Uncertainty and Investment Options

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Investment expenditures are the most volatile component of spending over the business cycle.

This paper looks at the role of uncertainty about government policy in destabilizing investment spending.

The main issue is that investors often have considerable discretion about the timing of investment spending.

Specifically, many types of investment can be postponed.

This is true for any project that is ‘proprietary’ to a single investor, so a rival cannot jump in if the initial investor hesitates.
Uncertainty means that the investor does not know how to exploit the project to best advantage.

Thus, uncertainty makes it advantageous to **postpone** if the investor expects to receive—within a moderate time frame—information that will reduce or eliminate the uncertainty.

Uncertainty about macroeconomic policy—fiscal policy, trade policy, and monetary policy—can make a wait-and-see strategy attractive for firms.

Hence the impact on **aggregate** investment can be substantial.
The sluggish recovery of investment in the U.S. after the financial crisis may be an example of this effect.

Fact 1: After falling dramatically in 2009, investment in the U.S. has remained low, prolonging the downturn.
Moreover, many large firms hold substantial reserves of liquid assets.
(Apple is the most famous example, but it is true more broadly.) For these firms, difficulty in obtaining bank loans is not an issue.
Real Gross Private Domestic Investment
Percentage change from previous peak, Seasonally Adjusted

- 1973 cycle
- 1981 cycle
- 1990 cycle
- 2001 cycle
- Current cycle

Quarters from previous peak
Motivation

Fact 2: The U.S. has been running large deficits, and the debt/GDP ratio is growing rapidly.

The costs of Social Security and Medicare will rise as baby boomers retire, and the ratio of retirees to working population grows.

The U.S. is on an unsustainable fiscal course: major reforms will be needed in the not-too-distant future.
Federal Debt: Total Public Debt as Percent of Gross Domestic Product (GFDEGDQ188S)
Federal Debt Held by the Public as Percent of Gross Domestic Product (FYGFGDQ188S)
Federal Debt Held by Foreign & International Investors as Percent of Gross Domestic Product (HBFIGDQ188S)

Shaded areas indicate US recessions. 2013 research.stlouisfed.org
Fact 3: There is no consensus on what the fiscal reforms should look like. Some groups advocate large cuts in entitlements and other spending, while others advocate substantial tax increases.

Fact 4: The shape of the fiscal reforms will have a substantial impact on the profitability of investment.

Baker, Bloom and Davis (2013) construct an Uncertainty Index based on:

— frequency of news media references to economic policy uncertainty
— number of federal tax code provisions set to expire
— extent of forecaster disagreement about inflation and about federal government purchases.
Hypothesis:

Fiscal uncertainty has contributed to depressed investment. The high degree of uncertainty about tax changes increases the option value of waiting to make investment decisions that are difficult/expensive to reverse.
Related literature

Much has been written about the effects of uncertainty on investment.

Most closely related to the idea here are:

Older models of delay to resolve project-specific uncertainty:

- Cukierman (*JPE* 1980),
- Bernanke (*QJE* 1983),
- McDonald and Siegel (*QJE* 1986).

Recent models of uncertainty at the firm level:

- Bloom (*Econometrica* 2009),
- Arellano, Bai and Kehoe (2011),

Empirical work in Fernandez-Villaverde, et. al. (2011) and in

- Baker, Bloom, and Davis (2012) provide some confirmation for
  the idea that policy uncertainty depresses investment.
Overview of the talk

1. Sketch the model.
2. Describe the firm’s decision after $T$, when the shock is realized.
3. Describe the firm’s decision before $T$. State the main result.
4. Extend the model to allow a stochastic arrival date $T$.
5. Look at an example.
6. Conclude.
1. The model

The investment technology has three key features.

1. Investment requires a ‘project’ as well as cash, so it involves exercising a **one-time option**.

Projects arrive exogenously, at a fixed rate.

A project can be thought of as particular investment opportunity:
— for a chain store, the location for a new retail outlet,
— for a manufacturing firm, the opportunity to build a new plant,
— for a real estate developer, a vacant parcel of land.

It is important that projects are exclusive to one investor.
Otherwise Bertrand competition would make delay impossible.
1. The model

2. The initial decision about how much cash to invest in a project is **irreversible**. (Costly reversibility would suffice.) Uncertainty about tax policy increases the option value of waiting, leading to a temporary decline in investment.

3. Projects are storable: they not depreciate (or not too quickly). When the uncertainty is resolved, there is a temporary investment boom, as projects that had been shelved are exploited. The size of the boom depends on the realized tax rate, with lower rates generating larger booms. If the uncertainty is small in magnitude, the effect is small.
1. The model

Time is continuous.
Dividends are discounted at the rate $\rho > 0$.
Liquid assets earn interest at the rate $0 \leq r < \rho$, so the firm prefers to pay out dividends as quickly as possible.
The firm cannot borrow, and capital cannot be sold off.
The revenue function $\pi(k)$ (net of variable input costs), is strictly increasing and strictly concave.
Capital depreciates at the rate $\delta > 0$.
Revenue is taxed at the flat rate $\tau$.
At $t = 0$, a tax change is announced for $T > 0$.
The new tax rate $\hat{\tau}$ will be drawn from a known distribution $F$. 
1. The model

The firm’s investment (a flow) is \( I = ni \), where \( n \) is the number of projects used, and \( i \) is the intensity of investment per project. The total cost (a flow) is \( ng(i) = ng(I/n) \), where \( g \) is strictly convex. Thus, investment at the rate \( I \) is less costly if it is spread over a larger number of projects, and the number of available projects affects the profitability of investment.

Discrete investment will be described below.
1. A two-period example

The firm receives a unit mass of projects in each period. It chooses investment intensities \( i_1, i_2 \geq 0 \) and investment scales

\[
0 \leq n_1 \leq 1, \\
0 \leq n_2 \leq 2 - n_1.
\]

The capital stock evolves as usual,

\[
k_t = (1 - \delta) k_{t-1} + l_t, \quad t = 1, 2,
\]

where \( l_t \equiv n_t i_t \). The total cost of investment in each period is

\[
TC_t = n_t g(i_t) = n_t g(l_t / n_t), \quad t = 1, 2.
\]

For \( n_1 = n_2 = 1 \), \( TC \) is as usual with convex costs.

For \( n_1 = 0 \) and \( n_2 = 2 \), because \( g \) is strictly convex,

\[
TC_2 = 2g(l_2 / 2) < g(l_2).
\]
2. The transition after $T$

First consider the firm’s continuation value after the tax change, given the new tax rate $\hat{\tau}$, capital stock $k_T$, and stocks $a_T, m_T \geq 0$ of liquid assets and projects.

If $a_T, m_T > 0$, it can make a discrete adjustment (DA) of $I = \hat{n}\hat{i}$ to its capital stock by implementing a mass of ideas $0 \leq \hat{n} \leq m_T$, each at intensity $\hat{i} \geq 0$.

The DA must be financed from liquid assets, requiring $\hat{n}g(\hat{i}) \leq a_T$.

The firm can also pay a discrete dividend $0 \leq \hat{D} \leq a_T - \hat{n}g(\hat{i})$.

The post-DA stocks are then

\[
\hat{k}_T = k_T + \hat{n}\hat{i}, \\
\hat{a}_T = a_T - \hat{D} - \hat{n}g(\hat{i}), \\
\hat{m}_T = m_T - \hat{n}.
\]
2. The transition after $T$

Given $\hat{\tau}$ and $(k_T, a_T, m_T)$, the firm chooses $(\hat{D}, \hat{i}, \hat{n})$ and $\{(D, i, n)\}_T^\infty$ to solve

$$v(k_T, a_T, m_T; \tau) = \max \left[ \dot{D} + \int_T^\infty e^{-\rho(t-T)} D(t) dt \right]$$

$$\dot{k} = ni - \delta k,$$

$$\dot{a} = ra + (1 - \hat{\tau}) \pi(k) - D - \mu g(i),$$

$$\dot{m} = \mu - n,$$

$$0 \leq D, i, n, a, m, \text{ all } t,$$

where the post-DA values $(\hat{k}_T, \hat{a}_T, \hat{m}_T)$ above are the initial conditions for the transition after $T$. 
Two facts about the solution:

1. For any $\hat{t}$ and any $k_0 > 0$ there is a unique solution. It converges to a steady state (SS).

   Projects and cash are not held in SS.

2. The discrete investment $\hat{n} \hat{i}$ with cost $\hat{n}g(\hat{i})$ exhausts $m_T$ or $a_T$ or both.
Which stock is exhausted by the DA?

For low tax rates (relative to others in $F$), projects are quite valuable.

Hence the DA uses a higher investment intensity.

The stock of assets is exhausted and projects remain.

For slightly higher tax rates, the DA exhausts both stocks.

For even higher tax rates, the intensity is lower. The DA uses all projects.

Excess cash remains and is paid as a lump sum dividend.

For very high tax rates, projects are remain and are carried forward.

Excess cash remains and is paid as a lump sum dividend.

Figure 2 illustrates
Figure 2: the Discrete Adjustment

Region A

Region B

Region C

\( a_T \)

\( m_T \)

\( q_aT \)

\( D \)

\( \hat{n} \)

\( \hat{m}_T \)

realized tax rate \( \tau \)
Let $\tau_0$ denote the initial tax rate is $\tau_0$, and suppose $k_0 \leq k^{ss}(\tau_0)$, and $a_0 = m_0 = 0$.

A new tax rate $\hat{\tau}$ will be chosen at $T > 0$, drawn from a known distribution $F$, and there will be no subsequent changes in policy. The firm’s problem is to choose $\{D, i, n\}_0^T$ to solve

$$\max \int_0^T e^{-\rho t} D(t) dt + e^{-\rho T} E_{\hat{\tau}} [\nu(k_T, a_T, m_T; \hat{\tau})],$$

subject to the laws of motion for the three stocks, where the expectation uses $F$. 
3. The transition before $T$

The main result is the following proposition.

**Proposition 4:** Unless $F$ puts unit mass at a single point, the firm accumulates both projects and liquid assets before $T$.

**Proof:** Suppose the contrary. Let $\bar{i}$ be the intensity before $T$. After $T$, the intensity $i$ varies with the realization of the tax rate. Since $g(i)$ is strictly convex, total investment costs could be reduced by smoothing intensity around $T$, for each $\hat{\tau}$. Storing liquid assets and projects accomplishes this.
4. Stochastic arrival date $T$

Suppose the arrival date $T$ is also uncertain.

In particular, assume the arrival is Poisson, with hazard rate $\theta$.

The date $T$ is now a stopping time.

After the arrival, the firm’s problem is the same as before.

Before the arrival, the firm’s problem is to choose $\{D, i, n\}_0^\infty$ to solve

$$\max \int_0^\infty e^{-(\rho+\theta)t} \left\{ D(t) + \theta E_{\hat{\tau}} [\nu(k(t), a(t), m(t); \hat{\tau})] \right\} dt,$$

with the same laws of motion as before for $k, a, m$.

Before $T$, the firm’s assets converge to SS levels $(k^*, a^*, m^*)$.

If $\theta$ is close to zero, $a^* = m^* = 0$, and $k^* \approx k^{ss}(\tau)$.

If $F$ puts unit mass on $\hat{\tau}$, and $\theta$ is very large, $(k^*, a^*, m^*) \approx (k^{ss}(\hat{\tau}), 0, 0)$. 
The following result is the analog of Proposition 4.

**Proposition 5:** Unless $F$ puts unit mass on a single point, for all $\theta$ sufficiently large, the pre-reform SS has $a^*, m^* > 0$.

**Proof:** Mimic the proof of Prop. 4, using expected values. For $\theta$ sufficiently large, there is an expected gain from the same perturbation.
5. An example

The example uses the revenue and cost functions

\[ \pi(k) = A k^\alpha, \quad g(i) = g_1 i + \frac{1}{2} g_2 i^2, \]

and the parameter values

\[ A = 1, \quad \alpha = 0.70, \quad g_1 = 1, \quad g_2 = 1.5, \]
\[ \delta = 0.10, \quad \mu = 1, \quad \rho = 0.04, \quad r = 0.03. \]

The initial tax rate is \( \tau_0 = 0.20 \), and the post-reform rate is

\[ \hat{\tau} = \begin{cases} \tau^L = 0.22, & \text{with probability 0.49,} \\ \tau^H = 0.42, & \text{with probability 0.51.} \end{cases} \]

The firm has an initial capital stock that is \( 3/4 \) of the steady state value for \( \tau_0 \), and the reform is anticipated \( T = 1/2 \) year in advance.
In the option model, project accumulation begins immediately, at $t = 0$. Liquid asset accumulation begins at $t = 1/3$.

Figure 3 shows the transition paths for the capital stock
— with full information at $t = 0$ about the tax change,
— in the option model,
— in the benchmark model, where projects cannot be accumulated.

The option model gives the firm about 42% of the gains (above the benchmark) it would get with full information.
Figure 3: capital stock

- full information
- option model
- benchmark model

$\tau^L = 0.22$

$\tau^H = 0.42$

$\tau_0 = 0.2$
The positive predictions of this model are stark: policy uncertainty produces sharp swings in both investment and dividends.

To embed it in a macro model, one could dampen the swings by letting stored projects depreciate. Two interpretations are:

— market changes might make the investment less profitable,
— rival firms might get access to the project and exploit it.

Projects with high depreciation rates are less storable.

For a given policy reform, only projects with sufficiently low depreciation rates would be delayed. The rest would be exploited immediately.
6. Conclusions

What are the welfare implications?

Here the decline in investment is largely offset by a boom after the uncertainty is resolved.

Like grocery store sales during a blizzard, the decline one day is, to a large extent, offset by an increase following day.

But the same is true of investment over the business cycle, so the welfare costs here might be similar to the costs of cyclical fluctuations.
6. Conclusions

In the model here, the uncertainty was about a tax rate and the decision was about a business investment.

The same logic applies more broadly:

to uncertainty about financial regulation, trade policy, energy policy;
to business hiring decisions and household purchases of housing and other durables.