Notes on Firms and Productivity Growth

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Conference in Honor of Tom Cooley

NYU - Stern

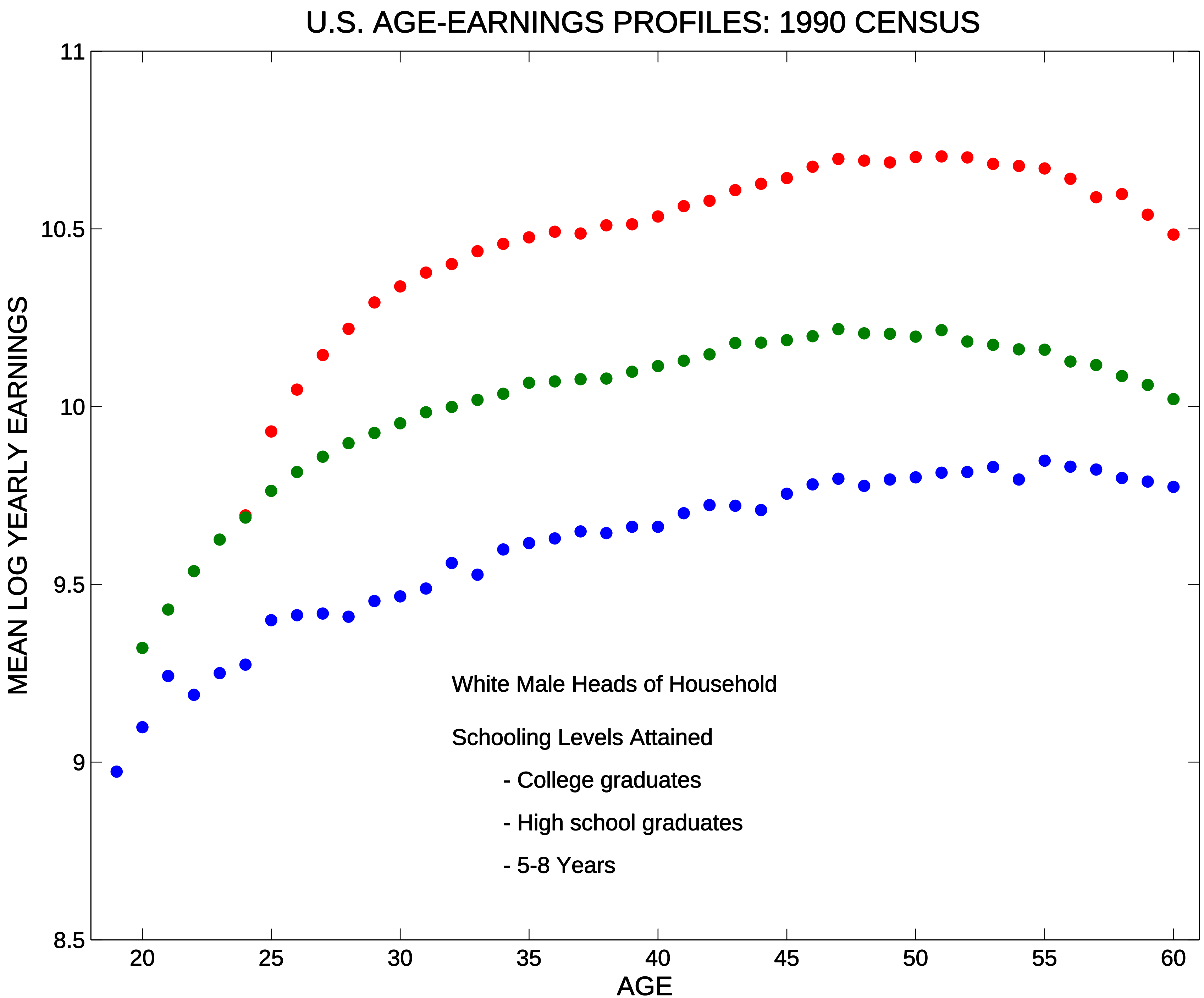
October 4, 2013

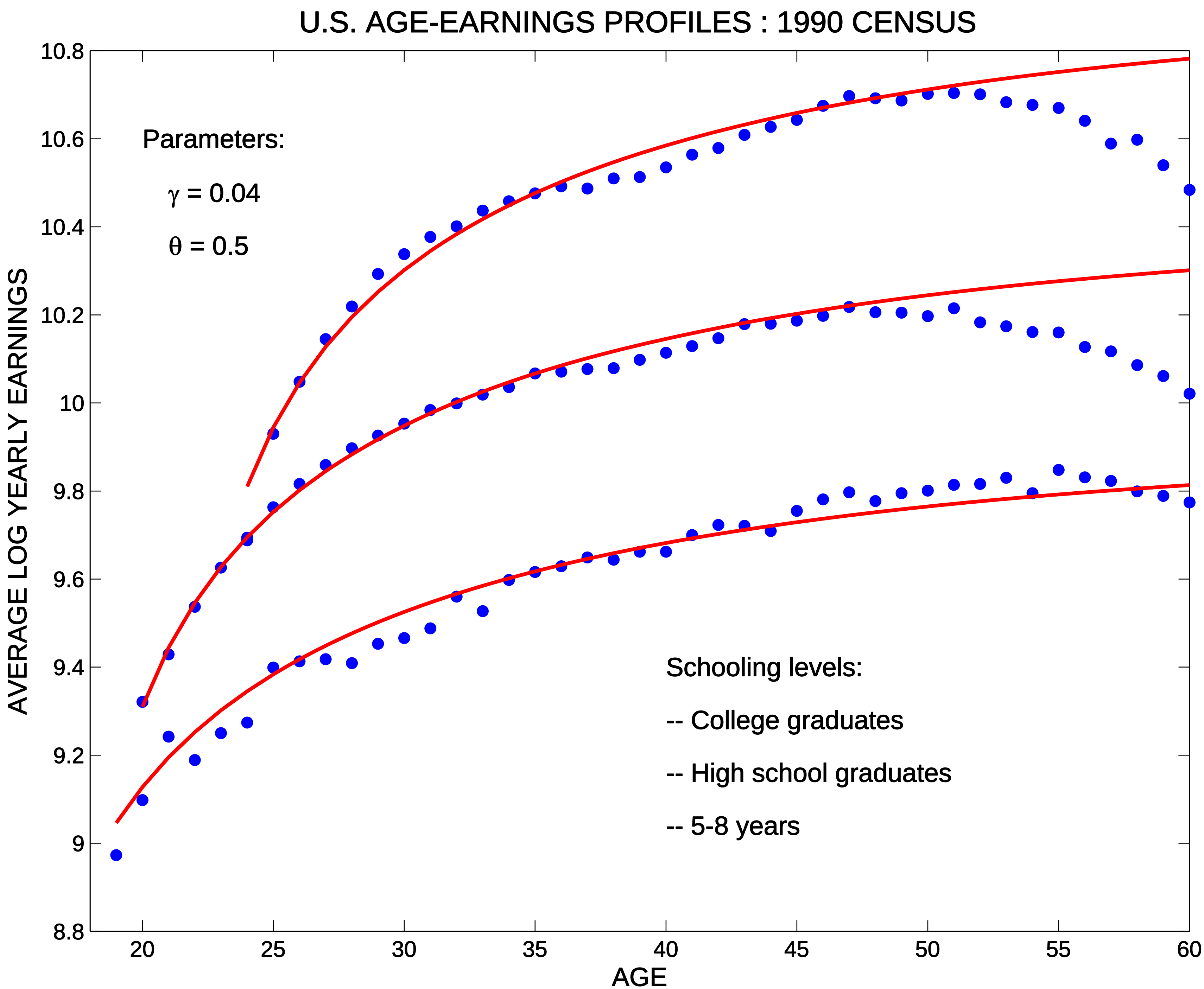
- Productivity growth means that some person knows how to do something today that he did not know how to do yesterday

- Identify individual productivity levels with earnings data, cross-sections, panel, aggregate time series

- Theoretical model: Description of how learning takes place

- Lucas (2009), Lucas/Moll (2013), Perla/Tonetti (2013): stochastic process models where learning depends on own effort, chance meetings with others


U.S. AGE-EARNINGS PROFILES: 1990 CENSUS
MEAN LOG YEARLY EARNINGS
AGE
11
10.5
10
9.5
9
8.5
20
25
30
35
40
45
50
55
60
White Male Heads of Household
Schooling Levels Attained
- College graduates
- High school graduates
- 5-8 Years



U.S. AGE-EARNINGS PROFILES : 1990 CENSUS
Parameters:
γ = 0.04
θ = 0.5
Schooling levels:
-- College graduates
-- High school graduates
-- 5-8 years
AVERAGE LOG YEARLY EARNINGS
AGE
10.8
10.6
10.4
10.2
10
9.8
9.6
9.4
9.2
9
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60

- Can also get cross-sections, panel data on size, productivity of business firms: also revealing productivity growth

- Recent work by Rossi-Hansberg/Wright (2007), Luttmer (2007,2010,2011), Gabaix (2011)

- Observe careers of firms as well as careers of individuals

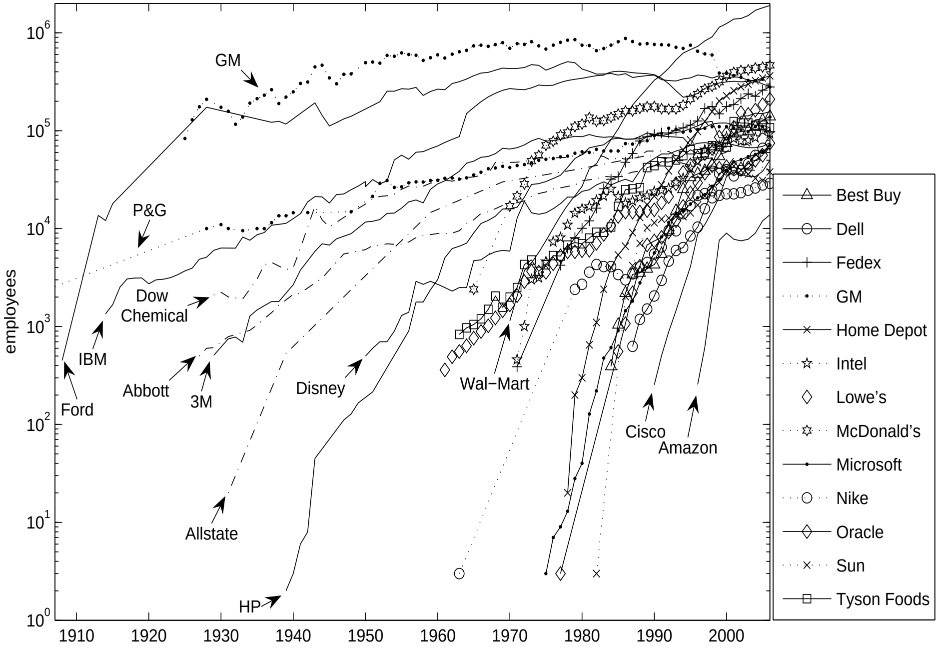

FIGURE I Selected Growth Histories

- How are these two kinds of evidence related? The people who show up in earnings data are also the people who comprise firms: can't just add them up

- To integrate these sources like to have model of how firm is built up from individuals

- Long, stimulating history, based on OLG models: Prescott/Boyd (1987), Chari/Hopenhayn (1991), Jovanovic (2013)

- Today want to outline approach based on Garicano/Rossi-Hansberg (2006, 2012) model of heirarchical firms, combined with Kortum (1997)-like dynamics as in Lucas (2009)

- Begin with version of G/R-H: A single firm, or an entire planned economy

- Continuum of agents, each identified with productivity type z. Cdf F, density f

- Each agent draws “problem” y : cdf G, density g

- If agent z draws $y \leq z$ he produces y; if he draws $y > z$ he produces nothing

- Under autarchy, then, total production is

$$Y = \int_0^\infty \left(\int_0^z yg(y)dy \right) f(z)dz$$

- Now introduce heirarchy. Type z can produce on his own, as above.

- **Or** he can be a non-producing manager/superviser who can enable $\kappa > 1$ workers each to produce at any level $y \le z$

- This choice—produce or manage—is made **before** problems are drawn

- Will assume that for some x_1, types $z \in (0, x_1)$ are workers; types $z > x_1$ are managers

- Moreover, each worker z is matched to a manager of type $\varphi(z)$ to whom he turns if he draws a problem $y > z$ that he can't solve

- Each manager z is matched to a manager of type $\varphi(z)$ to whom he passes all problems passed up to him that he is unable to solve

- This matching is assortative: $\varphi'(z) > 0$

- **Important**: Nobody knows how hard the problems are that he can't solve. All he knows is what he can solve. Those he can't are just passed up to the next level or layer. The really hard problems just get passed up the line, seeking solution

- First step:

$$F(\varphi(y)) - F(x_1) = \frac{1}{\kappa} \int_0^y [1 - G(z)] \, f(z) dz, \quad \text{all } y \in (0, x_1] \qquad \text{(1a)}$$

- Then successive management layers:

$$F(\varphi(y)) - F(x_{i+1}) = \int_{x_i}^{y} \frac{1 - G(z)}{1 - G(\varphi^{-1}(z))} f(z)dz, \tag{1b}$$

for all $y \in [x_i, x_{i+1}]$ and $i = 1, ...$

- Consolidate to get

$$1 - F(x_1) - \int_{x_1}^{\infty} \frac{1 - G(z)}{1 - G(\varphi^{-1}(z))} f(z)dz$$
$$= \frac{1}{\kappa} \int_0^{x_1} [1 - G(z)] f(z)dz \tag{2}$$

- Choose x_1 so that every problem is solved, every manager fully utilized

[Digression: Algorithmic treatment of these eqs:

- Distributions F, G taken as given.
- Need to solve for x_1 and matching function φ.
- Given x_1, invert (1a) to get function

$$\varphi(y) = F^{-1}\left(F(x_1) + \frac{1}{\kappa}\int_0^y [1 - G(z)]\, f(z)dz\right)$$

on interval $(0, x_1]$. Call $x_2 = \varphi(x_1)$.

- Now go to (1b) for $i = 1$. Have

$$\varphi(y) = F^{-1}\left(F(x_2) + \frac{1}{\kappa}\int_{x_1}^{y} \frac{1 - G(z)}{1 - G(\varphi^{-1}(z))} f(z)dz\right)$$

- Have already solved for $\varphi^{-1}(z)$ on $(x_1, x_2]$, so rhs gives $\varphi(y)$ on $(x_1, x_2]$

- Keep going, repeating (1b). Eventually have function $\varphi(z, x_1)$ for real line, given x_1

- Solve (2) for x_1. If $\kappa > 1$ is large enough, have finite solution x_1

End digression]

- Total production of [firm, economy] is

$$\begin{aligned} Y &= \int_0^{x_1} \int_0^{\infty} y g(y) dy f(z) dz \\ &= F(x_1) \int_0^{\infty} z g(z) dz \end{aligned}$$

- Now let's add some dynamics to this system

- Begin with basic Kortum ODE, where everyone in F draws from others in F at Poisson arrival rate α :

$$\frac{\partial F(x,t)}{\partial t} = -\alpha F(x,t) \left[1 - F(x,t)\right]$$

which has the solution

$$F(z,t) = \left(1 + \left[\frac{1 - F(z,0)}{F(z,0)}\right] e^{\alpha t}\right)^{-1} \qquad (*)$$

- Seek a *balanced growth path* (BGP) solution to (*): a cdf $\Phi(z)$ (density ϕ) and growth rate $\nu > 0$ such that

$$F(e^{\nu t} z, t) = \Phi(z) \quad \text{for all} \;\; t \geq 0$$

- Can show that if $F(z, 0)$ has a Pareto tail:

$$\lim_{z \to \infty} \frac{1 - F(z, 0)}{z^{-1/\theta}} = \lambda \;\; \text{for some } \lambda, \theta > 0$$

  then solution to (*) converges to BGP where $\nu = \alpha\theta$ and

$$\Phi(z) = \frac{1}{1 + \lambda z^{-1/\theta}}$$

- What is happening to production as this convergence takes place?

- If problem distribution G stays fixed, then

$$Y(t) = F(x_1(t)) \int_0^\infty zg(z)dz \to \int_0^\infty zg(z)dz$$

because $x_1(t) \to 1$.

- As people get smarter, fewer managers are needed

- In language of Goldin and Katz, education (on the job, here) has won the race with technology (problems to solve, here)

- To get sustained growth, we need the problem distribution G to move along with type distribution F.

- The processes of discovery and diffusion of ideas that we try to capture by the law of motion for F must be closely related to the process generating problems

- Each new technology brings with it a new set of problems

- These issues need more thought. Here we assume, just to get started, that F and G are the same distributions, with the common law of motion described above

- Equations (1) and (2) then reduce to

$$F(\varphi(y)) - F(x_1) = \frac{1}{\kappa}\int_0^y [1 - F(z)]\, f(z)dz, \quad \text{all } y \in (0, x_1] \qquad (1a)$$

- Then successive management layers:

$$F(\varphi(y)) - F(x_{i+1}) = \int_{x_i}^{y} \frac{1 - F(z)}{1 - F(\varphi^{-1}(z))} f(z)dz, \tag{1b}$$

for all $y \in [x_i, x_{i+1}]$ and $i = 1, \ldots$

- Consolidate to get

$$1 - F(x_1) - \int_{x_1}^{\infty} \frac{1 - F(z)}{1 - F(\varphi^{-1}(z))} f(z)dz$$

$$= \frac{1}{\kappa} \int_0^{x_1} [1 - F(z)]\, f(z)dz \tag{2}$$

- In autarky, people of type z together produce $\int_0^z yf(y)dy$ and total production is

$$Y^a = \int_0^\infty \left(\int_0^z yf(y)dy \right) f(z)dz$$

- In planning problem, cutoff level x_1 is chosen so that all problems of people on $[0, x_1]$ are solved, with help from types $z > x_1$

- Total production of economy is $\int_0^{x_1} zf(z)dz$

- People on $[0, x_1]$ can produce

$$\int_0^{x_1} \left(\int_0^z yf(y)dy \right) f(z)dz$$

  on their own

- People on (x_1, ∞) cooperate with those on $[0, x_1]$ to "produce" the remaining

$$\int_0^{x_1} \left(z - \int_0^{z} y f(y) dy \right) f(z) dz$$

- Along BGP, **fraction** of people who are workers, managers at layers 1,2,...are constant

- Many directions to explore from here. Do not yet know what is possible, interesting. Consider

  1. Cohort structure, age-earnings profiles

  2. Distinct firms (as opposed to centralized planner)

  3. Learning complementarities

1 Cohort structure

- In order to get predictions on age-earnings profiles, need to introduce a cohort structure

- Here assume fixed life cycle, exogenous birth, death rates

- Constant age density $\pi(s)$, birth rate $\pi(0)$, $\pi'(s) < 0$

- Set up notation

$$H(z,s,t) = \text{Pr}\{\text{person of age } s \text{ born in } t \text{ has productivity} \leq z\}$$

$$F(z.t+s\} = \text{Pr}\{\text{selected at random at date } t+s \text{ has productivity} \leq z\}$$

$$\frac{\partial H(z,s,t)}{\partial s} = -\alpha H(z,s,t)\left[1 - F(z,t+s)\right]$$

- Solve for H to get

$$H(z,s,t) = \exp\left(-\alpha \int_0^s \left[1 - F(z,t+\tau)\right] d\tau\right)$$

- F is population weighted average

$$F(z, t+s) = \int_0^\infty \pi(\tau) H(z, \tau, t+s-\tau) d\tau$$

- Along a BGP have

$$\gamma = \alpha \int_0^\infty \pi(s) \left(1 - e^{-\gamma s}\right) ds$$

- Fixed point problem in γ

- Growth rate is $\nu = \theta\gamma$ (Compare to $\nu = \theta\alpha$ with infinitely lived agents)

- Here dynamics proceed independently of structure of firm—as in first example

- But heirarchy will evolve as types z change, people enter, move up, retire

- Typical individual career will start as worker, end as manager

- But stochastic exceptions are the rule—Just as in real world!

- Everyone in this economy is an employee, earnings are entire income

- All have position in census age-earnings profiles

- Worker-manager complementarity leads to new predictions

- To calculate, will need to decentralize planning problem, get equilibrium wages for all types z

2 Distinct firms

- In Garicano/Rossi Hansberg (2006), (2012) there are distinct firms, each headed by single person

- Each firm defined by type z of CEO

- Think of CEO as assembling personnel at different z levels, paying market equilibrium wage

- If there are any firms with finite z at head, economy will fall short of $Y = F(x_1) \int_0^\infty zg(z)dz$

- Some problems too hard for anyone in firm to solve:

- Is all of production natural monopoly?

- Need to assume span of control?

- Or simply assume that a fixed fraction of population consists of "entrepeneurs": people who won't work for anyone else

- Low z entrepeneurs work for themselves, no manager to help. Bear risk of drawing $y > z$. Imagine perfectly pooled within each type, with mean earnings $\int_0^z y f(y) dy$ per person

- But as z gets higher, want to "leverage" entrepeneurial time, hire subordinates

- Need theory to work out details of this process, structure of firms

3 Learning complementarities

- In models outlined above, learning process is autonomous, independent of way firms are organized

- Key feature of Prescott-Boyd, Chari-Hopenhayn, Jovanovic is that learning rates depend on knowledge of others in same firm: "Dynamic Coalitions: Engines of Growth"

- Implies another form of assortative matching: most talented rookies match up with best firms

- Easy to think of ways to modify search model to incorporate firm effects:

  - Meetings of others in same firm more likely than meetings with outsiders

  - Meetings related to management layers: z improves his own skills by interacting with $\varphi(z)$, who in turn improves *his* skills by interacting with $\varphi(\varphi(z))$, etc.

- But mathematical structure changes dramatically from earlier examples: evolution of productivities now dependent on organization of firms