \infty$.

This result holds in the case in which all households are identical. If there is heterogeneity, then the aggregate labor supply elasticity depends on the extent of heterogeneity. (See, for example, Cho (1994), and Chang and Kim (2006, 2007)).

Implication: rather than focusing on better estimates of $\gamma$ from micro data, macroeconomists should focus on measuring the nature and extent of heterogeneity.
These results assumed that all adjustment takes place on the extensive margin. In reality, we see adjustment along both margins.

Rogerson and Wallenius (2009) is one model in which there is simultaneous adjustment along both intensive and extensive margins.

In this model, $\gamma$ matters for the mix of intensive vs extensive adjustment.
• In the context of cross-country steady state comparisons, Rogerson and Wallenius (2009) find that small values of $\gamma$ lead to “insufficient” adjustment along the intensive margin relative to the data.

• In the context of business cycle analysis, Chetty et al (2011) find that small values of $\gamma$ lead to “excessive” adjustment along the extensive margin.

• In these two contexts, getting the right “mix” of intensive and extensive margin adjustment requires values of $\gamma$ close to one.

Message: Although our understanding of how $\gamma$ matters for macro has evolved over time, the value of $\gamma$ remains quite relevant.
This Paper (And Its Companion)

- Existing literature has (reasonably) focused on adjustment of market hours along the intensive margin to estimate $\gamma$.

- We argue that studying retirement, can also provide information on $\gamma$.

- One benefit of looking at an alternative setting: exploring implications of parameter estimates for behavior in different settings is an important source of validation.

- A second benefit is that standard estimates are sensitive to some model features (e.g., Imai and Keane (2004), Domeij and Floden (2006)). These features are less relevant in the retirement setting.
Characterizing Retirement in the Data

Question: What does the transition from full time work to retirement look like in the data?

We address this from three perspectives:

- cross-section data analysis using CPS
- panel data analysis using PSID
- results from Blau and Shvydko (2010) based on HRS.
Analysis Using the CPS

We merge data from the 2002-2004 surveys about hours worked in the previous year.

We group observations for each age from 60 to 70 into 10 bins with boundaries 2000, 1750, 1500, 1250,....0.

We do this for the total population as well as just males.
Results for Males
Results for Males, All Groups
Results for Males and Females:
Message from CPS Data:

As the population ages from 60 to 70 the key aggregate transition involves the fractions of individuals doing full time work and no work.

Issue: This does not necessarily mean that individuals are moving directly from full time work to no work—they could be transitioning from full time work to no work over many years.

To assess this we next look at the PSID.

While the PSID does allow us to follow individuals, the sample size is quite small.
Analysis Using the PSID

We restrict attention to male head of households who have observations on hours worked for all ages from 60 to 70.

Sample size is only 307.

Aggregate statistics are similar to those in the CPS.
Analysis Using PSID, cont’d

- We focus on individuals with at least 1750 hours at age 60 and less than 250 at ages 69 and 70. Sample size is 151.

**Question:** What fraction of these move from more than 1750 hrs at age 60 to less than 250 hrs at ages 69-70 with at most one intermediate value during the transition?

**Answer:** 72.2%

Other statistics for this sample:

- age 60: mean hours: 2152 median hours: 2048
- last yr at full time: mean hrs: 2085, median hrs: 2010
- age 69 (70) : mean hours: 5 (7) median hours: 0 (0)
Message from analysis of the PSID:

The typical transition from full time work to retirement is for an individual to move from full time work to no work with at most one intervening year of intermediate work.

Message from Blau and Shvydko:

Results are not driven by health shocks or details of private pension plans.
Simple Life-Cycle Labor Supply Problem

Preferences:

\[
\int_0^1 \left[ u(c(t)) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - h(t))^{1 - \frac{1}{\gamma}} \right] dt
\]

\[
c(t) = \left[ a g(t)^{1 - \frac{1}{\eta}} + (1 - a) h_n(t)^{1 - \frac{1}{\eta}} \right] \frac{n}{\eta - 1}
\]

\[
h(t) = h_m(t) + h_n(t), \quad h_m(t) \in \{0, \tilde{h}\}
\]

Budget Set:

\[
\int_0^1 g(t) dt = \int_0^1 w(t) h_m(t) dt + \int_R^1 b(t) dt
\]

where \( w(t) = A H_m(t)^\phi, \quad H_m(t) = \int_0^t h_m(s) ds \)
Symmetry and separability allow us to write life cycle optimization problem as:

\[
\max \ e[u(c_w) + \frac{\alpha}{1-\frac{1}{\gamma}}(1-\bar{h}-h_w)^{1-\frac{1}{\gamma}}] + (1-e)[u(c_r) + \frac{\alpha}{1-\frac{1}{\gamma}}(1-h_r)^{1-\frac{1}{\gamma}}]
\]

s.t. \( eg_w + (1 - e)g_r = I \)

\[
I = \int_0^e A(t \bar{h})^{\phi} \bar{h} dt + \int_0^1 b(t) dt
\]

\[
c_w = [ag_w^{1-\frac{1}{\eta}} + (1 - a)h_w^{1-\frac{1}{\eta}}]^{\frac{n}{\eta-1}}
\]

\[
c_r = [ag_r^{1-\frac{1}{\eta}} + (1 - a)h_r^{1-\frac{1}{\eta}}]^{\frac{n}{\eta-1}}
\]
Optimal Life-Cycle Labor Supply:

\[
g_w : u'(c_w)c_w^{\frac{1}{\eta}} a g_w^{\frac{-1}{\eta}} = \mu \tag{1}
\]

\[
g_r : u'(c_r)c_r^{\frac{1}{\eta}} a g_r^{\frac{-1}{\eta}} = \mu \tag{2}
\]

\[
h_w : u'(c_w)c_w^{\frac{1}{\eta}} (1 - a) h_w^{\frac{-1}{\eta}} = \alpha (1 - \bar{h} - h_w)^{-\frac{1}{\gamma}} \tag{3}
\]

\[
h_r : u'(c_r)c_r^{\frac{1}{\eta}} (1 - a) h_r^{\frac{-1}{\eta}} = \alpha (1 - h_r)^{-\frac{1}{\gamma}} \tag{4}
\]
Rearranging:

\[
1 \div 2 : \left[ \frac{g_w}{g_r} \right]^{\frac{1}{\eta}} = \frac{u'(c_w)}{u'(c_r)} \left[ \frac{c_w}{c_r} \right]^{\frac{1}{\eta}}
\]

\[
3 \div 4 : \left[ \frac{1 - h_r}{1 - \bar{h} - h_w} \right]^{\frac{1}{\gamma}} \left[ \frac{h_w}{h_r} \right]^{\frac{1}{\eta}} = \frac{u'(c_w)}{u'(c_r)} \left[ \frac{c_w}{c_r} \right]^{\frac{1}{\eta}}
\]

Combining and taking logs:

\[
\frac{\gamma}{\eta} = \frac{\log(1 - h_r) - \log(1 - \bar{h} - h_w)}{\log(g_w/g_r) - \log(h_w/h_r)}
\]
Intuition

Given values for how allocations change at retirement, why does a higher value of $\eta$ imply that $\gamma$ must also be higher?

Think of taking all of the extra time as leisure and then increasing the time spent in home production until reaching the optimum. The benefit is the added consumption flow from home production, the cost is the decreasing value of leisure.

As $\eta$ increases, the benefit increases, pushing the individual to spend more time in home production. In order to rationalize a given value for time spent in home production, we must increase the cost of giving up leisure. This means less curvature. With high curvature, extra leisure is not very valuable, so the cost is small.
Alternative Derivation

Consider someone who has evolved through their life cycle making various decisions, facing various constraints and has arrived to age $R - 1$ and has assets $A$.

Assume also, that for whatever reason, age $R - 1$ is the last period of working life for this individual.

We focus on a subset of the choices that this individual makes from this point forward.
From this period on, the individual will maximize:

\[
[u(c_{R-1}) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - h_{R-1})^{1-\frac{1}{\gamma}}] + \beta[u(c_R) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - h_R)^{1-\frac{1}{\gamma}}] \]

\[+ \beta^2 \ldots \]

where

\[
c_t = [a g_t^{1-\frac{1}{\eta}} + (1-a) h_{nt}^{1-\frac{1}{\eta}}] \frac{\eta}{\eta-1}
\]

\[h_t = h_{mt} + h_{nt}\]

subject to the budget constraint:

\[g_{R-1} + \frac{1}{1 + i_t} g_R + \ldots = A + w_{R-1} f(h_{mR-1}) + \ldots\]
There are many choices that this individual will make.

But we focus on optimal choices for $g_{R-1}, g_R, h_{nR-1},$ and $h_{nR}$ assuming that $h_{mR} = 0$, consistent with the assumption that period $R - 1$ is the last period of work for this individual.

These 4 first order conditions will closely resemble the equations from our earlier model.
If we assume that $\beta = \frac{1}{1+i_t}$ and carry out the same manipulations as before, these 4 first order conditions will yield:

$$\frac{\gamma}{\eta} = \frac{\log(1 - h_{nR}) - \log(1 - h_{mR-1} - h_{nR-1})}{\log(g_{R-1}/g_R) - \log(h_{nR-1}/h_{nR})}$$

which is basically the same equation as before except now it is clear that what matters is the values just before and just after retirement, so we do not need to assume that these are constant over time within each phase of life.
Note:

We require no assumptions about how anything evolved or was chosen over the life cycle up to period $R - 1$.

We do not require that retirement is optimal, though we require that it is anticipated.

We do not need to take a position on how $h_{mR-1}$ is chosen. We can allow for heterogeneity in preference parameters such as $\alpha$ and $\omega$, in initial assets and in wage opportunities.
How to Proceed
Ideally, we would like panel data on individuals that provides us with data on time allocations and consumption expenditure.

Unfortunately, such data does not exist.
Instead we look for “typical” or “average” values for the change in time use and consumption expenditures at retirement.
Change in hours of market work is easy—I earlier presented data on average hours in the year prior to retirement from the PSID.

Change in consumption expenditures—a sizeable literature has estimated this, and we draw on that literature.
Change in time spent in home production: ??
## Time Use by Age in the ATUS 2003-2011 (hours/week)

<table>
<thead>
<tr>
<th>Age</th>
<th>MW</th>
<th>HP</th>
<th>L</th>
<th>PC</th>
<th>Residual</th>
<th>#obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>26.17</td>
<td>20.18</td>
<td>47.65</td>
<td>64.27</td>
<td>9.73</td>
<td>1748</td>
</tr>
<tr>
<td>61</td>
<td>22.72</td>
<td>21.24</td>
<td>49.68</td>
<td>64.82</td>
<td>9.54</td>
<td>1694</td>
</tr>
<tr>
<td>62</td>
<td>22.08</td>
<td>22.02</td>
<td>49.56</td>
<td>64.18</td>
<td>10.16</td>
<td>1616</td>
</tr>
<tr>
<td>63</td>
<td>16.67</td>
<td>22.17</td>
<td>53.24</td>
<td>65.63</td>
<td>10.29</td>
<td>1609</td>
</tr>
<tr>
<td>64</td>
<td>15.96</td>
<td>23.79</td>
<td>53.88</td>
<td>64.65</td>
<td>9.72</td>
<td>1483</td>
</tr>
<tr>
<td>65</td>
<td>11.20</td>
<td>24.46</td>
<td>55.65</td>
<td>66.09</td>
<td>10.60</td>
<td>1520</td>
</tr>
<tr>
<td>66</td>
<td>11.28</td>
<td>24.27</td>
<td>56.93</td>
<td>65.97</td>
<td>9.55</td>
<td>1384</td>
</tr>
<tr>
<td>67</td>
<td>10.91</td>
<td>22.71</td>
<td>57.84</td>
<td>66.33</td>
<td>10.21</td>
<td>1292</td>
</tr>
<tr>
<td>68</td>
<td>9.18</td>
<td>23.46</td>
<td>58.31</td>
<td>66.46</td>
<td>10.59</td>
<td>1271</td>
</tr>
<tr>
<td>69</td>
<td>7.58</td>
<td>23.27</td>
<td>60.03</td>
<td>67.05</td>
<td>10.07</td>
<td>1176</td>
</tr>
</tbody>
</table>
We carry out two exercises to estimate how working time gets allocated to leisure and home production in retirement:

Method 1: We construct a synthetic cohort and examine how the marginal change in market work affects the marginal change in home production time as individuals age.

Method 2: We look at the relationship between market work and home production in the cross-section.
Method 1: Synthetic Panel Estimates

We run the following regression using the pooled cross-section data:

\[ \text{hw}_a = \beta_0 - f_{HP} \text{mw}_a + \varepsilon_a \]

Idea is that most of the decrease in market work reflects people moving from full time work to retirement, and so most of the change in home production time reflects the adjustment associated with retirement.
## Synthetic Cohort Estimates

<table>
<thead>
<tr>
<th>Age Range</th>
<th>$f_{HP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 – 65</td>
<td>.274 (.066)</td>
</tr>
<tr>
<td>60 – 66</td>
<td>.264 (.047)</td>
</tr>
<tr>
<td>61 – 65</td>
<td>.255 (.109)</td>
</tr>
<tr>
<td>61 – 66</td>
<td>.246 (.069)</td>
</tr>
</tbody>
</table>
Method 2: Cross-Section Analysis
We run the following regression for each age:

\[ h_{wi} = \beta_0 - f_{HPmwi} + \varepsilon_i \]

We include controls for education and gender, but the results are effectively identical without them.
Cross-Sectional Estimates of $f_{HP}$

<table>
<thead>
<tr>
<th></th>
<th>60</th>
<th>61</th>
<th>62</th>
<th>63</th>
<th>64</th>
<th>65</th>
<th>66</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.27(.01)</td>
<td>.26(.02)</td>
<td>.27(.02)</td>
<td>.28(.02)</td>
<td>.30(.02)</td>
<td>.26(.02)</td>
<td>.29(.02)</td>
</tr>
</tbody>
</table>
Benchmark Values

\[ \frac{g_r}{g_w} = 0.90, \ h_w = 0.1762, \ f_{HP} = 0.2674, \bar{h} = 0.4764 \]
Results for Benchmark Specification

Table 4
Implied Values of $\gamma$ For Benchmark Specification

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>1.00</th>
<th>1.25</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{HP}$</td>
<td>0.2674</td>
<td>1.07</td>
<td>1.34</td>
<td>1.61</td>
<td>2.14</td>
</tr>
</tbody>
</table>
Sensitivity to Other Values of $f_{HP}$

### Table 5
Effect of $f_{HP}$ on Implied Values of $\gamma$

<table>
<thead>
<tr>
<th>$f_{HP}$</th>
<th>$\eta = 1.0$</th>
<th>$\eta = 1.25$</th>
<th>$\eta = 1.5$</th>
<th>$\eta = 2.0$</th>
<th>$\eta = 2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.34</td>
<td>.85</td>
<td>1.06</td>
<td>1.27</td>
<td>1.70</td>
<td>2.12</td>
</tr>
<tr>
<td>.40</td>
<td>.72</td>
<td>.89</td>
<td>1.07</td>
<td>1.43</td>
<td>1.79</td>
</tr>
<tr>
<td>.45</td>
<td>.62</td>
<td>.78</td>
<td>.94</td>
<td>1.25</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Note: To get $\gamma/\eta = 1/3 (1/2)$ would require $f_{HP} = .67 (.53)$
Table 6
Implied Values of $\gamma$ With Reduction in Discretionary Time

<table>
<thead>
<tr>
<th></th>
<th>$\eta = 1.00$</th>
<th>$\eta = 1.25$</th>
<th>$\eta = 1.50$</th>
<th>$\eta = 2.00$</th>
<th>$\eta = 2.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{HP}} = .2674$</td>
<td>.92</td>
<td>1.14</td>
<td>1.37</td>
<td>1.83</td>
<td>2.29</td>
</tr>
<tr>
<td>$f_{\text{HP}} = .3414$</td>
<td>.71</td>
<td>.88</td>
<td>1.06</td>
<td>1.42</td>
<td>1.77</td>
</tr>
</tbody>
</table>
Income and Health Shocks

Theoretical analysis assumed perfect foresight, no shocks to either income or preferences.

The only relevant changes/shocks are those that occur between what we called periods $R - 1$ and $R$.

Income shocks during this period will only reinforce our findings.

Health shocks are potentially more problematic.
### Fraction Healthy by Age

<table>
<thead>
<tr>
<th>Age Range</th>
<th>Entire Sample</th>
<th>Healthy Subsample</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 – 65</td>
<td>-.274 (.066)</td>
<td>-.311 (.174)</td>
</tr>
<tr>
<td>60 – 66</td>
<td>-.264 (.047)</td>
<td>-.279 (.135)</td>
</tr>
<tr>
<td>61 – 65</td>
<td>-.255 (.109)</td>
<td>-.250 (.220)</td>
</tr>
<tr>
<td>61 – 66</td>
<td>-.246 (.069)</td>
<td>-.219 (.159)</td>
</tr>
</tbody>
</table>

### Estimates of $f_{HP}$
Non-Convexities in Utility From Leisure

Period Utility Function:

\[ \alpha \log(c) + (1 + I_R D)(1 - \alpha) \frac{1}{1 - \frac{1}{\gamma}} (1 - h)^{1 - \frac{1}{\gamma}} \]

Interpreting \( D \):

\[ \frac{\alpha}{1 - \frac{1}{\gamma}} ((1 - \bar{h} - h_w))^{1 - \frac{1}{\gamma}} = \frac{\alpha (1 + D)}{1 - \frac{1}{\gamma}} (\Delta \cdot (1 - \bar{h} - h_w))^{1 - \frac{1}{\gamma}} \]

\[ \Delta = (1 + D)^{-1/(1 - \frac{1}{\gamma})} \]
Resulting Expression:

\[
\frac{\gamma}{\eta} = \frac{\log(1 - h_r) - \log(1 - \bar{h} - h_w)}{\log(g_w/g_r) - \log(h_w/h_r) + \eta \log D}
\]

Table 6

Implied Values of $\gamma$ With Non-Convexities in Leisure

<table>
<thead>
<tr>
<th>$\eta = 1$</th>
<th>$\eta = 1.25$</th>
<th>$\eta = 1.5$</th>
<th>$\eta = 1.75$</th>
<th>$\eta = 2$</th>
<th>$\eta = 2.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta = 1$</td>
<td>1.05</td>
<td>1.31</td>
<td>1.57</td>
<td>1.84</td>
<td>2.10</td>
</tr>
<tr>
<td>$\Delta = 2/3$</td>
<td>1.03</td>
<td>1.16</td>
<td>1.27</td>
<td>1.36</td>
<td>1.44</td>
</tr>
</tbody>
</table>
Conclusion

- We have considered models in which utility from leisure is strictly concave, implying that all else equal, individuals prefer smooth leisure over time.

- Retirement generates a very dramatic change in hours of market work, and a small/moderate increase in home production time.

- It is hard to reconcile the small increase in home production time with a moderate elasticity of substitution between time and goods and low IES.

- Results seem robust to allowing for plausible nonconvexities in the utility from leisure at retirement.