Asset Pricing with Heterogeneous Agents (Preliminary)

Yili Chien, Harold Cole and Hanno Lustig

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Puzzles

- Asset pricing
  - High, volatile and counter-cyclical risk premia
  - Low and stable risk-free rate
  - Hard to account for with standard macro models
- Asset markets
  - Rich asset markets
  - Poor consumption smoothing
  - Skewed wealth distribution
- Can portfolio behavior help explain all this?

Heterogeneity of Portfolio Behavior

- No participation
  - Many households do not participate equity markets, 50% in US, 70% in Europe.
  - Many who hold equities only do so in a small way
- Among participants: many still deviate from optimal portfolios
  - Make only very infrequent adjustments - inertia
  - Adjust but based on past returns (miss market timing)
- "Sophisticated" investors earn higher return by increasing risk
- Want to include these different investors in our model.

Heterogeneity of Preferences and Beliefs

Heterogeneity in portfolio behavior could result from

- Heterogeneity in preferences
  - Survey data: a wide dispersion in attitudes towards risk
  - Risk attitudes can be influenced by wealth, education, gender, experiences, and personality.
- Heterogeneity in beliefs
  - Survey data : a significant heterogeneity in beliefs on asset return and volatility
  - Different forecasts also reflect heterogeneity in beliefs
- Can this explain why those who trade, trade differently?

What We Do

- Extend our methodology to compute the equilibria of economies with **more heterogeneities**
  - In addition to heterogeneity in trading technologies
  - Now includes heterogeneity in **preferences and beliefs**
- One quantitative experiment
  - Focus on heterogeneity in beliefs
  - Introduce recency bias: volatile beliefs

Environment

- Aggregate endowment $Y_t = \exp(z_t) Y_{t-1}$ comes in two forms
  - *tradeable output* $(1-\gamma) Y_t$ depends on z^t
  - *non-tradeable output* $\gamma Y_t \eta_t$ depends on η_t too
- Idiosyncratic shocks
  - η are i.i.d. across households and $E\{\eta_t | z^t\} = 1$
  - $\pi(z^t, \eta^t)$ is probability of observing event (z^t, η^t)
- For now, assume a continuum of ex ante identical households with CRRA utility
- Our methodology builds on Arrow-Debreu environment

Arrow-Debreu Economy

With standard Arrow-Debreu economy, household i chooses consumption sequence $c_t^i(z^t, \eta^t)$ to

$$\max_{\{c_t^i\}} \sum_t \sum_{(z^t,\eta^t)} \beta^t \frac{c_t^i(z^t,\eta^t)^{1-\alpha}}{1-\alpha} \pi(z^t,\eta^t)$$

$$\text{s.t.} \sum_t \sum_{(z^t,\eta^t)} \left\{\gamma Y(z^t)\eta_t - c_t^i(z^t,\eta^t)\right\} \widetilde{P}(z^t,\eta^t) + \omega_0 \geq 0.$$

- subject to present-value budget constraint
- $\widetilde{P}(z^t,\eta^t) = P_t(z^t)\pi(z^t,\eta^t)$ is the time zero state price.
- ω_0 is the initial wealth.
- $\gamma Y(z^t)\eta_t$ is risky "labor" income

Add Trading Technologies

- Trading Technologies: restrictions on asset holdings over time
- How to impose restrictions on asset holding in time zero trading problem?
  - Remember that "Arrow" = "Arrow-Debreu"
  - Net assets ($a_t(z^t, \eta^t)$) = net savings
  - $a_t(z^t, \eta^t)$ must be consistent with their consumption plan

$$S^i(z^t, \eta^t) = \sum_{\tau \geq t} \sum_{(z^\tau, \eta^\tau)} \widetilde{P}(z^\tau, \eta^\tau) \left(\gamma Y(z^\tau) \eta_\tau - c(z^\tau, \eta^\tau) \right)$$

$$= -a_t(z^t, \eta^t) \widetilde{P}(z^t, \eta^t)$$

  - Hence, any restriction on $a_t(z^t, \eta^t)$ will also limit $[\cdot]$.

Examples on Trading Technologies

- Debt bounds

$$a_t(z^t, \eta^t) \geq \underline{D}\left(z^t\right),$$

- No contingent claim on idiosyncratic shocks (Z-com traders)

$$a_t(z^t, \widehat{\eta}^t) = a_t(z^t, \eta^t), \text{ for all } \widehat{\eta}^t \text{ and } \eta^t$$

- Fix portfolio

$$\frac{a_t(z^t, \eta^t)}{R^p(z^t)} = \frac{a_t(\widehat{z}^t, \widehat{\eta}^t)}{R^p(\widehat{z}^t)} = \hat{a}_{t-1}(z^{t-1}, \eta^{t-1})$$

where R^p is the return on the fix portfolio

Lagrangian Example

Take a household i with debt bounds and subject to fix portfolio restrictions as an example:

$$
\begin{aligned}
L = & \max_{\{c,\hat{a}\}} \min_{\{\chi,\nu,\varphi\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} \frac{c_t^i(z^t,\eta^t)^{1-\alpha}}{1-\alpha}\pi(z^t,\eta^t) \\
& +\chi^i \left\{ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \widetilde{P}(z^t,\eta^t) \left[\gamma Y(z^t)\eta_t - c^i(z^t,\eta^t)\right] + \varpi(z^0) \right\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \nu^i(z^t,\eta^t) \left\{ \begin{array}{c} S^i(z^t,\eta^t) \\ -\widetilde{P}(z^t,\eta^t)\hat{a}(z^{t-1},\eta^{t-1})R^p(z^t) \end{array} \right\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \varphi^i(z^t,\eta^t) \left\{ \underline{D}_t^i(z^t)\widetilde{P}(z^t,\eta^t) - S^i(z^t,\eta^t) \right\}.
\end{aligned}
$$

where

$$
S^i(z^t,\eta^t) = \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)} \widetilde{P}(z^\tau,\eta^\tau)\left(\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right).
$$

Time and State Varied Multiplier

- Recursive multiplier (Marcet and Marimon(1998))

$$\zeta^i(z^t, \eta^t) = \zeta^i(z^{t-1}, \eta^{t-1}) + \nu^i\left(z^t, \eta^t\right) - \varphi^i(z^t, \eta^t)$$

- All traders have first-order conditions

$$\beta^t u'(c_t^i(z^t, \eta^t)) = \zeta^i(z^t, \eta^t) P_t(z^t).$$

where $\zeta^i(z^t, \eta^t)$ varies to satisfy constraints on net savings and $P_t(z^t)$ is the state price.

- Law of motion on multiplier + FOC in consumption $\Longrightarrow$ Euler equations

Consumption Allocations

- The FOCs with respect to consumption for all traders:

$$\beta^t c_t^i(z^t,\eta^t)^{-\alpha} = \zeta^i(z^t,\eta^t) P_t(z^t)$$

- together with resource constraints $C_t\left(z^t\right) = \sum_i c_t^i(z^t,\eta^t)\mu_i$
- imply the consumption share rules

$$c_t^i(z^t,\eta^t) = \left(\frac{\zeta^i(z^t,\eta^t)^{-1/\alpha}}{\sum_i \zeta^i(z^t,\eta^t)^{-1/\alpha}\mu_i} \right) C\left(z^t\right).$$

- $h_t(z^t) \equiv \sum_i \zeta^i(z^t,\eta^t)^{-1/\alpha}\mu_i$ **is the one key moment**

Price Aggregation

- FOCs and consumption share rules imply that

$$P_t(z^t) = \beta^t C(z^t)^{-\alpha} h_t(z^t)^{+\alpha}.$$

- Perturbed version of Breeden-Lucas stochastic discount factor

$$m_{t+1} \equiv \frac{P_{t+1}(z^{t+1})}{P_t(z^t)} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} \left(\frac{h_{t+1}}{h_t}\right)^{+\alpha}.$$

  - standard part from a representative CRRA agent
  - this is the new part

- Depends on common homogeneous preferences

Computation

- Stochastic discount factor

$$m_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{h_{t+1}}{h_t} \right)^{+\alpha}.$$

- How to compute?
  - Conjecture on state contingent prices, h_{t+1} / h_t
  - Use individual multiplier ζ^i as state variable
  - Compute the law of motion on ζ^i for each individual
  - Update h_{t+1} / h_t: moments of multiplier distribution
  - Equilibrium is fixed point $F\left[h_{t+1} / h_t\right] = \left[h_{t+1} / h_t\right]$.

Heterogeneity in Preferences and Beliefs

Previously, all households had same CRRA preferences, discount rates and beliefs. Now agent of type i has preferences

$$\sum_{t\geq 1,(z^t,\eta^t)}^{\infty} (\beta_i)^t \, u^i(c_t^i(z^t,\eta^t))\tilde{\pi}^i(z^t,\eta^t),$$

- $u^i(c_t^i(z^t,\eta^t))$ is strictly concave
- own discount rate β_i
- $\tilde{\pi}^i(z^t,\eta^t)$ subjective probability of agent i on event (z^t,η^t).

New Trick to Get Aggregation

How can we apply the price aggregation result without common homogeneous preferences?

Answer: Create reference economy

Reference economy has

- fraternal twin for each trader with nice preferences+beliefs
- has given (same) state prices
- uses social planning weights to allocate consumptions
- choose weights so consumptions are the same between twins.

$$\sum_i \left\{ \beta^t \sum_{(z^t,\eta^t)} \frac{1}{\bar{\zeta}^i(z^t,\eta^t)} \bar{u}(\bar{c}(z^t,\eta^t)) - P(z^t)\bar{c}(z^t,\eta^t) \right\} \mu_i .$$

Mapping to Reference Trader

- The FOC for a type i household

$$\beta_i^t u'(c_t(z^t,\eta^t))\tilde{\pi}^i(z^t,\eta^t) = \zeta^i(z^t,\eta^t)P_t(z^t)\pi(z^t,\eta^t)$$

- Consider a **reference trader**, who has standard CRRA preference and correct belief.

$$\beta^t \bar{c}^i(z^t,\eta^t)^{-\bar{\alpha}} = \bar{\zeta}^i(z^t,\eta^t)P_t(z^t),$$

- Define an **adjusted multiplier**, $\bar{\zeta}^i(z^t,\eta^t)$ such that $c^i = \bar{c}^i$

$$\left(\frac{\bar{\zeta}^i(z^t,\eta^t)P_t(z^t)}{\beta^t}\right)^{-1/\bar{\alpha}} \equiv u'^{-1}\left(\frac{\zeta^i(z^t,\eta^t)\pi(z^t,\eta^t)P_t(z^t)}{\beta_i^t\tilde{\pi}^i(z^t,\eta^t)}\right)$$

Mapping to Reference Trader

- With these **adjusted multiplier** for the reference traders:
  - If markets clear in the economy with reference traders, they do in the original one too.
  - The same price aggregation applies in reference economy
  - Need law of motion for individual multipliers from original economy
  - Need (change of variable) reference trader multipliers to compute new h
- Also works on recursive utility or robust utility
- Simple mapping trick with a wide range of applications

Computation

- Stochastic discount factor

$$m_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{\bar{h}_{t+1}}{\bar{h}_t} \right)^{+\alpha}.$$

- How to compute?
  - Conjecture on state contingent prices, $\bar{h}_{t+1} / \bar{h}_t$
  - Use ζ^i as state variable
  - Compute the law of motion on ζ^i for each individual
  - **Map ζ^i into $\bar{\zeta}^i$**
  - Update $\bar{h}_{t+1} / \bar{h}_t$: moments of adjusted multiplier distribution
  - Equilibrium is fixed point $F\left[\bar{h}_{t+1} / \bar{h}_t\right] = \left[\bar{h}_{t+1} / \bar{h}_t\right]$.

Some Remarks

- Key to our methodology: price aggregation
- Compute via simple iterative method
- No need to find prices to each market
- Allow rich asset markets
- Price aggregation applies to finite agent case

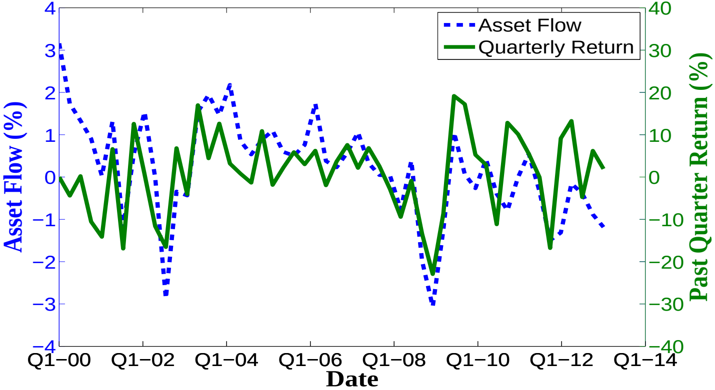
Quantitative Exercise: Heterogeneity in Beliefs

Why beliefs?

- Counter-cyclical market price of risk
- Survey evidence: Greenwood and Shleifer (2012)
  - expectations of returns are
  - **positively** correlated with past stock market returns and
  - **negatively** correlated with future returns
- Adjust portfolio based on past returns (miss market timing)

Quantitative Exercise: Heterogeneity in Beliefs

Evidence on missed market timing: US equity mutual funds



Volatile beliefs

- **Volatile beliefs**: traders form their belief $\tilde{\pi}(z^t, \eta^t)$
  - with probability κ on the ergodic transition $\pi(z_{t+1}|z_t)$ and
  - with probability $1-\kappa$ by the observed transition frequencies during the past 4 periods.
- Consistent with forecasting in a non-stationary world
  - Agent who thinks that the transition matrix might have changed a fixed number of periods ago.
  - Similar strategies are followed by many forecasting models which truncate the data or overweight recent observations.
- (Standard trader has $\kappa = 1$).

Bayesian Regime Switching

- Bayesian **regime-switching belief** traders believe that
  - There is high and a low regime
  - In high regime more likely to get high growth rate.
  - Regime switching governed by Markov transition matrix
  - Regime cannot be observed so infer from past history
- Recursive relationship for probability of high regime.
  - Use past history to update beliefs that regime is high, ω
  - High growth rates raise this probability.
- (Standard trader has $\pi^h = 1, \pi^l = 0$, and no persistence).

Calibration

- Preferences: CRRA with $\alpha = 5$ and $\beta = .95$
- Endowments:
  - Aggregate shock
    - iid version of Merha-Prescott
    - Two state Markov process
  - Idiosyncratic risk calibrated to Storeslatten et al (no CCV)
    - Two state Markov process
- Assets: equity, bond and aggregate-state contingent claim
- Non-negative net saving constraints for all agents

Calibration

- Fraction of traders
  - 50% does not participate equity market. (bond only)
  - 40% holds the market
  - 10% aggregate-complete (Z-com traders)
- Among the 10% of Z-com traders
  - Case 1: all of them have correct belief, $\kappa = 1$
  - Case 2: 1/2 have **volatile beliefs** with $\kappa = 0.75$
  - Case 3: 1/2 have Bayesian **regime-switching beliefs**.

Variation in Beliefs Results

	Baseline	Volatile	R-S
$\frac{\sigma(m)}{E(m)}$	0.41	0.41	0.42
$Std\left\{\frac{\sigma(m)}{E(m)}\right\}$	2.78	8.07	9.28
$E\left(R_f\right)$	1.93	2.04	2.04
$E\left(\omega_z\right)$	0.79	**0.89**	**0.88**
$E\left(\omega_{\tilde{z}}\right)$	-	0.69	0.70
$Corr\left(\omega_z, SR\right)$	0.93	**0.98**	**0.98**
$Corr\left(\omega_{\tilde{z}}, SR\right)$	-	-0.93	-0.87
$E\left(W_z/W\right)$	2.15	**2.33**	**2.36**
$E\left(W_{\tilde{z}}/W\right)$	-	1.91	1.86

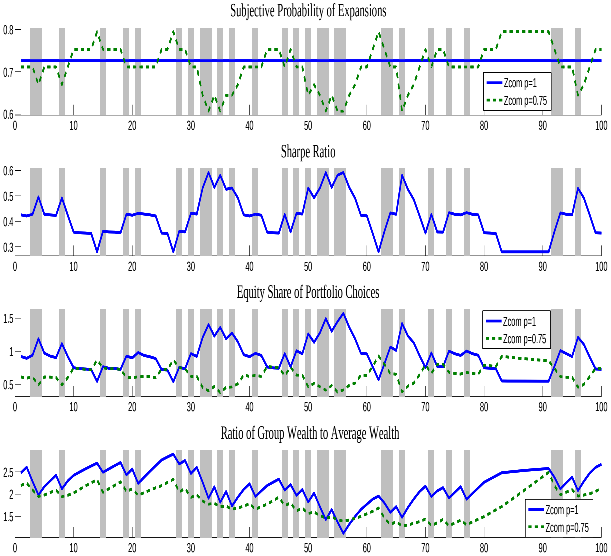
ω: equity share of portfolio, W: value of wealth

Variation in Beliefs Results

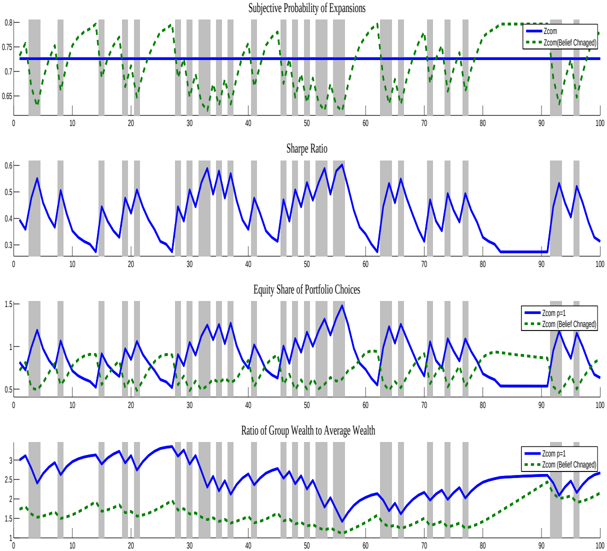
	Baseline	Volatile	R-S
$\frac{\sigma(\triangle\log(c_z))}{\sigma(\triangle\log(C))}$	2.99	2.92	2.92
$\frac{\sigma(\triangle\log(c_{\tilde{z}}))}{\sigma(\triangle\log(C))}$	-	3.07	3.12
$\frac{\sigma(\triangle\log(c_{cap}))}{\sigma(\triangle\log(C))}$	3.38	3.37	3.37
$\frac{\sigma(\triangle\log(c_{bond}))}{\sigma(\triangle\log(C))}$	3.59	3.57	3.57
$\frac{\sigma(\triangle\log(C_z))}{\sigma(\triangle\log(C))}$	6.98	7.08	7.12
$\frac{\sigma(\triangle\log(C_{\tilde{z}}))}{\sigma(\triangle\log(C))}$	-	7.02	7.00
$\frac{\sigma(\triangle\log(c_{cap}))}{\sigma(\triangle\log(C))}$	1.00	1.00	1.00
$\frac{\sigma(\triangle\log(c_{bond}))}{\sigma(\triangle\log(C))}$	0.91	0.92	0.92

c_i is individual consumption and C_i is group average

Volatile Beliefs



Regime-Switching Beliefs



Tentative Conclusion

- New methodology
  - Compute a G.E. model with a rich degree of heterogeneity
  - Heterogeneity in trading technologies, preferences and beliefs
- Volatile and R-S belief active traders
  - Volatility of market price of risk up a lot
  - Missed market timing
- Disciplined by trading behavior + belief survey data.

Can also report on risk aversion and discount differences.