Housing and the Labor Market: Time to Move and Aggregate Unemployment *

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July 30, 2010

Abstract

Conventional macro-search models (Mortensen and Pissarides) with unemployment benefits and taxes have been able to account for the variation in unemployment rates across countries but do not explain why geographical mobility is very low in some countries (on average, three times lower in Europe than in the U.S.). We build a model in which both unemployment and mobility rates are endogenous. Our findings indicate that an increase in unemployment benefits and in taxes does not generate a strong decline in mobility and accounts for only half to two-thirds of the difference in unemployment from the US to Europe. We find that with higher commuting costs the effect of housing frictions plays a large role and can generate a substantial decline in mobility. We show that such frictions can account for the differences in unemployment and mobility between the US and Europe.

*We thank Nobu Kiyotaki and seminar participants at Rogerson-Shimer-Wright NBER group meeting at the Atlanta Fed, INSEE-Crest, the ADRES-EDHEC conference in Paris (Labour Markets Outcomes, A Transatlantic Perspective), the Transatlantic conference IZA-SOLE in Bonn, Queens University, University of Toronto, University of Iowa, Warwick University, the first annual matching conference in Malaga, Spain, Northwestern University, UC Davis, UC Irvine.

1 Introduction

The Mortensen-Pissarides model has been shown to successfully explain cross-country differences in unemployment and unemployment spells with two labor market policies: unemployment benefits and taxes. Dale T. Mortensen and Christopher A. Pissarides (1999) highlights the fact that roughly half of the mileage between a US unemployment rate (6%) to a European one (11%) can be explained by each feature.

However, we posit that there are other policies that can affect the functioning of labor markets. In particular, policies that affect geographic mobility can affect the decision to accept a job, having a direct effect on employment and unemployment rates as well as the duration of unemployment. The policies we have in mind that can affect mobility or commuting decisions are housing regulations and taxes that affect commuting costs, such as gasoline taxes. Moreover, these policies differ substantially across countries.

Table 1 provides data on mobility for the United States and Europe. Lines 1 to 4 in the table show the fraction of individuals in each category having moved to a new residence from year to another. The last line in the table shows the fraction of moves that are between U.S. counties or "travel-to-work" areas in Europe. The residential mobility rate is roughly three times higher in the U.S. than the corresponding rates in Europe for all categories of the labor force. The largest share of the moves are within areas or counties, although there is more inter-area mobility in the U.S.

Table 1: Mobility

	US	Europe
Total	15.5%	4.95%
Employed workers	17.1%	5.38%
Unemployed workers	25.2%	10.94%
Out of labor force	11.3%	2.63%
Between counties / areas	42%	20.5%

Source: U.S.: Bureau of the Census, 2000; Europe: European Community Household Panel, 1999-2001.

In this paper we explore qualitatively and quantitatively, how low mobility can help explain a lower rate of employment in Europe. The mechanisms identified in this paper are as follows. Low mobility reflects the inability of housing markets to efficiently allocate workers across locations, for example due to housing market regulations. Therefore, job offers may be less attractive to workers, *ceteris paribus*, due to the difficulty to relocate. Given this, workers could choose to commute longer distances in order to avoid turning down offers. But transport costs, such as high gasoline taxes in Europe, may be an obstacle. Coupled with the fact that unemployment benefits are more generous in most European countries, more job offers are rejected, and commute distances are, on average, lower in Europe, and unemployment is higher. That is, there is a *complementarity* between various factors; in particular, difficulty to relocate has stronger effects on job acceptance if commute costs are larger. Table 2 highlights these facts, indicating higher gasoline taxes in Europe by a factor of three, shorter commute times and differences in unemployment benefits.

Table 2: Various Statistics

	US	Selected EU countries	Ratio
Mean housing regulation index (0-6)	2.97	3.70 (a)	107% (*)
Gasoline price (USD/gall.)	1.80	6.21(b)	3.45
Tax allowance (euro / kilometer)	0.21	0.514 (c)	2.45
Median commute time as a fraction of hours worked	10.2%	7.9%	0.77
Unemployment benefits	0.2	0.4	

(a) Denmark, Netherlands, Belgium, France, Ireland, Italy, Greece, Spain, Portugal, Austria, Finland, Germany. Without the UK, the mean is 3.70 (UK regulation index is 2.2).
(b) France, Belgium, Germany, Italy, Hungary, Netherlands, Norway (source Table xx and German Technical Corporation 2007).
(c) US : IRS 2007. EU : French Tax Authority, BO Impôts janvier 2007.
(*): difference divided by cross-country standard deviation: (3.70-2.97)/0.68=1.07

Our first task is to explore the potential causes of low mobility within a model. The second task is to provide a quantitative account of the consequences of low mobility. We build a modified version of Mortensen-Pissarides in order to capture geographical mobility. Workers receive offers characterized by a commute distance, and have the possibility to move conditional on receiving new location offers.[1] Within a very parsimonious setup, our model captures job acceptance decisions, decisions to move to another dwelling and job creation decisions, with search and matching frictions in the housing and labor market. We also show there is a complementarity between commuting and moving decisions, as well as unemployment benefits generosity and mobility costs. Since both of the latter components are higher in Europe than in the US, we provide an additional rationale of European unemployment. This feature is also present in Mortensen and Pissarides (1999).

[1] For ease of exposition we refer to locations as housing units, however, we do not explicitly model a housing sector.

In our model, a job location has an associated commuting time that may affect the job acceptance decision. Obstacles to mobility (that arise from rent controls, for example) will affect the reservation strategy of workers. Thus, aggregate unemployment affects the functioning of the housing market. The model can be thought of as the "dual" of typical search models of the labor market. In particular, in the standard search setup there is a non-degenerate distribution of wages but distance is degenerate. In our model, the distribution of wages is degenerate but there exists a non-degenerate distribution of distance from one's job.

The model is also used to provide a quantitative exercise to capture the effect these mechanisms have on unemployment, unemployment duration, and residential mobility. We explore the effects from changes in four factors: Benefits, labor taxes, commute costs and housing frictions. Our findings indicate that an increase in unemployment benefits and in taxes do not generate a strong decline in mobility and account for only two-thirds of the required increase in unemployment from the US to Europe. With higher commuting costs, like those in Europe, the effect of housing frictions play a large role and can generate a decline in mobility similar to that in Europe, roughly one-third of that in the U.S.

In a related model, Allen Head and Huw Lloyd-Ellis (2008) use a spatial model of housing and show that home owners are less mobile than renters, but that the effect of home ownership on unemployment is quantitatively small. In our paper there is no distinction between owning a home or renting and therefore we abstract from such differences. We view our paper as complementary to theirs as they focus mainly on renting vs. owning. Damien Gaumont, Martin Schindler and Randall Wright (2006) provide an example of how a non-degenerate wage distribution can arise from ex-ante homogeneous agents. In their model, when a worker chooses a job they also randomly choose a "cost to taking the job" that can be interpreted in the context of our model as a commuting cost.

Section 2 presents the model with labor market and housing frictions. Section 3 describes the optimal strategies and equilibrium as well as how frictions in the housing market affect mobility and unemployment rates. Section 4 extends the model to allow for "family shocks." Section 5 lays out the calibration strategy and parameters. Section 6 concludes.

2 Model

We begin by considering a simple model where a geographical mobility decision interacts with a job acceptance decision to expose the main logic of our framework. We initially assume away any "family shocks" in this section. In Section 4 we enrich the model to prepare the calibration exercise and include family and demographic shocks into the analysis.

2.1 Preferences and Search for Locations

Time is continuous and individuals discount the future at rate r. Individuals live in dwellings, defined as a bundle of services generating utility to an individual. The defining characteristic of a dwelling, however, is that the services it provides are attached to a fixed location. The services can, of course, depend on the quality of the dwelling and its particular location. Amenities such as space, comfort, proximity to theaters, recreation, shops and job increase the utility of a given dwelling. The dwelling may also be a factor of production of home-produced goods. In addition, the dwelling could be a capital asset. For these services, individuals pay a rent or a mortgage. To keep things tractable we do not model the market for houses or locations. Therefore, we do not keep track of individual house prices. Moving to a new location is costless and instantaneous once a location has been found.

In this paper we focus on one particular amenity, distance to work. Because a dwelling is fixed to a location, the commuting distance to one's job, ρ, becomes an important determinant of both job and location choice. We assume that space is symmetric, in the sense that the unemployed have the same chance of finding a job wherever their current residence. Therefore, ρ is a sufficient statistic determining both housing and job choice. We call this property isotropy of space: Wherever an individual is located, space looks the same. The implication is that there is no reason to move to a different location if unemployed.

Agents randomly receive opportunities to move to a new location that (possibly) allows them to obtain a shorter commute. These opportunities are assumed to be Poisson arrivals with parameter λ_H. The distribution of *new* vacancies is given as $G_N(\rho)$.

We make the simplifying assumption that the ease in which an agent can change locations can be captured in a single variable, λ_H. An interpretation is that it captures various frictions that makes it more difficult to relocate.[2] An increase in λ_H means there are more arrivals of opportunities to find a new location. As λ_H approaches infinity, housing frictions go to zero. The main idea behind λ_H is that agents may not move instantaneously to their preferred location. Such restrictions might arise from length of lease requirements or eviction policies. In the Appendix we discuss the relationship between housing market regulations and housing offers, λ_H, but for now we assume it represents housing market frictions. To simplify the analysis we assume that the rent or mortgage (we make no distinction between renting and owning) is such that utility across dwellings will be equalized to reflect any differences in amenities, a fact that results from the assumption that space (distance) is isotropic.

[2] For example, regulations in a housing market, such as rent controls or the inability to evict tenants.

2.2 Labor Market

Individuals can be in one of two states: employed or unemployed. While employed, income consists of an exogenous wage, w.[3] There is no on-the-job search, yet a match may become unprofitable, leading to a separation, which occurs exogenously with Poisson arrival rate s.

Unemployed agents receive income b, where b can be thought of as unemployment insurance or the utility from not working. While unemployed, job offers arrive at Poisson rate p, indexed by a distance to work, ρ, drawn from the cumulative distribution function F_J. Recall that we have imposed equal wages across all locations.

Let $E(\rho)$ be the value of employment for an individual residing at distance ρ from the job. Let U be the value of unemployment, which does not depend on distance, given the symmetry assumption made above. We can now express the problem in terms of the following Bellman equations:

$$(r+s)E(\rho) = w-\tau\rho+sU+\lambda_H\int \max\left[0,(E(\rho')-E(\rho))\right]dG_N(\rho') \tag{1}$$

$$(r+p)U = b+p\int \max[U,E(\rho')]dF(\rho'), \tag{2}$$

where τ is the per unit cost of commuting and ρ is the distance of the commute.[4] Eq. 1 states that workers receive a utility flow $w-\tau\rho$; may lose their job and become unemployed – in which case they stay where they are; they receive a housing offer from the distribution of new vacancies G_N, which happens with intensity λ_H, in which case they have the option of moving closer to their job. Eq. 2 states that the unemployed enjoy b; receive a job offer with Poisson intensity p, at a distance ρ', from the distribution $F(\rho)$. They have the option of rejecting the offer if the distance is too far.

3 Optimal Behavior, Equilibrium and Steady-states

3.1 Reservation Strategies

We now derive the job acceptance and moving strategies of individuals. Observe that E is downward sloping in ρ, with slope

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H P_W}, \tag{3}$$

[3]An exogenous wage greatly simplifies the analysis because the wage does not depend on commute distance. However, in the calibration we allow the wage to depend on taxes and benefits and examine alternative choices for the parameters as a robustness check on the importance of this assumption. In the Appendix we show how the model could be recast in terms of a wage-posting framework.

[4]It is possible to reinterpret commuting as any non-pecuniary aspect of the job.

where P_W is the probability of moving conditional on receiving a housing offer. Note that $0 < P_W < 1$ and possibly depends on ρ. The function $E(\rho)$ is monotonic so that there exists a well-defined reservation strategy for the employed, with a reservation distance denoted by $\rho^E(\rho)$, below which a *housing offer* is accepted. Note that there is state-dependence in the reservation strategy of the employed, $\rho^E(\rho)$, with presumably $d\rho^E(\rho)/d\rho > 0$. Evidently, the further away the tenants live from their job, the less likely they will be to reject a housing offer.

After some intermediate steps (described in the Appendix), we can show that the slope of $E(\rho)$ is given by

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho)}. \tag{4}$$

Next, in the absence of relocation costs (this case is studied in the Appendix), tenants move as soon as they get a dwelling offer closer to their current one, implying

$$\rho^E(\rho) = \rho.$$

Denote by ρ^U the reservation distance for the unemployed, below which any *job offer* is accepted, it is defined by

$$E(\rho^U) = U.$$

Using the fact that $E(\rho^U) = U$, we obtain

$$b + p\int_0^{\rho^u} [E(\rho') - U] dF(\rho') = w - \tau\rho^U + \lambda_H \int_0^{\rho^U} [E(\rho') - U] dG_N(\rho'). \tag{5}$$

Integrating Eq. 5 by parts gives the following implicit equation defining ρ^U:

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) - pF(\rho)}{r+s+\lambda_H G_N(\rho)} d\rho. \tag{6}$$

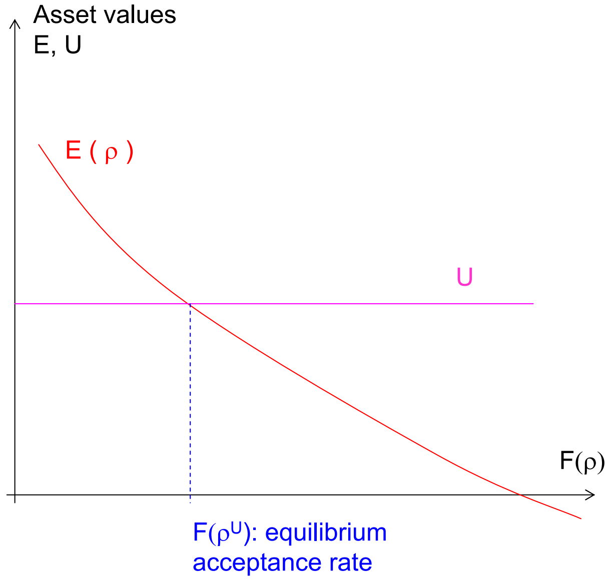
The determination of ρ^U is shown in Figure 1.

With this specification the model is quite parsimonious, since a single variable, ρ, determines:

1. *Job acceptance*: $F(\rho^U)$;

2. *Residential mobility rate*: $\int \lambda_H G_N(\rho)$ over the distribution of commute distance of employed workers $d\Phi$;

Figure 1: Determination of ρ_u



3.2 Free Entry

Assuming free entry of firms, and defining $\theta = \frac{V}{U}$ as labor market tightness, we have

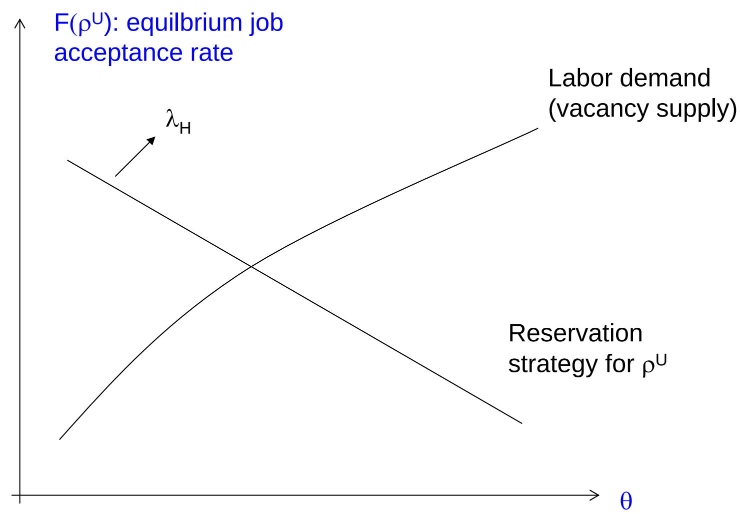
$$\frac{y-w}{r+s} = \frac{c}{q(\theta)P_F},$$

where P_F is the rate of acceptance of job offers by the unemployed, as expected from the viewpoint of the firm. We assume, still by symmetry, that the distribution of contacts between the firm and unemployed workers is given as $F(\rho)$, so that $P_F = F(\rho^U)$. This generates a positive link between θ and ρ^U since $q'(\theta) < 0$, characterized by:

$$q(\theta)F(\rho^U) = \frac{c(r+s)}{y-w}. \tag{7}$$

The intuition is quite simple. The firm's iso-profit curve at the entry stage depends negatively on both θ (as a higher θ implies more competition between the firm and the worker) and on ρ^U (as more of their offers will be rejected because of distance). The zero-profit condition thus implies a positive link between θ and ρ^U. Note that this relation is independent of λ_H. On the other hand, ρ^U is determined through (Eq. 6). It is decreasing in $p(\theta)$ and thus in θ, as can be seen in (Eq. A-4). When there are more job offers (higher θ) workers can wait for offers closer to their current residential location; they are pickier. The two curves are represented in (ρ^U, θ) space in Figure 2.

Figure 2: Vacancies and Reservation Strategy



3.3 Unemployment and the Beveridge Curve

Recapitulating, an increase in λ_H, the efficiency of the housing sector, raises the acceptance rate of job offers, increasing θ and thus increasing job offers by firms.

Letting $p(\theta) = \theta q(\theta)$, the steady state unemployment rate is given as

$$u = \frac{s}{s + p(\theta)F(\rho^U)}. \tag{8}$$

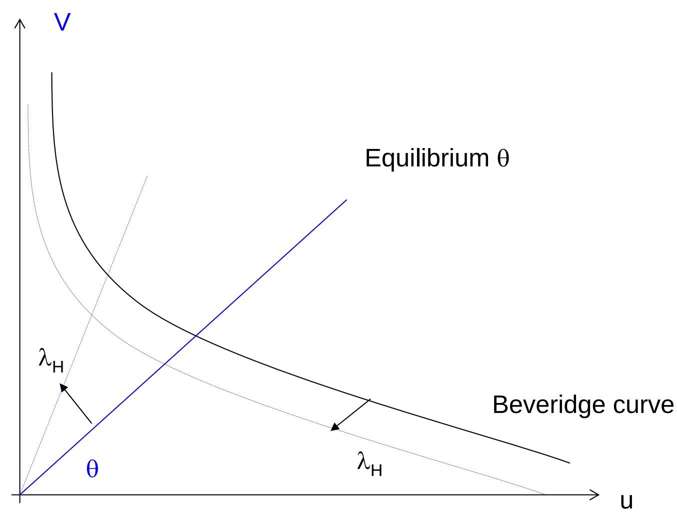
In terms of a Beveridge Curve representation (vacancy and unemployment space), increasing λ_H shifts the Beveridge curve inward (less structural mismatch) and also leads to a counter-clockwise rotation of θ. A graphical representation of this result is shown in Figure 3.

3.4 Housing Frictions and Mobility

3.4.1 The Effect of Regulations in the Housing Market

It is now possible to determine how housing frictions affect the decisions of workers and firms.

Figure 3: Beveridge Curve



Proposition 1 *An increase in λ_H makes the unemployed less choosy about jobs: $\partial\rho^U/\partial\lambda_H > 0$.*

Proof. See Appendix. ■

The proposition shows that an increase in the arrival rate of housing opportunities increases the probability the unemployed will accept jobs as they are willing to live farther away from their job initially because moving closer is relatively easier.

Next, differentiating (Eq. 7) and using Proposition 1, we can determine the effect of housing frictions on job creation:

Proposition 2 *An increase in λ_H increases job creation: $\partial\theta/\partial\lambda_H > 0$.*

Proof. Same as Proposition 1. ■

This is an indirect effect caused by more job creation through the higher job acceptance rate of workers. Another interpretation of this effect is that *firms don't like to create jobs where workers have no place to live.*

Using these results it is now possible to determine the effect of housing market frictions on unemployment.

Proposition 3 *An increase in λ_H has two effects on unemployment:*

- *it raises the job acceptance rate of workers (through a higher thresehold ρ^U);*

- *it raises θ (Proposition 2) and thus job creation.*

Proof. See Appendix. ■

Therefore, increases in the opportunity to move will decrease unemployment due to workers being more willing to accept jobs and because there are more vacancies created by firms, which increases market tightness.

3.4.2 Distribution of commute distance

Let $\Phi(\rho)$ be the steady-state distribution of employed workers living at a location closer than ρ. Φ is governed by the following law of motion, for all $\rho < \rho^U$:

$$(1-u)\frac{\partial\Phi(\rho)}{\partial t} = upF(\rho) + (1-u)(1-\Phi(\rho))\lambda_H G_N(\rho) - (1-u)\Phi(\rho)s \tag{9}$$

Eq. 9 states that the number of people residing in a location at a distance less than ρ from their job changes (either positively or negatively) due to:

- (+) the unemployed, u, receiving a job offer at rate p with a distance closer to ρ with probability $F(\rho)$;

- (+) the employed, $1-u$, who are further away from the current distance ρ (a fraction $1-\Phi(\rho)$), who receive an offer in the housing market with intensity λ_H closer to ρ with probability $G_N(\rho)$;

- (-) the employed, $1-u$, who receive an s-shock, that is, exogenous job destruction.

In steady state and for all $\rho < \rho^u M$:

$$\begin{aligned} \Phi(\rho) &= \frac{\lambda_H G_N(\rho) + pF(\rho)\frac{u}{1-u}}{\lambda_H G_N(\rho) + s} & (10) \\ &= \frac{\lambda_H G_N(\rho) + \frac{F(\rho)}{F(\rho^u)}s}{\lambda_H G_N(\rho) + s} \leq 1 & (11) \end{aligned}$$

The second line above is obtained by replacing u with its steady-state expression in (Eq. 8). Note that for $\rho = \rho^U$, $\Phi(\rho^U) = 1$ as no unemployed individual ever accepts a job offer farther away from a job than ρ^U.

3.4.3 Log Linearization

First, consider the special case: $\lambda_H \to \infty$. In the case where housing frictions go to zero, the model collapses to $\Phi(\rho) = 1$, meaning that all workers will be located epsilon-close to their job. The job acceptance decision is indeterminate since we now have

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} d\rho.$$

The intuition is straightforward: if $w > b$, all job offers are accepted, meaning that ρ^U goes to infinity. Therefore, we obtain the standard Pissarides value for tightness: $q(\theta^P) = \frac{c(r+\sigma)}{y-w}$ with $\theta^P > \theta^*$ where θ^* is equilibrium tightness in our mode. In addition,

$$\frac{q(\theta^P)}{q(\theta^*)} = F(\rho^U) < 1$$

and therefore, with $q(\theta + d\theta) = q(\theta) + q'(\theta)d\theta = q(\theta)(1 + \eta_q d\theta/\theta)$, we have

$$\frac{q(\theta^P)}{q(\theta^*)} = 1 + \eta_q d\theta/\theta^* = F(\rho^U)$$

hence

$$\frac{d\theta}{\theta^*} = \theta^P - \theta^* = \frac{1 - F(\rho^U)}{-\eta_q} > 0$$

The percentage change in tightness is of the order of magnitude of the rejection rate of job offers divided by the elasticity of matching. Since the percentage change in unemployment is the percentage change in tightness multiplied by $(1-u)\eta_p$, the overall change in unemployment due to imperfect housing markets is of the order of magnitude of the fraction of rejected offers $1 - F(\rho^U)$ if $\eta_p \simeq -\eta_q \simeq 0.5$.

4 Extension with family shocks

In reality, many residential moves occur due to changes in marital status, family size, schooling choices, neighborhood quality, and so on. To better capture these effects and to better fit the mobility data in the calibration section we now extend the model to include "family shocks."

In addition to the λ_H shock, individuals may receive a family shock that arrives according to a Poisson process with parameter δ. The shock changes the valuation of the current location, necessitating a move. Upon the arrival of the shock they make one draw from the existing stock of housing vacancies, distributed as $G_S(\rho)$.[5] Note that agents may sample from the existing stock of

[5] The one draw assumption is not very strong. It is equivalent to making up to N independent draws, in which case

houses at any time.

Bellman equations are now augmented by a new term (second line) starting with δ : when agents receive a family shock δ, they need to relocate and sample the existing stock G_S:

$$\begin{aligned}(r+s)E(\rho) &= w-\tau\rho+sU+\lambda_H\int\max\left[0,(E(\rho')-E(\rho))\right]dG_N(\rho') \quad (12)\\ &\quad +\delta\int\max[U-E(\rho),E(\rho'')-E(\rho)]dG_S(\rho'')\\ (r+p)U &= b+p\int\int\max[U,E(\rho'),E(\rho^{''})]dF_J(\rho')dG_S(\rho''), \quad (13)\end{aligned}$$

Given that we assume that households now have an option to sample into the existing stock of dwelling G_S, we must adapt the determination of the job acceptance decisions. The unemployed receive an offer at distance ρ' but also have the option to move instantaneously if they find a residence in the stock of existing vacant units at distance ρ''. To the extent that ρ' and ρ'' are independent draws, this means that there is a distribution, F, combining F_J and G_S such that the integral terms can be rewritten as $\int\max[U,E(\rho)]dF(\rho)$, where ρ is the minimum of the two draws: $\rho = Min(\rho',\rho'')$.[6]

The slope of E with respect to ρ is now

$$\frac{\partial E}{\partial\rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho)+\delta}. \quad (14)$$

leading to

$$\rho^U = \frac{w-b}{\tau}+\int_0^{\rho^U}\frac{\lambda_H G_N(\rho)+\delta G_S(\rho)-pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta}d\rho. \quad (15)$$

The determination of ρ^U is shown in Figure 1. Next, a job separation can now occur in two ways. First, due to the exogenous shock s. Second, workers may receive a family shock, δ, requiring them to redraw from the vacant housing stock distribution, G_S, but are unable to find a sufficiently close dwelling to the current job and (optimally) quit. That is, job separations are given by

$$\sigma = s+\delta(1-G_S(\rho^u)). \quad (16)$$

Therefore, we have an additional effect of ρ^U on unemployment, through quits.

The free entry condition becomes

$$q(\theta)F(\rho^U) = \frac{c(r+\sigma)}{y-w}. \quad (17)$$

it is like one single draw from a distribution $(G_S)^N$. See Lemma 1 in Quentin et al. (2006).

[6] We prove in the appendix that $1-F(\rho) = (1-F_J(\rho))(1-G_S(\rho))$.

where s has been replaced by σ, and similarly for the rate of unemployment,

$$u = \frac{\sigma}{\sigma + p(\theta)F(\rho^U)}. \tag{18}$$

Finally, the distribution of commute distance is also affected: in the law of motion of $\Phi(\rho)$, we have, for all $\rho < \rho^U$, two additional terms:

$$\begin{aligned}(1-u)\frac{\partial \Phi(\rho)}{\partial t} =& upF(\rho) + (1-u)(1-\Phi(\rho))\left\{\lambda_H G_N(\rho) + \delta G_S(\rho)\right\} \\ & - \delta(1-u)\Phi(\rho)(1-G_S(\rho)) - (1-u)\Phi(\rho)s\end{aligned} \tag{19}$$

- (+) the employed, $1-u$, who are further away from the current distance ρ (a fraction $1-\Phi(\rho)$), who face a δ-shock that brings them closer to ρ after sampling in the stock G_S;
- (-) the employed, $1-u$, who were at a distance less than ρ (a fraction $\Phi(\rho)$), receive a δ-shock that brings them further away from ρ after sampling in the stock G_S; note that a fraction of them would even quit if their new ρ is above ρ^U.

This leads to

$$\Phi(\rho) = \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) + \frac{F(\rho)}{F(\rho^u)}\sigma}{\lambda_H G_N(\rho) + \delta + \sigma} \leq 1 \tag{20}$$

5 Calibration

5.1 Calibration targets

In this section we will match the extended model of Section 4 to the data, in particular the mobility rates. We therefore need to calculate the mobility rate from the model. Denote by M_K^S the number of movers of status $S = (U,E)$ (unemployed, employed) and for reason $K = (J,D)$ (job-related or family-related), we have:

1. Job-related mobility of the employed (those with a job but relocate once they sample a better housing location):

$$\begin{aligned} M_J^E &= (1-u)\lambda_H \int_0^{\rho^U} G_N(\rho) d\Phi(\rho) && (21) \\ &= (1-u)\lambda_H \left[G_N(\rho^U) - \int_0^{\rho^U} g_N(\rho)\Phi(\rho) d\rho \right], && (22) \end{aligned}$$

   where the second line is found by integrating by parts and noticing that $\Phi(\rho^U) = 1$.

2. Job-related mobility of the unemployed (those who have a job offer, accept it with probability $G(\rho^U)$ and may relocate if they drew a location from G_S closer from their current ρ):

$$M_J^U = up\int_0^{\rho^U} G_S(\rho)dF_J(\rho)$$

3. Family-related mobility:

$$\begin{aligned} M_D^U &= u\delta \\ M_D^E &= (1-u)\delta \\ M_D^{E+U} &= \delta \end{aligned}$$

Note that in M_D^E, some workers quit their job (a fraction $1-G_S(\rho^U)$) since they did not find acceptable housing in the current stock.

5.2 Taxes, Benefits and Wages

So far, the model has abstracted from taxes. As shown in Mortensen and Pissarides (1999) and Edward C. Prescott (2004), taxes and benefits can explain much, if not all, of the variation in unemployment rates across countries. We therefore introduce a tax on labor denoted by t which will be set to 0.22 for the US and 0.4 for Europe. However, it is quite well known that taxes on labor lower wages and therefore that there is a "crowding out" effect: A one percentage point increase in taxes does not necessarily imply a one percentage point increase in labor costs. The net effect depends, in principle, on the elasticity of demand, supply and the bargaining power of workers. In the model developed so far, it is possible to make wages endogenous and introduce a bargaining game. However, the cost is to lose most of the simplicity of the model as the wage will then depend on commute distance. We take an alternative route here: Keep an exogenous wage, but argue that part of the effect of taxes is diluted due to a crowding out parameter denoted by ε.[7] In short, if taxes are t, the total labor cost is denoted by $w(1+\varepsilon t)$ and the net wage of workers is $w[1-(1-\varepsilon)t]$. It follows that the main equations of the model become:

$$q(\theta)F(\rho^U) = \frac{c(r+\sigma)}{y-w(1+\varepsilon t)} \tag{23}$$

$$\rho^U = \frac{w[1-(1-\varepsilon)t]-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho)+\delta G_S(\rho)-pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta} d\rho, \tag{24}$$

while the stock-flow equations and the rate of unemployment are unchanged. We set ε to be 0.35, implying that a 10% increase in labor taxes generates a 3.5% increase in labor costs and a 6.5%

[7] In the Appendix we derive results for endogenous wages in a wage posting model

decrease in the net wage of workers.[8]

Finally, it is unrealistic to assume that a change in unemployment benefits has no direct effect on wages, and only an indirect effect on the average wage in the economy through an increase in reservation wages. This is why in the calibration we allow for the direct effect by arguing that $w(b) = w_{US} + (1-\beta)(b - b_{US})$ where any additional dollar of unemployment compensation raises the wage by 1-β where β can be thought of as the bargaining power of workers: This is the same specification as that emerging from Nash-bargaining. We set $\beta = 0.5$ so that the bargaining power is symmetric. We set $w_{US} = 0.6$ and the output generated in the match is normalized to $y = 1$. Labor taxes in the US are given by $t = 0.22$ and unemployment benefits are $b = 0.25$. Labor taxes in Europe are $t = 0.4$ and unemployment benefits are equal to $b = 0.4$ (for a wage of 0.627). So, roughly speaking a replacement rate of 42% in the US and 64% in Europe.

5.3 Calibration

The time period is one month and the interest rate, r, is set to 0.0033, corresponding to an annual rate of 0.04. We calibrate to the mobility rate of the employed, 17.1% annually between March 1999 and March 2000, so the target is (17.1/12)%. The number for the employed that move comes from the Bureau of the Census.[9] Of the roughly 31 million persons who moved during that year, 22.3 million of them were employed, 1.5 million unemployed and 7.8 million out of the labor force.

We have three distributions to account for: G_N, new housing offers, G_S, the stock of houses and F, job offers. We assume that these distributions are represented by exponential functions with parameter α: $F = G_N = 1 - e^{-\alpha\rho}$ and $G_S = 1 - e^{-(\alpha/3)\rho}$.

To calculate α and τ, we proceed as follows. First, Table 3 shows the distribution of commute times from the Census 2000 as a fraction of total hours worked. The median commuter spends 0.083 of its working time to commute.

We assume that each hour of commute time has a utility cost estimated to be half of the hourly wage of workers (see Jos Van Ommeren, Gerard Van den Berg and C. Gorter (2000)). Hence, the total median cost for the median commuter should be 0.083/2 expressed as a fraction of the wage, or 0.083/2*(w/y) as a fraction of output (normalized to 1).

The total median cost is also calculated from the distribution of wage offers. Letting ρ^m be the median commute distance, $\rho^m = \ln 2/\alpha$, the total cost incurred for the median commuter is

[8] Assuming that ε is being approximated by $\varepsilon^{LS}/(\varepsilon^{LS} + \varepsilon^{LD})$ where ε^{LS} and ε^{LD} are the absolute elasticities of labor supply and labor demand, this would imply that $\varepsilon^{LD}/\varepsilon^{LS} = 2$.

[9] Why People Move: Exploring the March 2000 Current Population Survey, P23-204, Bureau of the Census, May, 2001.

Table 3: Commute time as a fraction of total hours worked

	US	Fr
Mean	0.102	0.079
10^{th} percentile	0.020	0.021
25^{th} percentile	0.041	0.031
Median	0.083	0.063
75^{th} percentile	0.125	0.094
90^{th} percentile	0.188	0.167

therefore given by

$$0.083/2 * (w/y) = \tau \rho^m$$

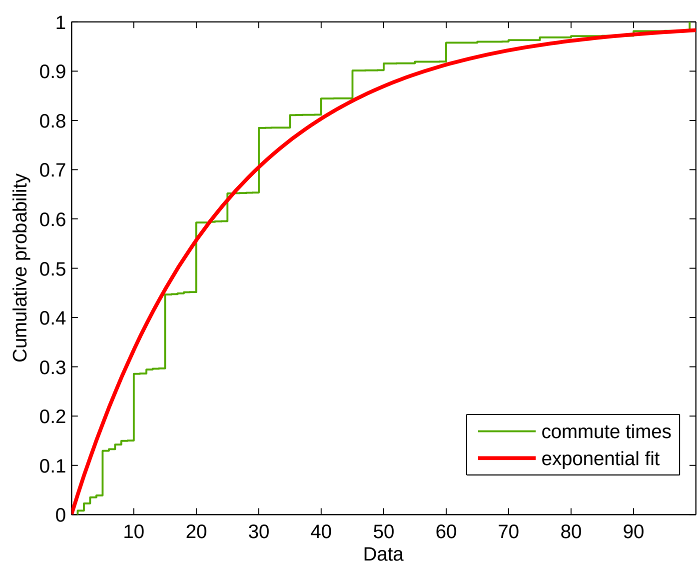
or

$$\tau = \frac{0.083/2 * (w/y)}{\ln 2/\alpha}$$

We then estimate α from the slope of the distribution F in the data. Inspection of Figure 4 shows that there is an optimal value of α that best approximates the c.d.f. We find empirically that it is equal to 9.77 after estimating $\ln(1 - F(\alpha\rho)) = \alpha\rho$ from the data. Hence, $\tau = 0.585(w/y)$.

Unsurprisingly, given that commute costs per kilometer are higher in France and the benefits are higher, the mean commute time as well as the median are two percentage points lower in France: the unemployed are more choosy.

Figure 4: Distribution of Commute Times in the U.S.



The program finds the parameters of the model given a target unemployment rate of 4.2% in the

U.S. (the average between March 1999 and March 2000), and a target job hiring rate of $p = 1/2.4$ monthly. The latter implies an average duration of unemployment of 2.4 months and therefore imposes a value for σ given that $u = \sigma/(\sigma + p)$ then $\sigma = p(u/(1-u)) = 0.0183$.

We match the mobility rate to a target value of 17.1% annually with (Eq. 22). The program finds the values of α and λ_H that are consistent with the target for mobility, given ρ^U, obtained from (Eq. 6).

We set $p(\theta) = A\theta^{0.5} * F(\rho^U)$. Setting $\theta = 1$ gives $A = 0.586$. Together with the free-entry condition, (Eq. 7), this fixes a value for recruiting costs c after normalizing $y = 1$.

To find the cost of commuting in Europe relative to the US, we use information from the tax authorities in the U.S. and France as well as gas prices given in Table 2. The Internal Revenue Service (IRS) in the U.S. and the tax authority BO Impôts in France provide standard mileage rates when using a car for business. For 2007, the allowance was \$0.485 per mile (0.20 euro per mile) in the U.S. and 0.514 euro per kilometer for a 6CV car.[10] However, since the typical US car consumes more gas per kilometer than in Europe, we assume that $\tau_{EU} = 1.5 * \tau_{US}$.

5.4 Findings

The findings for the benchmark economy are given in Table 4. The benchmark calibration is given in the first column. The other columns show the cumulative effect of institutional changes: higher benefits, b, (from 0.25 to 0.4); then, higher taxes (from 0.22 to 0.4); then a 1.5 increase in commute costs, and finally a decrease in the arrival rate of housing offers by a factor of 2.1 to exactly match the residential mobility rate (for job related reasons) in Europe (0.00082 per month). The fact that the arrival of housing offers needs to be divided by 2.1 suggests that the housing market in Europe is considerably more sclerotic than that of the U.S.

The combination of benefits and taxes more than doubles the unemployment rate, similar to that in Mortensen and Pissarides (1999). The inclusion of the housing market frictions and commute increases the unemployment rate by about 50%, from 9.6% to 13.3%.

It is also possible to calculate elasticities and slopes implied by the model. In particular, we can examine how changes in benefits, taxes, housing frictions and commuting costs affect unemployment and mobility. The elasticities and slopes are given in Table 5.

Our findings indicate that labor market institutions account for a large part of cross-country differences in unemployment but perform poorly in terms of explaining low mobility. Adding in housing frictions and commute costs delivers both low mobility and a quite sizeable increase in unemployment. Taxes and benefits alone generate a 4 percentage point increase in unemployment when European values are chosen instead of US values, a realistic increase in commute costs of

[10]The rate is progressive with power in France, ranging from 0.37 to 0.67 euro per kilometer.

Table 4: U.S. Calibration

	Benchmark	Higher b	Higher b, tax	$\tau * 1.5$	$\lambda_h/2.1$
θ_h	1.000	0.685	0.268	0.194	0.186
ρ^U	0.055	0.051	0.043	0.036	0.036
Unemp	0.042	0.054	0.096	0.129	0.133
Un. Dur.	2.400	3.125	5.802	8.015	8.336
Reject	0.836	0.848	0.869	0.889	0.891
Mobility (x 100)	0.244	0.227	0.197	0.169	0.082

Table 5: Elasticities and Slopes

	Unemployment		Mobility	
	Elasticity	Slope (%)	Elasticity	Slope (%)
Benefits	0.505	0.839	−0.066	−0.076
Taxes	0.270	0.511	−0.069	−0.090
Commuting cost (τ)	0.698	0.558	−0.330	−0.184
Housing (λ)	−0.093	−0.903	0.964	6.543

50% and a 30% increase in frictions in the housing markets raise unemployment by an additional 4 to 5 percentage points. Interestingly, housing frictions, per se, account for only a small portion of unemployment when commute costs are low. In other words, there is strong complementarity between the two parameters.

5.5 Robustness

Due to the fact that the wage is not endogenous we now provide several robustness exercises to show how the findings change with a change in the parameters. Table 6 shows how unemployment is affected by changes in β, ε and α. As with Table 4 the columns after the benchmark in column 1 show the cumulative effect of institutional changes. Changing β or α has only small effects on unemployment. However, changes in ε can have large effects on unemployment. When $\varepsilon = .15$ unemployment rises to over 20%. Note that this value of ε means that the wage of the worker falls by 85%.

Table 6: Robustness: Effects on Unemployment

	Benchmark	Higher b	Higher b, tax	$\tau * 1.5$	$\lambda_h/2.1$
Changes in β					
$\beta = 0.5$	0.042	0.054	0.0956	0.1272	0.1322
$\beta = 0.4$	0.042	0.054	0.0911	0.1212	0.1263
$\beta = 0.3$	0.042	0.054	0.0877	0.1166	0.1219
$\beta = 0.6$	0.042	0.054	0.1013	0.1349	0.1399
$\beta = 0.7$	0.042	0.054	0.1088	0.1449	0.1499
Changes in ε					
$\varepsilon = 0.15$	0.042	0.0597	0.1475	0.197	0.2026
$\varepsilon = 0.25$	0.042	0.0564	0.1124	0.1498	0.1551
$\varepsilon = 0.35$	0.042	0.054	0.0956	0.1272	0.1322
$\varepsilon = 0.45$	0.042	0.0522	0.0861	0.1143	0.1193
$\varepsilon = 0.55$	0.042	0.051	0.0805	0.1068	0.1118
Changes in α					
$\alpha = 9.77$	0.042	0.054	0.0956	0.1272	0.1322
$\alpha = 11$	0.042	0.0539	0.0953	0.1267	0.1317
$\alpha = 13$	0.042	0.0538	0.095	0.1259	0.1309
$\alpha = 9$	0.042	0.054	0.0957	0.1275	0.1325
$\alpha = 7$	0.042	0.0541	0.0961	0.1282	0.1334

6 Concluding Comments

In this paper we have taken seriously the idea that labor market frictions, and in particular the reservation strategies of unemployed workers when they decide whether to accept a job offer, depend strongly on the functioning of the housing market. This interconnection between two frictional markets (housing and labor) can be used to understand differences in the functioning of labor markets. This paper has offered such a model, based on decisions to accept or reject a job offer, given the commuting distance to jobs. The model is relatively parsimonious, thanks to simplifying assumptions such as the isotropy of space, an unrealistic assumption but which conveniently provides closed form solutions and makes it possible to explain quit, job acceptance and geographic mobility decisions with a decision rule based on a single dimension.

We have attempted to explain differences in mobility rates between Europe and the US. A calibration exercise of "Europe" and the US is able to account for differences in unemployment and mobility thanks to a parameter which captures the speed of arrival of housing offers to households.

In our calibration, we find that labor market institutions account for a large part of cross-country differences in unemployment but perform poorly in terms of explaining low mobility. In contrast, our "spatial block", that is housing frictions combined to higher commute costs, explain well low mobility and a quite sizeable increase in unemployment : while taxes and benefits generate a 4 percentage point increase in unemployment when European values are chosen instead of US values, a realistic increase in commute costs of 50% and a 75% increase in frictions in the housing markets raise unemployment by an additional 4 to 5 percentage points.

Interestingly enough, housing frictions, per se, account for only a small portion of unemployment when commute costs are low: there is a strong complementarity between the two parameters: when commuting is costly and when it is difficult to relocate in the future, then job rejection is much more frequent.

Future work should attempt to enrich the model to introduce more specific urban features such as anisotropy of space and the existence of centers in cities and suburbs, as well as different groups of the labor force. Our work is a first step in integrating housing and labor markets in a coherent macroeconomic model. In particular, since the model is simple, it can be extended to deal with new issues such as discrimination in the housing market, mobility allowances or "moving toward opportunity" schemes, spatial misatch issues and so on, as in the urban economics literature.

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A Appendix

A.1 Evidence on Mobility, Regulations and Why People Move

This section provides evidence on the relationship between housing market regulations and mobility across the EU. Table A-1 shows an index of housing market regulations derived by Simeon Djankov, Rafael La Porta, Florencio Lopez de Silanes and Andrei Shleifer (2003) and unemployment rates for several countries.

Table A-1: Mobility, Regulations and Unemployment

	Mobility rate outside the area within 3 yrs.	Housing market regulation index	Average unemployment rate 1995-2001
Denmark	0.054	3.6	5.3
Netherlands	0.029	3	4.2
Belgium	0.013	3.17	8.5
France	0.042	3.6	10.4
Ireland	0.010	3.2	7.9
Italy	0.011	4.24	10.7
Greece	0.011	4.31	10.5
Spain	0.009	4.81	14.5
Portugal	0.007	4.54	5.6
Austria	0.015	3.62	4.02
Finland	0.058	2.53	11.9
Germany	0.021	3.76	8.3
UK	0.072	2.22	6.5

Source: Mobility data from ECHP. Regulation indices from Djankov et al. (2003). Unempoyment rates from Eurostat.

A.2 Link between F, F_J and G_S

We start from the integrand $A = \int\int \max[U, E(\rho'), E(\rho'')] dF_J(\rho')dG_S(\rho'')$. We can rewrite A as

$$\begin{aligned} A &= \int\int \{I[\rho' > \rho_U]I[\rho'' > \rho_U]U \\ +I[\rho' &< \rho'']I[\rho' < \rho^U]E(\rho') \\ +I[\rho'' &< \rho']I[\rho'' < \rho^U]E(\rho'')\} \\ & \quad dF_J(\rho')dG_S(\rho''). \end{aligned}$$

This can be written as:

$$
\begin{aligned}
A &= U\int_{\rho^U} dF_J(\rho')\int_{\rho^U} dG_S(\rho'') \\
&\quad +\int_0^{\rho^U} E(\rho')\left(\int_{\rho'} dG_S(\rho'')\right)dF_J(\rho') \\
&\quad +\int_0^{\rho^U} E(\rho'')\left(\int_{\rho''} dF_J(\rho')\right)dG_S(\rho''),
\end{aligned}
$$

or

$$
\begin{aligned}
A &= U(1-F_J(\rho^U))(1-G_S(\rho^U)) \\
&\quad +\int_0^{\rho^U} E(\rho')\left(1-G_S(\rho')\right)dF_J(\rho') \\
&\quad +\int_0^{\rho^U} E(\rho'')\left(1-F_J(\rho'')\right)dG_S(\rho'').
\end{aligned}
$$

Letting $F = 1-(1-F_J(\rho))(1-G_S(\rho))$, gives

$$dF = (1-F_J)dG_S + (1-G_S)dF_J,$$

and we can thus rewrite as

$$
\begin{aligned}
A &= U(1-F_J(\rho^U))(1-G_S(\rho^U)) \\
&\quad +\int_0^{\rho^U} E(\rho)dF(\rho) \\
&= \int \max(U,E(\rho))dF(\rho)
\end{aligned}
$$

The last equality comes from the observation that $1-F(\rho^U) = (1-F_J(\rho^U))(1-G_S(\rho^U))$.

A.3 Proofs

Proof the slope of E(ρ)

We need to rewrite Eq. 1 and Eq. 2 as:

$$
\begin{aligned}
(r+s)E(\rho) &= w-\tau\rho+sU+\lambda_H\int_0^{\rho^U}[E(\rho')-E(\rho)]dG_N(\rho') && \text{(A-1)}\\
&\quad +\delta\int_0^{\rho^u}[E(\rho')-E(\rho)]dG_S(\rho'')+\delta\int_{\rho^u}^{+\infty}[U-E(\rho)]dG_S(\rho'')\\
(r+p)U &= b+p\int_0^{\rho^u}[E(\rho')]dF(\rho')+pU(1-F(\rho^u)). && \text{(A-2)}
\end{aligned}
$$

Proof of the determination of ρ^U

Rewrite the Bellman equations as:

$$
\begin{aligned}
rE(\rho) &= w-\tau\rho+s(U-E(\rho))+\lambda_H\int_0^{\rho^{E(\rho)}}(E'-E)dG_N(\rho')\\
&\quad +\int_0^{\rho^U}[E''(\rho)-E(\rho)]dG_S(\rho)+\int_{\rho^U}^{+\infty}[U-E(\rho)]dG_S(\rho)\\
rU &= b+p\int_0^{\rho^U}(E'-U)dF(\rho')
\end{aligned}
$$

Here, since $\rho^E(\rho)=\rho$, we apply this formula in $\rho=\rho^U$ of E in ρ^U, we have:

$$
\begin{aligned}
rE(\rho^U) &= w-\tau\rho^U+s(U-E(\rho^U))+\lambda_H\int_0^{\rho^U}(E'-E(\rho^U))dG_N(\rho')+\int_0^{\rho^U}[E''(\rho)-E(\rho)]dG_S(\rho)\\
&= rU=b+p\int_0^{\rho^U}(E'-U)dF(\rho')
\end{aligned}
$$

so

$$
\rho^U=\frac{\lambda_H}{\tau}\int_0^{\rho^U}(E'-U)dG_N(\rho')+\frac{\delta}{\tau}\int_0^{\rho^U}[E''(\rho)-E(\rho)]dG_S(\rho)-\frac{p}{\tau}\int_0^{\rho^U}(E'-U)dF(\rho')+\frac{w-b}{\tau} \tag{A-3}
$$

This equation simplifies a bit. Noting that

$$
\frac{\partial E}{\partial\rho}=\frac{-\tau}{r+s+\lambda_H G_N(\rho)+\delta}
$$

integration by parts leads to

$$
\int_0^{\rho^U}(E'-U)dH(\rho')=\int_0^{\rho^U}\frac{\tau H(\rho)}{r+s+\lambda_H G_N(\rho)}d\rho
$$

for all H distributions such that $H(0)=0$, we can thus rewrite ρ^U as in the text (equation Eq. 6).

Proof of Proposition 1

Fully differentiating ρ^u gives

$$d\rho^U\left(\frac{r+s+pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta}\right)=\left(\int_0^{\rho^U}\frac{G_N(\rho)(r+s)+pF(\rho)G_N(\rho)}{[r+s+\lambda_H G_N(\rho)+\delta]^2}d\rho\right)d\lambda_H \qquad \text{(A-4)}$$
$$-\left(\int_0^{\rho^U}\frac{F(\rho)d\rho}{r+s+\lambda_H G_N(\rho)+\delta}\right)dp+d\left(\frac{w-b}{\tau}.\right)$$

Eq. A-4 indicates that ρ^U depends positively on λ_H, negatively on p and positively on $w-b$. An increase in λ_H (more housing offers) will shift the curve in Figure 1 upward, raising ρ^U and thus the acceptance rate of the unemployed. The intuition is simply that they take the job offer knowing that they will be able to find a better location in the near future because housing offers arrive very frequently.

Proof of Proposition 3

Differentiating u, and letting ω_θ represent the partial derivative of θ to λ_H. gives:

$$\begin{aligned}\frac{du}{d\lambda_H} &= \frac{-\delta g_S(\rho^U)\omega_\rho(\sigma+p(\theta)F(\rho^U))-\sigma[-\delta g_S(\rho^U)\omega_\rho+p(\theta)f(\rho^U)\omega_\rho+p'(\theta)F(\rho^U)\omega_\theta]}{[\sigma+p(\theta)F(\rho^U)]^2}\\ &= \frac{-\delta g_S(\rho^U)p(\theta)F(\rho^U)\omega_\rho-\sigma\left[p(\theta)f(\rho^U)\omega_\rho+p'(\theta)F(\rho^U)\omega_\theta\right]}{[\sigma+p(\theta)F(\rho^U)]^2}.\end{aligned}$$

Next, note that

$$\frac{d\sigma}{d\lambda_H}=-\delta g_S(\rho^U)\frac{\partial\rho^U}{\partial\lambda_H}=-\delta g_S(\rho^U)\omega_\rho<0,$$

where ω_ρ is simply a convenient notation for the partial derivative of ρ^u to λ_H and

$$\begin{aligned}\frac{d[p(\theta)F(\rho^U)]}{d\lambda_H} &= p'(\theta)F(\rho^U)\frac{\partial\theta}{\partial\lambda_h}+p(\theta)f(\rho^U)\frac{\partial\rho^U}{\partial\lambda_H}\\ &= p(\theta)f(\rho^U)\omega_\rho+p'(\theta)F(\rho^U)\omega_\theta>0.\end{aligned}$$

A.4 Adding up moving costs

Let C be a relocation cost paid by workers. We ignore here G_S assumed to be degenerate and fix $\delta=0$. We have thus:

$$\begin{aligned}rE(\rho) &= w-\tau\rho+s(U-E(\rho))+\lambda_H\int\max\left[0,(E'-E-C)\right]dG_N(\rho')\\ rU &= b+p\int(E'-U)dF(\rho')\end{aligned}$$

E is downward sloping in ρ with slope

$$\frac{dE}{d\rho} = \frac{-\tau}{r+s+\lambda_H P_W}$$

where $P_W \geq 0$ is the probability to move conditional on receving a housing offer, with $0 < P_W < 1$ possibly depends on ρ. The function $E(\rho)$ is thus monotonic and there is thus a well-defined reservation strategy, with a reservation distance denoted by ρ^E for the employed above which a **housing offer** is rejected. Note in addition that there is NOW state-dependence in the reservation strategy of the employed: we have that $\rho^E(\rho)$ with presumably $d\rho^E/d\rho > 0$: the further away the tenants live from her job, the less likely they will reject an offer. (NB: to be shown in the general case). We can rewrite

$$P_W = G_N(\rho^E(\rho))$$

and obtain the Bellman equations as:

$$\begin{aligned} rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int_0^{\rho^E(\rho)} (E' - E - C) dG_N(\rho') \\ rU &= b + p \int_0^{\rho^U} (E' - U) dF(\rho') \end{aligned}$$

A.5 Housing Market Regulations and Housing Offers

We now provide a simple explanation for the link between housing market regulations mentioned in the Introduction and the parameter λ_H, the arrival of housing offers, that is, we describe an extension that endogenizes λ_H.

Landlords post vacancies and screen applicants. They offer a lease to the "best applicants", in a sense defined below. In case of a default on the rent by the tenant, however, landlords incur a loss due to the length of litigation and eviction procedures. The asset value of owning a dwelling with a tenant defaulting on the rent is denoted by Λ. Therefore, Λ will decline with more regulations in the rental housing market, due to the length of the procedures to recover the unpaid rent and the dwelling.

Potential tenants (applicants) are all ex-ante homogenous but ex-post may represent a default risk for the landlord. More precisely, at the time of the contact between the applicant, the landlord gets a random signal on the tenant and postulates that the particular applicant has a specific default rate δ_i (a Poisson rate in continuous time). It follows that for such a tenant, the value of a filled vacancy for the landlord is:

$$rH_F = R + \delta_i(\Lambda - H_F),$$

where R is the rent and H_F is the value of a filled vacancy.[11] Therefore,

$$H_F = \frac{R + \delta_i \Lambda}{r + \delta_i}. \tag{A-5}$$

The derivative of H_F with respect to δ_i is given as $\Lambda r - R$, which is negative since the landlord's value of a default, Λ, cannot be higher than the capitalized value of the rent R/δ_i. Therefore, H_F is decreasing in δ_i, from R/r to Λ. This implies that there will be a well defined reservation strategy by landlords given the signal they receive.

To derive this reservation strategy, we denote by H_V the value of a vacant housing unit prior to screening. This value is exogenous and is given, in the long-run, by the cost of construction of new housing units. Therefore, it is independent of rental regulations and, in particular, of Λ. At the time of contact with a tenant, landlords decide to offer a lease if the perceived value of default δ_i is below a cut-off value $\overline{\delta}$ with

$$\begin{aligned} \frac{R + \overline{\delta}\Lambda}{r + \overline{\delta}} &= H_V \\ \text{or } \overline{\delta} &= \frac{R - r\Lambda}{H_V - \Lambda} \end{aligned}$$

Note that $\overline{\delta}$ is increasing in Λ: the higher is Λ, the easier it is to accept a tenant since the risk of default is lower.

Hence, the screening rate, α, of tenants is prob $(\delta_i > \overline{\delta})$. Denoting by $L(\delta_i)$ the c.d.f. of default rates, we have

$$\alpha = 1 - L\left(\frac{R - r\Lambda}{H_V - \Lambda}\right)$$

The screening rate is therefore increasing when Λ is lower, that is, when rental housing market regulations are higher.

Finally, denote by ϕ the Poisson rate at which tenants receive dwelling offers. We assume that this contact rate is exogenous. It then follows that $\lambda_H = \phi\alpha$. Therefore,

Proposition A-1: *λ_H is decreasing in the amount of housing market regulations.*

[11] Recall that we assumed in the model that the flow of services of the dwelling to the tenant was exactly compensating the rent so we did not need to include the rent in the Bellman equations of workers. We also assume that workers do not benefit from defaulting on the rent: in case of default, they may have a disutility exerted by landlords or their lawyers so that, after default, the value of being in a dwelling is not higher than it was when paying. Therefore, the expression for the Bellman equation of tenants derived previously is unchanged.

A.6 Endogenous Wage: A Wage Posting Model

The main equations of the model are unchanged, that is, Eq. 1 and Eq. 2 lead to the reservation rule similar to Eq. 6, except that it now depends on the wage,

$$\rho^U(w) = \frac{w(1-t)-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) - pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta} d\rho. \tag{A-6}$$

The firm maximizes the ex-ante value of a vacancy,

$$w = ArgMax\, V(w) = \frac{-\gamma + q(\theta)F(\rho^U(w))J(w)}{r}.$$

The first order condition is given as

$$q(\theta) f(\rho^U) \frac{d\rho^U}{dw}[y-w] = 1. \tag{A-7}$$

Eq. A-7 states that the marginal gain of raising the wage is equal to the marginal increase of accepted job offers, $f(\rho^U)\frac{d\rho^U}{dw}$, times the probablility per unit of time to meet a worker, $q(\theta)$, times the value of a job, $J(w)$, has to equal to the loss of profits of higher wages, $-1/(r+\sigma)$.

Differentiating ρ^U from Eq. A-6, we obtain

$$\frac{d\rho^U(w)}{dw} = (1-t)A(p), \tag{A-8}$$

with

$$A(p) = \frac{1}{\tau} \frac{1}{1 - \frac{\lambda_H G_N(\rho^U)+\delta G_S(\rho^U)-pF(\rho^U)}{r+s+\lambda_H G_N(\rho^U)+\delta} - \int_0^{\rho^U} \frac{[\lambda_H G_N(\rho)+\delta G_S(\rho)-pF(\rho)]\delta g_S(\rho^U)}{[r+s+\lambda_H G_N(\rho)+\delta]^2} d\rho},$$

which is downward sloping. Substituting Eq. A-8 into Eq. A-7, we obtain

$$q(\theta) f(\rho^U)(1-t)A[y-w] = 1,$$

and therefore an expression for the optimal wage; in particular, the net wage and the gross wage:

$$\begin{aligned} \text{Gross wage} \quad &: \quad w = y - \frac{1}{q(\theta) f(\rho^U) A(1-t)} \\ \text{Net wage} \quad &: \quad w(1-t) = y(1-t) - \frac{1}{q(\theta) f(\rho^U) A}. \end{aligned}$$

The inclusion of taxes raises the gross wage and reduces the net wage. Finally, replacing the endogenous value of the wage into Eq. A-6, we obtain a value for ρ^U which is downward sloping in p and therefore θ.

$$\rho^U = \frac{\left[y(1-t) - \frac{1}{q(\theta)f(\rho^U)A}\right] - b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) - pF(\rho)}{r + s + \lambda_H G_N(\rho) + \delta} d\rho. \tag{A-9}$$

The free-entry of firms given their optimal wage is

$$V(w^*) = 0$$

leading to

$$\frac{y-w}{r+\sigma} = \frac{c}{q(\theta)F(\rho^U)},$$

Now, replacing for the wage, we obtain

$$\frac{1}{r+\sigma}\frac{1}{A(p)(1-t)} = c\frac{f(\rho^U)}{F(\rho^U)} \tag{A-10}$$

We assume here that the distribution f has the declining likelihood ratio property, that is $f(x)/F(x)$ is decreasing in x. It follows that the right-hand side is decreasing in ρ^U and $A(p)$ being decreasing in p, the right-hand side is increasing in p and θ.

Overall, equation Eq. A-10 defines a new relation between ρ^U and θ, which is negatively sloped.